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$y_{ij} \sim N(\beta_{0i} + \beta_{1i} x_j, \tau)$

$x_j = j - 5.5 \quad , \quad j = 1, 2, \ldots, 10$

$\beta_{0i} \sim N(\mu_0, \tau_0)$

$\beta_{1i} \sim N(\mu_1, \tau_1)$

$\tau \sim IG(5,4)$

$\tau_0 \sim IG(5,4)$

$\tau_1 \sim IG(5,4)$

$\mu_0 \sim N(12,1)$

$\mu_1 \sim N(1,1)$

$$\pi(\beta_0, \beta_1, \tau, \tau_0, \tau_1 \mid y, x) \propto \left\{\prod_{i=1}^{n}\left[\prod_{j=1}^{10} \tau^{-\frac{1}{2}} \exp\left[-\frac{1}{2\tau}\left(y_{ij} - (\beta_{0i} + \beta_{1i} x_j)\right)^2\right]\right]\right\}\left\{\prod_{i=1}^{n} \tau_0^{-\frac{1}{2}} \exp\left[-\frac{1}{2\tau_0}(\beta_{0i} - \mu_0)^2\right]\right\}$$

$$\cdot\left\{\prod_{i=1}^{n} \tau_1^{-\frac{1}{2}} \exp\left[-\frac{1}{2\tau_1}(\beta_{1i} - \mu_1)^2\right]\right\}\left\{\tau^{-6} \exp\left(-\frac{4}{\tau}\right)\right\}\left\{\tau_0^{-6} \exp\left(-\frac{4}{\tau}\right)\right\}\left\{\tau_1^{-6} \exp\left(-\frac{4}{\tau}\right)\right\}$$

$$\cdot\left\{\exp\left(-(\mu_0 - 12)^2\right)\right\}\left\{\exp\left(-(\mu_1 - 1)^2\right)\right\}$$

Looking at the relevant terms of the joint posterior, we can find the full conditionals of β_{0i} and β_{1i}

$$\pi(\beta_{0i} \mid y, x, \beta_{0(-i)}, \beta_1, \tau, \tau_0, \tau_1) \propto \exp\left[-\frac{1}{2\tau}\sum_{j=1}^{10}\left(-2y_{ij}\beta_{0i} + \beta_{0i}^2 + 2\beta_{0i}\beta_{1i}x_j\right)\right] \exp\left[-\frac{1}{2\tau_0}\left(\beta_{0i}^2 - 2\beta_{0i}\mu_0\right)\right]$$

$$\propto \exp\left[-\frac{1}{2\tau}\left(-2\beta_{0i}\sum_{j=1}^{10} y_{ij} + 10\beta_{0i}^2 + 2\beta_{0i}\beta_{1i}\overbrace{\sum_{j=1}^{10} x_j}^{0 \text{ since } X \text{ is mean centered}}\right) - \frac{1}{2\tau_0}\left(\beta_{0i}^2 - 2\beta_{0i}\mu_0\right)\right]$$

$$\propto \exp\left[-\frac{1}{2}\left[\left(10\tau^{-1} + \tau_0^{-1}\right)\beta_{0i}^2 - 2\left(\tau^{-1}\sum_{j=1}^{10} y_{ij} + \tau_0^{-1}\mu_0\right)\beta_{0i}\right]\right]$$

$$\boxed{\beta_{0i} \mid y, x, \beta_{0(-i)}, \beta_1, \tau, \tau_0, \tau_1 \sim N\left(\left(10\tau^{-1} + \tau_0^{-1}\right)^{-1}\left(\tau^{-1}\sum_{j=1}^{10} y_{ij} + \tau_0^{-1}\mu_0\right), \left(10\tau^{-1} + \tau_0^{-1}\right)^{-1}\right)}$$

$$\pi(\beta_{1i} \mid y, x, \beta_0, \beta_{1(-i)}, \tau, \tau_0, \tau_1) \propto \exp\left[-\frac{1}{2\tau}\sum_{j=1}^{10}\left(-2y_{ij}x_j\beta_{1i} + 2\beta_{0i}\beta_{1i}x_j + \beta_{1i}^2 x_j^2\right)\right] \exp\left[-\frac{1}{2\tau_1}\left(\beta_{1i}^2 - 2\beta_{1i}\mu_1\right)\right]$$

