Chapter 4: kinematics

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Kinematics

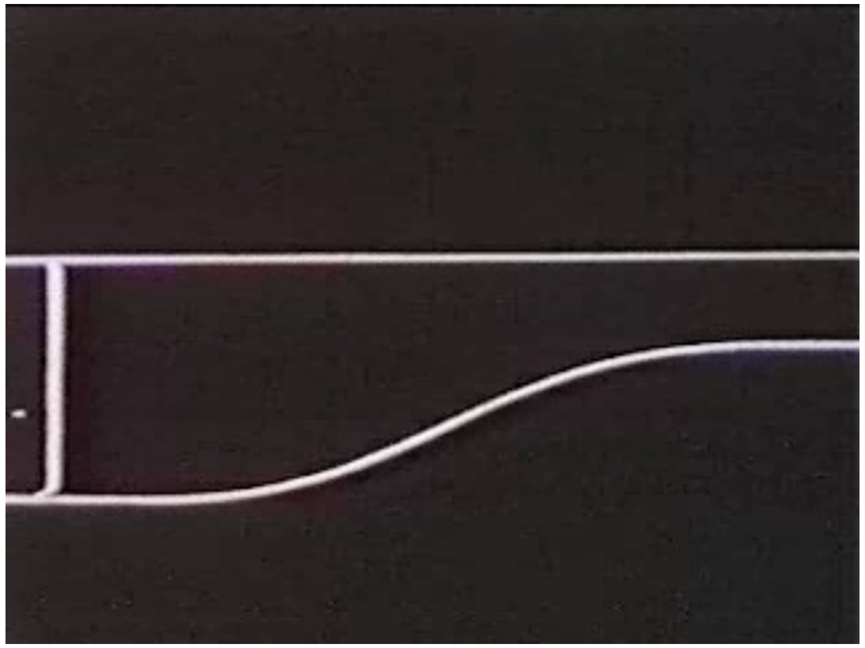
Kinematics

Fluid kinematics: how can we describe the flow field?

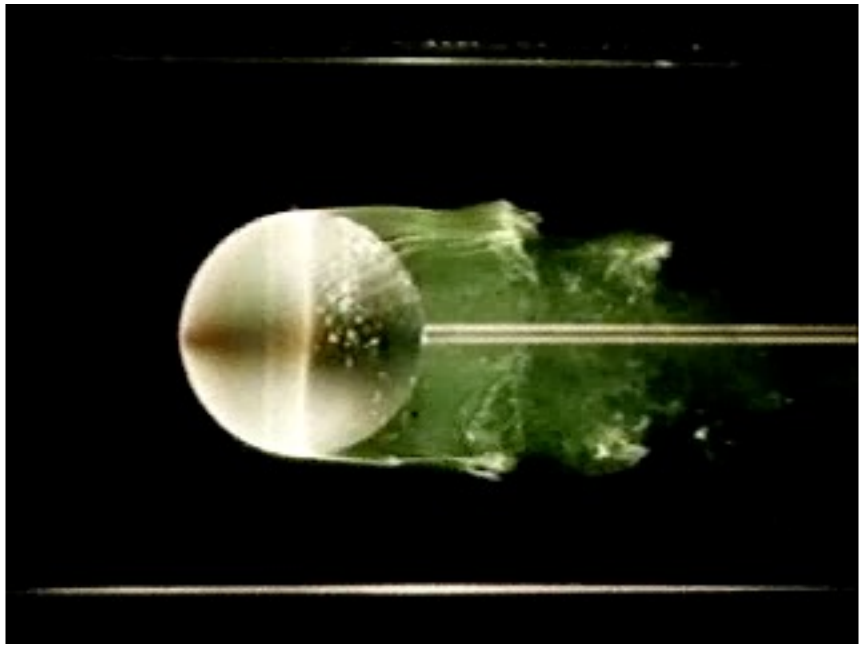
Kinematics of motion: various aspects of fluid motion without being concerned with the actual forces that produce the motion.
The forces (dynamics of motion) are not discussed in this chapter.

- Eulerian and Lagrangian descriptions of fluid motion
- Velocity field and acceleration field
- Control system and control volume
- Reynolds transport theorem

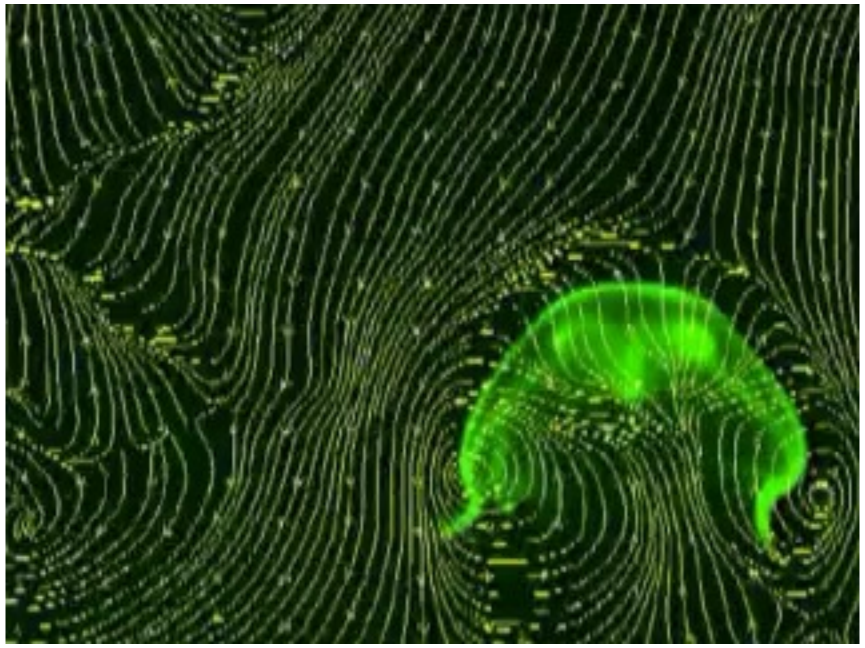
Kinematics



Movement and deformation of fluid particle



Flow pass a sphere



Flow near Jellyfish

Smoke stack

Kinematics

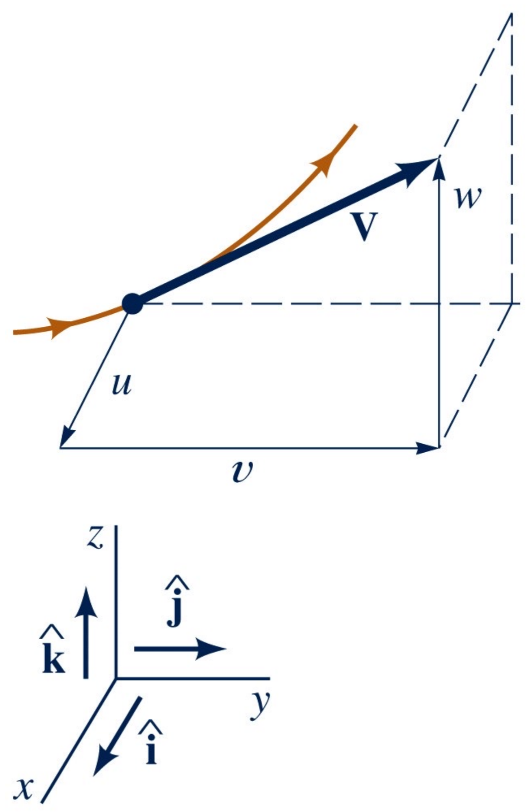
Field representation:

Kinematics

Field representation: at a given instant in time, a description of any fluid property (such as density, pressure, velocity, and acceleration) may be given as a function of the fluid's location and time.

Kinematics

Velocity field $$\vec{V} = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}$$



Kinematics

position vector $\vec{r}$

$$\frac{d\vec{r}}{dt} = \vec{V}$$

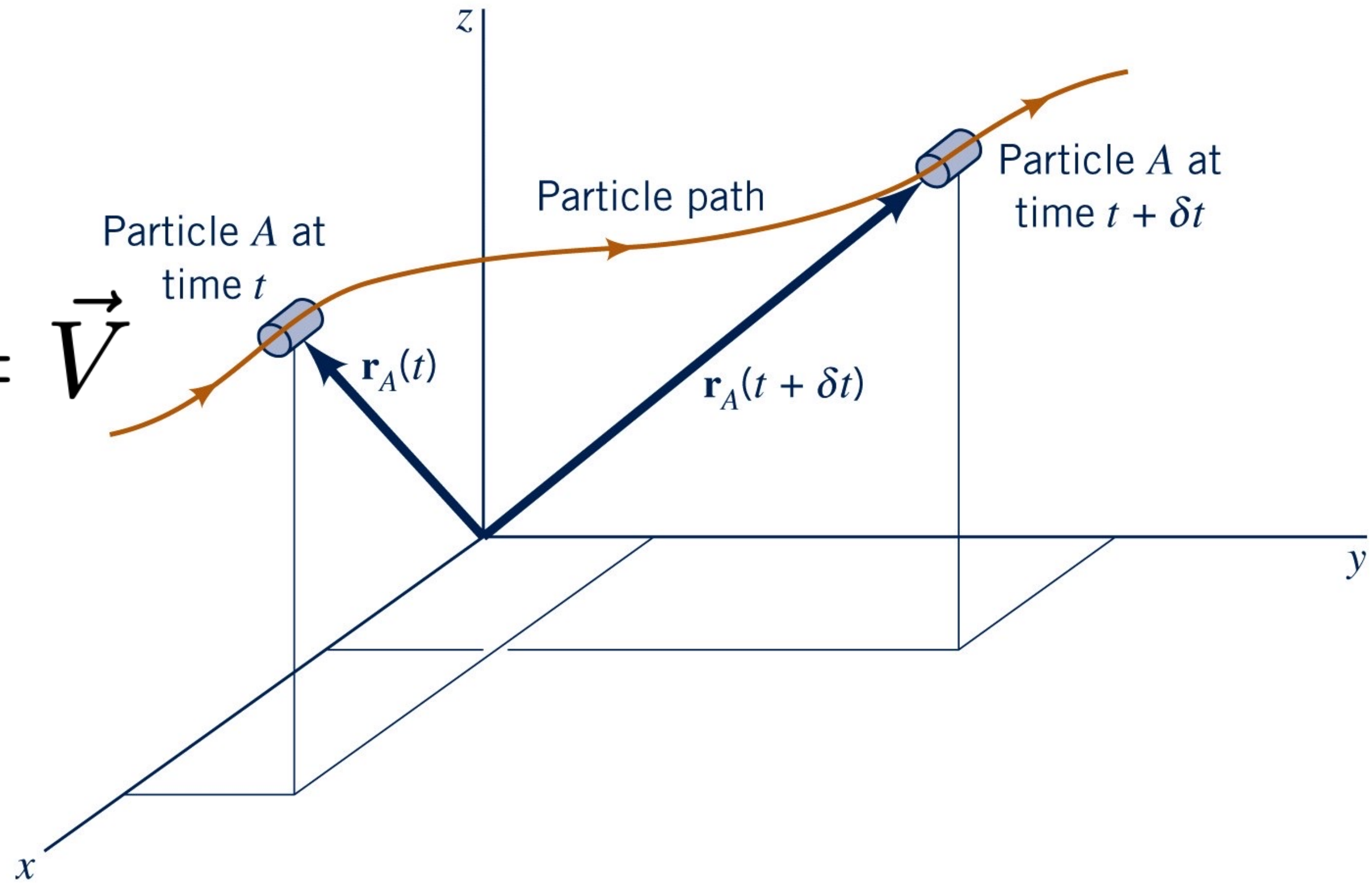
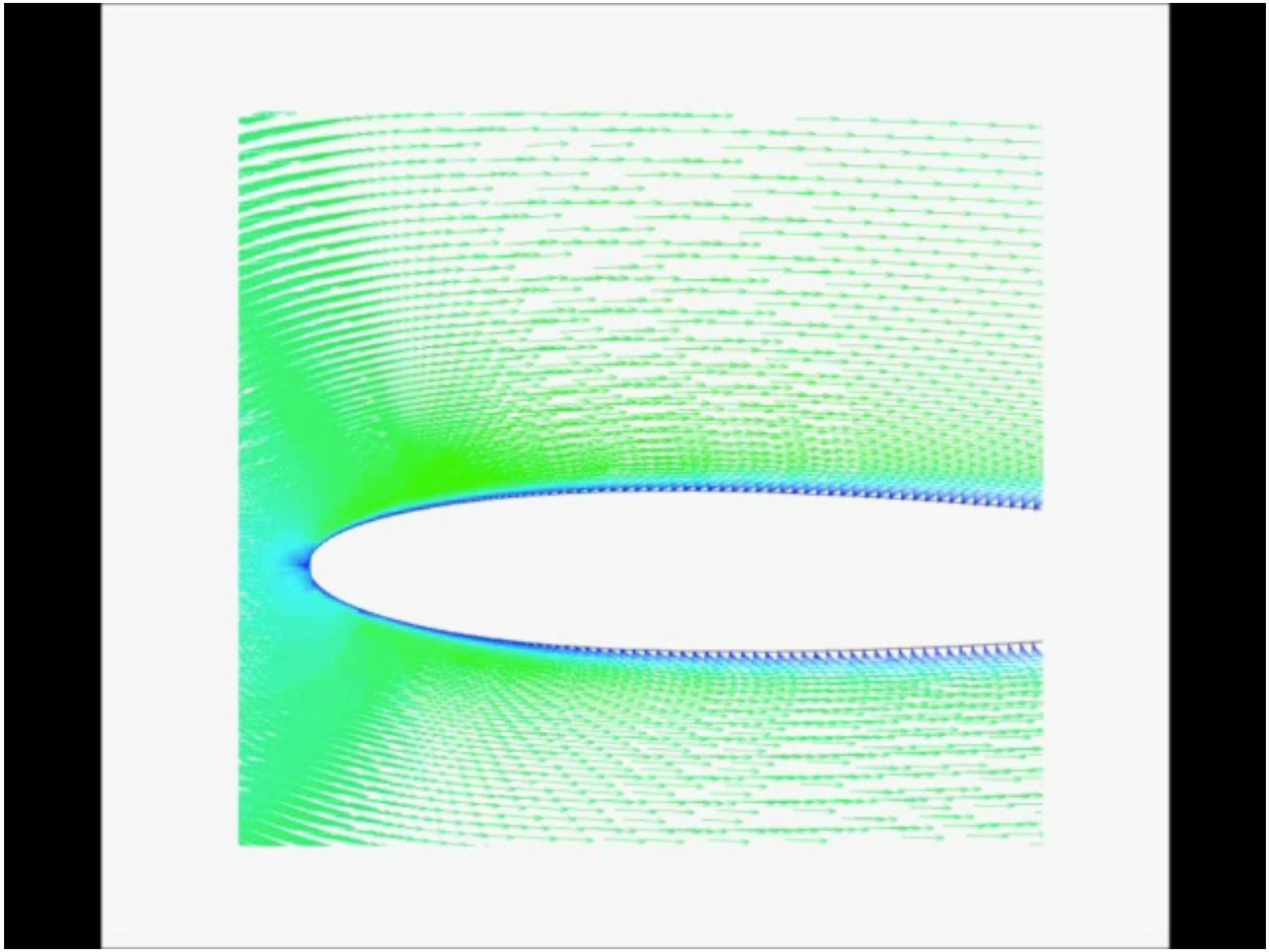


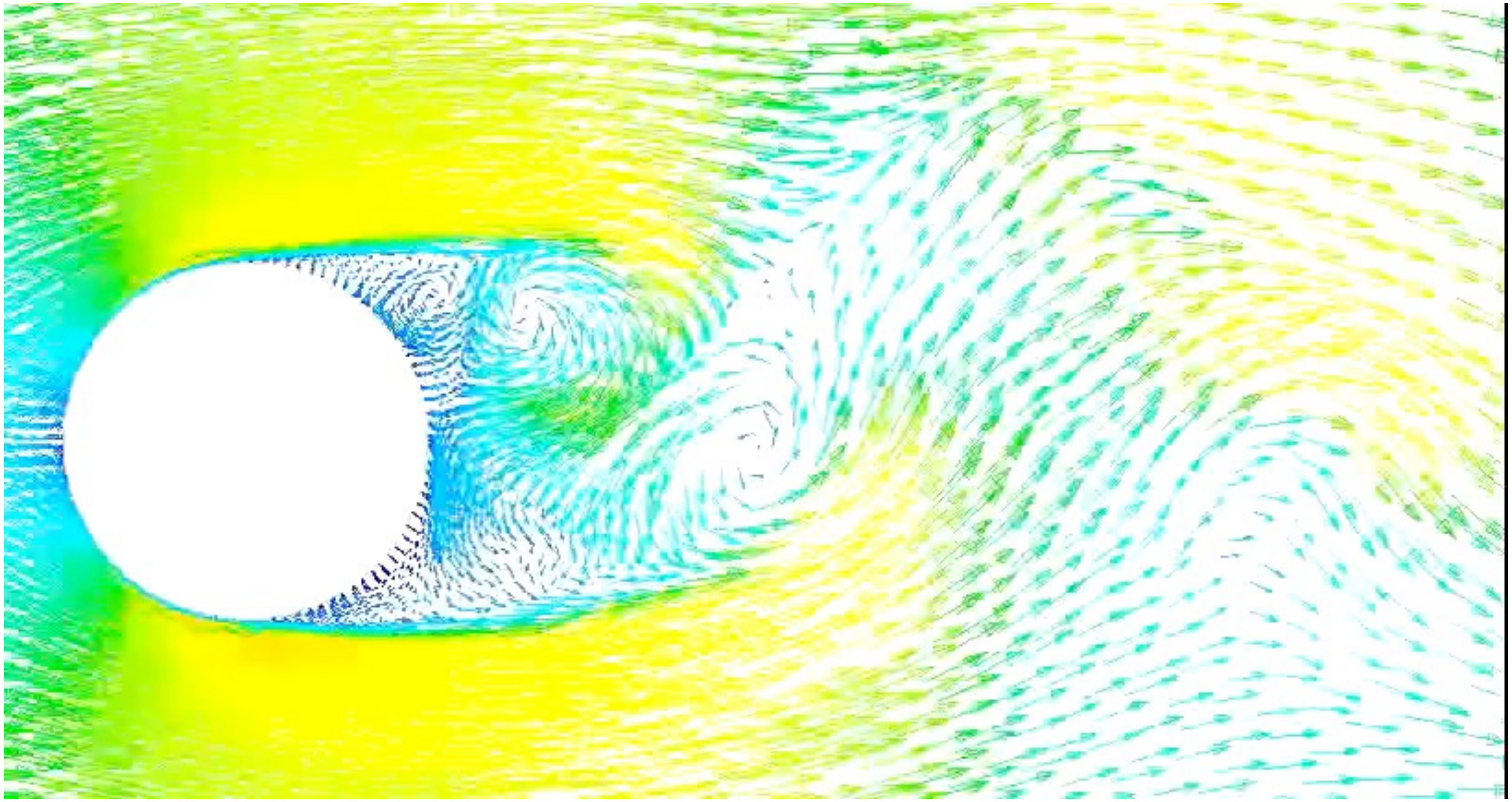
Figure 4.1

Velocity field about the front of an airfoil



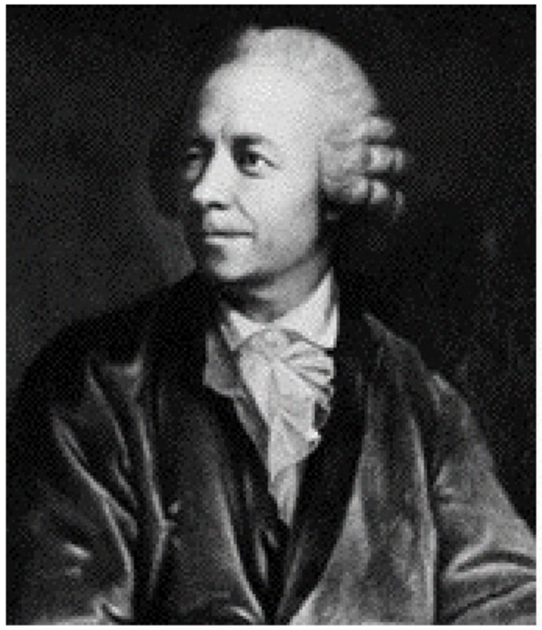
A field representation is often used to describe flows. In doing so, flow variables such a velocity and pressure are specified as functions of the spatial coordinates and time. The velocity field describes a flow by giving the point-by-point fluid velocity vectors throughout the flow field.

