4. Results & Derivation

■ MPF error with commutator scaling

Commutator factor for MPF: $\mu_{p,m}[p_0] = \max_{q:p\leq q\leq p_0} \left((\alpha_{\mathrm{com},q})^{\frac{1}{q+1}} \right)$

Theorem 1

$\epsilon \in (0,1)$: arbitrary fixed value, $p_0(N,\epsilon) = \lceil \log(3N/\epsilon) \rceil$: truncation order

When the time τ is small enough to satisfy $|\tau| \leq \dfrac{\mathrm{Const.}}{\mu_{p,m}[p_0(N,\epsilon)]}$, the MPF error is bounded by

$$\|e^{-iH\tau} - M(\tau)\| \leq \mathrm{Const.} \times \|c\|_1 (\mu_{p,m}[p_0(N,\epsilon)]\tau)^{m+1} + \|c\|_1 \|k\|_1 \epsilon$$

Theorem 2

$$\mu_{p,m}[p_0(N,\epsilon)] \leq \mathrm{Const.} \times (N^{\frac{1}{p+1}} g + g \log(N/\epsilon))$$

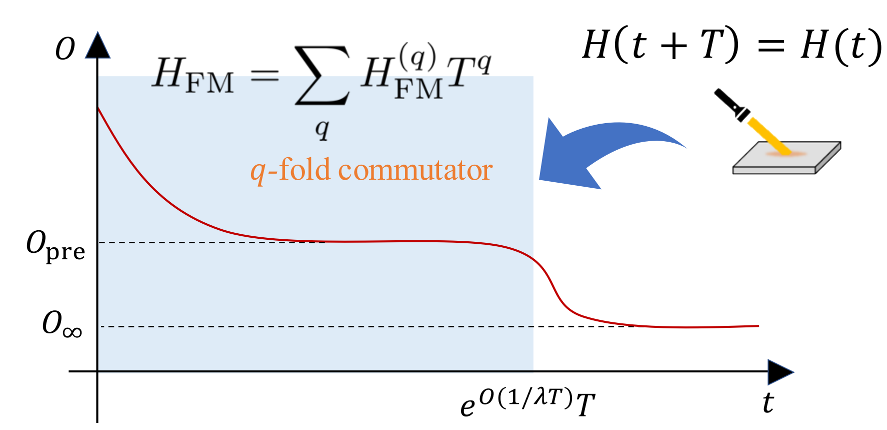
■ Derivation of the error bound

Theorem 1 Inspired by Floquet prethermalization T. Kuwahara, T. Mori, K. Saito, Ann. Phys. 367, 96 (2016)

$$\|e^{-iH\tau} - M(\tau)\| \leq$$

$$\left(\sum_{\{q_i\}} \prod_i \alpha_{\mathrm{com},q_i} \right) t^{p+J} + E^{(p+J+1)}(\{\alpha_{\mathrm{com},q_i}\}) t^{p+J+1} + \cdots + E^{(q)}(\{\alpha_{\mathrm{com},q_i}\}) t^q + \cdots$$

q_i-fold commutator



[$q \to \infty$] Divergent [Consider $q \leq \log(N/\epsilon)$] Convergence within error ϵ

Theorem 2 Use the bound on the nested commutator $\alpha_{\mathrm{com},q}$ with the locality

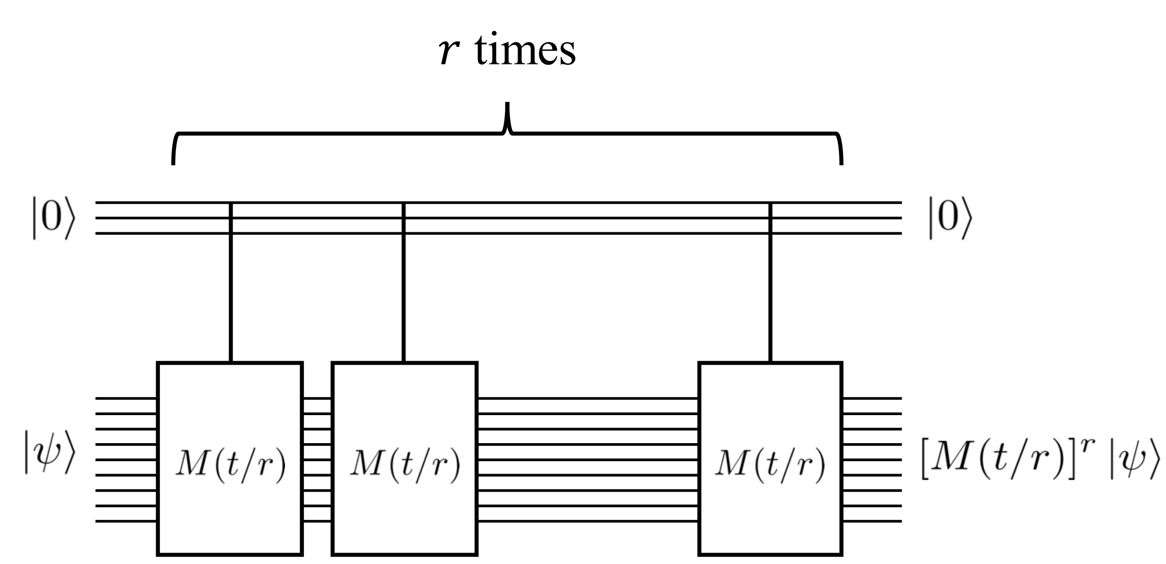
■ Derivation of the computational cost

Find Trotter number $r = t/\tau$ such that

$$r \times \|e^{-iHt/r} - M(t/r)\| \in \mathcal{O}\left(r \left(N^{\frac{1}{p+1}} g \frac{t}{r} \right)^{m+1} \right) \leq \varepsilon$$

with setting $m, J \in \mathcal{O}(\log(Ngt/\varepsilon))$, $\epsilon \in \tilde{\mathcal{O}}(\varepsilon/r)$

[Gate counts] $\|c\|_1 \|k\|_1 \, r \times (\text{Number of terms in } H)$



LCU + amplitude amplification