Biomedical Engineering
生醫工程

Jerry Tai
戴立嘉

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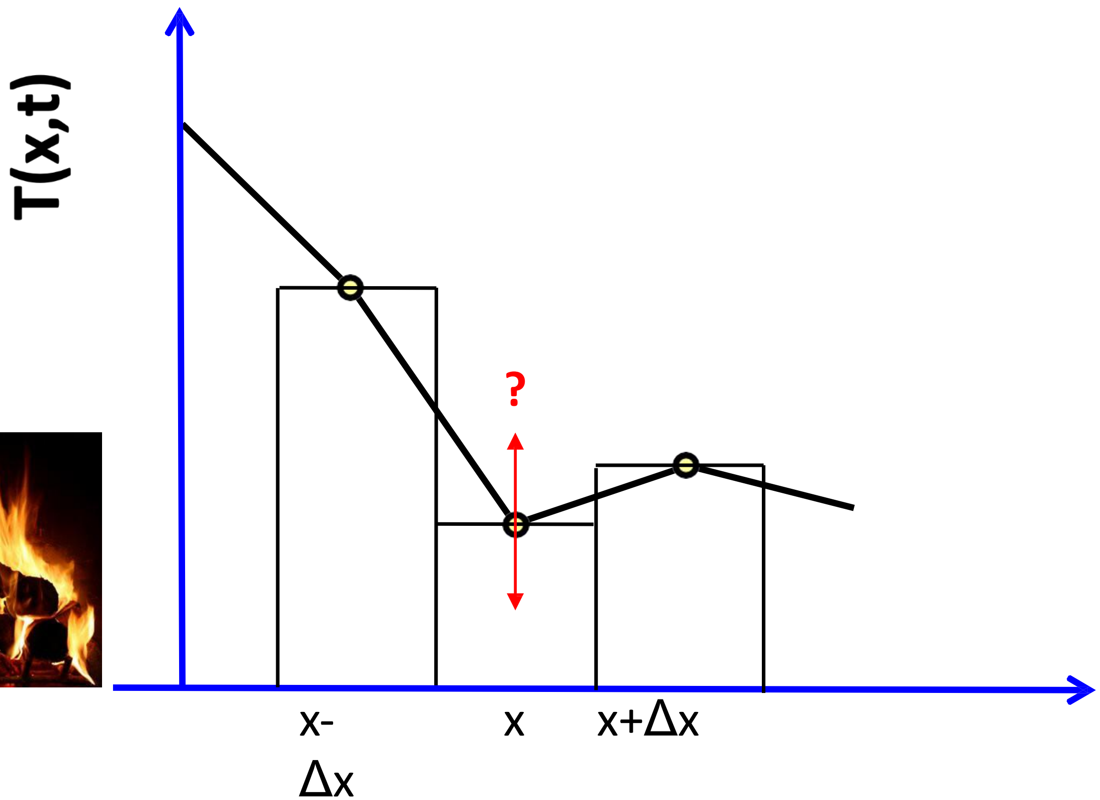
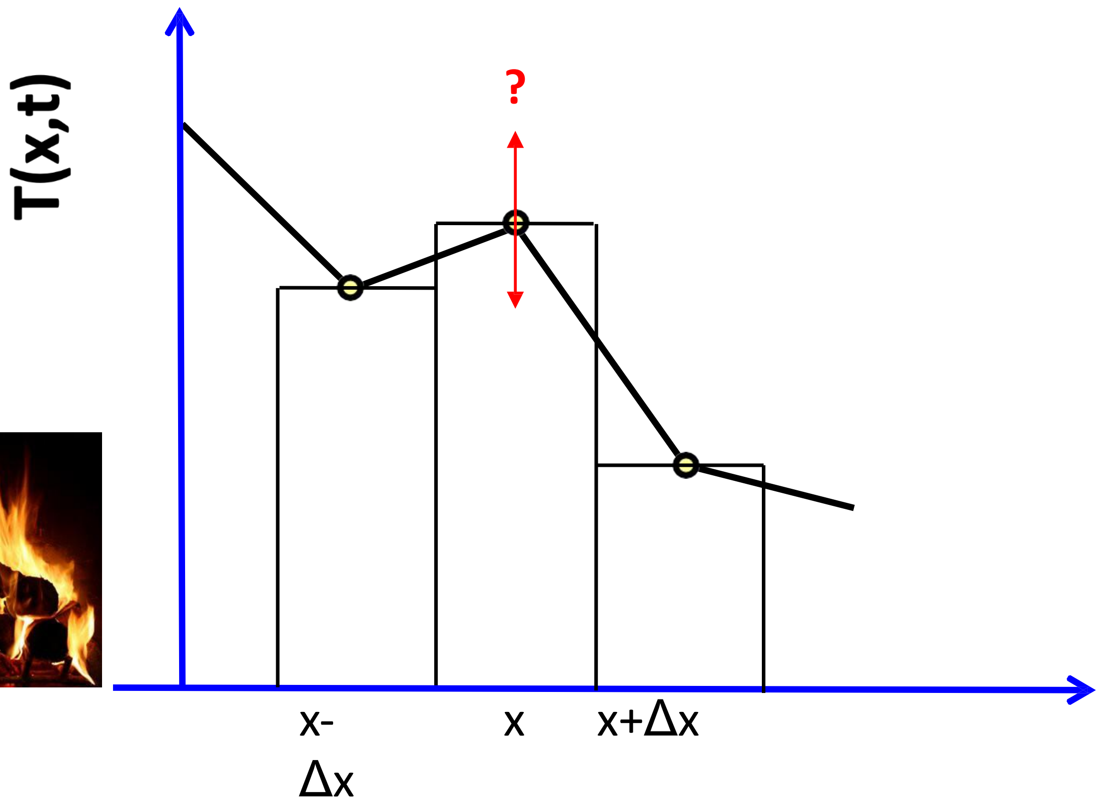
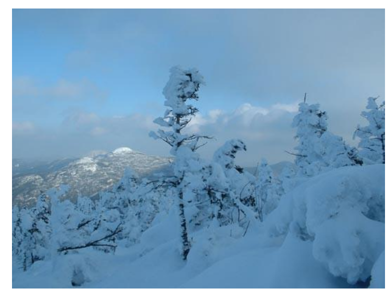
How do we expect one temperature point to change with time?