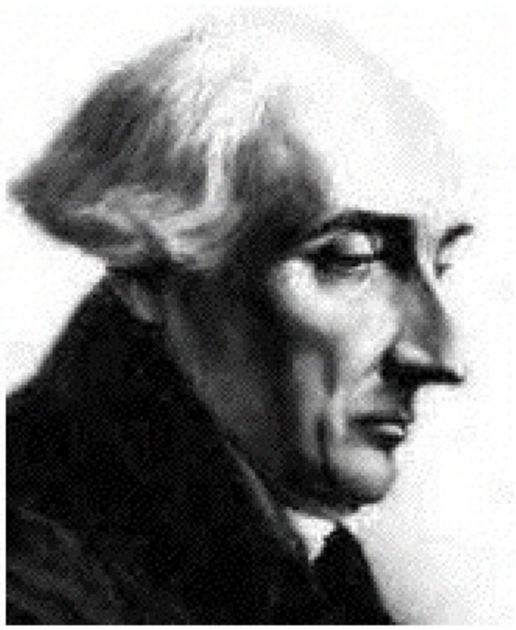
Unsteady, oscillating velocity field



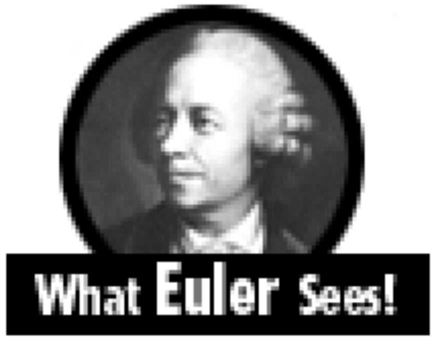
Considerable information about a flow field can be obtained from information about the velocity field. Typically, this information can be presented by showing arrows representing the fluid velocity (magnitude and direction) throughout the flow field.

Eulerian and Lagrangian Flow Descriptions





The particle-based representation of fluid flow is usually named for Joseph Louis Lagrange, whereas the field-based representation is named for Leonhard Euler.



Eulerian and Lagrangian Flow Descriptions

Eulerian method: field description

Lagrangian method: as in particle dynamics

Eulerian and Lagrangian Flow Descriptions

Eulerian method: field description

the fluid motion is given by completely prescribing the necessary properties (pressure, density, velocity, etc.) as functions of space and time.

$$\vec{V} = \vec{V}(x, y, z, t)$$

Lagrangian method: as in particle dynamics

involves following individual fluid particles as they move about and determining how the fluid properties associated with these particles change as a function of time. The fluid particles are “tagged” or identified, and their properties determined as they move.

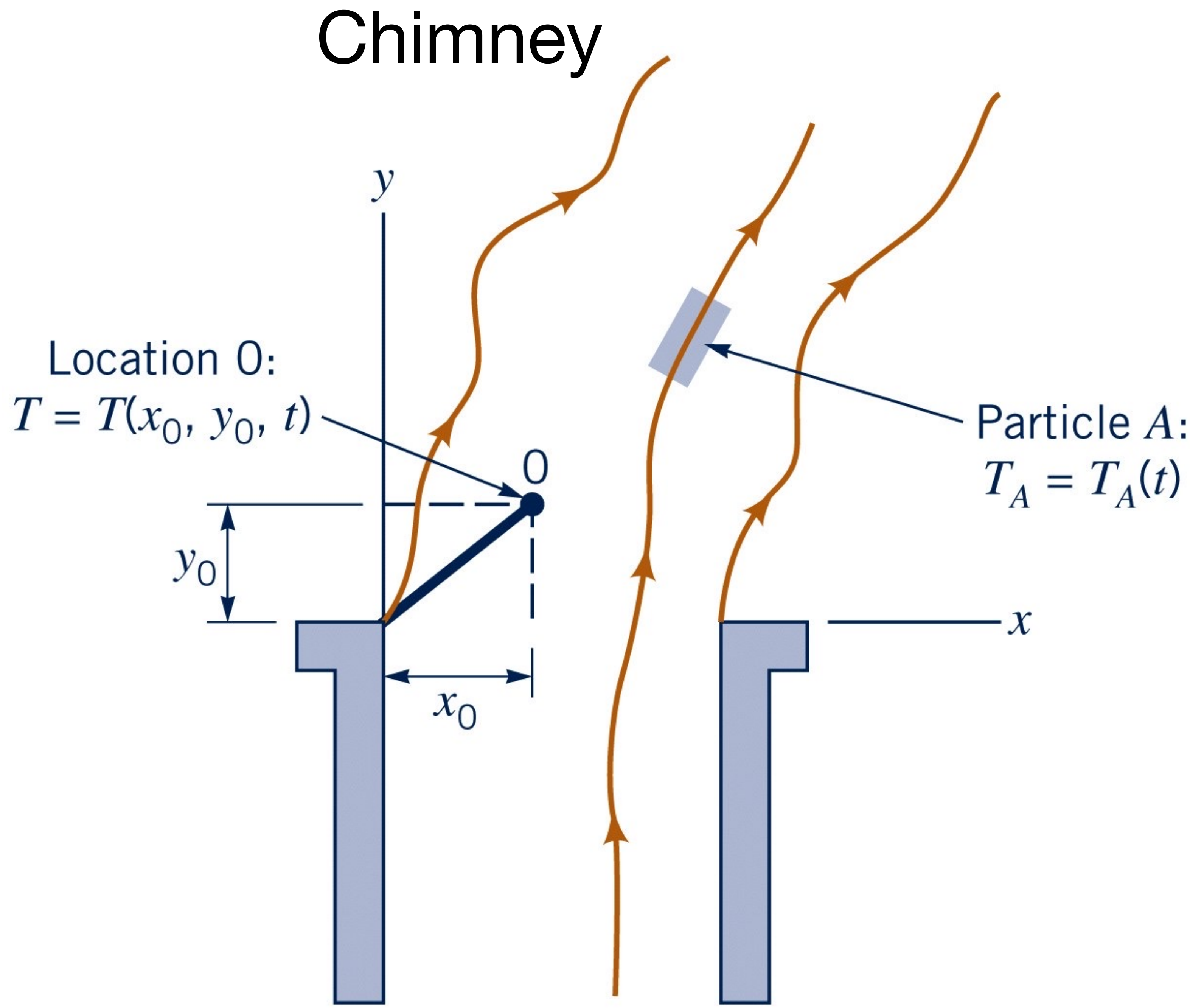
$$\vec{V} = \vec{V}(x_p, y_p, z_p, t)$$

Newton’s 2nd law is actually a Lagrangian law

Eulerian and Lagrangian Flow Descriptions

Euerian method:

Lagrangian method:

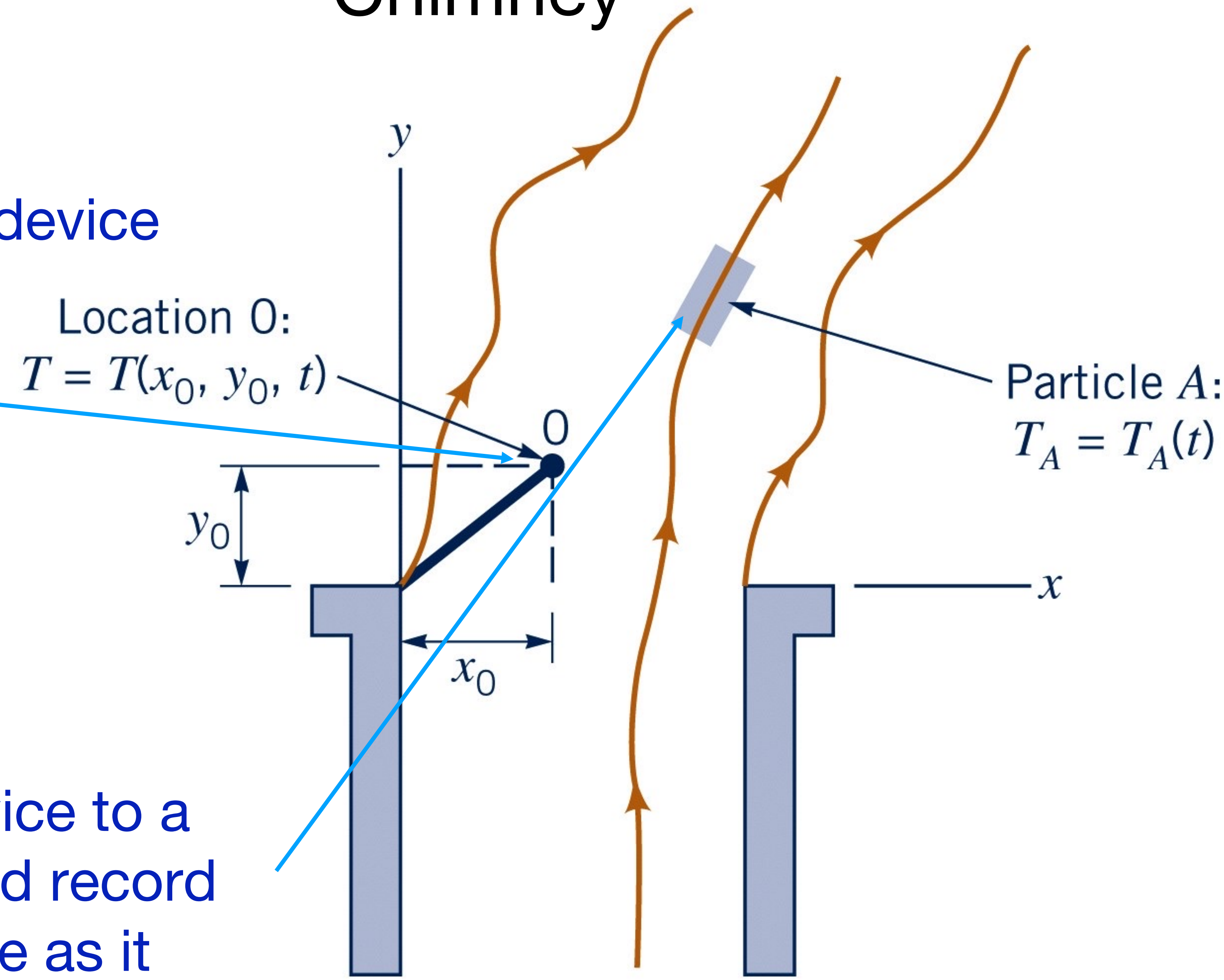


Eulerian and Lagrangian Flow Descriptions

Chimney

Eulerian method:

the Eulerian method: attach a device at ‘a position’ and record the temperature at that point as a function of time.



Lagrangian method:

attach the measuring device to a particular fluid particle and record that particle’s temperature as it moves about.

Eulerian and Lagrangian Flow Descriptions

Eulerian method:
field description

In fluid mechanics, it is easier to use Eulerian method to describe the flow, in either experiments and analytical investigations. But at some situations, Lagrangian method is more convenient.

Lagrangian method:
as in particle dynamics

One-, Two-, and Three-Dimensional Flows

Three-dimensional flow:

in almost any flow situation, the velocity field actually contains all three velocity components (u, v, and w).

Two-dimensional flow:

many situations one of the velocity components may be small (in some sense) relative to the two other components.

One-dimensional flow:

it is sometimes possible to further simplify a flow analysis by assuming that two of the velocity components are negligible.

Steady and Unsteady Flows

$$\frac{\partial \vec{V}}{\partial t} = 0$$ Steady flow

Steady and Unsteady Flows

$$\frac{\partial \vec{V}}{\partial t} = 0$$ Steady flow

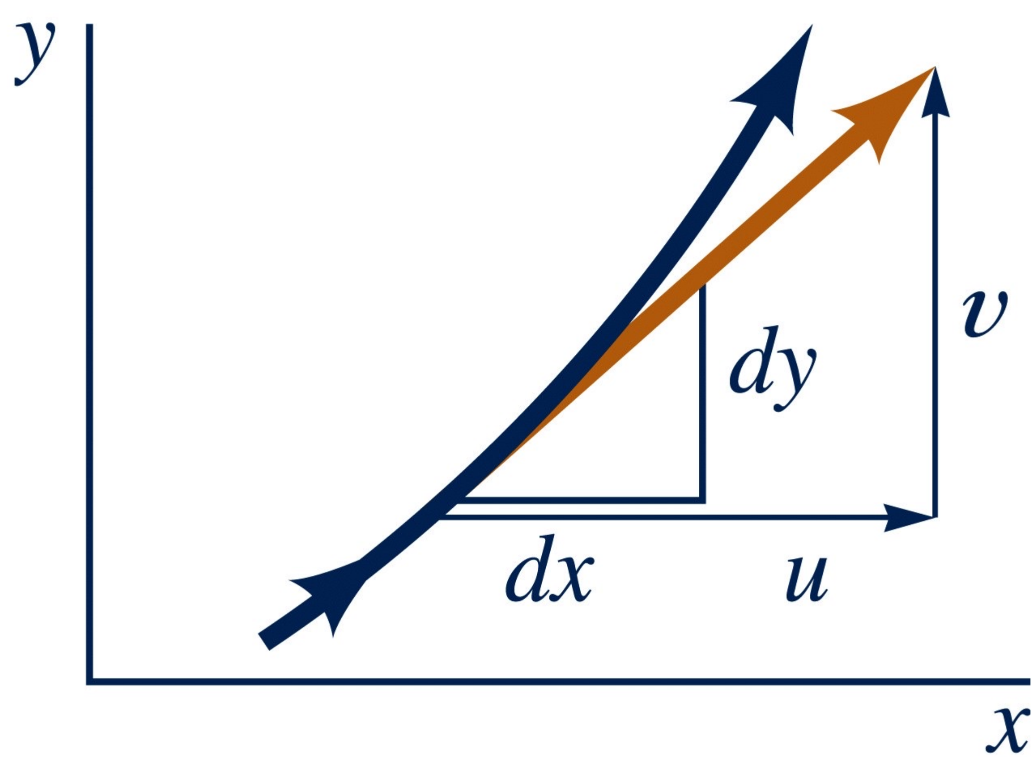
- The definition of steady or unsteady flow pertains to the behavior of a fluid property as observed at **a fixed point in space**.
- For steady flow, the values of **all fluid properties** (velocity, temperature, density, etc.) at any fixed point are independent of time.
- The value of those properties for a given fluid particle **may change with time as the particle flows along**, even in steady flow.

Streamlines, Streaklines, and Pathlines

Streamlines, Streaklines, and Pathlines

Streamline

is a line that is everywhere tangent to the velocity field. If the flow is steady, nothing at a fixed point changes with time, so the streamlines are fixed lines in space.



