

# Artificial Intelligence in Train Scheduling Problems

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- 1 Rail Data Feeds
- 2 Mixed Integer Linear Programming
- 3 Genetic Algorithms
- 4 Reinforcement Learning
- 5 Case Study : Manchester

# Project Motivation

Optimisation of train networks can provide return on all optimality factors; profit, timeliness or robustness. Train networks provide a unique problem where infrastructure expansions are complicated, expensive and disruptive. This leads to largely scheduling based problems, which this project is based upon.

# Brief Introduction

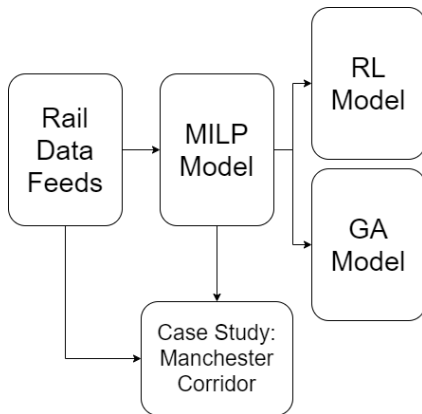


Figure: Project Outline

# Brief Introduction

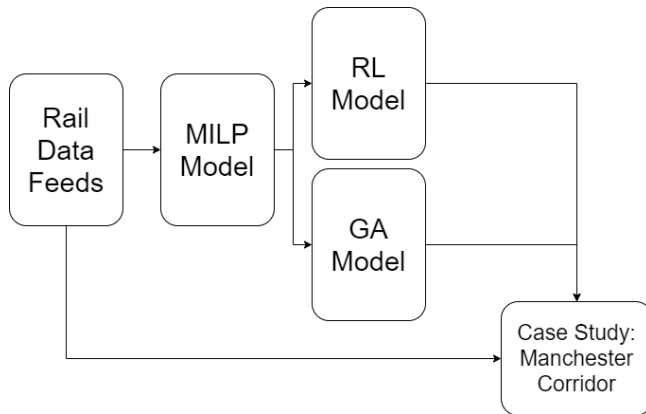


Figure: Intended Project Outline

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# RDF - Implementation

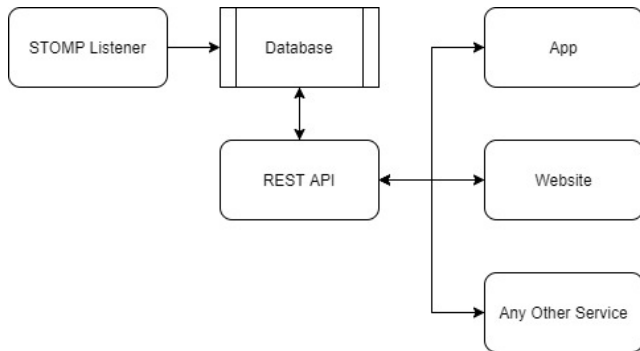


Figure: Rail Data Feeds Program Flow

# RDF - Purpose

- View the Network Live
- Save Network Statistics
- Create Real World Test Sets



# RDF - Outcomes

- Network Rail Listener
- Huxley Connector
- Network Rail Historic Database
- REST API
- Website

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# MILP Model - TSP

Objective Function

$$\sum_{n=1}^N A_{n,M}^S - D_{n,0}^S$$

Example Constraints:

$$D_{n,m}^r \leq A_{n+1,m}^r - H$$

$$A_{n,m+1}^r \geq D_{n,m}^r + E_m$$

# MILP Model - TRSP

Objective Function

$$\sum_{n=1}^N A_{n,M}^r - A_{n,M}^o$$

Example Constraints:

$$D_{n,m}^r = D_{n,m}^o$$

$$\delta_D > 1, \delta_t < D_{\delta_n}^{\delta_D-1} + E_{\delta_D-1} \rightarrow \dots$$

$$A_{\delta_n, \delta_D}^r \geq D_{\delta_n, \delta_D-1}^o + \delta_d + E_{\delta_d-1}$$

# MILP Model - TRSP (Passenger Weighted)

Objective Function

$$\sum_{n=1}^N P_{n,M}^c A_{n,M}^r$$

Example Constraints:

$$C - P_{n,m-1}^l < 100 \rightarrow P_c^{n,m} \leq 20\lambda_{n,m}$$

$$C - P_{n,m-1}^l \geq 100 \rightarrow P_c^{n,m} \leq 50\lambda_{n,m}$$

$$P_{n,m}^c \leq D + n, m^r - 50A_{n,m}^r$$

# MILP Model - Outcomes

- MiniZinc TRSP Standard Model
- Series of Associated FlatZinc Data Files
- Minizinc TRSP Passenger Weighted Model
- Series of Associated FlatZinc Data Files

