Rotation Curve of M33 Explained by Dark Matter Disc

aka

Rotation Curve of Pizza

Toshio FUKUSHIMA (NAOJ)

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ResearchGate Fukushima Click

Xvrot: Fortran 90 software

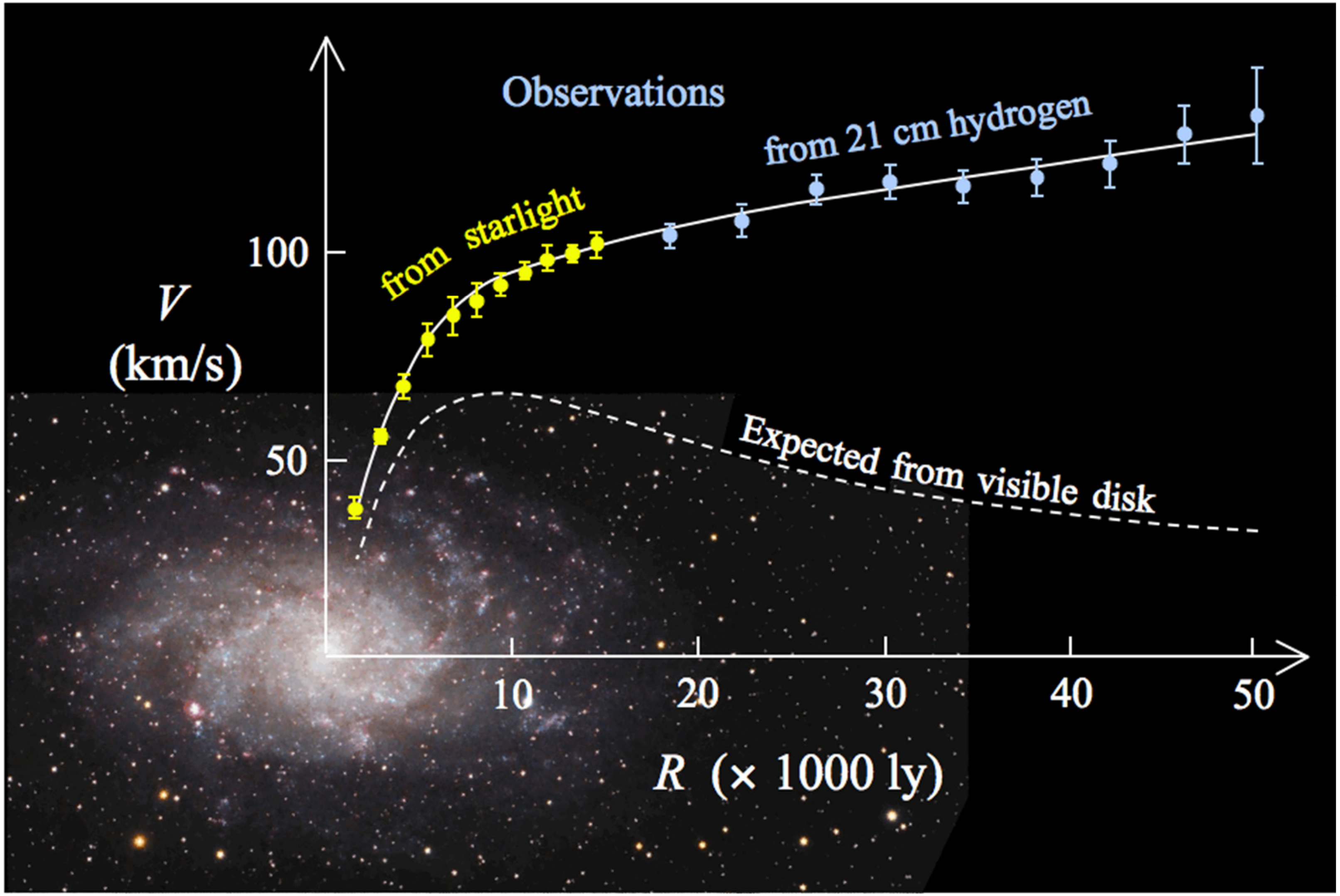
Pizza in Space

Spiral Galaxy M33

- Triangulum Galaxy = NGC598
  - 3rd Largest Member of Local Group
  - Companion to M31 (Andromeda Galaxy)
  - Size: 10 kpc radius
  - Mass: [6 (stars) + 3 (gas)] x 10^9 M_{sun}
  - Spiral with No Core/Bulge
  - Rising? Rotation Curve

Rotation Curve: M33

- Wiki



Cartesian Doubt

Descarte's Doubt Method

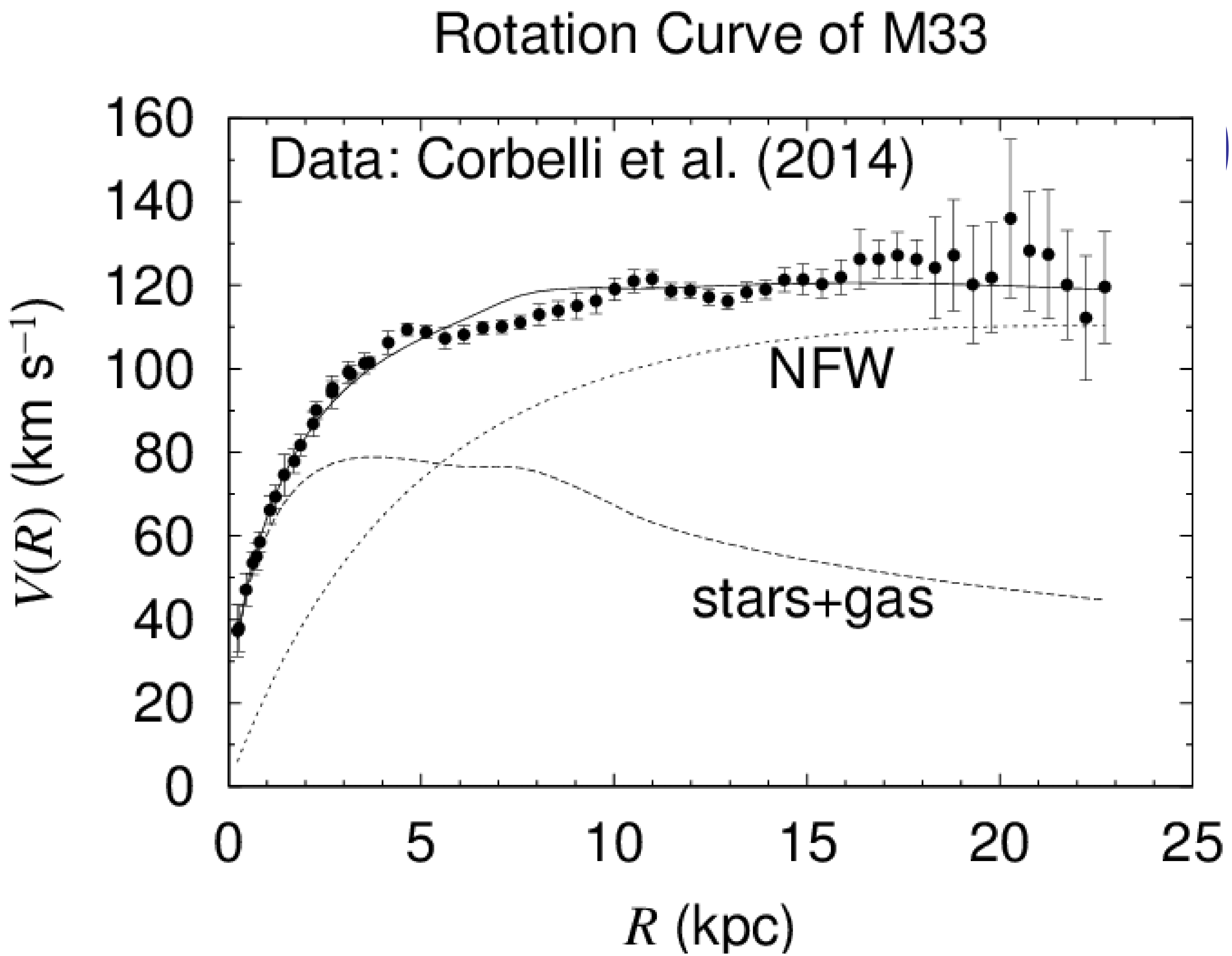
- Descarte (1641)
- 4 Steps Method
- 1. Accept Only Info You Know to be True
- 2. Break Down Truths into Smaller Units
- 3. Solve Simplest Problems First
- 4. Make Complete List of Other Problems

