Distance Based Neural Networks - 2

Contents of this lecture

- Kohonen net for data compression
- SOFM: Self-organizing feature map
- LVQ: Learning vector quantization
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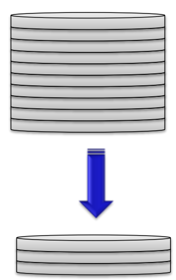
Data compression with Kohonen net: reducing the number of data

- The Kohonen neural network can be used for finding the representatives of clusters.
- This is useful for selecting important data from the training set.
- This is a kind of data compression
  - Reducing the number of data.
- Another well known algorithm is k-means, which is the off-line version of WTA learning.



Data compression with Kohonen net: reducing the data length

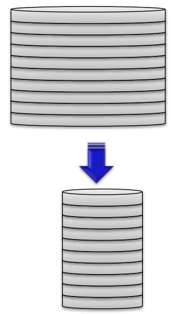
- This is well-known as vector quantization (VQ).
- Suppose that there are N n-dimensional patterns.
- The problem is to find M codes minimizing the approximation error

$$MSE = \sum_{i=1}^{N}\sum_{k=1}^{n}(x_k^i - c_k^{opt})^2$$

where c_k^{opt} is the $k-th$ element of the closest code (or winner for the current input pattern).

Data compression with Kohonen net: reducing the data length (cont.)

- If the error is sufficiently small, the N patterns can be represented by the M code words.
- Each pattern can be represented by an index with $\log_2 M$ bits.
- Many algorithms have been proposed to solve this problem.
  - LBG, ISODATA, K-means, etc.
  - *Winner-take-all is a on-line learning algorithm.*

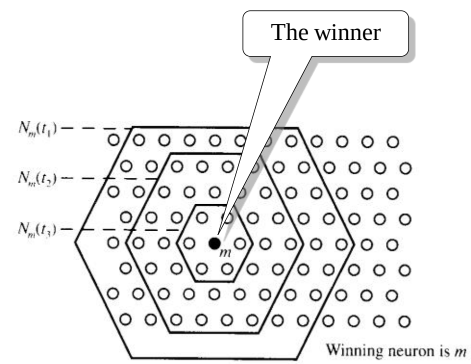


SOFM: Another way for reducing the data length

- The Kohonen network can also be extended to reducing the dimensionality of the pattern space.
- It is useful for visualizing the pattern space.
- A Kohonen network used for this purpose is called SOFM (self-organizing feature map).
- SOFM can reduce the dimensionality and may also preserve the topologic (neighborhood relation) structure of the problem space.

Structure of SOFM

- The neurons are usually arranged in a 2-dimensional planar array with hexagonal neighborhoods (see the figure in the next slide).
- During learning, the weights of all neurons in the neighborhood of the winner are updated.
- The amount of modification is inversely proportional to the distance between the neuron to be updated and the winner.
- The size of the neighborhood is reduced during learning.



The learning rule

- For any training example ***x***, find the winner.
- Suppose that the winner is the *m-th* neuron, the weight vectors of **all neurons close to the winner** are updated as follows

$$W_i = W_i + \alpha(x - W_i),\ \ for\ i \in N_m$$

  – N_m is the neighborhood of the *m-th* neuron.
  – The size of N_m is decreased as the training progresses.

The learning rate

- In general, the learning rate α is not a constant, it depends both on the training time and the distance between the neuron to be updated and the winner. For example

$$\alpha = \alpha(t)e^{-r/\sigma^2(t)}$$

where $\alpha(t)$ and $\sigma(t)$ are decreasing functions of the learning time t, and r is the distance between the current neuron and the winner.

Calibration: assign labels to the neurons

- After learning, each labeled pattern is provided to the neuron array.
- The winner is found.
- The winner is assigned with the same class label as the current input pattern
  – Example: when B is given as the input, the upper left neuron is the winner, and thus this neuron is labeled "B".

Example 1

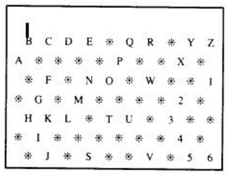
(*from "The self-organizing map" written by T. Kohonen, 1990*)

- There are 32 different 5-dimensional patterns labeled from A through 6
- We want to reduce the dimension from 5 to 2, and see the relation between patterns

	Item																															
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	1	2	3	4	5	6
Attribute																																
x_1	1	2	3	4	5	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
x_2	0	0	0	0	0	1	2	3	4	5	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
x_3	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	3	3	3	3	6	6	6	6	6	6	6	6	6	6
x_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	1	2	3	4	2	2	2	2	2	2
x_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6

Structure of the neural network

- The rectangular array consists of 70 neurons.
- Each neuron has 5 inputs.
- The neurons are trained using the 32 patterns selected each time at random.

