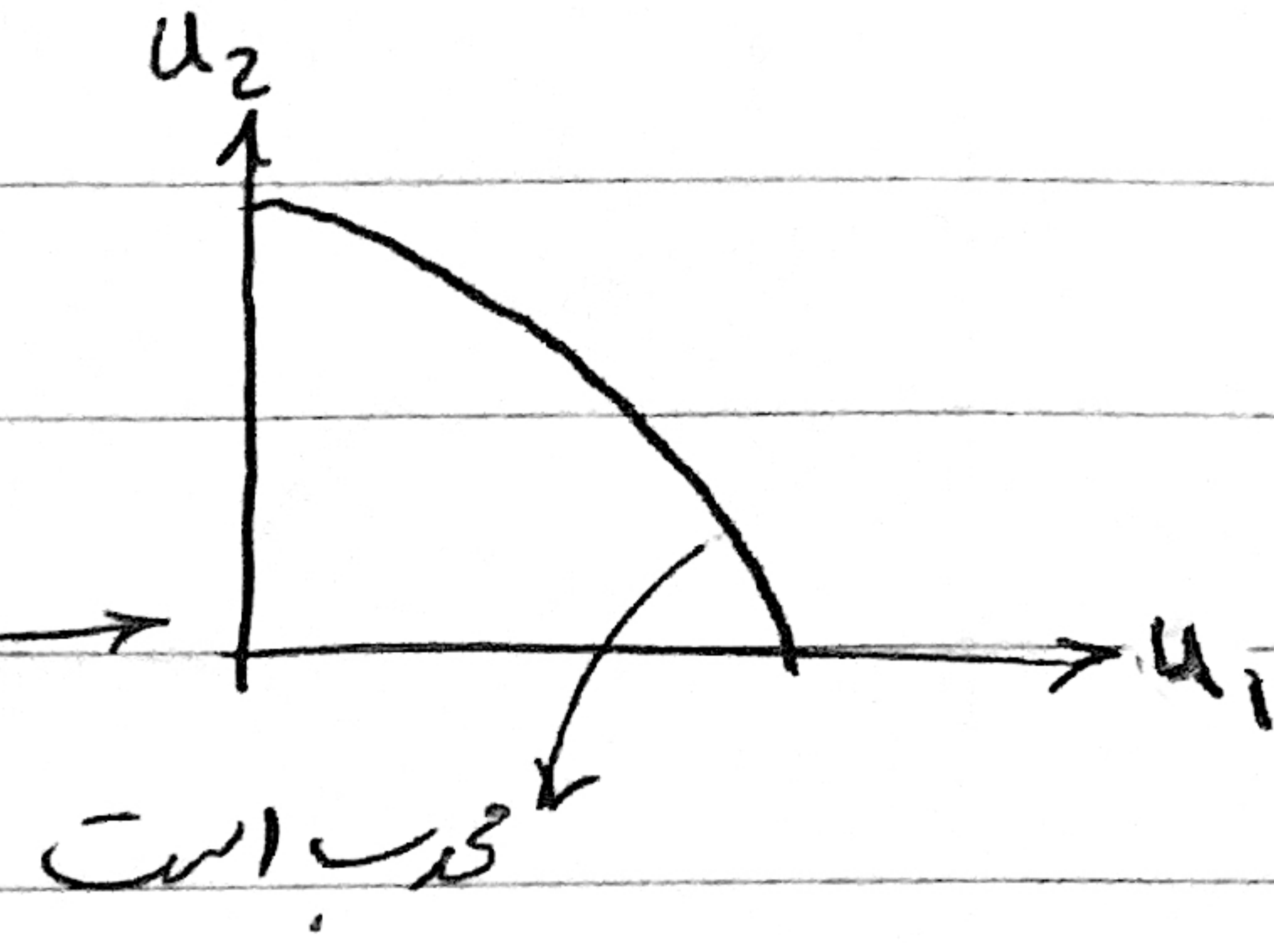
$$\begin{cases} u_1(x_1) = x_1^{1/8} \\ u_2(x_2) = x_2^{1/4} \\ x_1 + x_2 = 2 \end{cases} \longrightarrow \begin{cases} x_1 = u_1^8 \\ x_2 = u_2^4 \\ u_1^8 + u_2^4 = 2 \end{cases} \longrightarrow$$



$$d_1 = d_2 = 0 \longrightarrow \max u_1 u_2$$

$$\text{s.t. } u_1^8 + u_2^4 \quad 2$$

$$u_1, u_2 \geq 0$$

$$u_1^8 + u_2^4 = 2 \longrightarrow u_2^4 = 2 - u_1^8 \longrightarrow u_2 = (2 - u_1^8)^{1/4}$$

$$\max u_1 (2 - u_1^8)^{1/4} \longrightarrow \frac{\partial f}{\partial u_1} = (2-u_1^8)^{1/4} + \tfrac{1}{4} u_1 (-8u_1^7)(2-u_1^8)^{-3/4} = 0$$

$$u_1 > 0 \qquad = (2 - u_1^8) - 2u_1^8 = 0 \longrightarrow u_1^8 = 2/3$$

$$\frac{\partial^2 f}{\partial u_1^2} = -24 u_1^7 < 0 \rightarrow$$

مقدار منفی است پس
نقطه ماکزیمم است

$$\begin{cases} u_1 = (2/3)^{1/8}, \; x_1 = 2/3 \longrightarrow 33\% \\ u_2 = (4/3)^{1/4}, \; x_2 = 4/3 \longrightarrow 66\% \end{cases}$$

اثبات محدب به صورت ریاضی:

$$u_1^8 + u_2^4 = 2$$

gradient

$$[8u_1^7, 4u_2^3]$$

hessian

$$\begin{bmatrix} 56u_1^6 & 0 \\ 0 & 12u_2^2 \end{bmatrix} \geq 0 \quad \text{Positive Definite}$$