Lab 3

- Jennifer is a chemist working on a project to create molecule-size filters. She creates the filters by dissolving several chemical compounds in water in a cylindrical container, then adding a polymerizing agent, which creates a sponge-like structure. Different amounts of the chemicals in the solution affect the characteristics of the resulting structure, such as strength and pore size, but they also affect whether the solution polymerizes successfully at all.

  Knowing of your statistical prowess, Jennifer approaches you for help with some data analysis. She has experimented with several different solutions with mixed results. Two of them seem especially promising; we'll call them Solution A and Solution B. Here are her results from each of the two solutions:

Solution	Successful	Unsuccessful	Total
A	11	3	14
B	5	1	6

  Solution B appears to be performing better, but Jennifer is uncertain because of the small sample size.

  1. Using Beta(1/2,1/2) as a default prior, what are the posterior distributions for the probabilities of success p_A and p_B for each of the two distributions? Plot these distributions.

     Beta(1/2,1/2) is *Jeffrey's prior* for the Beta-Binomial model, a particular non-informative prior which has some nice properties.

  2. Jennifer doesn't want to waste time on a process that is successful less than 80% of the time. Given the data collected so far, what is the probability that solution A will be successful at least 80% of the time? What about solution B?

  3. Solution B has a higher success rate than solution A in the 20 trials Jennifer has done, but that's a pretty small sample size. What is the probability that solution B truly has a higher success rate than solution A? Compute this numerically—you can write down an integral which would give the exact answer if you could solve it, but it's intractable.

  4. What advice would you give Jennifer?