SFWR ENG 4E03

Assignment 1

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1. **A program is equally likely to have been written by one of two programmer, A or B. If the program is written by A , it is equally likely to have zero or one bugs. If the program is written by B, it is equally likely to have zero, one, two, three or four bugs.**

   a) What is the expected number of bugs in a randomly selected program?

$A = \{0, 1\}$

$B = \{0, 1, 2, 3, 4\}$

$P(A) = 0.5, P(B) = 0.5$

$E[X_A \cup X_B] = P(X_A)E[X_A] + P(X_B)E[X_B]$

$= 0.5(\frac{0+1}{2} + \frac{0+1+2+3+4}{5})$

$= 0.5(0.5 + 2)$

1.25

b) Suppose that there is exactly one bug in a program. What is the probability that this program was written by A?

let A represent the event that the program was written by A, and let B represent the event the program has was written by B , let C represent the event that the program has 1 bug.

probability of 1 bug

$P(C) = P(A \cap C) + P(B \cap C)$

$= 0.5 + 0.2 = 0.7$

probability A is author given program has 1 bug

$P(A|C) = \frac{P(A \cap C)}{P(C)}$

$= \frac{0.5}{0.7} = \frac{5}{7}$ $\therefore$ it is more likely that A authored the program given there is 1 bug

2. **Let a random variable X have density function:**

   $f_X(x) = \begin{cases} 3x^2 & 0 \le x \le k \\ 0 & \text{otherwise} \end{cases}$

Calculate k and $E[X]$

$\int_{-\infty}^{\infty} f_X(x)dx = 1$

$\int_{0}^{k} f_X(x)dx$

$\int_{0}^{k} 3x^2 dx$

$x^3|_0^k = 1$

$k^3 - 0 = 1$

$k = 1$

$$E[X] = \int_0^1 x(3x^2)dx$$

$$= 3\int_0^1 x^3 dx$$

$= \frac{3}{4}x^4|_0^1$

$$= \frac{3}{4}$$

3. **The probability that a given component is defective is 0.006. Each component is subjected to a test that correctly identifies a defective component, but with probability 0.02 identifies a good component as defective. Given a randomly chosen component is tested as defective, compute the probability it is actually defective**

Let C_D represent the event the component is defective, and let T_D represent the event a component tests defective. Conversely, let C'_D represent the event the component is *not* defective, and T'_D represent the event a component tests *not* defective.

$$P(C_D) = 0.006, P(C'_D) = 0.994$$

$$P(T_D|C_D) = 1$$

$$P(T_D|C'_D) = 0.02$$

probability a component tests defective

$$P(T_D) = (0.006) * (1) + (0.994)(0.02)$$

$$= 0.02588$$

$$P(C_D|T_D) = \frac{P(T_D|C_D)P(C_D)}{P(T_D)}$$

$$= \frac{1 * 0.006}{0.02588}$$

$$= 0.02588 = \frac{150}{647}$$

4. **Suppose that we draw a sequence of cards from a standard 52 card deck. Each time a card is drawn, it is put back in the deck, so a draw is always made from a full deck. Assume that each of the cards is equally likely to be drawn. Draws are made until a heart is revealed.**

   a) What is the probability that we will need at least three draws before we see a heart?

   Assuming that the card deck with replacement can be modelled using a Geometric Distribution with $k = 4$ *(4th draw),* $q = \frac{1}{4}$*(hearts are 1 of 4 suits in a card deck)***

   $P_x(k) = q(1-q)^{k-1} \; \forall k \in \mathbb{Z}^+$

   b) What is the expected number of draws until we see heart?

$$E[X] = \frac{1}{q}$$

$$E[X] = \frac{1}{\frac{1}{4}}$$

$$= 4$$

5. **A component has a lifetime T that is exponentially distributed with mean one year. (The lifetime is the time until the component fails.)**

a) Find the lifetime L which a typical component exceeds with probability 0.9

$$X \sim Exp(\lambda) \therefore E[X] = \frac{1}{\lambda}$$

$$E[X] = 1$$

$$\frac{1}{\lambda} = 1$$

$$\lambda = 1$$

$$P(X > L) = e^{-L\lambda}$$

$$= e^{1*-L} = e^{-L}$$

$$e^{-L} = 0.9$$

$$L = 0.1053605$$

b) If five components are sold to a manufacturer, find the probability that at least one of them will have a lifetime of less than L

Component failure is *independent* and equally likely:

$$P(failure\ before\ L) = 1 - P(machine\ exceeds\ L) = 1 - 0.9 = 0.1$$

All are equally likely to fail : $5 * (0.1) = 0.5$

c) Suppose that a component is still operating at time L. What is expected time starting at L, until it fails.

$$P(X > L)' = P(X < L)$$

$$= 1 - P(X > L)$$

$$= 1 - 0.1053605$$

$$= 0.8946395$$