Heat equation:

How do we expect one temperature point to change with time?





Heat equation:

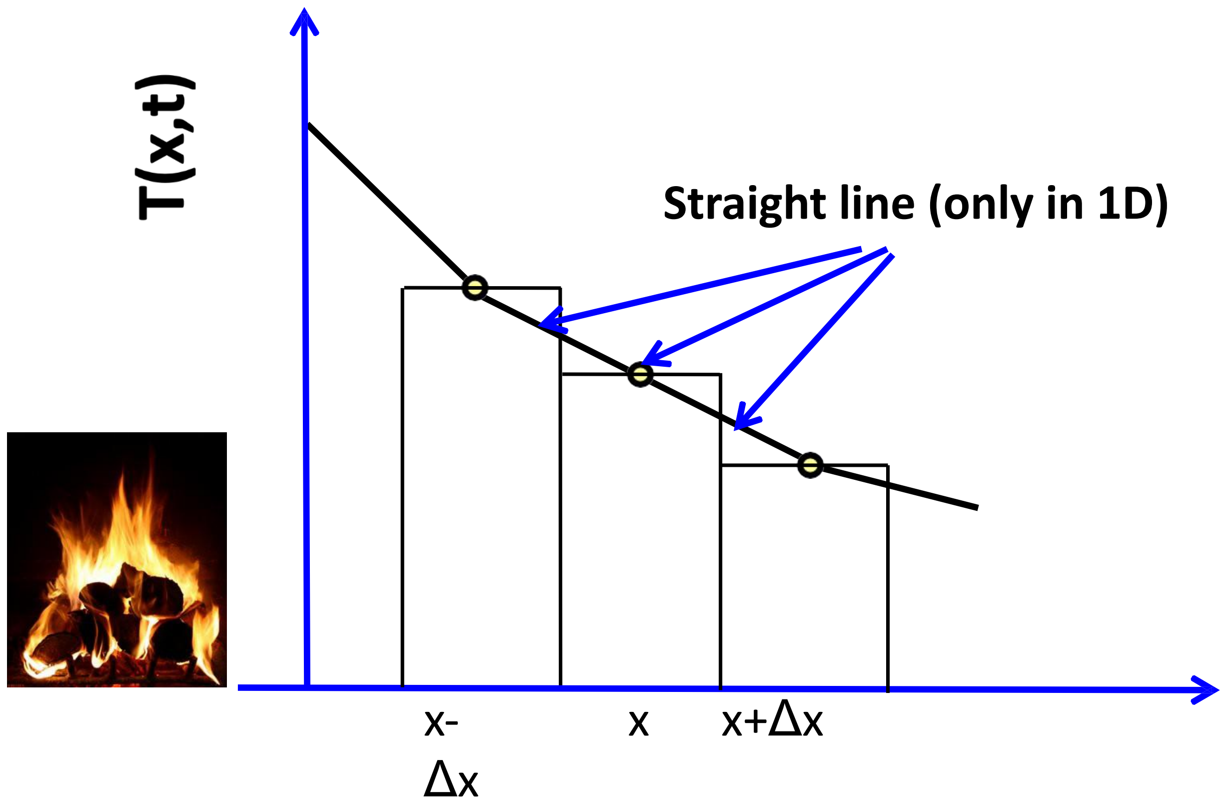
Generalization to 3D house T(x,y,z,t)

Heat equation in 3D: $$\frac{\partial T}{\partial t} = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] = k\vec{\nabla}^2 T = k\,\Delta T$$

where Δ is the Laplace operator.

The heat equation and the diffusion equation have identical forms. In one case, particles diffuse, in the other heat diffuses.

What is a stationary state?



