5. Comparison with previous results

[Low-Kliuchnikov-Wiebe 2019] G. H. Low, V. Kliuchnikov, and N. Wiebe, arXiv:1907.11679 (2019)

- Prove the good scaling in t & ε by the well-conditioned solution $\|c\|_1, \|k\|_1 \in \mathrm{poly}(J)$
- Prove that the error is given by commutators but NOT provide its explicit form

→ The good scaling in size N is not proven

[Aftab-An-Trivisa 2024] J. Aftab, D. An, and K. Trivisa, arXiv:2403.08922 (2024)

- Prove an explicit error bound of the MPF composed of nested commutators

(Simplified version)

$$\|e^{-iH\tau} - M(\tau)\| \leq \mathrm{Const.} \times \|c\|_1 (\mu_{p,m}[\infty]\tau)^{m+1} \quad \text{if} \quad |\tau| \leq \frac{\mathrm{Const.}}{\mu_{p,m}[\infty]}$$

- However, it involves q-fold nested commutators $\alpha_{\mathrm{com},q}$ with arbitrarily large q

→ Locality gives no meaningful upper bound on $\mu_{p,m}[\infty]$ due to $[q!(2kg)^q Ng]^{\frac{1}{q+1}} \to \infty$

We can only confirm $\mu_{p,m}[\infty] \leq \mathrm{Const.} \times Ng$ in general

→ The cost results in $r \in \tilde{\Theta}(Ngt)$ for generic local Hamiltonians (1-norm scaling)

Our result with commutator scaling: $r \in \tilde{\Theta}(N^{\frac{1}{p+1}} gt)$

Our contribution

Prove the MPF error composed of a finite number of nested commutators
& Complete the proof of the efficiency in N, t, ε for generic local Hamiltonians !