Biomedical Engineering
生醫工程

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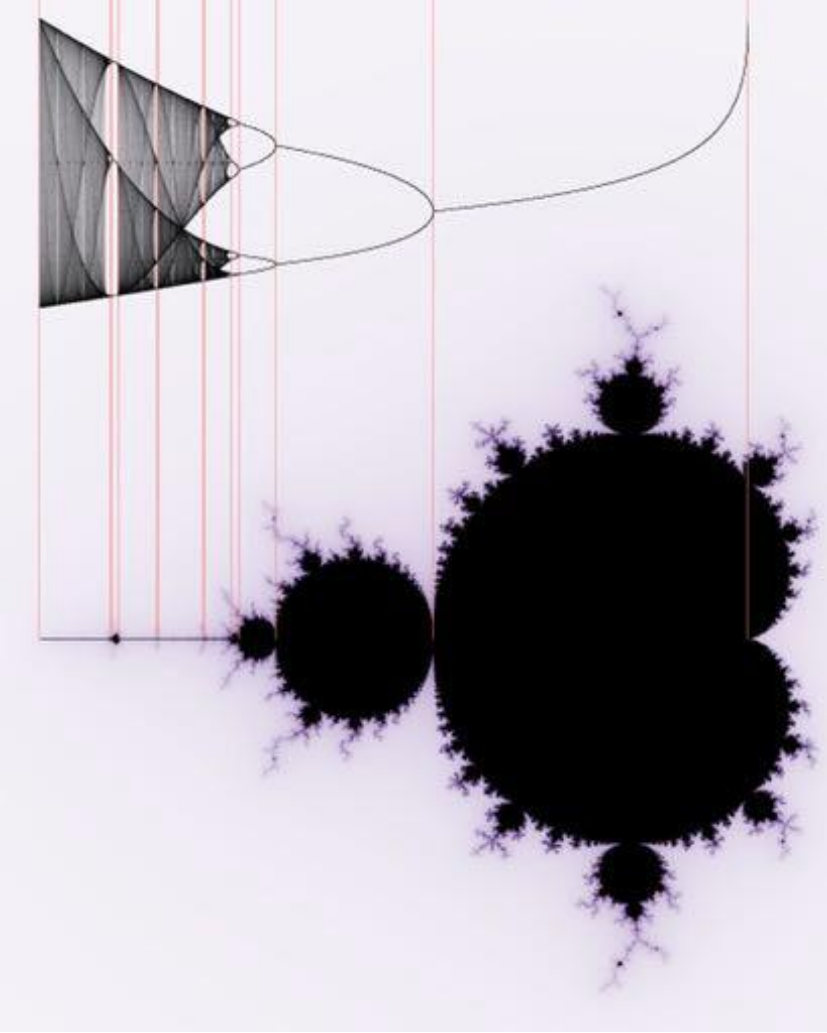
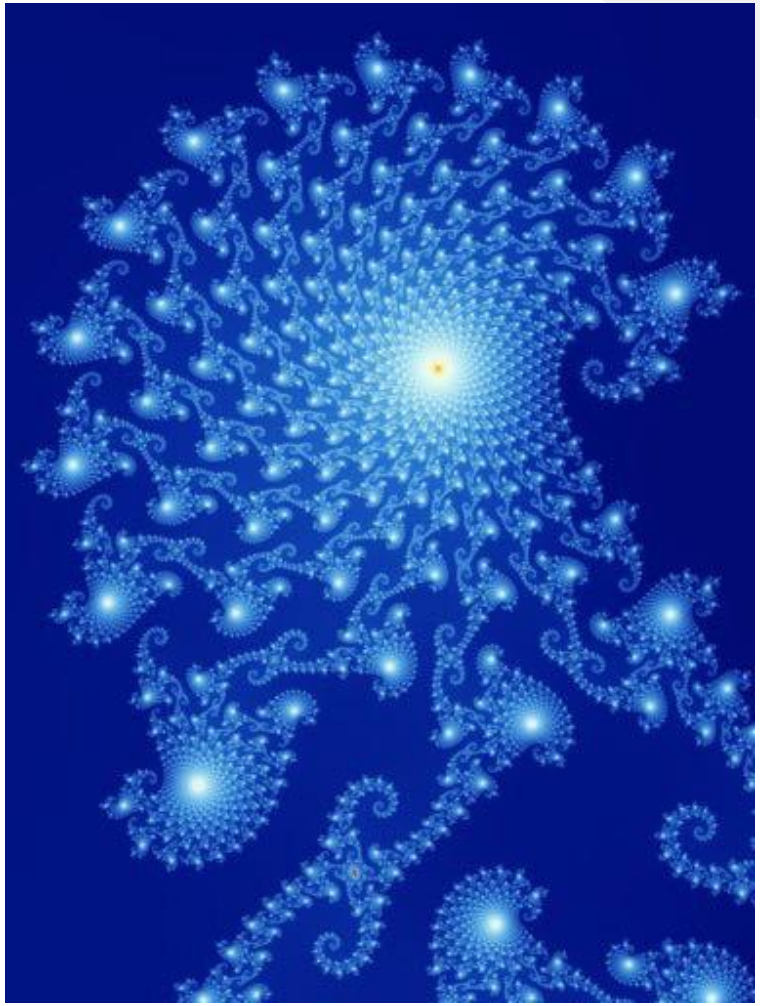
Spring 2024

Mandelbrot Set





Last Year's Guess-the-X-Position Data Set

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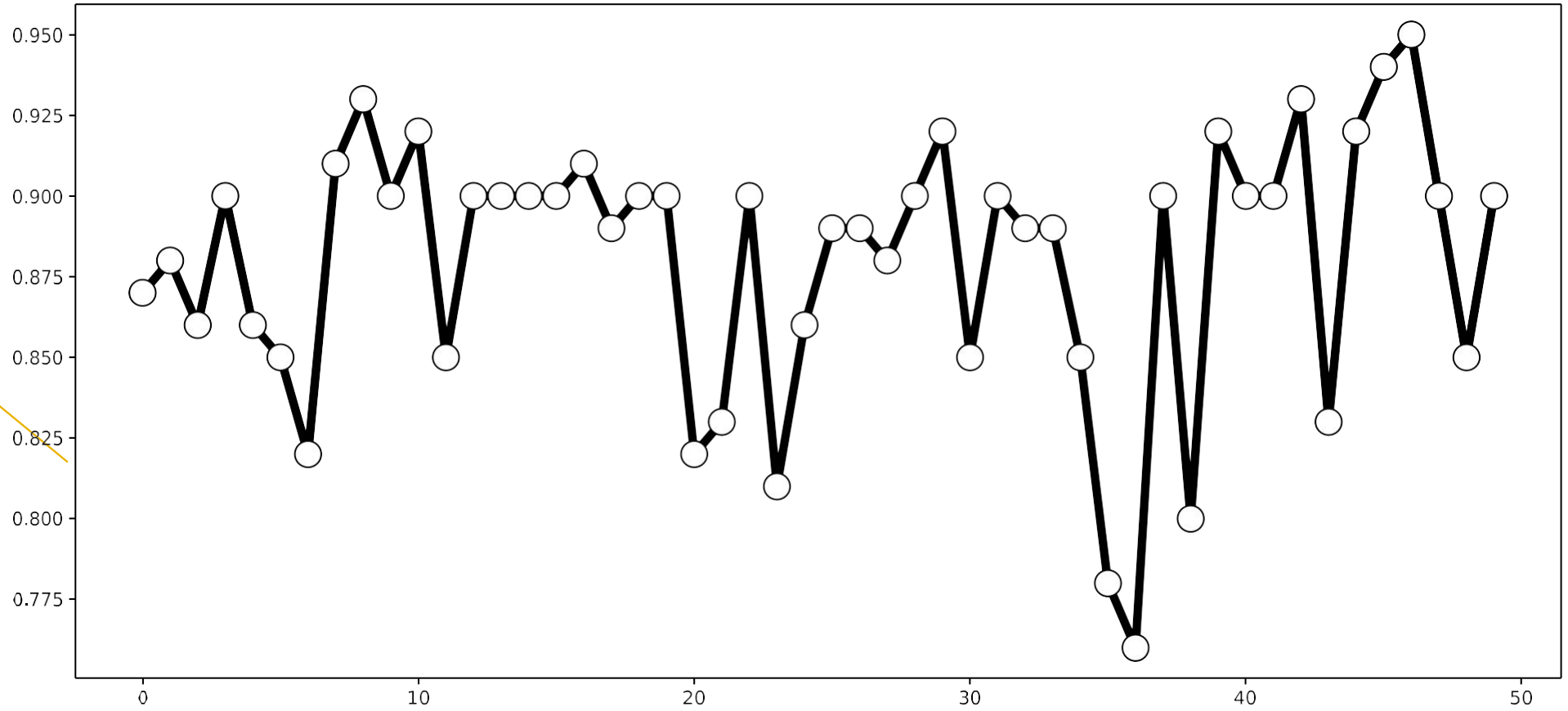
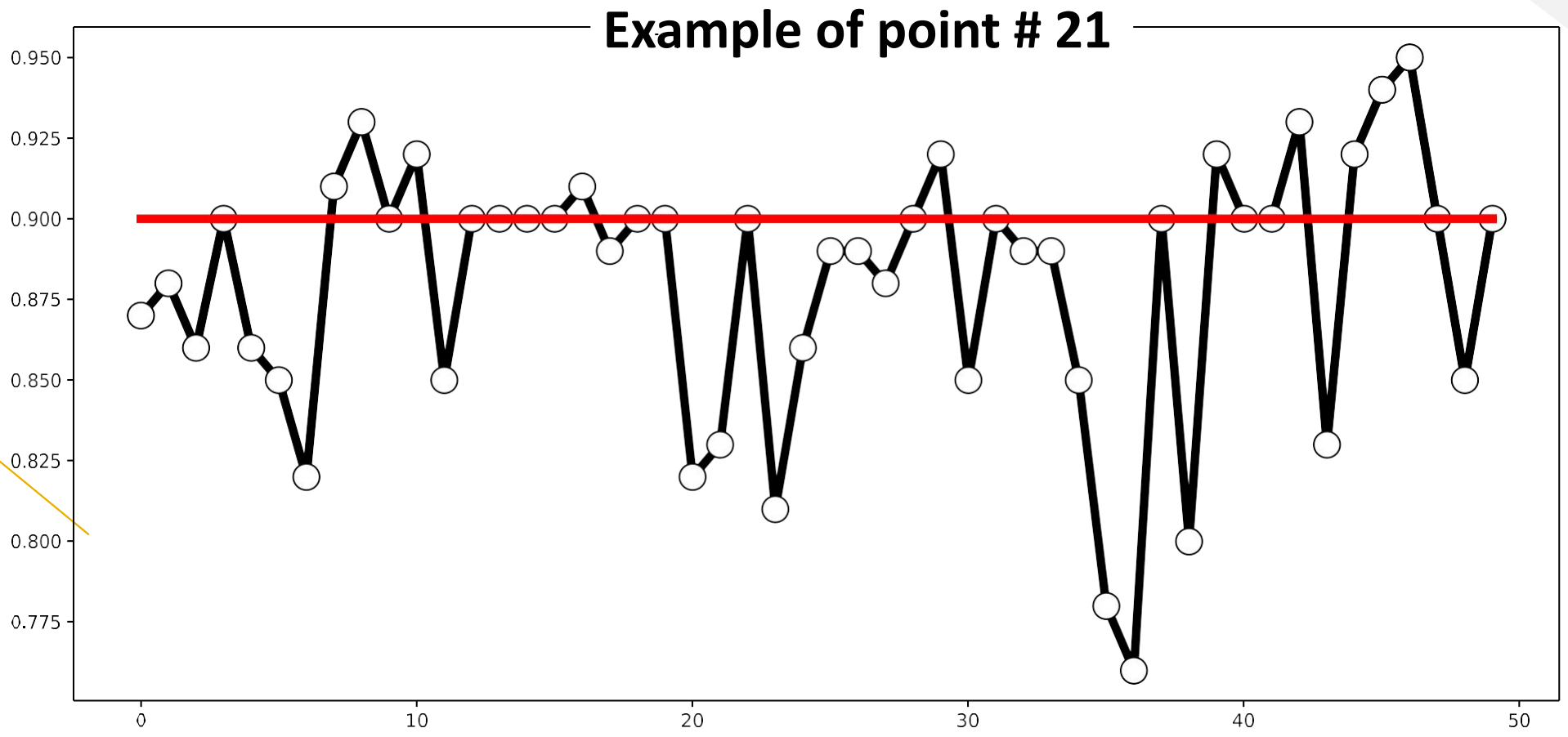
Example of point # 21



