A Survey on Weapon Target Allocation Models [Ghanbari et al., 2021]

- Two key components of command and control are: weapon target allocation (WTA) and threat evaluation.
- Resource allocation is stochastic/uncertain with regard to the WTA problem.
- The WTA component of the WTA problem can be considered in 3 parts: response planning, response execution, outcome assessment.
- There exists three basic models:

Basic Model 1: For maximizing damage to enemy (minimize expected target values F), we have

$$\min(\ F\ )\ =\ \sum_{i=1}^{|T|} V_i\ \Pi_{k=1}^{|W|}(1-P_{ik})^{x_ik}$$

This is the general WTA formula.

Basic Model 2: For allocation of available units to maximize expected total protection value J, we have

$$\max(\ J\ )\ =\ \sum_{j=1}^{|A|}\omega_j\ \Pi_{i\in G_j}(1-\pi_{ij}\Pi_{k=1}^{|W|}(1-P_{ik})^{x_ik}$$

Basic Model 3: This is the model for Dynamic WTA at stage t given the total expected combat value of surviving assets

$$\max(\ J_tX^t\ )\ =\ \sum_{j=1}^{|A(t)|}\omega_j\ \Pi_{i=1}^{|T(t)|}\left[1-\pi_{ij}\Pi_{h=t}^{S}\Pi_{k=1}^{|W(t)|}(1-p_{ik}(h))^{x_{ih}(h)}\right]$$

Variable Definitions	
Sets	
T_i	Set of detected threats $i = 1, 2, \cdots, I$.
w_k	Set of resources $k = 1, 2, \cdots, K$.
A_j	Set of assets $j = 1, 2, \cdots, J$.
S	Set of engagement stages, $s = 1, 2, \cdots, S$.
$A(t), T(t), W(t)$	Set of current "defended assets, hostile targets, and available weapons during stage t, respectively."
Parameters	
P_{ik}	Estimated effectiveness/probability that weapon $w_k \in W$ neutralizes threat $T_i \in T$ if assigned to it.
π_{ij}	Estimated probability threat $T_i \in T$ destroys asset $A_j \in A$.
V_{ik}	Threat value of the threat-asset pair (T_i, A_j).
ω_j	Protection value of asset A_j.
C_{ik}	Resource usage cost for assigning w_k to T_i.
Variables	
X_{ik}	Is 1 if resource w_k is assigned to T_i, 0 otherwise.
$[X^s_{ik}]_{I \times K}$	Decision matrix at stage s.
h	Index of stages $t, \cdots, S$.

- Dynamic WTA (DWTA) suffer from curse of dimensionality.
- WTA problem has two perspectives: *single platform perspective* and *force coordination perspective*. The former is single platform defending one asset against incoming threats, the latter is a command and control platform defending multiple assets.
- Within these perspectives exist two paradigms: *threat-by-threat* and *multi-threat*. The former being sequential targeting and the latter being parallel targeting.

- There also exists two different prioritizations of defense, as shown by basic models 1 and 2.
- Static WTA (SWTA) constraints: $X_{ik} \in \{0,1\} \forall i \in \{1,2,\cdots,|T|\}, \forall k \in \{1,2,\cdots,|W|\}$ given the equations: $\sum_{i=1}^{|T|} X_{ik} = 1 \quad \forall k \in \{1,2,\cdots,|W|\}$ if each firing unit must be assigned a target and $\sum_{i=1}^{|T|} X_{ik} \leqslant 1 \quad \forall k \in \{1,2,\cdots,|W|\}$ otherwise, with X_{ik} being a target i being assigned to a resource k in the constraint matrix X.
- Dynamic WTA (DWTA) problems have more constraints as follows:

Weapon multi-target constraint: This constraint describes multi-target systems. As each multi-target system can also be considered as separate systems, $n_k = 1 \forall k \in \{1,2,\cdots,W\}$.

$$\sum_{i=1}^{|T|} x_{ik}(t) \leqslant n_k \quad \forall t \in \{1,2,\cdots,S\}, \forall k \in \{1,2,\cdots,|W|\}$$

Strategy constraint: This constraint limits system-usage cost per target at stage t. m_i depends on performance of available resource k on target i. For missile systems, $m_i = 1$, and for artillery systems, $m_1 \geqslant 1$.

$$\sum_{k=1}^{|W|} x_{ik}(t) \leqslant m_i \quad \forall t \in \{1,2,\cdots,S\}, \forall i \in \{1,2,\cdots,|T|\}$$

Resource constraint: This constraint governs over ammunition availability.

$$\sum_{t=1}^{S} \sum_{i=1}^{|T|} x_{ik}(t) \leqslant N_k, \quad \forall k \in \{1,2,\cdots,|W|\}$$

Engagement feasibility constraint: This constraint is over the resource-target relationship: if a target i can be hit by a resource k at stage t, then $f_{ik}(t) = 1$, and $f_{ik}(t) = 0$ otherwise.

$$x_{ik}(t) \leqslant f_{ik}(t), \qquad \forall t \in \{1,2,\cdots,S\}, \forall i \in \{1,2,\cdots,|T|\}$$

Optimization of decision support system based on three-stage threat evaluation and resource management [Naseem et al., 2017]

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An approximate dynamic programming approach for comparing firing policies in a networked air defense environment [Summers et al., 2020]

- For the interception of theater ballistic missiles (TBMs), the U.S. Air Force noted that there exists two options for the WTAP: the Markov decision processes, an extension of the Markov chain, and approximate dynamic programming (ADP) for WTAPs involving TBMs.