Application to Rotation Curve of M33

  - 1. Accept Only Info You Know to be True
- Rotation Curve, Luminosity Profile
  - 2. Break Down Truths into Smaller Units
- Inner, and Outer Parts of Rotation Curve
  - 3. Solve Simplest Problems First
- Only Disc Mass Component
  - 4. Make Complete List of Other Problems
- Non-Axisymmetric Feature, ...

Standard Approach

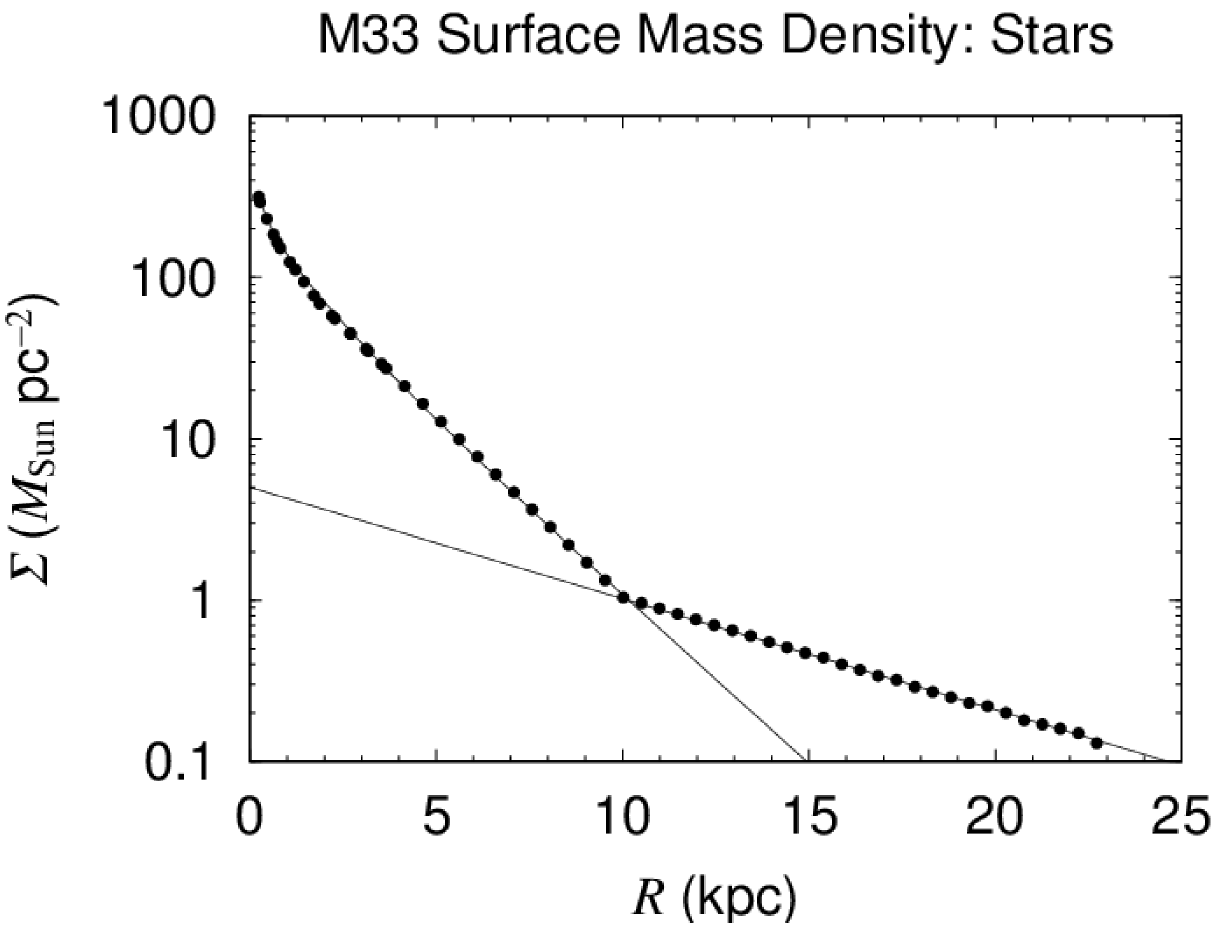
- Deconvolution Method
  - M33: Corbelli et al. (2014)
  - Milky Way: Sofue (2015)
- 1. Compute V(R) of Stars and Gas
- 2. Subtract them from Rotation Curve
- 3. Fit Spherically-Symmetric Model of Dark Matter Distribution to Residuals
  - Navarro, Frenk, & White (NFW) (1996)



Rotation Curve of M33
Data: Corbelli et al. (2014)
NFW
stars+gas
$V(R)$ (km s^{-1})
R (kpc)
0
20
40
60
80
100
120
140
160
0
5
10
15
20
25

