Biomedical Engineering
生醫工程

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Lecture 4: Random Walks and Diffusion Limited Aggregation

Motivation:

1) Diffusion limited aggregation:

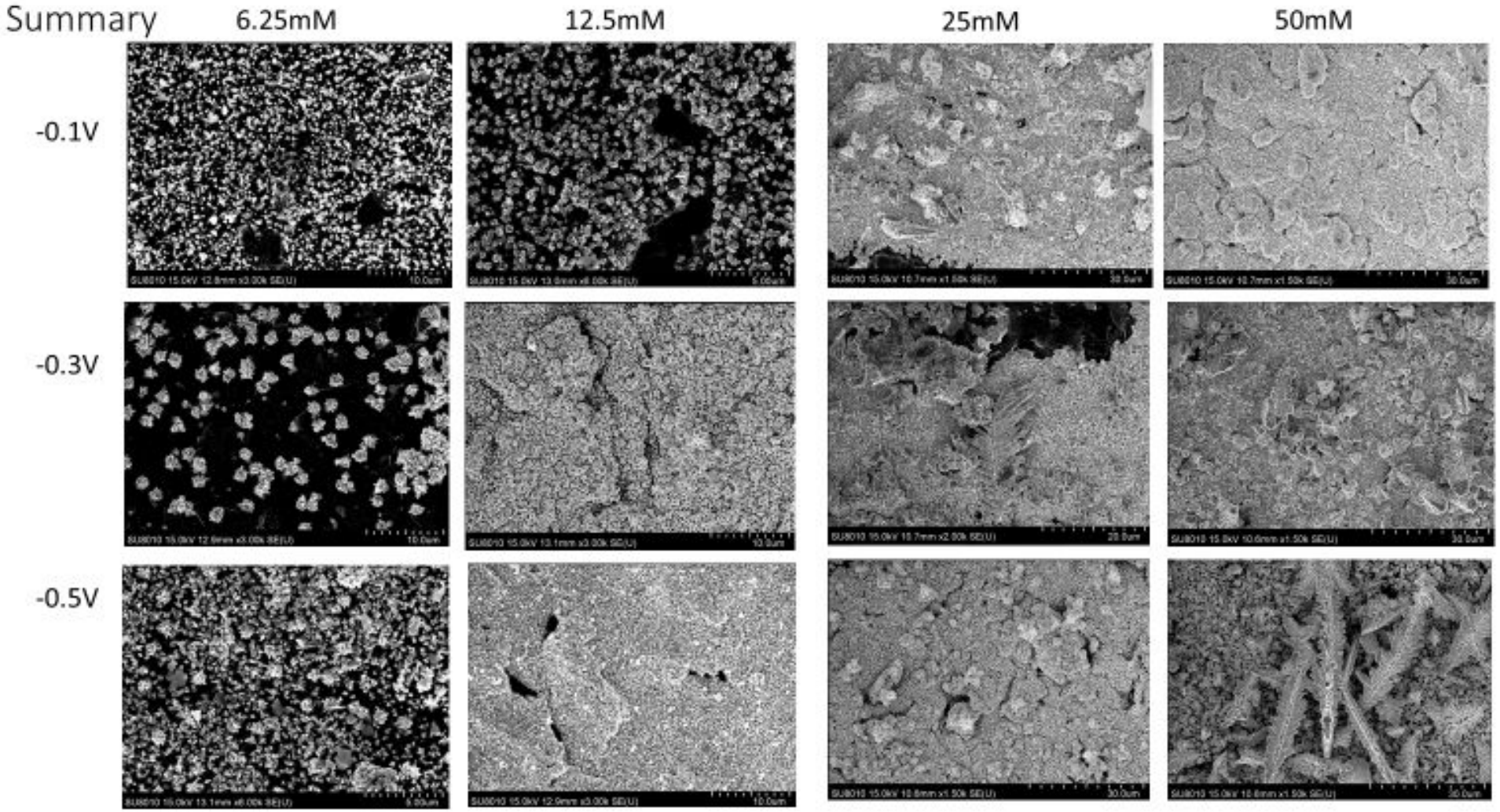
How did this structure grow?

1) This requires learning about random walks and Brownian motion

Diffusion Limited Aggregation (DLA)

A DLA cluster grown from a copper sulfate solution in an electro-deposition cell

Electroplating



Electrode:carbon
Source: DC
Time : 1hr
Solution: HAuCl4

Brownian Motion:

