1. $W \sim N(\mu, \sigma^2)$

$= W \sim N(10, 4^2)$

$U \sim N(0, 1)$ - Standard normal distribution.

and independent to W

$V = 0.5W + U$

Variance - Covariance Matrix for W and V

$$\begin{array}{ccc} & W & V \\ W & Var(W) & Cov(W, V) \\ V & Cov(V, W) & Var(V) \end{array}$$

- $Var(W) = 4^2 = 16$.
- $Cov(V, W) = E[VW] - E[V]E[W]$

$= E[(\frac{1}{2}W + U)W] - E[\frac{1}{2}W + U]E[W]$

$= \frac{1}{2}E[W^2] + \cancel{E[UW]} - \frac{1}{2}(E[W])^2 - \cancel{E[U]E[W]}$

$= \frac{1}{2}E[W^2] - \frac{1}{2}(E[W])^2$

$= \frac{1}{2}(\underbrace{E(W^2) - (E[W])^2}_{Var(W)}) = \frac{1}{2}Var(W) = 8$

- $Cov(W, V) = Cov(V, W) = 8$.
- $Var(V) = E(V^2) - (E[V])^2$

- $E[V] = E[\frac{1}{2}W + U] = \frac{1}{2}\underbrace{E[W]}_{10} + \underbrace{E[U]}_{0} = 5$
- $E[V^2] = E[\frac{1}{4}W^2 + WU + U^2]$

$= \frac{1}{4}E[W^2] + E[WU] + E[U^2]$

1 continue.

$$E[V^2] = \frac{1}{4}\left(\underbrace{Var(W) + (E(W))^2}_{E[W^2]}\right) + \underbrace{E[W]\cdot E[U]}_{E[WU] \because \text{both independent}}$$

$$+ \underbrace{var(U) + (E[U])^2}_{E[U^2]}$$

$$= \frac{1}{4}(4^2 + 10^2) + 10\cancel{0}$$

$$+ 1 + \cancel{0^2}$$

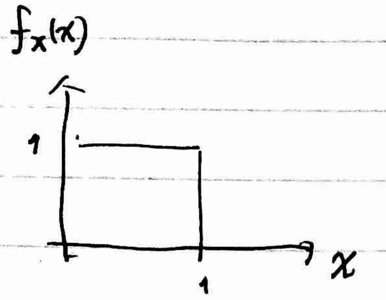
$$= 29 + 1 = 30$$

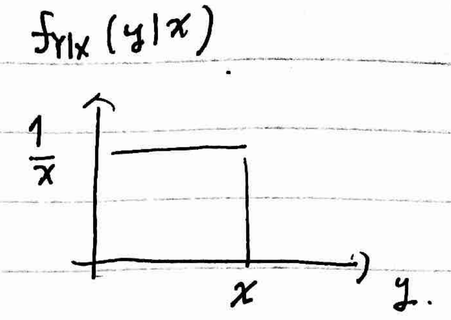
$\cdot$ $Var(V) = E[V^2] - (E[V])^2 = 30 - 5^2 = 5$

Variance - Covariance matrix for W and V

	W	V
W	$Var(W) = 16$	$cov(W.V) = 8$
V	~~Var~~($cov(V.W) = 8$	$Var(V) = 30$

2. (a)





$$E[Y|X] = \int_{-\infty}^{\infty} y\, f_{Y|X}(Y|X)\, dy$$

$$= \int_0^x y \cdot \frac{1}{x}\, dy = \frac{y^2}{2x}\Big|_0^x = \frac{x}{2}$$

(b) $$E(Y) = E(E(Y|X))$$

$$= E\left(\frac{X}{2}\right)$$

$$= \int_0^1 \frac{x}{2} \cdot 1 \cdot dx = \frac{1}{4}$$

(c) $$E[XY] = \frac{E[XY|X]}{f_X(x)} = \frac{\cancel{x}\, E[\cancel{X}Y|X]}{\cancel{f_X(x)}} = E[Y|X]$$

