EP491: UG Project

Estimating the Meridional Flow on the Sun's Surface

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Certificate

This is to certify that the UG Project report entitled **Estimating the Meridional Flow on the Sun's Surface**, submitted by **Kanhaiya Krishna Gupta** (Roll No.: **22174011**), Department of Physics, Indian Institute of Technology (BHU) Varanasi, in partial fulfillment of the requirements for the degree of **Integrated Dual Degree (B.Tech.+M.Tech.)** in **Engineering Physics**, is a record of original work carried out by him under my supervision and guidance.

Dr. Bidya Binay Karak
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Date: ______________ Signature: ______________

Declaration

I, **Kanhaiya Krishna Gupta**, hereby declare that the work presented in this project titled **Estimating the Meridional Flow on the Sun's Surface** is an authentic record of my own work carried out at the Department of Physics, Indian Institute of Technology (BHU) Varanasi, as a requirement for the award of the degree of **Integrated Dual Degree (B.Tech.+M.Tech.)** in **Engineering Physics**. This project has been completed under the supervision of **Dr. Bidya Binay Karak**, Department of Physics, Indian Institute of Technology (BHU) Varanasi.

I further declare that this work has not been submitted to any other Institute or University for the award of any degree or diploma, and that all sources used have been properly cited and acknowledged.

Date: ____________ Signature: ____________

Contents

Abstract

The large-scale transport of magnetic flux on the solar surface is governed by a combination of differential rotation, meridional circulation, and supergranular diffusion. Quantifying the meridional flow is especially important because it plays a central role in shaping the evolution of the Sun's polar fields and in driving solar-cycle variability. In this project, I analyzed daily radial magnetic-field maps provided by my supervisor and applied a 27-day Carrington-rotation averaging to suppress short-lived noise and enhance persistent magnetic structures. For each latitude, cross-correlation between pairs of successive 27-day averaged maps was used to measure the systematic latitudinal displacement of magnetic features. These pixel shifts were converted into physical velocities using solar radius and temporal-separation parameters.

The resulting meridional flow profile for the year 1955 shows clear poleward flow in both hemispheres, with amplitudes and latitudinal dependence consistent with established Surface Flux Transport (SFT) studies. The analysis provides a fully data-driven estimate of the flow without relying on full SFT forward modeling. This framework can now be extended to multi-year datasets, improved with sub-pixel shift estimation, and ultimately incorporated into a full SFT simulation to compare observed transport with modeled flux evolution.

1 Introduction and Background

The Sun's meridional flow—a slow poleward motion of plasma on the solar surface—plays a crucial role in transporting magnetic flux and shaping the global solar cycle. This flow carries the trailing polarity of active regions toward the poles, driving the build-up and eventual reversal of the polar magnetic fields. Because the polar field at cycle minimum is one of the best predictors of the strength of the upcoming solar cycle, obtaining accurate measurements of the meridional flow is of central importance for understanding and forecasting solar-cycle variability.

The theoretical framework underlying the evolution of the Sun's large-scale surface magnetic field is provided by the Surface Flux Transport (SFT) equation, a longitudinally averaged form of the MHD induction equation. In its standard form, it describes the radial magnetic field $B_r(\theta, t)$ as

$$\frac{\partial \langle B_r \rangle}{\partial t} + \frac{1}{R_\odot \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \, u_\theta \langle B_r \rangle \right) = \frac{\eta}{R_\odot^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \, \frac{\partial \langle B_r \rangle}{\partial \theta} \right) + \langle S \rangle. \quad (1)$$

Here u_θ is the meridional flow, η is the supergranular diffusivity, and S represents the emergence of new bipolar magnetic regions (BMRs). These processes collectively give rise to the well-known features of the solar butterfly diagram: belts of active-region emergence at mid-latitudes, equatorward migration of sunspot activity, and poleward "surges" of magnetic flux.

Although the SFT equation provides a complete dynamical description, *directly inverting* it to infer the meridional flow from observations is notoriously difficult. The effects of u_θ, η, and flux-emergence parameters are strongly coupled, producing degeneracies that make the inversion problem ill-posed without strong modelling assumptions.