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# GA - General Algorithm

```
 $P \leftarrow \text{generatePopulation}[100]$   
while endCondition = false do  
   $P \leftarrow \text{selection}(P)$   
   $P \leftarrow \text{crossover}(P)$   
   $P \leftarrow \text{mutation}(P)$   
   $B \leftarrow \text{fitness}(P)$   
  if  $B == \text{optimal}$  then  
    endCondition  $\leftarrow$  true  
  return  $B, P$ 
```

▷ Get 100 random variables



# GA - TSP Framework

|                 | Phenotype |      |      |        |             |              |
|-----------------|-----------|------|------|--------|-------------|--------------|
|                 | toc       | line | unit | origin | destination | intermediate |
| Bit Length      | 2         | 8    | 8    | 12     | 12          | $12n$        |
| Representations | 4         | 256  | 256  | 4096   | 4096        | $n(2^{12})$  |

# GA - Basic Scenario

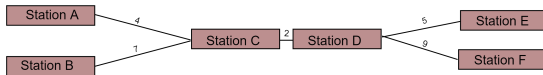
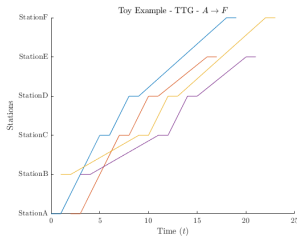
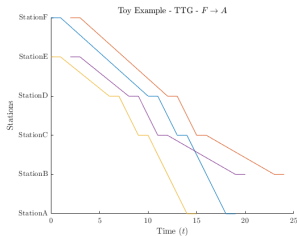