Motion of one particle in a solution

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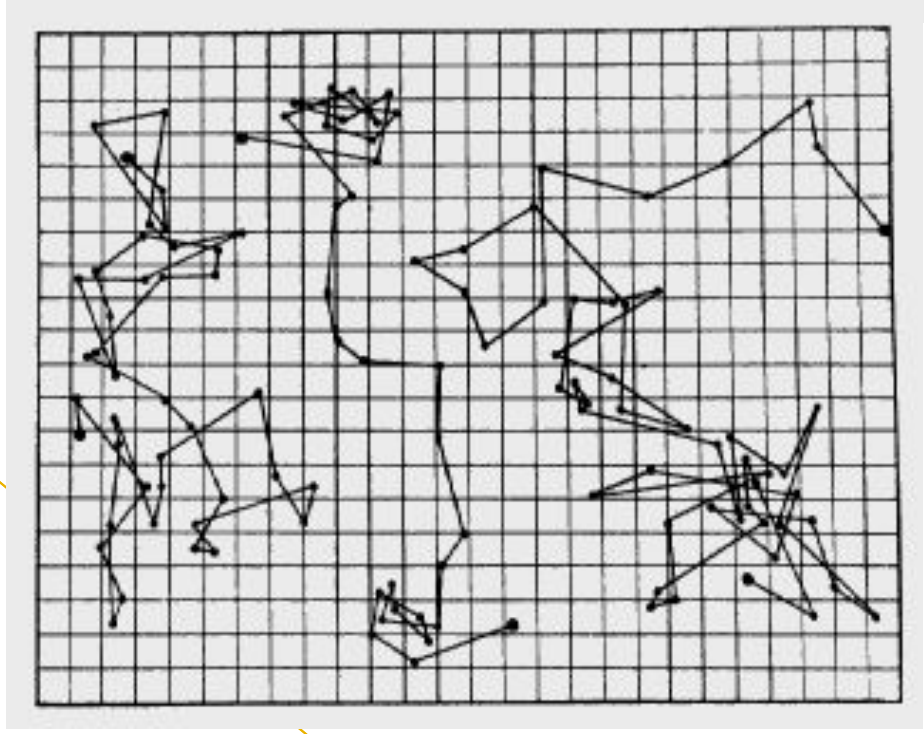
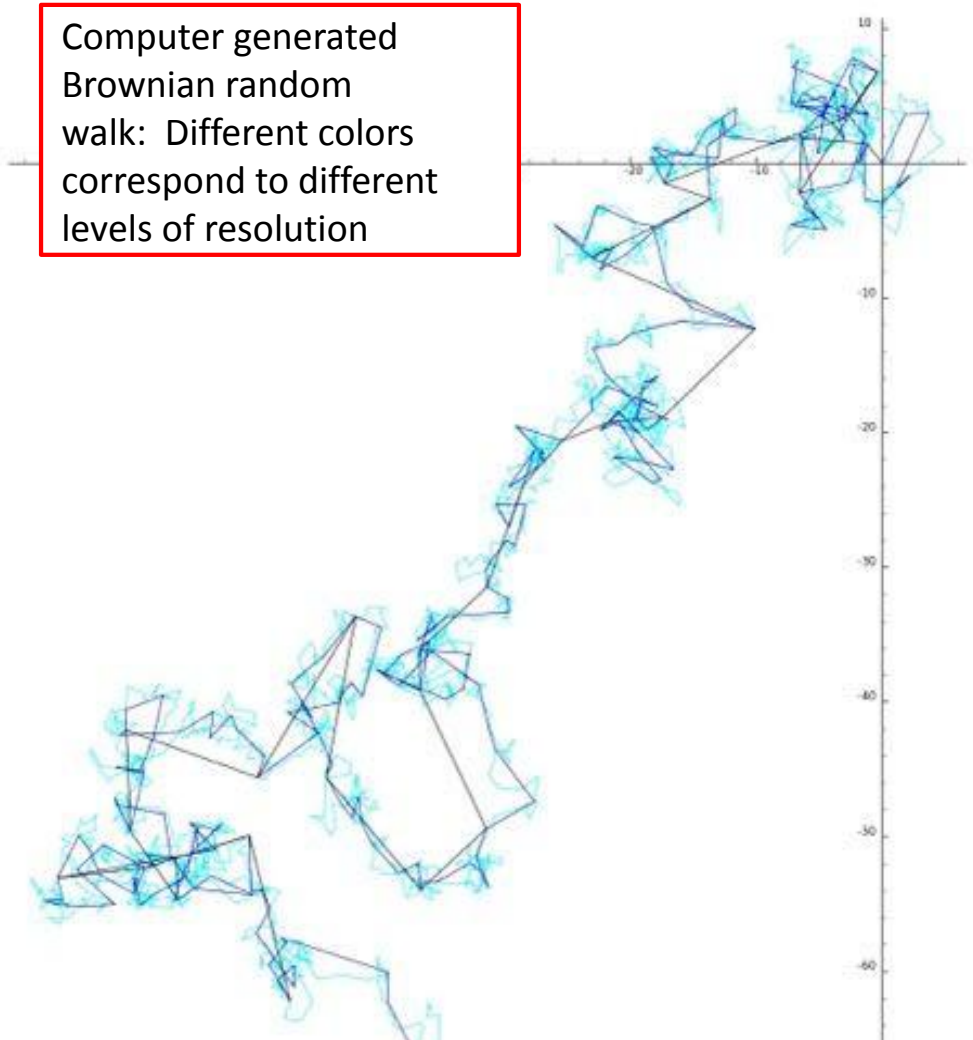
Experiment:



