Biomedical Engineering
生醫工程

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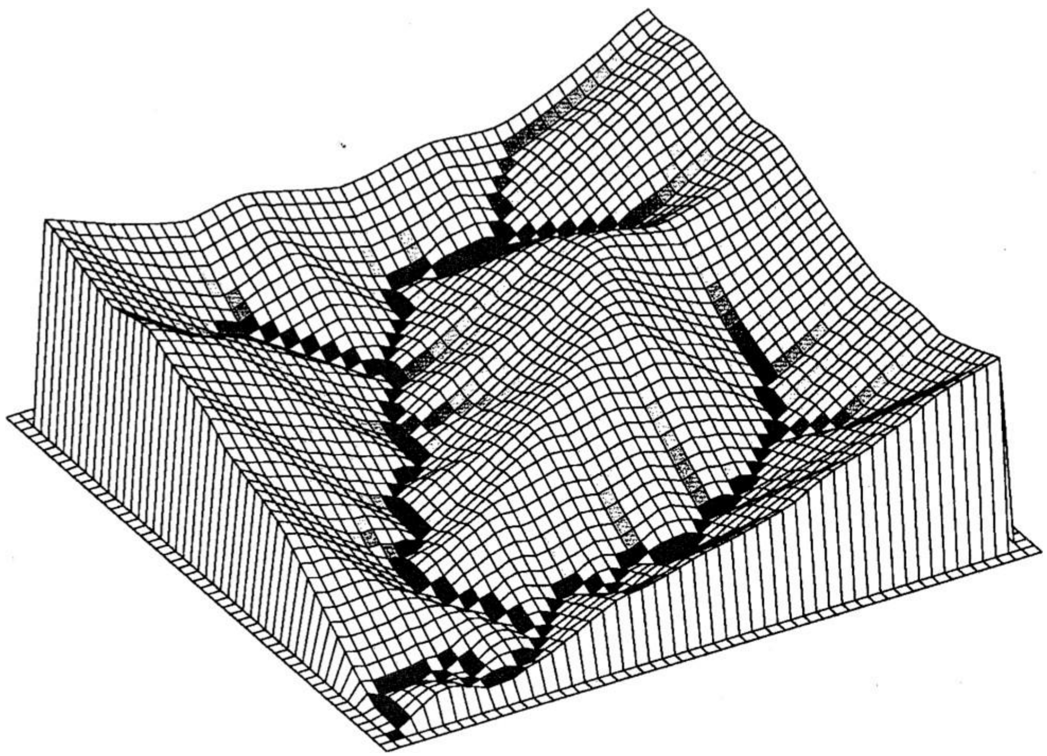
Classroom erosion experiment



Landscape Evolution

Controls on the spacing of first-order valleys

J. Taylor Perron,[1] William E. Dietrich,[2] and James. W. Kirchner[2,3,4]

Abstract. Many landscapes are composed of ridges and valleys that are uniformly spaced, even where valley locations are not controlled by bedrock structure. Models of long-term landscape evolution have reproduced this phenomenon, yet the process by which uniformly spaced valleys develop is not well understood, and there is no quantitative framework for predicting valley spacing. Here we use a numerical landscape evolution model to investigate the development of uniform valley spacing. We find that evenly spaced valleys arise from a competition between adjacent drainage basins for drainage area (a proxy for water flux) and that the spacing becomes more uniform as the landscape approaches a topographic equilibrium. Valley spacing is most sensitive to the relative rates of advective erosion processes (such as stream incision) and diffusion-like mass transport (such as soil creep), and less sensitive to the magnitude of a threshold that limits the spatial extent of stream incision. Analysis of a large number of numerical solutions reveals that valley spacing scales with a ratio of characteristic diffusion and advection timescales that is analogous to a Péclet number. We use this result to derive expressions for equilibrium valley spacing and drainage basin relief as a function of the rates of advective and diffusive processes and the spatial extent of the landscape. The observed scaling relationships also provide insight into the cause of transitions from rill-like drainage networks to branching networks, the spatial scale of first-order drainage basins, the contributing area at which hillslopes transition into valleys, and the narrow range of width-to-length ratios of first-order basins.

In-class Exercise: Write Python function that computes n!, the factorial of n

```
[1]: def fac(n):
```

In-class Exercise: Write Python function that computes n!, the factorial of n

[1]:
```
def fac(n):
    if (n<2):
        return 1
    else:
        return n*fac(n-1)
```