Last Year's Guess-the-X-Position Data Set



Problem A: One point measured many times. You do not know the exact answer.

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Example of point # 21

Problem B: How accurate did you or the whole class guess? You typically know the exact answer.

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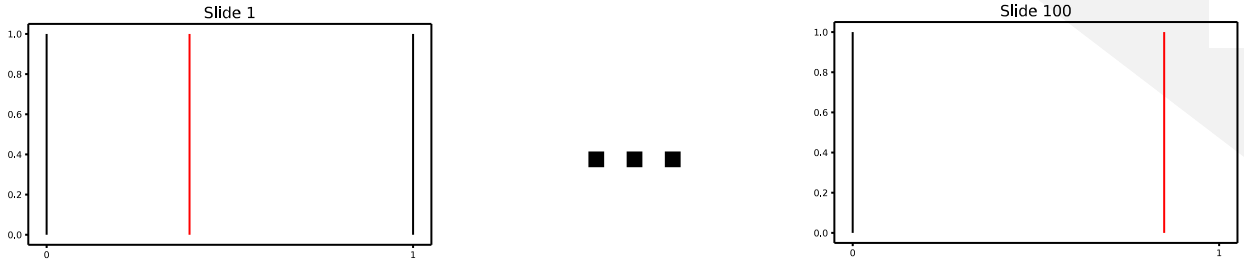
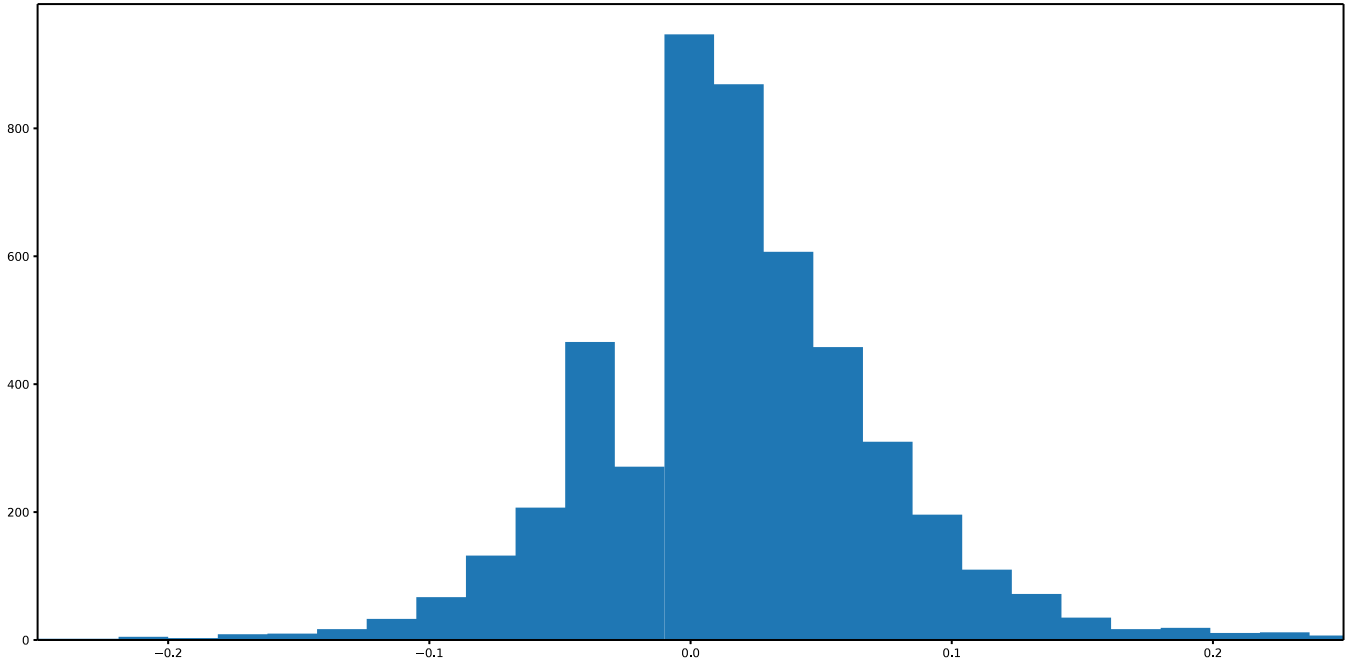
Use all points
1…100



You know the exact answer of all measurements:



Population

Variance

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$$

Standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}}$$

A Simple Quadratic Map

Start with $z_0=0$ and iterate using

$$z_{n+1} = z_n^2 + C$$

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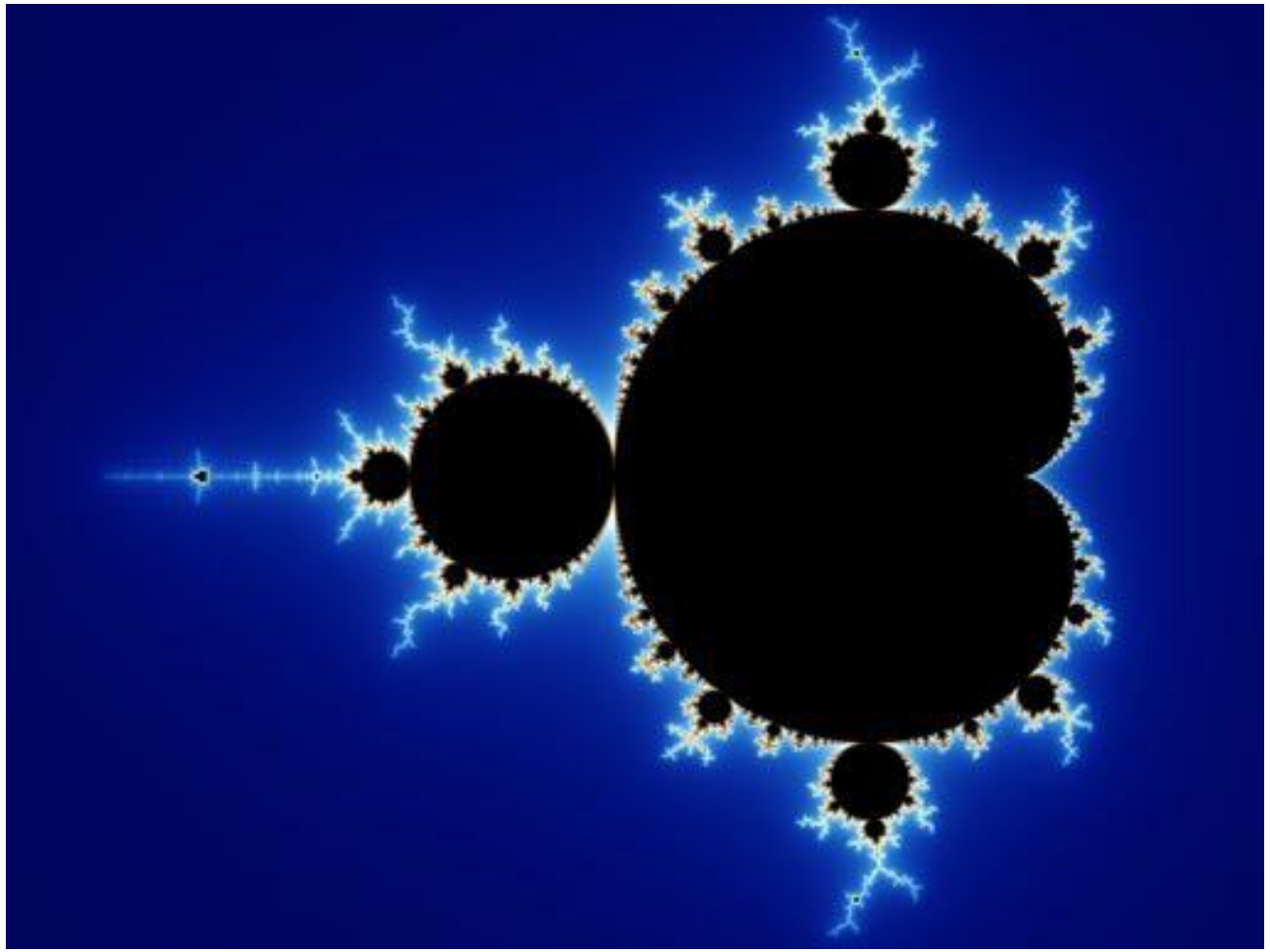
Review complex numbers