$$\frac{dy}{dx} = \frac{v}{u}$$

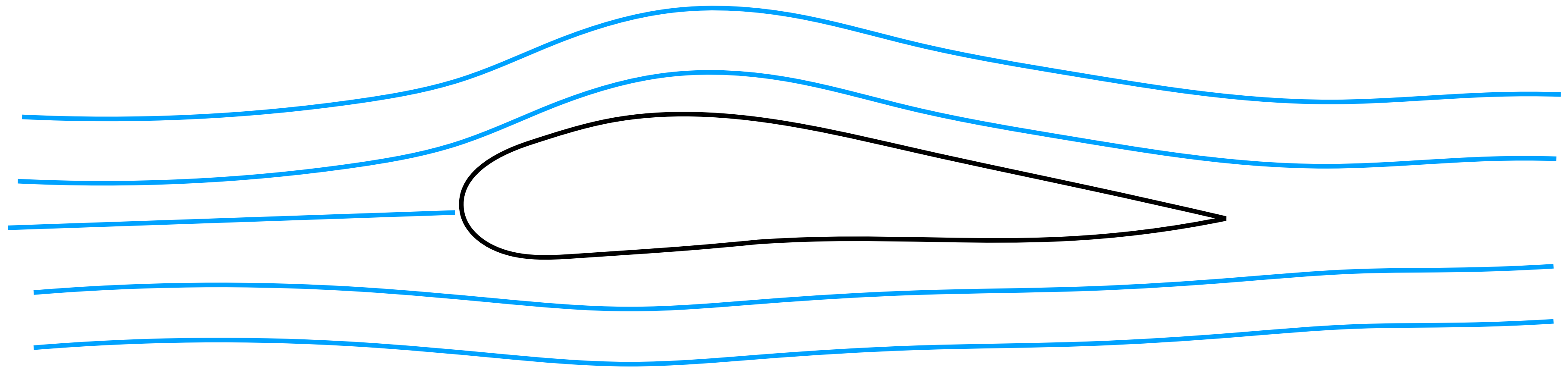
Also: (for 3D flow)

$$\frac{dz}{dx} = \frac{w}{u}, \quad \frac{dz}{dy} = \frac{w}{v}$$

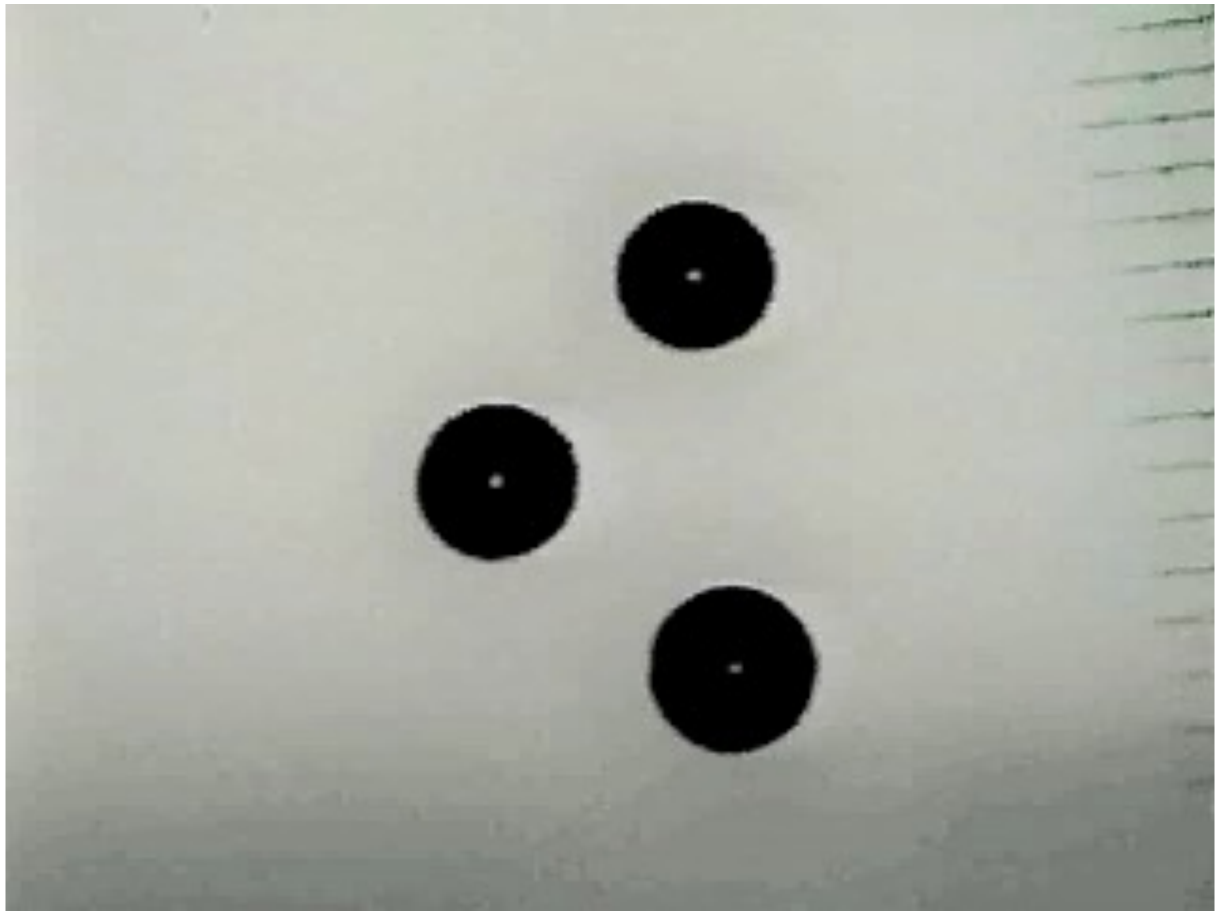
Equations of streamlines for 3D flows

Streamlines, Streaklines, and Pathlines

Streamline



Streamlines



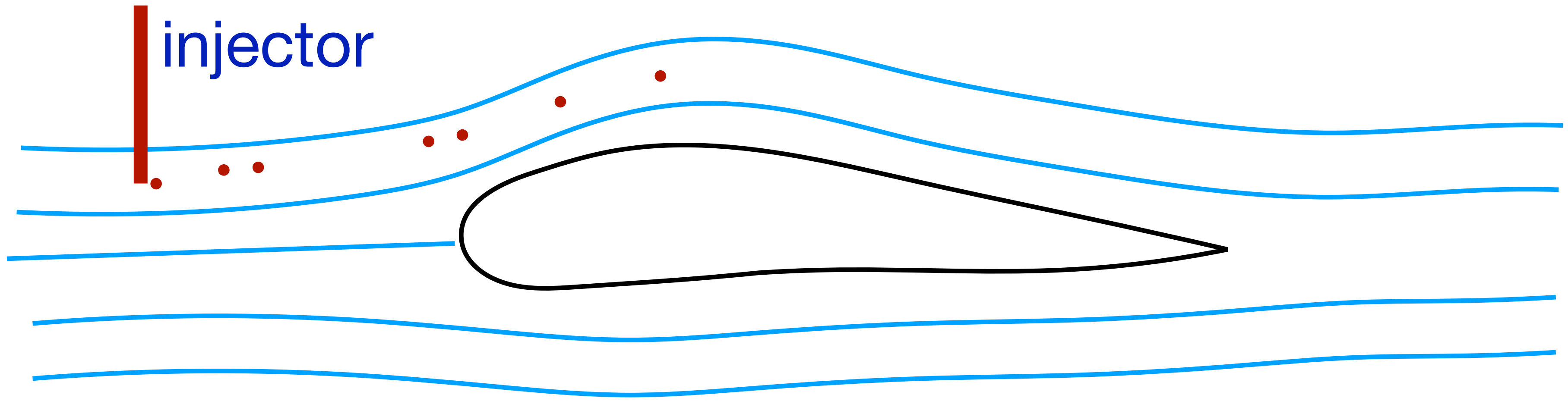
Streamlines created by injecting dye into water flowing steadily around a series of cylinders reveal the c

Steaklines

Streakline

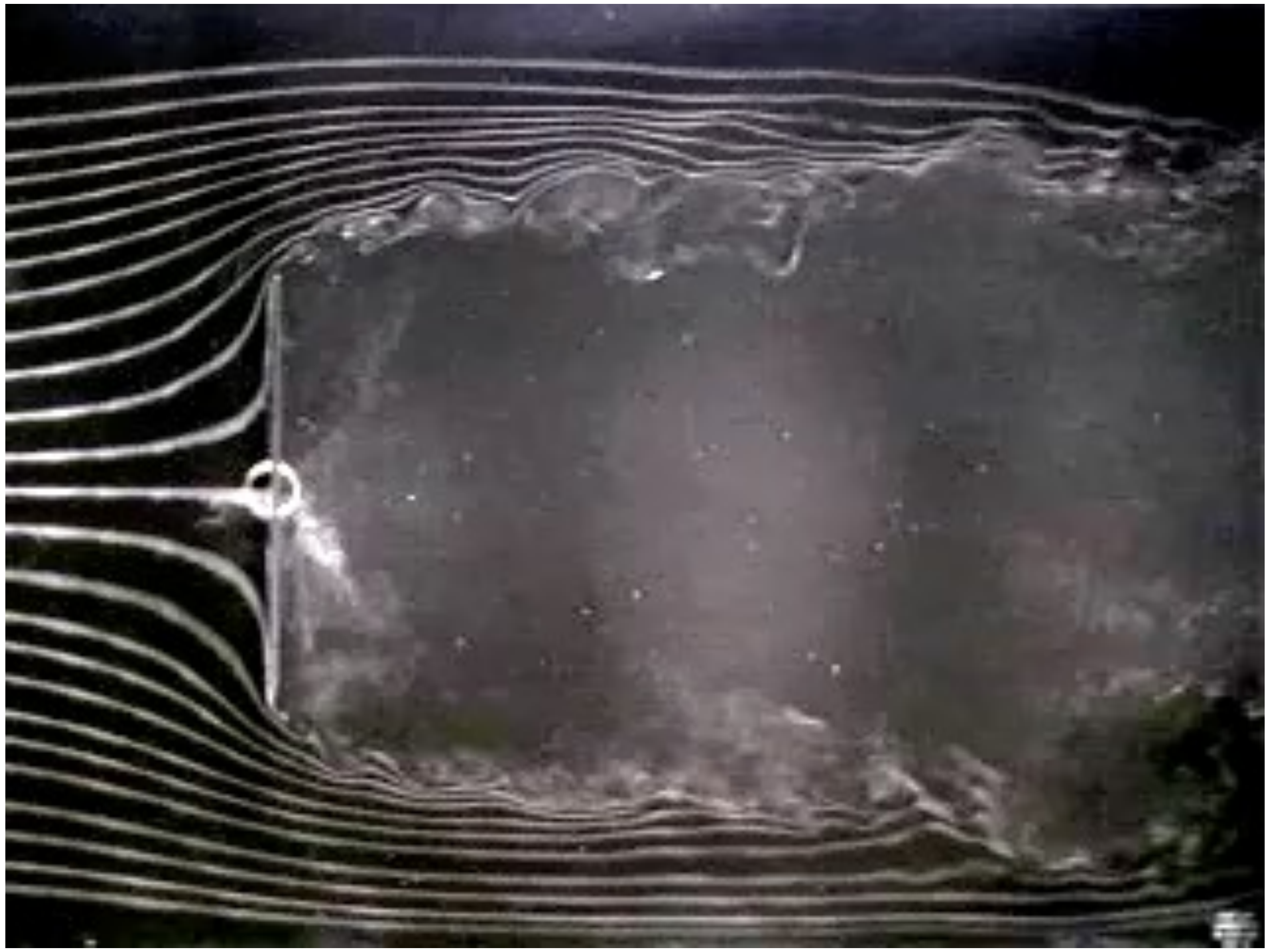
consists of all particles in a flow that have previously passed through a common point.

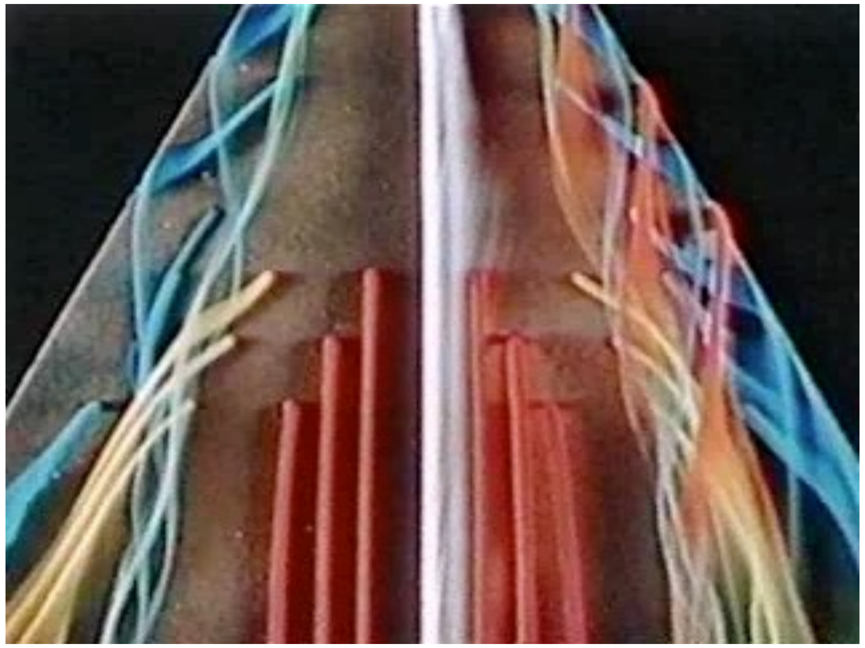
Place an injector, release particles; the particles that had a common starting point.



The streamline is often used in analytical work, while the streakline and pathline are often used in experimental work.

Streaklines





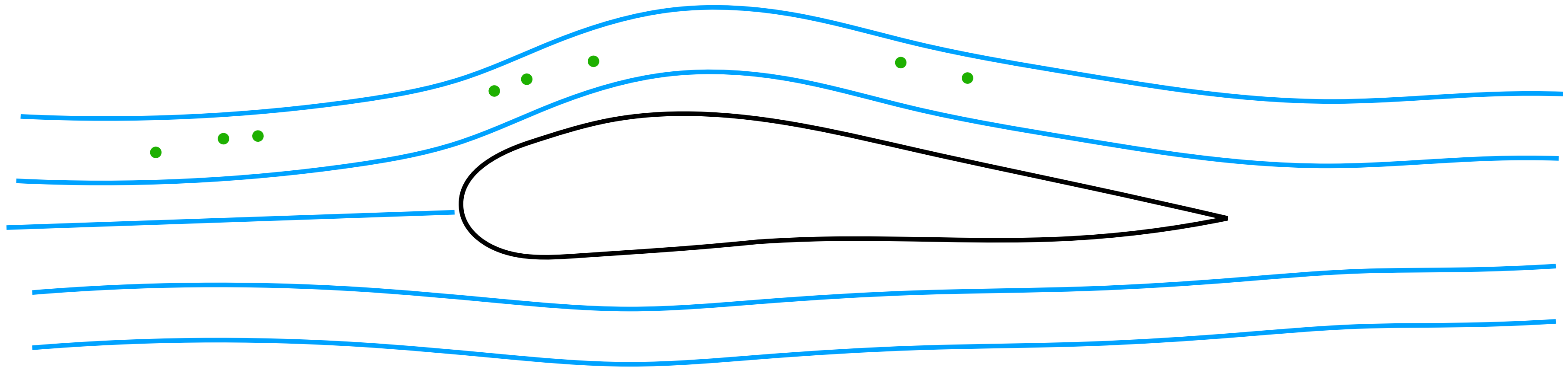
The flow past a flat plate normal to the upstream flow can be quite complex, even though the bounding geometry of the flow field is quite simple. Although the uniform upstream flow is steady and the plate is stationary, the resulting flow is unsteady. Because of this unsteadiness, the streaklines shown are not the same shape as the streamlines for this flow.

Pathlines

Pathline

Follow a particle, the path of that particle

The pathline is a Lagrangian concept that can be produced in the laboratory by marking a fluid particle, dying a small fluid element, and taking a time exposure photograph of its motion.



The streamline is often used in analytical work, while the streakline and pathline are often used in experimental work.

Pathlines

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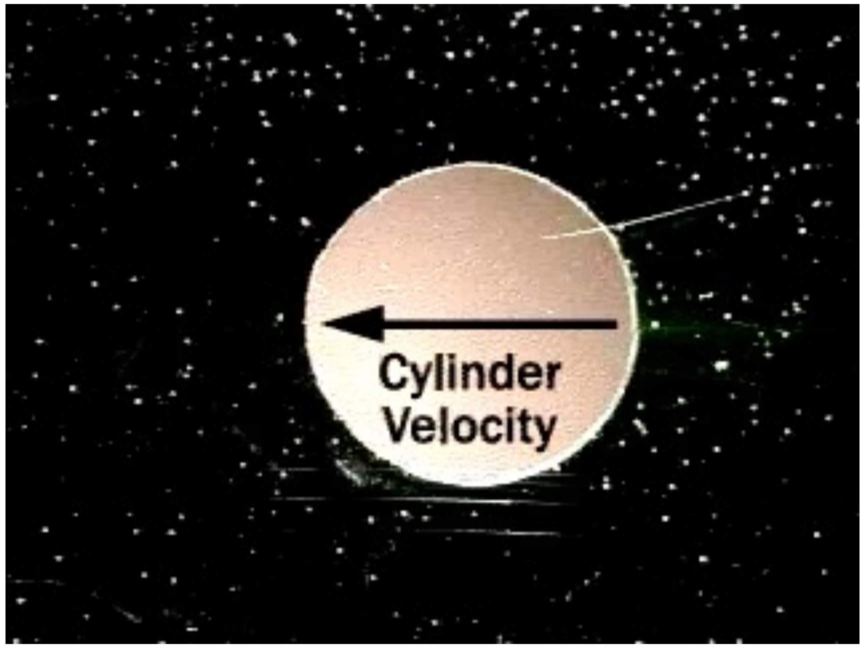
Follow a particle, the path of that particle

