Discrete Response Model
Lecture 2

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Odds Ratios

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Recall that a logistic regression model can be written as

$$log(\frac{\pi}{1-\pi}) = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p$$

where the left-side of the model is the log odds of a success.

Using a similar interpretation as the classical linear regression models, we can use β_r to interpret the effect that x_r has on the log odds of a success.

We can then form odds ratios by looking at these odds of success at different values of x_r.

For ease of presentation, consider the logistic regression with only one explanatory variable x:

$$\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 x$$

Or, rewrite this model as

$$Odds_x = e^{\beta_0 + \beta_1 x}$$

If we increase x by c-units, the odds of a success becomes

$$Odds_{x+c} = e^{\beta_0 + \beta_1 (x+c)}$$

Interpretation

To interpret the effect of increasing x by c-units, we can form an odds ratio:

$$OR = \frac{Odds_{x+c}}{Odds_x} = \frac{e^{\beta_0 + \beta_1 (x+c)}}{e^{\beta_0 + \beta_1 x}} = e^{c\beta_1}$$

Observe the beauty of the exponential functions.
➔ The x drops out!

- Regardless of the value of x, the estimated odds of a success change by the same amount for every c-unit increase in x.
- This is one of the main reasons why logistic regression is so popular for modeling binary response data.

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