Kelvin Niu

$$y_i | c_i = 0 \sim LN(\mu_1, \sigma_1^2)$$
$$y_i | c_i = 1 \sim LN(\mu_2, \sigma_2^2)$$

$$c_i = \begin{cases} 0, \text{if weekday} \\ 1, \text{if weekend} \end{cases}$$

$$\mu_2 \sim N(m, v)$$
$$\mu_1 = \mu_2 + \varepsilon$$

Question 1

$$\varepsilon \sim N(\mu_\varepsilon, \sigma_\varepsilon^2) I(\varepsilon > 0)$$
$$\sigma_1^{-2} \sim Ga(a,b)$$
$$\sigma_2^{-2} \sim Ga(a,b)$$

$$\pi(\mu_2, \sigma_1^{-2}, \sigma_2^{-2}, \varepsilon | y, c) \propto \prod_{c_i=0}\left[\frac{1}{\sigma_1}\right]\exp\left[-\frac{1}{2\sigma_1^2}\sum_{c_i=0}(\ln y_i - (\mu_2+\varepsilon))^2\right]\prod_{c_i=1}\left[\frac{1}{\sigma_2}\right]\exp\left[-\frac{1}{2\sigma_2^2}\sum_{c_i=1}(\ln y_i - \mu_2)^2\right]$$
$$\cdot v^{-\frac{1}{2}}\exp\left[-\frac{1}{2v}(\mu_2 - m)^2\right]\frac{1}{\sigma_\varepsilon}\exp\left[-\frac{1}{2\sigma_\varepsilon^2}(\varepsilon - \mu_\varepsilon)^2\right]I(\varepsilon>0)$$
$$\cdot (\sigma_1^{-2})^{a-1} e^{-b\sigma_1^{-2}} (\sigma_2^{-2})^{a-1} e^{-b\sigma_2^{-2}}$$

$$\pi(\mu_2 | \sigma_1^{-2}, \sigma_2^{-2}, \varepsilon, y, c) \propto \exp\left[-\frac{1}{2\sigma_1^2}\sum_{c_i=0}(-2\mu_2 \ln y_i + \mu_2^2 + 2\mu_2\varepsilon)\right]\exp\left[-\frac{1}{2\sigma_2^2}\sum_{c_i=1}(-2\mu_2\ln y_i + \mu_2^2)\right]$$
$$\cdot \exp\left[-\frac{1}{2v}(\mu_2^2 - 2\mu_2 m)\right]$$

$$\propto \exp\left[-\frac{1}{2}\left[\left(\sigma_1^{-2}\sum_{c_i=0}1 + \sigma_2^{-2}\sum_{c_i=1}1 + v^{-1}\right)\mu_2^2 - 2\left(\sigma_1^{-2}\sum_{c_i=0}\ln y_i - \sigma_1^{-2}\varepsilon\sum_{c_i=0}1 + \sigma_2^{-2}\sum_{c_i=1}\ln y_i + v^{-1}m\right)\mu_2\right]\right]$$

$$\boxed{\mu_2 | \sigma_1^{-2}, \sigma_2^{-2}, \varepsilon, y, c \sim N\left(\left(\sigma_1^{-2}\sum_{c_i=0}1 + \sigma_2^{-2}\sum_{c_i=1}1 + v^{-1}\right)^{-1}\left(\sigma_1^{-2}\sum_{c_i=0}\ln y_i - \sigma_1^{-2}\varepsilon\sum_{c_i=0}1 + \sigma_2^{-2}\sum_{c_i=1}\ln y_i + v^{-1}m\right), \left(\sigma_1^{-2}\sum_{c_i=0}1 + \sigma_2^{-2}\sum_{c_i=1}1 + v^{-1}\right)^{-1}\right)}$$

$$\pi(\sigma_1^{-2} | \mu_2, \sigma_2^{-2}, \varepsilon, y, c) \propto \sigma_1^{-\sum_{c_i=0}1}\exp\left[-\frac{1}{2\sigma_1^2}\sum_{c_i=0}(\ln y_i - (\mu_2+\varepsilon))^2\right](\sigma_1^{-2})^{a-1}e^{-b\sigma_1^{-2}}$$