Figure: Toy Example Map

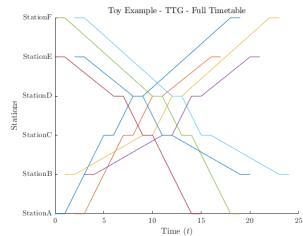
# GA - Basic Scenario



(a) Stage 1



(b) Stage 2



(c) Stage 3

# GA - Basic Scenario

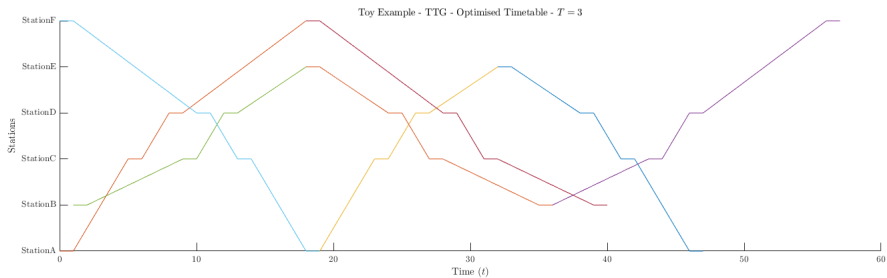


Figure: Toy Example Combined Stages

# GA - Medium Scenario

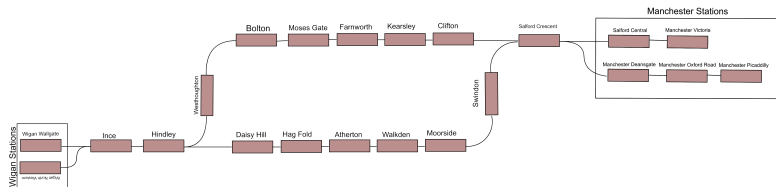


Figure: Small Example Map

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# SBB Challenge

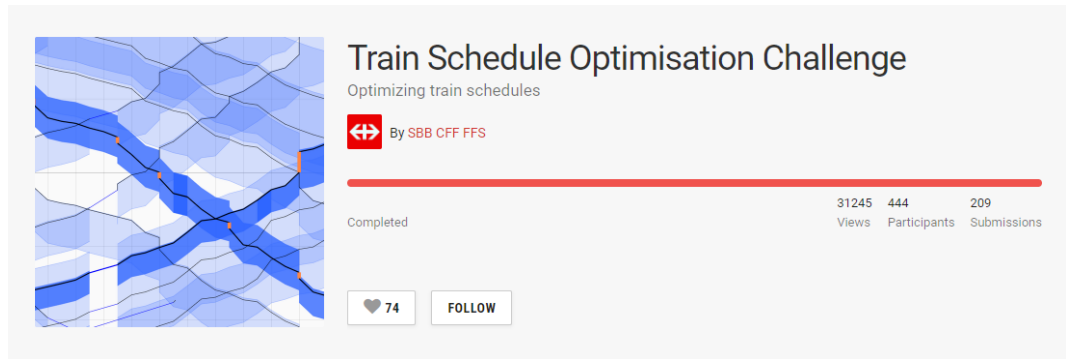


Figure: Crowd AI SBB Challenge

# SBB Challenge

| ID | Name                          | Trains | Routing | Difficulty |                 |
|----|-------------------------------|--------|---------|------------|-----------------|
| 01 | dummy                         | 4      | V.Few   | V.Simple   |                 |
| 02 | a_little_less_dummy           | 58     | Few     | Simple     |                 |
| 03 | FWA_0.125                     | 143    | Few     | Simple     |                 |
| 04 | V1.02_FWA_without_obstruction | 148    | Few     | Medium     |                 |
| 05 | V1.02_FWA_with_obstruction    | 149    | Medium  | Medium*    |                 |
| 06 | V1.20_FWA                     | 365    | High    | V.Hard     |                 |
| 07 | V1.22_FWA                     | 467    | High    | V.Hard     |                 |
| 08 | V1.30_FWA                     | 133    | V.High  | V.Hard     |                 |
| 09 | ZUEZGCH_06001200              |        |         | 287        | VV.High VV.Hard |

Table: Description of Problem Scenarios



# RL - Delayed Q-Learning

## Modelling a PACMDP Update Function

$$Q_{t+1}(s, a) = \frac{1}{m} \sum_{i=1}^m (r_{k_i} + \gamma V_{k_i}(s_{k_i})) + \epsilon_1$$

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Algorithm 1 Delayed Q-Learning

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```
function DLQ( $\gamma, S, A, M, \epsilon_1$ )  
  for all  $\langle s, a \rangle$  do  
     $Q(s, a) \leftarrow 1/(1 - \gamma)$   
     $U(s, a) \leftarrow 0$   
     $l(s, a) \leftarrow 0$   
     $t(s, a) \leftarrow 0$   
    LEARN( $s, a$ )  $\leftarrow$  TRUE  
   $t^* \leftarrow 0$   
  while LEARN( $s, a$ ) = TRUE do  
     $U(s, a) \leftarrow U(s, a) + r + \gamma \max_{a'} Q(s', a')$   
     $l(s, a) \leftarrow l(s, a) + 1$   
    if  $l(s, a) = m$  then  
      if  $Q(s, a) - U(s, a)/m \geq 2\epsilon_1$  then  
         $Q(s, a) \leftarrow U(s, a)/m + \epsilon_1$   
         $t^* \leftarrow t$   
      else if  $t(s, a) \geq t^*$  then  
        LEARN( $s, a$ )  $\leftarrow$  FALSE  
     $t(s, a) \leftarrow t$   
     $U(s, a) \leftarrow 0$   
     $l(s, a) \leftarrow 0$   
  if  $t(s, a) < t^*$  then LEARN( $s, a$ )  $\leftarrow$  TRUE
```

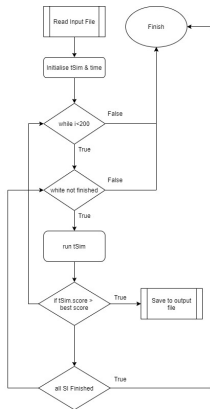
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Figure: Delayed Q-Learning

# RL SBB - Objective Function

$$\frac{1}{60} \times \left[ \sum_{S, \mathcal{R}, \mathcal{RS}} \text{weightIn}_{rs} \times \max \left( 0, (t_{S,r,rs}^{\text{entry}} - \text{inLat}_{S,rs}) \right) \right. \\ \left. + \text{weightOut}_{rs} \times \max \left( 0, (t_{S,r,rs}^{\text{exit}} - \text{outLat}_{S,rs}) \right) \right] \\ + \sum_{S, \mathcal{R}, \mathcal{RS}} p_{S,rs} \times \beta_{S,rs}$$

# RL SBB - Discrete Event Simulation



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## Algorithm 3 tSim Control Functions

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```
procedure RUN(self)
     $t \leftarrow \text{minTime}$ 
    while  $t < (\text{maxTime} + 10)$  do
        for  $e$  in event[ $t$ ] do
            runNextTrain[ $e$ ]
         $t \leftarrow t + 1$ 
    return calculateScore()

procedure RUNNEXTTRAIN(self,  $e$ )
    if  $e = \text{type}(\text{Node})$  then runNode()
    else if  $e = \text{type}(\text{Resource})$  then del e.resource
    else if  $e = \text{type}(\text{Station})$  then nextEvent  $\leftarrow$  newevent[ $e$ ]
```

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# RL SBB - Reinforcement Learning Applied Algorithm

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**Algorithm 5** tSim.QTable Class Functions

