2018 9th International Conference on Mechanical and Aerospace Engineering

Trajectory Estimation for a Ballistic Missile in Ballistic Phase using IR Images

Kyujin Moon, Hojun Kwon, Chang-Kyung Ryoo
Department of Aerospace Engineering
Inha University
Incheon, Korea
e-mail: joralmkj@gmail.com, gallantry@hanmail.net, ckryoo@inha.ac.kr

Hongchul Sim
3rd R&D Institute
Agency for Defense Development
Daejeon, Korea
e-mail: hcsim@add.re.kr

***Abstract*—It is possible to respond to threats of ballistic missile by using IR images from a geosynchronous satellite. However, IR images from the geosynchronous satellite are mainly used to detect ballistic missile, not to track. In this paper, we proposed a trajectory tracking algorithm for the ballistic missile using satellite IR image. The proposed algorithm estimates the trajectory of the ballistic missile by solving the optimization problem to find the initial value of the midcourse phase. For the ballistic missile flight and IR image measurement simulation, a pseudo 6-DOF was constructed and numerically showed the performance of the proposed algorithm.**

Keywords-geosynchronous satellite; trajectory estimation; ballistic missile; optimization; IR images

I. INTRODUCTION

The first ballistic missile, also known as V2, were developed by Germany in World War II[1,2]. After World War II, the technology of the ballistic missiles has developed rapidly due to an arms race between the United States and the Soviet Union. As technology has developed, the ballistic missiles have been widely deployed and operated on the battlefield as not only strategic weapons but also strategic weapons. It provide deterrence by punishment against nuclear proliferation. However, the Treaty on the Non-Proliferation of Nuclear Weapons was opened for signature in 1968 as fears of a nuclear war spread. Therefore, a MD(Missile Defense) system, one of the deterrence by denial, has emerged as an alternative.

The MD, defensive weapon system that destroys ballistic missiles or warheads before they reach their target, is being developed in many countries around the world, including the United States. In order to effectively respond to the ballistic missile threats, it is important to rapidly detect the launch and accurately track the trajectory of the ballistic missile. Various radars such as an early warning radar and a detection radar have been operated for these reasons, and studies have been carried out to track the ballistic missile targets[3-6].. However, detection and tracking are not possible with radar until the ballistic missiles rise above the horizon because of the curvature of the earth, thereby reducing response time. It is especially fatal for ROK(Republic of Korea), where has many mountainous terrain and threatened by short-range ballistic missiles with short flight times.

In order to overcome this problem, IR(Infra-Red) images from a geosynchronous satellite is needed that can be obtained always because the satellite is in GSO(GeoSynchronous Orbit). IR images of geosynchronous satellite are mainly used for detecting the launch of ballistic missiles quickly, not tracking, by measuring IR signal strength, so few studies has been done to track the ballistic missile targets. In this paper, we propose an algorithm for tracking trajectory of the ballistic missile using IR image measured from geosynchronous satellites.

This paper is composed as follows. In Section 2. The physical model, flight phases and coordinate systems are introduced to simulate the flight of the ballistic missiles and IR image. The algorithm to estimate the trajectory of the ballistic missile is explained in Section 3. Numerical simulation results are provided for verification of proposed algorithm in Section 4. Finally, the concluding remarks are in Section 5.

II. PHYSICAL MODEL AND COODINATE SYSTEM

A. *Pseudo 6-DOF Equations of Motion*

In this paper, we consider a pseudo 6-DOF(Degree of Freedom) simulation, expanded a 3-DOF simulation. Originally, in order to simulate a motion of an object in 3-D(Dimensional) space, a 6-DOF simulation consisting of force and moment equations have to be performed. It requires modelling for various subsystems of the ballistic missile such as actuator, autopilot and etc. However, we are only interested in briefly simulating the trajectory of a ballistic missile, not details, in the 3-D space. Thus, we constructs the pseudo 6-DOF simulation with assumption, the subsystems are well designed so that the ballistic missile follows guidance commands calculated by guidance computer with a constant time delay. Fig. 1 shows algorithm how to calculate an acceleration of the ballistic missile in the pseudo 6-DOF. It makes it possible to calculate attitude angles of the ballistic missile using Fig. 2 without the moment equations. The equations of motion for the pseudo 6-DOF are given by