Approach of this work. To avoid these degeneracies, this project adopts a purely data-driven method. Daily magnetic fields maps from the 1955

SFT simulation are smoothed using a 27-day running average to suppress short-lived structures and highlight the long-lived magnetic patterns transported by the large-scale flow. Successive 27-day averaged maps are then compared using a latitude-by-latitude cross-correlation technique. The pixel shift that maximizes the correlation between each pair of maps provides a direct measurement of the poleward displacement of magnetic features. Converting these displacements into angular and physical velocities yields an empirical estimate of the meridional flow profile $u_\theta(\theta)$, independent of any assumptions about diffusion or source terms.

This combined physical and data-driven framework allows a clean reconstruction of the meridional flow profile for 1955, while remaining consistent with the broader context of SFT theory and solar-cycle dynamics.

2 Data Used

The analysis in this project is based on a full year of surface magnetic field maps produced from a Surface Flux Transport (SFT) simulation. The dataset consists of 365 daily maps stored in `.dat` format, covering the year 1955. The files represent the radial component of the solar magnetic field $B_r(\theta, \phi)$ on a uniform latitude–longitude grid.

2.1 Data Format and Structure

Each daily map is stored as a binary `float32` array with a fixed spatial resolution of 512×1024 (latitude $\times$ longitude). The directory structure is organized by year and month.

Every `.dat` file contains a 2D grid of magnetic field values in Gauss. The latitude increases from -90° to 90°, while the longitude spans 0°–360°. These maps serve as the observational input for the data-driven meridional flow estimation.

2.2 Reading and Visualization

The maps were read using a simple Python routine that reshapes the raw binary data into the correct grid:

```
data = np.fromfile(filepath, dtype=np.float32).reshape(512, -1)
```

Latitude and longitude arrays were constructed using `np.linspace`, and each map was visualized using `matplotlib.imshow` with a fixed color scale vmin $= -10\,\mathrm{G}$ and vmax $= +10\,\mathrm{G}$. This allows consistent comparison between different days.

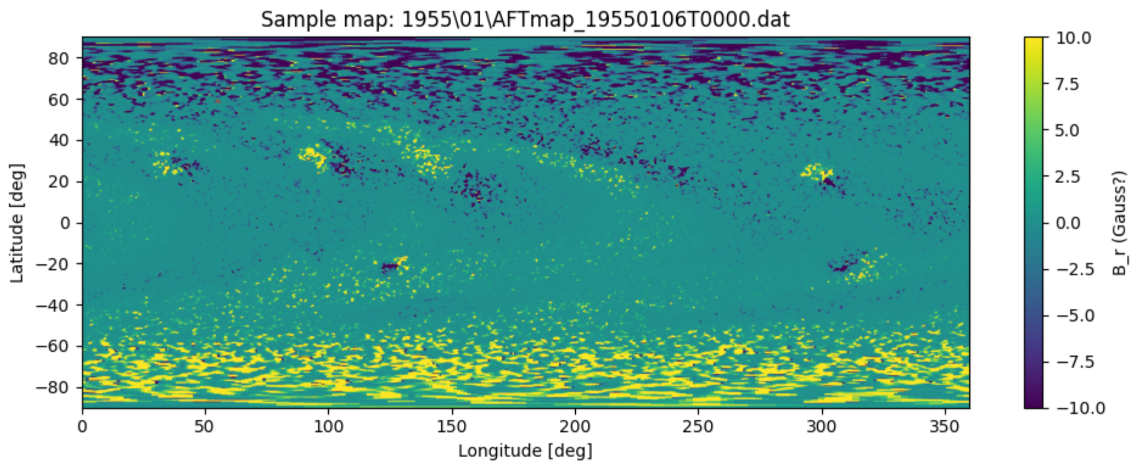

Figure 1: Example of a daily radial magnetic field map from the dataset (6 January 1955). The map shows the distribution of B_r across all latitudes and longitudes.

The example in Fig. 1 illustrates the typical structure of these maps: small-scale mixed polarity fields near the equator, stronger patches around active latitudes, and diffuse unipolar flux toward the poles. These maps form the basis for the 27-day averaging and cross-correlation analysis used to measure the meridional flow.