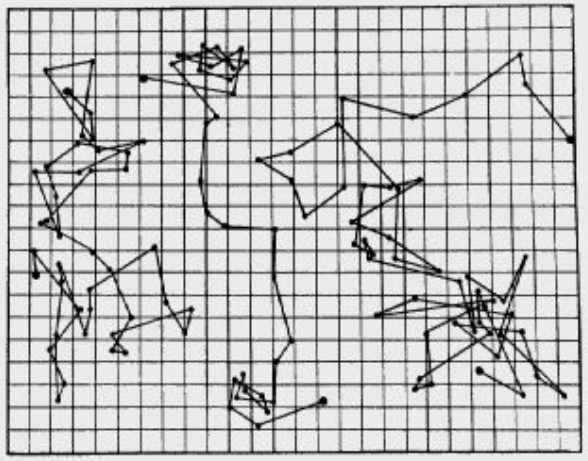
Reproduced from the book of Perrin, *Les Atomes,* three tracings of the motion of colloidal particles of radius 0.53µm, as seen under the microscope, are displayed. Successive positions every 30 seconds are joined by straight line segments (the mesh size is 3.2µm).

Computer simulations:

Computer generated Brownian random walk: Different colors correspond to different levels of resolution



Properties of Brownian Motion



Definition of Brownian motion: Random motion of one particle in solution due to collision with others.

- The Brownian motion's trail has the topological dimension 1. If confined to a finite space, it would fill the whole space with time. If confined, it has the fractal dimension 2.

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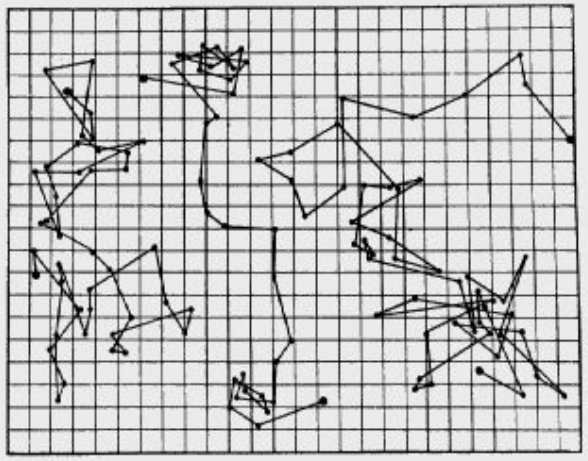

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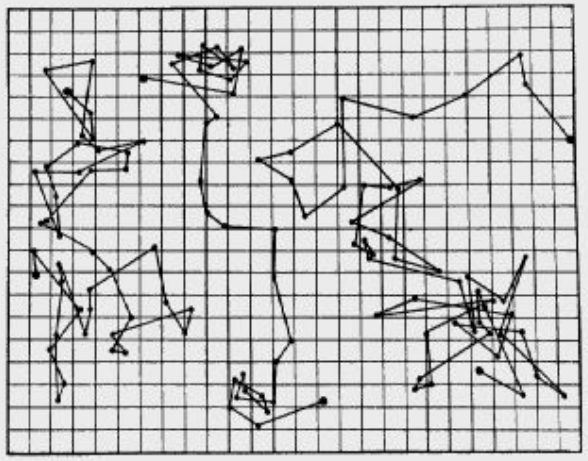

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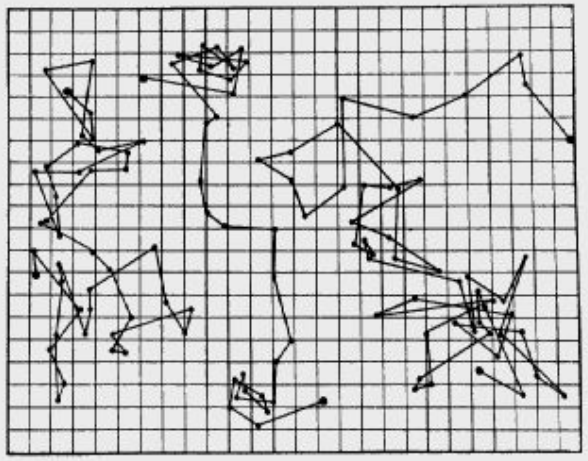

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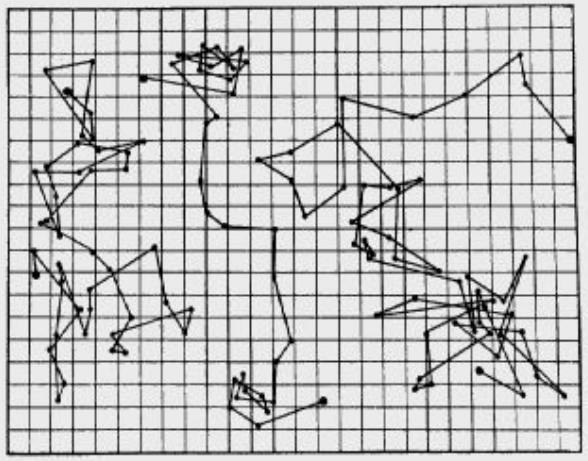

