Solution_Q6b

May 20, 2024

[1]:
```
import numpy as np
```

[2]:
```
def shapley_value(N, V):
    X = np.zeros(N)
    W = {}
    for keys, values in V.items():
        W[keys] = set()
        for k in keys:
            W[keys].add(int(k))

    for keys, values in V.items():
        if values != 0:
            for k in keys:
                X[int(k)-1] += values/len(keys)

            for k, v in W.items():
                if v.intersection(W[keys]) == W[keys]:
                    V[k] -= values

    return f'Sh({N}, V) = {X}'
```

[3]:
```
N = 3
V = {'1':6., '2':12., '3':18., ('1', '2'):30., ('1', '3'):60., ('2', '3'):90.,␣
 ↪('1', '2', '3'):120.}
```

[4]:
```
shapley_value(N, V)
```

[4]:
```
'Sh(3, V) = [22. 40. 58.]'
```

[5]:
```
N = 4
V = {'1':6., '2':12., '3':0., '4':18.,
     ('1', '2'):24., ('1', '3'):48., ('1', '4'):60., ('2', '3'):12., ('2', '4'):
 ↪32., ('3', '4'):38.,
     ('1', '2', '3'):120., ('1', '2', '4'):89., ('1', '3', '4'):150., ('2',␣
 ↪'3', '4'):179.,
     ('1', '2', '3', '4'):240.}
```

[6]:
```
shapley_value(N, V)
```

[6]: 'Sh(4, V) = [48.33333333 49.33333333 70.66666667 71.66666667]'