# Example application: automatically generated matrix inverse problem

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To compute the matrix inverse, we can perform "complete Gaussian elimination" on the augmented matrix $[A|I]$, where $I$ is an identity.

**Problem:** Using the augmented matrix approach, compute $A^{-1}$, where

$$A = \begin{bmatrix} 3 & -6 & 0 \\ 1 & 4 & 1 \\ 3 & -3 & 3 \end{bmatrix}.$$

**Solution:**

$$\begin{bmatrix} 3 & -6 & 0 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 & 1 & 0 \\ 3 & -3 & 3 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 := R_2 - (\frac{1}{3})R_1$ $R_3 := R_3 - (1)R_1$

$$\begin{bmatrix} 3 & -6 & 0 & 1 & 0 & 0 \\ 0 & 6 & 1 & -\frac{1}{3} & 1 & 0 \\ 0 & 3 & 3 & -1 & 0 & 1 \end{bmatrix}$$

$R_3 := R_3 - (\frac{1}{2})R_2$

$$\begin{bmatrix} 3 & -6 & 0 & 1 & 0 & 0 \\ 0 & 6 & 1 & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & \frac{5}{2} & -\frac{5}{6} & -\frac{1}{2} & 1 \end{bmatrix}$$

$R_1 := R_1 - (-1)R_2$

$$\begin{bmatrix} 3 & 0 & 1 & \frac{2}{3} & 1 & 0 \\ 0 & 6 & 1 & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & \frac{5}{2} & -\frac{5}{6} & -\frac{1}{2} & 1 \end{bmatrix}$$

$R_1 := R_1 - (\frac{2}{5})R_3$ $R_2 := R_2 - (\frac{2}{5})R_3$

$$\begin{bmatrix} 3 & 0 & 0 & 1 & \frac{6}{5} & -\frac{2}{5} \\ 0 & 6 & 0 & 0 & \frac{6}{5} & -\frac{2}{5} \\ 0 & 0 & \frac{5}{2} & -\frac{5}{6} & -\frac{1}{2} & 1 \end{bmatrix}$$

$R_1 := R_1/3$ $R_2 := R_2/6$ $R_3 := R_3/\frac{5}{2}$ gives the final answer:

$$A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{2}{5} & -\frac{2}{15} \\ 0 & \frac{1}{5} & -\frac{1}{15} \\ -\frac{1}{3} & -\frac{1}{5} & \frac{2}{5} \end{bmatrix}.$$