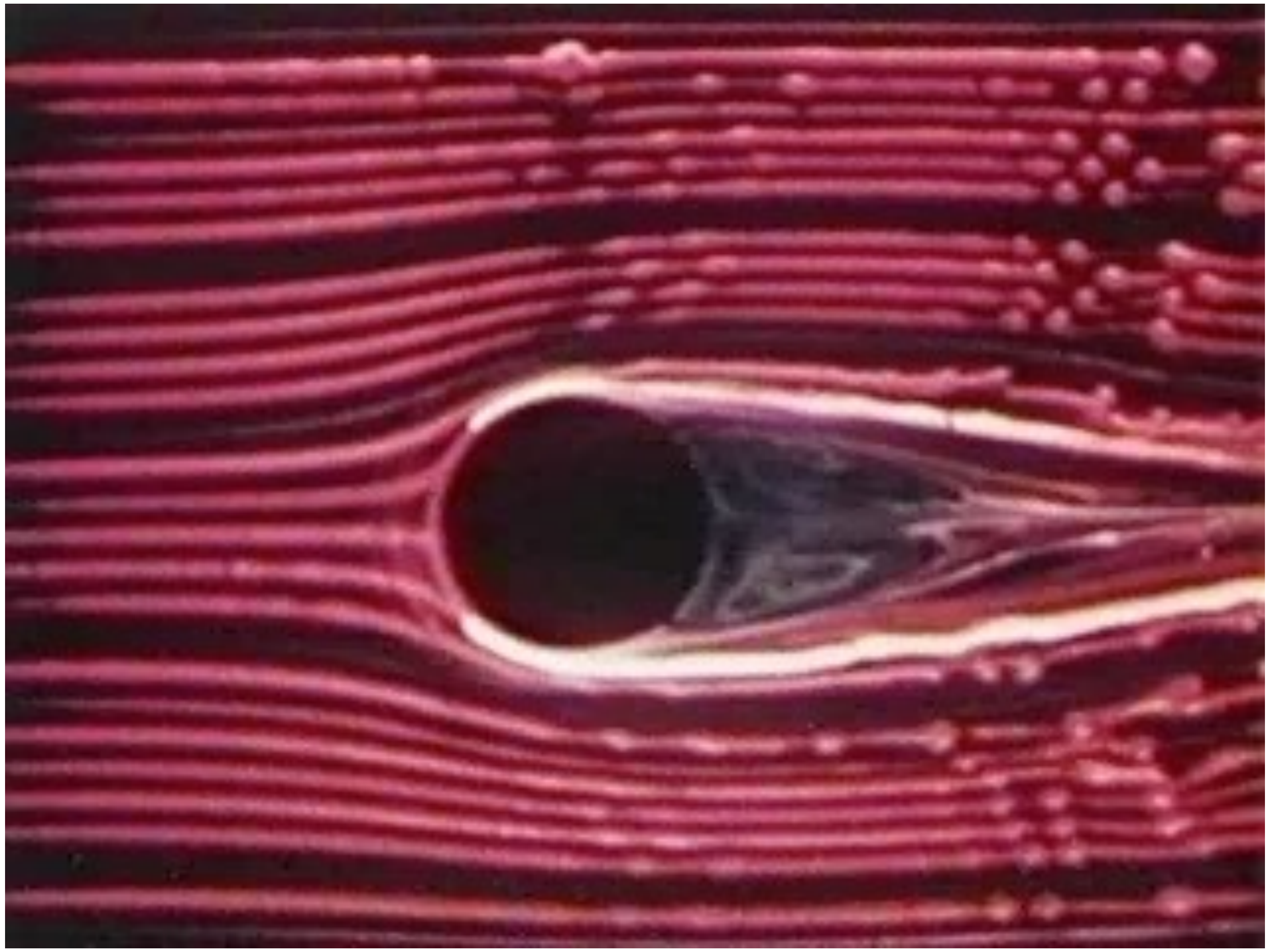
Pathline for steady flow



Pathline for unsteady flow

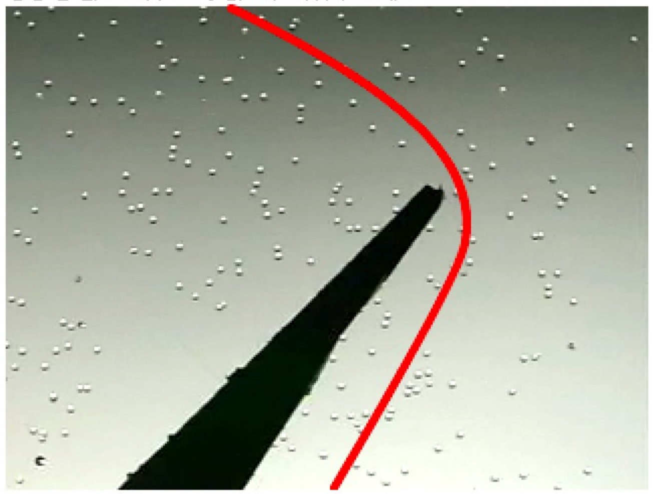
Streamlines, Streaklines, and Pathlines

For **steady** flow, streamlines, streaklines, and pathlines are the **same**

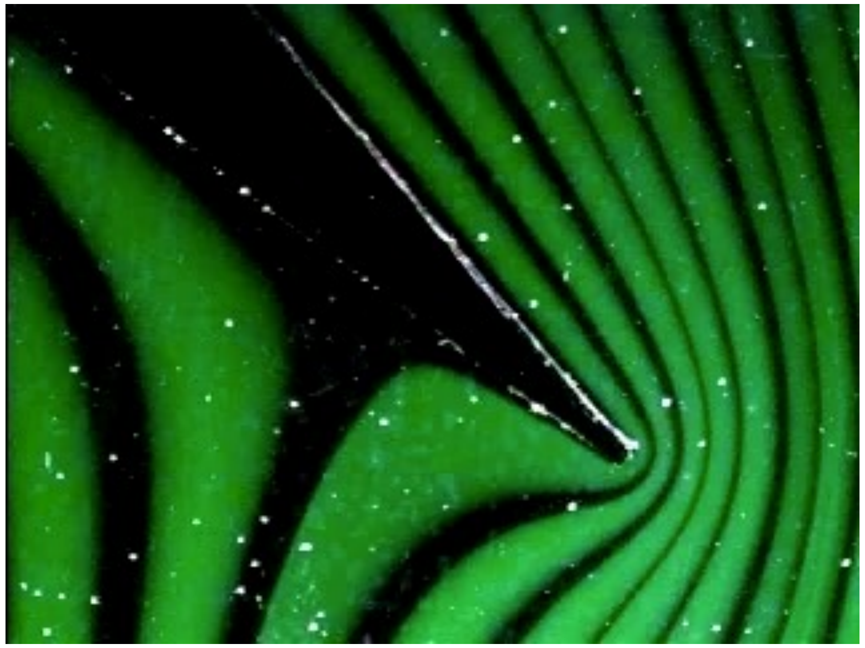


Streamlines, Streaklines, and Pathlines

For **steady** flow, streamlines, streaklines, and pathlines are the **same**



This video shows the motion of particles, i.e. pathlines, for flow around a pointed corner. Try following an individual particle or groups of nearby particles to see if you can build up a mental image of the flow. When you are ready, we'll show you an approximate pathline.



This video shows both pathlines, i.e. trajectories followed by individual particles, and streaklines, i.e. the lines illustrated by the continuous introduction of a marker (in this case a fluorescent dye). It's clear that these streaklines map out the pathlines in such a steady flow, including one that is close to the one we identified in the video without the dye.

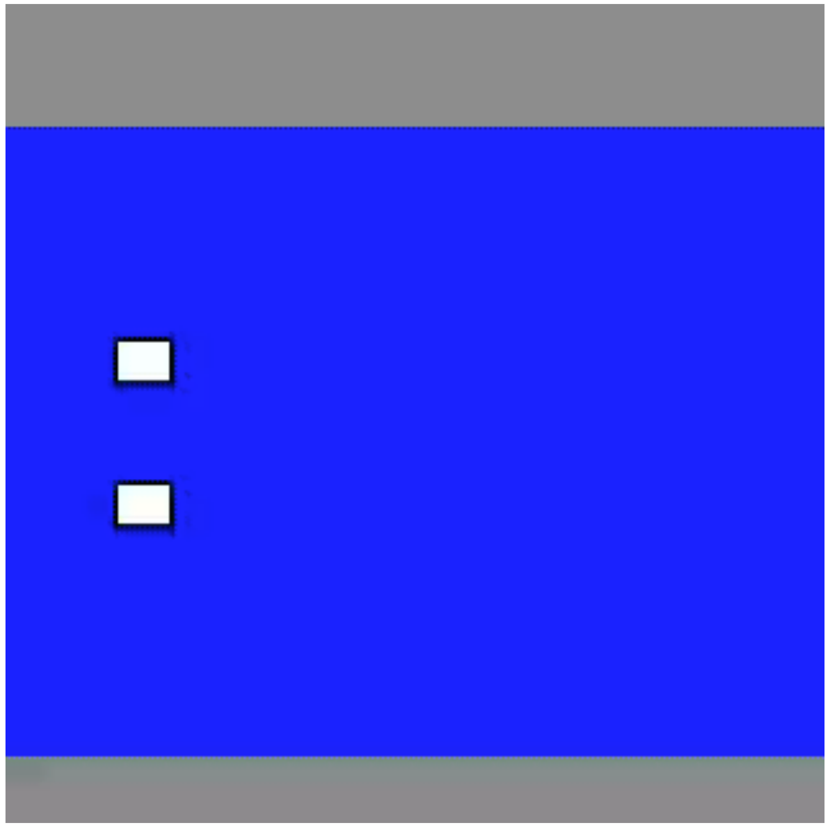
Streamlines, Streaklines, and Pathlines

For **steady** flow, streamlines, streaklines, and pathlines are the **same**

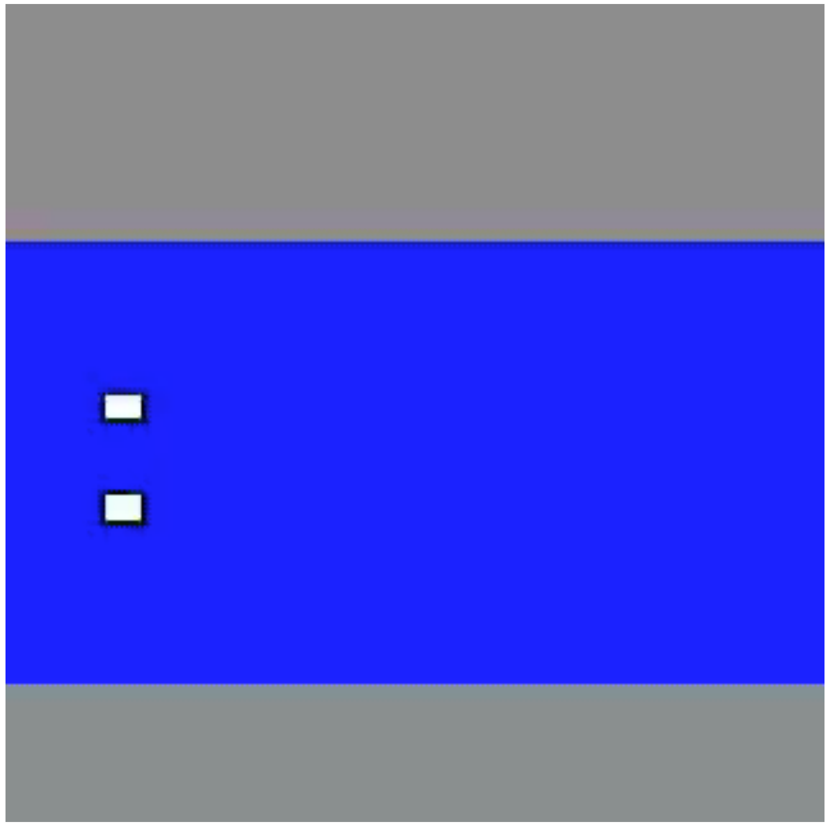
For **unsteady** flows none of these three types of lines need be the same

Streamlines, Streaklines, and Pathlines

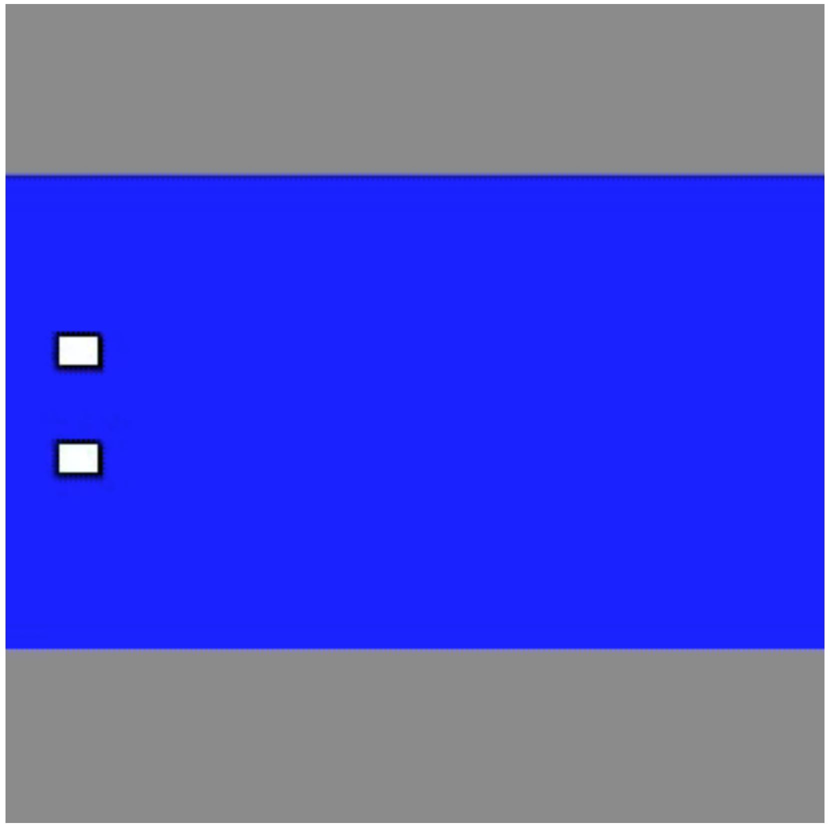
Streamlines,



Streaklines,



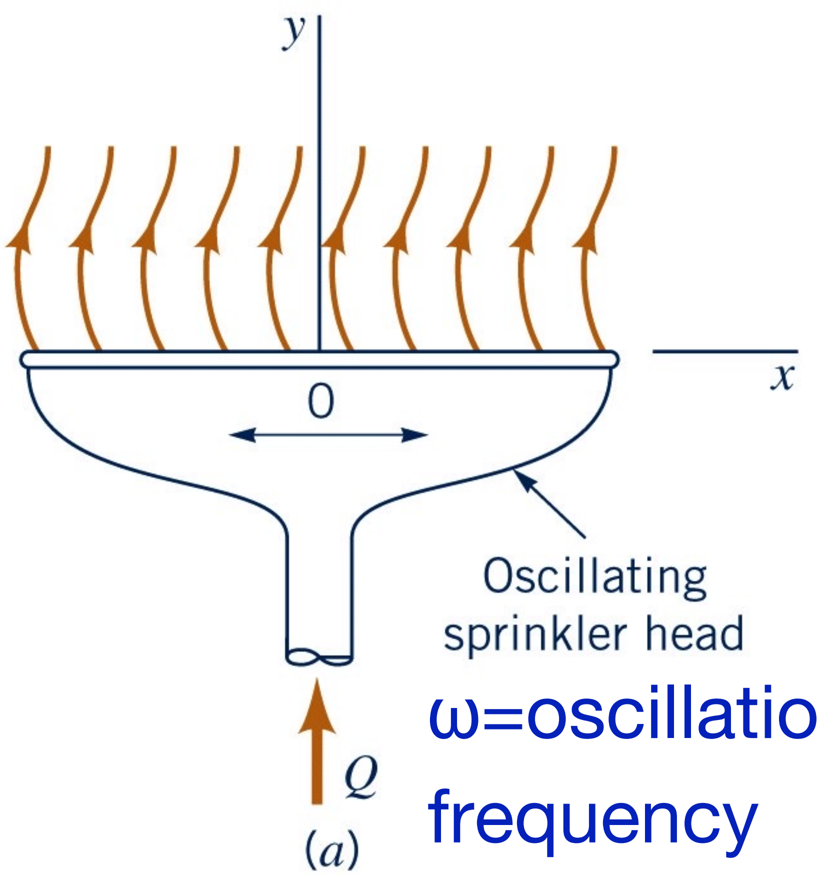
Pathlines



https://www.youtube.com/user/slffea

Example: oscillating sprinkler head

洒水器



ω=oscillation frequency

FIND **(a)** Determine the streamline that passes through the origin at $t = 0$; at $t = \pi/2\omega$.
(b) Determine the pathline of the particle that was at the origin at $t = 0$; at $t = \pi/2$.
(c) Discuss the shape of the streakline that passes through the origin.

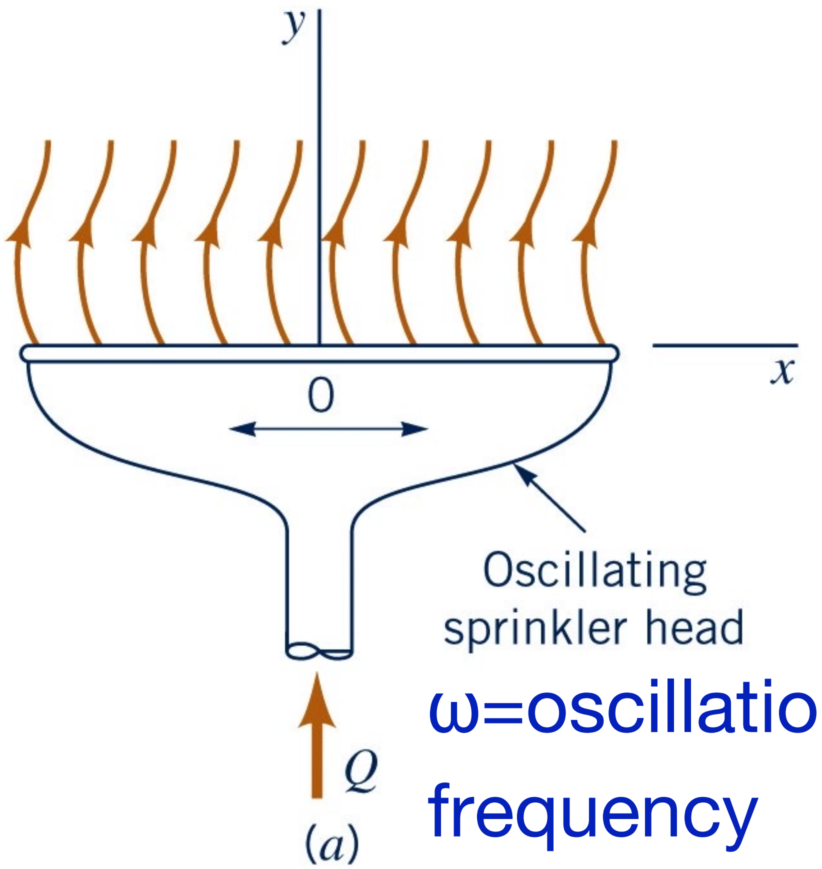
$$\vec{V} = u_0 sin[\omega(t - y/v_0)]\hat{i} + v_0\hat{j}$$

No x-dependence, implying?

Very long pipe

Example: oscillating sprinkler head

Streamlines



ω=oscillation

frequency

$$\vec{V} = u_0 sin[\omega(t - y/v_0)]\hat{i} + v_0\hat{j}$$

Example: oscillating sprinkler head

Streamlines

$$x = u_0/\omega \cos(\omega(t - y/v_0)) + C_1$$

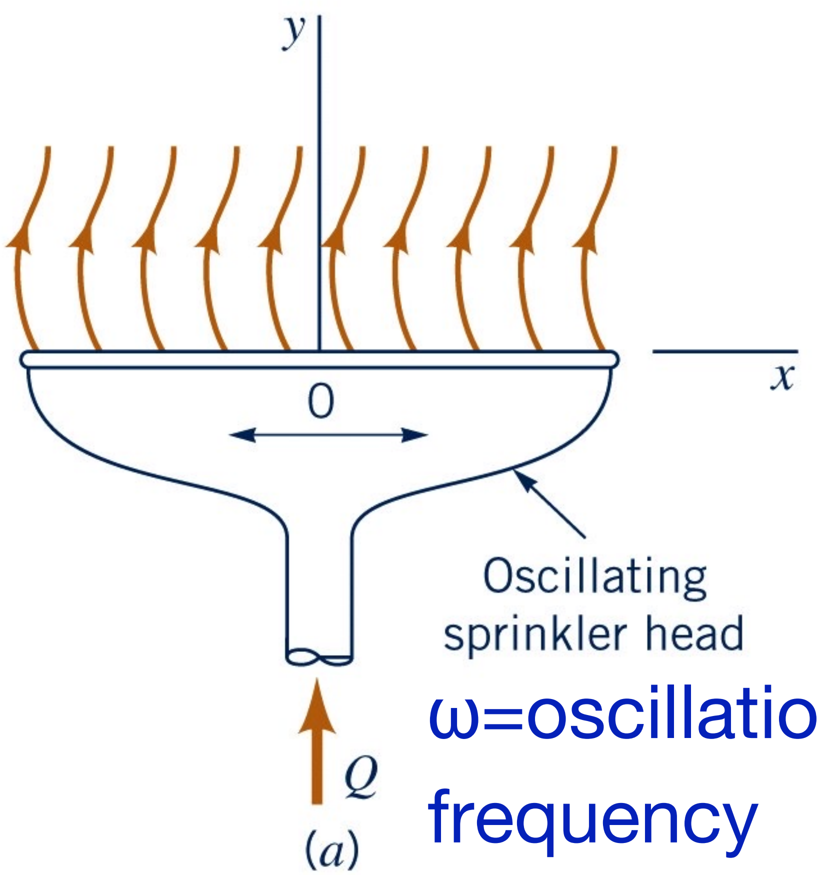
Example: oscillating sprinkler head

Streamlines

$$x = u_0/\omega \cos(\omega(t - y/v_0)) + C_1$$

FIND **(a)** Determine the streamline that passes through the origin at $t = 0$; at $t = \pi/2\omega$.

