Toy example

Suppose we are given a 3-D VAR(1) model:

$$r_t = \mathbf{\Phi} r_{t-1} + a_t \tag{1}$$

where $a_t \sim N(0, \Sigma)$, with

$$\mathbf{\Phi} = \begin{bmatrix} 0.6 & -0.3 & 0.1 \\ -0.4 & 0.2 & -0.4 \\ 0.4 & 0.3 & 0.9 \end{bmatrix}, \Sigma = \begin{bmatrix} 1 & 0.2 & 0 \\ 0.2 & 1 & 0.24 \\ 0 & 0.24 & 1 \end{bmatrix} \tag{2}$$

Solving for the eigenvalues and eigenvectors, we get the eigenvalue, eigenvector pair as:

- $\lambda_1 = -0.0325$, $v_1^T = [0, 1.4142, 1.4142]$
- $\lambda_2 = 0.5$, $v_2^T = [0.8660, 1.7321, 0.8660]$
- $\lambda_3 = 1.2324 > 1$ which we disregard as it corresponds to non-stationarity

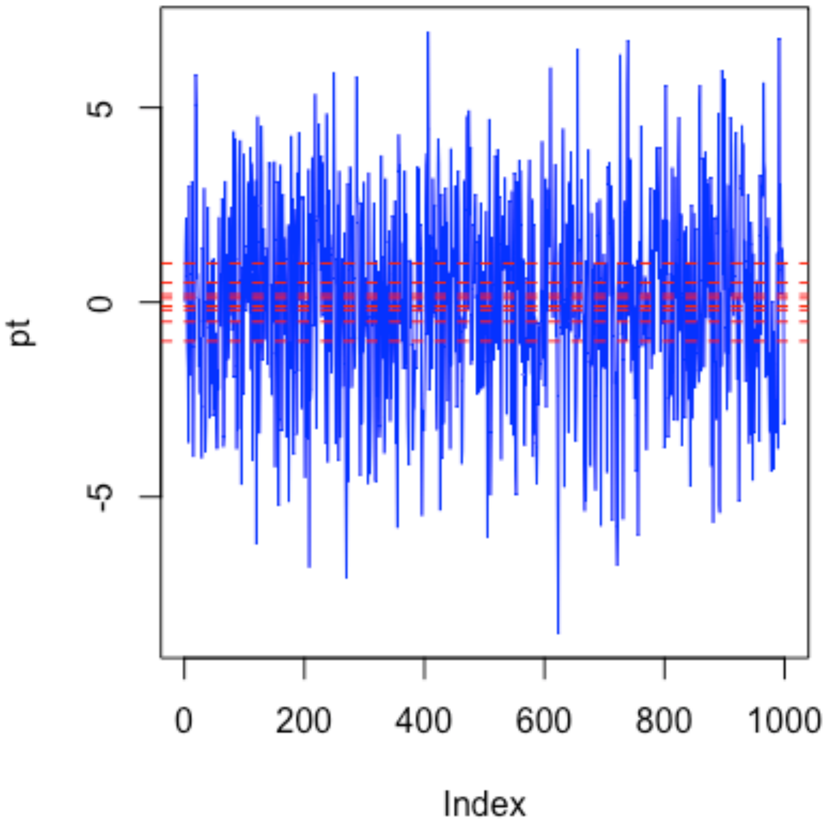
We can choose either λ_1 or λ_2 to construct our trading portfolio.

Suppose we select $\lambda_2 = 0.5$, then the mean-reversion trading portfolio is
$p_t = v_2^T r_t = 0.866 r_{1,t} + 1.7321 r_{2,t} + 0.866 r_{3,t}$

We will simulate this trading strategy using R studio.

In the simulation, we would need to simulate a correlated noise a_t using the covariance matrix given. We do this via a Cholesky Decomposition s.t. $\Sigma = \boldsymbol{L}\boldsymbol{L}^T$ and where $\boldsymbol{L}$ is the lower triangle matrix. We generate the correlated noise a_t by $a_t = \boldsymbol{L}Z$ where $Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$ is a vector of standard normal random variable.

The simulated result of our trading portfolio is as below:



which is a stationary time series and we can execute the mean-reversion trading strategy.

Refer to the R code uploaded for simulation.