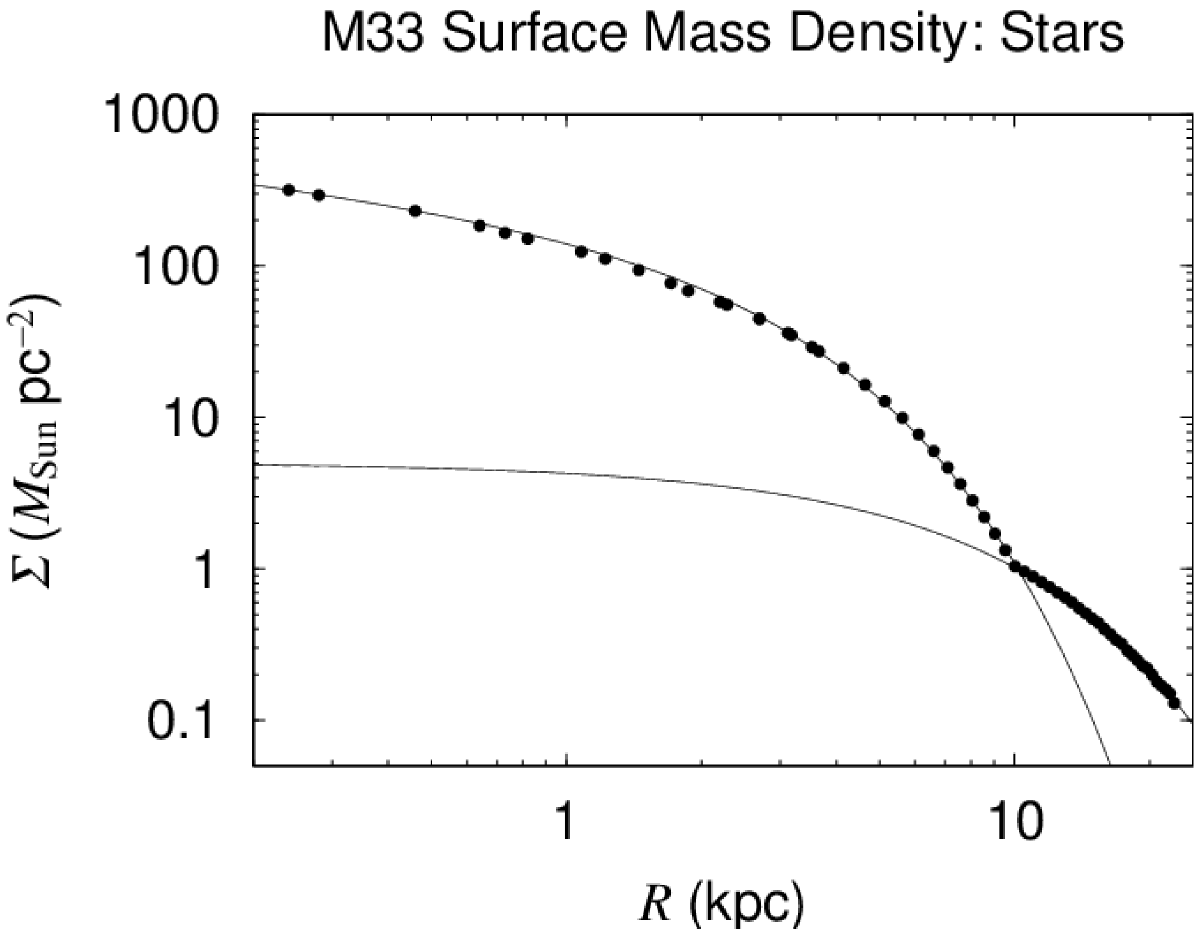
Stars Disc of M33

- 2 parts
- Power & Exp
- Exp.



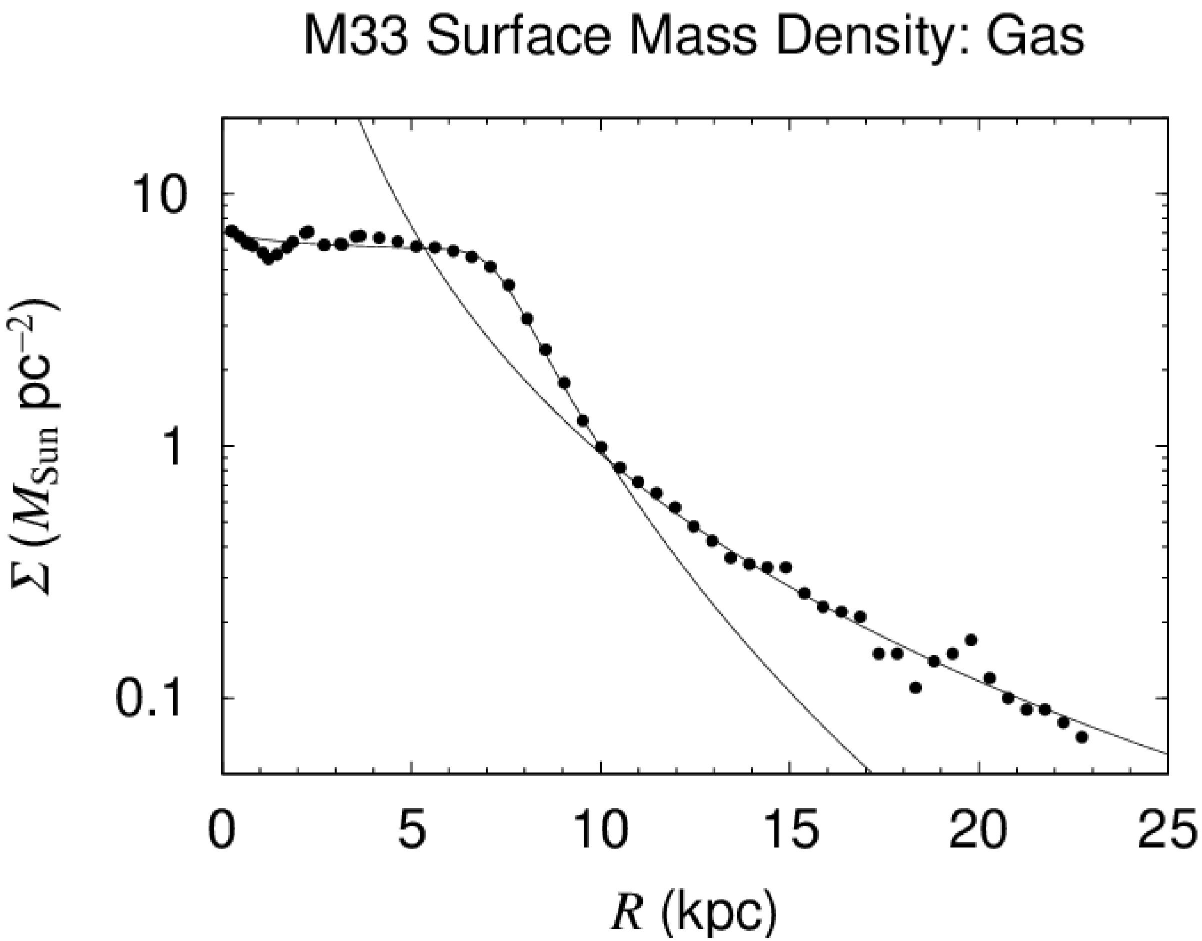
Stars Disc of M33

- 2 parts
- Power
  & Exp
- Exp.



