Discrete Response Model
Lecture 2

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The Notions of Deviance

Deviance, Residual Deviance, Null Deviance

- Deviance refers to the amount that a particular model deviates from another model as measured by **$-2\log(\Lambda)$**.

- Example: The **$-2\log(\Lambda) = 5.246$** value used for testing the change variable in the last example (given distance is in the model) is a measure of **how much the estimated probabilities of success for the null hypothesis model deviate from the estimated probabilities of success for the alternative hypothesis model**.

- Residual deviance denotes how much the null hypothesis model deviates from using the observed proportion of successes for each "observation" ($y_i = 0$ or 1 for Bernoulli response data or w_j/n_j for binomial response data) to estimate π (π_i or π_j).

Example

Deviance statistics are often calculated as an intermediary step for performing a LRT to compare two models. For example, consider testing:

$$H_0: \text{logit}(\pi^{(0)}) = \beta_0^{(0)} + \beta_1^{(0)} x_1$$

$$H_a: \text{logit}(\pi^{(a)}) = \beta_0^{(a)} + \beta_1^{(a)} x_1 + \beta_2^{(a)} x_2 + \beta_3^{(a)} x_3$$

The residual deviance for the model $\text{logit}(\pi^{(0)}) = \beta_0 + \beta_1 x_1$ tests

$$H_0: \text{logit}(\pi^{(0)}) = \beta_0^{(0)} + \beta_1^{(0)} x_1$$

H_a: Saturated model

with

$$-2\log(\Lambda) = -2\left[\sum_{i=1}^{n} y_i \log\left(\frac{\hat{\pi}_i^{(0)}}{y_i}\right) + (1 - y_i)\log\left(\frac{1-\hat{\pi}_i^{(0)}}{1-y_i}\right)\right] \quad (2)$$

Written in terms of binomial response data, we would have:

$$-2\log(\Lambda) = -2\left[\sum_{j=1}^{J} w_j \log\left(\frac{\hat{\pi}_j^{(0)}}{w_j / n_j}\right) + (n_j - w_j)\log\left(\frac{1-\hat{\pi}_j^{(0)}}{1 - w_j / n_j}\right)\right]$$

The residual deviance for the model

$$\text{logit}(\pi^{(a)}) = \beta_0^{(a)} + \beta_1^{(a)}x_1 + \beta_2^{(a)}x_2 + \beta_3^{(a)}x_3$$

tests

$$H_0: \text{logit}(\pi^{(a)}) = \beta_0^{(a)} + \beta_1^{(a)}x_1 + \beta_2^{(a)}x_2 + \beta_3^{(a)}x_3$$

H_a: Saturated model

with

$$-2\log(\Lambda) = -2\left[\sum_{i=1}^{n} y_i \log\left(\frac{\hat{\pi}_i^{(a)}}{y_i}\right) + (1 - y_i)\log\left(\frac{1-\hat{\pi}_i^{(a)}}{1 - y_i}\right)\right] \quad (3)$$

By finding (2) – (3) we obtain Equation (1); thus, we obtain the desired $-2\log(\Lambda)$ by subtracting the residual deviances of the models in our original hypotheses. The degrees of freedom for the test can be found from subtracting the corresponding degrees of freedom for (3) from (2).

Example in R

Below is the R code demonstrating the above discussion with respect to

$$H_0: \text{logit}(\pi) = \beta_0 + \beta_1 \text{distance}$$

$$H_a: \text{logit}(\pi) = \beta_0 + \beta_1 \text{distance} + \beta_2 \text{change}$$

```
mod.fit.Ho<-glm(formula = good ~ distance, family =
    binomial(link = logit), data = df)

dframe<-mod.fit.Ho$df.residual-mod.fit2$df.residual

stat<-mod.fit.Ho$deviance-mod.fit2$deviance
pvalue<-1-pchisq(q = stat, df = dframe)
data.frame(Ho.resid.dev = mod.fit.Ho$deviance,
    Ha.resid.dev = mod.fit2$deviance, df = dframe, stat =
    round(stat,4), pvalue = round(pvalue,4))
```

	Ho.resid.dev	Ha.resid.dev	df	stat	pvalue
1	775.745	770.4995	1	5.2455	0.022

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