№19. Кузьмин Александр

$$g_k(s_1 .. s_k) = \frac{e^{s_k}}{\sum_{i=1}^{k} e^{s_i}}$$

$$R^{(i)} = -\sum_{k=1}^{k} I\left(y^{(i)}=k\right) \lim g_k(s_1, s_2, \dots, s_k)$$

1) $$\frac{\partial g_k}{\partial s_i} = g_k\left(I(k=l) - g_i\right)$$

$$\frac{\partial g_k}{\partial s_i} = e^{s_k} \cdot \frac{-1}{\left(\sum_{j=1}^{k} e^{s_j}\right)^2} \cdot e^{s_i} + I(k=l)\, g_k =$$

$$= g_k\left(\frac{-e^{s_i}}{\sum_{j=1}^{k} e^{s_j}} + I(k=1)\right)$$ чтд

2) $$\frac{\partial R^{(i)}}{\partial g_k} = -\frac{I(y^{(i)})}{g_k}$$

$$\frac{\partial R^{(i)}}{\partial g_k} = -\sum_k \frac{I(y^{(i)}=k)}{g_k} = -\frac{I(y^{(i)})}{g_k}$$ чтд

3) $\frac{\partial R^{(i)}}{\partial s_l} = g_l - I(l = y^{(i)})$

$$\frac{\partial R^{(i)}}{\partial s_l} = -\sum_k I(y^{(i)} = k) \cdot \frac{\frac{\partial g_k}{\partial s_l}}{g_k} =$$

$$= -\sum_k I(y^{(i)} = k)(I(k = l) - g_l) =$$

$$= g_l - I(l = y^{(i)}) \text{ чтд.}$$

№20.

$$g_k(s_k) = \frac{1}{1+e^{-s_k}}, \quad g' = g(1-g) \Rightarrow (\ln g)' \text{ - лгд.}$$

$$R^{(i)} = -\sum_k \left(y_k^{(i)} \ln g_k + (1 - y_k^{(i)}) \ln(1 - g_k)\right)$$

$$\frac{\partial R^{(i)}}{\partial s_k} = g_k - y_k^{(i)}$$

$$\frac{\partial R^{(i)}}{\partial s_k} = -\left(y_k^{(i)}(1 - g_k) + (1 - y_k^{(i)}) \cdot \frac{-g_k(1-g_k)}{1-g_k}\right)$$

$$= -y_k^{(i)} + y_k^{(i)} g_k + g_k - y_k^{(i)} g_k = -y_k^{(i)} + g_k$$ чтд.

№15.

x_1	0	0	1	1	0	0	1	1	1	0
x_2	0	1	0	1	1	1	1	1	1	1
y	0	0	0	0	0	1	1	1	1	1

Оценить $P(Y=0 \mid X_1=1, X_2=1)$
$P(Y=1 \mid X_1=1, X_2=1)$

$$P(Y=0, X_1=1, X_2=1) = \frac{P(Y=0)P(X_1=1 \mid Y=0)P(X_2=1 \mid Y=0)}{P(X_1=1, X_2=1)} = 0{,}3$$

$$P(Y=1, X_1=1, X_2=1) = \frac{P(Y=1)P(X_1=1 \mid Y=1)P(X_2=1 \mid Y=1)}{P(X_1=1, X_2=1)} = 0{,}75$$

№9.

X_1	0	1	0	2	2	2	4	3
X_2	-1	0	0	0	1	0	1	2
y	0	0	0	0	0	1	1	1

$$\delta_y(x) = \mu^T_y \Sigma^{-1} x - \frac{1}{2}\mu^T_y \Sigma^{-1} \mu_y + \ln P_{ry}$$

класс 1: $P_{ry} = \frac{3}{8}$, $\hat{\mu}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\hat{D}_{\xi_1} = 1$, $\hat{D}_{\xi_2} = 1$

$\hat{cov}(\xi_1, \xi_2) = \frac{1}{3}$

$$\Sigma = \begin{pmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{pmatrix}, \quad \Sigma^{-1} = \begin{pmatrix} \frac{9}{8} & -\frac{3}{8} \\ -\frac{3}{8} & \frac{9}{8} \end{pmatrix}$$

$$\delta_1(x) = 3x_1 - 5{,}45089$$

класс 0: $P_{ry} = \frac{5}{8}$, $\hat{\mu}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\hat{D}_{\xi_1} = 1$ $\hat{D}_{\xi_2} = \frac{1}{2}$

$\hat{cov}(\xi_1, \xi_2) = \frac{2}{5}$

$$\Sigma = \begin{pmatrix} 1 & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{2} \end{pmatrix}, \quad \Sigma^{-1} = \begin{pmatrix} \frac{25}{17} & -\frac{20}{17} \\ -\frac{20}{17} & \frac{50}{17} \end{pmatrix}$$

$$\delta_0(x) = \frac{25}{17}x_1 - \frac{20}{17}x_2 - 1{,}2053$$

$$2.\ \delta_y(x) = -\frac{1}{2}\ln\det\Sigma_y - \frac{1}{2}(x-\mu_y)^T\Sigma_y^{-1}(x-\mu_y) + \ln$$

$$\delta_0(x) = \frac{-25}{34}x_1^2 + \frac{40}{34}x_1x_2 - \frac{90}{34}x_2^2 + \frac{50}{34}x_1 - \frac{40}{34}x_2 - 0{,}66589$$

$$\delta_1(x) = \frac{-9}{16}x_1^2 + \frac{6}{16}x_1x_2 + 3x_1 - \frac{9}{16}x_2^2 - 5{,}42193$$