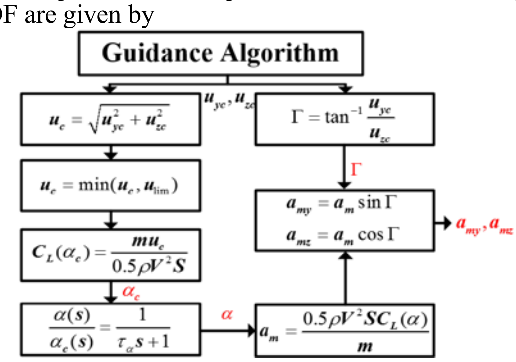

Figure 1. The flow diagram for the acceleration calculation

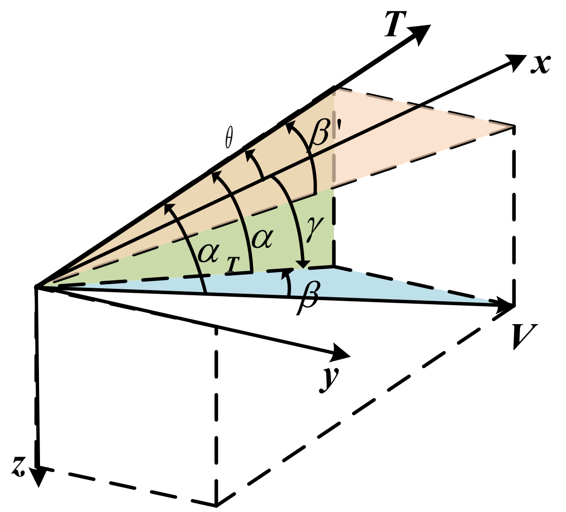

Figure 2. The geometry of the pseudo 6-DOF.

$$
\begin{aligned}
\dot{x} &= V\cos\gamma\cos\beta' \\
\dot{y} &= V\cos\gamma\sin\beta' \\
\dot{z} &= -V\sin\gamma \\
\dot{V} &= T\cos\alpha_T - D - g\sin\gamma \\
\dot{\gamma} &= \frac{a_{mz} + T\sin\alpha_T\cos\Gamma - g\cos\gamma}{V} \\
\dot{\beta} &= \frac{a_{my} + T\sin\alpha_T\sin\Gamma}{V}
\end{aligned}
\tag{1}
$$

where, x, y, z are the reference axes in the 3-D space, V is the missile speed, g is the gravitational acceleration, T, D are the specific thrust and drag, a_{my}, a_{mz} are the axial acceleration of the yaw and pitch plane, γ is the flight path angle, α, β are the angle of attack and sideslip angle and α_T is the total angle of attack. The drag is calculate by the following equation.

$$D = \frac{1}{2}\rho V^2 S C_D / m \tag{2}$$

where C_L, C_D are the lift and drag coefficient, S is the reference area, m is the mass of missile and ρ is the air density. The lift coefficient and drag coefficient are calculated using Missile Datcom and applied by scheduling.

B. Flight Phase of Ballistic Missile

The typical flight phase of the ballistic missile consists of three stages, as shown in Fig. 3, and each stage has the characteristics shown in Table I. Among three stages, the boost phase is the most important because it guides the ballistic missile to target using embedded program, called 'Pitch Program', following four steps[5].

Step 1. Vertical launch : vertical ascending to avoid structural failure and rolling toward the target during this phase.

Step 2. Kick turn : rapid pitch down toward the target before achieving high dynamic pressure.

Step 3. Gravity turn : maintain a zero angle of attack until the dynamic pressure is small to minimize lateral aerodynamic loads on the vehicle.

Step 4. Cut-off : cut-off the thrust if the fuel is burned out or the conditions are achieved to reach the target.

After the thrust cut-off, the ballistic missile is switched to the midcourse phase, as also called ballistic phase, that is only affected by gravity.

TABLE I. THE CHARACTERISTIC OF EACH FLIGHT PHASE

Phase	Characteristic
Boost	- Thrust, aerodynamics, gravity effect - Thrust cut-off for reaching a range shorter than reference range
Midcourse	- Divided into ascent and decent phase - Only gravity effect - Free(Ballistic) flight
Terminal	- Aerodynamics, gravity effect - Speed deceleration and radiance emission due to air resistance

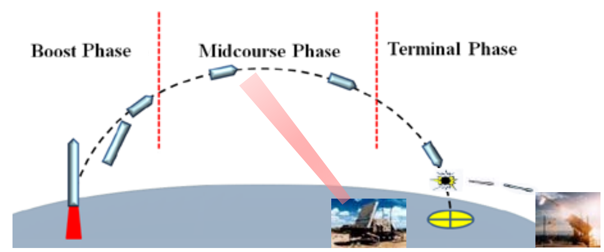

Figure 3. The typical flight phase of the ballistic missile

C. Coordinate System and Transformation for IR Image

In order to convert the trajectory of the ballistic missile represented in the 3-D space into 2-D IR image coordinates, it is necessary to define the coordinate systems and the transformation relation between the coordinate systems.

