1.5: Exponential and Logarithmic Functions

Learning Objectives

- Explain the difference between algebraic and transcendental functions.
- Identify the form of an exponential function.
- Explain the difference between the graphs of x^b and b^x.
- Recognize the significance of the number e.
- Identify the form of a logarithmic function.
- Explain the relationship between exponential and logarithmic functions.
- Describe how to calculate a logarithm to a different base.

In this section we examine exponential and logarithmic functions. We use the properties of these functions to solve equations involving exponential or logarithmic terms, and we study the meaning and importance of the number e.

Transcendental Functions

Thus far, we have discussed algebraic functions. Some functions, however, cannot be described by basic algebraic operations. These functions are known as **transcendental functions** because they are said to "transcend," or go beyond, algebra. The most common transcendental functions are trigonometric, exponential, and logarithmic functions. A trigonometric function relates the ratios of two sides of a right triangle. They are $\sin x, \cos x, \tan x, \cot x, \sec x, \text{ and } \csc x.$(We discuss trigonometric functions later in the chapter.) An exponential function is a function of the form $f(x)=b^x$, where the base $b>0,\ b\neq 1.$ A logarithmic function is a function of the form $f(x)=\log_b(x)$ for some constant $b>0,\ b\neq 1,$ where $\log_b(x)=y$ if and only if $b^y=x.$

Example 1.5.1: Classifying Algebraic and Transcendental Functions

Classify each of the following functions, a. through c., as algebraic or transcendental.

a. $f(x)=\dfrac{\sqrt{x^3+1}}{4x+2}$
b. $f(x)=2^{x^2}$
c. $f(x)=\sin(2x)$

Solution

a. Since this function involves basic algebraic operations only, it is an algebraic function.
b. This function cannot be written as a formula that involves only basic algebraic operations, so it is transcendental. (Note that algebraic functions can only have powers that are rational numbers.)
c. As in part b, this function cannot be written using a formula involving basic algebraic operations only; therefore, this function is transcendental.

Exercise 1.5.1:

Is $f(x)=x/2$ an algebraic or a transcendental function?

Answer

Algebraic

Exponential Functions

Exponential functions arise in many applications. One common example is **population growth.** For example, if a population starts with P_0 individuals and then grows at an annual rate of 2%, its population after 1 year is

$$P(1)=P_0+0.02P_0=P_0(1+0.02)=P_0(1.02).$$

Its population after 2 years is

$$P(2)=P(1)+0.02P(1)=P(1)(1.02)=P_0(1.02)^2.$$

In general, its population after t years is

$$P(t)=P_0(1.02)^t,$$

which is an exponential function. More generally, any function of the form $f(x)=b^x$, where $b>0$, $b\neq 1$, is an exponential function with **base** b and **exponent** x. Exponential functions have constant bases and variable exponents. Note that a function of the form $f(x)=x^b$ for some constant b is *not* an exponential function but a *power* function.

To see the difference between an exponential function and a power function, we compare the functions $y=x^2$ and $y=2^x$. In Table 1.5.1, we see that both 2^x and x^2 approach infinity as $x\to\infty$. Eventually, however, 2^x becomes larger than x^2 and grows more rapidly as $x\to\infty$. In the opposite direction, as $x\to-\infty$, $x^2\to\infty$, whereas $2^x\to 0$. The line $y=0$ is a horizontal asymptote for $y=2^x$.

Table 1.5.1

x	-3	-2	-1	0	1	2	3	4	5	6
x^2	9	4	1	0	1	4	9	16	25	36
2^x	1/8	1/4	1/2	1	2	4	8	16	32	64

In Figure 1.5.1, we graph both $y=x^2$ and $y=2^x$ to show how the graphs differ.

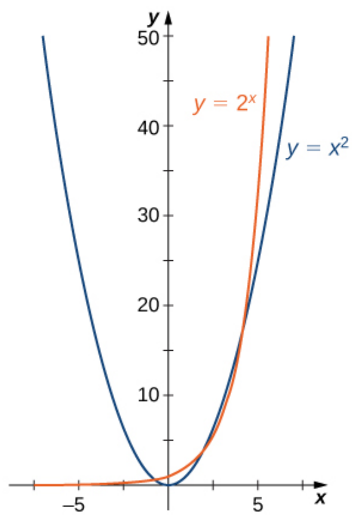

Figure 1.5.1: Both 2^x and x^2 approach infinity as $x\to\infty$, but 2^x grows more rapidly than x^2. As $x\to-\infty$, $x^2\to\infty$, whereas $2^x\to 0$.

