Linear stability analysis of galactic outflows in the cold dark matter halo

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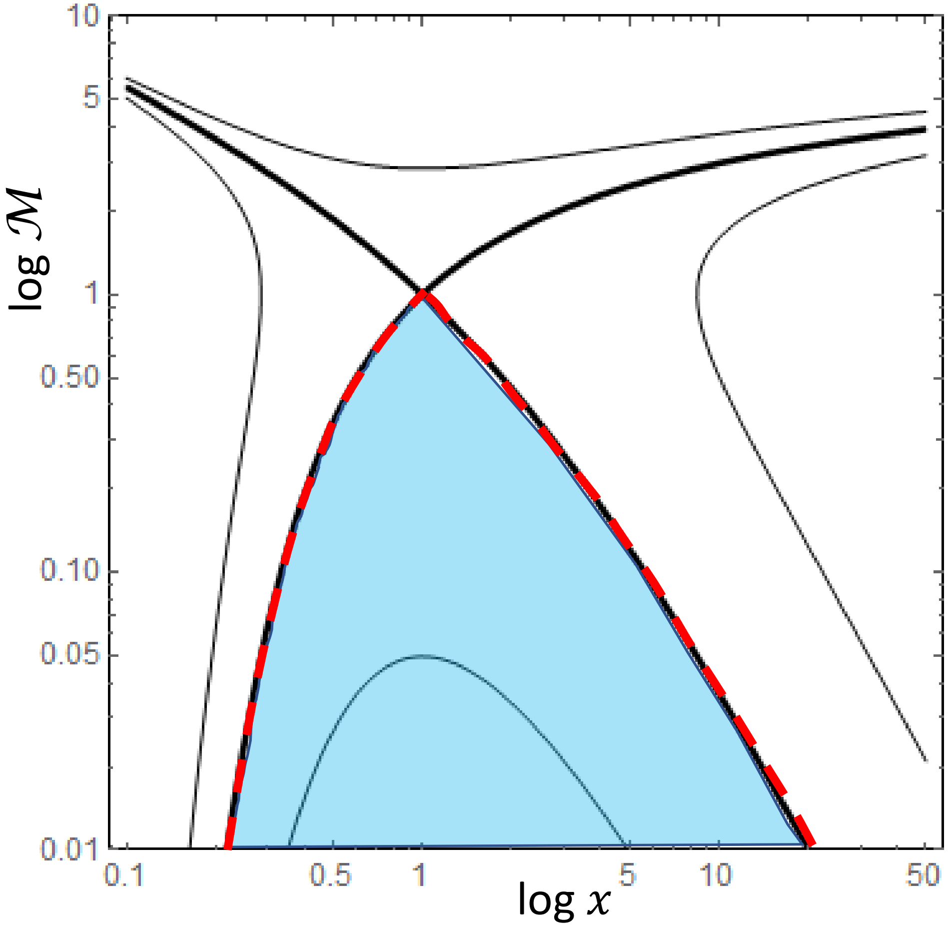
Introduction – Solution curves

- Transonic flow (bold line)
  - Flow continuously accelerates from subsonic to supersonic through a sonic point.
  - Transonic flow may form a terminal shock.

- Breeze (blue region)
  - Flow is always subsonic.

- Critical breeze (red dashed curve)
  - The flow accelerates until $\mathcal{M}$ =1 along the transonic solution and transitions to another transonic solution at the sonic points.



Purpose of this study

- We investigate the linear stability analysis of galactic breezes to explore the fundamental physical properties of the galactic outflows in a cold dark matter halo.

- We want to show the importance of the transonic outflows and the relevance between the density profile of dark matter halo (DMH) and the stability of the galactic winds.

