

## Lab 1

Table: Average Interval Length and Coverage for Frequentist and Uniform Bayes Intervals for Different Sample Sizes

<b>n = 30</b>	Frequentist	Uniform Bayes
Average Interval Length	0.3212	0.3080
Coverage	0.9539	0.9311
<b>n = 5</b>	Frequentist	Uniform Bayes
Average Interval Length	0.6533	0.6010
Coverage	0.8026	0.9672

- When  $n = 5$ , the coverage for the frequentist interval (0.8026) is much worse than for the uniform Bayes interval (0.9672). Perhaps this is because with such a small sample size, the estimator of  $p$  cannot be approximated with a normal distribution, resulting in a less accurate frequentist confidence interval.
- When  $n = 30$ , the coverage for the frequentist interval is as desired, near 95%. The uniform Bayes interval has a slightly worse coverage (0.9311), probably because the uniform prior is not a great choice for the prior.
- The average interval length is smaller for both intervals when the sample size is larger. This makes sense because with more data points, the true value of  $p$  can be narrowed down more.

### Code

```
n <- 5
p <- 0.7
i <- 10000

data <- rbinom(i, n, p)
freq_interval_lengths <- NULL
unif_interval_lengths <- NULL
freq_interval_captures <- NULL
unif_interval_captures <- NULL

for(i in 1:length(data)) {
  successes <- data[i]
  x_bar <- successes/n
  interval_freq <- c(x_bar - 1.96*sqrt((x_bar*(1 - x_bar))/n), x_bar + 1.96*sqrt((x_bar*(1 - x_bar))/n))
  interval_prior_unif <- qbeta(c(0.025, 0.975), 1 + successes, 1 + n - successes)
  freq_interval_lengths = cbind(freq_interval_lengths, interval_freq[2] - interval_freq[1])
  unif_interval_lengths = cbind(unif_interval_lengths, interval_prior_unif[2] - interval_prior_unif[1])
  freq_interval_captures = cbind(freq_interval_captures, p > interval_freq[1] & p < interval_freq[2])
  unif_interval_captures = cbind(unif_interval_captures, p > interval_prior_unif[1] & p <
interval_prior_unif[2])
}

freq_coverage <- mean(freq_interval_captures)
unif_coverage <- mean(unif_interval_captures)
average_freq_length <- mean(freq_interval_lengths)
average_unif_length <- mean(unif_interval_lengths)
```