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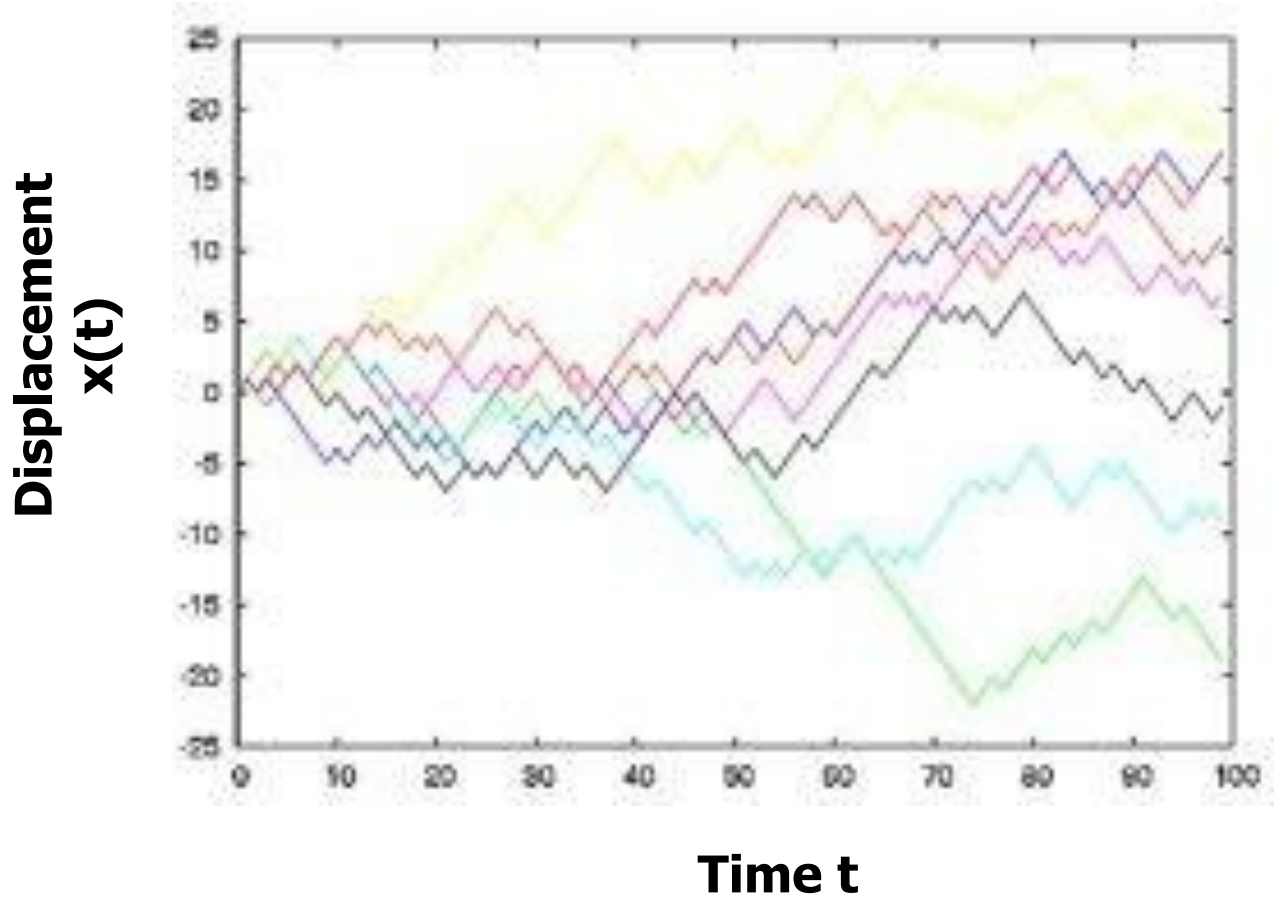
- How far does the particle diffusion in time interval t ?
- The average distances increase with the square root of time:

$$(r(t) - r(0))^2 \sim t$$

(Wait 4 times as long, get only twice as far!)

Construct a random walk in on a computer



Example in 1 dimension x(t)