1) Definition of coordinate systems

a) World coordinate system(E-frame: x_e, y_e, z_e)

It is a reference coordinate system for representing the position of an object relatively. In this paper, the ECEF(Earth Centered Earth Fixed) coordinate system in which is ordinary to represent the position of satellites is set to the world coordinate system[6].

b) Camera coordinate system(C-frame: x_c, y_c, z_c)

The origin is located at the center of the IR camera and the axes are fixed relative to the camera body. The z_c axis coincides with the optical axis of the IR camera. The y_c axis is vertically downwards and x_c axis is mutually perpendicular to y_c and z_c axes. In addition, the angle which the camera coordinate system is rotated by the x_c and y_c axes is defined as θ_{tilt} and θ_{pan}. θ_{tilt} and θ_{pan} are zero when y_e and z_c are parallel.

c) Image coordinate system(I-frame: x_i, y_i, z_i)

The origin is located at the left-down corner of the IR camera sensor and the axes are fixed relative to the sensor. It is determined by FOV(Field Of View), resolution, focal length, etc. Especially, a normalized image coordinate system is widely used with a focal length 1[7].

2) Transformation of the coordinate systems

Fig. 4 shows the geometric relationship between a satellite and a ballistic missile in the previously defined coordinate systems where p_m and p_c are the position of the ballistic missile and IR camera in the world coordinate system respectively.

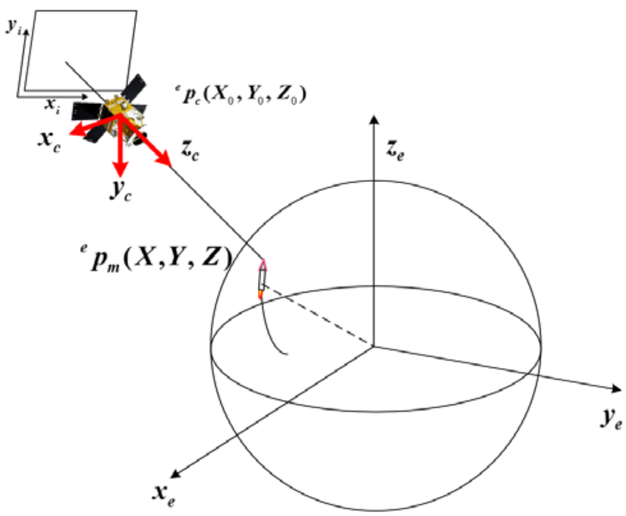

Figure 4. The geometry of the coordinate systems.

a) Transformation from E-frame to C-frame

To convert the location of the ballistic missile in the E-frame to C-frame, the DCM(Direction Cosine Matrix) represented as follows must be calculated.

$$C_e^c = R_z(\theta_{pan}) R_x(-\frac{\pi}{2} + \theta_{tilt}) \tag{3}$$

where R_x and R_z are the DCM for x-axis and z-axis respectively. The position in the C-frame is derived via rotating the position vector from the center of IR camera to ballistic missile using the DCM as following

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = C_e^c \left(\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \right) \tag{4}$$

where X_c, Y_c, Z_c is the position of ballistic missile in C-frame.

b) Transformation from C-frame to I-frame

The 2-D position on the I-frame can be obtained by projection the position of the ballistic missile in the C-frame to I-frame.

$$\begin{bmatrix} X_i \\ Y_i \end{bmatrix} = \begin{bmatrix} \frac{X_c}{Z_c} \times s + c_x \\ \frac{Y_c}{Z_c} \times s + c_y \end{bmatrix} \tag{5}$$

where X_i, Y_i are the position on the I-frame at Z_i = focal length, s is a scale factor calculated by FOV and resolution of IR camera and c_x, c_y are the location of the center of the image sensor.