Model – Basic equations

- Steady, spherically symmetric, isothermal outflows

➢ Basic equations

$$\frac{\partial}{\partial x}(\rho v x^2) = 0 \qquad v\frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\partial \Phi}{\partial x} \qquad p = c_s^2 \rho$$

➢ Equation of base flow

$$\left(\mathcal{M} - \frac{1}{\mathcal{M}}\right)\frac{d\mathcal{M}}{dx} = \frac{2}{x} - \frac{1}{c_s^2}\frac{d\Phi(x)}{dx}$$

$x = r/r_s$
r_s : scale radius of DMH
c_s : sound velocity (constant)
Φ : gravitational potential
x_0 : start point of flows
$\mathcal{M}_0$: Mach number at start point

$$\mathcal{M} = \begin{cases} \sqrt{-W_0\left(-\frac{\mathcal{M}_0^2 x_0^4}{x^4}\exp\left(-\mathcal{M}_0^2 + 2\left(\Phi(x) - \Phi(x_0)\right)\right)\right)} & \text{(for subsonic)} \\ \sqrt{-W_{-1}(-\frac{\mathcal{M}_0^2 x_0^4}{x^4}\exp\left(-\mathcal{M}_0^2 + 2\left(\Phi(x) - \Phi(x_0)\right)\right))} & \text{(for supersonic)} \end{cases}$$

➢ Lambert W function
defined the inverse function of $y = x\exp(x)$

Model – Non-perturbed state

RHS of equation of base flow

$$\frac{2}{x} - \frac{1}{c_s^2}\frac{d\Phi(x)}{dx} = \frac{2}{x} - K_{\mathrm{DMH}} f(x)$$

$f(x)$: function obtained by integrating DMH density profile

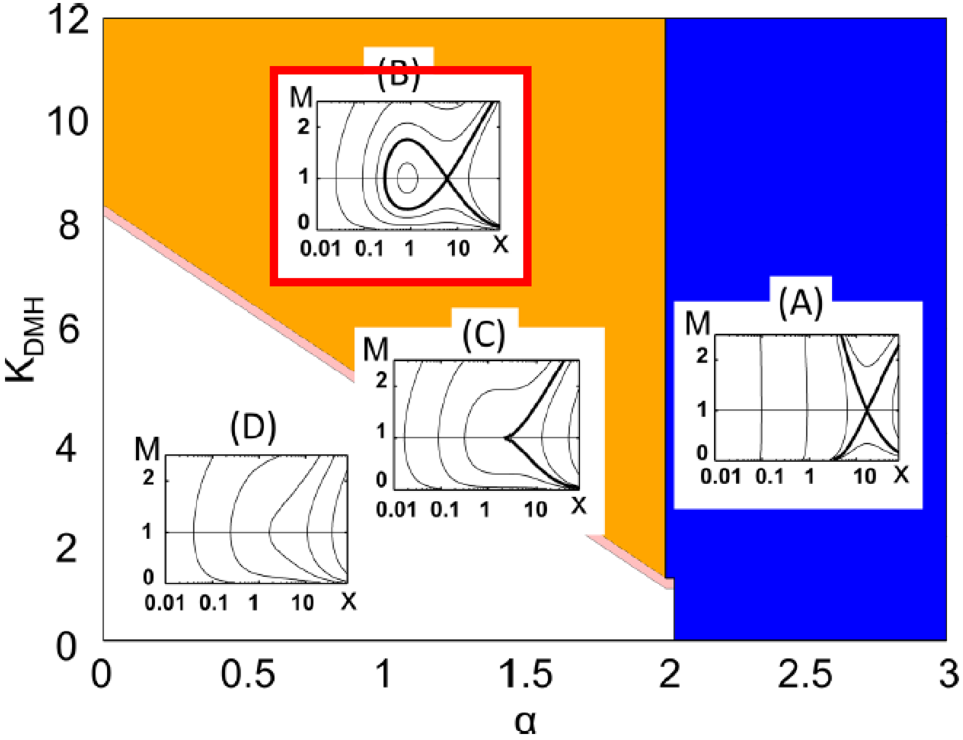
- DMH density profile

$$\rho(x) = \begin{cases} \dfrac{\rho_d}{x\ (x\ + 1)^2} & \text{(NFW profile)} \\ \dfrac{\rho_d}{(x\ + 1)^3} & \text{(core profile)} \end{cases}$$

- Parameter K

$$K_{\mathrm{DMH}} = \frac{2\pi\rho_d r_d^2 G}{c_s^2} \approx \frac{\text{gravitatioal potential}}{\text{thermal energy}}$$

Using K=10.0, to consider transonic solution and compare two type DM profile.