Example: oscillating sprinkler head



ω=oscillation frequency

$$\vec{V} = u_0 sin[\omega(t - y/v_0)]\hat{i} + v_0\hat{j}$$

Streamlines

$$\frac{dy}{dx} = \frac{v}{u} = \frac{v_0}{u_0 \sin(\omega(t - y/v_0))}$$

Seperate the variables

$$dy\, u_0 \sin(\omega(t - y/v_0)) = v_0 dx$$

Integrate

$$u_0(v_0/\omega)\cos(\omega(t - y/v_0)) = v_0 x + C$$

How to plot streamlines?

$$x = u_0/\omega \cos(\omega(t - y/v_0)) + C_1$$

Example: oscillating sprinkler head

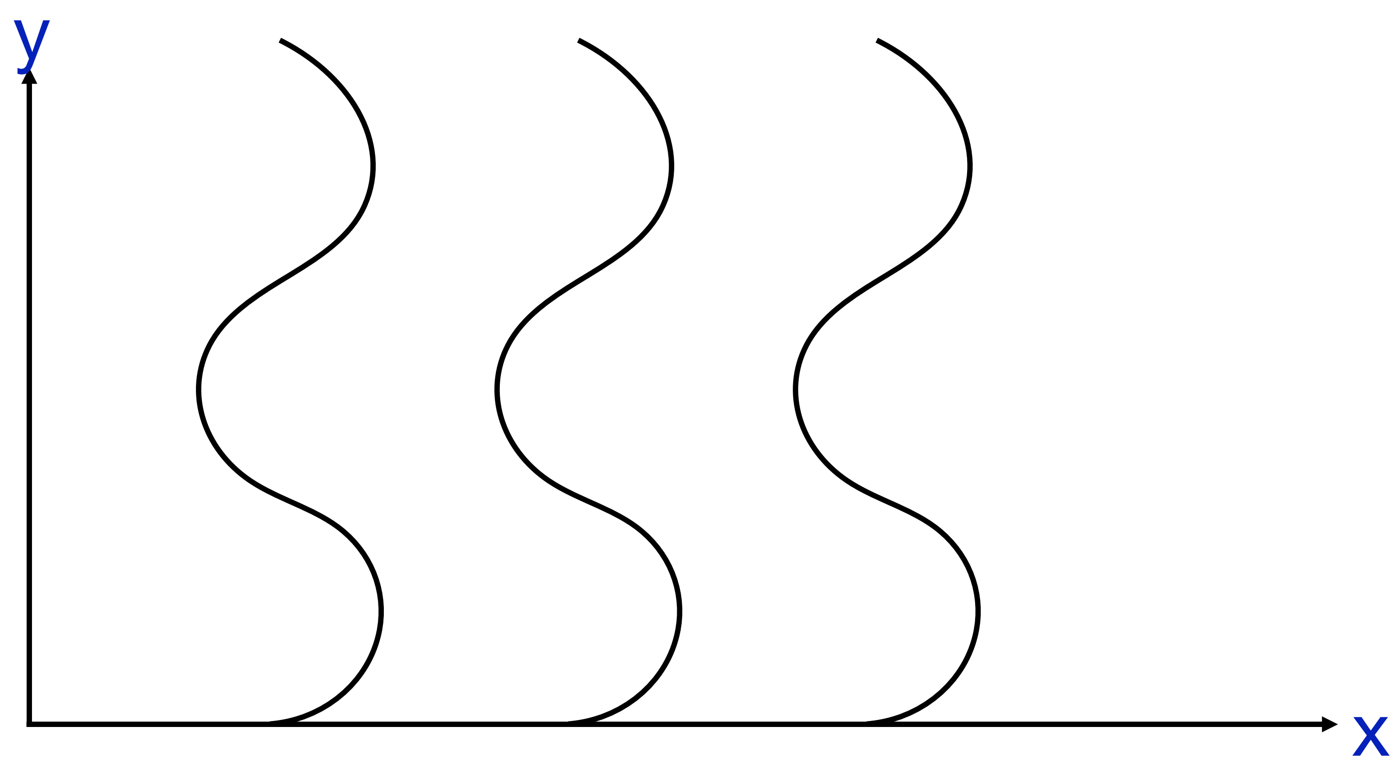
Streamlines

$$x = u_0/\omega \cos(\omega(t - y/v_0)) + C_1$$

A family of cos curves

at a given time

It has a time dependence



Note: different C1 value gives different SL

Example: oscillating sprinkler head

Streamlines

$$x = u_0/\omega \cos(\omega(t - y/v_0)) + C_1$$

Plot SL through the origin at t=0

t=0, x =y = 0 $\longrightarrow$ $C_1 = -u_0/\omega$

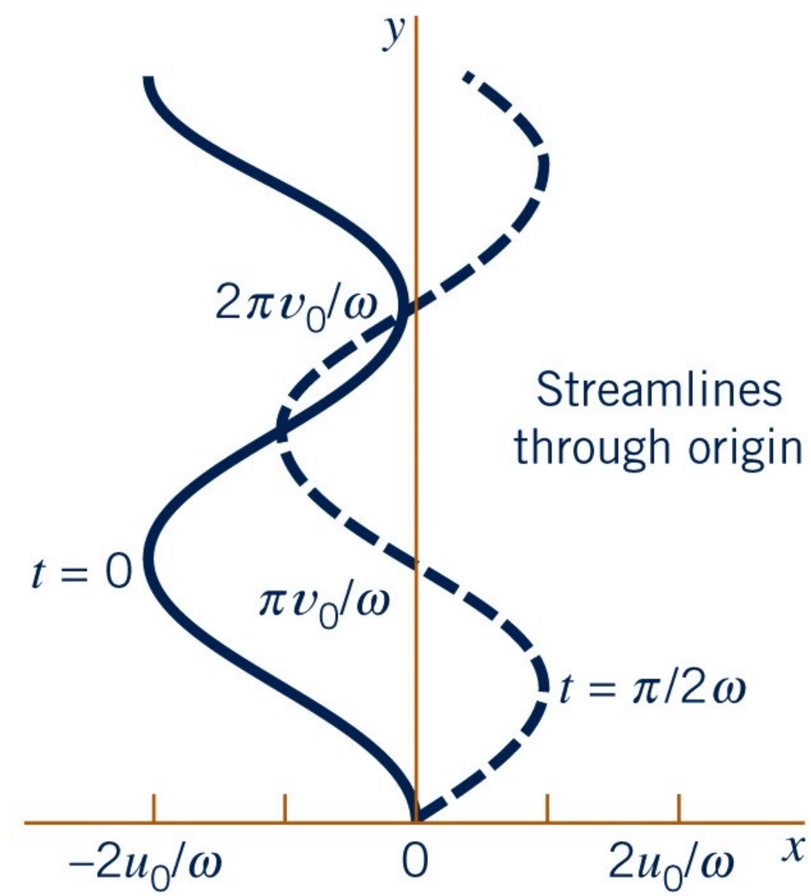
$$x = u_0/\omega \cos(\omega y/v_0) - u_0/\omega$$

Plot SL through the origin at t=π/2ω

t=π/2ω, x =y = 0 $\longrightarrow$ $C_1 = 0$

$$x = \frac{u_0}{\omega}\cos[\omega(\frac{\pi}{2\omega} - \frac{y}{v_0})] = \frac{u_0}{\omega}\sin(\frac{\omega y}{v_0})$$

Note: it has time-dependence



(b)

Example: oscillating sprinkler head

Pathlines (Lagrangian)

Example: oscillating sprinkler head

Pathlines (Lagrangian)

(b) Determine the pathline of the particle that was at the origin at $t = 0$; at $t = \pi/2$.

$$y = v_0 t + C_1$$

$$x = -[u_0 \sin(C_1 \omega / v_0)]t + C_2$$

Example: oscillating sprinkler head

Pathlines (Lagrangian)

$$\frac{dx}{dt} = u_0 \sin(\omega(t - y/v_0))$$
$$\frac{dy}{dt} = v_0$$

Velocity field: Eulerian. Lagrangian field?

Integrate

$$\boxed{y = v_0 t + C_1}$$

y varies with time, plug into x-eq

$$\frac{dx}{dt} = u_0 sin[\cancel{\omega(t} - \frac{\cancel{v_0 t} + C_1}{v_0})] = -u_0 \sin(C_1 \omega / v_0)$$ a constant

$$\boxed{x = -[u_0 \sin(C_1 \omega / v_0)]t + C_2}$$

Example: oscillating sprinkler head

Pathlines (Lagrangian)

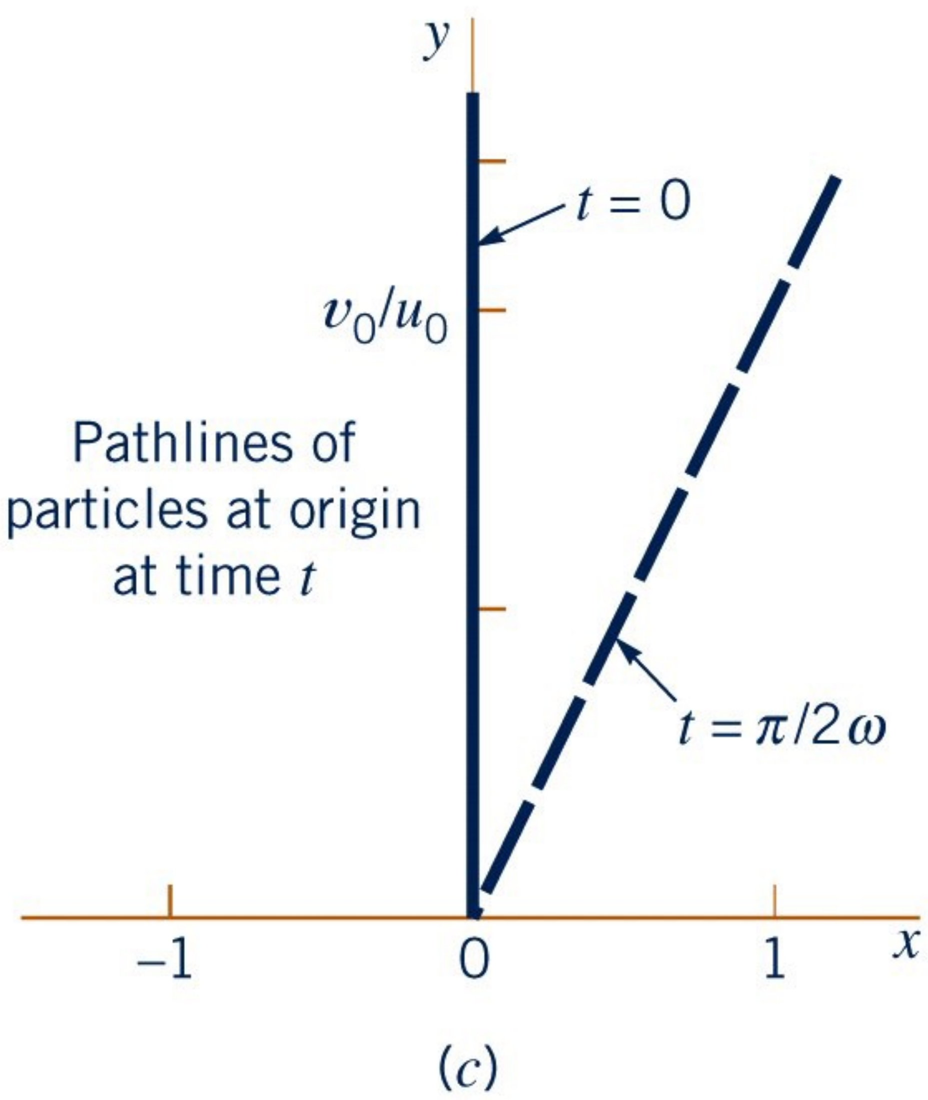
$$y = v_0 t + C_1$$

$$x = -[u_0 \sin(C_1 \omega / v_0)]t + C_2$$

For the particle at the origin at t=0

t=0, x =y = 0 $\longrightarrow$ $C_1 = C_2 = 0$

Pathline: $$y = v_0 t$$ $$x = 0$$



For the particle at the origin at t=π/2ω

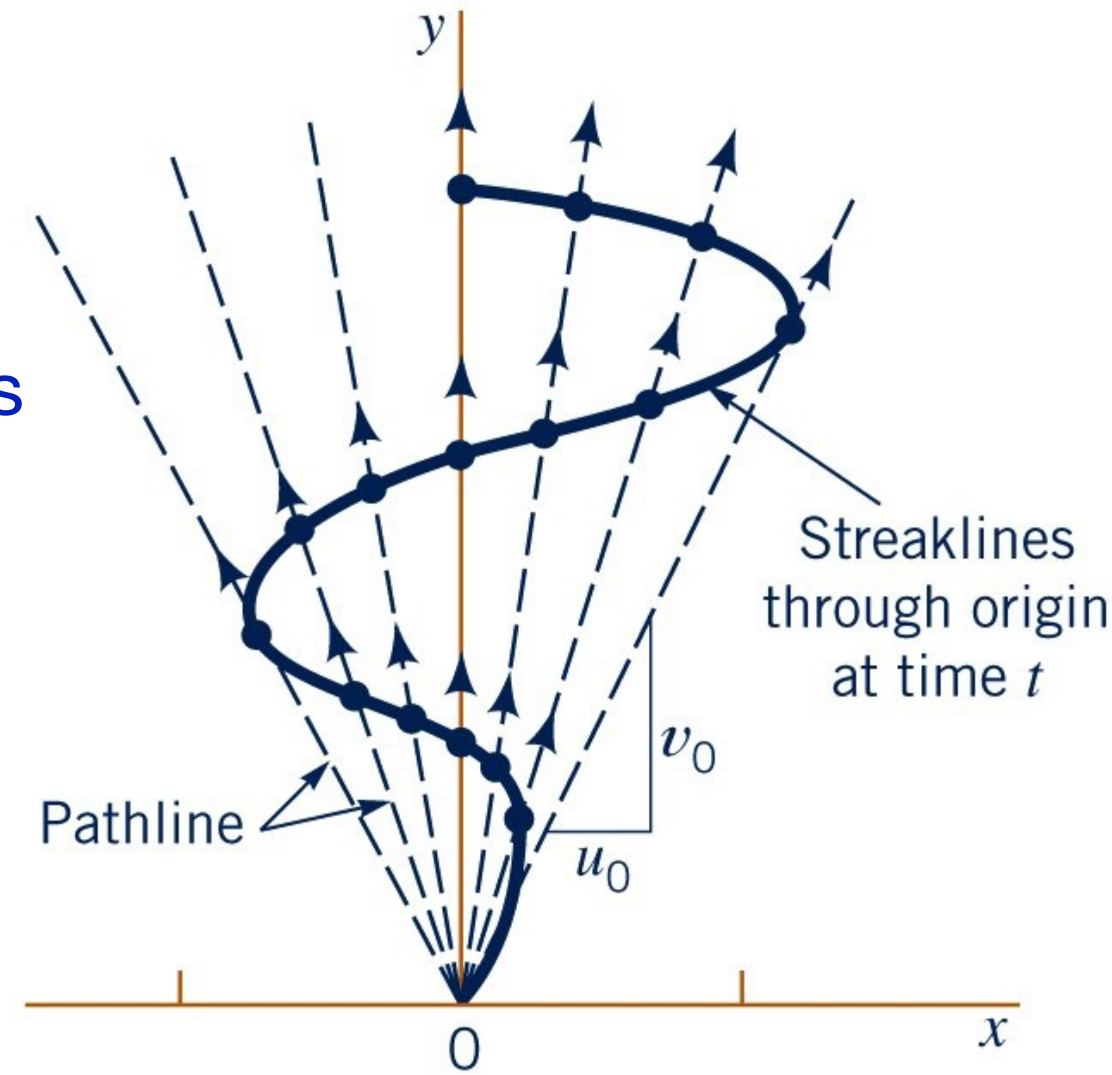
$$C_1 = -\pi v_0 / 2\omega, \; C_2 = -\pi u_0 / 2\omega$$

Pathline: $$x = u_0 \left(t - \frac{\pi}{2\omega}\right)$$ $$y = v_0 \left(t - \frac{\pi}{2\omega}\right)$$ $\Rightarrow$ $$y = \frac{v_0}{u_0} x$$

Example: oscillating sprinkler head

Streaklines

- Each particle that flows through the origin travels in a straight line (pathlines)
- Particles passing through the origin at different times are located on different rays from the origin and at different distances from the origin
- The net result is that a stream of dye continually injected at the origin (a streakline) would have the shape shown in Figure
- Because of the unsteadiness, the streakline will vary with time, although it will always have the oscillating, sinuous character shown.



(d)

Acceleration of a fluid element

As we start from Newton's 2nd law, **acceleration** is important!

Consider a fluid particle moving along its pathline

Acceleration of the particle is time rate of change of its velocity:

Note: the velocity may be a function of both **position and time**, its value may change because of the change in time as well as a change in the particle's position

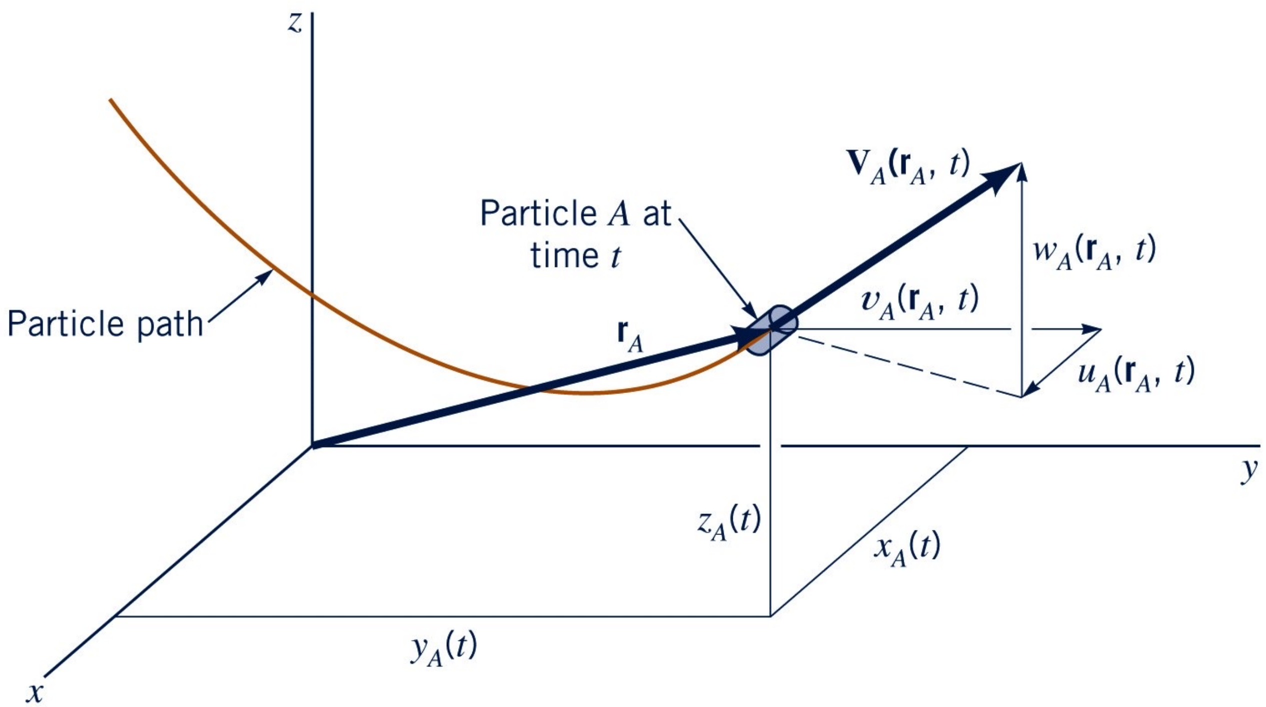
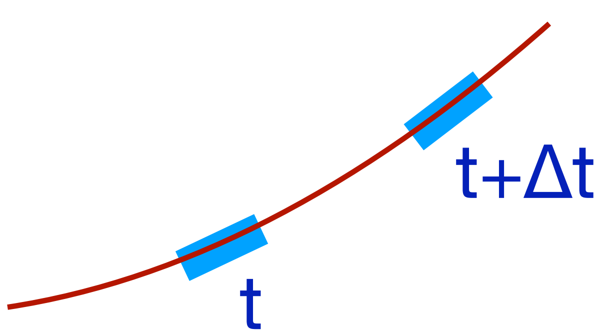


Figure 4.4

How to calculate acceleration?

