HW 3 Problems

Assessing whether perchlorate is carcinogenic based on rats (hypothesis testing)

Data:
Control (y): $0/30$
Exposed (x): $2/30$
$y_i \sim Bern(\theta_1)$
$x_i \sim Bern(\theta_2)$

Hypotheses:
H_0: $\theta_1 = \theta_2$
H_1: $\theta_1 \neq \theta_2$
Let $M = \begin{cases} 1, if\ H_1\ is\ true \\ 0, if\ H_0\ is\ true \end{cases}$ and let us set a prior on $M \sim Bern(0.5)$ that doesn't favor either hypotheses.

$\Pr(M = 1|data) = \frac{1}{1+BF}$ where $BF = \frac{L(data|M=0)}{L(data|M=1)}$ is the Bayes factor in favor of H_0 over H_1.

To do:
Write down simpler expressions for $\Pr(M = 1|data)$ and BF
Trying 2 different priors, report the $\Pr(M = 1|data)$ and BF
Run a simulation study → 100 reps under H_0 and H_1 → compare Bayes and freq (Fisher's exact test) results