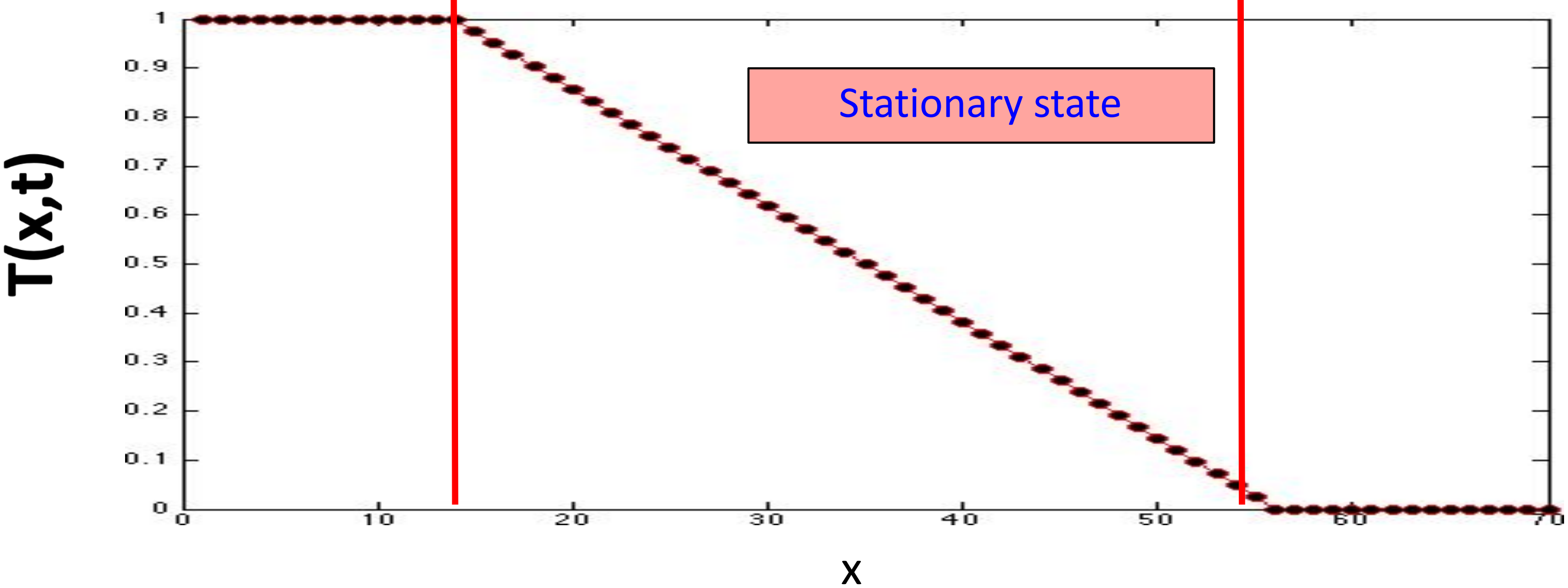
Stationary state:

$$\frac{\partial T}{\partial t} = 0$$

$$\frac{\partial^2 T}{\partial x^2} = 0$$

Stationary state: Temperature distribution in the wall falls of linearly





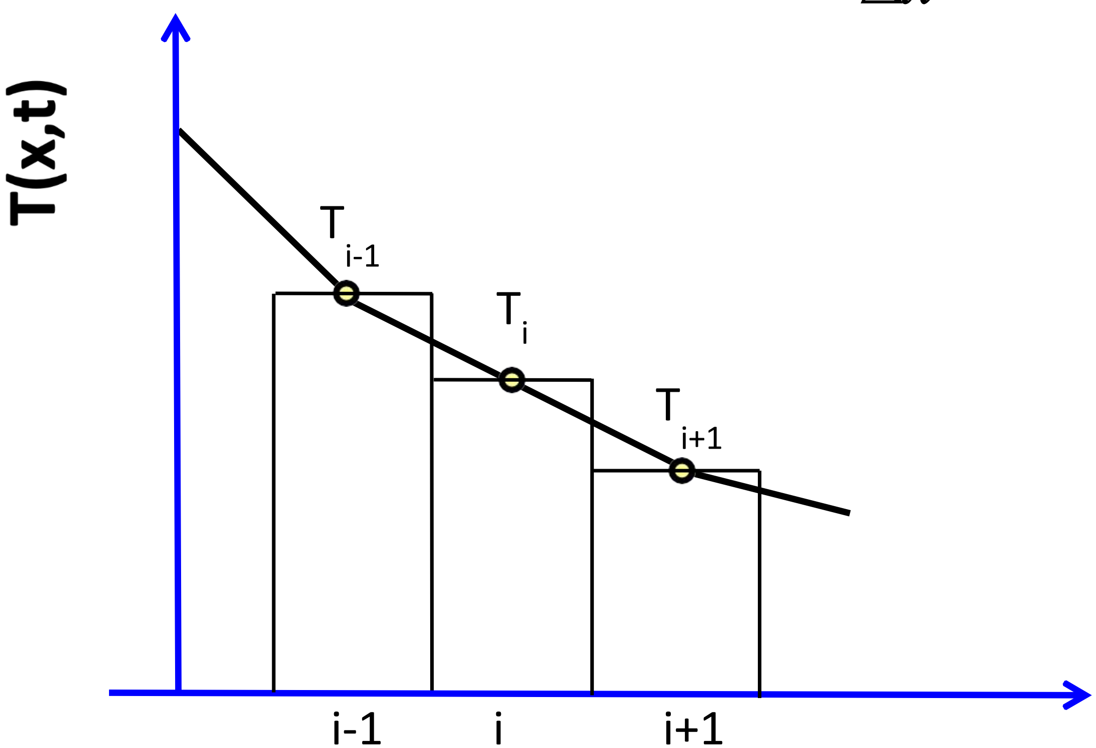
Numerical methods to find the stationary state