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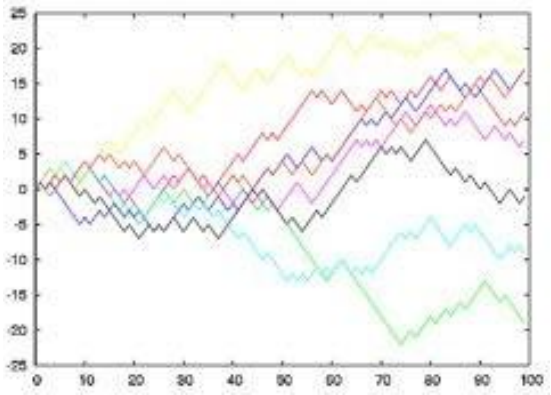
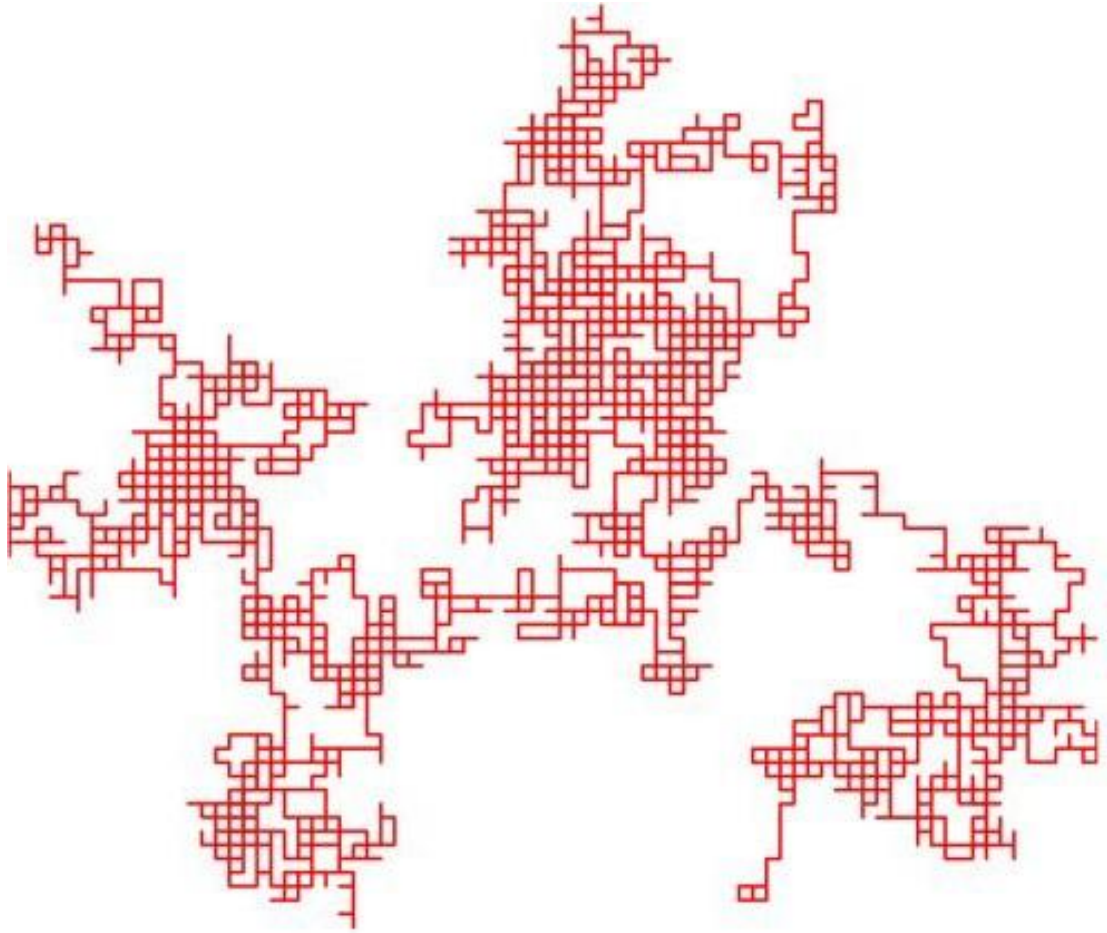
How to generate random numbers?

What happens inside Python's random() function?

Construct a 2D random walk





Example in 2 dimension x(t)

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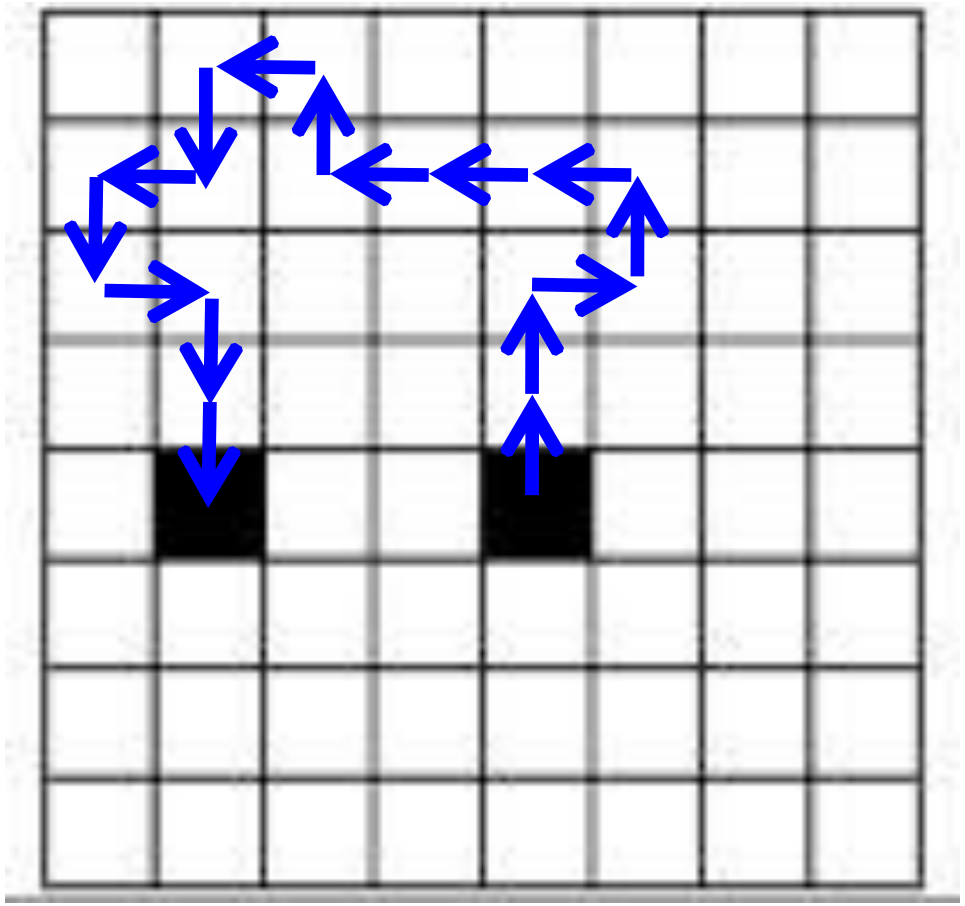
Classroom experiment:
Random walk in 2D