$$\propto \exp\left[-\frac{1}{2\tau}\left(-2\beta_{1i}\sum_{j=1}^{10} y_{ij}x_j + 2\beta_{0i}\beta_{1i}\overbrace{\sum_{j=1}^{10} x_j}^{0} + \beta_{1i}^2\sum_{j=1}^{10} x_j^2\right) - \frac{1}{2\tau_1}\left(\beta_{1i}^2 - 2\beta_{1i}\mu_1\right)\right]$$

$$\propto \exp\left[-\frac{1}{2}\left[\left(\tau^{-1}\sum_{j=1}^{10} x_j^2 + \tau_1^{-1}\right)\beta_{1i}^2 - 2\left(\tau^{-1}\sum_{j=1}^{10} y_{ij}x_j + \tau_1^{-1}\mu_1\right)\beta_{1i}\right]\right]$$

$$\boxed{\beta_{1i} \mid y, x, \beta_0, \beta_{1(-i)}, \tau, \tau_0, \tau_1 \sim N\left(\left(\tau^{-1}\sum_{j=1}^{10} x_j^2 + \tau_1^{-1}\right)^{-1}\left(\tau^{-1}\sum_{j=1}^{10} y_{ij}x_j + \tau_1^{-1}\mu_1\right), \left(\tau^{-1}\sum_{j=1}^{10} x_j^2 + \tau_1^{-1}\right)^{-1}\right)}$$

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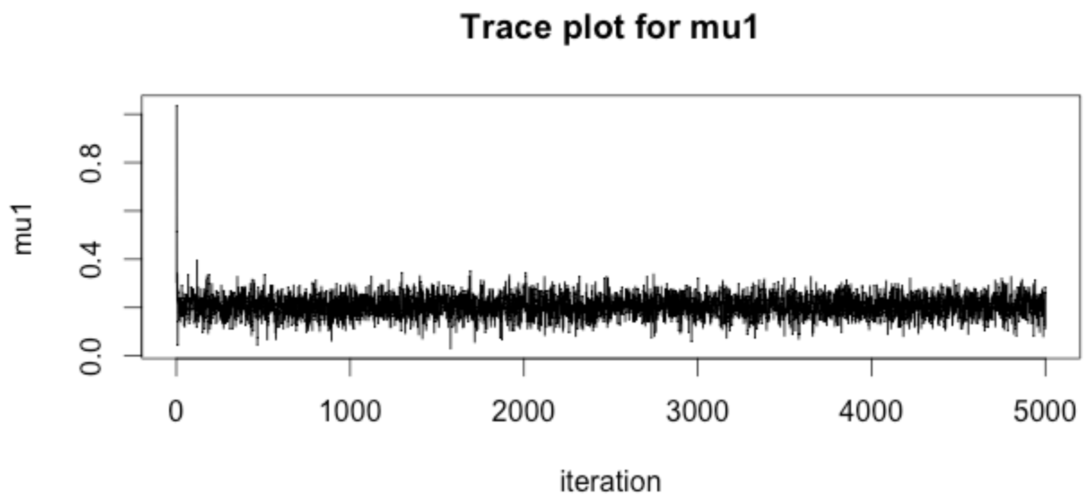
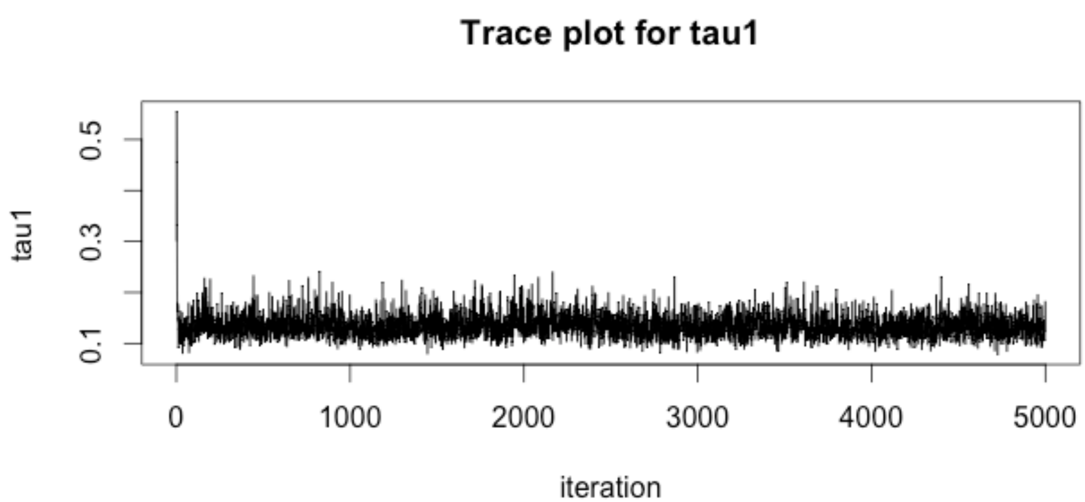
Lab 7