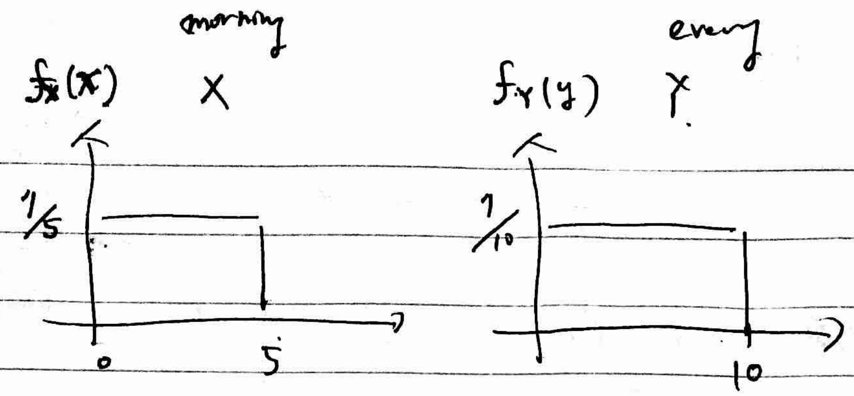
$$= \frac{x}{2}$$

(d) $$Cov(X, Y) = E[XY] - E[X]E[Y]$$

$$= \frac{x}{2} - \frac{1}{4} \cdot \frac{1}{2}$$

$$= \frac{x}{2} - \frac{1}{8}$$

3 (a)



X, Y are independent.

$$E[X] = \underline{\frac{5}{2}}\ //, \quad E[Y] = \underline{5}\ //$$

$$E[X+Y] = E[X] + E[Y] = \frac{5}{2} + 5 = \frac{15}{2}$$

$$\underline{Z = X+Y}.$$

$$E[\underbrace{Z+Z+Z+Z+Z}_{5\text{ days}}] = 5E[Z] = 5E[X+Y] = 5\cdot\frac{15}{2}$$

$$= \underline{\frac{75}{2}}\ //$$

3 (b)

$$var[5Z] = 25\, var(Z)$$

$$var(Z) = var(X) + var(Y)$$

$$var(X) = E[X^2] - (E[X])^2 = \frac{25}{3} - \left(\frac{5}{2}\right)^2 = \frac{25}{12}$$

$$var(Y) = E[Y^2] - (E[Y])^2 = \frac{100}{3} - 5^2 = \frac{25}{3}$$

$$var(Z) = \underline{\frac{125}{12}}\ //$$

$$var(5Z) = 25\cdot\frac{125}{12} = \underline{260.4167}\ //$$

3(c) $E[5(\underset{\text{evening}}{Y} - \underset{\text{morning}}{X})]$

$= 5E(Y) - 5E(X)$

$= 5 \cdot 5 - 5 \cdot \frac{5}{2} = \frac{25}{2}$

3 d) $var(5(Y - X))$

$= 5^2 var(Y - X)$

$= 5^2 var(Y) + 5^2 var(X)$

$= 25 \cdot \frac{25}{3} + 25 \cdot \frac{25}{12} = 260,4167$

$=$

4. $Y = aX + b$, $a \neq 0$.

$$\text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y}$$

square both sides $\quad (\text{corr}(X, Y))^2 = \frac{(\text{cov}(X, Y))^2}{\sigma_X^2 \sigma_Y^2}$

$$= \frac{(\text{cov}(X, Y))^2}{Var(X)\, Var(Y)}$$

First, focus on the numerator

$$(\text{cov}(X, Y))^2 = (E[XY] - E[X]E[Y])^2.$$

$$\begin{cases} E[XY] = E[aX^2 + bX] = a\,E[X^2] + b\,E[X] \\ E[X]E[Y] = E[X]\,E[aX+b] = a(E[X])^2 + b\,E[X] \end{cases}$$

Plug them in $\quad (E[XY] - E[X]E[Y])^2$

$$= a^2 \left(E[X^2] - (E[X])^2\right)^2 = a^2 (Var[X])^2$$

Denominator $\quad Var(X)\, Var(Y) = Var(X)\, Var(aX+b) = a^2 (Var(X))^2$

Therefore $\quad (\text{Corr}(X, Y))^2 = \frac{a^2 (Var(X))^2}{a^2 (Var(X))^2} = 1$

Therefore when $Y = aX + b$, $a \neq 0$, $\underline{\text{Corr}(X, Y) = 1 \text{ or } -1}$ //