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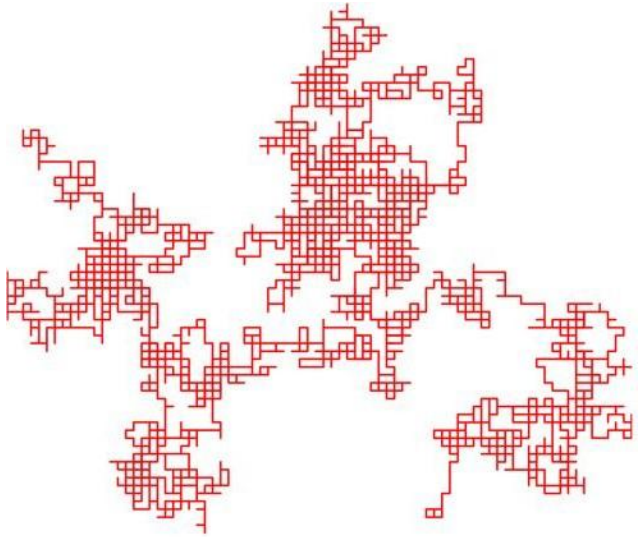
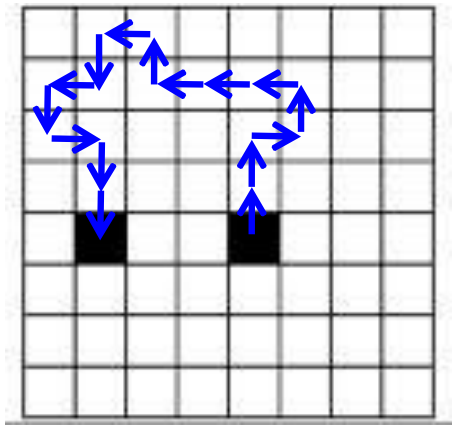
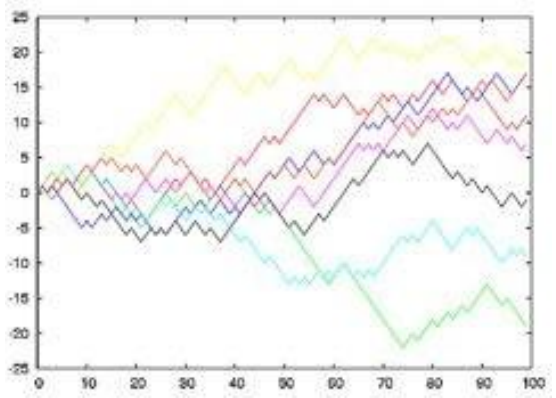
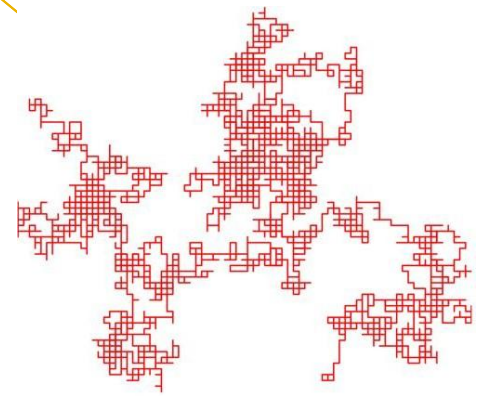