Gas Disc of M33

- 2 parts
- Double Power
- Single Power



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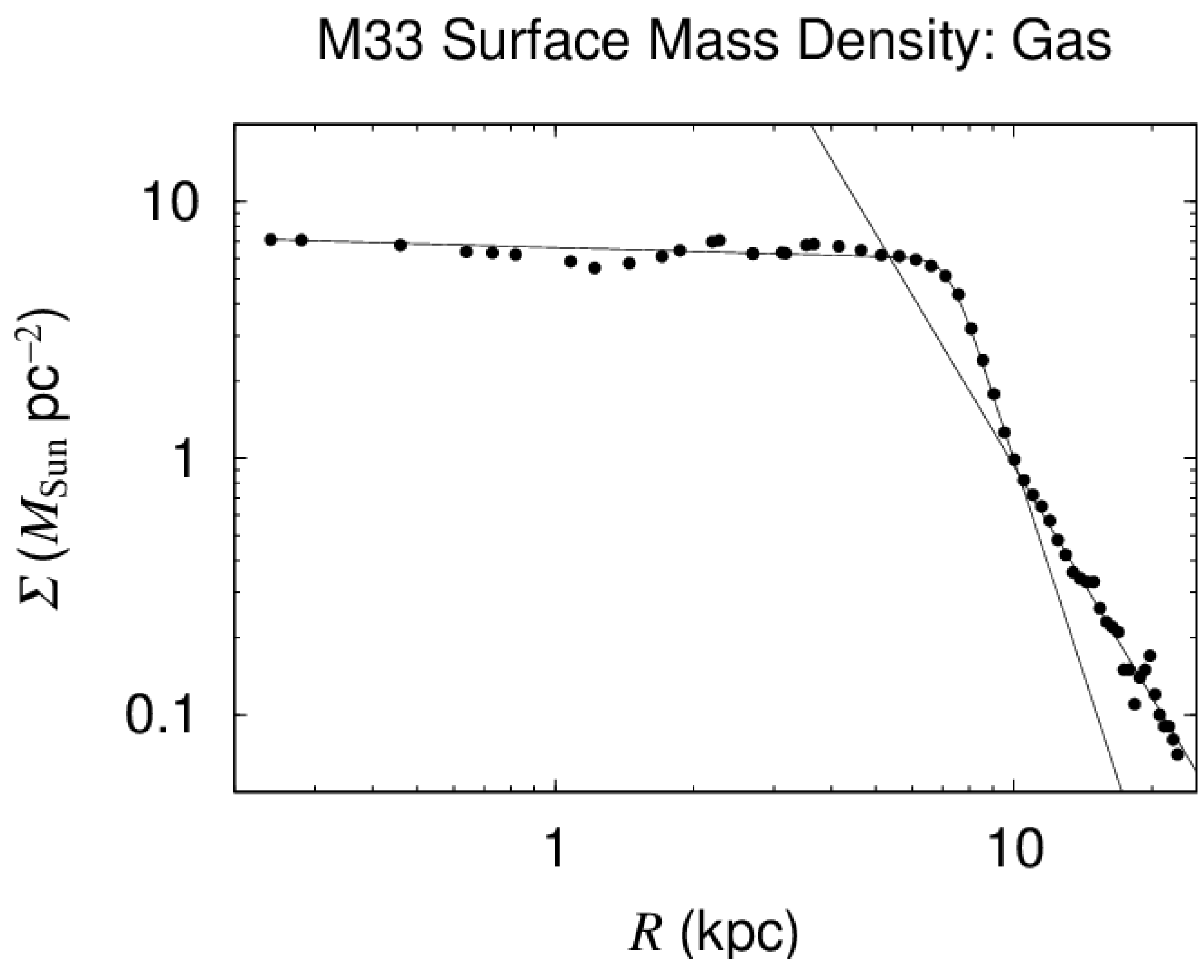


Piecewise Density F.

- No Existing Formulation is Applicable
  - (Infinite) Exponential Disc Model
  - (Infinite) Power-Law Disc Model, …
- Demand for Gravitational Field Computation of General Thin Disc
  - Arbitrary Size and Shape (Finite, Hole, …)
  - Arbitrary Density F. (Double-Power, …)
  - @ Arbitrary Point

The Force is Always with You, Potential

New Method of Grav. Field Computation

- Assumptions
  - Axisymmetric, Infinitely-Thin, Piecewise
- Strategy
  - Potential: Numerical Integration of Ring P.
  - Acceleration: Numerical Differentiation
- Integral Expression

$$\Phi\left(R,z\right)=\sum_{j=1}^{J}\Phi_{j}\left(R,z\right)$$

$$\Phi_{j}\left(R,z\right)=\int_{R_{j-1}}^{R_{j}}\Psi\left(R';R,z\right)dR'$$

Integrand Expression

- Ring Potential (Kellogg 1929)

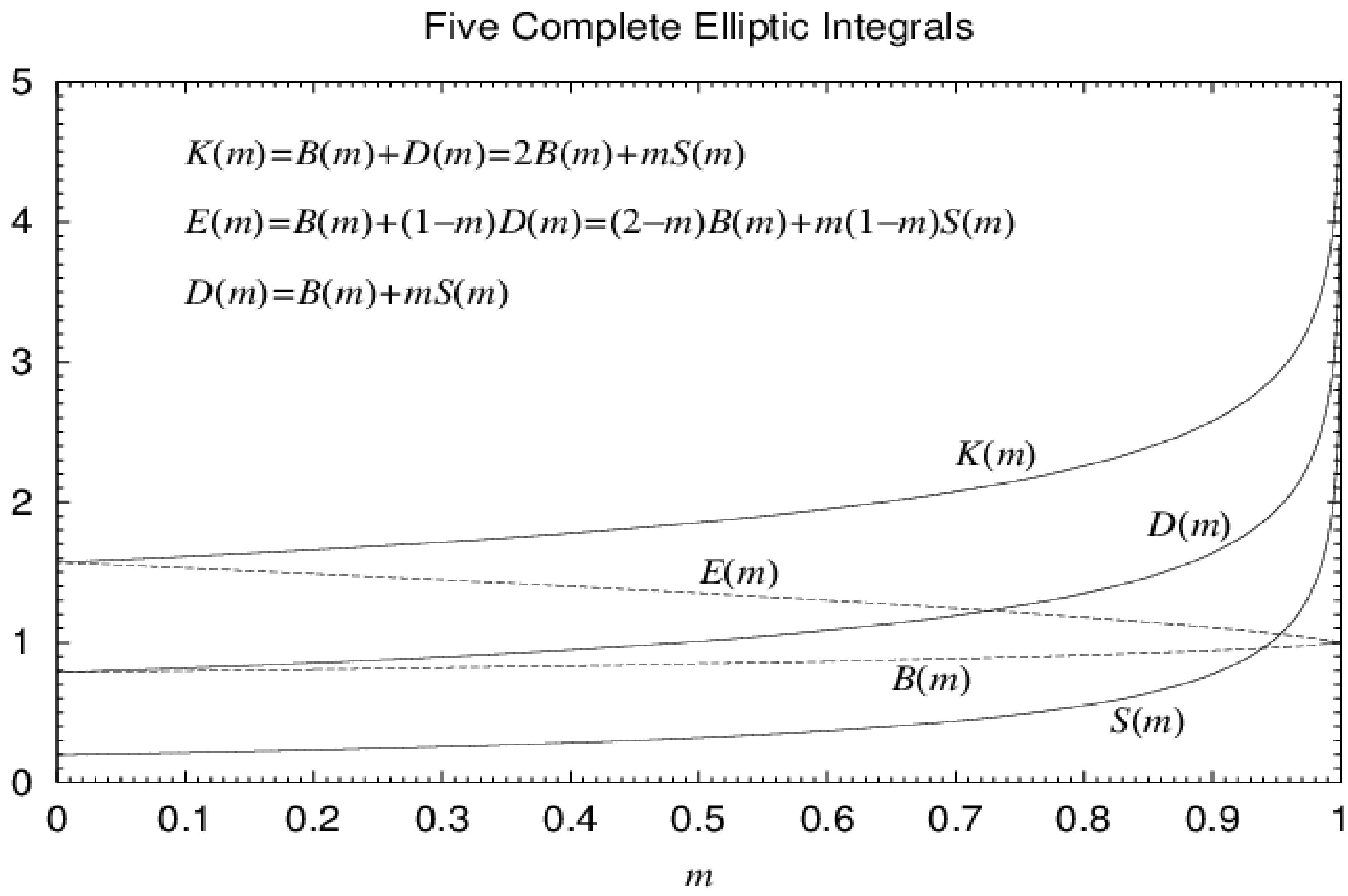
$$\Psi\left(R';R,z\right)=\frac{-4G\Sigma\left(R'\right)K\left(m\left(R';R,z\right)\right)R'}{P\left(R';R,z\right)}$$