Laplace equation in regular form:

$$\frac{\partial^2 T}{\partial x^2} = 0$$

Laplace equation in discretized form:

$$\frac{T(x+\Delta x) - 2T(x) + T(x-\Delta x)}{\Delta x^2} = 0$$



Convert between real and index space:

$x_i = i * \Delta x$

$T_i = T(x)$

$$T_{i+1} - 2T_i + T_{i-1} = 0$$

Must specify the values at the boundaries (Boundary value problem)

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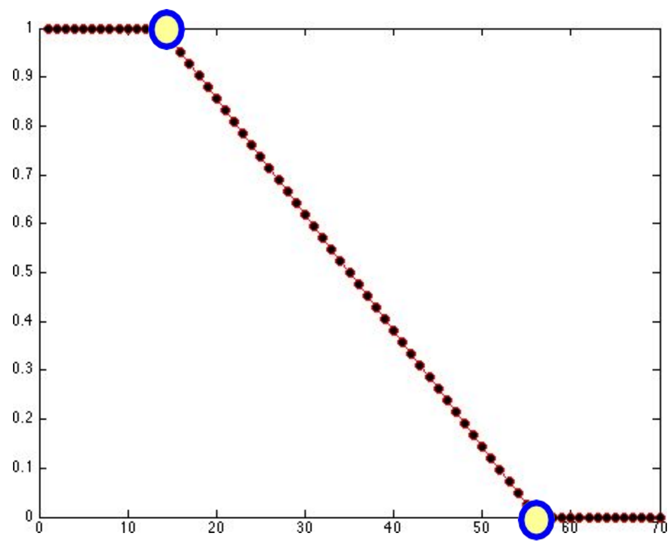

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Domain:

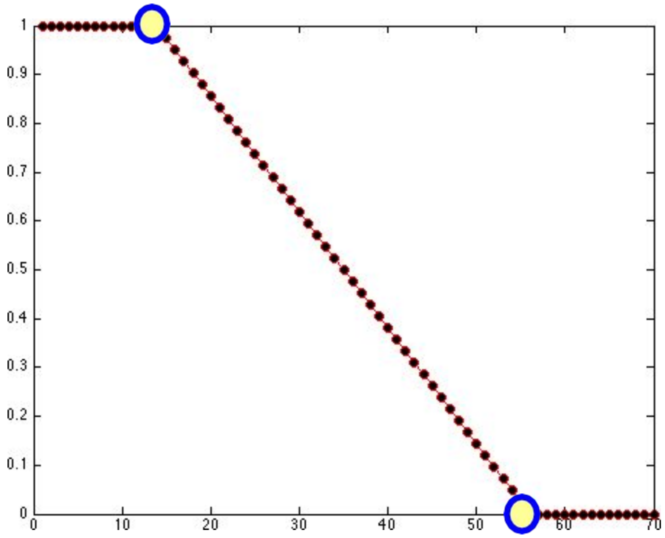
$$0 \le x \le L$$

$$0 \le i \le N$$
$$\Delta x = L / N$$

Boundary values:

$$T(x=0) = 1 \qquad T(x=L) = 0$$

$$T_0 = 1 \quad T_N = 0$$



Convert in the following way:

