Least Squares

Dr François Pitié

Problem 1. Which of these models are linear?

$$y = w_0 + w_1 x^2 + \epsilon \tag{1}$$

$$y = w_0 x^{w_1} + w_2 + \epsilon \tag{2}$$

$$y = \exp(w_0 + w_1 x) + \epsilon \tag{3}$$

$$\log(y) = w_0 + w_1 x + \epsilon \tag{4}$$

Problem 2. For n real numbers $x_1, \cdots, x_n$, what is the value $\hat{x}$ that minimises the sum of squared distances from x to each x_i:

$$\hat{x} = \arg\min_x \sum_{i=1}^{n} (x_i - x)^2$$

Problem 3. For a linear model $\mathbf{y} = \mathbf{X}\mathbf{w} + \boldsymbol{\epsilon}$, derive, in a matrix form, the expression of the least square error. That is, for $E(\mathbf{w}) = \boldsymbol{\epsilon}^\top \boldsymbol{\epsilon}$ derive the expression of $\min_{\mathbf{w}} E(\mathbf{w})$.

Problem 4. An autoregressive model is when a value from a time series is regressed on previous values from that same time series.

$$x_t = w_0 + \sum_{i=1}^{p} w_i x_{t-i} + \varepsilon_t$$

write the design matrix for this problem.

Problem 5. Consider the linear model $y = w_0 + w_1 x$. We want to bias w_1 towards the value $\hat{w_1}$. Write a loss function that achieves this.