

Homework 4

Answer Key

1. Best Game in the Casino

You flip a fair coin 3 times, and get a different amount of money depending on how many heads you get. For 0 heads, you get \$0. For 1 head, you get \$2. For 2 heads, you get \$4. Your expected winnings from the game are \$6.

- (a) How much do you get paid if the coin comes up heads 3 times?

$$E(X) = 6$$

$$P(0) = 1/8$$

$$P(1) = 3/8$$

$$P(2) = 3/8$$

$$P(3) = 1/8$$

$$E(X) = (P(0) * 0) + (P(1) * 2) + (P(2) * 4) + (P(3) * x)$$

$$6 = (1/8 * 0) + (3/8 * 2) + (3/8 * 4) + (1/8 * x)$$

$$6 = \$0 + 6/8 + 12/8 + 1/8x$$

$$24/4 = 9/4 + 1/8x$$

$$15/4 = 1/8x$$

$$120/4 = x$$

$$x = \$30$$

- (b) Write down a complete expression for the cumulative probability function for your winnings from the game.

$$F(x) = \begin{cases} 1/8, & x < 2 \\ 1/2, & x < 4 \\ 7/8, & x < 30 \\ 1, & x \leq 30 \end{cases}$$

2. Processing Pasta

A certain manufacturing process creates pieces of pasta that vary by length. Suppose that the length of a particular piece, L , is a continuous random variable with the following probability density function.

$$f(l) = \begin{cases} 0, & l \leq 0 \\ l/2, & 0 < l \leq 2 \\ 0, & 2 < l \end{cases}$$

- (a) Write down a complete expression for the cumulative probability function of L .

$$F(l) = \begin{cases} 0, l \leq 0 \\ l/2, 0 < l < 2 \\ 1, l \leq 2 \end{cases}$$

(b) What is the expected length of the pasta, $E(L)$?

$$= 0 + 2 / 2 = 1$$

3 The Warranty is Worth It

Suppose the life span of a particular (shoddy) server is a continuous random variable, with a uniform probability distribution between 0 and 1 year. The server comes with a contract that guarantees you money if the server lasts less than 1 year. In particular, if the server lasts t years, the manufacturer will pay you $x = 100(1 - t)^{1/2}$. Let X be the random variable representing the payout from the contract.

(a) Given that the server lasts 6 months without failing, what is the conditional probability that it will last another 3 months?

```
#Divide probability of serving last 9 months by 6 months.
#SD = 1.5 months
prob_6 = 0.5 # Mean, no SD
prob_9 = 0.0227 # 2 SD's away from mean.
prob_9 / prob_6
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```
## [1] 0.0454
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(b) Write down an expression for the cumulative probability function of the payout from the contract. That is, what is $F(x)$, the probability that X is less than x ? (Hint: make sure that $F(0) = 0$, $F(\$100) = 1$).

$$\Phi(x) = e^{-\frac{1}{2}x^2} / \sqrt{2\pi}$$

$$n = 100(1 - \Phi(x))^{1/2}$$

(c) Compute the expected payout from the contract, $E(X)$.

$$E(X) = .5 = t$$

$$x = 100(1-t)^{1/2}$$

$$x = 100(1-.5)^{1/2}$$

$$x = 100(.5)^{1/2}$$

$$x = 100(.71)$$

$$x = \$71$$

The Baseline for Measuring Deviations

(a) Compute the value of t that minimizes $E(Y)$. (Hint: Your answer should take less than 5 characters to write)

(b) What is the value of $E(Y)$ for this choice of t ?