Igarashi et al. (2013)

Model - Perturbation

Applying the solar wind model given by Velli (2001),

- Perturbation $y^{\pm} = \widehat{\mathcal{M}} \pm \hat{p} = y^{\pm}(x)\exp\{(-i\omega + \gamma)t\}$

$y^{\pm}$ are the conserved quantities (Riemann invariants) along characteristics.

- Perturbation equation

$$(\mathcal{M} \pm 1){y^{\pm}}' + (-i\omega + \gamma)y^{\pm} + \frac{1}{2}\left(y^{\mp} + y^{\pm}\right)\frac{\mathcal{M}'}{\mathcal{M}}(\mathcal{M} \mp 1) = 0$$

Assumption

① : long wavelength approximation ($\omega = 0$)
② : $y^{+}(x_0) = y^{-}(x_0) \neq 0$
③ : $|y^{+}(x_1)|, |y^{-}(x_1)| \to 0 \ \ (x_1 \gg 1)$

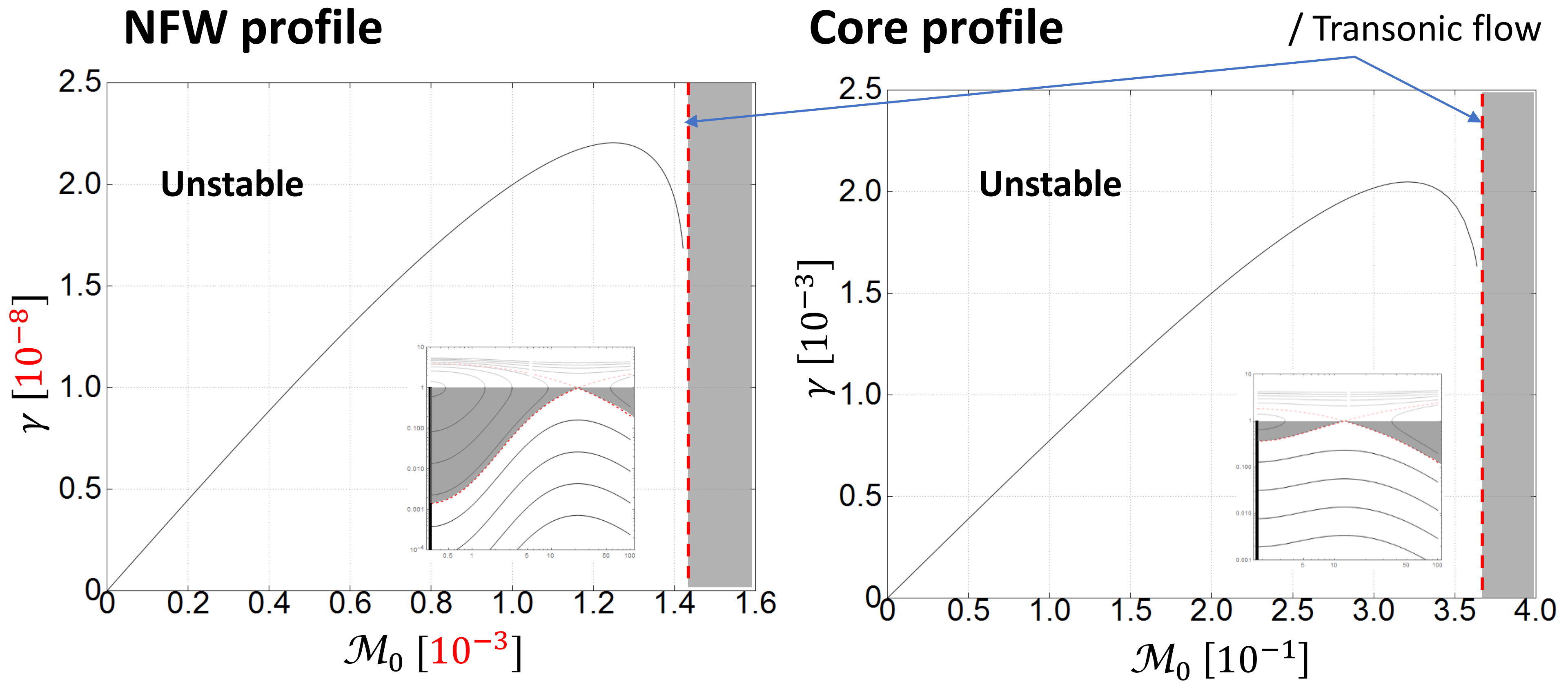
Subscribe
0 : start point of flows
1 : very large distance from galactic center