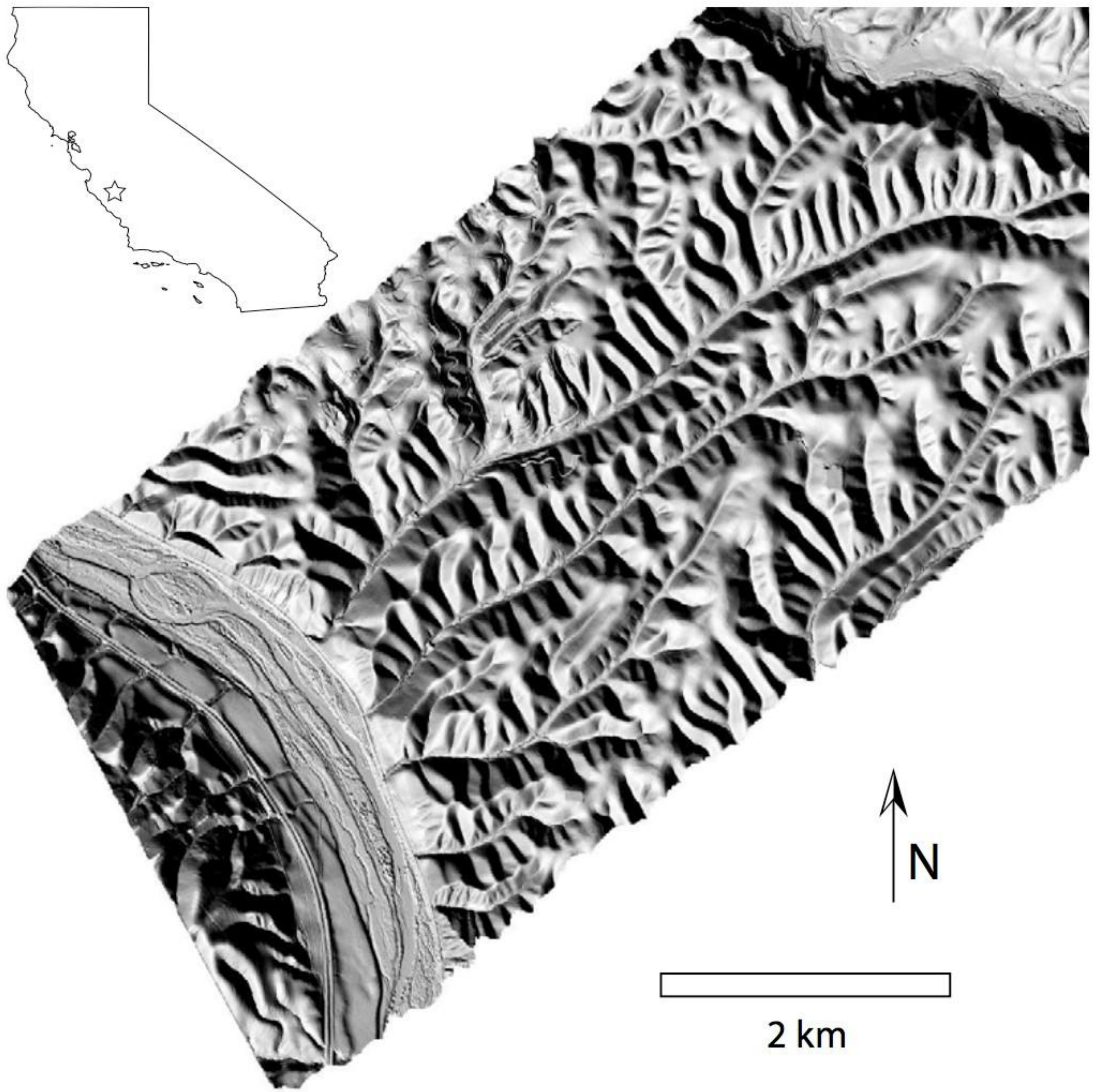


Figure 2. Shaded relief map of a portion of the Gabilan Mesa, California, at approximately 35.9°N, 120.8°W. The tributary valleys that drain into the NE-SW-trending canyons show a remarkably uniform spacing. The topographic data, with a horizontal resolution of 1 m, were collected and processed by the National Center for Airborne Laser Mapping (NCALM, http://www.ncalm.org). The Salinas River and U.S. Highway 101 are visible to the southwest.