Using these conditionals, 5000 iterations of Gibbs sampling were run to approximate the posterior distributions of the parameters.

We are interested in the slope of a new patient $\beta_{1(74)} \sim N(\mu_1, \tau_1)$. Looking at the trace plots for μ_1 and τ_1, I threw away the first 10 values of each as burn-ins.



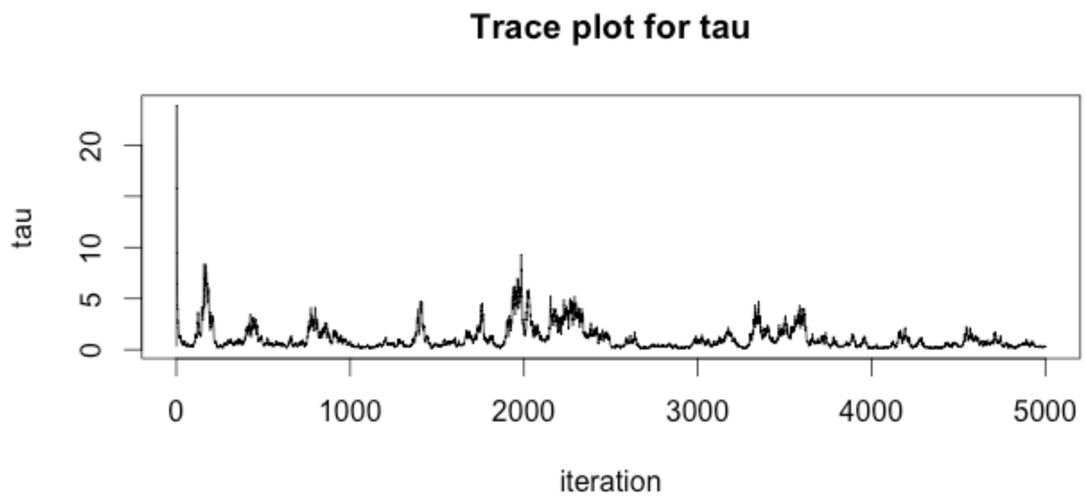


I then iterated through each of the remaining values for μ_1 and τ_1, and drew a slope $\beta_{1(74)}$ from $N(\mu_1, \tau_1)$. Finally, I estimated the posterior probability that a new patient will have a slope exceeding 0.5 by taking the proportion of these simulated slopes that were greater than 0.5

$$P(\beta_{1(74)} > 0.5 \mid y, x, \beta_0, \beta_1, \tau, \tau_0, \tau_1) \approx 0.2114$$

Extra Credit

Looking at the trace plots for the parameters, we see that τ appears to have the highest autocorrelation since its trace plot oscillates slower and each of its iterations seems to be strongly correlated with the previous.



To get less correlated draws of τ, we could do "thinning" and only keep every 10^{th} iteration of τ. In this way each draw is less dependent on the previous.