The Mandelbrot Set:
A Quadratic Map in the Complex Plane



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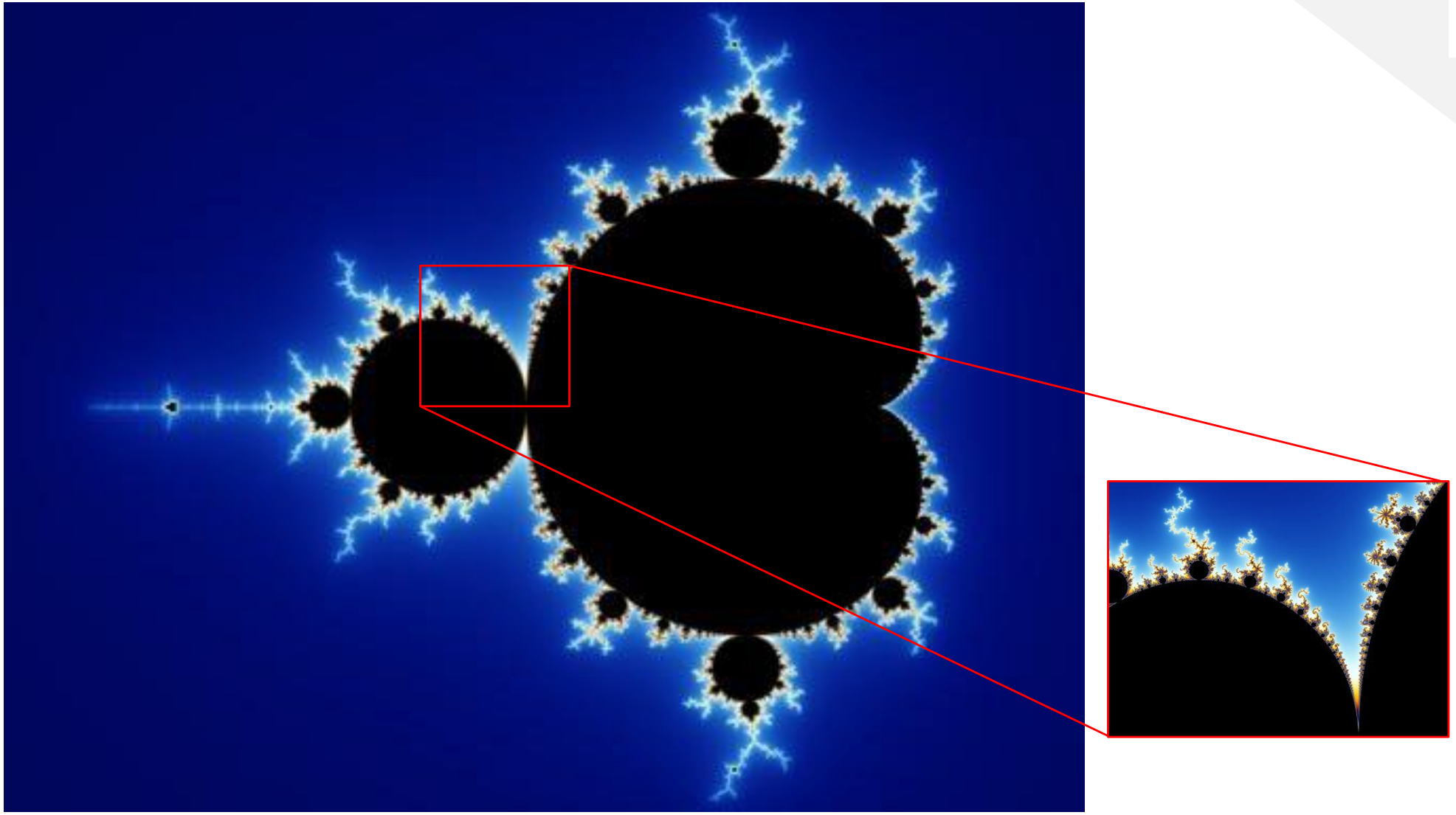
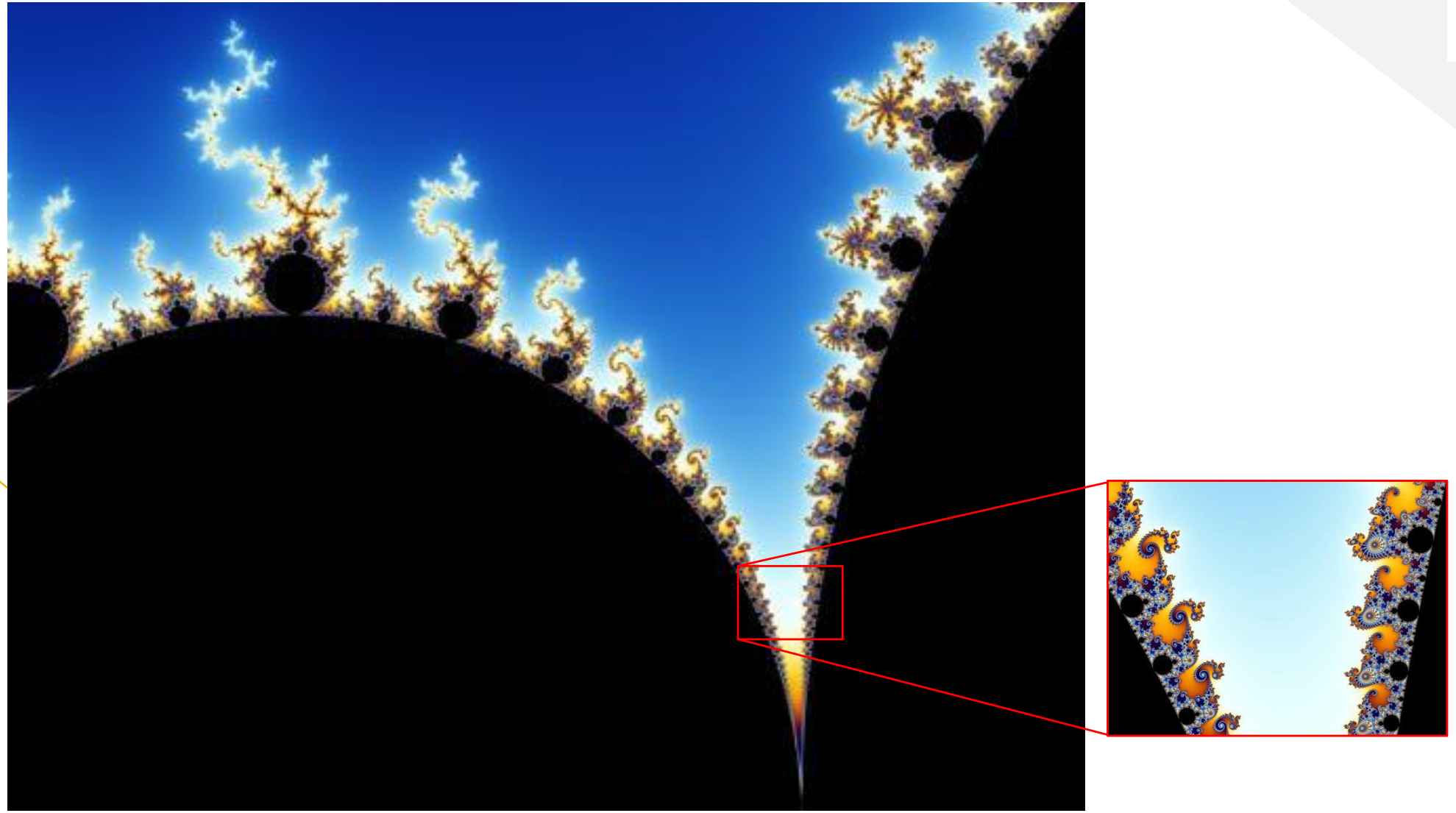
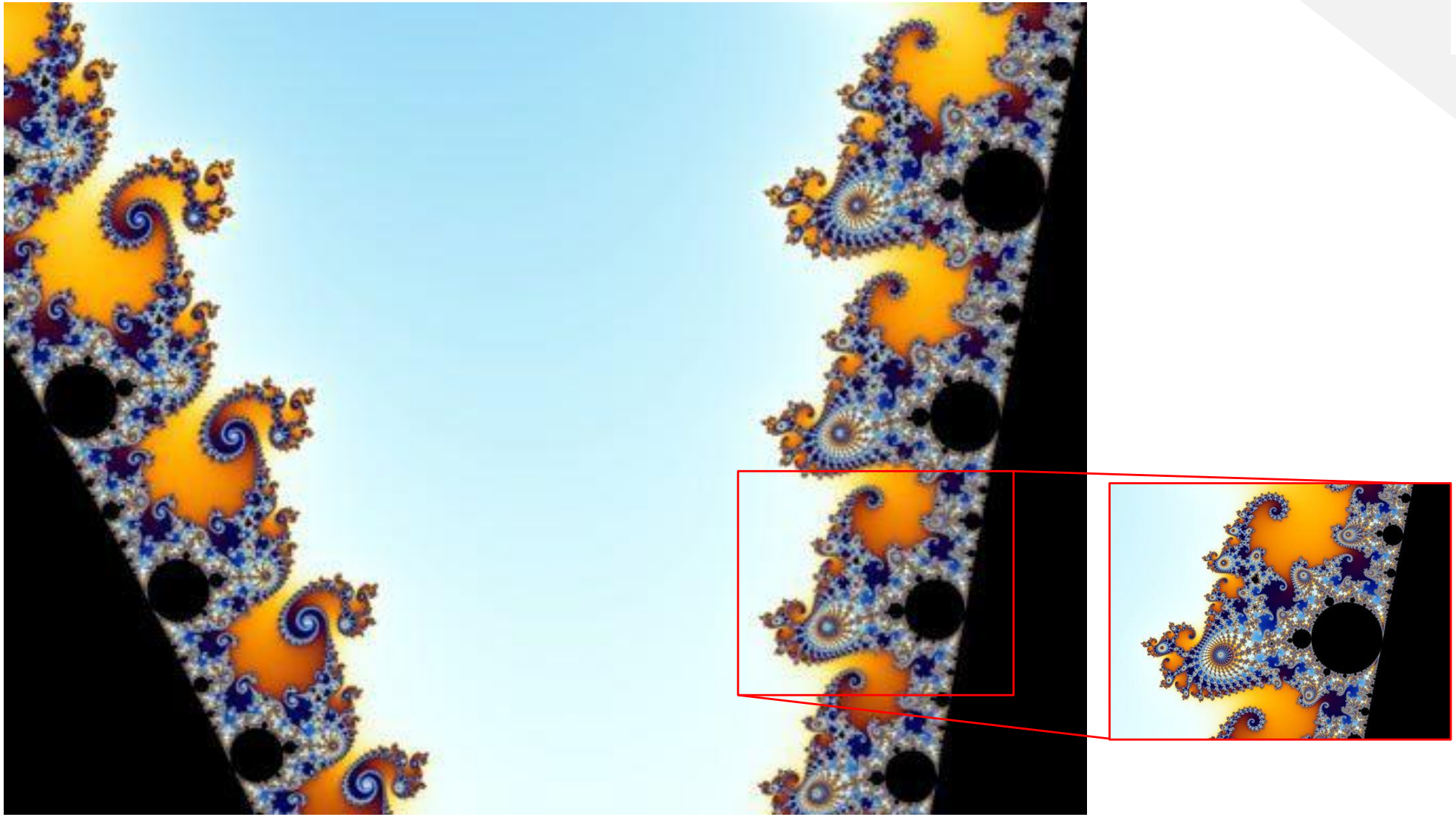
Mandelbrot Set: Zoom level 1



