Lab 3 (from Douglas N. VanDerwerken)

- Suppose $X|(\tau^2) \sim N(0, 1/\tau^2)$ and $\tau^2 \sim \text{Gamma}(\text{shape} = \nu/2, \text{rate} = \nu/2)$. Derive the marginal distribution of X. Show ALL work.

- Let $\nu = 1$. Draw a sample of 10000 from the marginal distribution of X by drawing 10000 τ^2's and then 10000 X's given the τ^2's. Plot the sample (either histogram or density is fine). Give two names for the actual marginal distribution $p(X)$ when $\nu = 1$.

- Use the Kolmogorov-Smirnov test (`ks.test` in R) to test whether your observed distribution is equal to a t distribution with 1 degree of freedom. Report the p-value. What is the conclusion of the test?

- Now, repeat the above sampling and `ks.test` 1000 times, using 100 draws from $p(X)$ each time (instead of 10000 draws as above). Record the p-value at each iteration. (Do not report, but this will be used for the next step. The p-value can be grabbed using this R code: `ks.test(x,'pt',1)$p`.) Plot a histogram of the 1000 p-values and include this in report. What distribution should this be? Hint: it's a Beta(a, b) for some a, b in $\{1, 2, 3, ...,\}$.

- Does the Central Limit Theorem hold for the mean of a sample from $p(X)$ when $\nu = 1$? What about $\nu = 2$? $\nu = 3$? Why or why not? A quick explanation will do; an involved proof is NOT required.