Evaluating Exponential Functions

Recall the properties of exponents: If x is a positive integer, then we define $b^x=b\cdot b\cdots b$ (with x factors of b). If x is a negative integer, then $x=-y$ for some positive integer y, and we define $b^x=b^{-y}=1/b^y$. Also, b^0 is defined to be 1. If x is a rational number, then $x=p/q$, where p and q are integers and $b^x=b^{p/q}=\sqrt[q]{b^p}$. For example, $9^{3/2}=\sqrt{9^3}=\left(\sqrt{9}\right)^3=27$. However, how is b^x defined if x is an irrational number? For example, what do we mean by $2^{\sqrt{2}}$? This is too complex a question for us to answer fully right now; however, we can make an approximation.

Table 1.5.2: Values of 2^x for a List of Rational Numbers Approximating $\sqrt{2}$

x	1.4	1.41	1.414	1.4142	1.41421	1.414213
2^x	2.639	2.65737	2.66475	2.665119	2.665138	2.665143

In Table 1.5.2, we list some rational numbers approaching $\sqrt{2}$, and the values of 2^x for each rational number x are presented as well. We claim that if we choose rational numbers x getting closer and closer to $\sqrt{2}$, the values of 2^x get closer and closer to some number L. We define that number L to be $2^{\sqrt{2}}$.

It's often strange for students when they first encounter this conversation that b^x has never been defined for them for irrational values of x; however, there was no need for such discussion until calculus. For example, up to this point in your mathematical career, you needed to compute values like

$$2^{10}, 2^{-2}, \text{ and } 8^{4/3}.$$

However, do you recall the need to compute 2^{π}?

Example 1.5.2: Bacterial Growth

Suppose a particular population of bacteria is known to double in size every 4 hours. If a culture starts with 1000 bacteria, the number of bacteria after 4 hours is $n(4)=1000\cdot 2$. The number of bacteria after 8 hours is $n(8)=n(4)\cdot 2=1000\cdot 2^2$. In general, the number of bacteria after $4m$ hours is $n(4m)=1000\cdot 2^m$. Letting $t=4m$, we see that the number of bacteria after t hours is $n(t)=1000\cdot 2^{t/4}$. Find the number of bacteria after 6 hours, 10 hours, and 24 hours.

Solution

The number of bacteria after 6 hours is given by

$$n(6)=1000\cdot 2^{6/4}\approx 2828 \text{ bacteria.}$$

The number of bacteria after 10 hours is given by

$$n(10)=1000\cdot 2^{10/4}\approx 5657 \text{ bacteria.}$$

The number of bacteria after 24 hours is given by $n(24)=1000\cdot 2^6=64,000$ bacteria.

Exercise 1.5.2

Given the exponential function $f(x)=100\cdot 3^{x/2}$, evaluate $f(4)$ and $f(10)$.

Answer

$f(4)=900$

$f(10)=24,300$

Graphing Exponential Functions

For any base $b>0$, $b\neq 1$, the exponential function $f(x)=b^x$ is defined for all real numbers x and $b^x>0$. Therefore, the domain of $f(x)=b^x$ is $(-\infty,\infty)$ and the range is $(0,\infty)$. To graph b^x, we note that for $b>1$, b^x is increasing on $(-\infty,\infty)$ and $b^x\to\infty$ as $x\to\infty$, whereas $b^x\to 0$ as $x\to-\infty$. On the other hand, if $0<b<1$, $f(x)=b^x$ is decreasing on $(-\infty,\infty)$ and $b^x\to 0$ as $x\to\infty$ whereas $b^x\to\infty$ as $x\to-\infty$ (Figure 1.5.2).

