

The Weapon-Target Assignment Problem

[Kline et al., 2019]

- As \uparrow quantity and quality of missiles, effective allocation research emerged.
- Weapon Target Assignment (WTA) aka Missile Allocation Problem (MAP) \Leftrightarrow minimize probability of a missile destroying a protected assignment
- Sometimes offense perspective OR defense perspective
- WTA \rightarrow Static WTA (SWTA) or Dynamic WTA (DWTa)
- SWTA:
input: num. of incoming missiles (targets), num. of interceptors (weapons), probabilities of destroying targets
output: how many of each weapon type to shoot at each target
- DWTa includes time as a dimension. Two variants: two-stage and shoot-look-shoot
 - Two-stage DWTa:
stages/input: 1. SWTA and 2. probability distribution of various kinds of targets
output: 1. allocation of weapons and 2. how many weapons to reserve to minimize prob. of destruction
 - Shoot-kill-Shoot DWTa:
replicates SWTA too, but enables observation of leakers: target that maybe survived the initial engagement for a subsequent engagement
solution: allocation of weapons and reservation of weapons to reengage any leakers
- WTA is NP-Complete, so majority of solutions seek near-optimal solutions in real-time, or fast-enough solutions before the adversary reaches their goals.
- These solutions use heuristics or have exact solutions applied to variants of the WTA problem

Formulations

Notation:

- p_{ij} : the probability weapon i destroys target j
- $q_{ij}(= 1 - p_{ij})$: the probability weapon i fails to destroy target j
- V_j : the destructive value of target j

- x_{ij} : the number of weapons of type i assigned to target j
- K : the number of protected assets
- a_k : the value of asset k
- n : the number of targets
- m : the number of weapon types
- w_i : the number of weapons of type i
- c_{ij} : a cost parameter for assigning a weapon of type i to target j
- \mathcal{F} : the set of feasible assignments
- γ_{jk} : the probability target j destroys asset k
- s_j : the maximum number of weapons that can be assigned to target j
- t : the number of stages

SWTA's main formulation:

$$\begin{aligned}
\min \quad & \sum_{j=1}^n V_j \prod_{i=1}^m q_{ij}^{x_{ij}} \\
\text{s.t.} \quad & \sum_{j=1}^n x_{ij} \leq w_i, \text{ for } i = 1, \dots, m \\
& x_{ij} \in \mathbb{Z}_+, \text{ for } i = 1, \dots, m, j = 1, \dots, n
\end{aligned}$$

In English: find the best assignment of number of weapons, across all types, that minimizes the survival rate of the targets, with higher emphasis of those with more destructive value. The “such that (s.t.)” requirements indicate that we must respect our supply capacity of weapons and that we must allocate at least weapon of each type to all targets.

Other formulations: They make simplifying assumptions, such as assuming that the probability of using any weapon to destroy a target is the same or that there is only a capacity of one weapon of each type. Maybe there is one weapon type per target. All of these different assumptions simplify optimization in some way, but obviously each has its pros and cons in terms of optimizability and applicability to real-world scenarios.

DWTA's main formulation: - I'm not going to really elaborate on the math; it gets substantially more complicated. I think an important takeaway is that because SWTA was already intractable with its large amount of permutations to begin with, DWTA is certainly intractable as well, given how it's basically just a stack of SWTAs. - Feel free to look at the math yourself, but be willing to spend a lot of attention and time. I just did not think it was worth it. - Another important takeaway: it's a must to use approximation methods.

Exact Algorithms for SWTA

Maximum Marginal Return (MMR)

1. Assign $x_{ij} = 1$ where $\{i, j\} \in \operatorname{argmax}(V_j p_{ij})$
2. $V_j \leftarrow V_j(1 - p_{ij})$
3. $p(i, \cdot) \leftarrow 0$ and $p(\cdot, j) \leftarrow 0$
4. Repeat until all weapons have been assigned

Note that it's best to divide the weapons evenly across all targets when there is only one probability of kill, regardless of weapon or target.

Exact Algorithms for DWTA

Heruistics for SWTA

Heruistics for DWTA

Discussion

Applying reinforcement learning to the weapon assignment problem in air defence [Mouton et al., 2011]

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Optimization of Weapon-Target Pairings Based on Kill Probabilities [Bogdanowicz et al., 2013]

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A New Approach to Weapon-Target Assignment in Cooperative Air Combat [Chang et al., 2017]

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A Coordinated Air Defense Learning System Based on Immunized Classifier Systems [Nantogma et al., 2021]

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The state-of-the-art review on resource allocation problem using artificial intelligence methods on various computing paradigms [Joloudari et al., 2022]

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References

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