- Growth rate of the instability

$$\gamma = \frac{2\left(|y^{+}|_{x_0}^{2} - |y^{+}|_{x_1}^{2}\right)}{\int_{x_0}^{x_1} \mathcal{M}^{-1}[(\mathcal{M}+1)|y^{+}|^{2} - (\mathcal{M}-1)|y^{-}|^{2}]dx}$$

Estimate γ for $\mathcal{M}_0$, Mach number at the start point, of each breeze.

Result – the growth rate γ



- $\gamma > 0$ → Unstable
- These galactic breezes are always unstable.
- γ for NFW profile is much smaller than that for core profile.

$$\boxed{\gamma \sim \begin{cases} 10^{-8} & \text{(NFW)} \\ 10^{-3} & \text{(core)} \end{cases}}$$

Discussion - Growth time of perturbation

$$y^{\pm} = y^{\pm}(x)\exp(\gamma \boxed{t}) \rightarrow t = \frac{t'}{\boxed{t_s}} \rightarrow t_{\rm grow} = \frac{(r_s/c_s)}{\gamma} \qquad \gamma \sim \begin{cases} 10^{-8} & \text{(NFW)} \\ 10^{-3} & \text{(core)} \end{cases}$$

Sound crossing time

$M_{\rm halo}$	z	r_s[kpc]	Growth time $t_{\rm grow}$ [yr]	
			NFW	Core
$10^8 M_\odot$	0	2.4	7.8×10^{14}	7.8×10^{9}
	9	0.2	7.7×10^{13}	7.7×10^{8}
$10^{10} M_\odot$	0	11	3.5×10^{15}	3.5×10^{10}
	9	1.1	3.5×10^{14}	3.5×10^{9}
$10^{12} M_\odot$	0	51	1.6×10^{16}	1.6×10^{11}
	9	5.1	1.6×10^{15}	1.6×10^{10}

$c_s \cong 300$ km/s

NFW model:

- The growth times are far longer than the age of the universe.
- So, the breeze is mathematically unstable, but it is virtually stable.
- On the other hand, very small $\mathcal{M}_0$ ($\sim 10^{-3\sim-4}$), Mach number at the start point, indicates that the breeze hardly exists in actual galaxies.

Core model:

- In less-massive high-z galaxies, the breezes are physically unstable; therefore, they could not exist during the galactic evolution.

<u>**Transonic galactic outflows play an essential role in galactic evolution.**</u>

Summary

- We investigate the stability of galactic breezes.
- In NFW profile, the breeze is mathematically unstable but virtually stable. Besides, for very small $\mathcal{M}_0$, the breeze hardly exits in actual galaxies.
- In Core profile, the breezes in less-massive high-z galaxies is physically unstable.
- Transonic outflows are essential to galactic evolution.

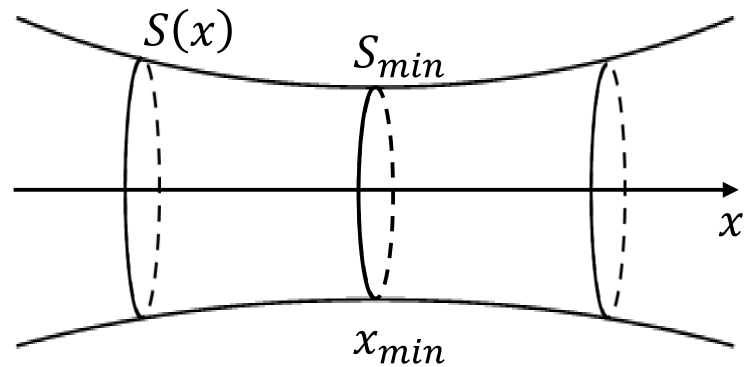
Future work

- Liner stability analysis for transonic outflows and inflows with a termination shock.
- Comparing with observations.