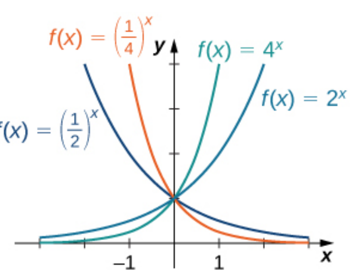

Figure 1.5.2: If $b>1$, then b^x is increasing on $(-\infty,\infty)$. If $0<b<1$, then b^x is decreasing on $(-\infty,\infty)$.

Note that exponential functions satisfy the general Laws of Exponents. To remind you of these laws, we state them as rules.

Theorem 1.5.1: Laws of Exponents

For any constants $a>0$, $b>0$, and for all x and y,

1. $$b^x \cdot b^y = b^{x+y}$$
2. $$\frac{b^x}{b^y} = b^{x-y}$$
3. $$(b^x)^y = b^{xy}$$
4. $$(ab)^x = a^x b^x$$
5. $$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

Example 1.5.3: Using the Laws of Exponents

Use the Laws of Exponents to simplify each of the following expressions.

a. $\dfrac{(2x^{2/3})^3}{(4x^{-1/3})^2}$
b. $\dfrac{(x^3y^{-1})^2}{(xy^2)^{-2}}$

Soution

a. We can simplify as follows:

$$\frac{(2x^{2/3})^3}{(4x^{-1/3})^2} = \frac{2^3(x^{2/3})^3}{4^2(x^{-1/3})^2} = \frac{8x^2}{16x^{-2/3}} = \frac{x^2x^{2/3}}{2} = \frac{x^{8/3}}{2}.$$

b. We can simplify as follows:

$$\frac{(x^3y^{-1})^2}{(xy^2)^{-2}} = \frac{(x^3)^2(y^{-1})^2}{x^{-2}(y^2)^{-2}} = \frac{x^6y^{-2}}{x^{-2}y^{-4}} = x^6x^2y^{-2}y^4 = x^8y^2.$$

Exercise 1.5.3

Use the Laws of Exponents to simplify $\dfrac{6x^{-3}y^2}{12x^{-4}y^5}$.

Hint

$x^a/x^b = x^{a-b}$

Answer

$x/(2y^3)$

The Number e

A special type of exponential function appears frequently in applications related to STEM fields. To describe it, consider the following example of exponential growth, which arises from **compounding interest** in a savings account. Suppose a person invests P dollars in a savings account with an annual interest rate r, compounded annually. The amount of money after 1 year is

$$A(1) = P + rP = P(1+r) \ .$$

The amount of money after 2 years is

$$A(2) = A(1) + rA(1) = P(1+r) + rP(1+r) = P(1+r)^2 \quad .$$

More generally, the amount after t years is

$$A(t)=P(1+r)^t\ .$$

If the money is compounded 2 times per year, the amount of money after half a year is

$$A\left(\frac{1}{2}\right)=P+\left(\frac{r}{2}\right)P=P\left(1+\left(\frac{r}{2}\right)\right)\ .$$

The amount of money after 1 year is

$$A(1)=A\left(\frac{1}{2}\right)+\left(\frac{r}{2}\right)A\left(\frac{1}{2}\right)=P\left(1+\frac{r}{2}\right)+\frac{r}{2}\left(\left(P(1+\frac{r}{2}\right)\right)=P\left(1+\frac{r}{2}\right)^2.$$

After t years, the amount of money in the account is

$$A(t)=P\left(1+\frac{r}{2}\right)^{2t}.$$

More generally, if the money is compounded n times per year, the amount of money in the account after t years is given by the function

$$A(t)=P\left(1+\frac{r}{n}\right)^{nt}.$$

What happens as $n\to\infty$? To answer this question, we let $m=n/r$ and write

$$\left(1+\frac{r}{n}\right)^{nt}=\left(1+\frac{1}{m}\right)^{mrt},$$

and examine the behavior of $(1+1/m)^m$ as $m\to\infty$, using a table of values (Table 1.5.3).