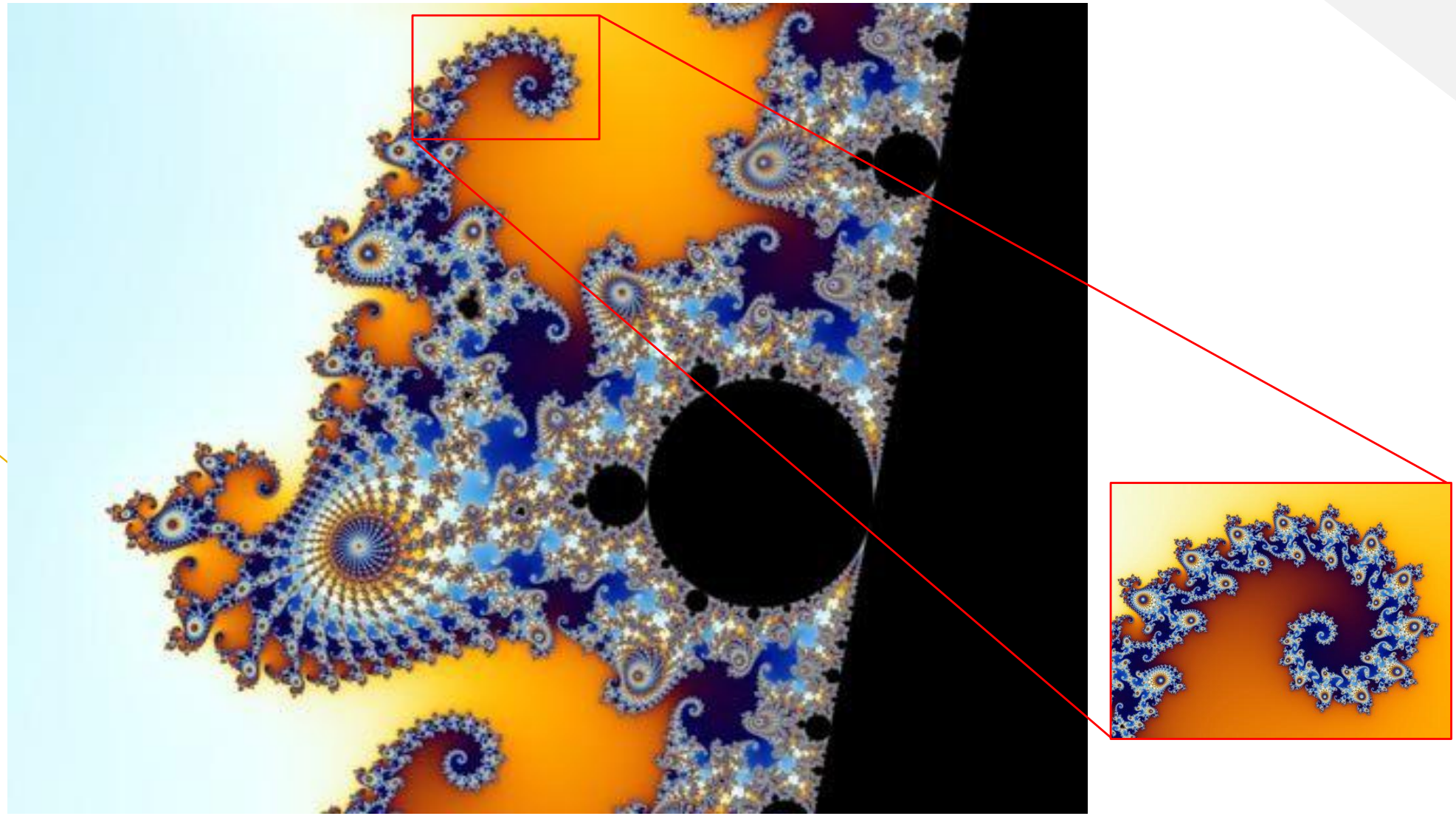
Mandelbrot Set: Zoom level 2



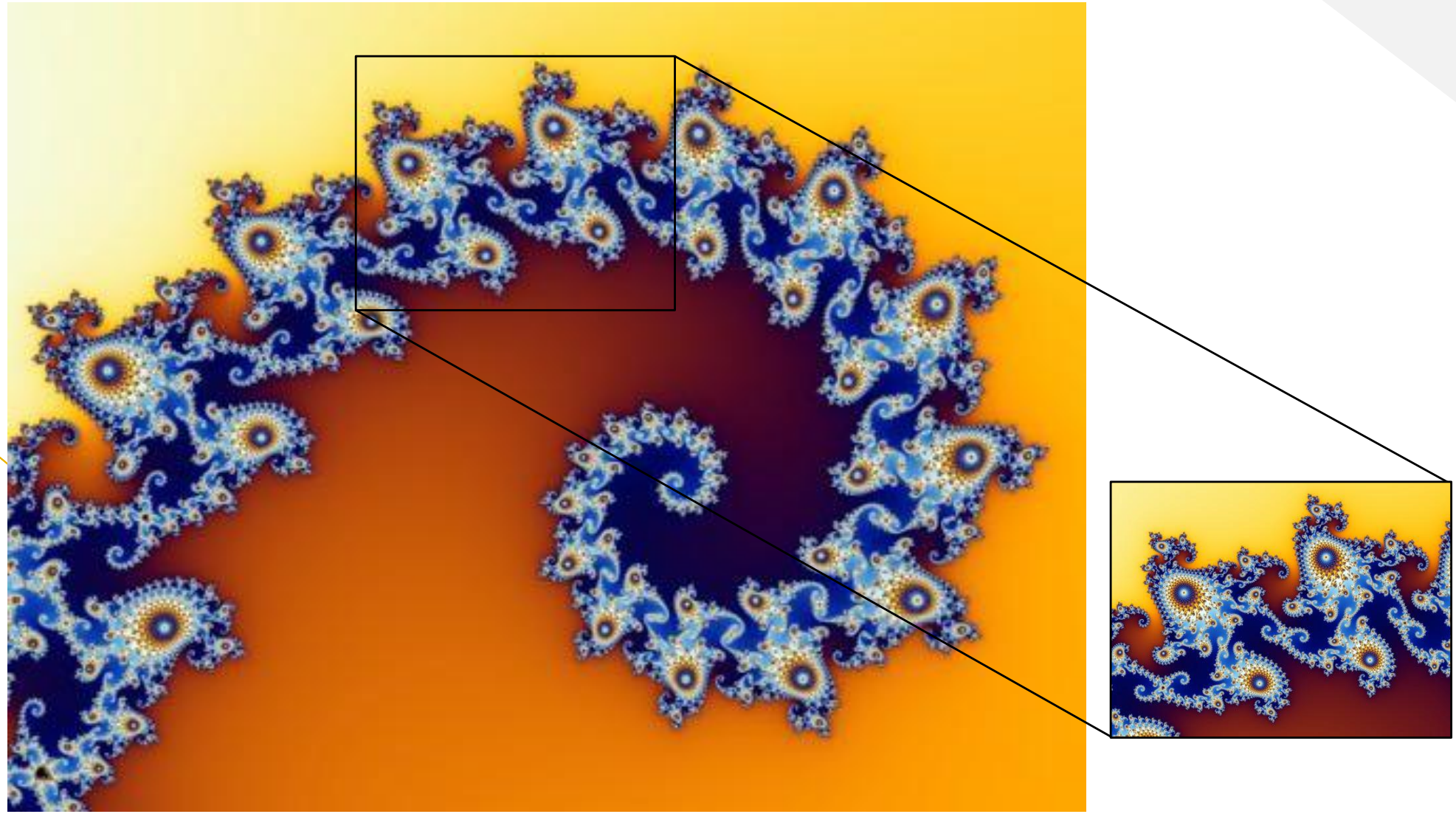
Mandelbrot Set: Zoom level 3



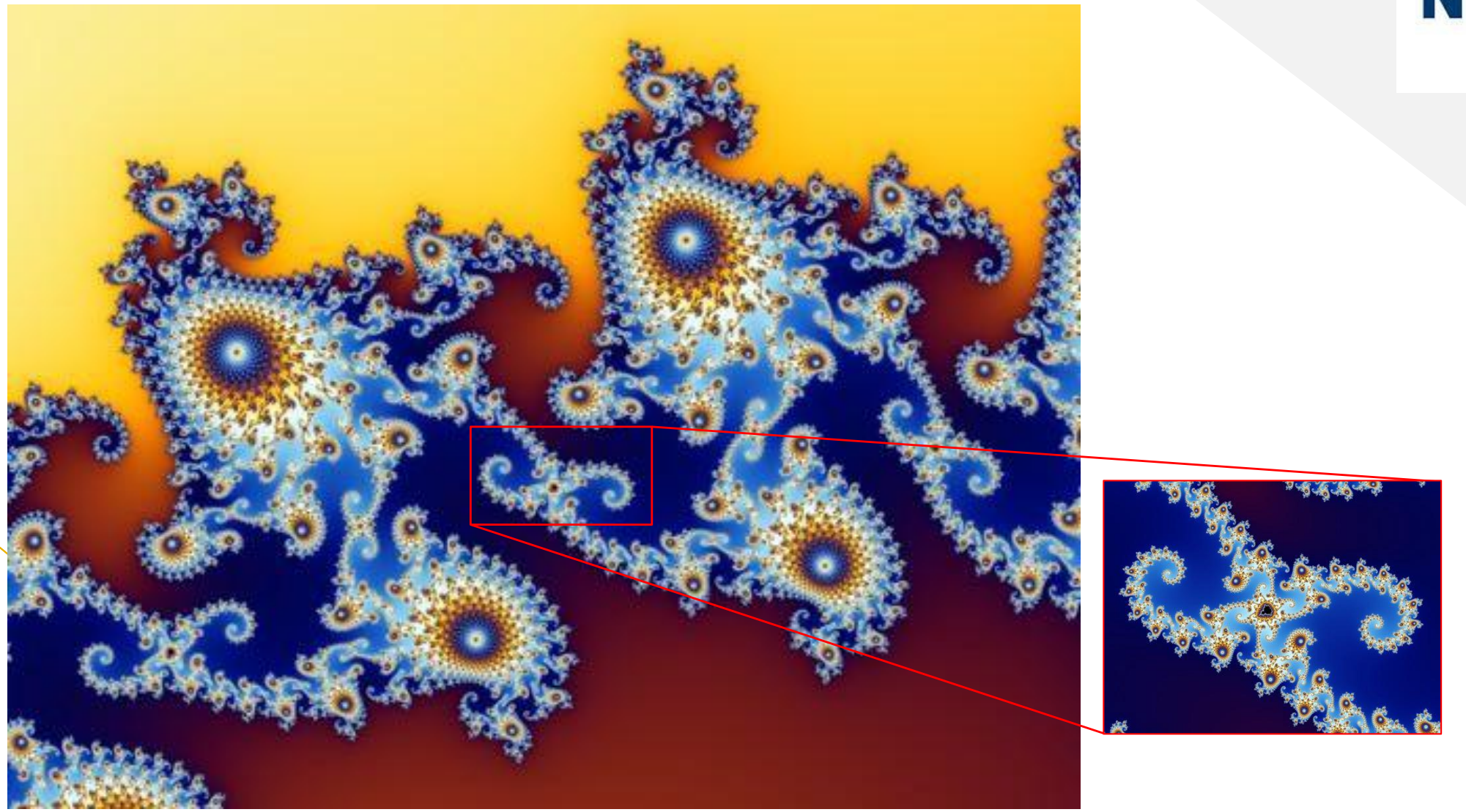
Mandelbrot Set: Zoom level 4



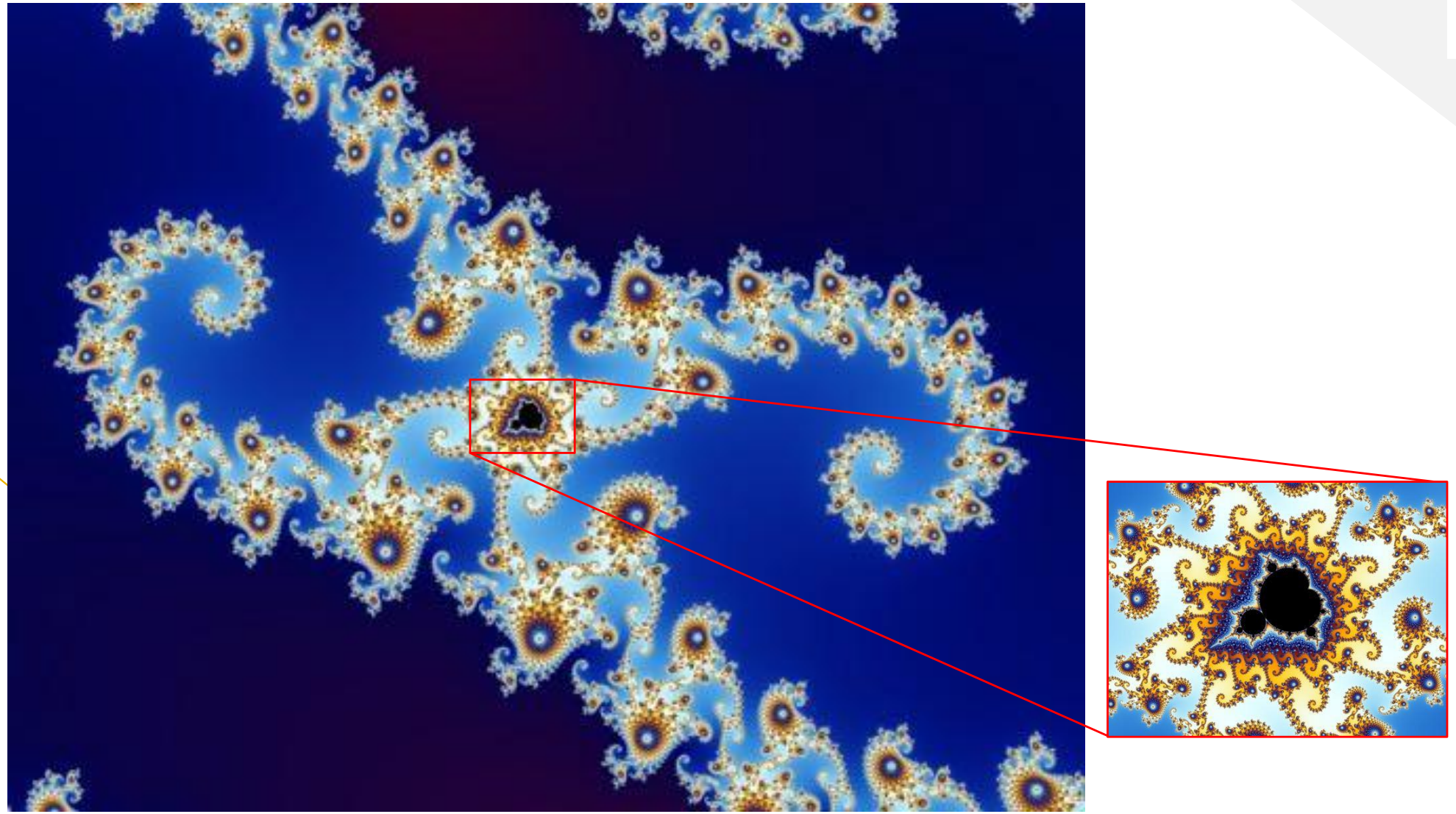
Mandelbrot Set: Zoom level 5



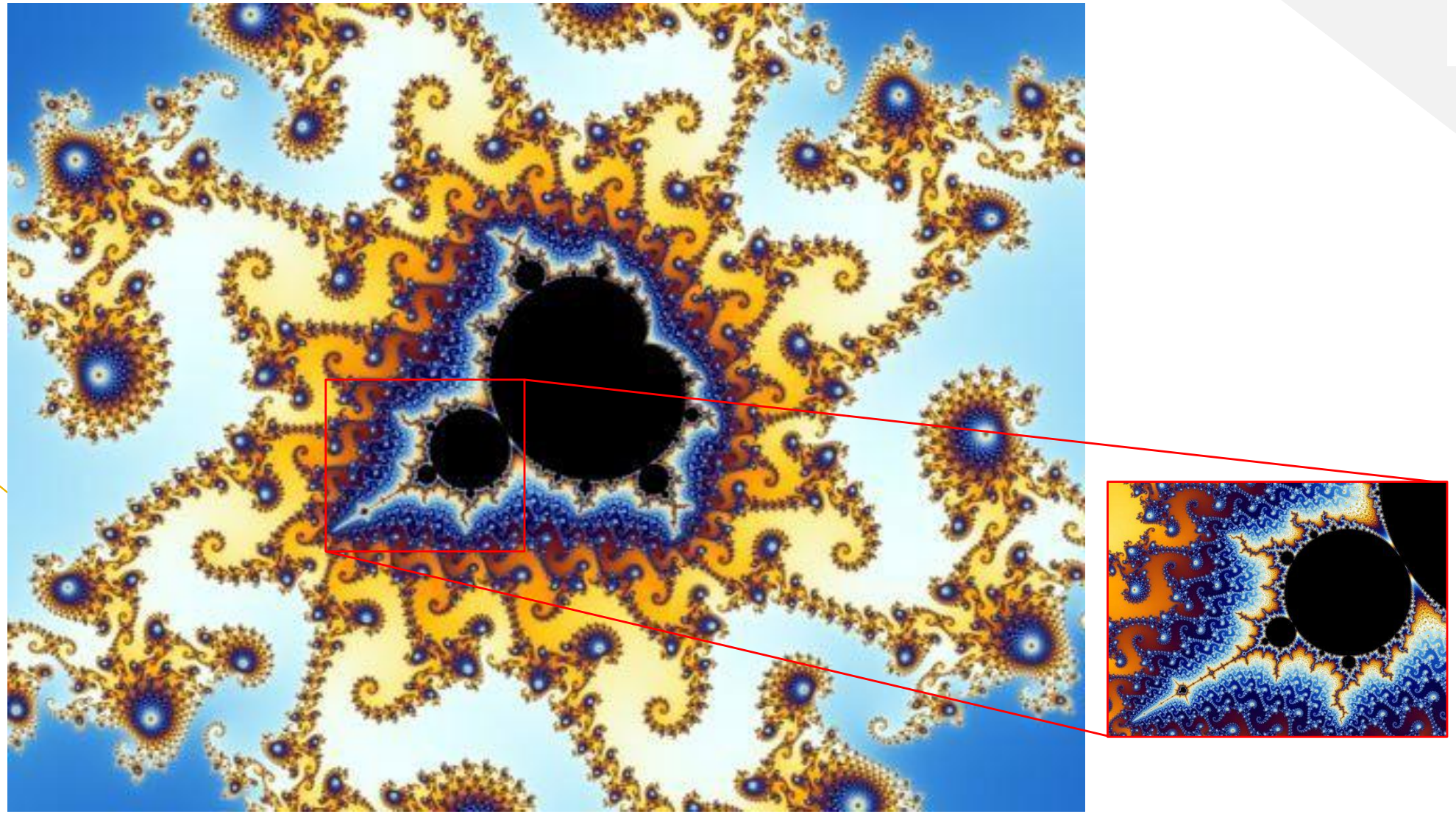
