Lab Assignment 5: Gibbs Sampling

Your friend, "Sloppy" Jo(e) is an aspiring pollster, and she (he) agrees to conduct a poll for you, free of charge. You give the following instructions: "Please ask about 25 people whether they are in favor of more gun control, and report back to me the number who are in favor." After a few days Sloppy returns with the poll results: there were *y* = 20 in favor. "And how many people did you ask?" you inquire. "Ummm, I dunno. You didn't ask me to record that. All I know is that it was about 25."

Assume $(y|N,\beta) \sim Binomial(N,\beta)$. Furthermore, assume a uniform prior on β and a Poisson prior on *N*. Do the following:

1. Derive the joint distribution of (y, N, β)
2. Derive full conditionals $(N|\beta, y)$ and $(\beta|N, y)$.
3. Use these to sample (using Gibbs sampling) from the joint posterior $(\beta, N|y)$, using a starting value of $(\beta^{(0)}, N^{(0)}) = (.05, 50)$.
4. Show the 2D trace plot for the first 10 draws of the Gibbs sampler, $\left(\beta^{(0:9)}, N^{(0:9)}\right)$. I want to see both the points and the connecting lines.
5. Give the central 90% posterior credible interval for β, accurate to (and rounded to) the nearest 1% for both upper and lower limits.
6. What is the probability that exactly 20 people were polled? Base answer on at least 10,000 draws (post-burn-in), and round to nearest one-tenth of 1% (answer need not be accurate to the nearest one tenth of 1%).

Hints: I will take off points if rounding is not as specified. By "accurate to and rounded to the nearest 1%", I mean that your reported answer should be something like (0.74, 0.94) and the exact true answer, when rounded to the nearest 1% should be the same. When I say "round to nearest one-tenth of 1%," I am looking for a number like 11.3% (or 0.113).

When finding full conditional for *N*, you may want to find the distribution of *N-y* and add this to the given value for *y*.