CSCI 416 - Intro to Machine Learning Homework 9/30 Kyle Chen

(Due September 30)

Question 1

(1)

$$\frac{\partial}{\partial u} f(u, v) = 16uv^4 + 6$$

(2)

$$\frac{\partial}{\partial v} f(u, v) = 32u^2v^3 + 12v^2$$

(3)

$$\frac{\partial}{\partial u} g(u, v, w) = \frac{x}{u} + yvw^3$$

(4)

$$\frac{\partial}{\partial v} g(u, v, w) = yuw^3$$

(5)

$$\frac{\partial}{\partial w} g(u, v, w) = 3yuvw^2$$

(6)

$$\frac{\partial}{\partial u} h(u, v) = \sum_{i=1}^{m} (x^{(i)})^2 u + x^{(i)} y^{(i)} v$$

(7)

$$\frac{\partial}{\partial v} h(u, v) = \sum_{i=1}^{m} x^{(i)} y^{(i)} u + (y^{(i)})^2 v$$

Question 2

(1) Negative.

(2) Negative.

(3) Positive.

(4) Negative.

(5) $u = 1, v = 2$

Question 3

(1)
$$\begin{bmatrix} 3 & -1 \\ 2 & 5 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} u & a \\ v & b \end{bmatrix} = \begin{bmatrix} 3u - v & 3a - b \\ 2u + 5v & 2a + 5b \\ -2u + 2v & -2a + 2b \end{bmatrix}$$

(2) Yes, product $AB \in \mathbb{R}^{2\times 4}$

(3) No.

(4) $y^T A$ is a row vector, $y^T A \in \mathbb{R}^{1\times 2}$

(5) Ax is a column vector, $Ax \in \mathbb{R}^3$

(6) Since we know that the zero vector 0 is a row vector, we can proceed as follows:

$$\begin{aligned}
(Bx + y)^T A^T &= 0 \\
(Bx + y)^T A^T \cdot (A^T)^- 1 &= 0 \cdot (A^T)^- 1 \\
(Bx + y)^T (A^T \cdot (A^T)^- 1) &= 0 \cdot (A^T)^- 1 \\
(Bx + y)^T (I^{n\times n}) &= 0 \\
(Bx + y)^T &= 0 \\
((Bx + y)^T)^T &= 0^T \\
Bx + y &= 0^T \\
Bx &= 0^T - y \\
Bx &= -y \\
B^{-1} \cdot Bx &= B^{-1} \cdot (-y) \\
(B^{-1} \cdot B)x &= -B^{-1} y \\
I^{n\times n} x &= -B^{-1} y \\
x &= -B^{-1} y
\end{aligned}$$

and we are done.

Question 4

```
Part 1:
 A =
 [[-2 -3]
 [ 1  0]]
B =
 [[-1  1]
 [ 1  0]]
x =
 [[-1]
 [ 1]]
Part 2:
 C =
 [[-0.          1.        ]
 [-0.33333333 -0.66666667]]
Part 3:
 AC =
 [[ 1.00000000e+00 -1.11022302e-16]
 [ 0.00000000e+00  1.00000000e+00]]
CA =
 [[1. 0.]
 [0. 1.]]
Part 4:
 Ax =
 [[-1]
 [-1]]
Part 5:
 A^(T) A =
 [[5 6]
 [6 9]]
Part 6:
 Ax - Bx =
 [[-3]
 [ 0]]
Part 7:
 ||x|| =
 [[1.41421356]]
Part 8:
 ||Ax - Bx|| =
 3.0
Part 9:
 The first column of A is:
 [[-2]
 [ 1]]
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Part 10:
 New B matrix is:
 [[-1  1]
 [ 1  0]]
Part 11:
 The element-wise product between the first and second columns of A is:
 [[6]
 [0]]
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