$x_i = i * \Delta x$

$T_i = T(x_i)$

Left end: x=0 T_0

Right end: x=L T_N

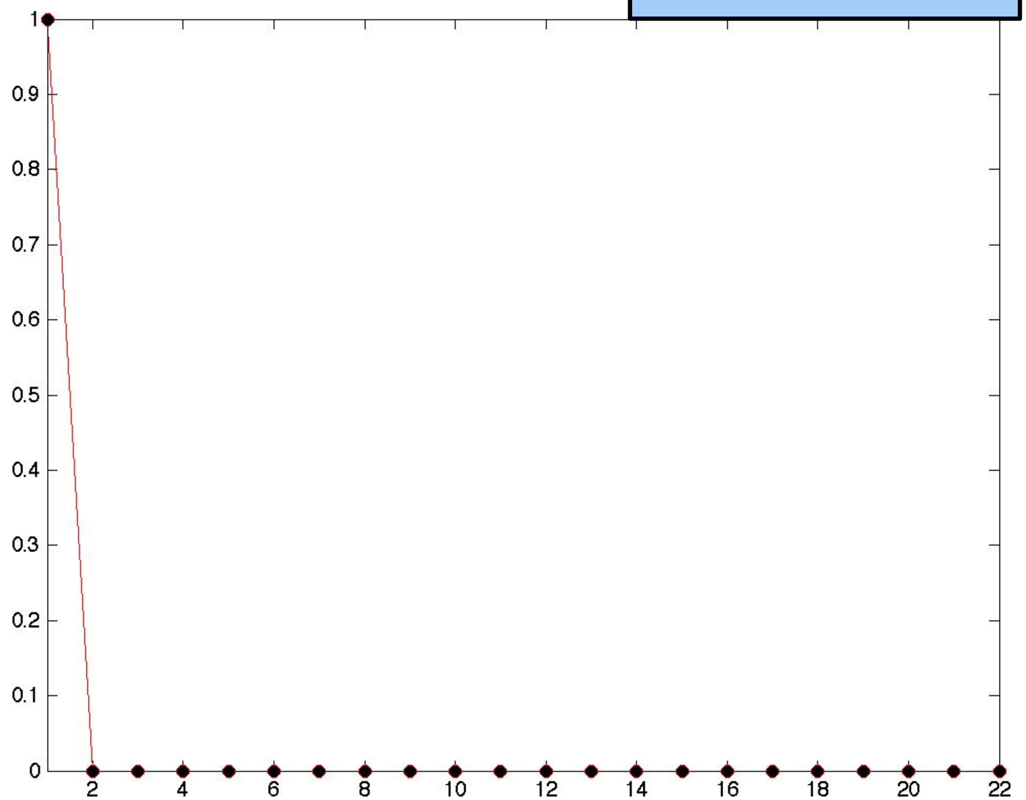
How do we solve this boundary value problem?

Fix values at boundary

$$T_0 = 1 \quad T_N = 0$$

Pick initial values:

$$T_{0<i<n} = 0$$



Split interval (0,L) into N sections.
So we have N+1 temperature points.
Since Python starts counting indices with 0, the range of i will be from 0 to N.

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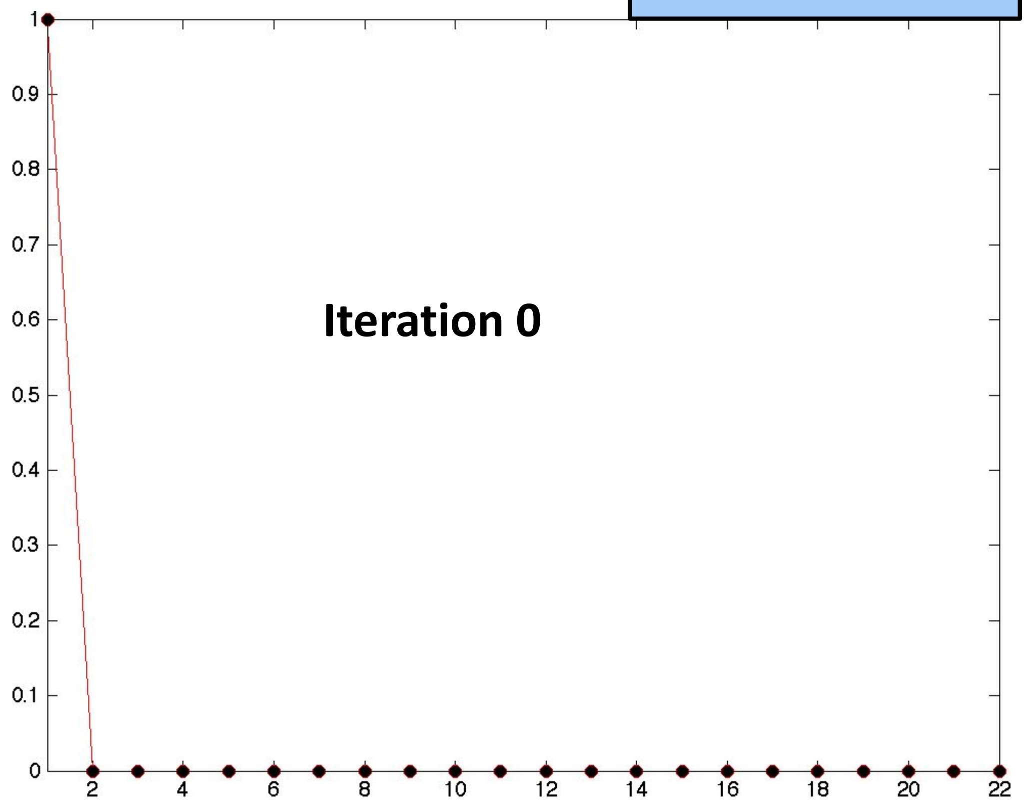
Solve it iteratively:

