The Meat

a) No. The distribution can take any form to form an average of 2 pounds / per month. because the problem doesn't say anything about the distribution.
The distribution might be skewed and might not be normal.

b) Yes. According to Central Limit Theorem, regardless of the population distribution, sample mean of a random sample of n observation from the population follows approximately normal distribution. for sufficiently large n.

In this case $n = 100$ which sufficiently large. (larger than 30)

c) Since the population standard deviation is unknown, we use t-distribution to construct the confidence interval

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$= 2.45 \pm t_{\alpha/2} \frac{2}{\sqrt{100}} \quad (1-\alpha = 0.95) \quad df = n-1 = 100-1 = 99$$

$[2.053, \ 2.847]$

$t_{\alpha/2} = qt(1-0.05/2, 100-1) = 1.984217$

GRE Scores.

Calculate critical value for ~~sample~~ sample size $n = 10$, $n = 200$

Given, we don't know the standard deviation of the population.
We use t-distribution to construct C.I.

i) $n = 10$, ~~$t_{\alpha/2}$~~ 95% CI $\rightarrow$ $1-\alpha = 0.95$ $\alpha = 0.05$.

Critical Value $t_{\alpha/2} = qt(1 - \alpha/2, df)$

$= qt(1 - 0.025, 10 - 1)$

$= 2.26$.

$$95\% \text{ CI} = \left[\bar{X} - 2.26 \cdot \frac{S}{\sqrt{10}}, \bar{X} + 2.26 \cdot \frac{S}{\sqrt{10}} \right]$$

Note that since $n < 30$, in other words, n is small,
it might not be advisable to ~~use~~ assume normal distribution ~~for~~
However, t-distribution is known to be fairly robust for even
small samples in the real world. I would keep the C.I..

ii) $n = 200$, critical value $t_{\alpha/2} = qt(1 - \alpha/2, df)$

$= qt(1 - 0.025, 200 - 1)$

$= 1.97$

$$95\% \text{ CI} = \left[\bar{X} - 1.97 \cdot \frac{S}{\sqrt{200}}, \bar{X} + 1.97 \cdot \frac{S}{\sqrt{200}} \right]$$

Given the larger sample size $n = 200$, CI is much narrower
than the one with $n = 10$.

Maximum Likelihood Estimation for an Exponential Distribution.

a)

$$L(\lambda) = P(X_1 = x_1 \cap X_2 = x_2 \cap \cdots \cap X_n = x_n \mid \lambda)$$

$$= P(X_1 = x_1 \mid \lambda) \cdot P(X_2 = x_2 \mid \lambda) \cdots P(X_n = x_n \mid \lambda) \quad (\because X_i \text{ independent})$$

$$= \lambda e^{-\lambda x_1} \cdot \lambda e^{-\lambda x_2} \cdots \lambda e^{-\lambda x_n}$$

$$= \prod_{i=1}^{n} \lambda e^{-\lambda x_i}$$

b) Take the log of $L(\lambda)$

$$\sum_{i=1}^{n} \left(\log \lambda + \log e^{-\lambda x_i} \right)$$

$$= \sum_{i=1}^{n} \left(\log \lambda \cancel{+} - \lambda x_i \right)$$

$$= n \log \lambda - \sum_{i=1}^{n} \lambda x_i$$

c) Take the derivative of the log of ~~the~~ likelihood with respect to λ and set equal to zero.

$$\frac{n}{\lambda} - \sum_{i=1}^{n} x_i = 0$$

$$\hat{\lambda}_{MLE} = \frac{n}{\sum_{i=1}^{n} x_i} = 1 \,/\, \text{The mean time between arrivals.}$$