Table 1.5.3: Values of $\left(1+\frac{1}{m}\right)^m$ as $m\to\infty$

m	10	100	1000	10,000	100,000	1,000,000
$\left(1+\frac{1}{m}\right)^m$	2.5937	2.7048	2.71692	2.71815	2.718268	2.718280

Looking at this table, it appears that $(1+1/m)^m$ is approaching a number between 2.7 and 2.8 as $m\to\infty$. In fact, $(1+1/m)^m$ does approach some number as $m\to\infty$. We call this number e. To six decimal places of accuracy,

$$e\approx 2.718282.$$

Leonhard Euler

The letter e was first used to represent this number by the Swiss mathematician Leonhard Euler during the 1720s. Although Euler did not discover the number, he showed many important connections between e and logarithmic functions. We still use the notation e today to honor Euler's work because it appears in many areas of mathematics and because we can use it in many practical applications.

Returning to our savings account example, we can conclude that if a person puts P dollars in an account at an annual interest rate r, compounded continuously, then $A(t)=Pe^{rt}$. This function may be familiar. Since functions involving base e arise often in applications, we call the function $f(x)=e^x$ the **natural exponential function**. Not only is this function interesting because of the definition of the number e, but also, as discussed next, its graph has an important property.

Since $e>1$, we know $f(x)=e^x$ is increasing on $(-\infty,\infty)$. In Figure $1.5.3$, we show a graph of $f(x)=e^x$ along with a **tangent line** to the graph of f at $x=0$. We give a precise definition of tangent line in the next chapter; but, informally, we say a tangent line to a graph of f at $x=a$ is a line that passes through the point $(a,f(a))$ and has the same "slope" as f at that point (as we will learn, using the word "slope" for a nonlinear curve is generally frowned upon, but it will have to suffice for now). The function $f(x)=e^x$ is the only exponential function b^x with tangent line at $x=0$ that has a slope of 1. As we see later in the text, having this property makes the natural exponential function the most simple exponential function to use in many instances.

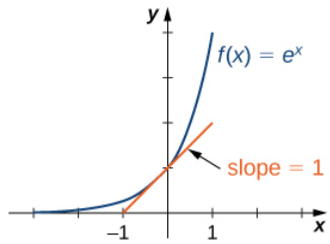

Figure 1.5.3: The graph of $f(x) = e^x$ has a tangent line with slope 1 at $x = 0$.

Example 1.5.4: Compounding Interest

Suppose $\$500$ is invested in an account at an annual interest rate of $r = 5.5\%$, compounded continuously.

a. Let t denote the number of years after the initial investment and $A(t)$ denote the amount of money in the account at time t. Find a formula for $A(t)$.
b. Find the amount of money in the account after 10 years and after 20 years.

Solution

a. If P dollars are invested in an account at an annual interest rate r, compounded continuously, then $A(t) = Pe^{rt}$. Here $P = \$500$ and $r = 0.055$. Therefore, $A(t) = 500e^{0.055t}$.

b. After 10 years, the amount of money in the account is

$$A(10) = 500e^{0.055\cdot 10} = 500e^{0.55} \approx \$866.63.$$

After 20 years, the amount of money in the account is

$$A(20) = 500e^{0.055\cdot 20} = 500e^{1.1} \approx \$1,502.08.$$

Exercise 1.5.4

If $\$750$ is invested in an account at an annual interest rate of 4%, compounded continuously, find a formula for the amount of money in the account after t years. Find the amount of money after 30 years.

Hint

$A(t) = Pe^{rt}$

Answer

$A(t) = 750e^{0.04t}$. After 30 years, there will be approximately $\$2,490.09$

Logarithmic Functions

Using our understanding of exponential functions, we can discuss their inverses, which are the logarithmic functions. These come in handy when we need to consider any phenomenon that varies over a wide range of values, such as the pH scale in chemistry or decibels in sound levels.

The exponential function $f(x) = b^x$ is one-to-one, with domain $(-\infty, \infty)$ and range $(0, \infty)$. Therefore, it has an inverse function, called the **logarithmic function** with base b. For any $b > 0$, $b \neq 1$, the logarithmic function with base b, denoted $\log_b$, has domain $(0, \infty)$ and range $(-\infty, \infty)$,and satisfies

$$\log_b(x) = y$$

if and only if $b^y = x$.

For example,

$$\log_2(8) = 3$$

since $2^3 = 8$,

$$\log_{10}\left(\frac{1}{100}\right) = -2$$

since $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$,

$$\log_b(1) = 0$$

since $b^0 = 1$ for any base $b > 0$.

