Discrete Response Model Lecture 1

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Wald Confidence Interval

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Because $\hat{\theta} \sim N\left(\theta, Var(\hat{\theta})\right)$, we can rewrite this as a standardized statistic:

$$\frac{\hat{\theta} - \theta}{\sqrt{Var(\hat{\theta})}} \sim N(0,1)$$

This logic is covered many times at w203. Please make sure you understand this logic before proceeding to the next slide.

Concept Check (1 minute):

This logic is covered in w203. Make sure you understand the logic used here before proceeding to the next slide.

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Wald Confidence Interval

Also, because we have a probability distribution here, we can quantify with a level of certainty that observed values of the statistic are within a particular range:

$$P\left(Z_{\alpha/2} < \boxed{\frac{\hat{\theta} - \theta}{\sqrt{\mathrm{Var}(\hat{\theta})}}} < Z_{1-\alpha/2} \right) \approx \underbrace{1 - \alpha}$$

where $Z_{1-\alpha/2}$ is the $1 - \alpha/2$ quantile from a standard normal. For example, if $\alpha = 0.05$, we have $\underbrace{Z_{0.975} = 1.96}$.

Wald Confidence Interval

```
> qnorm(p = 1-0.05/2, mean = 0, sd = 1)
[1] 1.959964
```

Note that I specially chose $Z_{\alpha/2}$ and $Z_{1-\alpha/2}$ for symmetry. Of course, $Z_{\alpha/2} = -Z_{1-\alpha/2}$.

If we rearrange items within the P(·), we obtain

$$P\left(\hat{\theta} - Z_{1-\alpha/2}\sqrt{\mathrm{Var}(\hat{\theta})} < \theta < \hat{\theta} + Z_{1-\alpha/2}\sqrt{\mathrm{Var}(\hat{\theta})}\right) \approx 1 - \alpha$$

Thus, if α is chosen to be small, we are fairly certain the expression within P(·) will hold true. When we substitute the observed values of $\hat{\theta}$ and $\mathrm{Var}(\hat{\theta})$ into the expression, we obtain the $(1 - \alpha)100\%$ "Wald" confidence interval for θ as

$$\hat{\theta} - Z_{1-\alpha/2}\sqrt{\mathrm{Var}(\hat{\theta})} < \theta < \hat{\theta} + Z_{1-\alpha/2}\sqrt{\mathrm{Var}(\hat{\theta})}$$

Notice this interval follows the typical form of a confidence interval for a parameter:

Estimator ± (distributional value)*(standard deviation of estimator)

Wald Confidence Interval

```
> qnorm(p = 1-0.05/2, mean = 0, sd = 1)
[1] 1.959964
```

Because $\hat{\pi}$ is a maximum likelihood estimator, we can use a Wald confidence interval for π:

$$\hat{\pi} \pm Z_{1-\alpha/2}\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}$$

Because $\hat{\pi}$ is close to 0 or 1, two problems may occur:

1) Calculated limits may be less than 0 or greater than 1, which is outside the boundaries for a probability.
2) When $\hat{\pi}$ = 0 or 1, $\hat{\pi}(1-\hat{\pi})/n$ = 0 for n > 0. This leads to the lower and upper limits to be exactly the same (0 for $\hat{\pi}$ = 0 or 1 for $\hat{\pi}$ = 1).

Example: Field Goal Kicking

Suppose $\sum_{i=1}^{n} y_i$ = w = 4 and n = 10. The 95% confidence interval is

$$\hat{\pi} \pm Z_{1-\alpha/2}\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}} = 0.4 \pm 1.96\sqrt{\frac{0.4(1-0.4)}{10}}$$

$$0.0964 < \pi < 0.7036$$

Below is the implementation in R:

```
w<-4
n<-10
alpha<-0.05
pi.hat<-w/n

var.wald<-pi.hat*(1-pi.hat)/n
lower<-pi.hat - qnorm(p = 1-alpha/2) * sqrt(var.wald)
upper<-pi.hat + qnorm(p = 1-alpha/2) * sqrt(var.wald)
round(data.frame(lower, upper), 4)

#Quicker
round(pi.hat + qnorm(p = c(alpha/2, 1-alpha/2)) * sqrt(var.wald), 4)
```

```
> round(pi.hat + qnorm(p = c(alpha/2, 1-alpha/2)) * sqrt(var.wald), 4)
[1] 0.0964 0.7036
```

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