An Implementation of DVCM in Water Hammer Event

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Outline

Introduction of DVCM

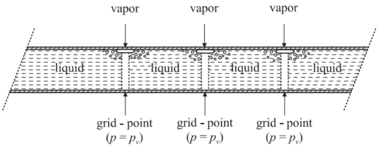

Fig. 9. Definition sketch for discrete vapor cavity model (adapted from Tijsseling, 1993, Fig. 4.9).

Sketch of DVCM Model

The sketch of DVCM is shown in the figure above. The DVCM model is a 1D model that can simulate the water hammer event in the pipeline. The model allows the discrete cavitation bubbles to be generated and collapsed at the grid points.

Details of DVCM

The basic equations of MOC(method of characteristics) are farmiliar to us. The equations are shown below:

$$\begin{cases} H_j^t - H_{j-1}^{t-\Delta t} + \frac{a}{gA}((Q_u)_j^t - Q_{j-1}^{t-\Delta t}) + \frac{f\Delta x}{2gDA^2}(Q_u)_j^t|Q_{j-1}^{t-\Delta t}| = 0 \\ H_j^t - H_{j+1}^{t-\Delta t} - \frac{a}{gA}(Q_j^t - (Q_u)_{j+1}^{t-\Delta t}) - \frac{f\Delta x}{2gDA^2}Q_j^t|(Q_u)_{j+1}^{t-\Delta t}| = 0 \end{cases} \tag{1}$$

Details of DVCM

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H represents the head of the water, Q represents the downstream flow rate of the water, Q_u represents the upstream flow rate. The footnote represents the number of node, the headnode represents the time.

As we did in the previous assignment, we consider that $Q_u = Q$, and then solve the equations of the head $H_j^{t+\Delta t}$ and flow rate $Q_j^{t+\Delta t}$ these two unknowns.

Details of DVCM

But in the DVCM model, the cavitation bubbles are considered. Once if the head drops below the vapor pressure H^*,the cavitation happens and 2 equations are attached to the original equations:

$$\begin{cases} \dot{V} = -(Q_u) + Q \\ H = H^* \end{cases}$$

Since when $Q_u \neq Q$ and in which V represents the volumn of the cavitation bubble.

Details of DVCM

The discrete form of the equations would be more usebul as shown in the following:

$$\begin{cases} V_j^{t+\Delta t} = V_j^t + \psi(-(Q_u)_j^{t+\Delta t} + Q_j^{t+\Delta t})\Delta t + (1-\psi)(-(Q_u)_j^t + Q_j^t)\Delta t \\ H_j^{t+\Delta t} = H^* \end{cases} \tag{2}$$

As we have learnt in mathmatical analysis, ψ is the weight factor generated by the mean value theorem of integrals.

Therefore, the unknowns are $(Q_u)_j^{t+\Delta t}$ and $Q_j^{t+\Delta t}$ in (1) and (2) instead of $H_j^{t+\Delta t}$ and $Q_j^{t+\Delta t}$(equals to $(Q_u)_j^{t+\Delta t}$).

After solving the Q and (Q_u), we can get the ΔV and then update the V. Once updated V is less than 0, the cavitation bubble collapses and the equations go back to the original form.

Sumarry of Details of DVCM

In sumarry, when $H_j^t > H^*$ the normal equations are:

$$\begin{cases} H_j^t - H_{j-1}^{t-\Delta t} + \frac{a}{gA}((Q_u)_j^t - Q_{j-1}^{t-\Delta t}) + \frac{f\Delta x}{2gDA^2}(Q_u)_j^t|Q_{j-1}^{t-\Delta t}| = 0 \\ H_j^t - H_{j+1}^{t-\Delta t} - \frac{a}{gA}(Q_j^t - (Q_u)_{j+1}^{t-\Delta t}) - \frac{f\Delta x}{2gDA^2}Q_j^t|(Q_u)_{j+1}^{t-\Delta t}| = 0 \\ Q_u = Q \end{cases} \tag{1}$$

And once $H_j^t < H^*$ the equations are added:

$$\begin{cases} V_j^{t+\Delta t} = V_j^t + \psi(-(Q_u)_j^{t+\Delta t} + Q_j^{t+\Delta t})\Delta t + (1-\psi)(-(Q_u)_j^t + Q_j^t)\Delta t \\ H_j^{t+\Delta t} = H^* \end{cases} \tag{2}$$

And when $V_j^{t+\Delta t} < 0$, the equations go back to (1).

