Определение 1. *Говорят, что в области G задано скалярное поле f(векторное поле $\bar{a}$), если $\forall M \in G$ поставлено в соотв. число $f(M)$ (вектор $\bar{a}(M)$)*

1. *Градиент скалярного поля:* $\overline{\nabla} f = grad\ f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$
2. *Производная по направлению:* $\frac{\partial f}{\partial \bar{l}} = \lim\limits_{t \to 0} \frac{f(M) - f(M_0)}{t}$, *где* $\overline{M_0 M} = t\bar{l},\ \ t > 0\ \frac{\partial f}{\partial \bar{l}}(grad\ f, \bar{l})$
3. *Дивергенция* $div\ \bar{a} = (\overline{\nabla}, \bar{a}) = \left(\frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}\right)$
4. *Ротор:* $rot\bar{a} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix} = [\overline{\nabla} \times \bar{a}]$
5. *(Векторный оператор) Набла:* $\overline{\nabla} = \left(\frac{\partial}{\partial x}\bar{i} + \frac{\partial}{\partial y}\bar{j} + \frac{\partial}{\partial z}\bar{k}\right)$
6. *оператор Лапласа (Лапласиан):* $\Delta = (\overline{\nabla} \cdot \overline{\nabla}) = \left(\frac{\partial^2}{\partial x} + \frac{\partial^2}{\partial y} + \frac{\partial^2}{\partial z}\right)$
7. *Градиент* $\bar{a}$ *по* $\bar{b}$: $(\bar{b}\overline{\nabla})\bar{a} = b_x \frac{\partial \bar{a}}{\partial x} + b_y \frac{\partial \bar{a}}{\partial y} + b_z \frac{\partial \bar{a}}{\partial z}$

Правило Лейбница: (как $(fg)' = fg' + g'f$): $\overline{\nabla}(p, q) = \nabla(\mathbf{p}, q) + \nabla(p, \mathbf{q})$
Примеры: (жирным выделено то, по чему мы берем оператор Набла)

1. $div(f\bar{a}) = (\nabla, f\bar{a}) = (\nabla, \mathbf{f}\bar{a}) + (\nabla, f\bar{\mathbf{a}}) = (\nabla\mathbf{f}, \bar{a}) + f \cdot (\nabla, \bar{\mathbf{a}}) = (grad\ f, \bar{a}) + f div(\bar{a})$;
2. $div[\bar{a}, \bar{b}] = (\nabla, [\bar{a}, \bar{b}]) = (\nabla, \bar{a}, \bar{b}) = (\nabla, \bar{\mathbf{a}}, \bar{b}) + (\nabla, \bar{a}, \bar{\mathbf{b}}) = (\bar{b}, p\nabla, \bar{\mathbf{a}}) - (\bar{a}, [\nabla, \bar{\mathbf{b}}]) = (\bar{b}, rot\bar{a}) - (\bar{a}, rot\bar{b})$.
3. $rot[\bar{a}, \bar{b}] = [\nabla, [\bar{a}, \bar{b}]] = [\nabla, [\bar{\mathbf{a}}, \bar{b}]] + [\nabla, [\bar{a}, \bar{\mathbf{b}}]] = ((\nabla\bar{b})\bar{a} - \bar{b}(\nabla, \bar{a})) + (\bar{a}(\nabla, \bar{b}) - (\nabla, \bar{a})\bar{b}) = ((\bar{b}\nabla)\bar{a} - \bar{b} div\bar{a}) + (\bar{a} div\bar{b} - (\bar{a}\nabla)\bar{b})$

Пример задачи: $div\ grad\ f(r) = div\ (f'(r)\frac{\bar{r}}{r})$

$$\frac{\partial(grad\ f(r)|_x)}{\partial x} = \frac{\partial}{\partial x}(f'(r)\frac{x}{r}) = \frac{f'(r)}{r} + x\frac{\partial}{\partial x}\left(\frac{f'(r)}{r}\right) = \frac{f'(r)}{r} + x\frac{\partial r}{\partial x}\frac{\partial}{\partial r}\left(\frac{f'(r)}{r}\right) =$$

$$= \frac{f'(r)}{r} + \frac{x^2}{r}\left(\frac{f''(r)r - f'(r)}{r^2}\right) = \frac{f'(r)}{r} + \frac{x^2}{r^2}\left(f'' - \frac{f'}{r}\right)$$

$$\to div\ grad\ f(r) = 3\frac{f'(r)}{r} + \left(f'' - \frac{f'}{r}\right)\left(\frac{x^2 + y^2 + z^2}{r^2}\right)$$