2. Intro: Multi-product formula (MPF)

Cost of existing algorithms

	pth-order PF	LCU	QSVT
Pros & Cons	Good scaling in N / Bad scaling in ε	Exponentially good scaling in ε Good scaling in t / Bad scaling in N	
Origin	Commutator scaling A. M. Childs, et al., PRX **11**, 011020 (2021)	Rapid convergence of polynomial approximation	

(Well-conditioned) MPF

G. H. Low, et al., arXiv:1907.11679 (2019)

→ Promising candidate simultaneously efficient in N, t, ε

$$M(\tau) = \underbrace{\sum_{j=1}^{J} c_j}_{\text{Linear combination of}} \underbrace{[T_p(\tau/k_j)]^{k_j}}_{p\text{th-order PF}} = e^{-iH\tau} + \mathcal{O}(\tau^{m+1}).$$

Smaller than $\mathcal{O}(\tau^{p+1})$
by Richardson extrapolation

Suppose we have 2 approximations, $f_j(\tau) = f_{\text{true}} + a_j\tau^2 + \mathcal{O}(\tau^3)$.

Then, we obtain better approximation $c_1 f_1(\tau) + c_2 f_2(\tau) = f_{\text{true}} + \mathcal{O}(\tau^3)$ with c_j s.t. $c_1 + c_2 = 1$, $c_1 a_1 + c_2 a_2 = 0$.

Open problem

Does the cost of MPF has good scaling in any of N, t, ε ?