Furthermore, since $y = \log_b(x)$ and $y = b^x$ are inverse functions,

$$\log_b(b^x) = x$$

and

$$b^{\log_b(x)} = x.$$

The most commonly used logarithmic function is the function $\log_e$. Since this function uses natural e as its base, it is called the **natural logarithm**. Here we use the notation $\ln(x)$ or $\ln x$ to mean $\log_e(x)$. For example,

$$\ln(e) = \log_e(e) = 1$$
$$\ln(e^3) = \log_e(e^3) = 3$$
$$\ln(1) = \log_e(1) = 0.$$

Since the functions $f(x) = e^x$ and $g(x) = \ln(x)$ are inverses of each other,

$$\ln(e^x) = x \text{ and } e^{\ln x} = x,$$

and their graphs are symmetric about the line $y = x$ (Figure $1.5.4$).

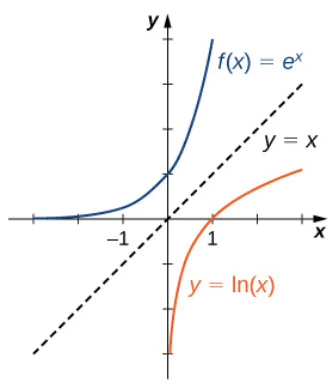

Figure $1.5.4$: The functions $y = e^x$ and $y = \ln(x)$ are inverses of each other, so their graphs are symmetric about the line $y = x$.

In general, for any base $b > 0$, $b \neq 1$, the function $g(x) = \log_b(x)$ is symmetric about the line $y = x$ with the function $f(x) = b^x$. Using this fact and the graphs of the exponential functions, we graph functions $\log_b$ for several values of $b > 1$ (Figure $1.5.5$).

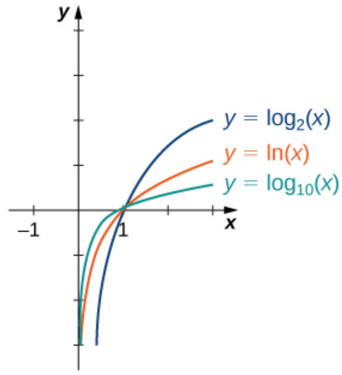

Figure 1.5.5: Graphs of $y = \log_b(x)$ are depicted for $b = 2,\ e,\ 10$.

Before solving some equations involving exponential and logarithmic functions, let's review the basic properties of logarithms.

Theorem 1.5.2: Laws of Logarithms

If $a,\ b,\ c > 0,\ b \neq 1$, and r is any real number, then

- **Product property**

$$\log_b(ac) = \log_b(a) + \log_b(c) \tag{1.5.1}$$

- **Quotient property**

$$\log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c) \tag{1.5.2}$$

- **Power property**

$$\log_b(a^r) = r\log_b(a) \tag{1.5.3}$$

Example 1.5.5: Solving Equations Involving Exponential Functions

Solve each of the following equations for x.

a. $5^x = 2$
b. $e^x + 6e^{-x} = 5$

Solution

a. Applying the natural logarithm function to both sides of the equation, we have

$$\ln 5^x = \ln 2.$$

Using the power property of logarithms,

$$x\ln 5 = \ln 2.$$

Therefore,

$$x = \frac{\ln 2}{\ln 5}.$$

b. Multiplying both sides of the equation by e^x,we arrive at the equation

$$e^{2x} + 6 = 5e^x.$$

Rewriting this equation as

$$e^{2x} - 5e^x + 6 = 0,$$

we can then rewrite it as a quadratic equation in e^x:

$$(e^x)^2 - 5(e^x) + 6 = 0.$$

Now we can solve the quadratic equation. Factoring this equation, we obtain

$$(e^x - 3)(e^x - 2) = 0.$$

Therefore, the solutions satisfy $e^x = 3$ and $e^x = 2$. Taking the natural logarithm of both sides gives us the solutions $x = \ln 3, \ln 2$.

