

# Least Squares

Dr François Pitié

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**Problem 1.** Which of these models are linear?

$$y = w_0 + w_1x^2 + \epsilon \quad (1)$$

$$y = w_0x^{w_1} + w_2 + \epsilon \quad (2)$$

$$y = \exp(w_0 + w_1x) + \epsilon \quad (3)$$

$$\log(y) = w_0 + w_1x + \epsilon \quad (4)$$

**Problem 2.** For  $n$  real numbers  $x_1, \dots, x_n$ , what is the value  $\hat{x}$  that minimises the sum of squared distances from  $x$  to each  $x_i$ :

$$\hat{x} = \arg \min_x \sum_{i=1}^n (x_i - x)^2$$

**Problem 3.** For a linear model  $\mathbf{y} = \mathbf{X}\mathbf{w} + \boldsymbol{\epsilon}$ , derive, in a matrix form, the expression of the least square error. That is, for  $E(\mathbf{w}) = \boldsymbol{\epsilon}^\top \boldsymbol{\epsilon}$  derive the expression of  $\min_{\mathbf{w}} E(\mathbf{w})$ .

**Problem 4.** An autoregressive model is when a value from a time series is regressed on previous values from that same time series.

$$x_t = w_0 + \sum_{i=1}^p w_i x_{t-i} + \varepsilon_t$$

write the design matrix for this problem.

**Problem 5.** Consider the linear model  $y = w_0 + w_1x$ . We want to bias  $w_1$  towards the value  $\hat{w}_1$ . Write a loss function that achieves this.