III. Trajectory Estimation

As mentioned earlier, the ballistic missile only affected by gravity in the midcourse phase. Therefore, it follows the trajectory that satisfies Kepler's second law. It means that the trajectory of the ballistic missile is determined by the initial conditions of midcourse phase. Finally we can estimate the trajectory of the ballistic missile by solving the following parameter optimization problem; Find X, minimizes the following cost function

$$J(X) = \frac{1}{2} \sum \left[\left(\hat{X}_{i,m} - X_{i,s} \right)^2 + \left(\hat{Y}_{i,m} - Y_{i,s} \right)^2 \right] \tag{6}$$

where $X = [p_m, \dot{p}_m]$, $X_{i,m}$, $Y_{i,m}$ are the position measured by IR camera, $\hat{X}_{i,m}$, $\hat{Y}_{i,m}$ are the estimated position on the I-frame using measurement and $X_{i,s}$, $Y_{i,s}$ are the position acquired through simulation with optimization parameter X [7]. In the case of the position measured with IR camera, it is denoted as an integer in pixel units so that the solution is likely to converge to a local minimum due to the effect that the ballistic missile position is rapidly changed. That is why the estimated position are used rather than measurement directly. In this paper, the polynomial curve fitting algorithm is applied for estimation. Fig. 5 shows the overall process of the mentioned earlier[5].

IV. Simulation Results

Numerical simulation is performed to verify the performance of the proposed algorithm. The scenario is constructed to track a ballistic missile using two geosynchronous satellites since it is known that it is possible for the estimation of 3-D trajectory using two 2-D image theoretically. The simulation scenario details are shown in Table II. First, Fig. 6, 7 are the results of ballistic missile flight simulation. Figures show that the characteristics of the flight phase of the ballistic missile mentioned in section 2 are well represented. The results of estimating the trajectory using IR image are shown in Table III and Fig. 8~10. The trajectory of the ballistic missile was estimated with an impact error about 30 kilometer using only IR image. It is caused by a combination of effects such as short measuring time, low resolution, initial value setting, etc. In the simulation scenario, the distance represented by a pixel is about 500 meter. It is critical for the ballistic trajectory significantly changed by the initial value, especially speed. It is expected to be improved by improving the IR camera specification, integrating it with the radar system and so on.

TABLE II. THE SIMULATION SCENARIO DETAILS

Satellite 1 position [LLA]	[0 deg 4 deg 36000 km]
Satellite 2 position [LLA]	[0 deg 0 deg 36000 km]
IR camera resolution	1280 x 960
IR camera FOV	2° x 1.6°
Frame rate	10 Hz

LLA(Latitude, Longitude, Altitude), ARA(Azimuth, Range, Altitude)

TABLE III. THE SIMULATION RESULTS

Cost	1.051 pixel	
Impact error	34.001 km	
x_e **error**	y_e **error**	z_e **error**
1103.129 m	2439.670 m	3157.716
$\dot{x}_e$ **error**	$\dot{y}_e$ **error**	$\dot{z}_e$ **error**
2.695 m/s	82.851 m/s	84.579 m/s

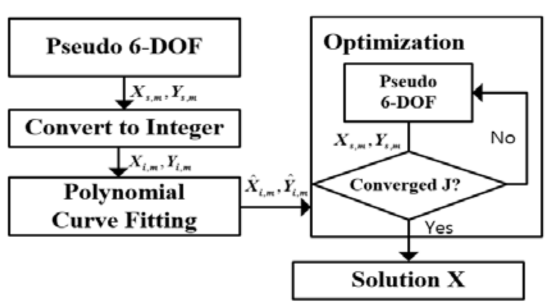

Figure 5. The geometry of the pseudo 6-DOF.

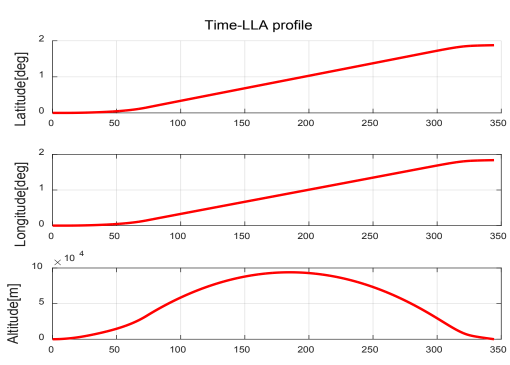

Figure 6. The result of simulation(LLA trajectory profile).

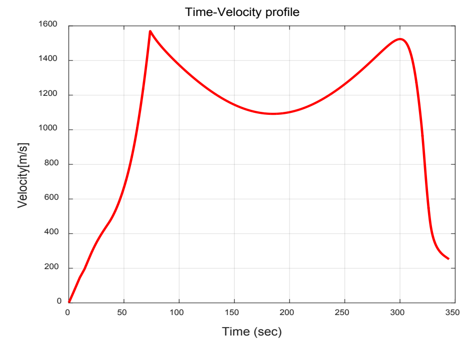

Figure 7. The result of simulation(Velocity profile).