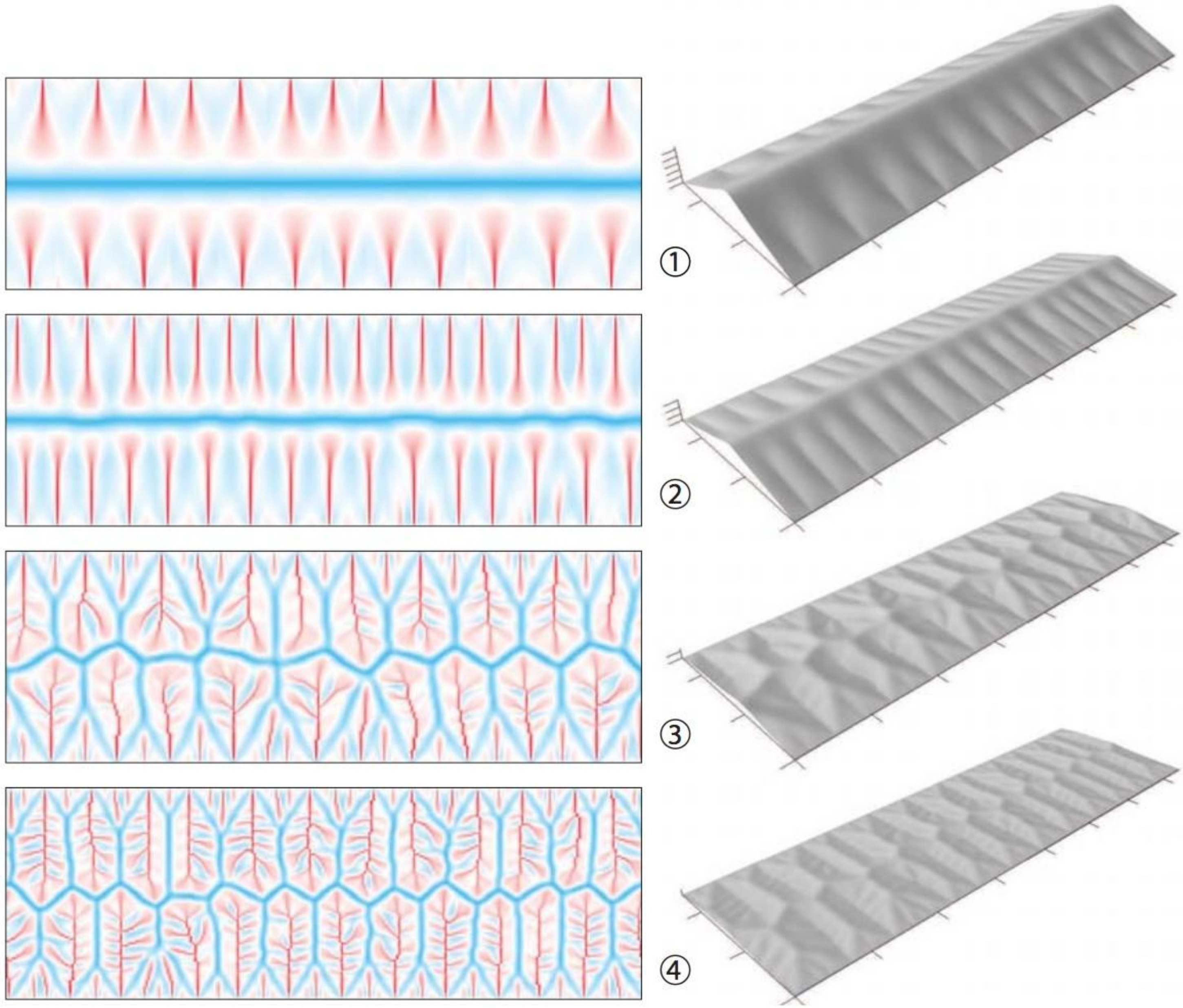
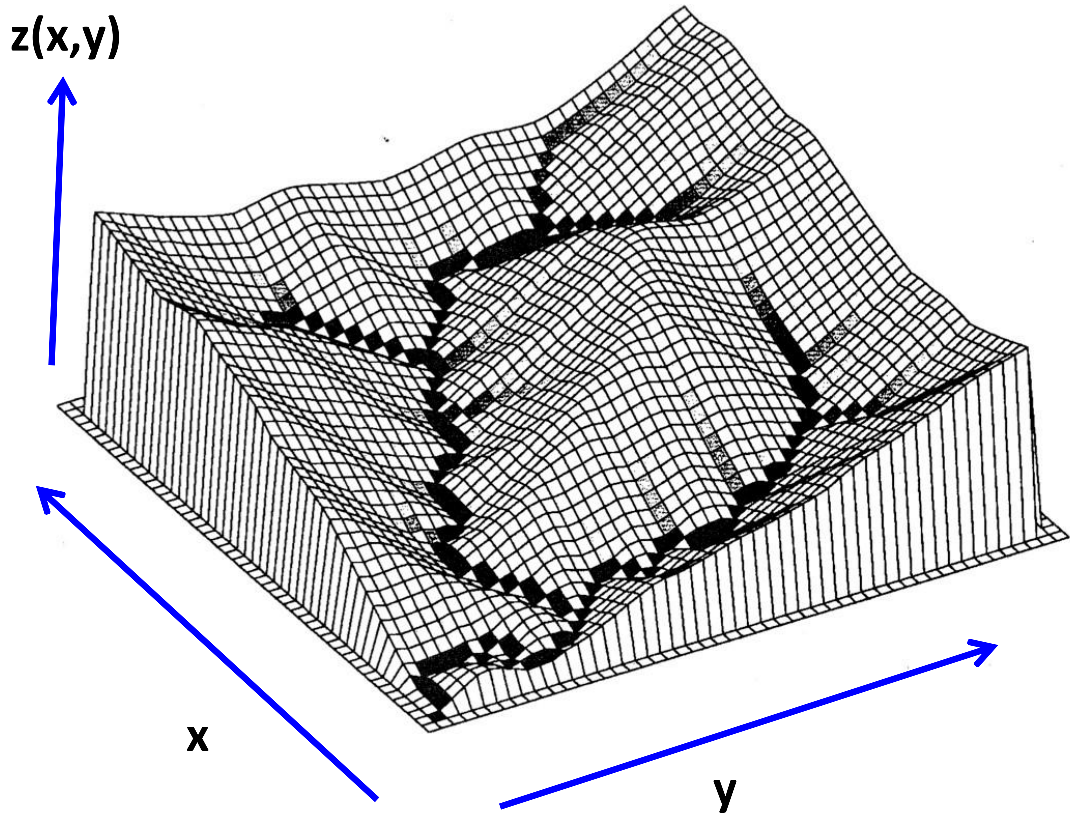


Figure 12. Four representative solutions to Equation 13 showing the variety of observed behavior. Locations of the solutions are indicated on the scaling plots in Figures 10 and 11. Colors in the image maps (left) show the Laplacian of elevation ($\nabla^2 z$) normalized to the maximum and minimum values in the grid. Concave-up areas (red) are valleys; concave-down areas (blue) are hillslopes. Axis tick intervals in the perspective views (right) are 200 m in the horizontal and 5 m in the vertical. 4× vertical exaggeration.