Mandelbrot Set: Zoom level 6



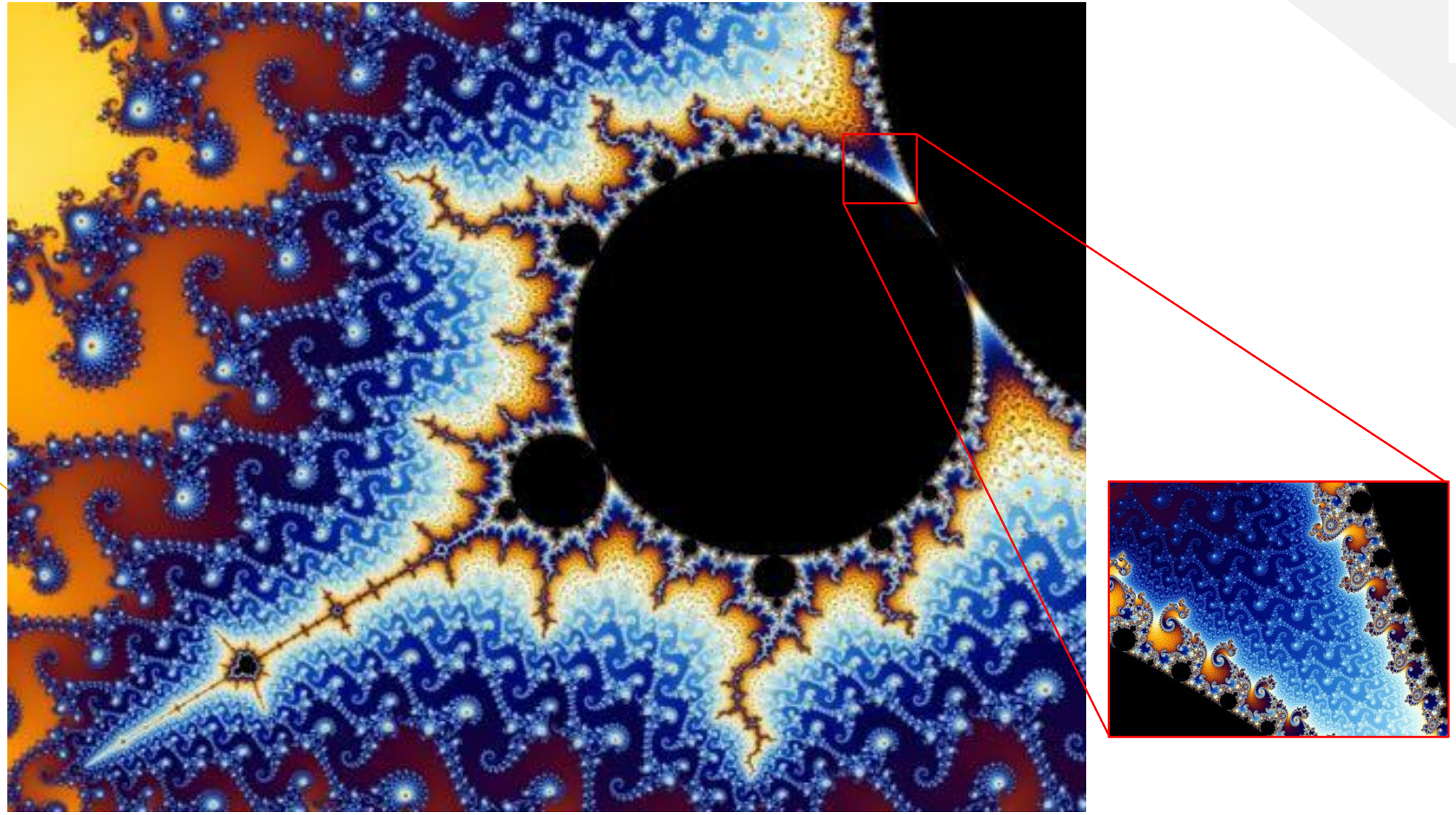
Mandelbrot Set: Zoom level 7



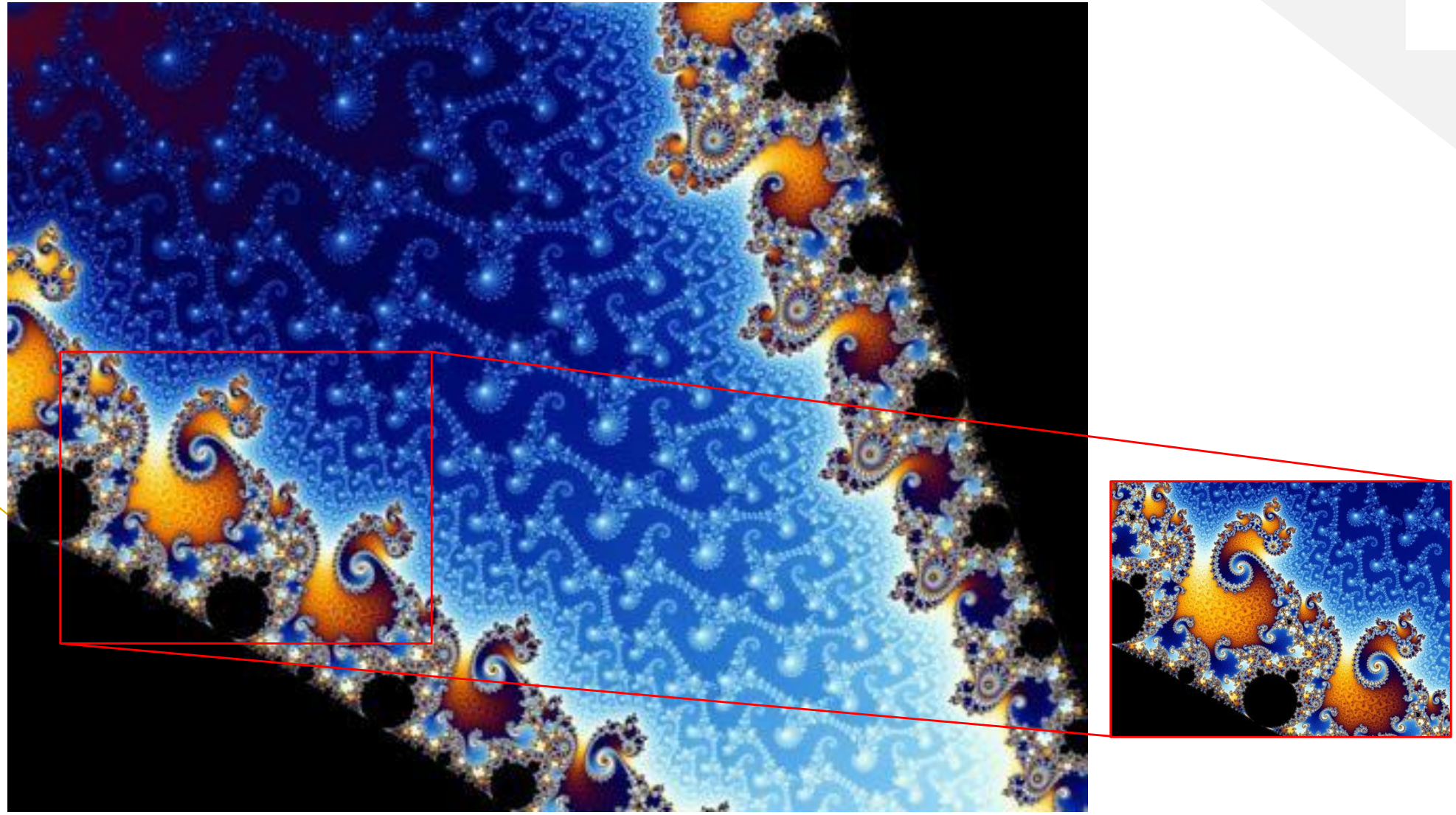
Mandelbrot Set: Zoom level 8



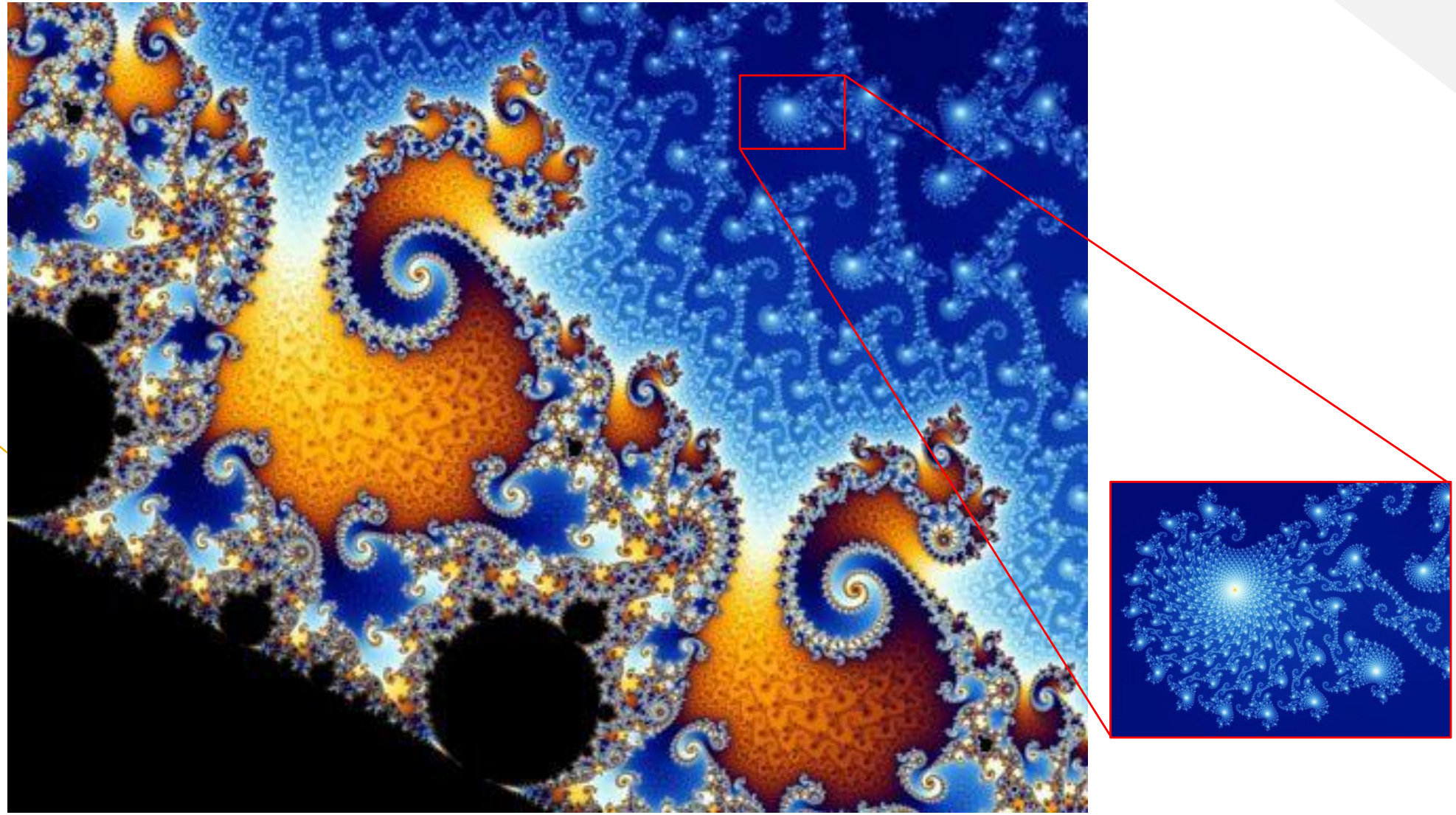
Mandelbrot Set: Zoom level 9



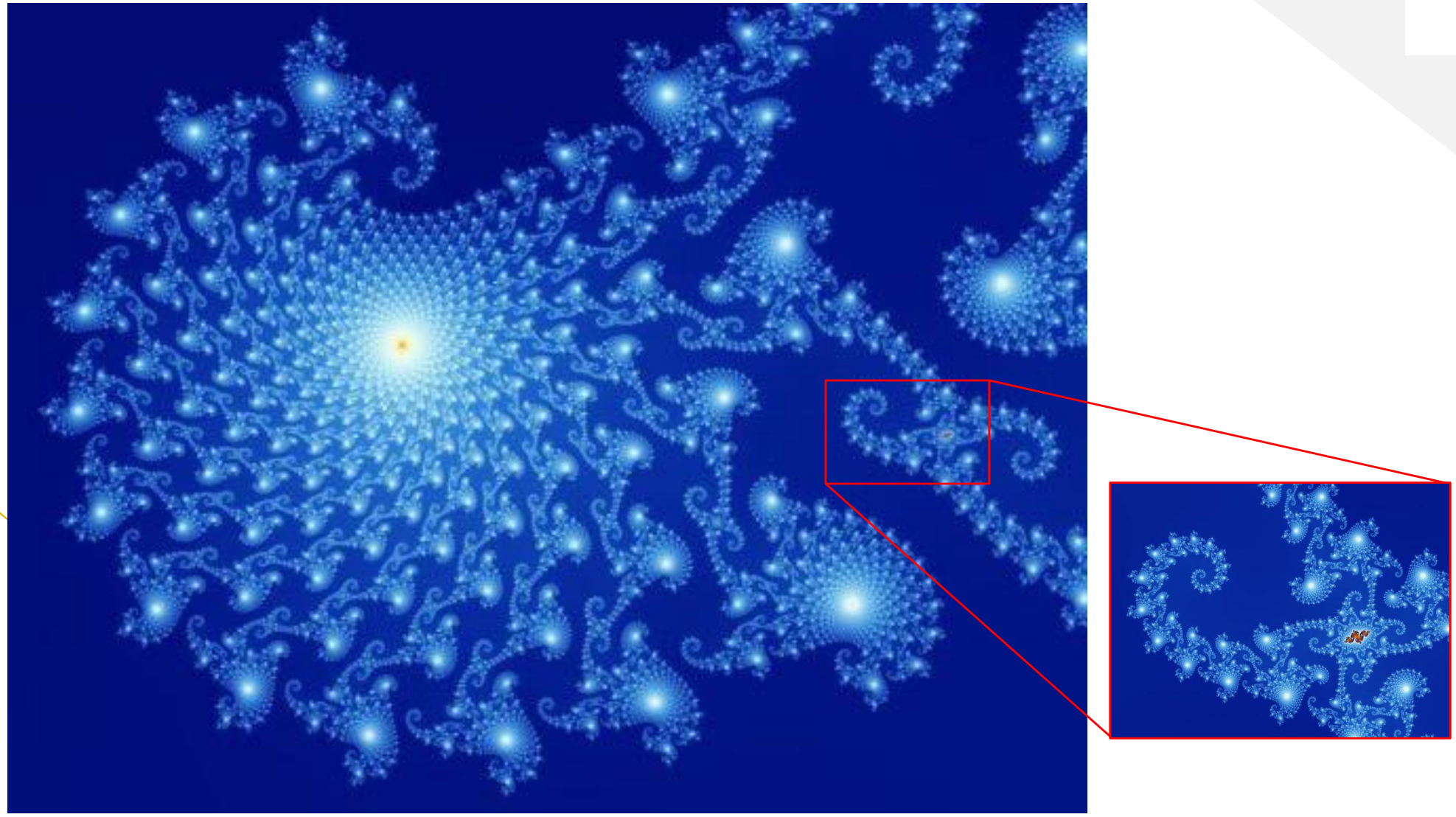
Mandelbrot Set: Zoom level 10



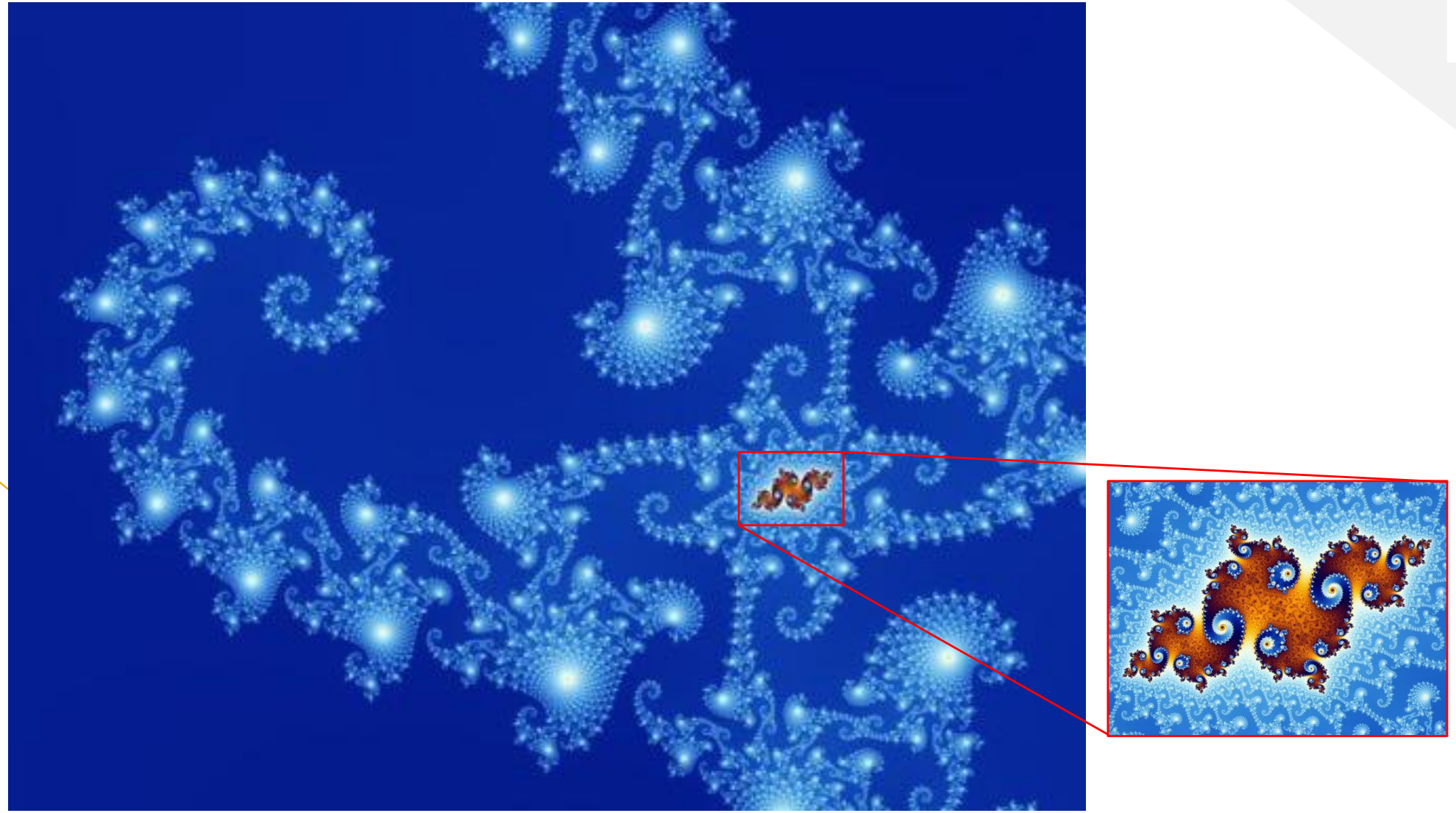