Exercise 1.5.5

Solve

$$e^{2x}/(3 + e^{2x}) = 1/2.$$

Hint

First solve the equation for e^{2x}

Answer

$x = \dfrac{\ln 3}{2}$.

Example 1.5.6: Solving Equations Involving Logarithmic Functions

Solve each of the following equations for x.

a. $\ln\left(\dfrac{1}{x}\right) = 4$
b. $\log_{10}\sqrt{x} + \log_{10} x = 2$
c. $\ln(2x) - 3\ln(x^2) = 0$

Solution

a. By the definition of the natural logarithm function,

$$\ln\left(\frac{1}{x}\right) = 4$$

if and only if $e^4 = \dfrac{1}{x}$.

Therefore, the solution is $x = 1/e^4$.

b. Using the product (Equation 1.5.1) and power (Equation 1.5.3) properties of logarithmic functions, rewrite the left-hand side of the equation as

$$\begin{aligned} \log_{10}\sqrt{x} + \log_{10} x &= \log_{10} x\sqrt{x} \\ &= \log_{10} x^{3/2} \\ &= \frac{3}{2}\log_{10} x. \end{aligned}$$

Therefore, the equation can be rewritten as

$$\frac{3}{2}\log_{10} x = 2$$

or

$$\log_{10} x = \frac{4}{3}.$$

The solution is $x = 10^{4/3} = 10\sqrt[3]{10}$.

c. Using the power property (Equation 1.5.3) of logarithmic functions, we can rewrite the equation as $\ln(2x) - \ln(x^6) = 0$.

Using the quotient property (Equation 1.5.2), this becomes

$$\ln\left(\frac{2}{x^5}\right) = 0$$

Therefore, $2/x^5 = 1$, which implies $x = \sqrt[5]{2}$. We should then check for any extraneous solutions.

Exercise 1.5.6

Solve $\ln(x^3) - 4\ln(x) = 1$.

Hint

First use the power property, then use the product property of logarithms.

Answer

$x = \dfrac{1}{e}$

When evaluating a logarithmic function with a calculator, you may have noticed that the only options are $\log_{10}$ or log, called the **common logarithm**, or ln, which is the natural logarithm. However, exponential functions and logarithm functions can be expressed in terms of any desired base b. If you need to use a calculator to evaluate an expression with a different base, you can apply the **Change of Base Formulas** first. Using this change of base, we typically write a given exponential or logarithmic function in terms of the natural exponential and natural logarithmic functions.

Theorem 1.5.3: Change of Base Formula

Let $a > 0,\ b > 0$, and $a \neq 1,\ b \neq 1$.

1. $a^x = b^{x \log_b a}$ for any real number x.

   If $b = e$, this equation reduces to $a^x = e^{x \log_e a} = e^{x \ln a}$.

2. $\log_a x = \dfrac{\log_b x}{\log_b a}$ for any real number $x > 0$.

   If $b = e$, this equation reduces to $\log_a x = \dfrac{\ln x}{\ln a}$.

Proof

For the first Change of Base Formula, we begin by making use of the power property of logarithmic functions. We know that for any base $b > 0,\ b \neq 1$, $\log_b(a^x) = x\log_b a$. Therefore,

$$b^{\log_b(a^x)} = b^{x \log_b a}.$$

In addition, we know that b^x and $\log_b(x)$ are inverse functions. Therefore,

$$b^{\log_b(a^x)} = a^x.$$

Combining these last two equalities, we conclude that $a^x = b^{x \log_b a}$.

To prove the second property, we show that

$$(\log_b a) \cdot (\log_a x) = \log_b x.$$

Let $u = \log_b a, v = \log_a x$, and $w = \log_b x$. We will show that $u \cdot v = w$. By the definition of logarithmic functions, we know that $b^u = a,\ a^v = x$, and $b^w = x$. From the previous equations, we see that

$$b^{uv} = (b^u)^v = a^v = x = b^w.$$

Therefore, $b^{uv} = b^w$. Since exponential functions are one-to-one, we can conclude that $u \cdot v = w$.

Q.E.D.

Example 1.5.7: Changing Bases

Use a calculating utility to evaluate $\log_3 7$ with the Change of Base Formula presented earlier.