$$m\left(R';R,z\right)\equiv\frac{4RR'}{\left[P\left(R';R,z\right)\right]^2}$$

$$P\left(R';R,z\right)\equiv\sqrt{\left(R'+R\right)^2+z^2}$$

- K(m): Complete Elliptic Integral of 1st Kind
  - Fukushima (2015): Precise and Fast Comp.

Complete Elliptic Integrals



Singularity Problem

- Blow-Up Logarithmic Singularity of K(m)
- Integrable in Principle, but …
- Happens if m=1
  - When R=R′ & z=0: Somewhere inside Disc
- Troublesome Even if m~1
  - Sharp Peak of Integrand

Divide
&
Rule

Split Quadrature

- Splitting Integration Interval at Peak

$$\Phi_j(R,z) = \int_{R_{j-1}}^{R} \Psi(R';R,z)\,dR' + \int_{R}^{R_j} \Psi(R';R,z)\,dR'$$

- Double Exponential Quadrature Rule
  - Takahashi & Mori (1973)
  - Program: intde & intdei (Ooura 2006)
- Simple but Works
  - Fukushima (2014)

Acceleration Vector

- Definition

$$\mathbf{A} = A_R \mathbf{e}_R + A_z \mathbf{e}_z$$

$$A_R \equiv -\left(\frac{\partial \Phi(R,z)}{\partial R} \right), A_z \equiv -\left(\frac{\partial \Phi(R,z)}{\partial z} \right)$$

- Numerical Differentiation
  - Primitive but Works
  - Somewhat Costly and Inaccurate
- Ridder's Method (Ridder 1982)
  - Program: dfridr (Numerical Recipe in F77)