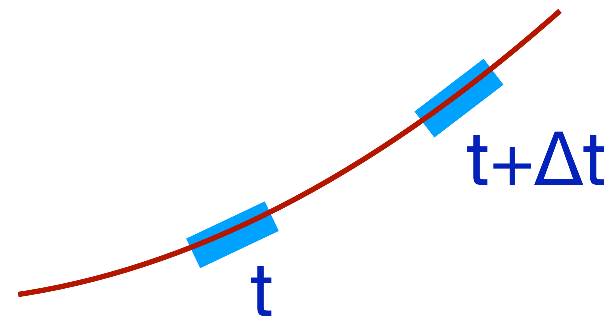


Also:
x+Δx
y+Δy
z+Δz

Acceleration of a fluid element

the velocity may be a function of both **position and time**

Consider a fluid particle moving along its pathline



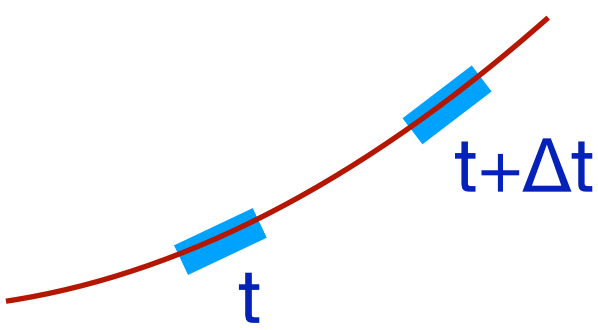
Also:
x+Δx
y+Δy
z+Δz

$$\vec{V} = \vec{V}(x, y, z, t)$$ Eulerian field

Acceleration of a fluid element

the velocity may be a function of both **position and time**

Consider a fluid particle moving along its pathline



Also:
x+Δx
y+Δy
z+Δz

$$\vec{V} = \vec{V}(x, y, z, t)$$ Eulerian field

Acceleration has to be same regrades the field you used

$$\vec{a} = \lim_{\Delta \to 0} \frac{\vec{V}(x+\Delta x, y+\Delta y, z+\Delta z, t+\Delta t) - \vec{V}(x(t), y(t), z(t), t)}{\Delta t}$$

Note: x, y, z are function of t. $$\vec{V} = \vec{V}(x(t), y(t), z(t), t)$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \vec{V}}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \vec{V}}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial \vec{V}}{\partial t}$$

Acceleration of a fluid element

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \vec{V}}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \vec{V}}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial \vec{V}}{\partial t}$$

Acceleration of a fluid element

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \vec{V}}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \vec{V}}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial \vec{V}}{\partial t}$$

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$u = \frac{\partial x}{\partial t} \qquad v = \frac{\partial y}{\partial t} \qquad w = \frac{\partial z}{\partial t}$$

$$\vec{a} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial \mathrm{y}} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}$$

It looks complicated. It is the same $\frac{d\vec{V}}{dt}$, but is expressed in Eulerian frame

Acceleration of a fluid element

$$\vec{a} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial \mathrm{y}} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t} \quad = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Components

Acceleration of a fluid element

$$\vec{a} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial \mathrm{y}} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t} \quad = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Components

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Acceleration of a fluid element

Important to note the difference:

$$\frac{\partial \vec{V}}{\partial t} \qquad\qquad \frac{d\vec{V}}{dt}$$

Acceleration of a fluid element

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$$\frac{\partial \vec{V}}{\partial t} \qquad\qquad \frac{d\vec{V}}{dt}$$

$\downarrow$ Part of the acceleration $\downarrow$ Local acceleration

$\downarrow$ Acceleration

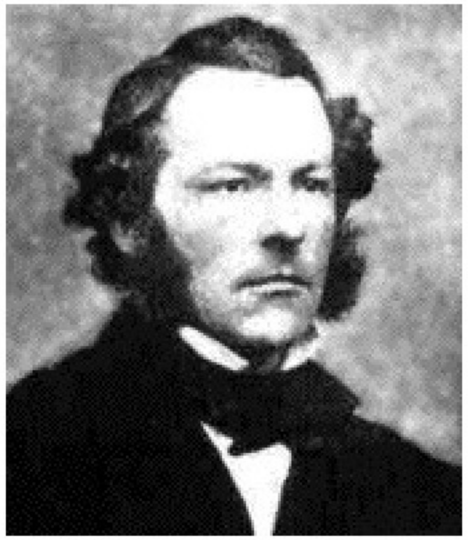
If the flow is steady, local acce. = 0. Total acce. =0 ?

Acceleration of a fluid element

In fluid mechanics,

$$\frac{d}{dt} = \frac{D}{Dt} \qquad \frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

- Material derivative
- Substantial derivative
- Lagrangian derivative
- Derivative following the fluid



Sir George Gabriel Stokes

The notation using capital D for the material derivative, i.e D/Dt, is due to G. G. Stokes. It is a notation that is still in use to this day.

Acceleration of a fluid element

Define a new operator $\vec{V} \cdot \nabla$

Acceleration of a fluid element

Define a new operator

$$\vec{V} \cdot \nabla = (u\hat{i} + v\hat{j} + w\hat{k}) \cdot (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k})$$

$$= u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

It is a scalar operator

$\vec{V} \cdot \nabla$ can be applied to any scalar or vector field

Acceleration of a fluid element

$\vec{V} \cdot \nabla$ is applied to velocity field, it yields the convective acceleration

$$(\vec{V} \cdot \nabla)\vec{V} =$$

Acceleration of a fluid element

$\vec{V} \cdot \nabla$ is applied to velocity field, it yields the convective acceleration

$$(\vec{V} \cdot \nabla)\vec{V} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

Contains the 3 components of the convective acce.

$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}$$

Total acce.   Local acce.   Convective acce.

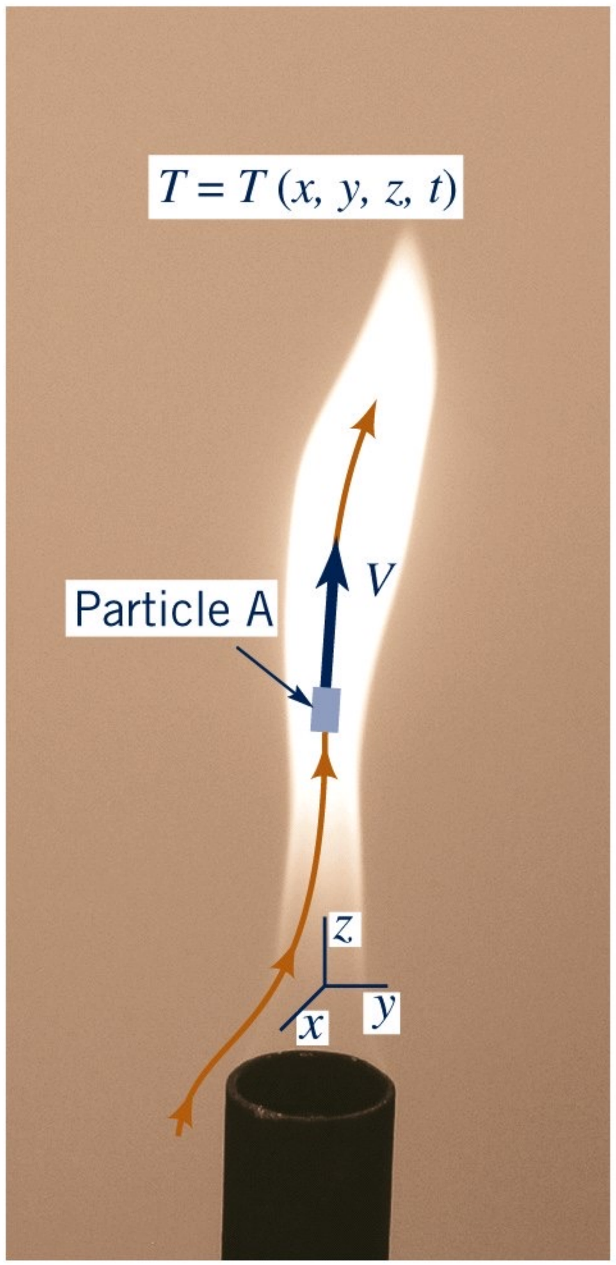
How many terms?

$(\vec{V} \cdot \nabla)\vec{V}$ 9 terms are nonlinear

Why nonlinear? Nonlinearity makes the solution very complicated…

Apply to other fields

$\vec{V} \cdot \nabla$ Can be applied to other field, such as temperature in a flame T(x,y,z,t)



Consider a fluid particle (A)

the rate of change of temperature

$$\frac{dT_A}{dt} =$$

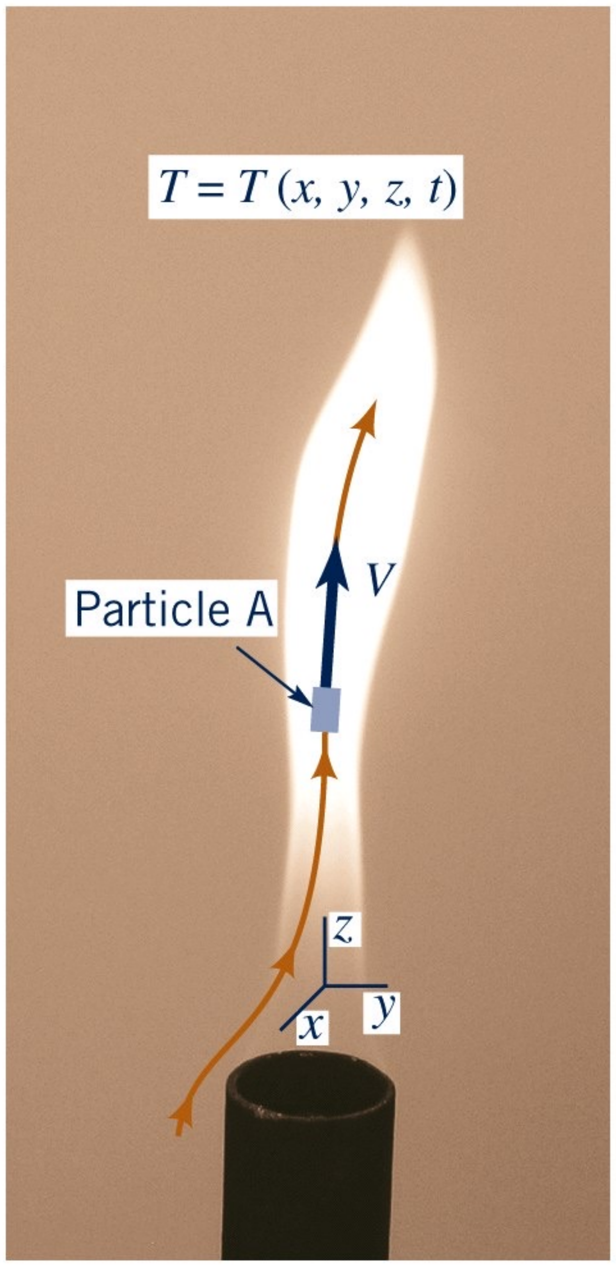
This can be written as

$$\frac{DT}{Dt} =$$

Unnumbered 4 p167

Apply to other fields

$\vec{V} \cdot \nabla$ Can be applied to other field, such as temperature in a flame T(x,y,z,t)



Unnumbered 4 p167

Consider a fluid particle (A)

the rate of change of temperature

$$\frac{dT_A}{dt} = \frac{\partial T_A}{\partial t} + \frac{\partial T_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial T_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial T_A}{\partial z}\frac{dz_A}{dt}$$

This can be written as

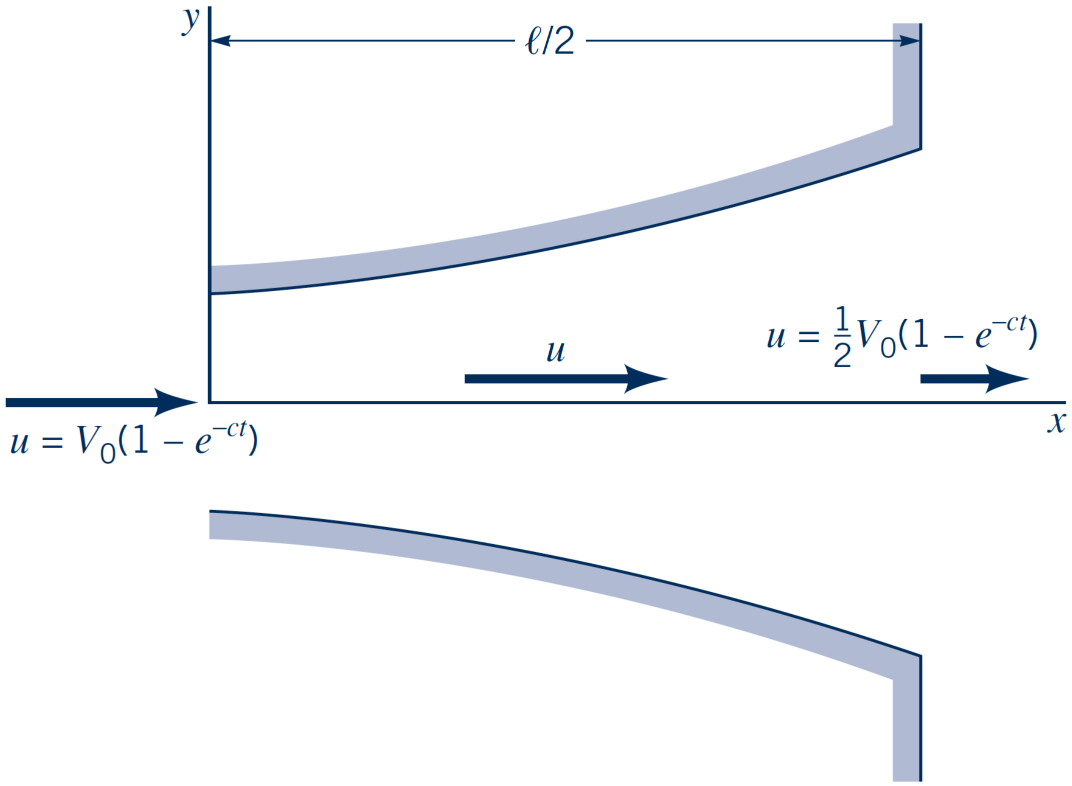
$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T$$

As in the determination of the acceleration, the material derivative operator, $D(\ )/Dt$, appears.

Example

$$V = u\hat{i} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{i}$$

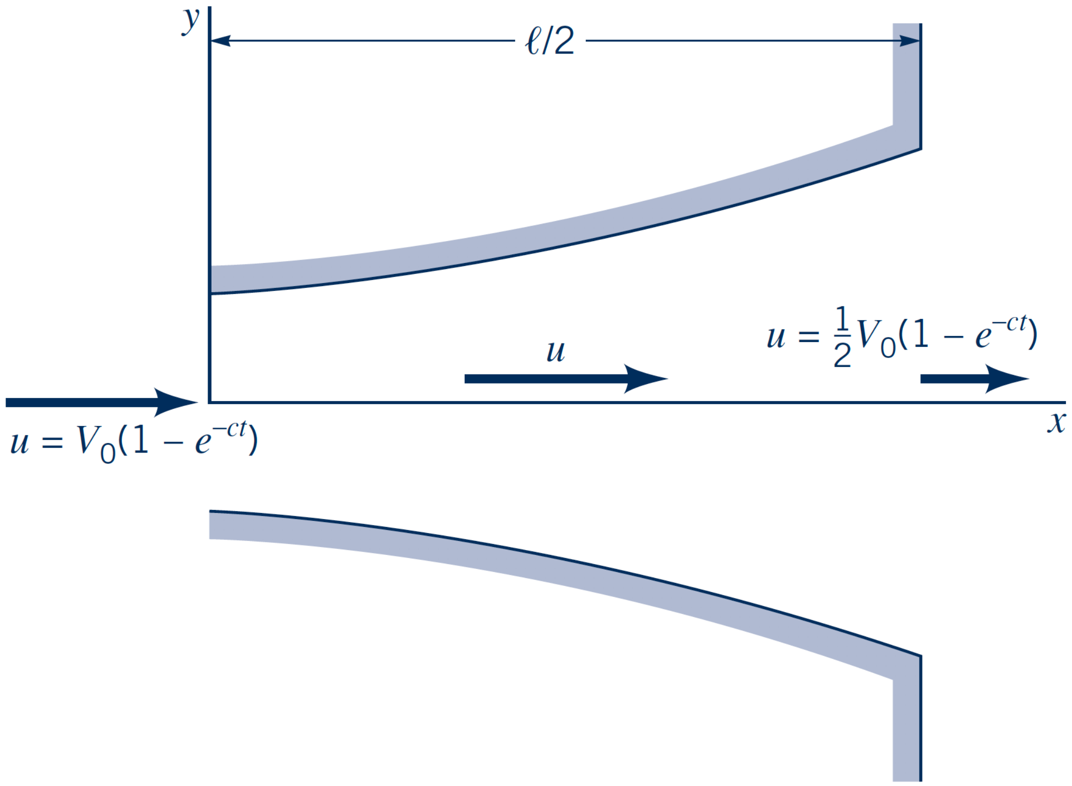
calculate acce.