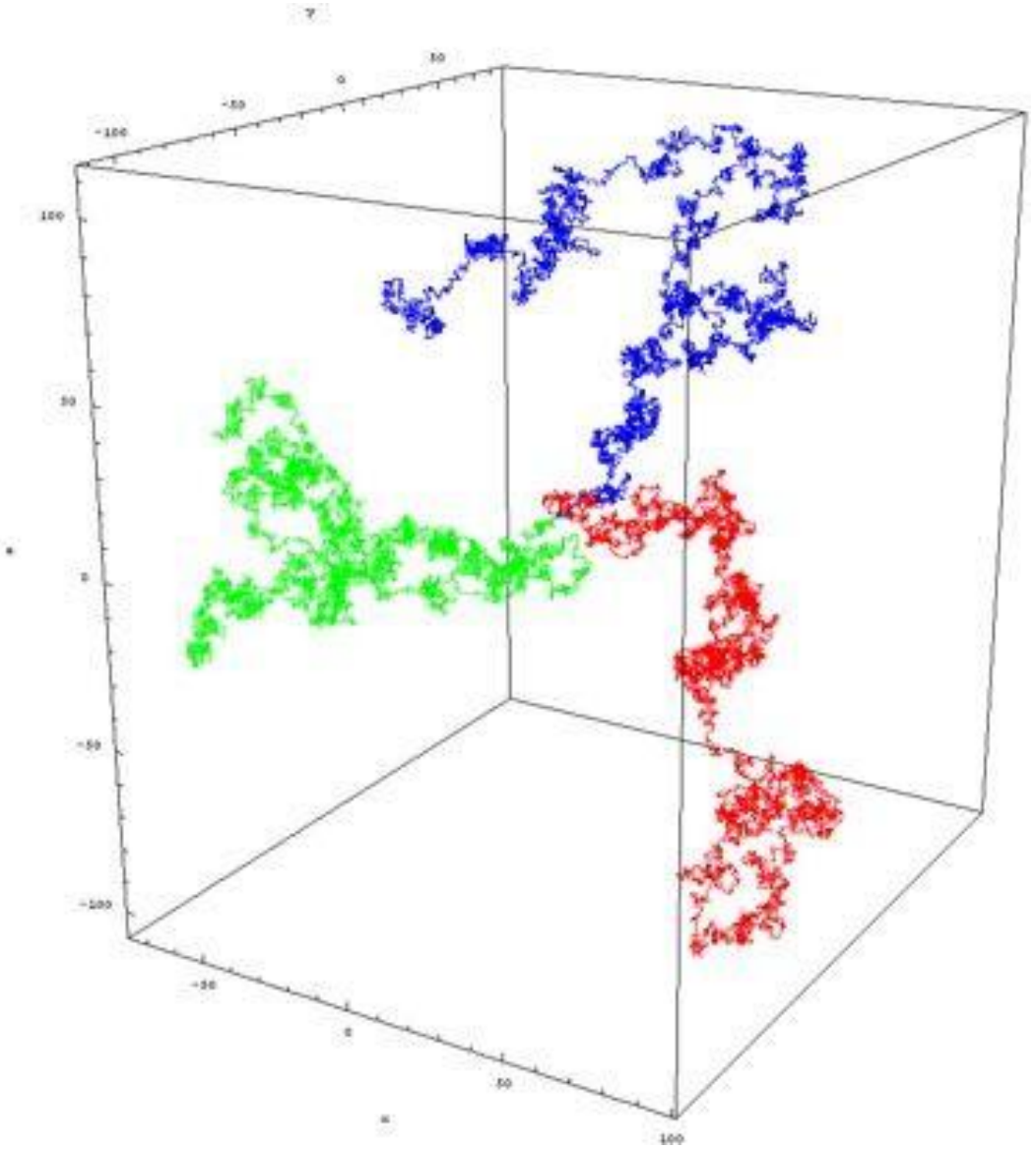


```
>> random_numbers_lecture
0.906
0.127
0.913
0.632
0.098
0.278
0.547
0.958
0.965
0.158
```

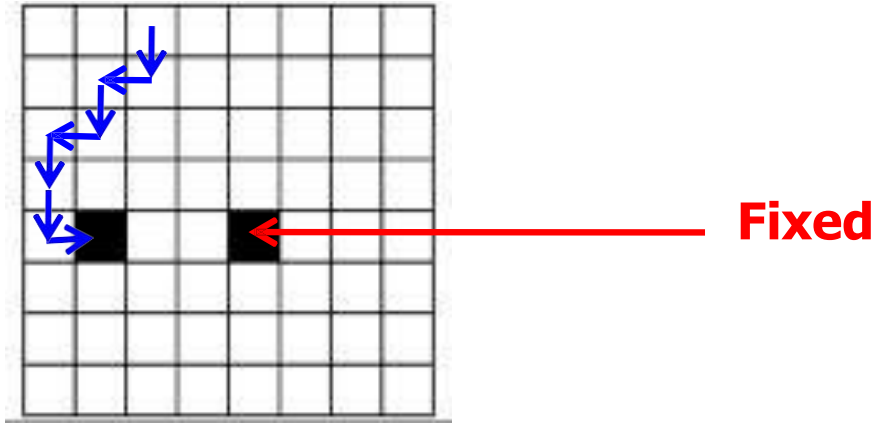
Construct a 3D random walk on a computer







