Linear Algebra

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Problem 1.

Assume
$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix}$$
, is a vector of size $p \times 1$, What is the size of

1. **aa**[⊤]

2.
$$\mathbf{a}^{\mathsf{T}}\mathbf{a}$$

3.
$$\mathbf{a}\mathbf{a}^{\mathsf{T}}\mathbf{a}\mathbf{a}^{\mathsf{T}}$$

4.
$$\mathbf{a}^{\mathsf{T}}\mathbf{a}\mathbf{a}^{\mathsf{T}}\mathbf{a}$$

Problem 2.

Given no assumptions about matrices A, B and vectors a and b, compute the gradient

1.
$$E(\mathbf{w}) = \mathbf{w}^{\mathsf{T}} \mathbf{w}$$

2.
$$E(\mathbf{w}) = (\mathbf{w} - \mathbf{a})^{\mathsf{T}} \mathbf{A} (\mathbf{w} - \mathbf{a})$$

3.
$$E(\mathbf{w}) = (\mathbf{A}\mathbf{w} - \mathbf{b})^{\mathsf{T}}(\mathbf{A}\mathbf{w} - \mathbf{b})$$

4.
$$E(\mathbf{w}) = (\mathbf{w} - \mathbf{B}\mathbf{w})^{\mathsf{T}} \mathbf{A} (\mathbf{w} - \mathbf{a})$$