Conservation of Mass

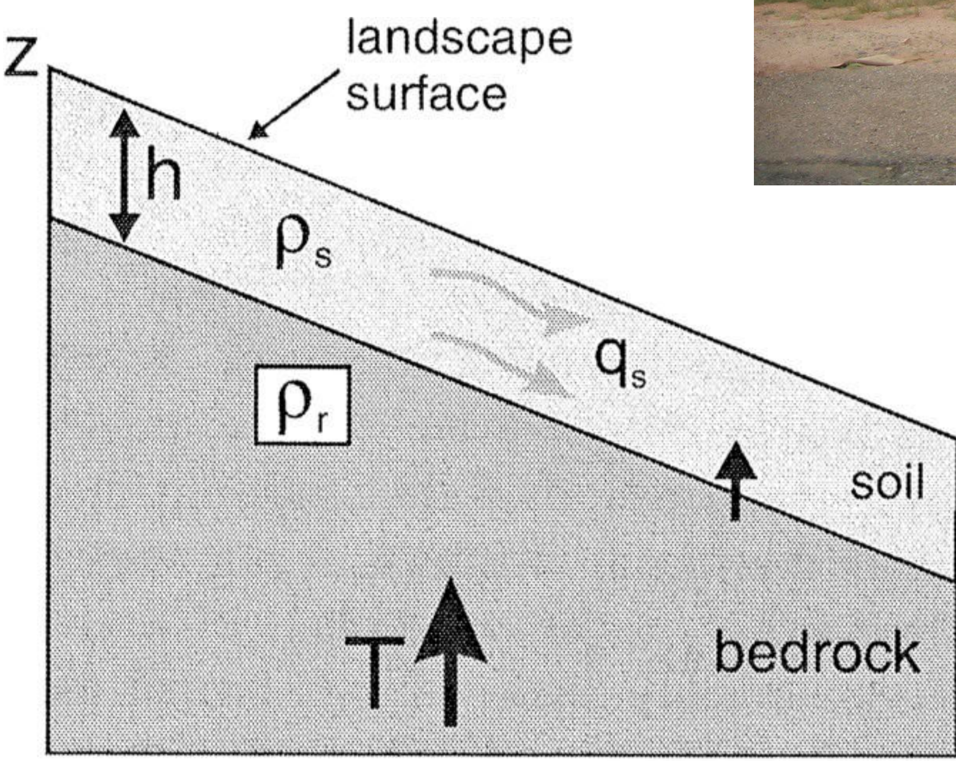
$$\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \vec{\nabla} \circ \vec{q}_s$$

z(x,y) … is the surface elevation



Soil Production

$$\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \vec{\nabla} \circ \vec{q}_s$$



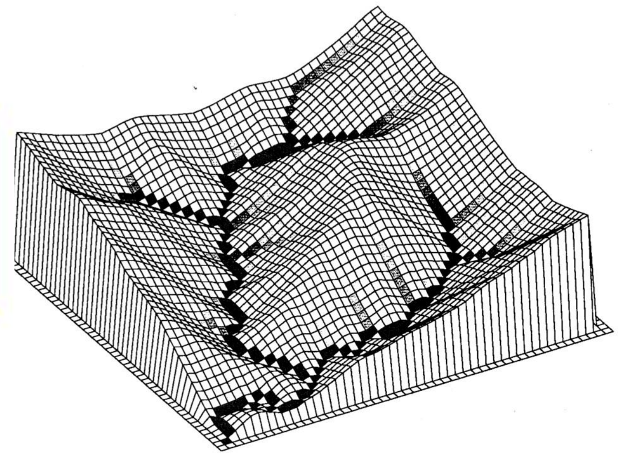
U … rate to convert bedrock to soil

Depends strongly on thickness of soil layer.