- For the latter option, there exist two different algorithmic approaches: approximate value iteration and approximate policy iteration (API); the paper uses the latter.

- The API algorithmic strategy maps the system state – includes incoming target amount, current asset health, and interceptor health –
to reaction fire against incoming targets, specifically how many interceptors to assign to each incoming target.

- MDP Formula:
Let $\Gamma = \{1, 2, \cdots, T\},\ T \leqslant \infty$, where the number of decision epochs T is random and follows a geometric distribution with parameter $0 \leqslant \gamma < 1$.
Asset status component $a_t = (a_{ti})_{i \in A} \equiv (a_{t1}, a_{t2}, \cdots, a_{t|A|}$, where set of all assets $A = \{1, 2, \cdots, |A|\}$ and health of asset $a_{ti} \in \{0, 0.25, 0.5, 0.75, 1\}$, with 0 being destroyed and 1 being undamaged.
Let resource inventory component $R_t = (R_{ti})_{i \in A} \equiv (R_{t1}, R_{t2}, \cdots, R_{t|A|})$, with status element $R_{ti} \in \{0, 1, \cdots r_i\}$, with

- The MDP formula is too complex to be summarized, please refer to pages 7-10 of the paper.

- There are two value function approximations for API: least squares policy evaluation (LSPE) and least squares temporal difference (LSTD).

API-LSPE Algorithm

Step 0: Initialize θ^0 .

Step 1:

for $n = 1$ to N (Policy Improvement Loop).

Step 2:

for $k = 1$ to K (Policy Evaluation Loop).

i. Generate a random post-decision state $S^x_{t-1,k}$.
ii. Record basis function evaluation $\phi(S^x_{t-1,k})$.
iii. Simulate transition to next pre-decision state $S_{t,k}$ using equation (6).
iv. Determine decision $x_{t,k} = X^{\pi_{adp}}(S_{t,k}|\theta^{n-1})$ using equations (5), (7), (9).

v. Record cost $C(S_{t,k}, x_{t,k})$.

end for

Update θ^n and policy:

$\hat{\theta} = [(\Phi_{t-1})^T(\Phi_{t-1}]^-1(\Phi_{t-1})^T C_t$

$\theta^n = a_n\hat{\theta} + (1-a_n)\theta^{n-1}$

end for

Return $X^{\pi_{adp}}(\cdot|\theta^N)$ and θ^N

End.

API-LSTD Algorithm

Step 0: Initialize θ^0 .

Step 1:

for $n = 1$ to N (Policy Improvement Loop)

Step 2:

for $k = 1$ to K (Policy Evaluation Loop).

i. Generate a random post-decision state $S^x_{t-1,k}$.

ii. Record basis function evaluation $\phi(S^x_{t-1,k})$.

iii. Simulate transition to next pre-decision state $S_{t,k}$ using equation (6).

iv. Determine decision $x_{t,k} = X^{\pi_{adp}}(S_{t,k}|\theta^{n-1})$ using equations (5), (7), (9).

v. Record cost $C(S_{t,k}, x_{t,k})$.

vi. Record next post-decision state $S^x_{t,k}$ with $x_{t,k}$ and equation (5).

vii. Record basis function evaluation $\phi(S^x_{t,k})$.

end for

Update θ^n and policy:

$\hat{\theta} = [(\Phi_{t-1} - \gamma\Phi_t)^T(\Phi_{t-1} - \gamma\Phi_t]^-1(\Phi_{t-1} - \gamma\Phi_t)^T C_t$

$\theta^n = a_n\hat{\theta} + (1-a_n)\theta^{n-1}$

end for

Return $X^{\pi_{adp}}(\cdot|\theta^N)$ and θ^N

End.

With equations $5, 6, 7, 9$ being expressed in page 10-11.

- θ is a parameter vector created from the set of basis functions $\phi_f(S_t))_{f\in F}$, with F being the set of basis functions that reduces the size of the state variable to the most significant factors.

- We also have the basic function vector $\Phi_{t-1} \rightarrow \begin{bmatrix} \phi(S_{t-1,1}) \\ \vdots \\ \phi(S^x_{t-1\,K}) \end{bmatrix}$ and cost vector $C_t \rightarrow$

$$\begin{bmatrix} C(S_{t,1}) \\ \vdots \\ C(S_{t,K}) \end{bmatrix}.$$

- We can estimate θ with $a_n = \frac{a}{a+n-1},\ a \in (0, \infty)$
- Of the two ADP methods and given the standard methods of engagement – them being the one-target-per-interceptor, or "Match" Policy, and the one-target-per-two-interceptors, or "Overmatch" Policy – the LSPE algorithm outperforms the standard methods of engaging TBMs when the duration of the engagement is inherently short or the targets have high hit likelihood – i.e. are high quality. However, the Match Policy outperforms the LSPE algorithm if the engagement is long and the attacker's weapons are of lower quality. The LSTD algorithm follows this same trend.

Threat Evaluation In Air Defense Systems Using Analytic Network Process [Unver and Gürbüz, 2019]

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References

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