

# Linear Algebra

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## Problem 1.

Assume  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix}$ , is a vector of size  $p \times 1$ ,

What is the size of

1.  $\mathbf{a}\mathbf{a}^\top$
2.  $\mathbf{a}^\top\mathbf{a}$
3.  $\mathbf{a}\mathbf{a}^\top\mathbf{a}\mathbf{a}^\top$
4.  $\mathbf{a}^\top\mathbf{a}\mathbf{a}^\top\mathbf{a}$

## Problem 2.

Given no assumptions about matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and vectors  $\mathbf{a}$  and  $\mathbf{b}$ , compute the gradient  $\frac{\partial E(\mathbf{w})}{\partial \mathbf{w}}$  for

1.  $E(\mathbf{w}) = \mathbf{w}^\top \mathbf{w}$
2.  $E(\mathbf{w}) = (\mathbf{w} - \mathbf{a}^\top)\mathbf{A}(\mathbf{w} - \mathbf{a})$
3.  $E(\mathbf{w}) = (\mathbf{A}\mathbf{w} - \mathbf{b})^\top (\mathbf{A}\mathbf{w} - \mathbf{b})$
4.  $E(\mathbf{w}) = (\mathbf{w} - \mathbf{B}\mathbf{w})^\top \mathbf{A}(\mathbf{w} - \mathbf{a})$