Appendix

Acceleration mechanism (1)

Accelerate mechanism : Laval nozzle



Fluid equation

$$(\mathcal{M}^2 - 1)\frac{1}{v}\frac{dv}{dx} = \frac{1}{S}\frac{dS(x)}{dx}$$

$\mathcal{M}$: Mach number $(= v/c_s)$
c_s : sound velocity
S : cross section of nozzle

For subsonic $(\mathcal{M} < 1)$,
If $dS/dx < 0$, fluid is accelerated.
If $dS/dx > 0$, fluid is decelerated.
For supersonic $(\mathcal{M} > 1)$, change of cross section affects inverted effect for fluid.

$dS/dx = 0$ at $x = x_{min}$, if $\mathcal{M} = 1$ then, dv/dx doesn't decided uniquely.

This point, $(x, \mathcal{M}) = (x_{min}, 1)$ called transonic point.

Acceleration mechanism (2)

For galactic wind,

Fluid equation

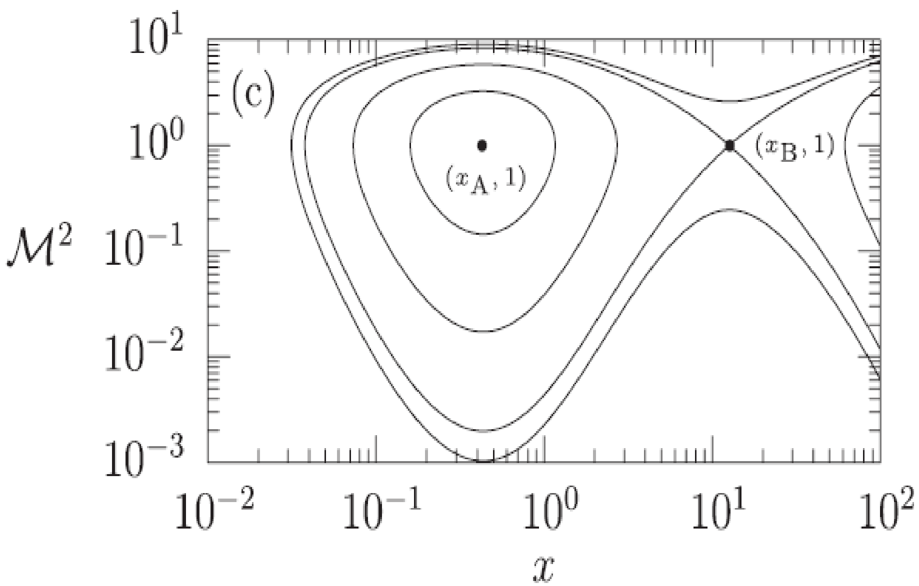
$$(\mathcal{M}^2 - 1)\frac{1}{v}\frac{dv}{dx} = \frac{1}{S}\frac{dS(x)}{dx}$$

$$\left(\mathcal{M} - \frac{1}{\mathcal{M}}\right)\frac{d\mathcal{M}}{dx} = \underbrace{\boxed{\frac{2}{x}}}_{\text{Expansion}} - \underbrace{\boxed{\frac{1}{c_s^2}\frac{d\Phi(x)}{dx}}}_{\text{Contraction}}$$

$\Phi(x)$: gravitational potential

Right hand side shows magnitude correlation between thermal energy and gravitational potential.

Balance of two energies decides the acceleration of wind.



(Tsuchiya et al. 2013)

Previous work (1) – Velli (2001)

- This work investigated the instability of breeze for steady, spherically symmetric, isothermal solar wind.