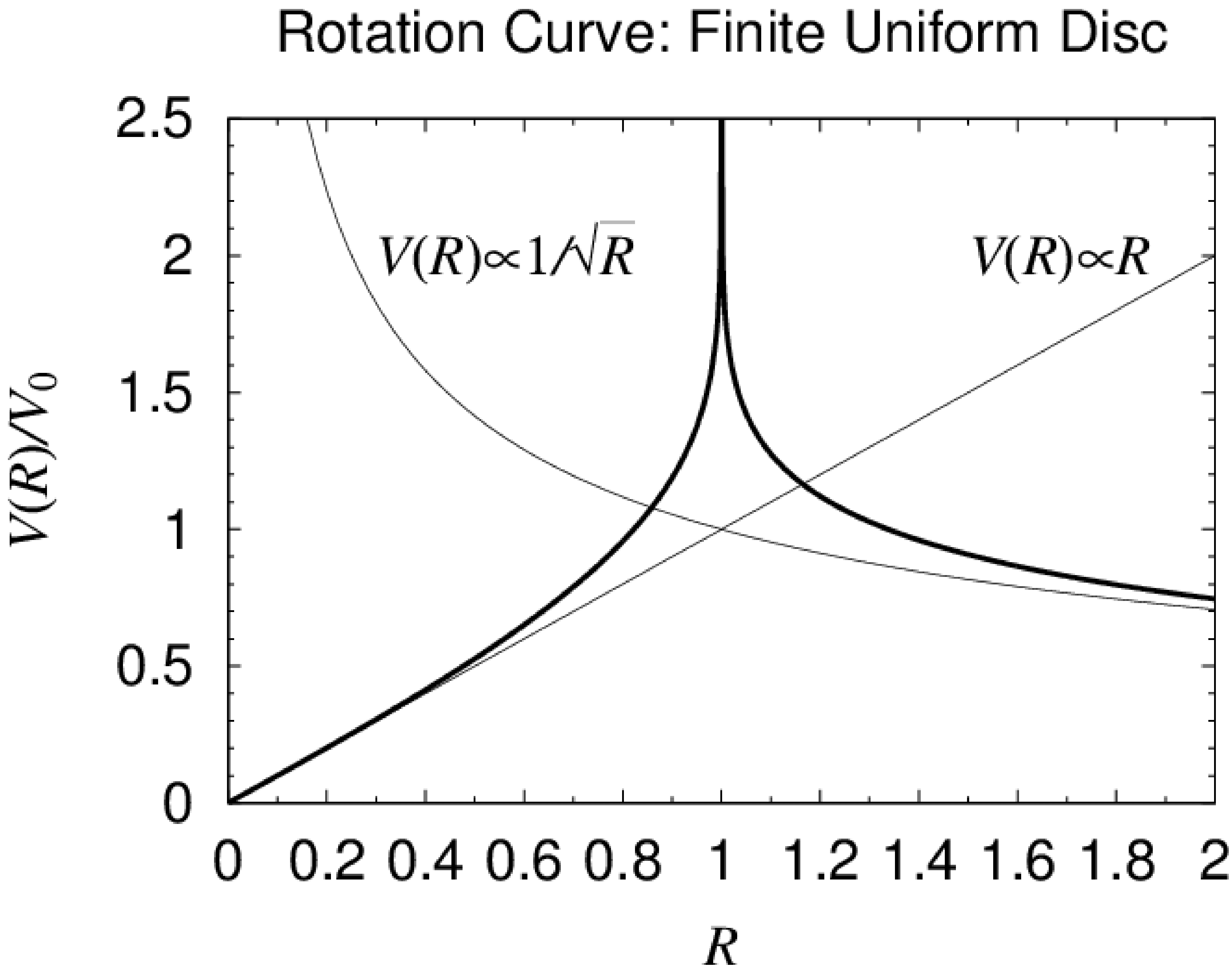
Numerical Tools

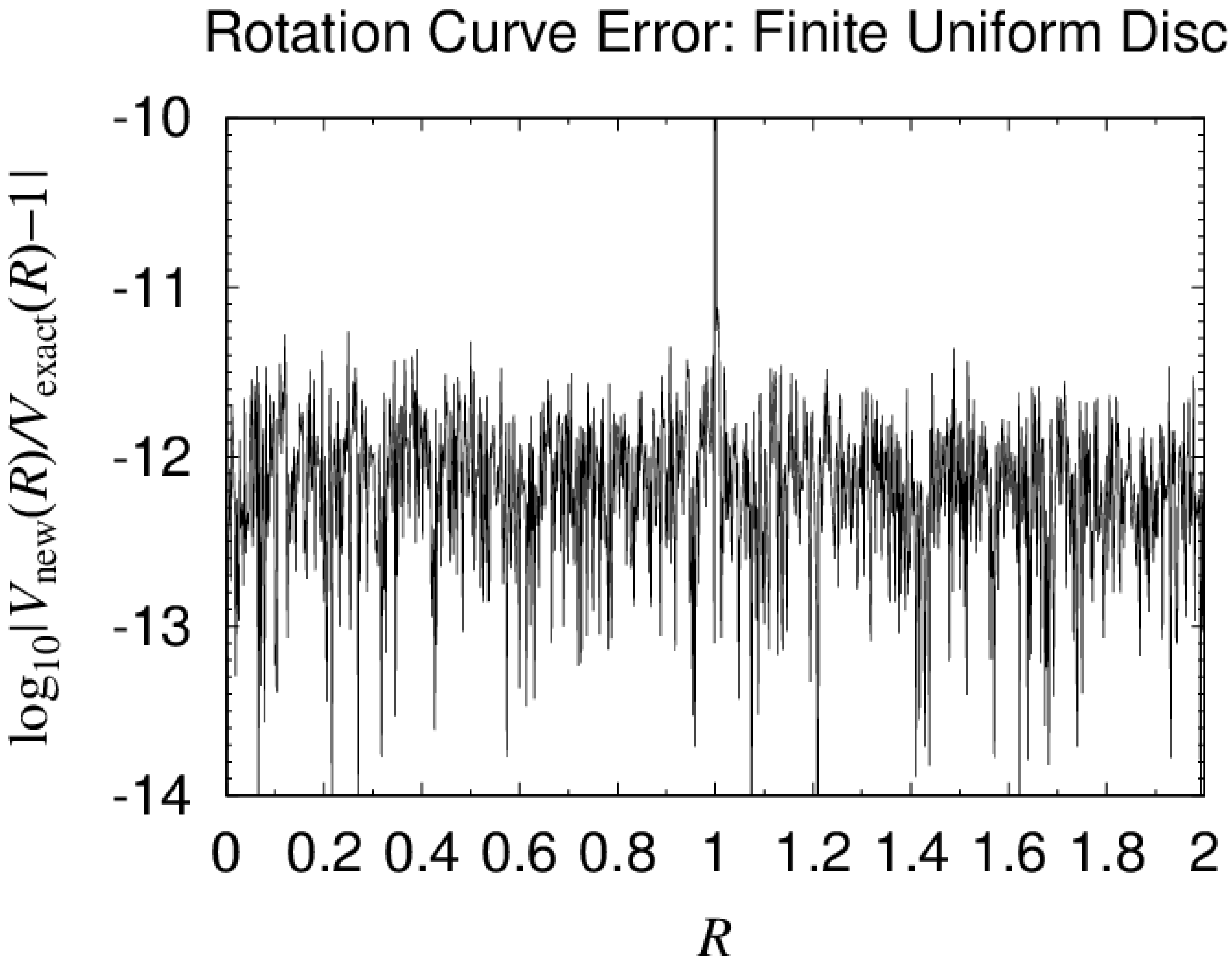
- Complete Elliptic Integral, K(m): ceik
  - Fukushima (2015)
  - https://www.researchgate.net/profile/Toshio_Fukushima/
- Numerical Quadrature: intde
  - Ooura (2006)
  - http://www.kurims.kyoto-u.ac.jp/ooura/intde.html
- Numerical Differentiation: dfridr
  - Press et al. (1992, Sect. 5.7)
  - http://apps.nrbook.com/fortran/index.html