$$\propto (\sigma_1^{-2})^{a+\frac{1}{2}\sum_{c_i=0}1 - 1}\exp\left[-\sigma_1^{-2}\left(b + \frac{1}{2}\sum_{c_i=0}(\ln y_i - (\mu_2+\varepsilon))^2\right)\right]$$

$$\boxed{\sigma_1^{-2} | \mu_2, \sigma_2^{-2}, \varepsilon, y, c \sim Ga\left(a + \frac{1}{2}\sum_{c_i=0}1, b + \frac{1}{2}\sum_{c_i=0}(\ln y_i - (\mu_2+\varepsilon))^2\right)}$$

$$\pi(\sigma_2^{-2} | \mu_2, \sigma_1^{-2}, \varepsilon, y, c) \propto \sigma_2^{-\sum_{c_i=1}1}\exp\left[-\frac{1}{2\sigma_2^2}\sum_{c_i=1}(\ln y_i - \mu_2)^2\right](\sigma_2^{-2})^{a-1}e^{-b\sigma_2^{-2}}$$

$$\propto (\sigma_2^{-2})^{a+\frac{1}{2}\sum_{c_i=1}1 - 1}\exp\left[-\sigma_2^{-2}\left(b + \frac{1}{2}\sum_{c_i=1}(\ln y_i - \mu_2)^2\right)\right]$$

$$\boxed{\sigma_2^{-2} | \mu_2, \sigma_1^{-2}, \varepsilon, y, c \sim Ga\left(a + \frac{1}{2}\sum_{c_i=1}1, b + \frac{1}{2}\sum_{c_i=1}(\ln y_i - \mu_2)^2\right)}$$

$$\pi(\varepsilon \mid \mu_2, \sigma_1^{-2}, \sigma_2^{-2}, y, c) \propto \exp\left[-\frac{1}{2\sigma_1^2} \sum_{c_i=0} \left(-2\varepsilon \ln y_i + 2\varepsilon \mu_2 + \varepsilon^2\right)\right] \exp\left[-\frac{1}{2\sigma_\varepsilon^2}\left(\varepsilon^2 - 2\varepsilon \mu_\varepsilon\right)\right] I(\varepsilon > 0)$$

$$\propto \exp\left[-\frac{1}{2}\left[\left(\sigma_1^{-2} \sum_{c_i=0} 1 + \sigma_\varepsilon^{-2}\right)\varepsilon^2 - 2\left(\sigma_1^{-2} \sum_{c_i=0} \ln y_i - \sigma_1^{-2} \mu_2 \sum_{c_i=0} 1 + \sigma_\varepsilon^{-2} \mu_\varepsilon\right)\varepsilon\right]\right] I(\varepsilon > 0)$$

$$\boxed{\varepsilon \mid \mu_2, \sigma_1^{-2}, \sigma_2^{-2}, y, c \sim N\left(\left(\sigma_1^{-2} \sum_{c_i=0} 1 + \sigma_\varepsilon^{-2}\right)^{-1}\left(\sigma_1^{-2} \sum_{c_i=0} \ln y_i - \sigma_1^{-2} \mu_2 \sum_{c_i=0} 1 + \sigma_\varepsilon^{-2} \mu_\varepsilon\right), \left(\sigma_1^{-2} \sum_{c_i=0} 1 + \sigma_\varepsilon^{-2}\right)^{-1}\right) I(\varepsilon > 0)}$$