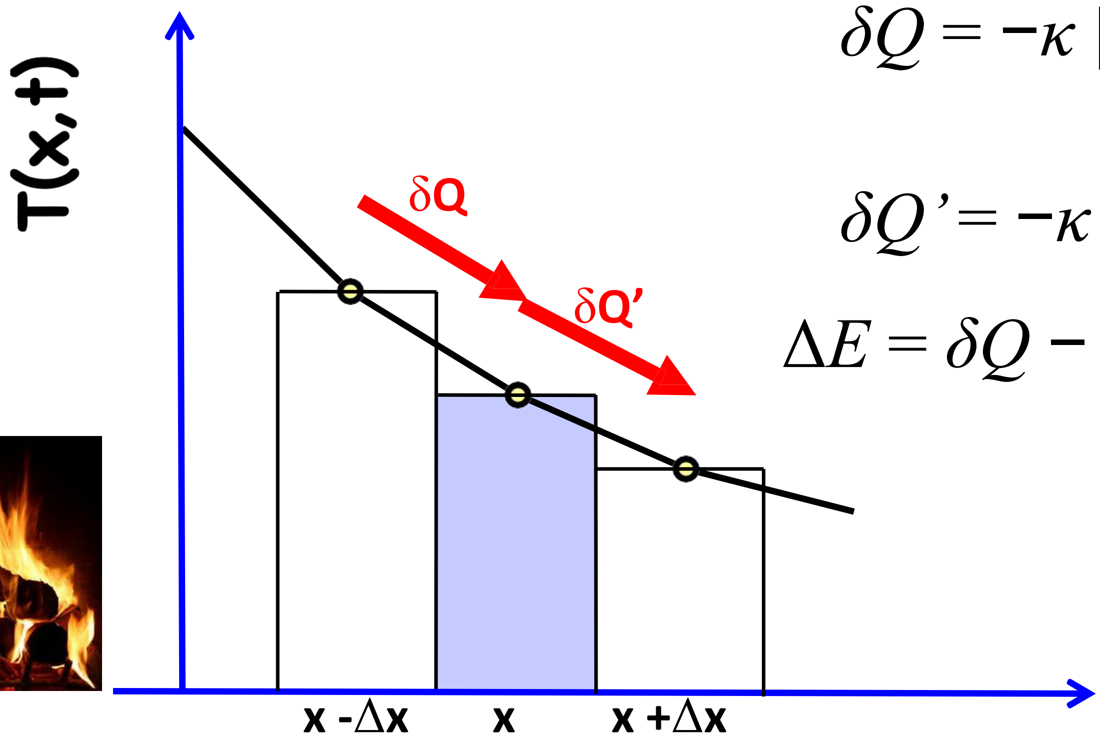
Reminder: What is Flux

$$\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \vec{\nabla} \circ \vec{q}_s$$

$$\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \left(\frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y} \right)$$



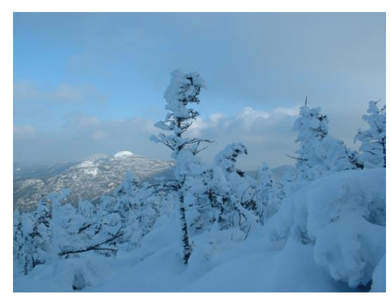
z(x,y) … is the surface elevation



$$\delta Q = -\kappa \, [T(x) - T(x - \Delta x)] \, \Delta t / \Delta x$$

$$\delta Q' = -\kappa \, [T(x) - T(x + \Delta x)] \, \Delta t / \Delta x$$

$$\Delta E = \delta Q - \delta Q' = c_P m \, \Delta T$$



Flux type I: Mass Movement of Sediments

$$\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \vec{\nabla} \circ \vec{q}_s$$

$$\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \left(\frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y} \right)$$

z(x,y) … is the surface elevation

$$\vec{q}_s = \vec{q}_m + \vec{q}_c$$

Mass movement by sediments

(without water)

- Different processes e.g. landslides, fine-scale transport by rain splash
- Creep motion from frost heaving, wetting/drying, bioturbation (plant/animals)
- Smooths out landscape, landslides represented implicitly

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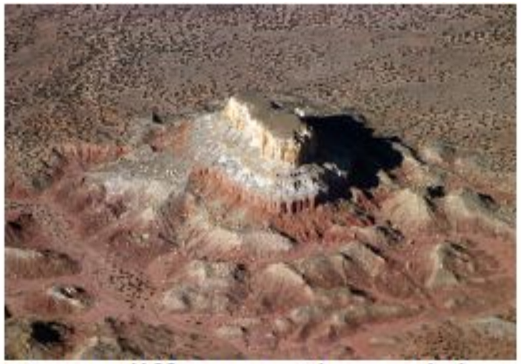


Steep cliff are not in model.

z(x,y) … is the surface elevation

$$\vec{q}_s = \vec{q}_m + \vec{q}_c$$

$$\vec{q}_m = -D \vec{\nabla} z$$

$$q_{mx} = -D \frac{\partial z}{\partial x}$$

$$q_{my} = -D \frac{\partial z}{\partial y}$$

Mass movement by sediments

(without water)

- Different processes e.g. landslides, fine-scale transport by rain splash
- Creep motion from frost heaving, wetting/drying, bioturbation (plant/animals)
- Smooths out landscape, landslides represented implicitly

Assumption: Flux increase linearly with the slope. No explicit landslides!!!

Flux type II: Fluvial Erosion

$$\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \vec{\nabla} \circ \vec{q}_s$$

$$\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \left(\frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y} \right)$$

z(x,y) … is the surface elevation

$$\vec{q}_s = \vec{q}_m + \vec{q}_c$$

$$\vec{\nabla} \circ \vec{q}_c = \varepsilon = \begin{cases} k_1(\tau - \tau_c) & if \quad \tau > \tau_c \\ 0 & if \quad \tau \le \tau_c \end{cases}$$

Channelized erosion

(transport of material by water)

Flow of water and debris leads to a shear stress, τ

$$\varepsilon = k_1 \left[\rho_w g \left(\frac{N k_3^{1-b}}{k_2 k_4} \right)^{\frac{3}{5}} A^{\frac{3}{5} a(1-b)} |\nabla z|^{\frac{7}{10}} - \tau_c \right]$$

Collecting area (nonlocal term)

Gradient

$$|\vec{\nabla} z| = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)}$$

Combine both terms into a single landscape evolution equation

$$\boxed{\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \vec{\nabla} \circ \vec{q}_s}$$

$$\frac{\partial z}{\partial t} = D\nabla^2 z - K(A^m |\nabla z|^n - \theta_c) + E\,,$$

with

$$K = k_1 \rho_w \rho_s g \left(\frac{N k_3^{1-b}}{k_2 k_4} \right)^{\frac{3}{5}}$$

$$\theta_c = \frac{k_1 \rho_s}{K} \tau_c$$

$$m = \frac{3}{5} a(1-b)$$

$$n = \frac{7}{10}\,.$$

Combine both terms into a single landscape evolution equation: Simplest form

$$\boxed{\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \vec{\nabla} \circ \vec{q}_s}$$

$$\frac{\partial z}{\partial t} = D\nabla^2 z - K(A^m |\nabla z|^n - \theta_c) + E\,,$$

$$\frac{\partial z}{\partial t} = D\nabla^2 z - K \cdot A \cdot |\overrightarrow{\nabla z}|$$