Check, Check, Check

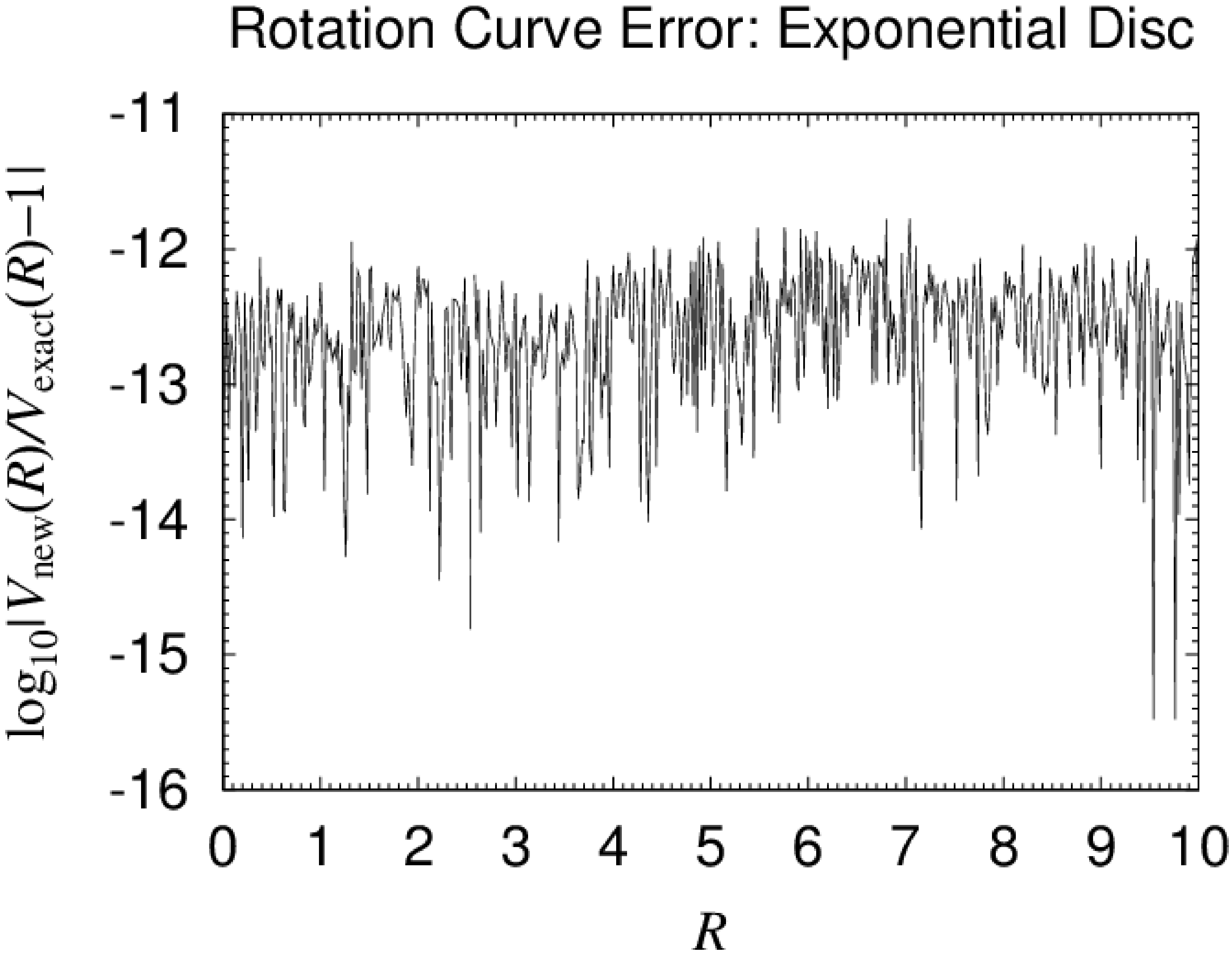
Validation

- Test 1: Finite Uniform Disc
  - Durand (1953), Fukushima (2010)
  - Complete Elliptic Integrals of All Three Kind
- Test 2: Infinite Exponential Disc
  - Freeman (1970)
  - Modified Bessel Functions
- Check: Rotation Curve Computation
- Confirmed 11-12 Digits Accuracy


Rotation Curve: Finite Uniform Disc
2.5
2
1.5
1
0.5
0
V(R)/V0
V(R)∝1/√R
V(R)∝R
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
R

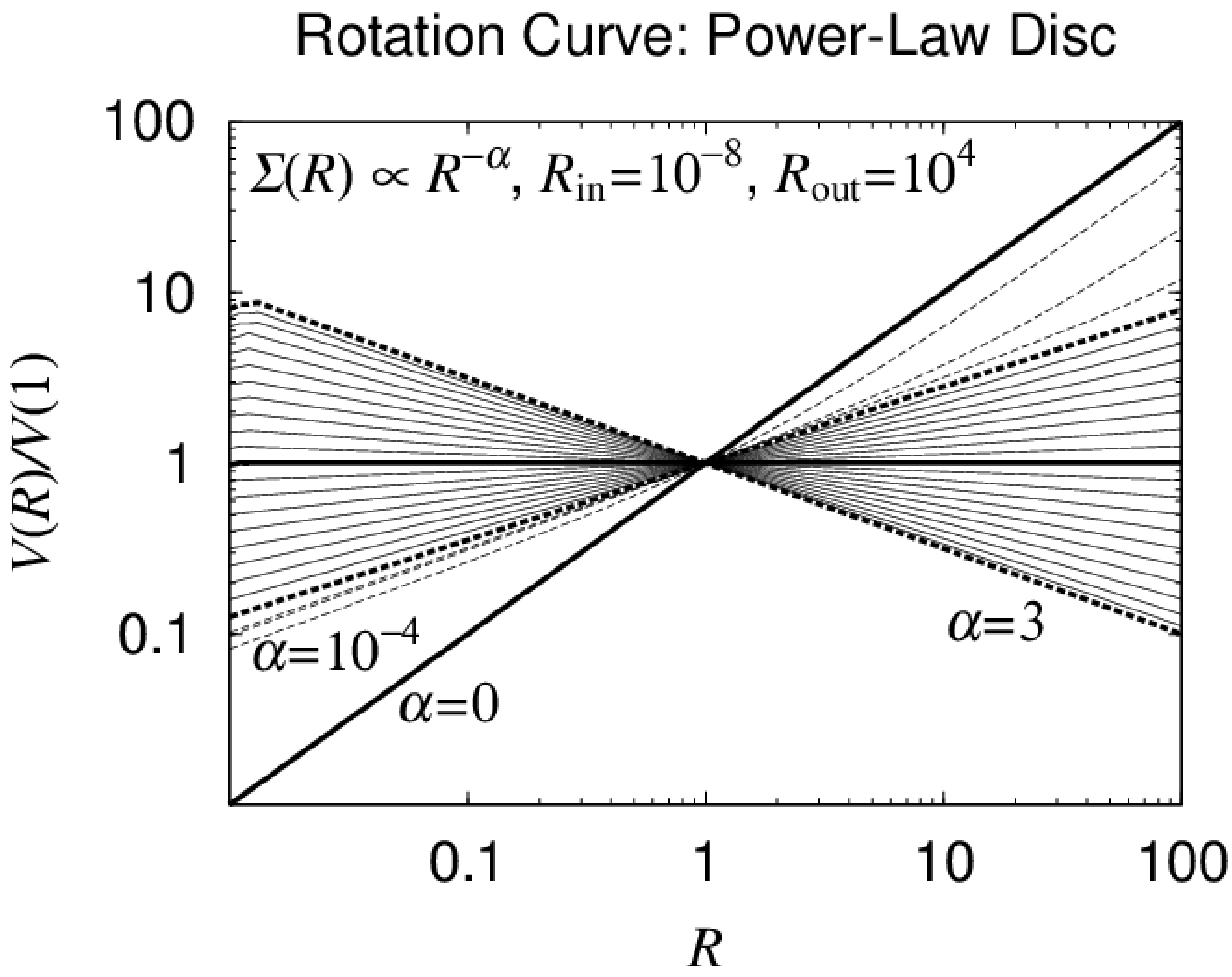


Rotation Curve Error: Finite Uniform Disc
$\log_{10}|V_{\rm new}(R)/V_{\rm exact}(R)-1|$
-10
-11
-12
-13
-14
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
R


Rotation Curve Error: Exponential Disc
-11
-12
-13
-14
-15
-16
0
1
2
3
4
5
6
7
8
9
10
R
$\log_{10}|V_{\rm new}(R)/V_{\rm exact}(R)-1|$