Fundamental equations of flow

Mass conservation $$\frac{\partial}{\partial r}(\rho v r^2) = 0$$

Equation of motion $$v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{GM_{\odot}}{r^2}$$

$$p = c_s^2 \rho$$

Perturbations are linear combination of Mach number $\widehat{\mathcal{M}}$ and pressure $\hat{p}$.

$$y^{\pm} = \widehat{\mathcal{M}} \pm \hat{p} = y^{\pm}(r)\exp\{(-i\omega + \gamma)t\}$$

Equations of perturbation

$$(\mathcal{M} \pm 1){y^{\pm}}' + (-i\omega + \gamma)y^{\pm} + \frac{1}{2}\left(y^{\mp} + \mathrm{y}^{\pm}\right)\frac{\mathcal{M}'}{\mathcal{M}}(\mathcal{M} \mp 1) = 0$$

Previous work (2) - Velli (2001)

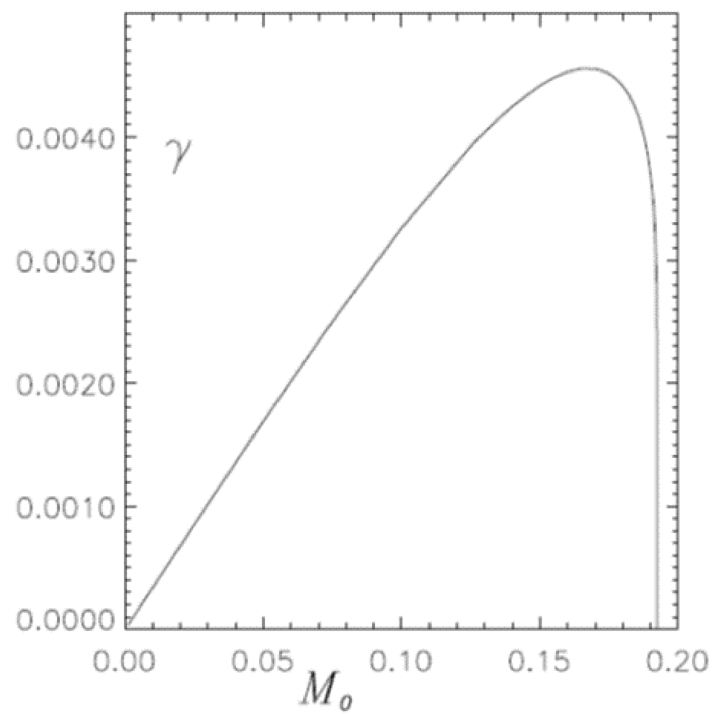
Assumption

① : long wavelength approximation $(\omega = 0)$
② : $y^{+}(0) = y^{-}(0) \neq 0$
③ : $|y^{+}(r_1)|, |y^{-}(r_1)| \rightarrow 0 \ \ (r_1 \gg 1)$

Instability growth rate

$$\gamma = \frac{2\left(|y^{+}|_0^2 - |y^{+}|_{r_1}^2\right)}{\int_{r_0}^{r_1} \mathcal{M}^{-1}[(\mathcal{M}+1)|y^{+}|^2 - (\mathcal{M}-1)|y^{-}|^2]}$$

$\gamma > 0$ → flow is stable.
$\gamma < 0$ → flow is instable.



Breeze is instable for solar wind.

(Velli 2001)

Model – gravitational source

- Gravitational source : DMH only

DMH density profile

$$\rho(x;\alpha,\beta,\gamma) = \frac{\rho_d}{x^\alpha (x^\beta + 1)^\gamma}$$

ρ_d : scale density
α, β, γ : power low index

In this work, density profile assumes NFW profile and core profile.

