3. Results: Simultaneous efficiency of MPF in size, time, accuracy

Setup

Generic k-local Hamiltonian $H = \sum_{\gamma=1}^{\Gamma} H_\gamma, \quad H_\gamma = \sum_{X \subset \Lambda} h_X^\gamma$

Extensiveness: $g = \max_{i \in \Lambda} \left(\sum_\gamma \sum_{X \ni i} \|h_X^\gamma\| \right)$

$$g \in \begin{cases} \mathcal{O}(1) & \text{(finite-range interaction)} \\ \mathcal{O}(1) & \text{(long-range int., } \nu > d) \\ \mathcal{O}(\log N) & \text{(long-range int., } \nu = d) \\ \mathcal{O}(N^{1-\nu/d}) & \text{(long-range int., } \nu < d) \end{cases} \quad U(r) \sim r^{-\nu}$$

Nested commutators

$$\alpha_{\mathrm{com},q} = \sum_{\gamma_0, \cdots, \gamma_q = 1}^{\Gamma} [H_{\gamma_q}, \cdots, [H_{\gamma_1}, H_{\gamma_0}]] \xrightarrow{\text{Locality of Hamiltonian}} \alpha_{\mathrm{com},q} \leq q!(2kg)^q N g$$

c.f. PF error with commutator scaling: $\|e^{-iH\tau} - T_p(\tau)\| \leq \text{Const.} \times \alpha_{\mathrm{com},p} \tau^{p+1} \in \mathcal{O}((N^{\frac{1}{p+1}} g\tau)^{p+1})$

Cost

	Number of $\mathcal{O}(1)$-qubit gates		Ancilla qubits
	for finite-range int.	for long-range int.	
pth-order PF A. Childs, et al. (2021)	$Ng\left(\frac{Ngt}{\varepsilon}\right)^{\frac{1}{p}}$	$N^k g\left(\frac{Ngt}{\varepsilon}\right)^{\frac{1}{p}}$	0
LCU/QSVT A. Gilyén, et al. (2019)	$N(Ngt + \log(1/\varepsilon))$	$N^k(Ngt + \log(1/\varepsilon))$	$\log N$
HHKL J. Haah, et al. (2021)	$Ngt \times \mathrm{polylog}(Ngt/\varepsilon)$	$Ngt\left(\frac{Ngt}{\varepsilon}\right)^{\frac{2d}{\nu-d}}$ Only for $\nu > 2d$ ← Needs Lieb-Robinson bound	$\log\log(Ngt/\varepsilon)$
pth-order MPF (this work)	$N^{1+\frac{1}{p+1}} gt \times \mathrm{polylog}(Ngt/\varepsilon)$	$N^{k+\frac{1}{p+1}} gt \times \mathrm{polylog}(Ngt/\varepsilon)$	$\log\log(Ngt/\varepsilon)$

Good N-scaling like PF & Good t-, ε-scaling like LCU/QSVT & Versatility to generic local Hamiltonians !