HM4

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1. Best Game in hte Casino (a) Probability distribution: {0 head: 1/8, 1 head: 3/8, 2 heads: 3/8, 3 heads: 1/8 }

$$\begin{aligned} E[x] &= \sum aP(X=a) \\ 6 &= 0*1/8 + 2*3/8 + 4*3/8 + t*1/8 \\ t &= 30 \end{aligned}$$

b.

$$F(x) = P(X \le x)$$

$$x < \$2, F(x) = f(0) = 0$$

$$\$2 \le x < \$4, F(x) = f(0) + f(1) = 1/8$$

$$\$4 \le x < \$6, F(x) = f(0) + f(1) + f(2) = 1/8 + 3/8 = 1/2$$

$$\$6 \le x < \$30, F(x) = f(0) + f(1) + f(2) = 1/8 + 3/8 + 3/8 = 7/8$$

$$\$30 \le x, F(x) = 1$$

Thus,

$$F(x) = \begin{cases} 0 & \text{for} \, x < 2; \\ 1/8 & \text{for} \, 2 \le x < 4; \\ 1/2 & \text{for} \, 4 \le x < 6; \\ 7/8 & \text{for} \, 6 \le x < 30; \\ 1 & \text{for} \, x \ge 30. \end{cases}$$

2. Processing Pasta (a)

$$F(l) = \begin{cases} 0 & \text{for} \, l \le 0; \\ l^2/4 & \text{for} \, 0 < l \le 2; \\ 1 & \text{for} \, l > 2. \end{cases}$$

(b) Definiton of E[x] is

$$E[x] = \int_a^b x f(x) dx$$

$$\begin{aligned} E[L] &= \int_0^2 l f(l) dl \\ &= \int_0^2 \frac{l^2}{2} dl \\ &= \frac{l^3}{3} \Big|_0^2 \\ &= \frac{8}{3} \end{aligned}$$

3. The Warranty is Worth It Two random variable T (life span) and X (payout)

Probability density function for T

$$f(T) = \begin{cases} 0 & \text{for} t < 0; \\ 1 & \text{for} 0 \le t \le 1; \\ 0 & \text{for} t > 1. \end{cases}$$

Cumulative probability function for T

$$F(T) = \begin{cases} 0 & \text{for} t < 0; \\ t & \text{for} 0 \le t \le 1; \\ 1 & \text{for} t > 1. \end{cases}$$

a. Definiton of E[x] is

$$E[x] = \int_a^b xf(x)dx$$

$$\begin{aligned} E[X] &= E(g(T)) \\ &= \int_0^1 g(t)f(g(t)) \\ &= \int_0^1 g(t) \\ &= \int_0^1 100(1-t)^{\frac{1}{2}} \\ &= \frac{200}{3} \end{aligned}$$

(b)-i. For X = x, \$0 <= x <= \$100,

$$100(1-t)^{\frac{1}{2}} = x$$

$$t = 1 - \frac{x^2}{10000}$$

(b)-ii. when T <= t, the payment X <= x (b)-iii. Use cumulative probability desnsity function for T above. For 0 <= t <= 1,

$$F(T \le t) = t = 1 - \frac{x^2}{10000}$$

Cumulative probability function for X

$$\begin{aligned} Fx(X) &= 1 - Ft(g^{-}1(t)) \\ &= 1 - Ft(1 - \frac{x^2}{10000}) \\ &= \frac{x^2}{10000} \end{aligned}$$

$$Fx(X) = \begin{cases} 0 & \text{for} x < 0; \\ \frac{x^2}{10000} & \text{for} 0 \le x \le 100; \\ 1 & \text{for} x > 100. \end{cases}$$

(b)-iv.

$$Fx(X)' = fx(X) = \frac{x}{5000}$$

(b)-v.

$$E[x] = \int_0^{100} xf(x)dx = \int_0^{100} \frac{x^2}{5000}dx = \frac{200}{3}$$

4. The Baseline for Measuring Deviations (a) For Y = (X-t)^2 and random variable X and a real number t

$$\begin{aligned} E[Y] &= E((X-t)^2) \\ &= E(X^2) - 2tE(x) + t^2 \end{aligned}$$

b.

$$E[Y]' = 2t - 2E(X) = 0$$

When t = E(X), E[Y] is minimum

c.

$$E(Y) = E(X^2) - 2(E(X))^2$$