∂t

Diffusion term = mass flow (analogue to heat flow)

A = Collecting area
= all water that passes through that point.
(nonlocal term)

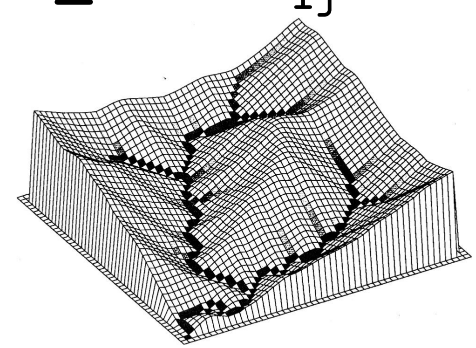
Slope of hillside at this point (x,y)

Recursive method compute the collection/drainage/contributing area

It rains equally on our landscape. Need to calculate the amount of water the flow through each grid cell. Assume amount is proportional to collection area.

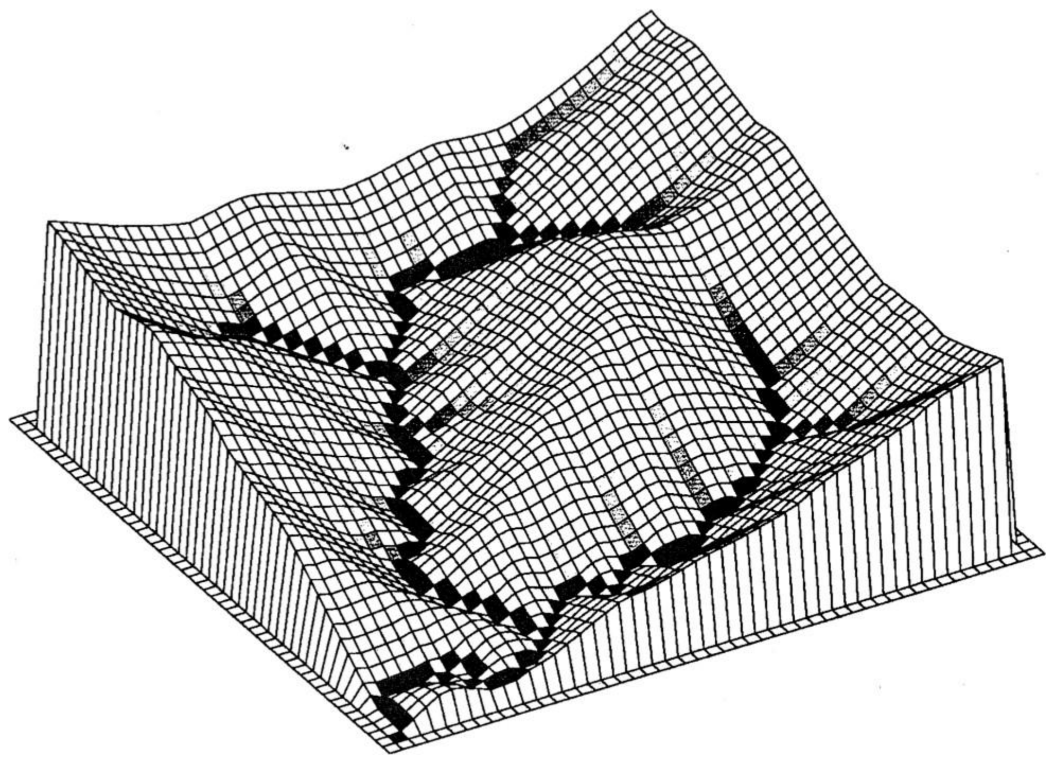
A recursive scheme to compute the collection area (pseudo-code):

```
function area=drainage_area(x,y)
% start with area of cell and then recursively
% add area of all uphill neighbors
area = grid_cell_size^2;
for all neighbors n_ij  {(x-1,y) (x+1,y) (x,y-1) (x,y+1)}
      if uphill(n_ij)
            area = area + fraction(n_ij)*drainage_area(n_ij);
      end
end
```