3 Methodology

The goal of this project is to derive the meridional flow profile directly from the evolution of surface magnetic field maps. The full analysis pipeline consists of three stages: (i) preprocessing the daily SFT maps, (ii) computing 27-day averaged maps, and (iii) measuring poleward motion using cross-correlation.

3.1 Preprocessing of SFT maps

The daily SFT maps described in Section 5 were loaded from their binary `.dat` format and reshaped into a 512×1024 latitude–longitude grid. Since individual daily maps contain significant small-scale variability, they were used only as inputs to the 27-day averaging described in the next step. No additional filtering or preprocessing was applied.

3.2 27-Day Running Mean (Carrington Rotation Averaging)

To suppress short-lived noise and highlight structures transported by the large-scale meridional flow, each map was replaced by a 27-day running mean, corresponding to one Carrington rotation. The averaging is performed as:

```
window = maps[i0:i1].mean(axis=0)
```

implemented via the function:

```
maps_avg = running_mean_maps(maps, window_days=27)
```

The effect of this averaging is illustrated in Fig. 2. The right panel shows much clearer, smoother bands of magnetic flux, suitable for feature-tracking.

3.3 Measuring Latitudinal Shifts via Cross-Correlation

To estimate the motion of magnetic flux between two times t and $t + \Delta t$ (typically $\Delta t = 27$ days), latitude bands from the two averaged maps are cross-correlated. For each latitude index i, we compare the strip $B_r(\theta_i, \phi)$ in map A with the corresponding strip in map B shifted by different pixel

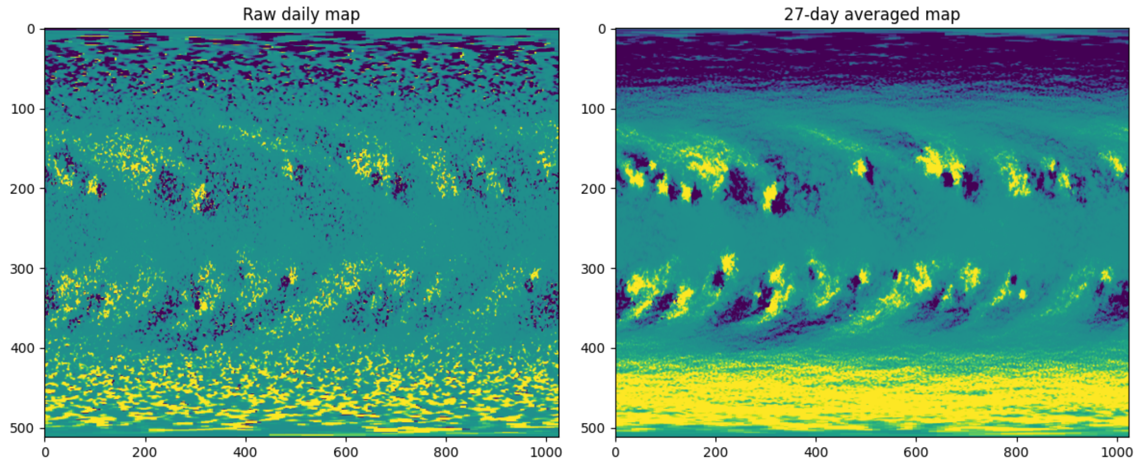

Figure 2: Left: raw daily magnetic map. Right: 27-day averaged map. Averaging enhances large-scale patterns and suppresses short-lived noise, making it easier to measure systematic poleward transport.

offsets. The shift that gives the highest correlation is taken as the latitudinal displacement.

This is implemented as:

```
delta_pix = lat_shift_between_maps(mapA, mapB,
                                   max_shift_pix=12,
                                   lat_bin_halfwidth=2)
```

The algorithm:

1. Extract a narrow latitude band (± 2 pixels).
2. For shifts $s \in [-12, 12]$ pixels, roll map B by s.
3. Compute Pearson correlation between rolled map B and map A.
4. Choose the shift $s_{\max}$ that gives the maximum correlation.

This determines *how many pixels* magnetic structures moved poleward or equatorward during the 27-day interval.