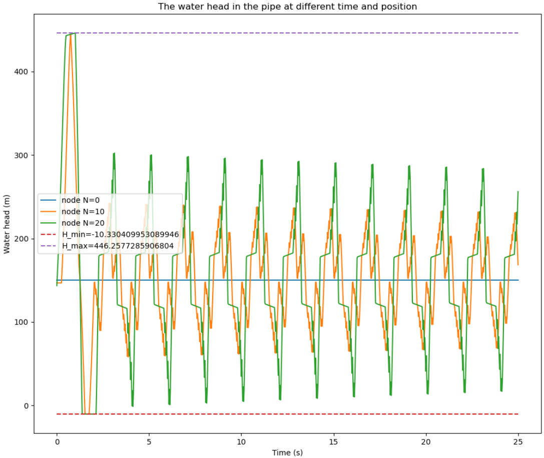
TALK IS CHEAP, SHOW ME THE CODE

Some key parts of the code are shown below:

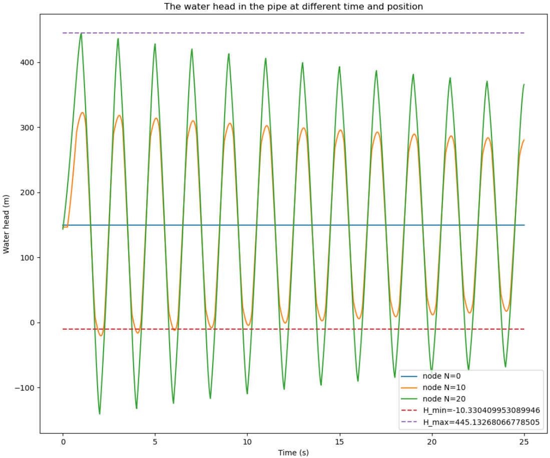
```
    # The iteration of the middle nodes
    elif j != 0 and j != N:
        C_P = H[i-1,j-1]+Qu[i-1,j-1]*(B+S-R*abs(Qu[i-1,j-1]))
        C_M = H[i-1,j+1]-Qu[i-1,j+1]*(B-S-R*abs(Qu[i-1,j+1]))
        H[i,j] = (C_P+C_M)/2
        Qu[i,j] = (C_P-C_M)/(2*B)
        Q[i,j] = Qu[i,j]
        if H[i,j] < H_min:
            E[i, j] = 0  (constant) H: NDArray[float64]
            H[i,j] = H_m
            Qu[i,j] = (-H[i,j]+H[i-1,j-1]+B*Q[i-1,j-1])/(B+R*abs(Q[i-1,j-1]))
            Q[i,j] = (H[i,j]-H[i-1,j+1]+B*Qu[i-1,j+1])/(B+R*abs(Qu[i-1,j+1]))
            E[i,j] = E[i-1,j] + (1-psi)*(Q[i,j]*dt - Qu[i,j]*dt) + psi*(Q[i-1,j]*dt - Qu[i-1,j]*dt)

elif E[i-1,j]>0:# If the cavitation happens, E[i,j] > 0
    # The cavitation happens at the middle of the pipe
    H[i,j] = H_min
    Qu[i,j] = (-H[i,j]+H[i-1,j-1]+B*Q[i-1,j-1])/(B+R*abs(Q[i-1,j-1]))
    Q[i,j] = (H[i,j]-H[i-1,j+1]+B*Qu[i-1,j+1])/(B+R*abs(Qu[i-1,j+1]))
    E[i,j] = E[i-1,j] + (1-psi)*(Q[i,j]*dt - Qu[i,j]*dt) + psi*(Q[i-1,j]*dt - Qu[i-1,j]*dt)
    if E[i,j] < 0:
        E[i,j] = 0
```

Key parts of the code to implement DVCM

Results of DVCM



Compared to Normal MOC



Physics Features to Be Explained

- The cavitation bubbles are generated and collapsed in the DVCM model so in some periods of time ,the pressure is a line with value H^*.
- The water head in the DVCM model is lower than the normal MOC model after the first oscillation. This could be explained that the generation and the diminishment of the cavitation bubbles consume the energy of the water. (Seems Good News? Less Pressure, Less Damage? Not Sure Yet!)
- The water head in the DVCM model is more fluctuated than the normal MOC model due to the turbulence of the cavitation bubbles.

Future the Work

Congratulation: We have set the primary step on the moon!

- What do the experimenters say?
- Considering whether the DVCM can be developed by the conservation of mass and energy.
- What about other improved 1D models (DGCM discrete gas cavity model)?
- What about high dimensional models? Seems to be hard, limited desire to try. (QAQ)