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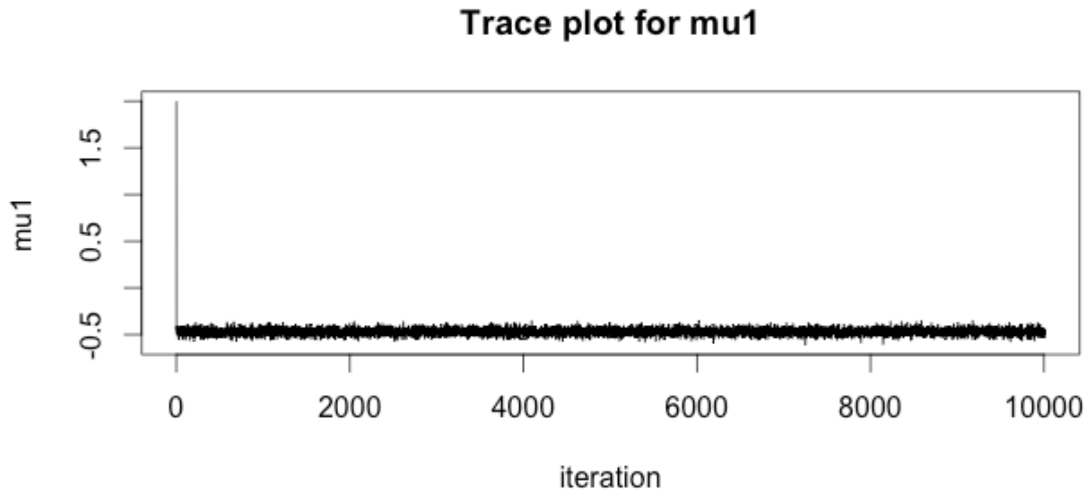
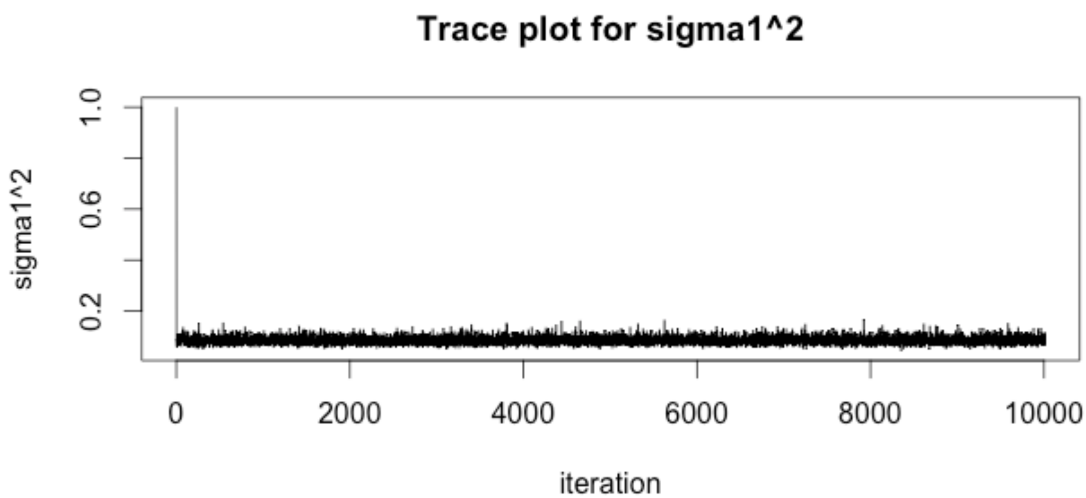
Lab 9

Question 2





The trace plots suggest that the samplers for μ_1 and σ_1^2 have converged. 10 burn-ins were thrown out for the analysis to follow.

Question 3

Table: Posterior point estimates and credible intervals for parameters

	Mean	95% CI
μ_1	-0.4688	[-0.5350, -0.4026]
σ_1^2	0.0847	[0.0612, 0.1179]
μ_2	-1.3885	[-1.5054, -1.2761]
σ_2^2	0.0924	[0.0537, 0.1567]

<u>Question 4</u>
To estimate the desired probabilities, I looked at the samples from the marginal posterior distributions (obtained from Gibbs) and calculated the proportion of iterations for which $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$.
$P(\mu_1 > \mu_2 \mid y, c) \approx \mathbf{1}$
$P(\sigma_1 > \sigma_2 \mid y, c) \approx \mathbf{0.4297}$

<u>Question 5</u>
For each iteration of μ_1 and σ_1^2, I drew a sample representing a weekday from a log normal distribution with parameters μ_1 and σ_1^2. Similarly, for each iteration of μ_2 and σ_2^2, I drew a sample representing a weekend from a log normal distribution with parameters μ_2 and σ_2^2. To estimate the probability that a randomly chosen future weekday has a higher pollution level than a randomly chosen weekend, I calculated the proportion of the drawn samples for which the weekday sample was greater than the weekend sample:
0.9833