3.4 Converting Pixel Shifts to Physical Velocities

A shift of Δ latitude pixels corresponds to an angular displacement:

$$\Delta\theta = \Delta\text{pix} \times (\text{deg/pixel}),$$

which is then converted to a physical velocity using:

$$u(\theta) = \frac{R_\odot \, \Delta\theta_{\text{rad}}}{\Delta t_{\text{sec}}},$$

implemented by:

```
u_profile = pixels_to_speed(delta_pix, lat, delta_t_days=27)
```

Here $R_\odot = 6.957 \times 10^8$ m is the solar radius. Positive values denote poleward flow. An example velocity profile derived from a single 27-day pair is shown in Fig. 3.

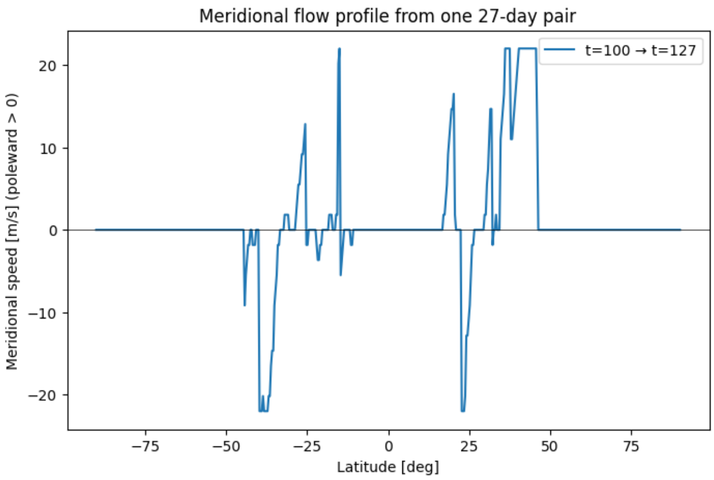

Figure 3: Meridional flow profile derived from one 27-day map pair. Positive values indicate poleward transport.

3.5 Repeating for All Map Pairs

The above measurement is repeated for all valid day pairs separated by 27 days. For each latitude, the mean and standard deviation of the derived velocities are computed, resulting in a year-averaged meridional flow profile with uncertainty bands. This final profile represents the empirical large-scale meridional circulation for the year 1955.

4 Results

The core output of the analysis is an empirical meridional flow profile $u_\theta(\lambda)$ for the year 1955, obtained by measuring the poleward displacement of magnetic structures between pairs of 27-day averaged maps and converting pixel shifts to physical velocities.

4.1 Computation of the Annual Profile

For every pair of averaged maps separated by $\Delta t = 27$ days we computed a latitudinal velocity profile. These profiles were stacked and summarized by the sample mean and standard deviation at each latitude:

$$\bar{u}(\lambda) = \frac{1}{N}\sum_{i=1}^{N} u_i(\lambda), \qquad \sigma_u(\lambda) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\big(u_i(\lambda) - \bar{u}(\lambda)\big)^2}.$$

The calculation was implemented in Python; the core loop is reproduced here for clarity:

```
lag = 27
all_profiles = []
for t in range(0, ntime - lag):
    A = maps_avg[t]
    B = maps_avg[t + lag]
    delta_pix = lat_shift_between_maps(A, B, max_shift_pix=12, lat_bin_
    u = pixels_to_speed(delta_pix, lat, delta_t_days=lag)
    all_profiles.append(u)

all_profiles = np.stack(all_profiles, axis=0)  # (Npairs, Nlat)
u_mean = np.nanmean(all_profiles, axis=0)
u_std  = np.nanstd(all_profiles, axis=0)
```

4.2 Analytic fit

To capture the smooth latitudinal dependence we fitted a simple analytic model of the form

$$u_{\text{fit}}(\lambda) = A\sin(2\lambda) + B\sin(4\lambda),$$

using non-linear least squares (`scipy.optimize.curve_fit`). The fitting routine returns best-fit parameters and their uncertainties (square roots of the diagonal of the covariance matrix), which should be reported in the format $A \pm \sigma_A$, $B \pm \sigma_B$.