Example

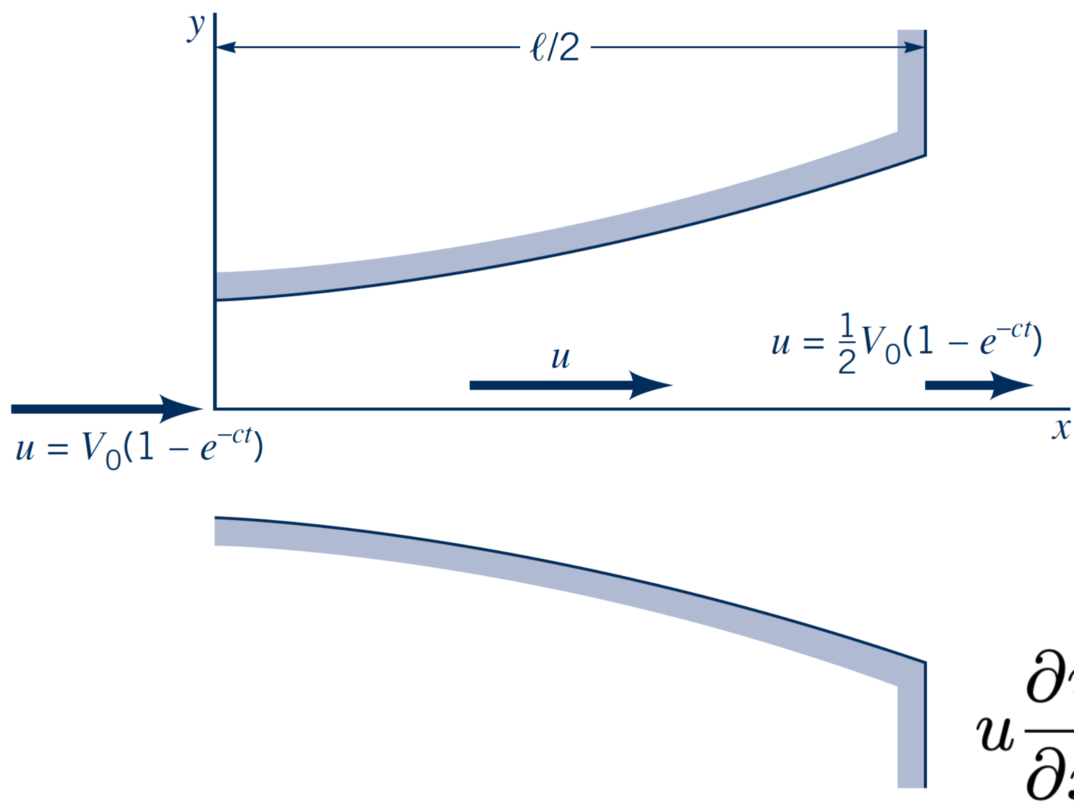
$$V = u\hat{i} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{i}$$

calculate acce.



Example

$$V = u\hat{i} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{i}$$ calculate acce.



1D, unsteady flow

$$\vec{a} = a_x\hat{i} = (\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x})\hat{i}$$

$$\frac{\partial u}{\partial t} = V_0(1 - x/\ell)ce^{-ct}$$

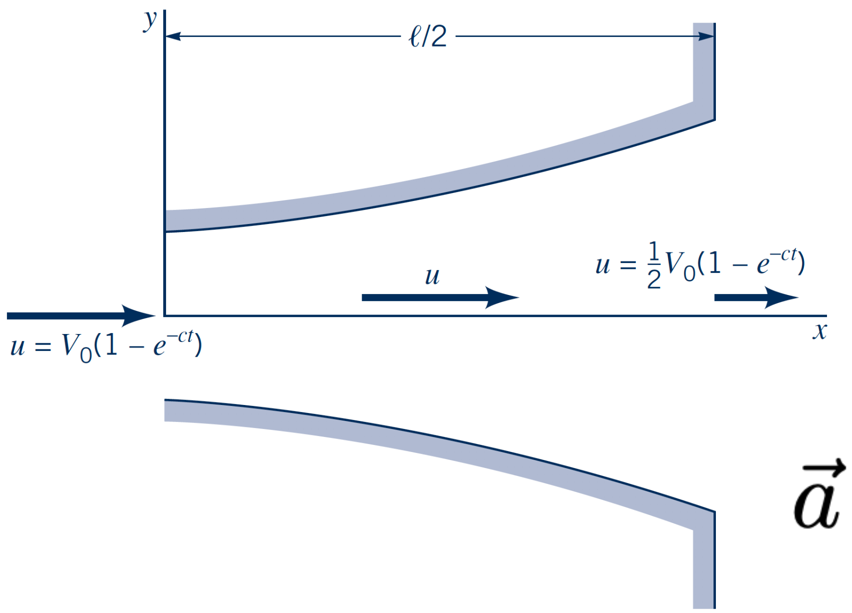
Local acce.

$$u\frac{\partial u}{\partial x} = V_0(1 - e^{-ct})(1 - x/\ell)V_0(1 - e^{-ct})(-1/\ell)$$

$$= -\frac{V_0^2}{\ell}(1 - e^{-ct})^2(1 - x/\ell)$$ Convective acce.

$$\boxed{\vec{a} = V_0(1 - x/\ell)[ce^{-ct} - (V_0/\ell)(1 - e^{-ct})^2]\hat{i}}$$

Example



$$V = u\hat{i} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{i}$$

$$\vec{a} = V_0(1 - x/\ell)[ce^{-ct} - (V_0/\ell)(1 - e^{-ct})^2]\hat{i}$$

What happens at a very long time

$t \to \infty$ $\vec{a}$ is a constant; steady acceleration

$\vec{V}$ is a constant; steady velocity

Flow is steady, but deceases with x

Steady flow can have non-zero acce. $\vec{a} \neq 0$

Control volume and system

Control volume and system

Definition:

a **system**

A **control volume**

Control volume and system

Definition:

a **system** is a collection of matter of fixed identity (always the same atoms or fluid particles), which may move, flow, and interact with its surroundings.

A **control volume** is a volume in space (a geometric entity, independent of mass) through which fluid may flow

Control volume and system

Definition:

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A **control volume** is a volume in space (a geometric entity, independent of mass) through which fluid may flow

Both control volume and system concepts can be used to describe fluid flow

In Fluid Mechanics:

System = closed system = control mass

Open system = control volume

Control volume approach

From thermodynamics

Control volume

System = control mass

Control volume approach

From thermodynamics

Control volume

$$\left(\frac{dm}{dt}\right)_{CV} = \dot{m}_{in} - \dot{m}_{out}$$

System = control mass

$$\left(\frac{dm}{dt}\right)_{sys} = 0$$

Simple Reynolds Transport Theorem

Simple Reynolds Transport Theorem

The governing laws of fluid motion are stated in terms of fluid systems, not control volumes

To use the governing equations in a control volume approach to problem solving, we must rephrase the laws in an appropriate manner

Simple Reynolds Transport Theorem

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To use the governing equations in a control volume approach to problem solving, we must rephrase the laws in an appropriate manner

Reynolds Transport Theorem

Provides an analytical tool to shift between CV and SYS

Simple Reynolds Transport Theorem

Control volume equation

Simple Reynolds Transport Theorem

Control volume equation

$$\left(\frac{dm}{dt}\right)_{CV} = \dot{m}_{in} - \dot{m}_{out}$$

$$\left(\frac{dm}{dt}\right)_{CV} - \dot{m}_{in} + \dot{m}_{out} = 0 \quad = \left(\frac{dm}{dt}\right)_{sys}$$

$m \rightarrow B$ more general

Simple Reynolds Transport Theorem

B	$b = B/m$
m	1
$m\mathbf{V}$	$\mathbf{V}$
$\frac{1}{2}mV^2$	$\frac{1}{2}V^2$

More general $m \longrightarrow \text{any properties}$

Extensive property: B　　B = m b　　广延量

Intensive property: b　　强度量

$$B_{\text{sys}} = \lim_{\delta \forall \to 0} \sum_i b_i(\rho_i\, \delta \forall_i) = \int_{\text{sys}} \rho b\, d\forall$$

Extensive property: B

Intensive property: b

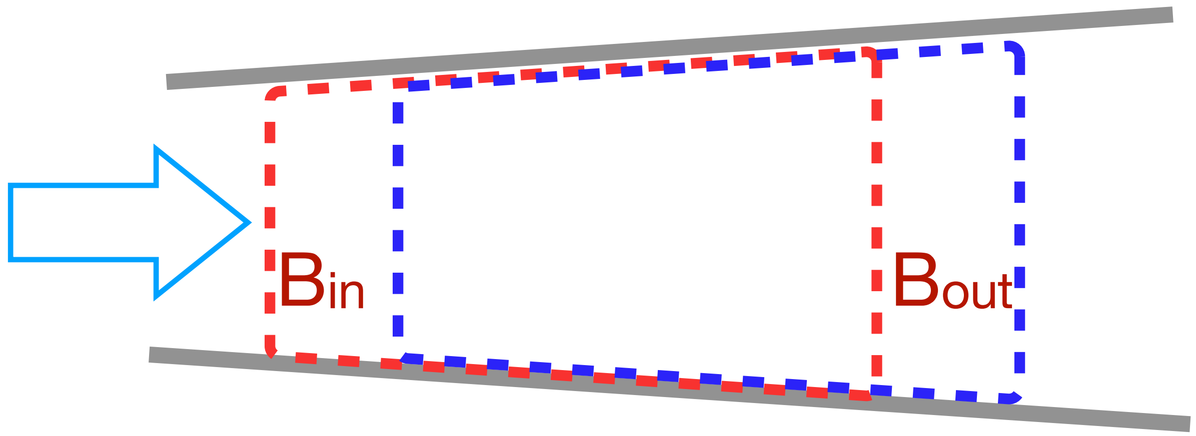
B = m b

the time rate of change of an extensive property

$$\frac{dB_{\text{sys}}}{dt} = \frac{d\left(\int_{\text{sys}} \rho b\, d\forall\right)}{dt}$$

$$\frac{dB_{\text{cv}}}{dt} = \frac{d\left(\int_{\text{cv}} \rho b\, d\forall\right)}{dt}$$

Simple Reynolds Transport Theorem

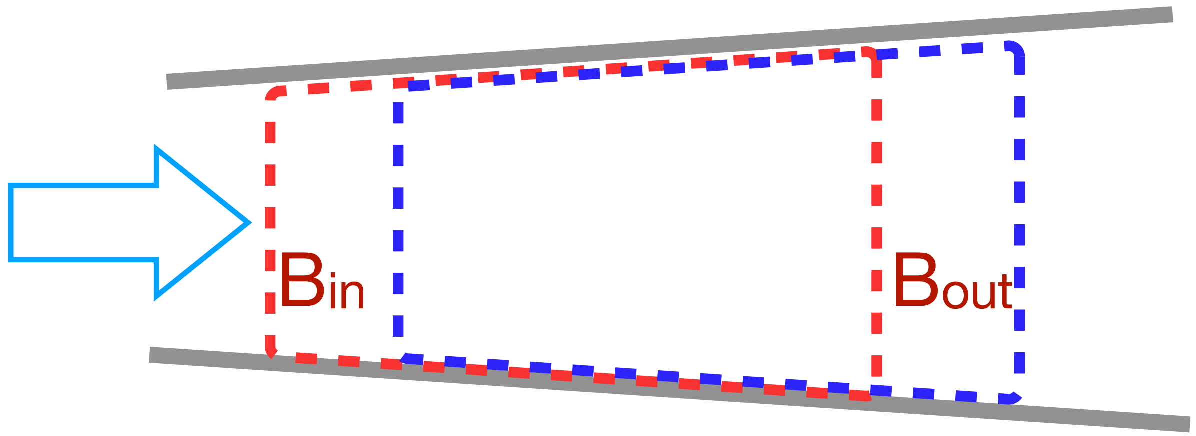


Control volume

System at time t

System at time t+δt

Simple Reynolds Transport Theorem

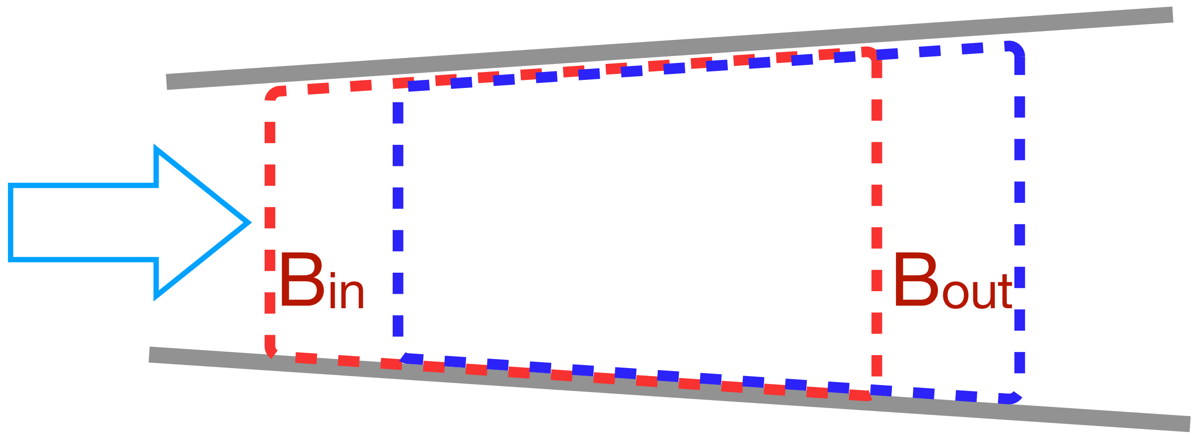


Control volume

System at time t

System at time t+δt

Simple Reynolds Transport Theorem



Control volume

System at time t

System at time t+δt

At time t $$B_{sys} = B_{CV}$$

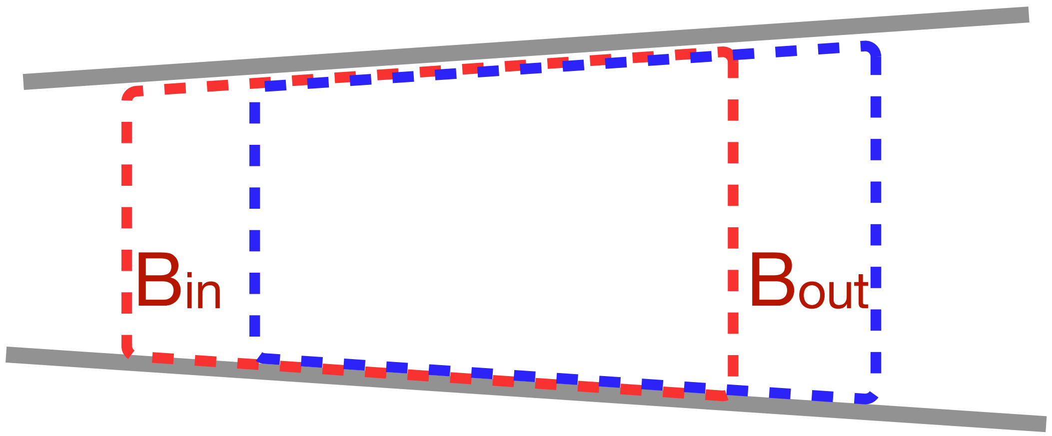
At time t + δt $$B_{sys} = B_{CV} - B_{in} + B_{out}$$

The rate of change $$\frac{\delta B_{sys}}{\delta t} = \frac{\delta B_{CV}}{\delta t} - \frac{\delta B_{in}}{\delta t} + \frac{\delta B_{out}}{\delta t}$$

The system is flowing:
Lagrangian field

$$\left(\frac{DB}{Dt}\right)_{sys} = \left(\frac{\partial B}{\partial t}\right)_{CV} + \dot{B}_{out} - \dot{B}_{in}$$

Simple Reynolds Transport Theorem



Control volume

System at time t

System at time t+δt

$$\left(\frac{DB}{Dt}\right)_{sys} = \left(\frac{\partial B}{\partial t}\right)_{CV} + \dot{B}_{out} - \dot{B}_{in}$$

$$\dot{B}_{in} = b_{in}\dot{m}_{in} = b_{in}\rho_{in}A_{in}V_{in}$$

$$\dot{B}_{out} = b_{out}\dot{m}_{out} = b_{out}\rho_{out}A_{out}V_{out}$$

This is the simple Reynolds Transport Theorem, which tracks down the variation of B in the **system** by tracking what happens in the **C.V.** and what **flow in and out.**

