

HW 11 Problems

Bayes approach under ignorability/MAR (missing data imputation)

$$y_i = x_i' \beta + e_i$$

$$e_i \sim N(0, \sigma^2)$$

$x = \begin{pmatrix} x_1' \\ \dots \\ x_n' \end{pmatrix}$ is the design matrix with some elements missing

$(x_{i2}, \dots, x_{ip}) \sim N_{p-1}(\mu, \Sigma)$ where p is the number of β 's

We can set priors for $\beta, \sigma^2, \mu, \Sigma$

$(\beta, \sigma^2) \sim N - IG \rightarrow$ jointly conjugate without missingness

$(\mu, \Sigma) \sim MVN - IW \rightarrow$ jointly conjugate without missingness

Since the observed data likelihood in calculating the posterior is ugly, we can use data augmentation

$$\pi(\beta, \sigma^2, \mu, \Sigma, x_{mis} | y, x_{obs}) \propto \pi(\beta, \sigma^2) \pi(\mu, \Sigma) L(y | x, \beta, \sigma^2) L(x | \mu, \Sigma)$$

To do:

Derive details of above

Implement for $p = 3$ with attached data and summarize posterior of β relative to complete case MLE