Solution – Linear Algebra Tutorial

Problem 1:

$$a = (P \times 1), \quad a^T = (1 \times P)$$

1. $aa^T \Longrightarrow (P \times 1)(1 \times P) = (P \times P)$
2. $a^T a \Longrightarrow (1 \times P)(P \times 1) = (1 \times 1)$
3. $aa^T aa^T \Longrightarrow (P \times 1)(1 \times P)(P \times 1)(1 \times P) = (P \times P)$
4. $a^T aa^T a \Longrightarrow (1 \times P)(P \times 1)(1 \times P)(P \times 1) = (1 \times 1)$

Problem 2:

1. $E(w) = w^T w = 2w$

2. $E(w) = (w-a)^T A(w-a) \qquad Assume\ A\ is\ symmetric$
$= (w^T - a^T)(Aw - Aa)$
$= w^T Aw - w^T Aa - a^T Aw + a^T Aa$
$= 2Aw - Aa - A^T a$
$= 2A(w-a)$

3. $E(w) = (Aw-b)^T (Aw-b) \qquad Assume\ A\ is\ symmetric$
$= (A^T w^T - b^T)(Aw - b)$
$= (A^T w^T Aw - A^T w^T b - b^T Aw + b^T b)$
$= 2A^T Aw - A^T b - A^T b$
$= 2A^T Aw - 2A^T b$
$= 2A^T (Aw - b)$

4. $E(w) = (w - Bw)^T A(w-a) \qquad Assume\ A\ is\ symmetric$
$= (w^T - B^T w^T)(Aw - Aa)$
$= w^T Aw - w^T Aa - B^T w^T Aw + B^T w^T Aa$
$= 2Aw - Aa - 2B^T Aw + B^T Aa$
$= 2Aw - 2B^T Aw + B^T Aa - Aa$
$= 2Aw(1 - B^T) + Aa(B^T - 1)$