Kelvin Niu

Question 1

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \sim N\Big(\underbrace{(0,0,0)}_{\mu}, \underbrace{\begin{bmatrix} 1 & 0.9 & 0.1 \\ 0.9 & 1 & 0.1 \\ 0.1 & 0.1 & 1 \end{bmatrix}}_{\Sigma} \Big)$$

In general, if $\underbrace{\begin{bmatrix} X \\ Y \end{bmatrix}}_{m\text{-dim},\ n\text{-dim}} \sim N\left(\begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}, \begin{pmatrix} \Sigma_X & \Sigma_{XY} \\ \Sigma_{YX} & \Sigma_Y \end{pmatrix} \right)$

then $X \mid Y \sim N\left(\mu_X + \Sigma_{XY}\Sigma_Y^{-1}(Y-\mu_Y),\ \Sigma_X - \Sigma_{XY}\Sigma_Y^{-1}\Sigma_{YX} \right)$

$$X \mid (Y,Z) \sim N\left(0 + [0.9\ 0.1]\begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}^{-1}\left(\begin{bmatrix} Y \\ Z \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right),\ 1 - [0.9\ 0.1]\begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix} \right)$$

$$\sim \boxed{N(0.8990Y + 0.0101Z,\ 0.1899)}$$

$$Y \mid (X,Z) \sim N\left(0 + [0.9\ 0.1]\begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}^{-1}\left(\begin{bmatrix} X \\ Z \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right),\ 1 - [0.9\ 0.1]\begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix} \right)$$

$$\sim \boxed{N(0.8990X + 0.0101Z,\ 0.1899)}$$

$$Z \mid (X,Y) \sim N\left(0 + [0.1\ 0.1]\begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}^{-1}\left(\begin{bmatrix} X \\ Y \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right),\ 1 - [0.1\ 0.1]\begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \right)$$

$$\sim \boxed{N(0.0526X + 0.0526Y,\ 0.9895)}$$

Question 3

Using the same formula from Question 1,

$$(X,Y) \mid Z \sim N\left(\begin{bmatrix}0\\0\end{bmatrix} + \begin{bmatrix}0.1\\0.1\end{bmatrix} 1^{-1}(z-0), \begin{bmatrix}1 & 0.9\\0.9 & 1\end{bmatrix} - \begin{bmatrix}0.1\\0.1\end{bmatrix} 1^{-1} \begin{bmatrix}0.1 & 0.1\end{bmatrix}\right)$$

$$\sim \boxed{N\left(\begin{bmatrix}0.1\\0.1\end{bmatrix} z, \begin{bmatrix}0.99 & 0.89\\0.89 & 0.99\end{bmatrix}\right)}$$

$$Z \mid (X,Y) \sim N\left(0 + \begin{bmatrix}0.1 & 0.1\end{bmatrix} \begin{bmatrix}1 & 0.9\\0.9 & 1\end{bmatrix}^{-1}\left(\begin{bmatrix}X\\Y\end{bmatrix} - \begin{bmatrix}0\\0\end{bmatrix}\right), 1 - \begin{bmatrix}0.1 & 0.1\end{bmatrix} \begin{bmatrix}1 & 0.9\\0.9 & 1\end{bmatrix}^{-1} \begin{bmatrix}0.1\\0.1\end{bmatrix}\right)$$

$$\sim \boxed{N(0.0526X + 0.0526Y, 0.9895)}$$

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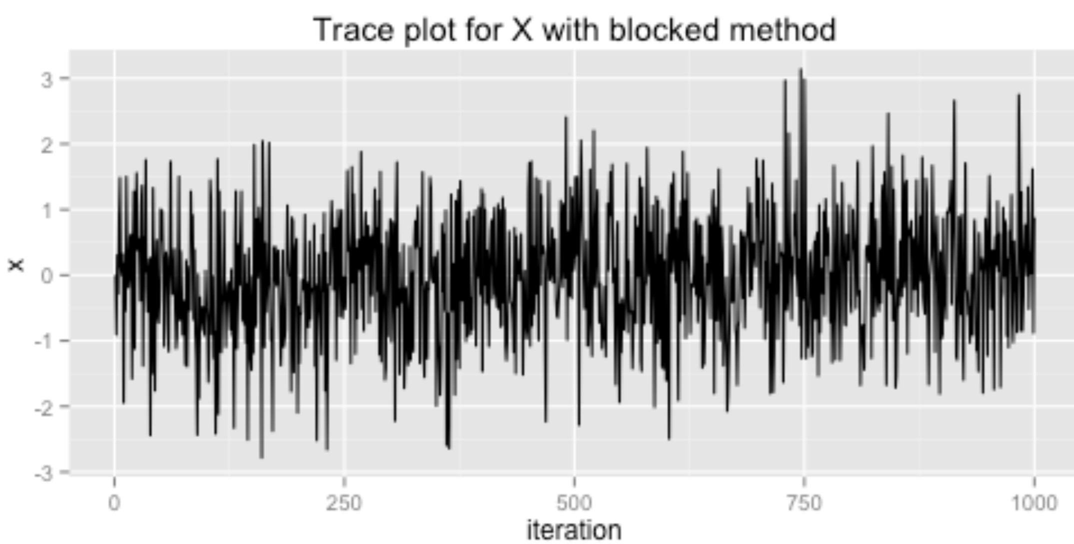
Lab 6

Question 2

The trace plot of X is centered at around 0. It seems to fluctuate rather slowly, staggering and taking several iterations to move up and down. The value at each iteration appears to be highly influenced by the previous iteration.

Question 3



The trace plot for X is again centered at around 0. However, it fluctuates much quicker, with the value of each iteration appearing to be less influenced by the previous iteration.

Question 4

Because each iteration in the regular updating is highly correlated with the previous iteration, it is less efficient than the block updating. It takes more iterations under regular updating to get an accurate approximation of the marginal posterior distribution of X. Looking at the covariance matrix, we see that X and Y have the highest covariance of 0.9. Perhaps by updating X and Y together, we are avoiding most of the correlation found in regular updating.