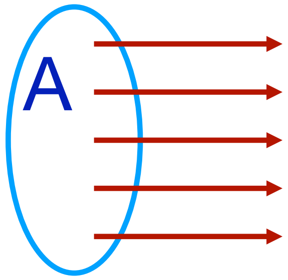
$$\left(\frac{DB}{Dt}\right)_{sys} = \left(\frac{\partial B}{\partial t}\right)_{CV} + (b\rho VA)_{out} - (b\rho VA)_{in}$$

simple Reynolds Transport Theorem

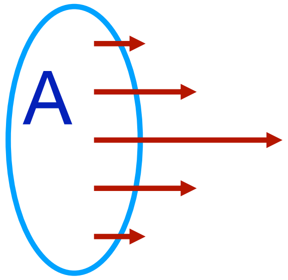
Simple Reynolds Transport Theorem

Reynolds Transport Theorem

A more comprehensive version of RTT

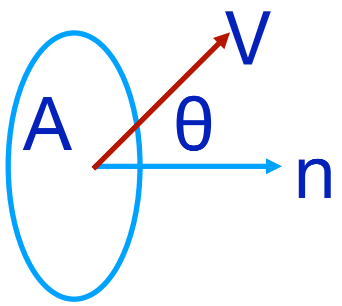


Uniform flow



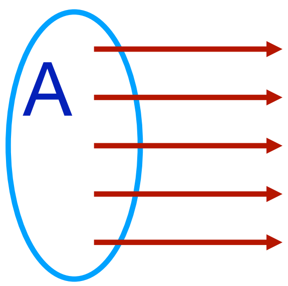
V ρ - not uniform

Velocity is not perpendicular to A



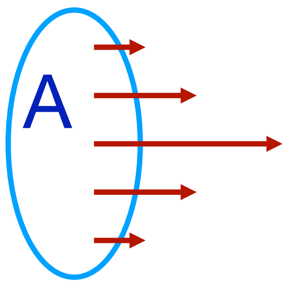
Reynolds Transport Theorem

A more comprehensive version of RTT



Uniform flow

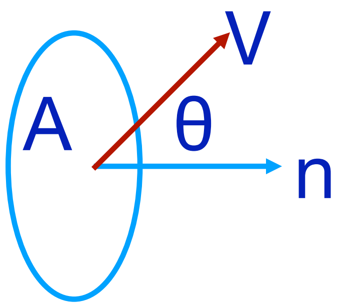
$$\dot{m} = \rho V A$$



V ρ - not uniform

$$\dot{m} = \int_A \rho V dA$$

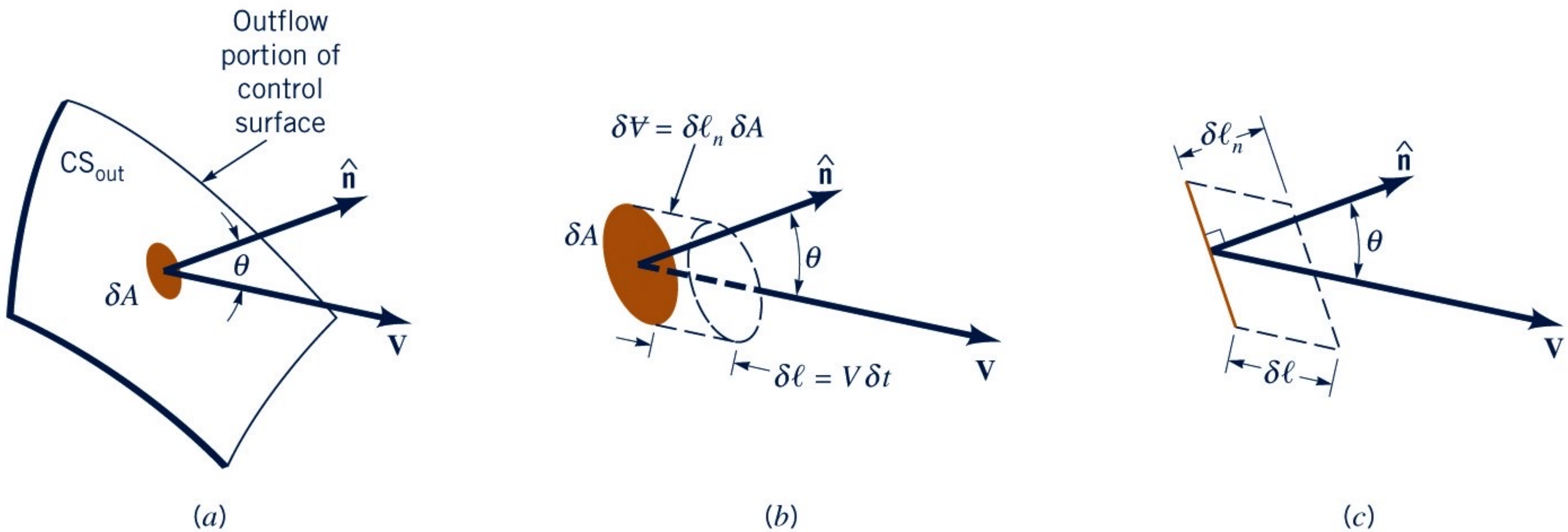
Velocity is not perpendicular to A



$$\dot{m} = \int_A \rho V \cos\theta dA$$

Reynolds Transport Theorem

Outflow across a typical portion of the control surface

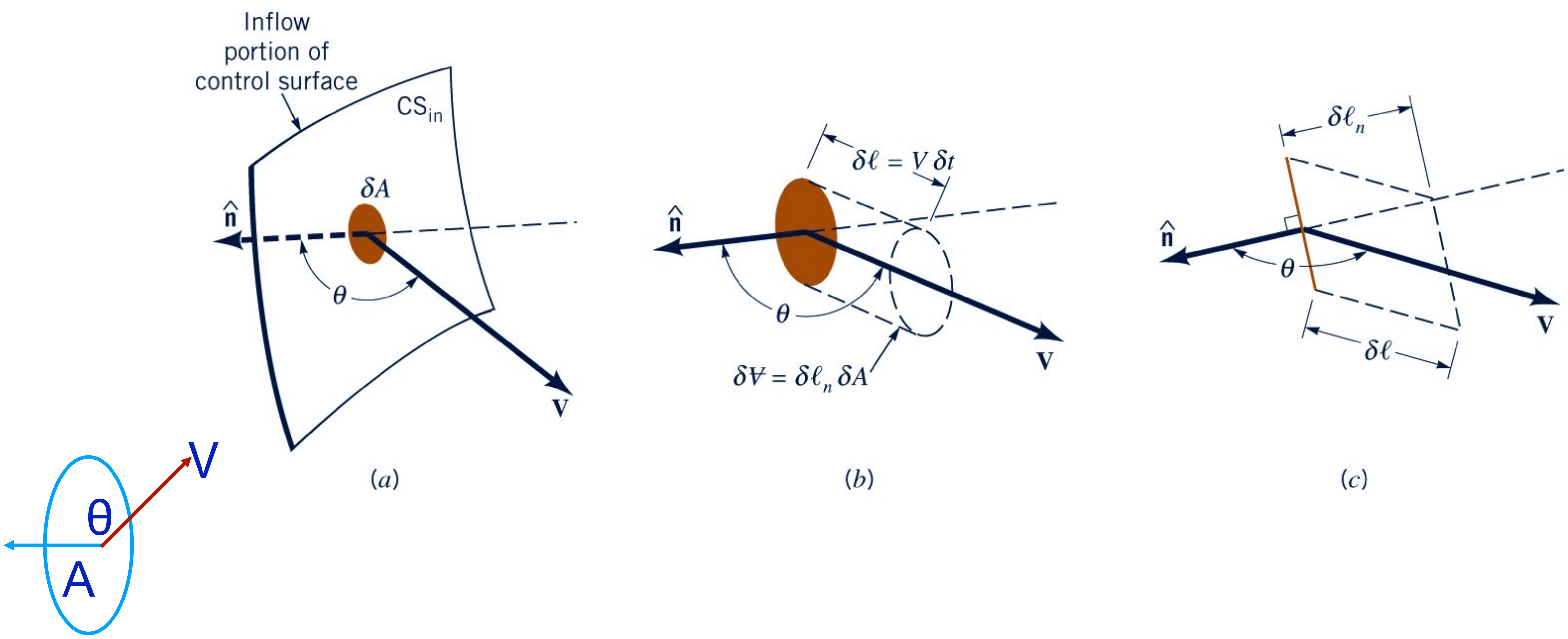


$$\dot{B}_{out} = \int_A b\rho \underline{V \cos\theta} dA \qquad = \int_A b\rho \vec{V} \cdot \hat{n} dA$$

Express in elegant way

Reynolds Transport Theorem

Inflow across a typical portion of the control surface



$$\dot{B}_{in} = -\int_A b\rho\vec{V}\hat{n}dA \qquad -\dot{B}_{in} = \int_A b\rho\vec{V}\hat{n}dA$$

Reynolds Transport Theorem

$$\dot{B}_{\text{out}} - \dot{B}_{\text{in}} = \int_{\text{cs}_{\text{out}}} \rho b \mathbf{V} \cdot \hat{\mathbf{n}}\, dA - \left(- \int_{\text{cs}_{\text{in}}} \rho b \mathbf{V} \cdot \hat{\mathbf{n}}\, dA \right)$$

$$= \int_{\text{cs}} \rho b \mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

Reynolds Transport Theorem

$$\dot{B}_{\text{out}} - \dot{B}_{\text{in}} = \int_{\text{cs}_{\text{out}}} \rho b \mathbf{V} \cdot \hat{\mathbf{n}}\, dA - \left(- \int_{\text{cs}_{\text{in}}} \rho b \mathbf{V} \cdot \hat{\mathbf{n}}\, dA \right)$$

$$= \int_{\text{cs}} \rho b \mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

$$\boxed{\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t} \int_{\text{cv}} \rho b \, d\forall + \int_{\text{cs}} \rho b \, \mathbf{V} \cdot \hat{\mathbf{n}} \, dA}$$

Both the material derivative and the Reynolds transport theorem equations represent ways to transfer from the Lagrangian viewpoint to the Eulerian viewpoint

Moving control volume

Relative velocity = absolute velocity of C.S. - the control volume velocity

$$\mathbf{W} = \mathbf{V} - \mathbf{V}_{\mathbf{CV}}$$

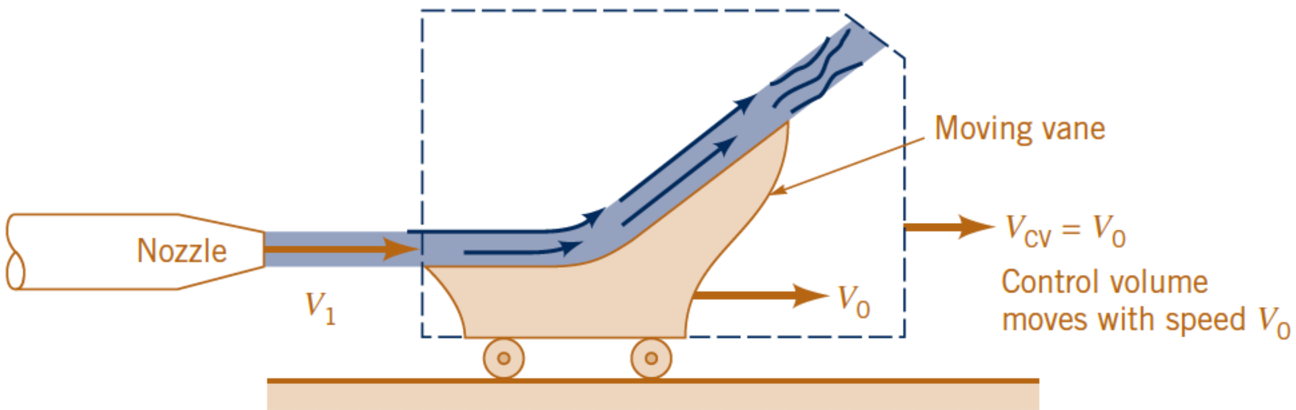
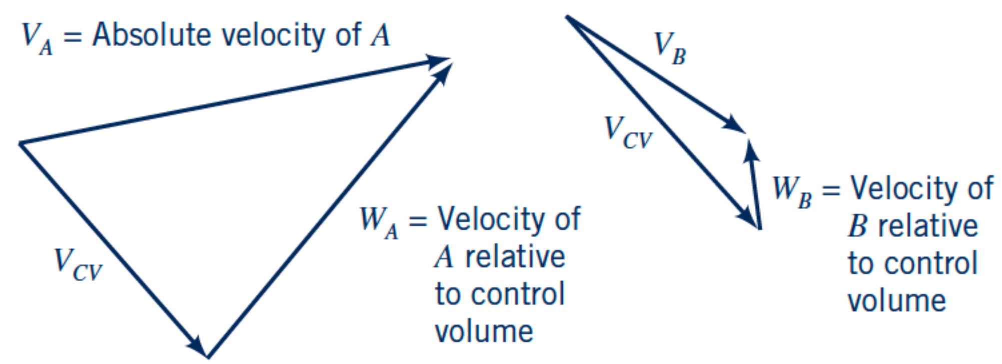


Figure 4.20 **Example of a moving control volume.**

simply replacing the absolute velocity, **V**, in the RTT equation by the relative velocity, **W**

$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t} \int_{\text{cv}} \rho b \, d\forall + \int_{\text{cs}} \rho b \, \mathbf{W} \cdot \hat{\mathbf{n}} \, dA$$



Control volume approach

Typical control volumes: (a) fixed control volume, (b) fixed or moving control volume, (c) deforming control volume

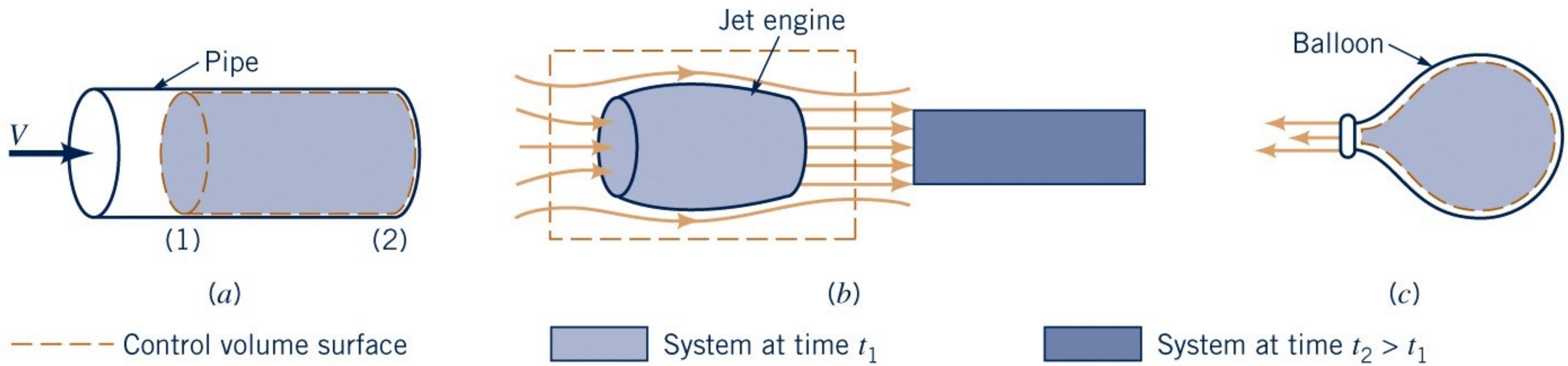


Figure 4.10

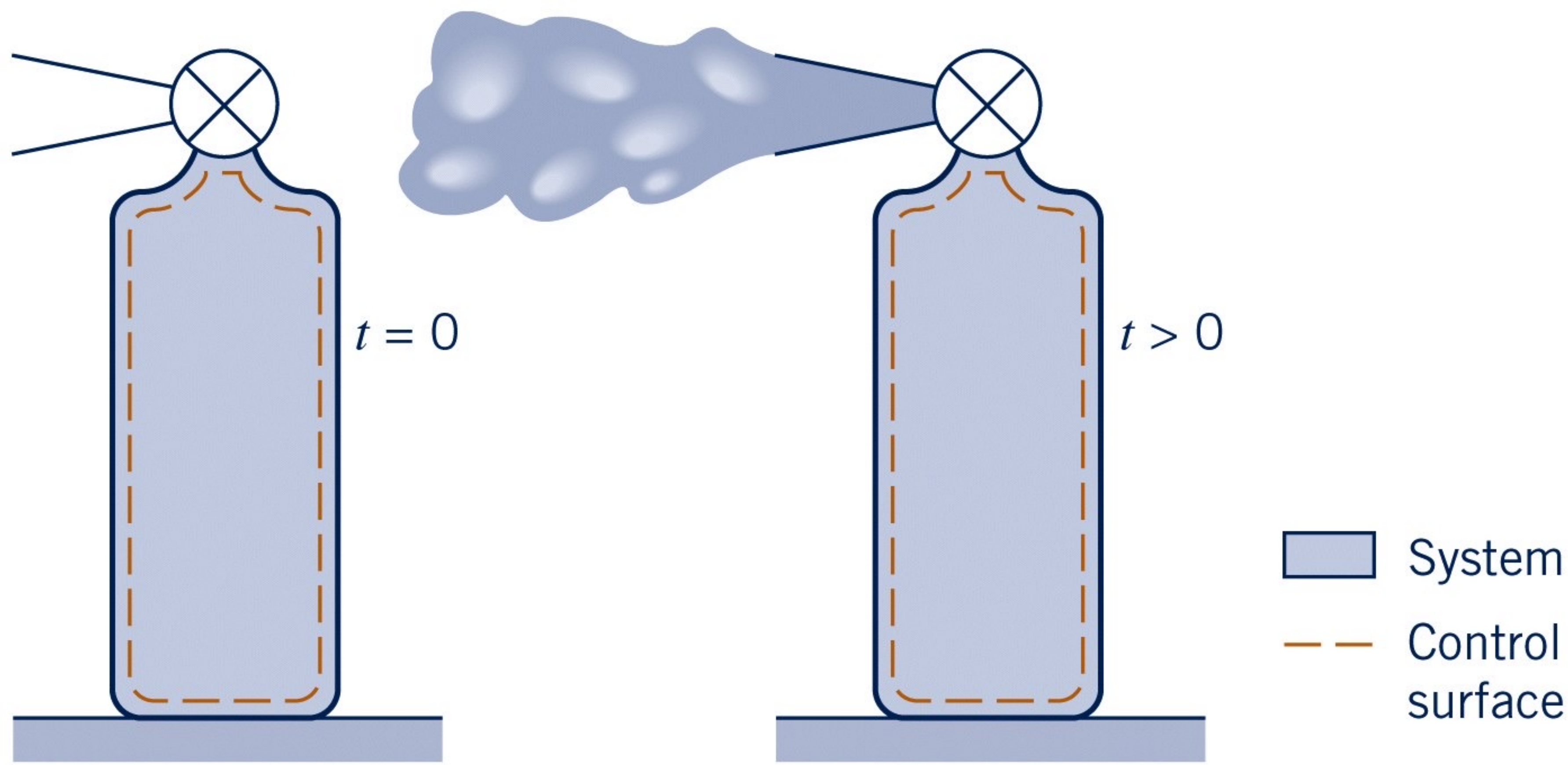
The governing laws of fluid motion are stated in terms of fluid systems, not cor

To use the governing equations in a control volume approach to problem solvi

Simple Reynolds Transport Theorem

(*a*)

Figure E4.7a



(*b*) (*c*)

Figure E4.7bc

RTT

Whiteboard

Fluid Mechanics – study guide

Book: 'Fundamentals of fluid mechanics', by Munson *et al.*
Chapter: 4
Instructor: Chao Sun

Chapter 4 is dedicated to fluid kinematics, i.e. the description of fluid motion without considering the forces/pressures necessary to drive the flow. One distinguishes Lagrangian and Eulerian flow descriptions. The Lagrangian method follows fluid particles as they 'go with the flow'. The Eulerian method considers the *velocity field* in a fixed point is space, i.e. $\mathbf{V} = u(x,y,z,t)\hat{\mathbf{i}} + v(x,y,z,t)\hat{\mathbf{j}} + w(x,y,z,t)\hat{\mathbf{k}}$. Most fluid dynamics considerations involve the Eulerian method.

When we apply Newton's third law we need to know the acceleration of a fluid particle (Lagrangian method) from the velocity field (Eulerian method). This is done by applying the *material derivative* $\mathbf{a} = D\mathbf{V}/Dt$, as explained in section 4.2. It is very important to understand the physical origin of the time and space derivatives in this term.

In many cases it is convenient to describe the motion of a finite volume of fluid. This can be done by a *Control Volume* (fixed volume in space, Eulerian) or by a *System* (fixed collection of matter, Lagrangian). Again, one can relate both situations by a material derivative, now using the *Reynolds Transport Theorem.*

After studying this chapter you will be able to:

- explain the difference between Eulerian and Lagrangian flow descriptions
- follow the derivation of the material derivative, eq. (4.5), and interpret the physical meaning of time and space derivatives
- apply the material derivative to derive fluid particle acceleration and temperature variations from a velocity field.
- analyze simple flow situations by using Control Volumes and the Reynolds Transport Theorem

Most important Examples: 4.1,4.2, 4.4, 4.7,4.8

Backup Whiteboard

Backup Whiteboard

Backup Whiteboard