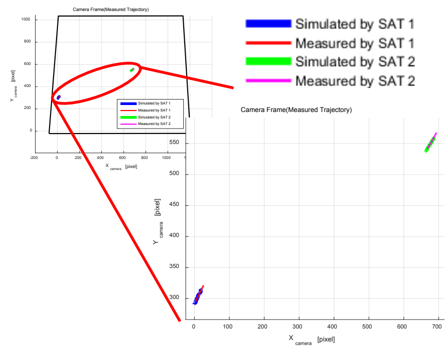

Figure 8. The result of simulation(Trajectory in I-frame).

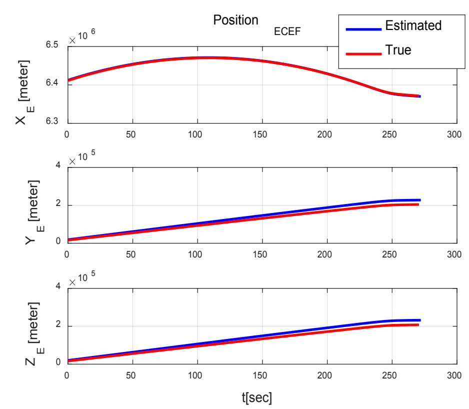

Figure 9. The result of simulation(Position prifile in E-frame).

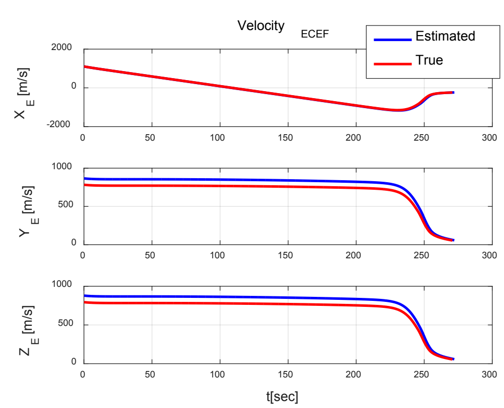

Figure 10. The result of simulation(Veloicity prifile in E-frame).

V. CONCLUSION

In this paper, we proposed the algorithm for estimating the ballistic missile trajectory using IR image. The trajectory of the ballistic missile is estimated by solving the simple optimization problem that find the initial conditions of the midcourse phase via image comparing the measured estimated IR image and simulated trajectory. The simulation results indicate that the trajectory of the ballistic missile is estimated using IR image only. Due to the low IR camera specifications, the performance of algorithm is limited. However, it is expected to be effective in using it as an auxiliary tracking system for integrating with the radar system or replacing the radar if it is not available since ROK has many mountainous terrain. It is also expected that algorithm performance will improve as the IR camera specifications are improved.

ACKNOWLEDGMENT

This study has been done under the support of ADD(Agency of Defense Development) of Korea.

REFERENCES

[1] George M. Siouris, Missile guidance and control systems. Springer Science & Business Media, 2004.

[2] P. Zarchan, Tactical and strategic missile guidance. American Institute of Aeronautics and Astronautics, Inc, 1994.

[3] J. Dituri, Ballistic missile trajectory estimation. Monterey, CA:Naval Progaduate School, 2006.

[4] A. Benavoli, L. Chisci and A.Farina, "Tracking of a ballistic missile with a-priori information," IEEE transactions on aerospace and electronic systems, vol. 43, no. 3, July. 2007, pp. 1000-1016.

[5] S. Park, J. Jung, C. K. Ryoo. and K. Choi, "Detection and classification of a ballistic missile in ascent phase," Proc. International Conference on Control, Automation and Systems (ICCAS 2011), ICROS, Oct. 2011, pp. 1235-1238.

[6] M. Yu, L. Gong, H. Oh, W. H. Chen and J. Chambers, "Multiple model ballistic missile tracking with state-dependent transitions and gaussian particle filtering," IEEE Transactions on Aerospace and Electronic Systems., in press.

[7] K. Moon, W. Park, and C. K. Ryoo, "Performance improvement of low-cost star-tracker simulator via image distortion compensation," Society of Instrument and Control Engineers Annual Conference(SICE 2015), SICE, July. 2015, pp. 842-847.