Literature Review

A Survey on Weapon Target Allocation Models [Ghanbari et al., 2021]

- Two key components of command and control are: weapon target allocation (WTA) and threat evaluation.
- Resource allocation is stochastic/uncertain with regard to the WTA problem.
- The WTA component of the WTA problem can be considered in 3 parts: response planning, response execution, outcome assessment.
- There exists three basic models:

Basic Model 1: For maximizing damage to enemy (minimize expected target values F), we have

$$\min(\ F\ )\ =\ \sum_{i=1}^{|T|} V_i\ \Pi_{k=1}^{|W|}(1-P_{ik})^{x_i k}$$

This is the general WTA formula.

Basic Model 2: For allocation of available units to maximize expected total protection value J, we have

$$\max(\ J\ )\ =\ \sum_{j=1}^{|A|} \omega_j\ \Pi_{i\in G_j}(1-\pi_{ij}\Pi_{k=1}^{|W|}(1-P_{ik})^{x_i k}$$

Basic Model 3: This is the model for Dynamic WTA at stage t given the total expected combat value of surviving assets

$$\max(\ J_t X^t\ )\ =\ \sum_{j=1}^{|A(t)|} \omega_j\ \Pi_{i=1}^{|T(t)|}\left[1-\pi_{ij}\Pi_{h=t}^{S}\Pi_{k=1}^{|W(t)|}(1-p_{ik}(h))^{x_{ih}(h)}\right]$$

Variable Definitions	
Sets	
T_i	Set of detected threats $i = 1, 2, \cdots, I$.
w_k	Set of resources $k = 1, 2, \cdots, K$.
A_j	Set of assets $j = 1, 2, \cdots, J$.
S	Set of engagement stages, $s = 1, 2, \cdots, S$.
$A(t), T(t), W(t)$	Set of current "defended assets, hostile targets, and available weapons during stage t, respectively."
Parameters	
P_{ik}	Estimated effectiveness/probability that weapon $w_k \in W$ neutralizes threat $T_i \in T$ if assigned to it.
π_{ij}	Estimated probability threat $T_i \in T$ destroys asset $A_j \in A$.
V_{ik}	Threat value of the threat-asset pair (T_i, A_j).
ω_j	Protection value of asset A_j.
C_{ik}	Resource usage cost for assigning w_k to T_i.
Variables	
X_{ik}	Is 1 if resource w_k is assigned to T_i, 0 otherwise.
$[X_{ik}^s]_{I \times K}$	Decision matrix at stage s.
h	Index of stages $t, \cdots, S$.

- Dynamic WTA (DWTA) suffer from curse of dimensionality.
- WTA problem has two perspectives: *single platform perspective* and *force coordination perspective.* The former is single platform defending one asset against incoming threats, the latter is a command and control platform defending multiple assets.
- Within these perspectives exist two paradigms: *threat-by-threat* and *multi-threat.* The former being sequential targeting and the latter being parallel targeting.

- There also exists two different prioritizations of defense, as shown by basic models 1 and 2.
- Static WTA (SWTA) constraints: $X_{ik} \in \{0,1\} \forall i \in \{1,2,\cdots,|T|\}, \forall k \in \{1,2,\cdots,|W|\}$ given the equations: $\sum_{i=1}^{|T|} X_{ik} = 1 \forall k \in \{1,2,\cdots,|W|\}$ if each firing unit must be assigned a target and $\sum_{i=1}^{|T|} X_{ik} \leqslant 1 \forall k \in \{1,2,\cdots,|W|\}$ otherwise, with X_{ik} being a target i being assigned to a resource k in the constraint matrix X.

- Dynamic WTA (DWTA) problems have more constraints as follows:

Weapon multi-target constraint: This constraint describes multi-target systems. As each multi-target system can also be considered as separate systems, $n_k = 1 \forall k \in \{1,2,\cdots,W\}$.

$$\sum_{i=1}^{|T|} x_{ik}(t) \leqslant n_k \forall t \in \{1,2,\cdots,S\}, \forall k \in \{1,2,\cdots,|W|\}$$

Strategy constraint: This constraint limits system-usage cost per target at stage t. m_i depends on performance of available resource k on target i. For missile systems, $m_i = 1$, and for artillery systems, $m_1 \geqslant 1$.

$$\sum_{k=1}^{|W|} x_{ik}(t) \leqslant m_i \forall t \in \{1,2,\cdots,S\}, \forall i \in \{1,2,\cdots,|T|\}$$

Resource constraint: This constraint governs over ammunition availability.

$$\sum_{t=1}^{S} \sum_{i=1}^{|T|} x_{ik}(t) \leqslant N_k, \forall k \in \{1,2,\cdots,|W|\}$$

Engagement feasibility constraint: This constraint is over the resource-target relationship: if a target i can be hit by a resource k at stage t, then $f_{ik}(t) = 1$, and $f_{ik}(t) = 0$ otherwise.

$$x_{ik}(t) \leqslant f_{ik}(t), \qquad \forall t \in \{1,2,\cdots,S\}, \forall i \in \{1,2,\cdots,|T|\}$$

Optimization of decision support system based on three-stage threat evaluation and resource management [Naseem et al., 2017]

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An approximate dynamic programming approach for comparing firing policies in a networked air defense environment [Summers et al., 2020]

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Threat Evaluation In Air Defense Systems Using Analytic Network Process [Unver and Gürbüz, 2019]

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The SSA-BP-based potential threat prediction for aerial target considering commander emotion [Wang et al., 2022]

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References

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