Solution

Use the second equation with $a=3$ and $b=e$: $\log_3 7=\dfrac{\ln 7}{\ln 3}\approx 1.77124$.

Exercise 1.5.7

Use the Change of Base Formula and a calculating utility to evaluate $\log_4 6$.

Hint

Use the change of base to rewrite this expression in terms of expressions involving the natural logarithm function.

Answer

$\log_4 6=\dfrac{\ln 6}{\ln 4}\approx 1.29248$

Example 1.5.8: The Richter Scale for Earthquakes

In 1935, Charles Richter developed a scale (now known as the Richter scale) to measure the magnitude of an earthquake. The scale is a base-10 logarithmic scale, and it can be described as follows: Consider one earthquake with magnitude R_1 on the Richter scale and a second earthquake with magnitude R_2 on the Richter scale. Suppose $R_1>R_2$, which means the earthquake of magnitude R_1 is stronger, but how much stronger is it than the other earthquake?

Figure 1.5.6: (credit: modification of work by Robb Hannawacker, NPS)

A way of measuring the intensity of an earthquake is by using a seismograph to measure the amplitude of the earthquake waves. If A_1 is the amplitude measured for the first earthquake and A_2 is the amplitude measured for the second earthquake, then the amplitudes and magnitudes of the two earthquakes satisfy the following equation:

$$R_1-R_2=\log_{10}\left(\frac{A1}{A2}\right).$$

Consider an earthquake that measures 8 on the Richter scale and an earthquake that measures 7 on the Richter scale. Then,

$$8-7=\log_{10}\left(\frac{A1}{A2}\right).$$

Therefore,

$$\log_{10}\left(\frac{A1}{A2}\right)=1,$$

which implies $A_1/A_2=10$ or $A_1=10A_2$. Since A_1 is 10 times the size of A_2, we say that the first earthquake is 10 times as intense as the second earthquake. On the other hand, if one earthquake measures 8 on the Richter scale and another measures 6, then the relative intensity of the two earthquakes satisfies the equation

$$\log_{10}\left(\frac{A1}{A2}\right)=8-6=2.$$

Therefore, $A_1=100A_2$.That is, the first earthquake is 100 times more intense than the second earthquake.

How can we use logarithmic functions to compare the relative severity of the magnitude 9 earthquake in Japan in 2011 with the magnitude 7.3 earthquake in Haiti in 2010?

Solution

To compare the Japan and Haiti earthquakes, we can use an equation presented earlier:

$$9-7.3=\log_{10}\left(\frac{A1}{A2}\right).$$

Therefore, $A_1/A_2=10^{1.7}$, and we conclude that the earthquake in Japan was approximately 50 times more intense than the earthquake in Haiti.

Exercise 1.5.8

Compare the relative severity of a magnitude 8.4 earthquake with a magnitude 7.4 earthquake.

Hint

$R_1-R_2=\log_{10}(A1/A2)$.

Answer

The magnitude 8.4 earthquake is roughly 10 times as severe as the magnitude 7.4 earthquake.

Key Concepts

- Trigonometric, exponential, and logarithmic functions are examples of transcendental functions.
- The exponential function $y=b^x$ is increasing if $b>1$ and decreasing if $0<b<1$. Its domain is $(-\infty,\infty)$ and its range is $(0,\infty)$.
- The logarithmic function $y=\log_b(x)$ is the inverse of $y=b^x$. Its domain is $(0,\infty)$ and its range is $(-\infty,\infty)$.
- The natural exponential function is $y=e^x$ and the natural logarithmic function is $y=\ln x=\log_e x$.
- Given an exponential function or logarithmic function in base a, we can make a change of base to convert this function to any base $b>0$, $b\neq 1$. We typically convert to base e.

Glossary

base
: the number b in the exponential function $f(x)=b^x$ and the logarithmic function $f(x)=\log_b x$

exponent
: the value x in the expression b^x

natural exponential function
: the function $f(x)=e^x$

natural logarithm
: the function $\ln x=\log_e x$

number e
: as m gets larger, the quantity $(1+(1/m)^m$ gets closer to some real number; we define that real number to be e; the value of e is approximately 2.718282

transcendental function
: a function that cannot be expressed by a combination of basic arithmetic operations