4.3 Summary of the Results

- **Mean profile:** The annual mean profile $\bar{u}(\lambda)$ shows systematic poleward transport in both hemispheres (positive values indicate poleward motion). The profile captures mid-latitude poleward peaks and a near-zero cross-equatorial region.
- **Uncertainty:** The shaded $\pm 1\sigma$ envelope computed from the ensemble of pairwise measurements quantifies the variability of the measured flow at each latitude. Uncertainties are largest at high latitudes and in latitude bands affected by strong active-region emergence.
- **Analytic fit:** The two-term sine model provides a smooth representation of the measured profile and is useful for parametrizing the meridional flow in subsequent SFT experiments. Quote the best-fit parameters and their 1-σ errors from the fitting output (e.g. printed values returned by `curve_fit`). Example printout in the code reports:

```
Fit parameters: A=3.64±0.18, B=0.49±0.18
```

4.4 Figures and Numerical Outputs

- **Raw vs averaged maps:** A side-by-side comparison demonstrating the effect of 27-day averaging (e.g. Fig. 2).
- **Example single-pair profile:** Meridional flow estimated from one representative 27-day pair (e.g. Fig. 3).
- **Annual mean and fit:** The main science figure showing $\bar{u}(\lambda)$ (solid line), the $\pm 1\sigma$ band (shaded), and the analytic fit (dashed). This figure should be captioned with the fitting parameters and the number of pairs used to compute the mean.

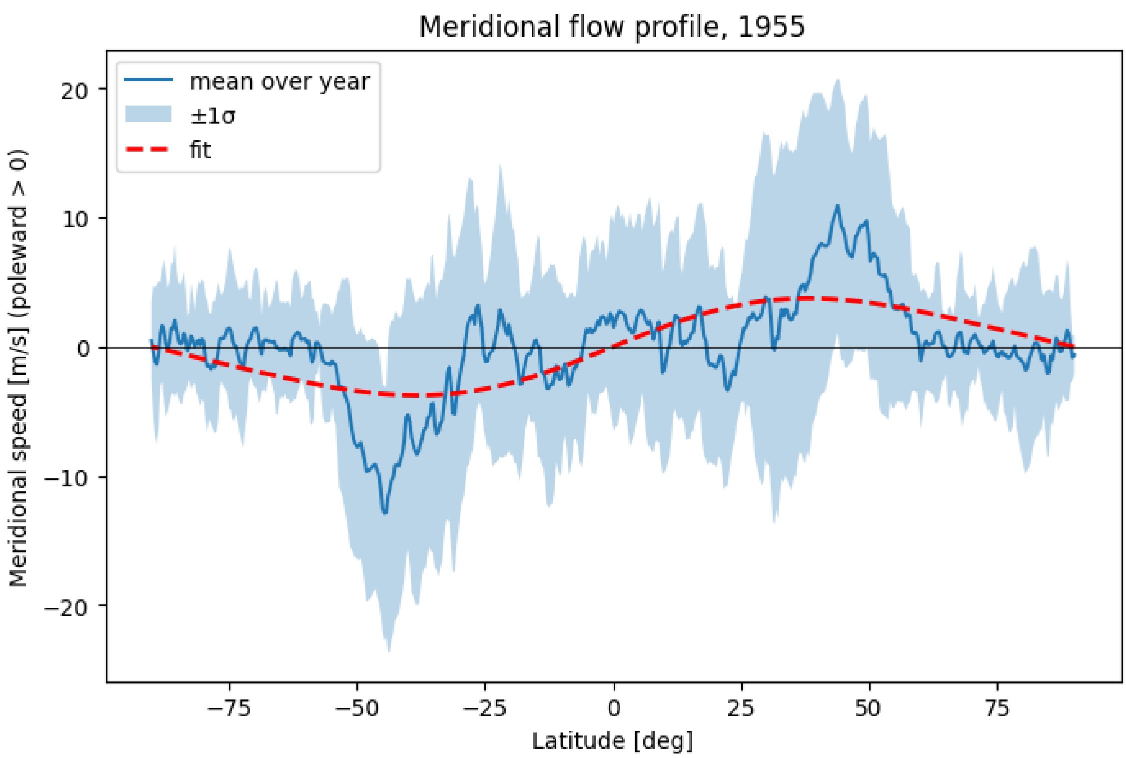

Figure 4: Annual meridional flow profile for 1955. Solid blue line: mean $\bar{u}(\lambda)$. Shaded region: $\pm 1\sigma$ across all 27-day pairs. Dashed red line: analytic fit $A\sin(2\lambda) + B\sin(4\lambda)$.