$$T_i^{new} = (T_{i+1} + T_{i-1})/2$$



Converge iteratively to the stationary state

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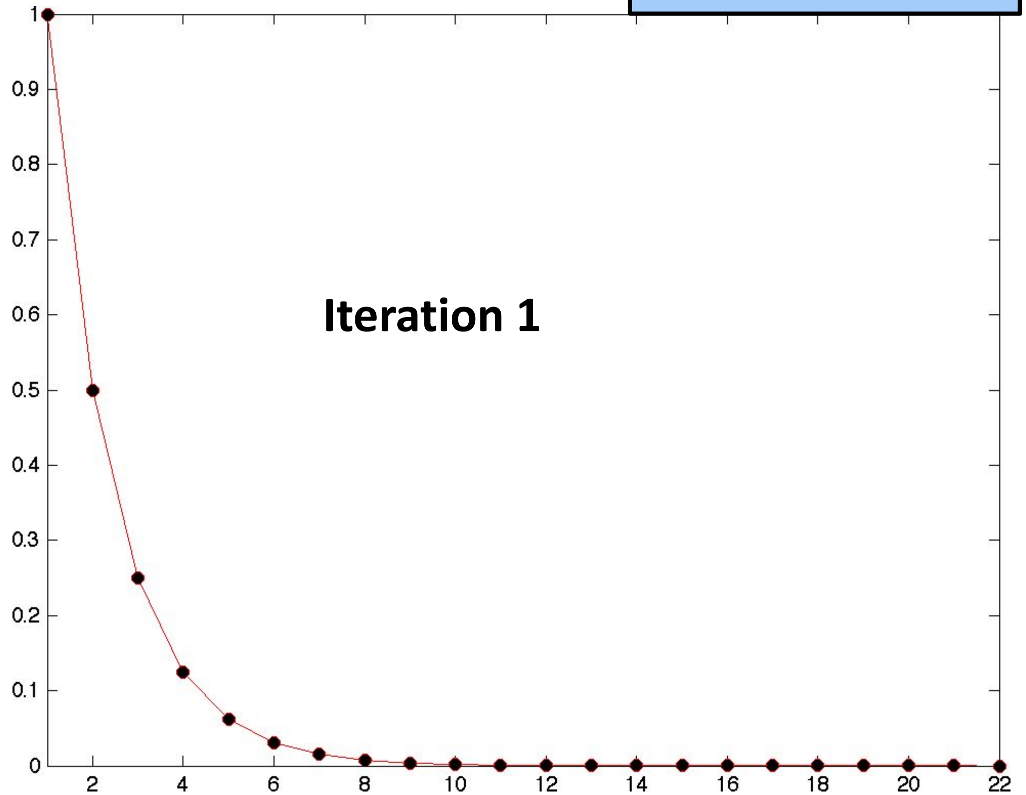

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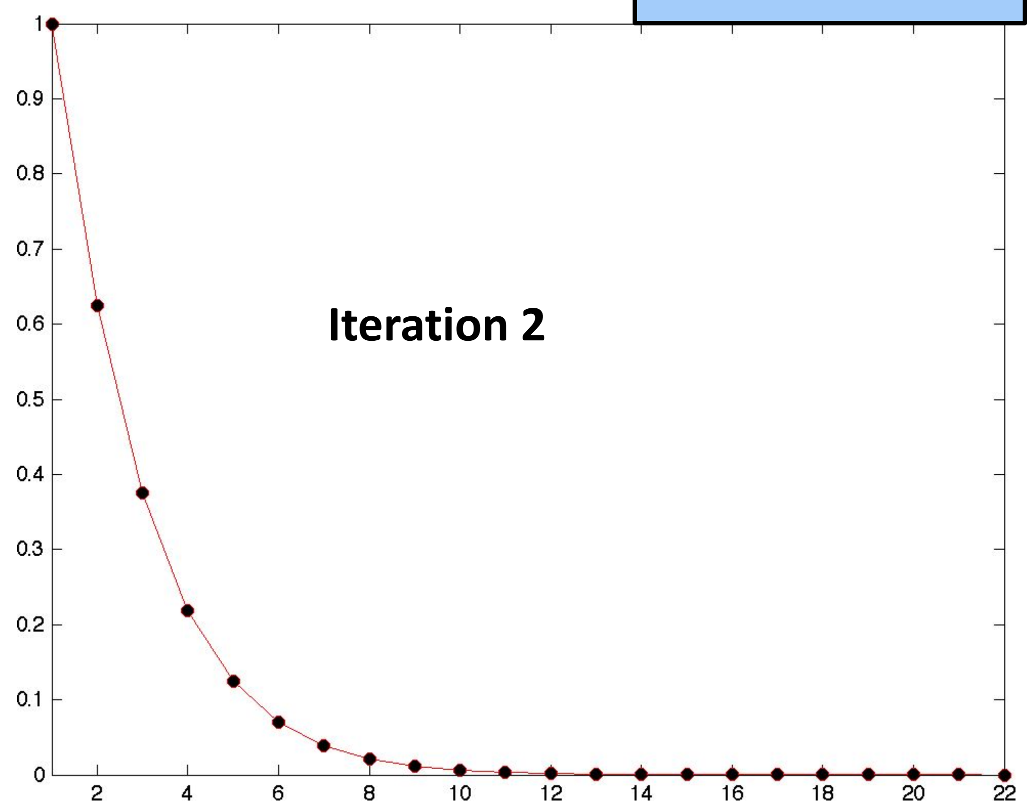