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```
procedure GETACTION(self,o,s)      ▷ Getting Action for any option or state
  if  $length(o) = 1$  return  $o[0]$  then
  if randj $\epsilon$  then                ▷ Uniform Random Package, for better convergence
     $a \leftarrow rand(o)$ 
  else
     $m \leftarrow -\infty$                                 ▷ Maximum Q-Value
     $a \leftarrow \text{null}$                                 ▷ Action to be performed
    for all  $o$  do
       $v \leftarrow qval(s, o[i])$                         ▷ qval is predicted reward of action o[i]
      if  $v > m$  then
         $a \leftarrow c$ 
         $m \leftarrow v$ 
  return  $a$ 
procedure UPDATE(self, $S_n, S_{n-1}, A, r$ )                ▷ Updating Q-Values
   $p \leftarrow qval(S_{n-1}, A)$                         ▷ Previous Value
   $m \leftarrow 0$ 
  if  $qval(S_n) > 0$  then                                ▷ If exists, update max
     $m \leftarrow max(qval(S_n))$ 
   $v \leftarrow (1 - \alpha) * p + \alpha * (r + \gamma * m)$   ▷ New Value
  return  $v$                                              ▷ Update Q-Table with new Q-Value
```

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# RL SBB - Results

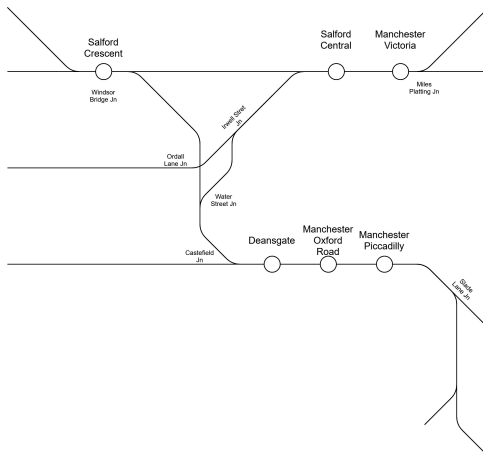
| ID | Solving System |       |               |          |               |          |
|----|----------------|-------|---------------|----------|---------------|----------|
|    | 01             |       | 02            |          | 03            |          |
|    | <i>ObjVal</i>  | Time  | <i>ObjVal</i> | Time     | <i>ObjVal</i> | Time     |
| 01 | 0              | 5.452 | 0             | 4.296    | 0             | 7.590    |
| 02 | 0.43           | 87    | 0.43          | 72.087   | 0.43          | 77.042   |
| 03 | 0.86           | 45    | 0.96          | 198.273  | 0.11          | 200.234  |
| 04 | 24.94          | 430   | 24.94         | 439.121  | 024.94        | 435.945  |
| 05 | n/a            | 1800  | n/a           | 1800     | n/a           | 1800     |
| 06 | 207.12         | 1355  | 207.12        | 1174.690 | 207.12        | 1750.154 |
| 07 | 456.60         | 1800  | 442.33        | 1800     | 467.10        | 1800     |
| 08 | 153.23         | 1800  | 122.70        | 1800     | 233.6         | 1800     |
| 09 | 138.95         | 1800  | 38.25         | 1800     | 43.95         | 1800     |

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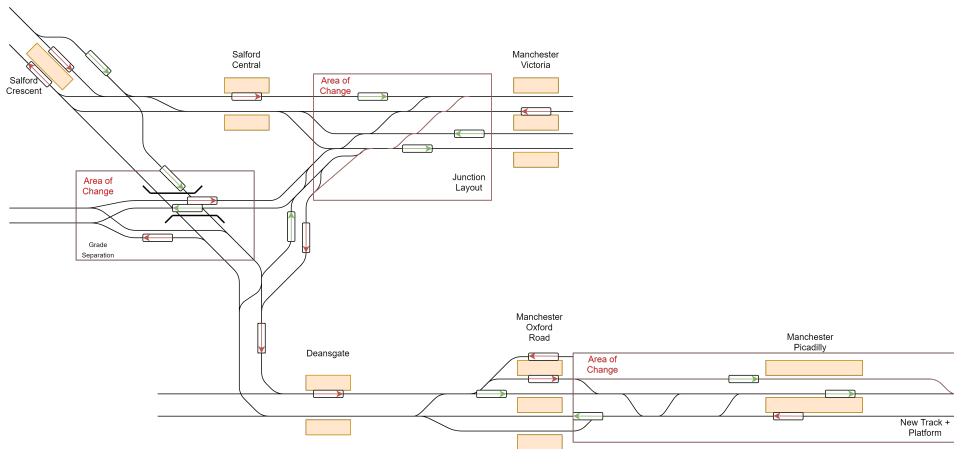
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# Manchester Case Study - Method

- Junction  
TPH  
Analysis
- Station  
Timing  
Data
- Previous  
Case  
Studies



# Manchester Case Study - Key Outcomes





# Final Words - Questions

## Brief Discussion and Q&A



(a) Full Research Paper



(b) Full Slides



(c) Full Poster