Literature Review

The Weapon-Target Assignment Problem [Kline et al., 2019]

- As $\uparrow$ quantity and quality of missiles, effective allocation research emerged.
- Weapon Target Assignment (WTA) aka Missile Allocation Problem (MAP) $\Leftrightarrow$ minimize probabiilty of a missile destroying a protected assignment
- Sometimes offense perspective OR defense perspective
- WTA $\rightarrow$ Static WTA (SWTA) or Dynamic WTA (DWTA)
- SWTA:
  input: num. of incoming missiles (targets), num. of interceptors (weapons), probabilities of destroying targets
  output: how many of each weapon type to shoot at each target
- DWTA includes time as a dimension. Two variants: two-stage and shoot-look-shoot
  - Two-stage DWTA:
    stages/input: 1. SWTA and 2. probability distribution of various kinds of targets
    output: 1. allocation of weapons and 2. how many weapons to reserve to minimize prob. of destruction
  - Shoot-kill-Shoot DWTA:
    replicates SWTA too, but enables observation of leakers: target that maybe survived the initial engagement for a subsequent engagement
    solution: allocation of weapons and reservation of weapons to rengage any leakers
- WTA is NP-Complete, so majority of solutions seek near-optimal solutions in real-time, or fast-enough solutions before the adversary reaches their goals.
- These solutions use heuristics or have exact solutions applied to variants of the WTA problem

Formulations

Notation:

- p_{ij} : the probability weaponi destroys target j
- $q_{ij} (= 1 - p_{ij})$: the probability weapon i fails to destroys target j
- V_j : the destructive value of target j

- x_{ij} : the number of weapons of type i assigned to target j
- K : the number of protected assets
- a_k : the value of asset k
- n : the number of targets
- m : the number of weapon types
- w_i : the number of weapons of type i
- c_{ij} : a cost parameter for assigning a weapon of type i to target j
- $\mathcal{F}$: the set of feasible assignments
- γ_{jk} : the probability target j destroys asset k
- s_j : the maximum number of weapons that can be assigned to target j
- t : the number of stages

SWTA's main formulation:

$$\begin{array}{rl} \min & \sum_{j=1}^{n} V_j \prod_{i=1}^{m} q_{ij}^{x_{ij}} \\ \text{s.t.} & \sum_{j=1}^{n} x_{ij} \leq w_i, \text{ for } i = 1, \dots, m \\ & x_{ij} \in \mathbb{Z}_+, \text{for } i = 1, \dots, m, j = 1, \dots, n \end{array}$$

In English: find the best assignment of number of weapons, across all types, that minimizes the survival rate of the targets, with higher emphasis of those with more destructive value. The "such that (s.t.)" requriements indicate that we must respect our supply capacity of weapons and that we must allocate at least weapon of each type to all targets.

Other formulations: They make simplifying assumptions, such as assuming that the probability of using any weapon to destroy a target is the same or that there is only a capacity of one weapon of each type. Maybe there is one weapon type per target. All of these different assumptions simplify optimization in some way, but obviously each has its pros and cons in terms of optimizability and applicability to real-world scenarios.

DWTA's main formulation: - I'm not going to really elaborate on the math; it gets substantially more complicated. I think an important takeaway is that because SWTA was already intractable with its large amount of permutations to begin with, DWTA is certainly intractable as well, given how it's basically just a stack of SWTAs. - Feel free to look at the math yourself, but be willing to spend a lot of attention and time. I just did not think it was worth it. - Antoher important takeaway: it's a must to use approximation methods.

Exact Algorithms for SWTA

Maximum Marginal Return (MMR)

This algorithm assums that the probabiilty of kill for any weapon target to target j is the same.

Then, the optimal solution is:

1. Assign $x_{ij} = 1$ where $\{i, j\} \in argmax(V_j p_{ij})$
2. $V_j \leftarrow V_j(1 - p_{ij})$
3. $p(i, \cdot) \leftarrow 0$ and $p(\cdot, j) \leftarrow 0$
4. Repeat until all weapons have been assigned

Note that it's best to divide the weapons evenly across all targets when there is only one probability of kill, regardless of weapon or target.

On Solving the Original SWTA Formulation

When you try exhaustive searches: a problem with 9 weapons and 8 targets take 13 min to run to completion, and adding one additional target takes 43.7 min. to run to completion. Thus, there is a combinatorial explosion in run time as a function of the problem size.

Brief Mention of Other Algorithms

- Branch-and-bound
- Lower bounding strategies (generalized network flow, MMR, and minimum cost flow)
- Linear integer programming
- Joint Munition Effectiveness Manual (JMEM)

Exact Algorithms for DWTA

- Mathematics-based (Burr et al. (1985) & Soland (1987) & Hosein (1989))
- Concave Adaptive Value Estimation (CAVE) with modified MMR (aka MMR Plus Algorithm)
- Dynamic Programming

Heruistics for SWTA

All the above solutions focus on the optimal solution, but heuristic algorithms focus on real-time solutions.

- Genetic algorithms
- Very Large Scale Neighborhood (VLSN) search metahueristic

- Ant Colony Optimization
- Integer relaxed NLP and rounding schemes
- Neural networks
- Network-flow-based construction heruistic
- Simulated Annealing (SA)
- Variable Neighbor Search (VNS)
- Tabu Search
- Particle Swarm Optimization (PSO)
- Lagrangian relaxation Branch and Bound
- Hungarian Algorithm

Heruistics for DWTA

- ALIAS algorithm
- Decomposition algorithm
- Virtual permutations
- Rule-based
- Hungarian Algorithm
- Neuro-dynamic programming to obtain near optimal policies. Optimal policies are obtained through dynamic programming

Discussion

The branch and bound algorithm and the genetic algorithm are both widely used in the literature. Bogdanowicz (2012)'s algorithm or Xin et al. (2010)'s rule-based hueristic efficiently exploit the special structure of the problem.

Applying reinforcement learning to the weapon assignment problem in air defence [Mouton et al., 2011]

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Optimization of Weapon-Target Pairings Based on Kill Probabilities [Bogdanowicz et al., 2013]

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A New Approach to Weapon-Target Assignment in Cooperative Air Combat [Chang et al., 2017]

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A Coordinated Air Defense Learning System Based on Immunized Classifier Systems [Nantogma et al., 2021]

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The state-of-the-art review on resource allocation problem using artificial intelligence methods on various computing paradigms [Joloudari et al., 2022]

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