Discrete Response Model
Lecture 1

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Hypothesis Tests for π

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Hypothesis:

$H_0: \pi = \pi_0$ vs. $H_a: \pi \neq \pi_0$

Test statistic: $$Z_0 = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

Reject H_0 if $|Z_0| > Z_{1-\alpha/2}$

This type of test is a type of "score test."

- Another way to perform a hypothesis test of $H_0: \pi = \pi_0$ vs. $H_a: \pi \neq \pi_0$ is a likelihood ratio test (LRT), which is very frequently used in the analysis of categorical data (beyond testing for π).

Hypothesis Tests for π (cont.)

The LRT statistic, Λ, is the ratio of two likelihood functions. The numerator is the likelihood function maximized over the parameter space restricted under the null hypothesis. The denominator is the likelihood function maximized over the unrestricted parameter space.

$$\Lambda = \frac{\text{Max. lik. when parameters satisfy } H_0}{\text{Max. lik. when parameters satisfy } H_0 \text{ or } H_a}$$

Wilks (1935, 1938) shows that –2log(Λ) can be approximated by a χ^2_u for a large sample and under H_o where u is the difference in dimension between the alternative and null hypothesis parameter spaces.

Example: Field Goal Kicking (cont.)

Suppose the hypothesis test H_0:$\pi = 0.5$ vs. H_a:$\pi \neq 0.5$ is of interest. Remember that w = 4 and n = 10.

The numerator of Λ is the maximum possible value of the likelihood function under the null hypothesis. Because $\pi = 0.5$ is the null hypothesis, the maximum can be found by just substituting $\pi = 0.5$ in the likelihood function.

$$L(\pi = 0.5 \mid y_1,\ldots,y_n) = \underbrace{0.5^w}\underbrace{(1-0.5)^{n-w}}$$

Then

$$L(\pi = 0.5 \mid y_1,\ldots,y_n) = 0.5^4(0.5)^{10-4} = 0.0009766$$

$$\Lambda = \frac{\text{Max. lik. when parameters satisfy } H_0}{\text{Max. lik. when parameters satisfy } H_0 \text{ or } H_a}$$

$$= \frac{0.0009766}{0.001194} = 0.8179$$

Then $-2\log(\Lambda) = -2\log(0.8179) = 0.4020$ is the test statistic value. The critical value is…

Example: Field Goal Kicking (cont.)

Then $-2\log(\Lambda) = -2\log(0.8179) = 0.4020$ is the test statistic value.

The critical value is $\chi^2_{1,\,0.95} = 3.84$ using $\alpha = 0.05$

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> qchisq(p = 0.95, df = 1)
[1] 3.841459
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Therefore, there is not sufficient evidence to reject the hypothesis that $\pi = 0.5$.

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