

3. Results: Simultaneous efficiency of MPF in size, time, accuracy

■ Setup

Generic k -local Hamiltonian

$$H = \sum_{\gamma=1}^{\Gamma} H_{\gamma}, \quad H_{\gamma} = \sum_{X \subset \Lambda} h_X^{\gamma}$$

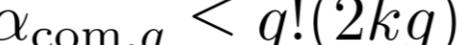
$$\text{Extensiveness: } g = \max_{i \in \Lambda} \left(\sum_{\gamma} \sum_{X \ni i} \|h_X^{\gamma}\| \right)$$

$$g \in \begin{cases} \mathcal{O}(1) & (\text{finite-range interaction}) \\ \mathcal{O}(1) & (\text{long-range int., } \nu > d) \\ \mathcal{O}(\log N) & (\text{long-range int., } \nu = d) \\ \mathcal{O}(N^{1-\nu/d}) & (\text{long-range int., } \nu < d) \end{cases} \quad U(r) \sim r^{-\nu}$$

■ Nested commutators

$$\alpha_{\text{com},q} = \sum_{\gamma_0, \dots, \gamma_q=1}^{\Gamma} [H_{\gamma_q}, \dots, [H_{\gamma_1}, H_{\gamma_0}]]$$

Locality of Hamiltonian



$$\alpha_{\text{com},q} \leq q!(2kg)^q Ng$$

c.f. PF error with commutator scaling: $\|e^{-iH\tau} - T_p(\tau)\| \leq \text{Const.} \times \alpha_{\text{com},p} \tau^{p+1} \in \mathcal{O}((N^{\frac{1}{p+1}} g \tau)^{p+1})$

■ Cost

	Number of $\mathcal{O}(1)$ -qubit gates for finite-range int.	Number of $\mathcal{O}(1)$ -qubit gates for long-range int.	Ancilla qubits
<i>p</i> th-order PF A. Childs, et al. (2021)	$Ng \left(\frac{Ngt}{\varepsilon} \right)^{\frac{1}{p}}$	$N^k g \left(\frac{Ngt}{\varepsilon} \right)^{\frac{1}{p}}$	0
LCU/QSVT A. Gilyén, et al. (2019)	$N(Ngt + \log(1/\varepsilon))$	$N^k(Ngt + \log(1/\varepsilon))$	$\log N$
HHKL J. Haah, et al. (2021)	$Ngt \times \text{polylog}(Ngt/\varepsilon)$	$Ngt \left(\frac{Ngt}{\varepsilon} \right)^{\frac{2d}{\nu-d}}$ ← Needs Lieb-Robinson bound	$\log \log(Ngt/\varepsilon)$
<i>p</i>th-order MPF (this work)	$N^{1+\frac{1}{p+1}} gt \times \text{polylog}(Ngt/\varepsilon)$	$N^{k+\frac{1}{p+1}} gt \times \text{polylog}(Ngt/\varepsilon)$	$\log \log(Ngt/\varepsilon)$

Good N -scaling like PF & Good t -, ε -scaling like LCU/QSVT & Versatility to generic local Hamiltonians !