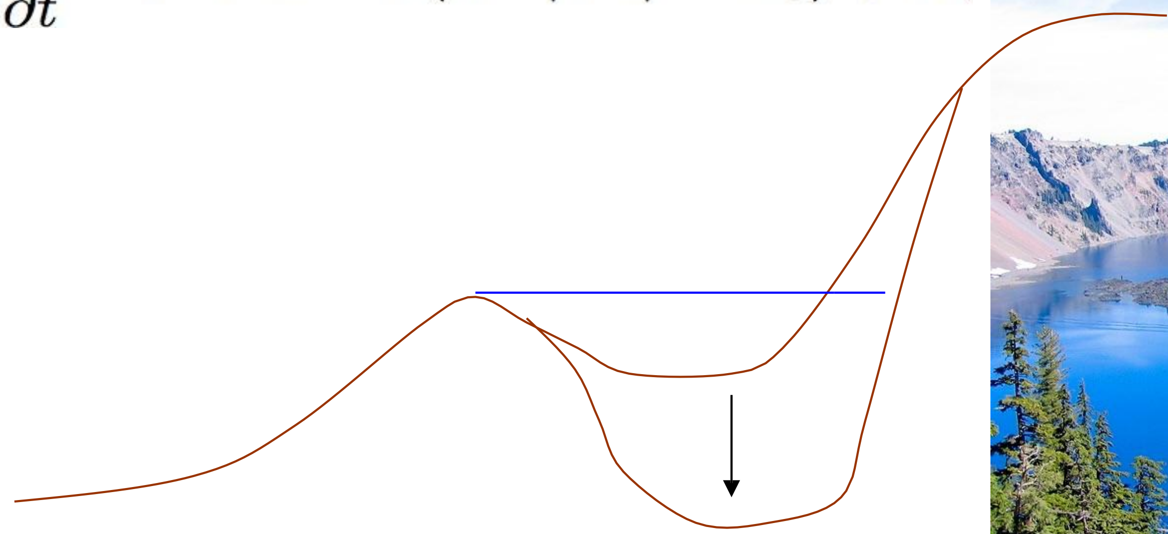
Base case: no more uphill neighbors (peaks and local maxima).

Discuss a non-recursive approach to the calculating the collection area



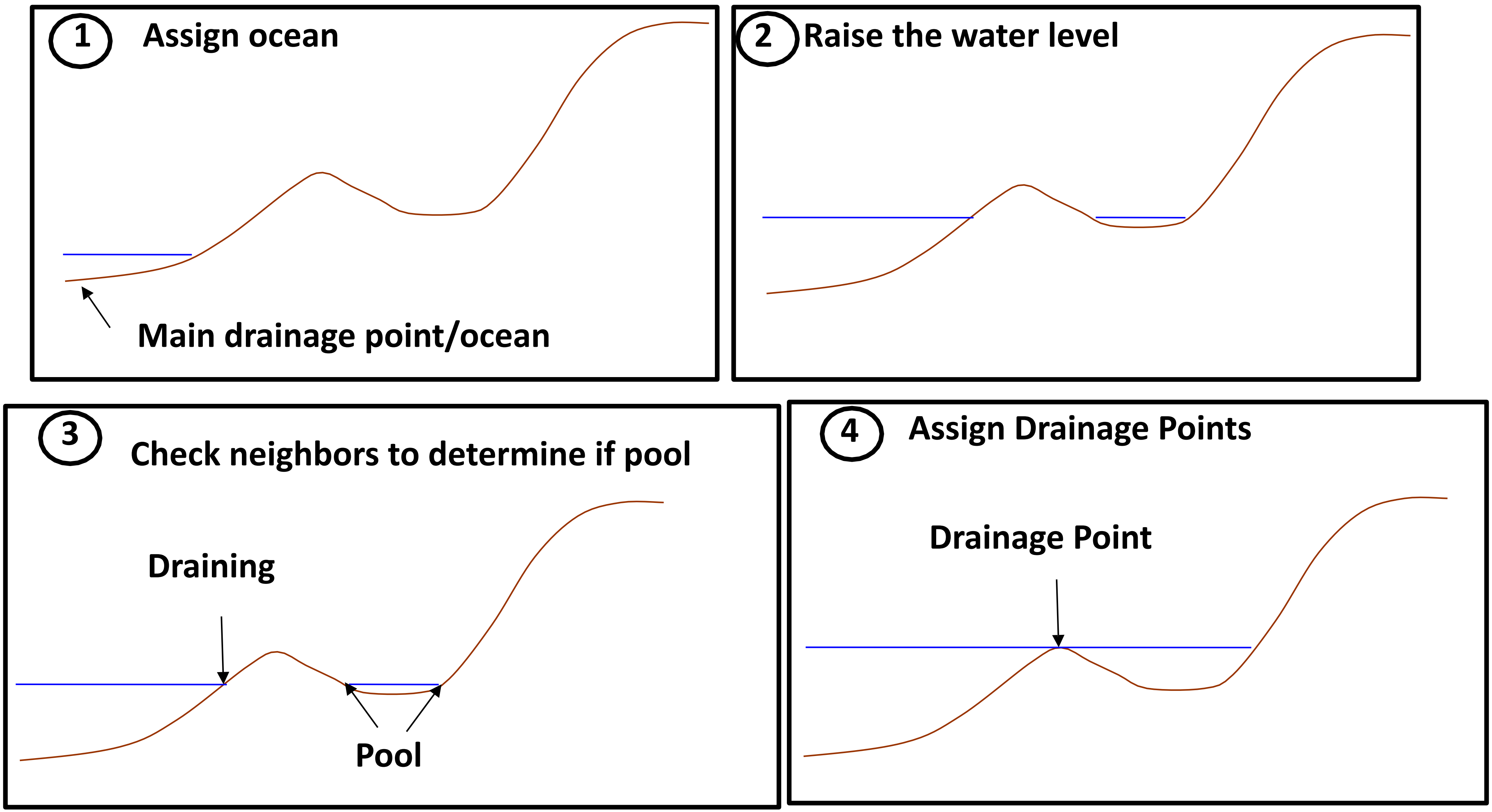
Landscape Evolution:
What about pools and lakes?

$$\frac{\partial z}{\partial t} = D\nabla^2 z - K(A^m\,|\nabla z|^n - \theta_c) + E\,,$$



Small perturbations in our landscape become large holes!