4.5 Numeric Results

The meridional flow profile obtained for the year 1955 yields the following quantitative values:

- **Peak poleward speed (Northern Hemisphere):**

$$+10.93\,\mathrm{m\,s^{-1}} \quad \text{at latitude } +43.86^\circ.$$

- **Peak poleward speed (Southern Hemisphere):**

$$-12.88\,\mathrm{m\,s^{-1}} \quad \text{at latitude } -44.56^\circ.$$

  (Negative values denote poleward transport in the south.)

- **Global mean of the absolute flow speed:**

$$\langle |u| \rangle = 2.61\,\mathrm{m\,s^{-1}}.$$

- **Global RMS meridional speed:**

$$u_{\mathrm{RMS}} = 3.88\,\mathrm{m\,s^{-1}}.$$

- **Number of independent 27-day pairs used:**

$$N = 338.$$

These values summarize the large-scale poleward transport of magnetic flux across latitudes during 1955 and form the basis for the fitted analytical flow model.

5 Future Scope

The present study demonstrates a fully data-driven method for estimating the solar meridional flow from sequences of magnetic maps. Several extensions and improvements are possible:

- **Sub-pixel correlation.** The current method identifies integer-pixel shifts. Using parabolic or Gaussian sub-pixel interpolation would improve accuracy, especially at high latitudes where the signal is weak.
- **Alternative smoothing windows.** While a 27-day average is physically motivated, testing shorter (13-day) and longer (54-day) windows could balance noise reduction against temporal resolution.
- **Analytical flow-model fitting.** The fitted two-term sinusoidal model can be refined by including asymmetric terms or higher harmonics, enabling more realistic SFT modelling.
- **Multi-year reconstruction.** Extending the analysis to multiple years would allow investigation of solar-cycle variability in meridional flow and comparison with historical observations.
- **Active-region masking.** Removing newly emerging regions before cross-correlation may reduce noise and isolate the large-scale, long-lived magnetic patterns.
- **Comparison with full SFT inversion.** The empirical flow profile may be compared with meridional-flow estimates obtained by solving the full SFT equation, helping quantify parameter degeneracies.
- **Advanced tracking techniques.** Optical-flow methods, feature tracking, or machine-learning-based flow estimation could provide more stable and high-resolution measurements.

Overall, these extensions would help generalize and strengthen the empirical reconstruction of meridional flow and its role in the solar cycle.

6 Conclusion

In this project, I developed a data-driven method to estimate the Sun's meridional flow by tracking the poleward motion of magnetic features in daily SFT-simulation maps. The analysis consisted of reading and preprocessing the SFT maps, applying a 27-day running average to suppress short-lived variability, and computing latitude-dependent displacements using cross-correlation. These displacements were converted into physical velocities, and the resulting profiles were averaged over all 27-day pairs to obtain a yearly mean flow for 1955.

The final profile shows clear poleward flow in both hemispheres, with peak speeds of approximately $+11\,\mathrm{m\,s^{-1}}$ in the north and $-13\,\mathrm{m\,s^{-1}}$ in the south. These values fall within the expected range reported in the literature and agree qualitatively with the meridional flow shapes discussed in recent SFT reviews. The fitted analytic model provides a smooth parameterization suitable for incorporation into SFT simulations.

This empirical approach is valuable because it avoids the degeneracies inherent in direct inversion of the SFT equation, relying instead on the observed advection of magnetic-field patterns. The method therefore provides an independent constraint on the large-scale flow that governs polar-field formation, a key element in solar cycle prediction. The framework developed here can be extended to multiple years to study temporal changes in meridional flow and its influence on the solar dynamo.

7 Technical Implementation

All code used for data processing, map averaging, cross-correlation tracking, and meridional-flow reconstruction is available in the project repository: `https://github.com/K-K-Gupta/UG-Project`. The repository includes the full Jupyter notebook, scripts, and generated figures for complete reproducibility.

References

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