$\rightarrow$NFW : $\alpha = 1, \beta = 1, \gamma = 2$
core : $\alpha = 0\ \beta = 1, \gamma = 3$

Gravitational potential and force for each profile

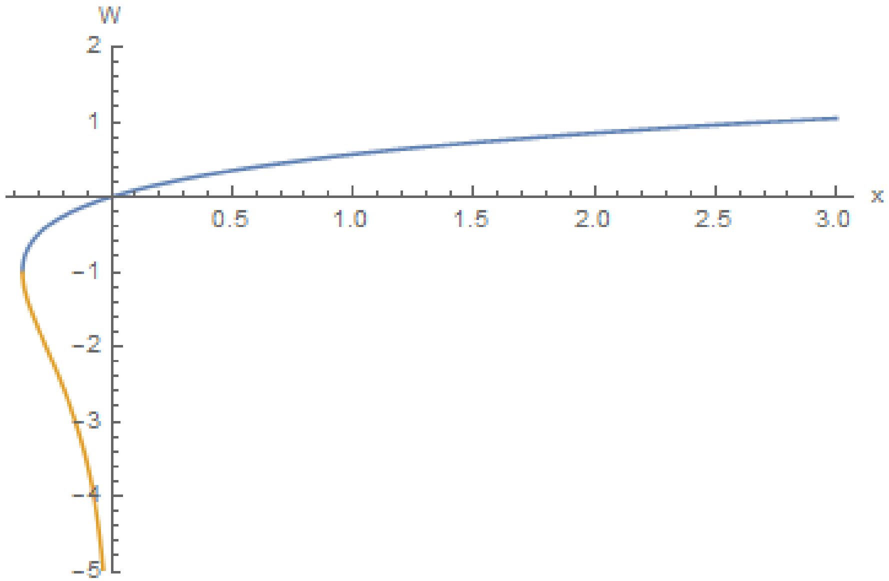
NFW profile	**Core profile**
$\frac{d\Phi(x)}{dx} = 4\pi\rho_d r_d^2 G \left(\ln(x+1) - \frac{x}{x+1}\right)$ $\Phi(x) = -4\pi\rho_d r_d^2 G \frac{\log(x+1)}{x}$	$\frac{d\Phi(x)}{dx} = \frac{4\pi\rho_d r_d^2 G}{x^2} \left(\ln(x+1) - \frac{x(3x+2)}{2(x+1)^2}\right)$ $\Phi(x) = -4\pi\rho_d r_d^2 G \left(\frac{\log(x+1)}{x} - \frac{1}{2(x+1)}\right)$

Function of Mach number (1)

- To formulate Mach number of x, using Lambert W function.

  - Lambert W function
    defined the inverse function of $y = x\exp(x)$

$$y = \begin{cases} W_0(x) \\ W_{-1}(x) \end{cases}$$



Each branched functions
correspond velocities of flows.

$W_0 \Rightarrow$ subsonic flow
$W_{-1} \Rightarrow$ supersonic flow

Function of Mach number (2)

- Mass conservation

$$\frac{\partial}{\partial x}(\rho v x^2) = 0 \quad \rightarrow \quad p\mathcal{M}x^2 = p_0\mathcal{M}_0 x_0^2$$

(Subscribe 0 show quantities at start point of flows)

- Equation of motion

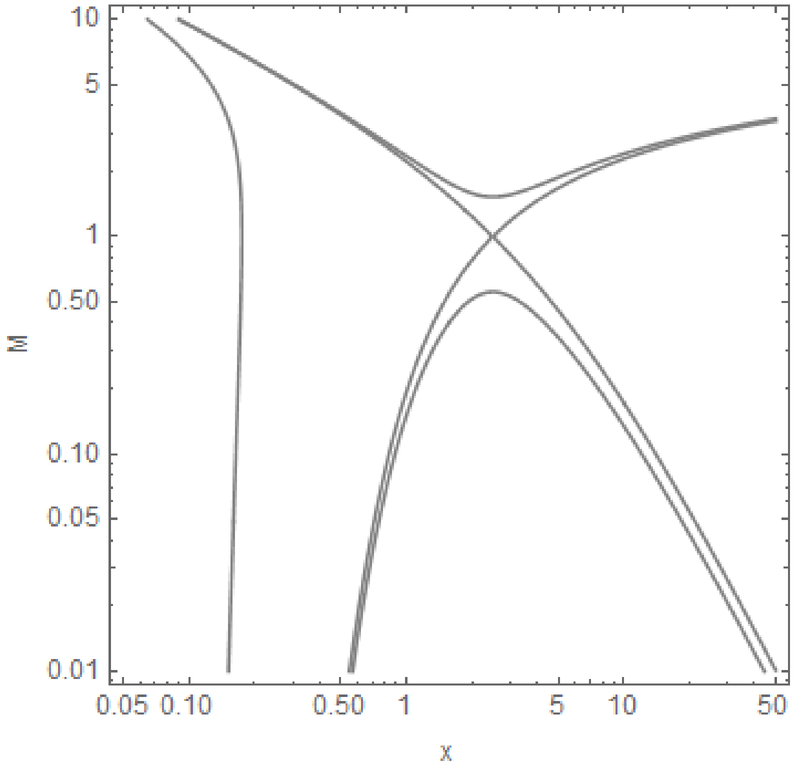
$$v\frac{dv}{dx} = -\frac{1}{\rho}\frac{dp}{dx} - \frac{d\Phi}{dx} \quad \rightarrow \quad p = p_0 \exp\left(-\frac{1}{2}(\mathcal{M}^2 - \mathcal{M}_0^2) - \frac{1}{c_s^2}\big(\Phi(x) - \Phi(x_0)\big)\right)$$

