Paper Summary

The authors focuses on the Trotter errors in digitized adiabatic evolution in the presence of counter-diabatic (CD) potentials. The long-time error suppression, called the self healing, has recently been discovered in the standard digitized adiabatic evolution. The authors have found that this characteristic healing takes place also in finite-time dynamics under digitized adiabatic evolution accelerated by a CD drive. They begin with analytical calculation of the overlap amplitude, in which the non-adiabatic contributions vanish by the CD potential. After some approximations such as the perturbation theory and the extraction of the dominant Fourier mode, they clarify how the infidelity depends on the gap Δ_{i0}, the final time T, and the discretization step Δt. They also numerically demonstrate the self healing in non-interacting and interacting spins. They show that the Trotter error decays with respect to the final time T in both cases, confirming the presence of healing. A similar behavior is observed also when the exact CD potential is replaced by the order-l nested commutator approximations.

Comments and questions

Digitized adiabatic quantum dynamics is one of the significant topics in quantum computation, exemplified by adiabatic state preparation. Trotterization of the time-dependent dynamics and the acceleration by the CD drive are both central tools in this field. Thus, I believe that the authors' result implying the self-healing in digitized adiabatic evolution under CD drives is practically important and will attract readers of Physical Review A. However, before acceptance, it seems to be necessary to conduct more elaborate analysis analytically and numerically for supporting their results. I attach some comments and questions below.

- About $\mathcal{S}_{ij}$, $\mathcal{Q}_{ij}$, and $\mathcal{R}_{ij}$
  Throughout the manuscript, I feel that the description of $\mathcal{S}_{ij}$, $\mathcal{Q}_{ij}$, and $\mathcal{R}_{ij}$ is ambiguous. These quantities are explained only with texts in both the main text and Appendix A. They should be described exactly by some equations (e.g., by instantaneous energy eigenstates), not just in the order description. It is difficult for readers to grasp how they behave along with the non-adiabatic errors under the exact CD potential. I'm also wondering how they are modified when the nested commutator approximation is applied.
  Regarding the quantity $\mathcal{R}_{ij}$, the authors numerically estimate the dominant mode $\bar{q}$ and the gap Δ through the fitting. However, they can be calculated numerically from $\mathcal{R}_{ij}$ following Eqs. (19)-(24) without fitting. For confirming the authors' analysis, I feel that examining whether the result computed by $\mathcal{R}_{ij}$ matches the one by the fitting is significant.
- About the nested commutator approximation
  I am not convinced about the validity of the nested commutator approximation. In Fig. 3, although the results with different approximation orders l are described, the result under the exact CD potential is omitted. All these results should be compared in the same figure, and it should be examined how the results with the nested commutator approximation reproduces the exact result. In particular, I'm curious about whether the infidelity scales as $\mathcal{O}\left(T^{-2}\right)$ in the approximate cases.

I am also concerned about whether the authors' analysis from Eq. (17) to Eq. (22) explains the results in Fig. 3. The authors say that the single mode approximation like Eq. (24) breaks. If so, is it possible to reproduce Fig. 3 by considering Eq. (19) with a few modes q?

- Scaling in system size

  One of the significant properties of the product formula is the commutator scaling (Ref. [59] in the manuscript), in which the error has a characteristic scaling in system size N. This is also proven for the product formula in time-dependent cases [arXiv:2410.14243]. Although this manuscript focuses only on the dependency on time, is it possible to investigate the scaling of the error (i.e., the infidelity) in system size both analytically and numerically?

- (Minor comment) Equation (19)

  The right hand side should be dependent on the indices i and j. So, $R(\Delta t)$ seems to be replaced by $R_{ij}(\Delta t)$.

- (Minor comment) On some equations

  Some of the equations in the manuscript are not precise. For instance, the equality in Eq. (20) is incorrect because Eq. (20) is just the result of the perturbation theory and it does not exactly give the overlap defined by Eq. (15). It should be replaced by "$P_i(t,\lambda) \simeq$" or $P_i(t,\lambda) = (\cdots) + \mathcal{O}(\cdot)$ with the error term $\mathcal{O}(\cdot)$. Similarly, Eqs. (23) and (24) should be modified since they are the result of approximation by a dominant term.