Finding pools and lakes



How do these different points erode?

Pools - No fluvial erosion! Only diffusive processes are permittedto erode these points.

Draining Area - Erodes with both fluvial and diffusive processes.

Drainage Points -

Erodes with both fluvial and diffusive processes. However, it’s drainage area is considered to be that of the entire pool.

Landscape Evolution

One particular landscape evolution model involving

(A) Mass flow of sediments

(A) Fluvian Erosion

Combine both terms into a single landscape evolution equation

$$\boxed{\frac{\partial z}{\partial t} = \frac{\rho_{rock}}{\rho_{soil}} U - \vec{\nabla} \circ \vec{q}_s}$$

$$\frac{\partial z}{\partial t} = D\nabla^2 z - K(A^m |\nabla z|^n - \theta_c) + E \,,$$

We calculate finite difference solutions to Equation 13 on a rectangular grid $z_{i,j}$ with grid spacings Δx and Δy and dimensions $N_x \times N_y$, such that

$$z_{i,j} = z(x_i, y_j) \tag{14a}$$

$$x_i = i\Delta x \tag{14b}$$

$$y_j = j\Delta y \tag{14c}$$

$$i = 0 \ldots N_x - 1 \tag{14d}$$

$$j = 0 \ldots N_y - 1 \,. \tag{14e}$$

Combine both terms into a single landscape evolution equation

$$\frac{\partial z}{\partial t} = D\nabla^2 z - K(A^m |\nabla z|^n - \theta_c) + E,$$

$$\Delta z = \Delta t \left[\Phi(z) + \Psi(z)\right], \tag{15}$$

with

$$\Phi(z) = D\left(\frac{z_{i+1,j} - 2z_{i,j} + z_{i-1,j}}{\Delta x^2} + \frac{z_{i,j+1} - 2z_{i,j} + z_{i,j-1}}{\Delta y^2}\right)$$

Combine both terms into a single landscape evolution equation

$$\frac{\partial z}{\partial t} = D\nabla^2 z - K(A^m |\nabla z|^n - \theta_c) + E,$$

$$\Delta z = \Delta t \left[\Phi(z) + \Psi(z)\right], \qquad (15)$$

with

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$$\Psi(z) = E - \frac{w_{i,j}}{\delta} K \left[A^m_{i,j}\left(\frac{\sqrt{{s_1}^2 + {s_2}^2} + \sqrt{{s_3}^2 + {s_4}^2}}{2}\right)^n - \theta_c\right]$$

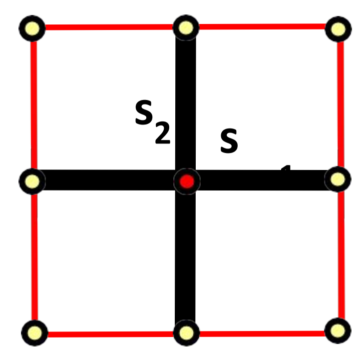
$$s_1 = \frac{z_{i+1,j} - z_{i-1,j}}{2\Delta x}$$

$$s_2 = \frac{z_{i,j+1} - z_{i,j-1}}{2\Delta y}$$

$$s_3 = \frac{z_{i+1,j-1} - z_{i-1,j+1}}{2\sqrt{\Delta x^2 + \Delta y^2}}$$

$$s_4 = \frac{z_{i+1,j+1} - z_{i-1,j-1}}{2\sqrt{\Delta x^2 + \Delta y^2}}.$$

$$A^m |\nabla z|^n$$



Combine both terms into a single landscape evolution equation

$$\frac{\partial z}{\partial t} = D\nabla^2 z - \underbrace{K(A^m\,|\nabla z|^n - \theta_c) + E}_{},$$

$$\Delta z = \Delta t\,[\Phi(z) + \Psi(z)]\,, \qquad (15)$$

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$$\Phi(z) = D\left(\frac{z_{i+1,j} - 2z_{i,j} + z_{i-1,j}}{\Delta x^2} + \frac{z_{i,j+1} - 2z_{i,j} + z_{i,j-1}}{\Delta y^2}\right)$$

$$\Psi(z) = E - \frac{w_{i,j}}{\delta}K\left[A^m_{i,j}\left(\frac{\sqrt{{s_1}^2 + {s_2}^2} + \sqrt{{s_3}^2 + {s_4}^2}}{2}\right)^n - \theta_c\right]$$

$$s_1 = \frac{z_{i+1,j} - z_{i-1,j}}{2\Delta x}$$

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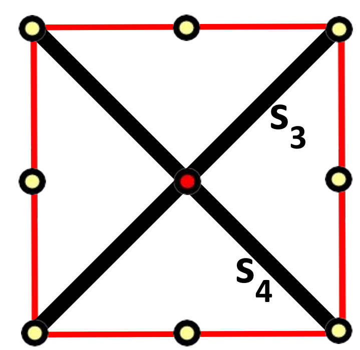
$$s_3 = \frac{z_{i+1,j-1} - z_{i-1,j+1}}{2\sqrt{\Delta x^2 + \Delta y^2}}$$

$$s_4 = \frac{z_{i+1,j+1} - z_{i-1,j-1}}{2\sqrt{\Delta x^2 + \Delta y^2}}\,.$$

$$A^m\,|\nabla z|^n$$



How do we compute the collection area?