Linear Algebra

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Problem 1.

Assume $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix}$, is a vector of size $p \times 1$,

What is the size of

1. $\mathbf{a}\mathbf{a}^\top$
2. $\mathbf{a}^\top\mathbf{a}$
3. $\mathbf{a}\mathbf{a}^\top\mathbf{a}\mathbf{a}^\top$
4. $\mathbf{a}^\top\mathbf{a}\mathbf{a}^\top\mathbf{a}$

Problem 2.

Given no assumptions about matrices $\mathbf{A}$, $\mathbf{B}$ and vectors $\mathbf{a}$ and $\mathbf{b}$, compute the gradient $\frac{\partial E(\mathbf{w})}{\partial \mathbf{w}}$ for

1. $E(\mathbf{w}) = \mathbf{w}^\top\mathbf{w}$
2. $E(\mathbf{w}) = (\mathbf{w} - \mathbf{a}^\top)\mathbf{A}(\mathbf{w} - \mathbf{a})$
3. $E(\mathbf{w}) = (\mathbf{A}\mathbf{w} - \mathbf{b})^\top(\mathbf{A}\mathbf{w} - \mathbf{b})$
4. $E(\mathbf{w}) = (\mathbf{w} - \mathbf{B}\mathbf{w})^\top\mathbf{A}(\mathbf{w} - \mathbf{a})$