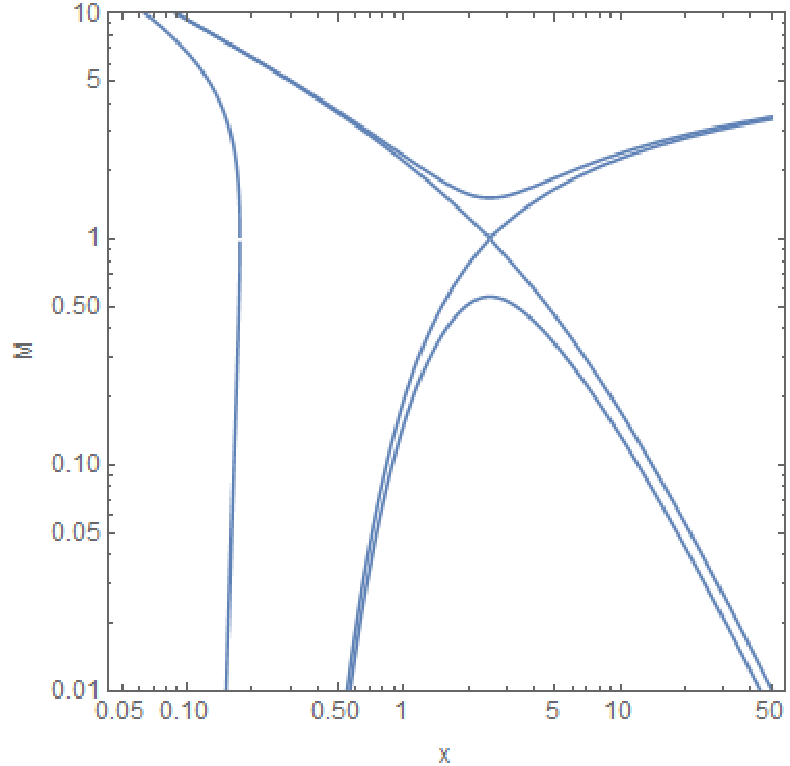
To combine above two functions, formulating Mach number function using Lambert W.

$$\mathcal{M} = \begin{cases} \sqrt{-W_0\left(-\dfrac{\mathcal{M}_0^2 x_0^4}{x^4}\exp\left(-\mathcal{M}_0^2 + 2\big(\Phi(x) - \Phi(x_0)\big)\right)\right)} & \text{(for subsonic)} \\ \sqrt{-W_{-1}(-\dfrac{\mathcal{M}_0^2 x_0^4}{x^4}\exp\left(-\mathcal{M}_0^2 + 2\big(\Phi(x) - \Phi(x_0)\big)\right))} & \text{(for supersonic)} \end{cases}$$

Function of Mach number (3)

M-x phase diagram for solar wind
(left : differential equation of Mach number, right : function using Lambert W)





We obtain the function of Mach number for arbitrary gravitational potential.