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Let $X_1, \ldots, X_{100}$ be the measurements of fine particulate air pollution

$X_i \sim LN(\mu, \sigma^2)$

$\mu \sim N(\mu_0, \sigma_0^2)$

$\sigma^{-2} \sim Ga(a, b)$

$$\pi(\mu, \sigma^{-2} | x) = L(x | \mu, \sigma^{-2}) \pi(\mu) \pi(\sigma^{-2})$$

$$\propto \left[\prod_{i=1}^{n} \frac{1}{x_i \sigma} \exp\left[-\frac{1}{2\sigma^2}(\ln x_i - \mu)^2\right]\right] \left[\frac{1}{\sigma_0} \exp\left[-\frac{1}{2\sigma_0^2}(\mu - \mu_0)^2\right]\right] \left[(\sigma^{-2})^{a-1} e^{-b\sigma^{-2}}\right]$$

$$\propto \left[\prod_{i=1}^{n} x_i^{-1}\right] \sigma^{-n-2a+2} \exp\left[-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (\ln x_i - \mu)^2\right] \sigma_0^{-1} \exp\left[-\frac{1}{2\sigma_0^2}(\mu - \mu_0)^2\right] e^{-b\sigma^{-2}}$$

$$\pi(\mu | \sigma^{-2}, x) \propto \exp\left[-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (-2\mu \ln x_i + \mu^2) - \frac{1}{2\sigma_0^2}(\mu^2 - 2\mu\mu_0)\right]$$

$$\propto \exp\left[-\frac{1}{2\sigma^2}\left(-2\mu \sum_{i=1}^{n} \ln x_i + n\mu^2\right) - \frac{1}{2\sigma_0^2}(\mu^2 - 2\mu\mu_0)\right]$$

$$\propto \exp\left[-\frac{1}{2}\left[(n\sigma^{-2} + \sigma_0^{-2})\mu^2 - 2\left(\sigma^{-2} \sum_{i=1}^{n} \ln x_i + \sigma_0^{-2}\mu_0\right)\mu\right]\right]$$

$$\boxed{\mu | \sigma^{-2}, x \sim N\left((n\sigma^{-2} + \sigma_0^{-2})^{-1}\left(\sigma^{-2} \sum_{i=1}^{n} \ln x_i + \sigma_0^{-2}\mu_0\right), (n\sigma^{-2} + \sigma_0^{-2})^{-1}\right)}$$

$$\pi(\sigma^{-2} | \mu, x) \propto (\sigma^{-2})^{\frac{n}{2} + a - 1} \exp\left[-\sigma^{-2}\left(b + \frac{1}{2}\sum_{i=1}^{n} (\ln x_i - \mu)^2\right)\right]$$

$$\boxed{\sigma^{-2} | \mu, x \sim Ga\left(a + \frac{n}{2}, b + \frac{1}{2}\sum_{i=1}^{n} (\ln x_i - \mu)^2\right)}$$

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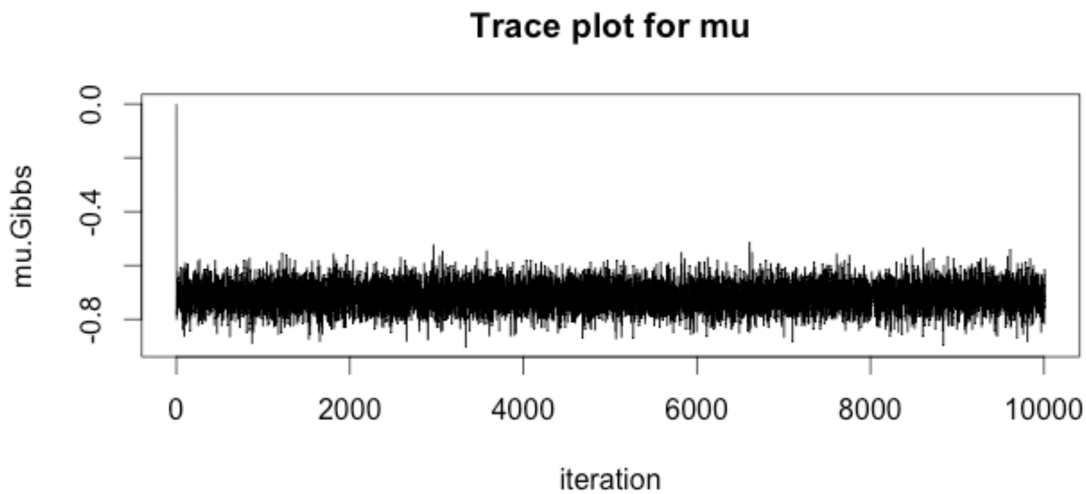
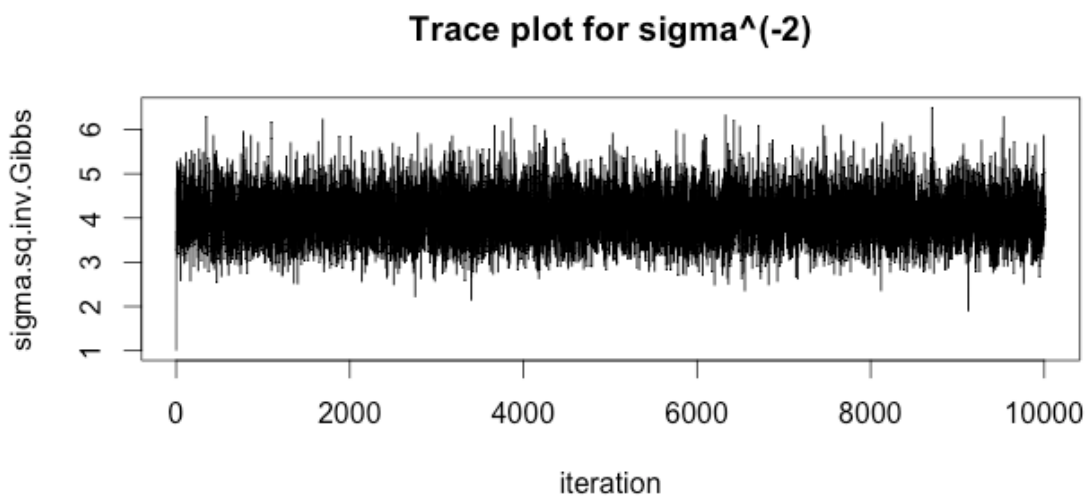
Lab 8

Using the full conditionals and weak priors $\mu \sim (0, 100)$ and $\sigma^{-2} \sim Ga(0.001, 0.001)$, I ran Gibbs sampling with 10,000 iterations to estimate the marginal posterior distributions of μ and σ^{-2}. Examination of the trace plots below suggests that convergence has been reached and the space has been explored reasonably well.





After throwing out 10 burn-ins for both μ and σ^{-2}, I obtained the following estimates of μ and σ^{-2}

Table: Estimates of the marginal posterior distributions of μ and σ^{-2} from Gibbs sampling

	Mean	95% CI
μ	-0.7150	[-0.8132, -0.6161]
σ^{-2}	4.0380	[2.9688, 5.2499]

I then calculated a mean and variance for each iteration using the formulas

$$Mean = e^{\mu+\frac{\sigma^2}{2}}$$

$$Var = \left(e^{\sigma^2} - 1\right)e^{2\mu+\sigma^2}$$

Next, using all the means and variances from all the iterations, I determined 95% credible intervals for the mean and variance of the pollution levels

Table: 95% credible intervals for mean and variance of pollution levels

	95% CI
Mean	[0.5027, 0.6201]
Var	[0.0586, 0.1388]