Least Squares

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Problem 1. Which of the following models with input x_1, x_2, parameters w_1, w_2 and noise $\epsilon \sim \mathcal{N}(0, \sigma^2)$, are linear in the parameters and can be used as such for Least Squares:

1. $y = w_0 + w_1 x^2 + \epsilon$
2. $y = w_0 x^{w_1} + w_2 + \epsilon$
3. $y = \exp(w_0 + w_1 x) + \epsilon$
4. $\log(y) = w_0 + w_1 x + \epsilon$

Problem 2. Which of the following models with input x_1, x_2, parameters w_1, w_2 and noise $\epsilon \sim \mathcal{N}(0, \sigma^2)$, are linear in the parameters and can be used as such for Least Squares:

1. $y = \sin(x_1 w_1 + w_2) + \epsilon$
2. $y = \log(x_1) w_1 + \log(x_2) w_2 + \epsilon$
3. $y = w_1 x_1^2 + \epsilon$
4. $y = {w_1}^2 x_1 + \epsilon$

Problem 3. For n real numbers $x_1, \cdots, x_n$, what is the value $\hat{x}$ that minimises the sum of squared distances from x to each x_i:

$$\hat{x} = \arg\min_x \sum_{i=1}^{n} (x_i - x)^2$$

Problem 4. For a linear model $\mathsf{y} = \mathsf{X}\mathsf{w} + \boldsymbol{\epsilon}$, derive, in a matrix form, the expression of the least square error. That is, for $E(\mathsf{w}) = \boldsymbol{\epsilon}^\top \boldsymbol{\epsilon}$ derive the expression of $\min_{\mathsf{w}} E(\mathsf{w})$.

Problem 5. An autoregressive model is when a value from a time series is regressed on previous values from that same time series.

$$x_t = w_0 + \sum_{i=1}^{p} w_i x_{t-i} + \varepsilon_t$$

write the design matrix for this problem.

Problem 6. Consider the linear model $y = w_0 + w_1 x$. We want to bias w_1 towards the value $\hat{w_1}$. Write a loss function that achieves this.