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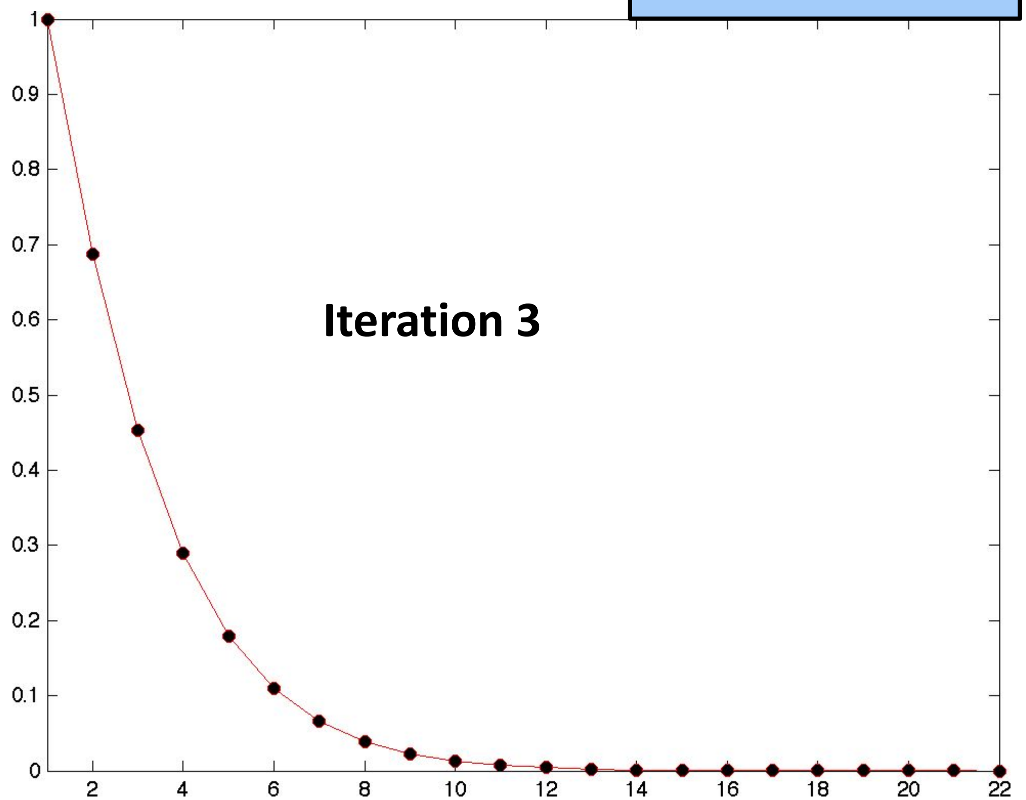

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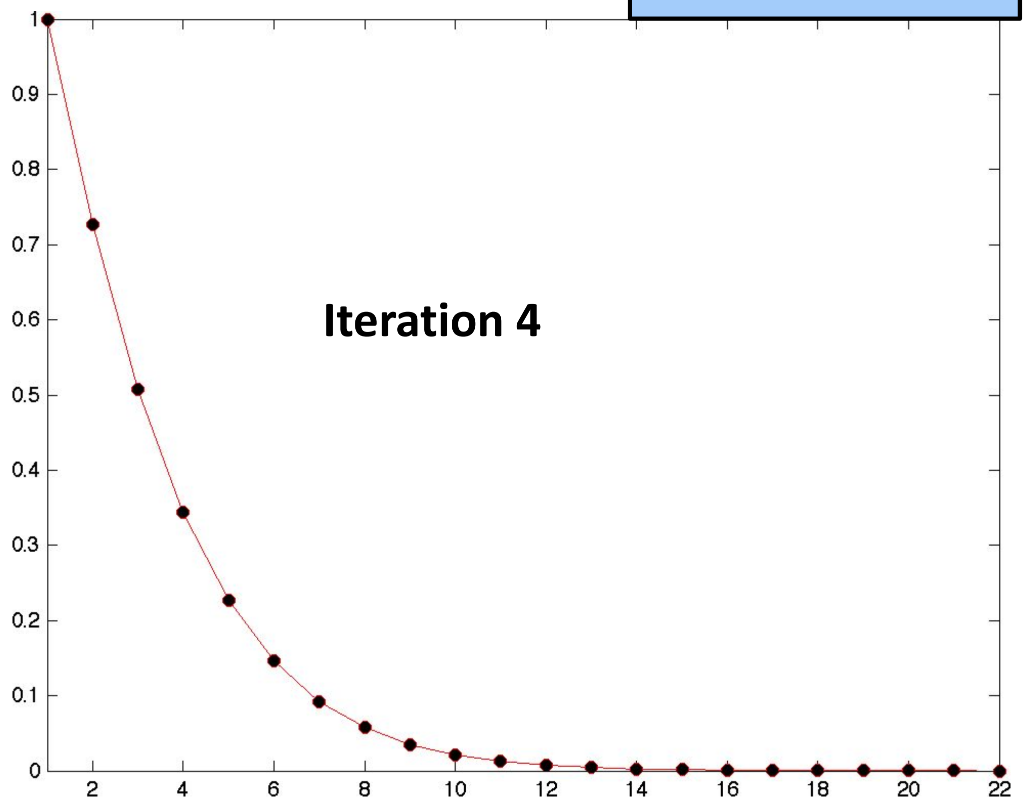