Study diffusion limited aggregation in a computer simulations



Study diffusion limited aggregation in a computer simulations

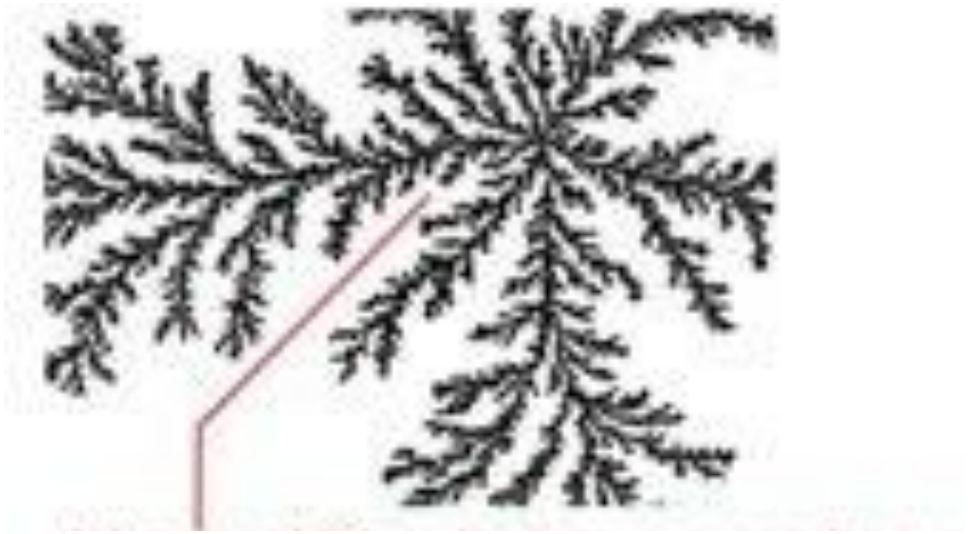
Fractal dimension D=1.71

Diffusion limited aggregation compared to a quartz crystal

Fractal dimension D=1.71

What is missing to make a perfect crystal?

Study diffusion limited aggregation with computer simulations



It is very difficult for a random walker to get here avoiding all side branches

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Diffusion Limited Aggregation Along a Sticky Wall?

Diffusion Limited Aggregation With Different Sticking Probabilities

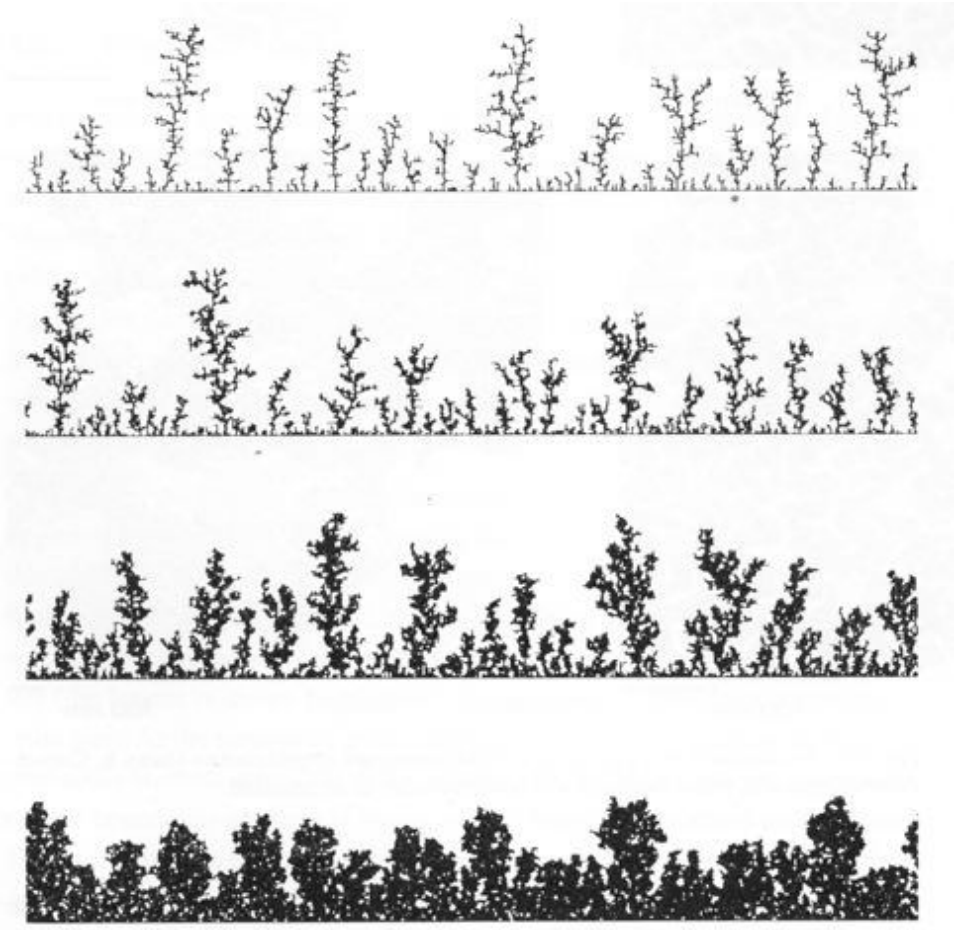

Fig. 1.8: Samples of Diffusion Limited Aggregation (DLA) simulations from a line showing variations of growth from dendritic to moss-like.

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Modified Diffusion Limited Aggregation Process leads to Better Packing?

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Summary

- Diffusion is related for random walks of particles

$$(r(t) - r(0))^2 \sim t$$

- A special process called diffusion limited aggregation leads to fractal structures that resemble experiments