الف)

نش خالص نداریم.

نش مقلوط:

$$\delta^* = (p, q, 1-p-q)$$

روش سوم:

$$u_1(row_1, \delta^*) = u_1(row_2, \delta^*)$$

$$P + 3q + 2(1-p-q) = 2q + 5(1-p-q) \rightarrow P+q = \frac{3}{4}$$

$$u_1(row_2, \delta^*) = u_1(row_3, \delta^*)$$

$$2q + 5(1-p-q) = 6p + q + 4(1-p-q) \rightarrow P = \frac{1}{7}$$

$$q = \frac{17}{28}$$

$$\delta^* = \left(\frac{1}{7}, \frac{17}{28}, \frac{1}{4}\right)$$

ب) هر ESS حتماً نش است بنابراین نش لازم است پس ESS بودن δ^* بزرگ نشد

نشدی $\rightarrow u(\delta^*, \delta^*) = u(\delta, \delta^*)$ نش ضعیف

شرط دوم ESS: $\delta = (p, q, 1-p-q)$; $u(\delta^*, \delta) > u(\delta, \delta)$

$$\left(\frac{1}{7}, \frac{17}{28}, \frac{1}{4}\right) \begin{pmatrix} 1 & 3 & 2 \\ 0 & 2 & 5 \\ 6 & 1 & 4 \end{pmatrix} \begin{pmatrix} p \\ q \\ 1-p-q \end{pmatrix} = u(\delta^*, \delta)$$

$$\left(p \quad q \quad 1-p-q\right) \begin{pmatrix} 1 & 3 & 2 \\ 0 & 2 & 5 \\ 6 & 1 & 4 \end{pmatrix} \begin{pmatrix} p \\ q \\ 1-p-q \end{pmatrix} = u(\delta, \delta)$$

به ازای شکل مقعر $\delta = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$; $u(\delta^*, \delta) > u(\delta, \delta)$ بنابراین δ^* ESS است

	c_1	c_2	c_3
r_1	2, -2	1, -1	0, 0
r_2	2, -2	0, 0	3, -3
r_3	-1, 1	3, -3	-3, 3

$$\begin{cases} BR(c_1) = r_1, r_2 \\ BR(c_2) = r_3 \\ BR(c_3) = r_2 \end{cases}$$

$$\begin{cases} BR(r_1) = c_3 \\ BR(r_2) = c_2 \\ BR(r_3) = c_3 \end{cases}$$

نش خالص ندارد .

به سراغ نش مخلوط می‌رویم .

با درهیچ‌حالتی BR نش بازی نمی‌شود

درنتیجه آنرا از c_1 بازی نشود، r_1 هم بازی نمی‌شود.

ماتریس بصورت روبرو ساده می‌شود.

	c_2	c_3
r_2	0, 0	3, -3
r_3	3, -3	-3, 3

$$\delta_1^* = (0, p, 1-p)$$
$$\delta_2^* = (0, q, 1-q)$$

روش سوم :

$$u_1(r_2, \delta_2^*) = u_1(r_3, \delta_2^*)$$
$$3(1-q) = 3q - 3(1-q) \rightarrow q = \frac{2}{3}$$

$$u_2(c_2, \delta_1^*) = u_2(c_3, \delta_1^*)$$
$$-3(1-p) = -3p + 3(1-p) \longrightarrow p = \frac{2}{3}$$

تعادل نش : $\left[(0, \frac{2}{3}, \frac{1}{3}), (0, \frac{2}{3}, \frac{1}{3}) \right]$

$$\eta_1^0 = (1,1,1) \ , \ \eta_2^0 = (2,1,0)$$

$$\mu_1^0 = (\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}), \ \mu_2^0 = (\tfrac{2}{3},\tfrac{1}{3},0)$$

$$EU_1(r_1) = \tfrac{1}{3}(2+1+0) = 1$$

$$EU_1(r_2) = \tfrac{1}{3}(2+0+3) = \tfrac{5}{3} \ \checkmark$$

$$EU_1(r_3) = \tfrac{1}{3}(-1+3-3) = \tfrac{-1}{3}$$

$$\Rightarrow S_1^0 = r_2$$

$$EU_2(c_1) = \tfrac{2}{3} \times (-2) + \tfrac{-2}{3} = \tfrac{-6}{3}$$

$$EU_2(c_2) = \tfrac{-2}{3} \ \checkmark$$

$$EU_2(c_3) = \tfrac{-3}{3}$$

$$\Rightarrow S_2^0 = c_2$$

$$\eta_1' = (1,2,1) \ , \ \eta_2' = (2,2,0)$$

$$\mu_1' = (\tfrac{1}{4},\tfrac{1}{2},\tfrac{1}{4}) \ , \ \mu_2' = (\tfrac{1}{2},\tfrac{1}{2},0)$$