Mandelbrot Set: Zoom level 11



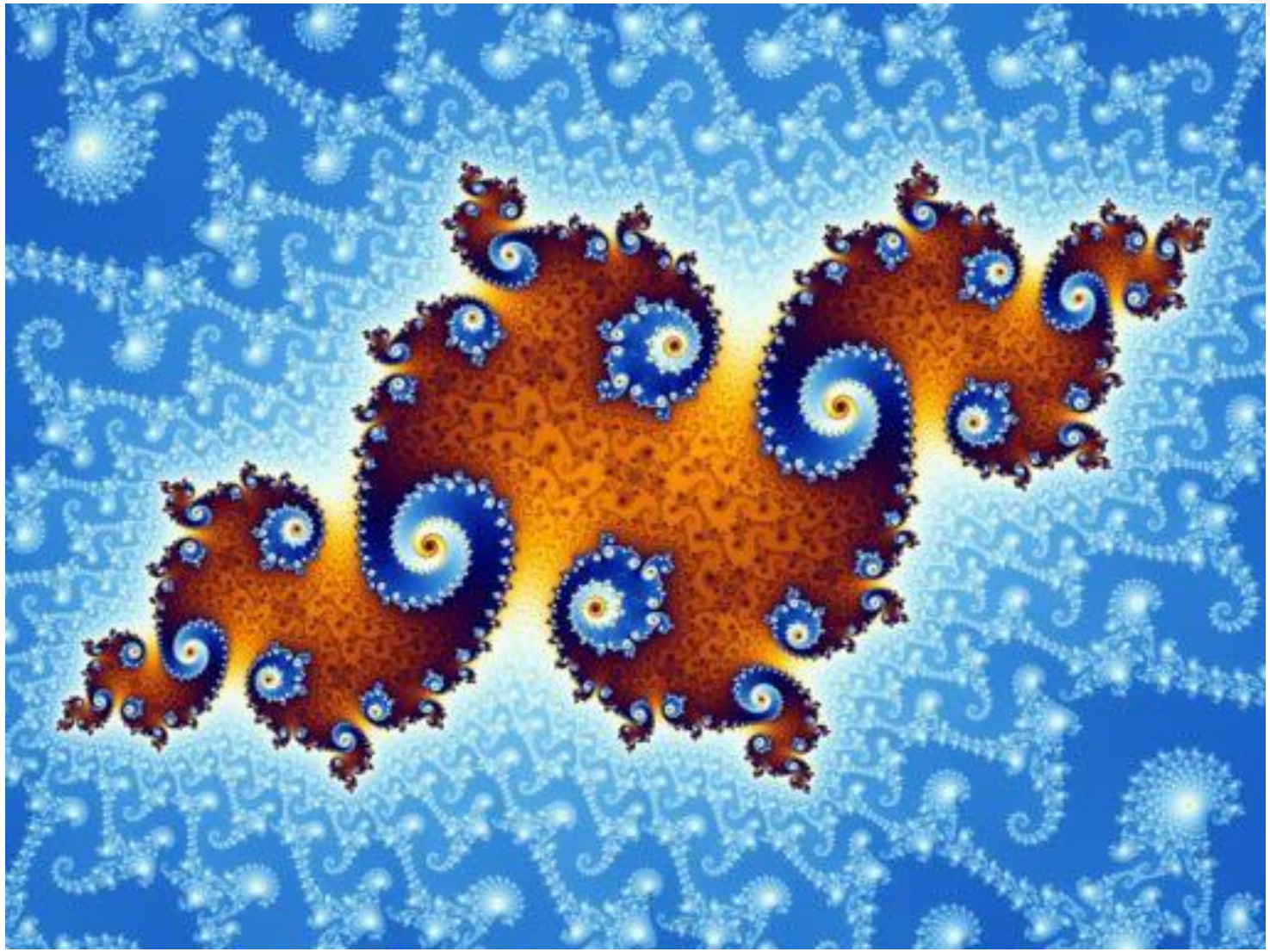
Mandelbrot Set: Zoom level 12



Mandelbrot Set: Zoom level 13



Mandelbrot Set: Zoom level 14



Logistic Map and the Mandelbrot Set are related

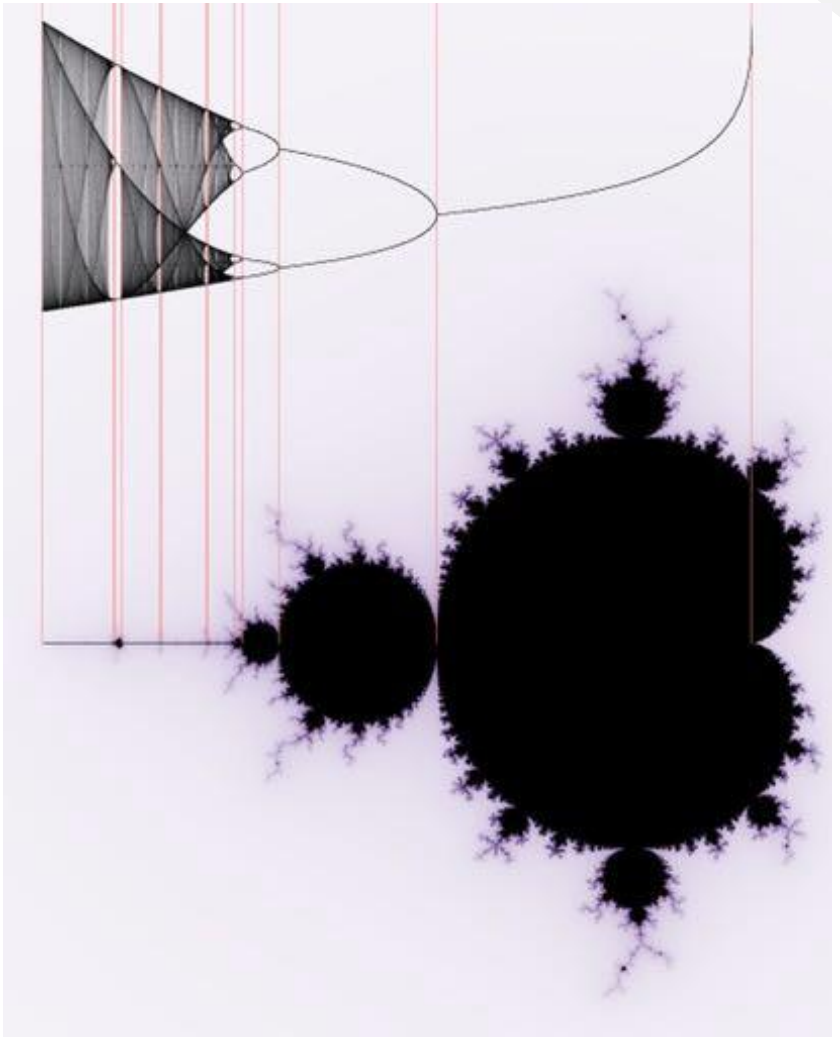
$$x_{n+1} = r\,x_n(1-x_n)$$

$$z_{n+1} = z_n^{\,2} + C$$

Relate the constants C and r by assuming the transformation

$$z_n = a\,x_n + b$$

holds for any x_n and z_n. Then solve for a and b, which leads to a relation for C and r.



Programming Examples

- fill a vector with ‘n’ random numbers, use the Python function `random.random()`
- ***sum*** of all elements
- the ***average*** of all elements
- ***maximum*** of all elements
- find element ***closest to 0.5***
- statistical ***variance***

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Lecture Summary

- Remember how complex numbers work.
- Mandelbrot set emerged from a quadratic map that is applied to complex numbers.
- This mapping function also has fixed points for certain values of the constant C.