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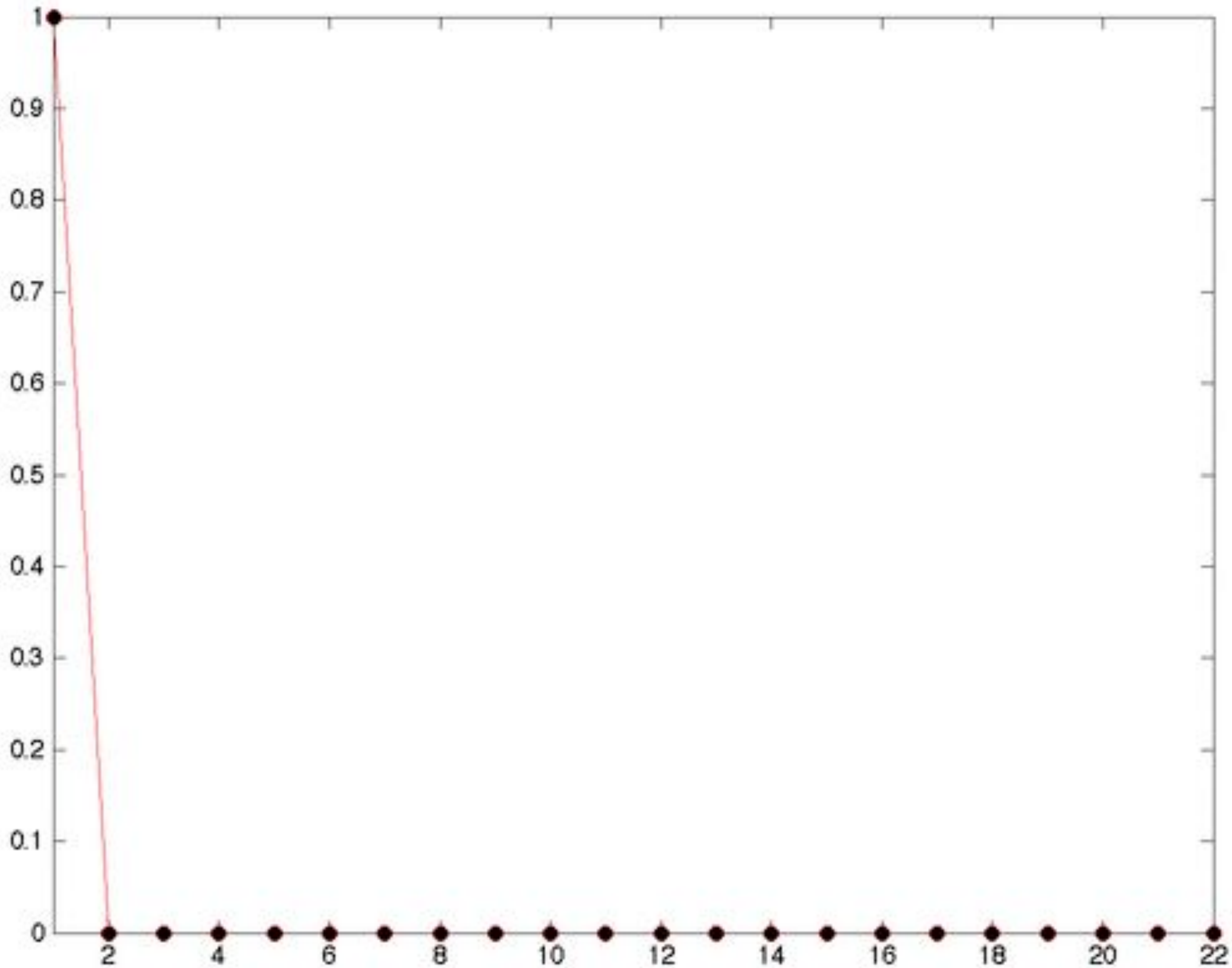

Summary: Heat Equation

(1) Heat equation as example of a partial differential equation.

(2) Stationary temperature distributions

(3) Discretized the domain to construct a numerical algorithm to obtain the stationary state (Not a time dependent solution!)