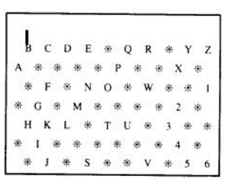


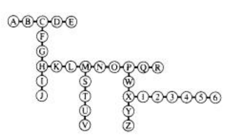
The learning parameters

- The learning rate
  - During the first 1,000 learning cycle, the learning rate decreases linearly with time from 0.5 to 0.04.
  - During the subsequent 10,000 learning cycles, the learning rate decreases from 0.04 to 0 linearly with time.
- The neighborhood size
  - During the first 1,000 learning cycles, the neighborhood size decreased from 6 to one linearly with time.
  - The neighborhood size is kept the same during the subsequent learning cycles.

Post learning calibration

- After 10,000 training cycles, the neurons are calibrated by providing each data point once.
- The neighborhood relation between the patterns is visible in the 2-D space.





Learning vector quantization (LVQ)

- Kohonen network was originally proposed for self-organization learning.
- If the class labels (or teacher signals) of the training patterns are known, we can also use the same network for supervised learning.
- The basic idea is to find some representatives for each class, rather than for each cluster.

The LVQ learning rule

- In LVQ, each neuron is assigned a class label
- For any input pattern ***x***, find the winner.
- If the class label of the winner is the same as that of the input pattern, update the weight of the winner as follows:

$$\boxed{W_i = W_i + \alpha(x - W_i)}$$

The LVQ learning rule

- If the class label of the winner is different from that of the input pattern, update the weight of the winner as follows:

$$\boxed{W_i = W_i - \alpha(x - W_i)}$$

Varieties

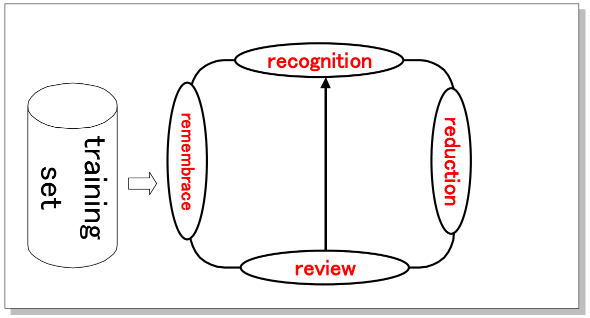
- Update the weights only when the winner gives the correct answer.
- Update the weights only when the winner gives the wrong answer.
- Update the weights for both cases.

Problems in using LVQ

- LVQ itself does not tell us how many neurons should be used for a certain problem.
- If the number of neurons is too large, the compression ratio will be small.
- If the number of neurons is too small, the learning process may not converge.

→ *A systematic way for determining the network structure is required*

The R^4-rule



Zhao and Higuchi, IEEE Trans. On neural networks, 1996

The first R: Recognition

- The first task of *recognition* is to find the recognition rate of the whole system.
- At the same time, the *fitness* of each neuron is also evaluated.
- The fitness of a neuron is high if it is a *winning neuron* for many training examples.

Definition of the winner

- For a given input pattern $\boldsymbol{x}$, a neuron is the winner if
  – It belongs to the same class as $\boldsymbol{x}$.
  – It fires before any neuron of different classes.
  – Its fitness is larger than that of all other neurons satisfying the first two conditions.

The second R: Remembrance

- The task is to remember an unknown example by inserting a new neuron to the current network.
- The weight vector of the new neuron equals to the unknown example.
- The example to be remembered is selected at random from all unknown examples.

The third R: Review

- The task of review is to re-adjust the weights of the current network using LVQ.
- In our research, we have adopted the DSM learning algorithm.
- That is, for any training example ***x***, we update the weights of the related neurons **only if** the most active neuron has a different class label.

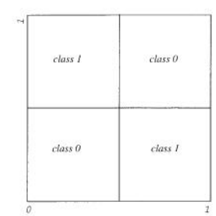
The fourth R: Reduction

- If the current network is good enough, we can select a neuron with a low fitness, and delete it from the network.
- In the reference, the neuron to be deleted is selected at random from all neurons whose fitness are smaller than a given threshold.
- We may also adopt some other selection methods proposed for genetic algorithms.

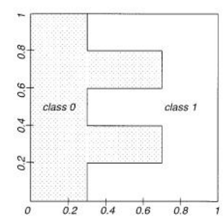
The whole learning process

- Step 1: Find the recognition rate and the fitness of each neuron using recognition.
- Step 2: If the recognition rate is smaller than the pre-specified rate, insert a new neuron using Remembrance; otherwise, delete a neuron using Reduction.
- Step 3: Re-adjust the network using Review.
- Step 4: Return to Step 1.

Examples



The generalized XOR problem



The straight line class boundaries problem

Examples (cont.)

	Generalized XOR		SLCB problem	
Methods	Size	Error rate	Size	Error rate
NNC	6400	0.91	6400	0.97
CNN	231	1.11	307	1.17
RNN	162	1.27	216	1.14
RCE	183	0.69	243	0.56
R^4-rule	4	0.13	10	0.48

Team Project V

- A program is given in the web for SOFM.
- Down –load this program, and check if it works for the example given in this lecture.
- Modify the program, and apply your program to IRIS dataset.
- Try to find a set of learning parameters that can result in good results (a 2-Dimensional map of the feature space).