$$EU_1(r_1) = \tfrac{2}{4} + \tfrac{1}{2} = 1$$

$$EU_1(r_2) = \tfrac{2}{4} + \tfrac{3}{4} = \tfrac{5}{4} \ \checkmark$$

$$EU_1(r_3) = \tfrac{-1}{4} + \tfrac{3}{2} - \tfrac{3}{4} = \tfrac{1}{2}$$

$$\Rightarrow S_1' = r_2$$

$$EU_2(c_1) = -2$$

$$EU_2(c_2) = \tfrac{-1}{2} \ \checkmark$$

$$EU_2(c_3) = \tfrac{-3}{2}$$

$$\Rightarrow S_2' = c_2$$

$$\eta_1^2 = (1,3,1), \quad \eta_2^1 = (2,3,0)$$

$$\mu_1^2 = \left(\frac{1}{5}, \frac{3}{5}, \frac{1}{5}\right), \quad \mu_2^2 = \left(\frac{2}{5}, \frac{3}{5}, 0\right)$$

$$\left. \begin{array}{l} EU_1(r_1) = 1 \\ EU_1(r_2) = 1 \\ EU_1(r_3) = 1 \end{array} \right\} \longrightarrow$$

به است که ممکن 2 S_2 را انتخاب کند.

$$\begin{array}{l} \textcircled{1} \quad r_1 \\ \textcircled{2} \quad r_2 \\ \textcircled{3} \quad r_3 \end{array}$$

$$\left. \begin{array}{l} EU_2(c_1) = \frac{-4}{5} - \frac{6}{5} \\ EU_2(c_2) = \frac{-2}{5} \quad \checkmark \\ EU_2(c_3) = -\frac{9}{5} \end{array} \right\} \rightarrow S_2^2 = C_2$$

$$\eta_1^3 = (1,4,1), \quad \eta_2^3 = \begin{array}{l} \textcircled{1} \ (3,3,0) \\ \textcircled{2} \ (2,4,0) \\ \textcircled{3} \ (2,3,1) \end{array}$$

$$\mu_1^3 = \left(\frac{1}{6}, \frac{4}{6}, \frac{1}{6}\right), \quad \mu_2^3 = \begin{array}{l} \textcircled{1} \ \left(\frac{1}{2}, \frac{1}{2}, 0\right) \\ \textcircled{2} \ \left(\frac{1}{3}, \frac{2}{3}, 0\right) \\ \textcircled{3} \ \left(\frac{2}{6}, \frac{3}{6}, \frac{1}{6}\right) \end{array}$$

$$\left. \begin{array}{l} EU_1(r_1) = \frac{2}{6} + \frac{4}{6} = 1 \\ EU_1(r_2) = \frac{2}{6} + \frac{3}{6} = \frac{5}{6} \\ EU_1(r_3) = \frac{-1}{6} + \frac{12}{6} - \frac{3}{6} = \frac{8}{6} \quad \checkmark \end{array} \right\} \rightarrow S_1^3 = r_3$$

$$\textcircled{1} \quad S_2^3 = C_2$$

ممکن است 1

$$\textcircled{2} \quad \left\{ \begin{array}{l} EU_2(c_1) = \frac{-2}{3} - \frac{4}{3} \\ EU_2(c_2) = \frac{-1}{3} \quad \checkmark \\ EU_2(c_3) = \frac{-6}{3} \end{array} \right\} \rightarrow S_2^3 = C_2$$

$$\textcircled{3} \quad \left\{ \begin{array}{l} EU_2(c_1) = \frac{-4}{6} - \frac{6}{6} + \frac{1}{6} = \frac{-9}{6} \\ EU_2(c_2) = \frac{-2}{6} - \frac{3}{6} = \frac{-5}{6} \quad \checkmark \\ EU_2(c_3) = \frac{-9}{6} + \frac{8}{6} = \frac{-6}{6} \end{array} \right\} \rightarrow S_2^3 = C_2$$

CamScanner

ج) جا توجه به اینکه بازی sum-zero دو نفره است، طبق قضیه‌ی p ارائه

متوسط زمانی که براس شود.