Derivation of $E[X]$, $Var[X]$ for Binomial Distribution.

- Definition: $E[X] = \sum_{x} x \, p_X(x)$.

- $$p_X(x) = \binom{n}{k} p^k (1-p)^{n-k}$$ for binomial distribution.

  as number of successes p in n independent trials.

- $$E[X] = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$$

- Let $U_1, \dots, U_n$ be independent Bernoulli's r.v.

  $E(U_i) = p$ and $Var(U_i) = p(1-p)$.

  $X = U_1 + \dots + U_n$.

- $$E[X] = E[U_1 + \dots + U_n]$$

  property of expectation. ~~since independent~~.

  $E[C_1X_1 + C_2X_2] = C_1E[X_1] + C_2E[X_2]$

  $$= E[U_1] + \dots + E[U_n]$$

  $$= p + \dots + p = \underline{np}$$

- $$Var[X] = Var[U_1 + \dots + U_n]$$

  Since independent.

  $$= Var[U_1] + \dots + Var[U_n]$$

  $$= p(1-p) + \dots + p(1-p)$$

  $$= \underline{np(1-p)}$$