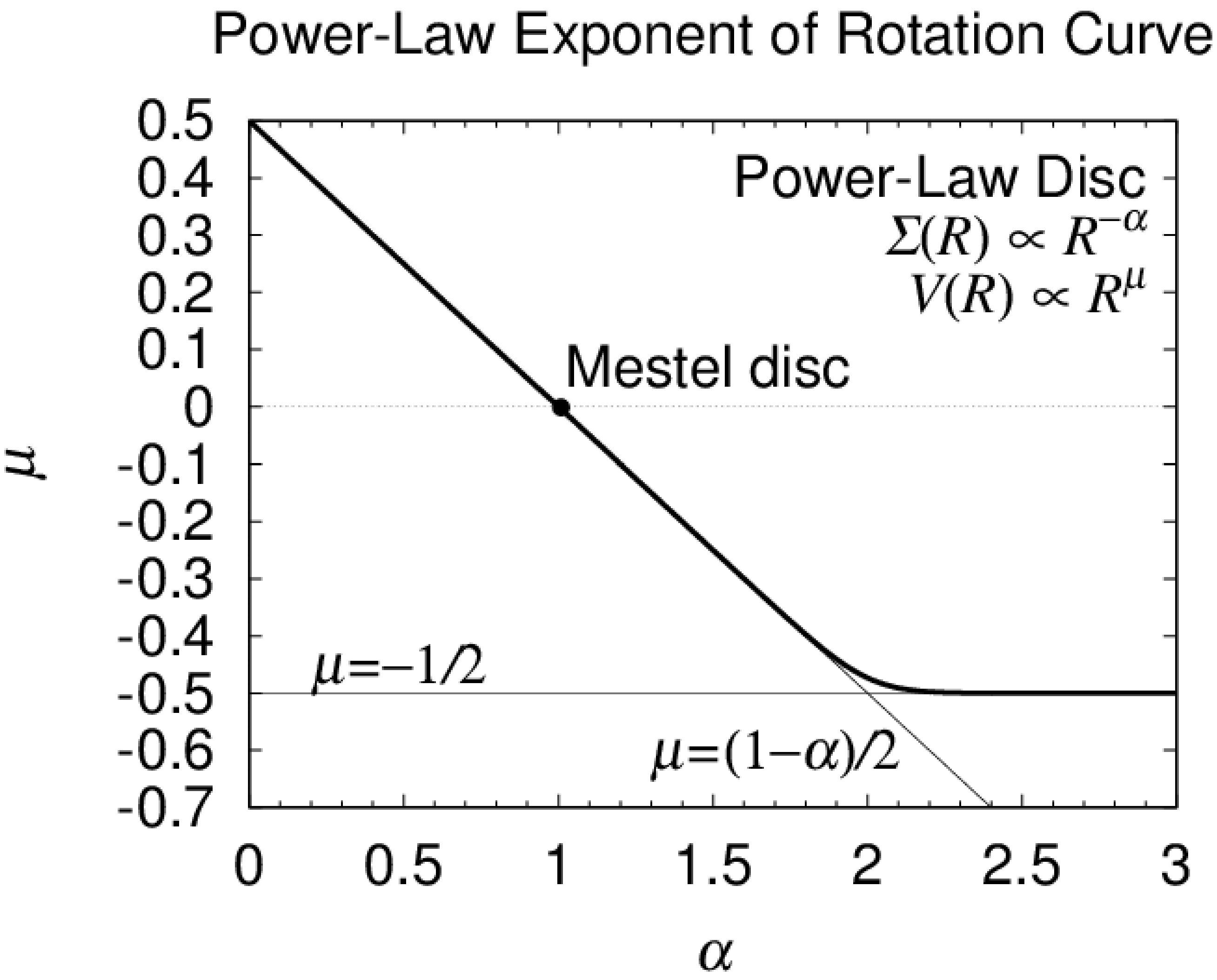
It’s Show Time

Case 1: Finite Power-Law Disc

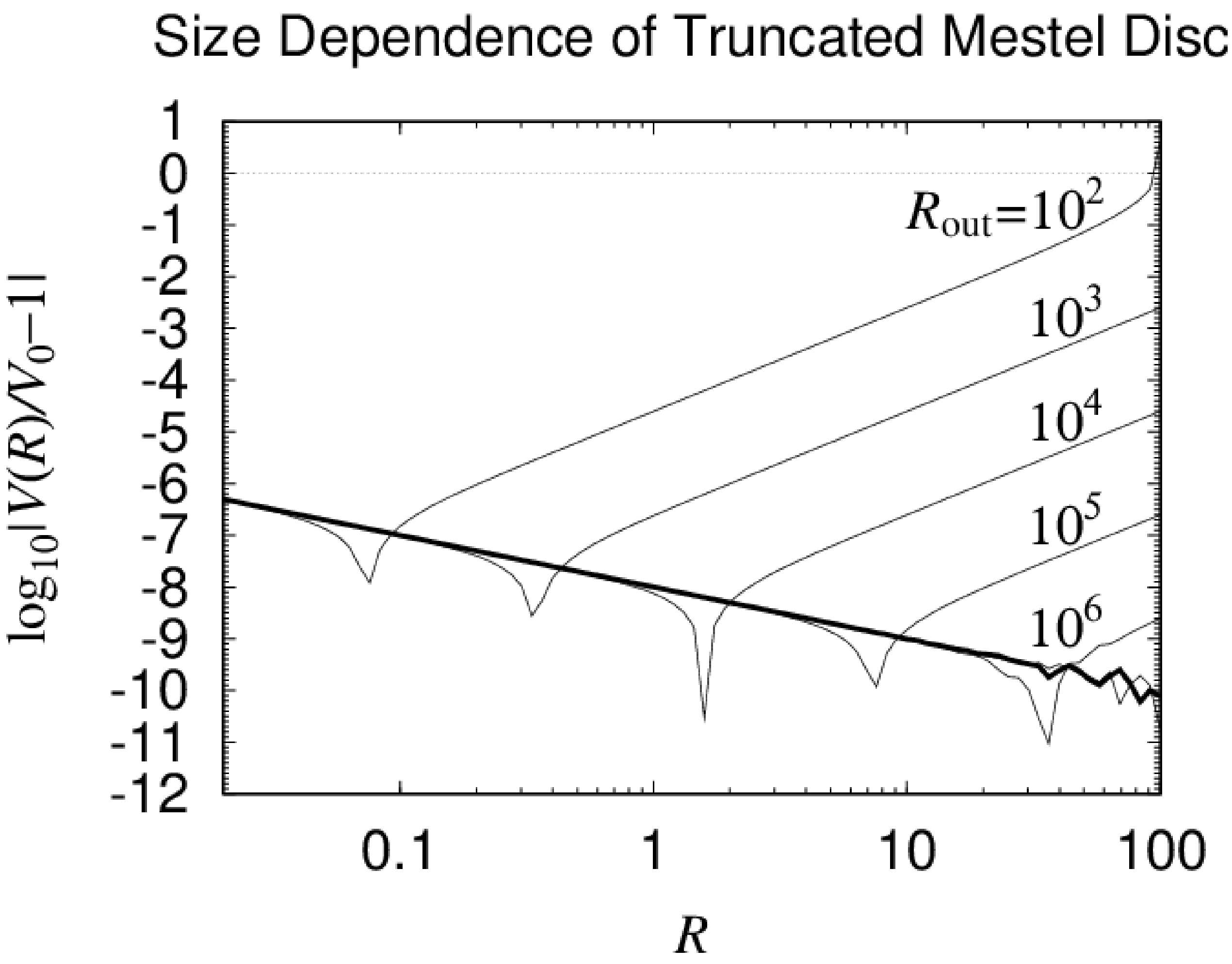


- Power-Law Density Profile Results Almost Power-Law Rotation Curve

Power-Law Index Relation

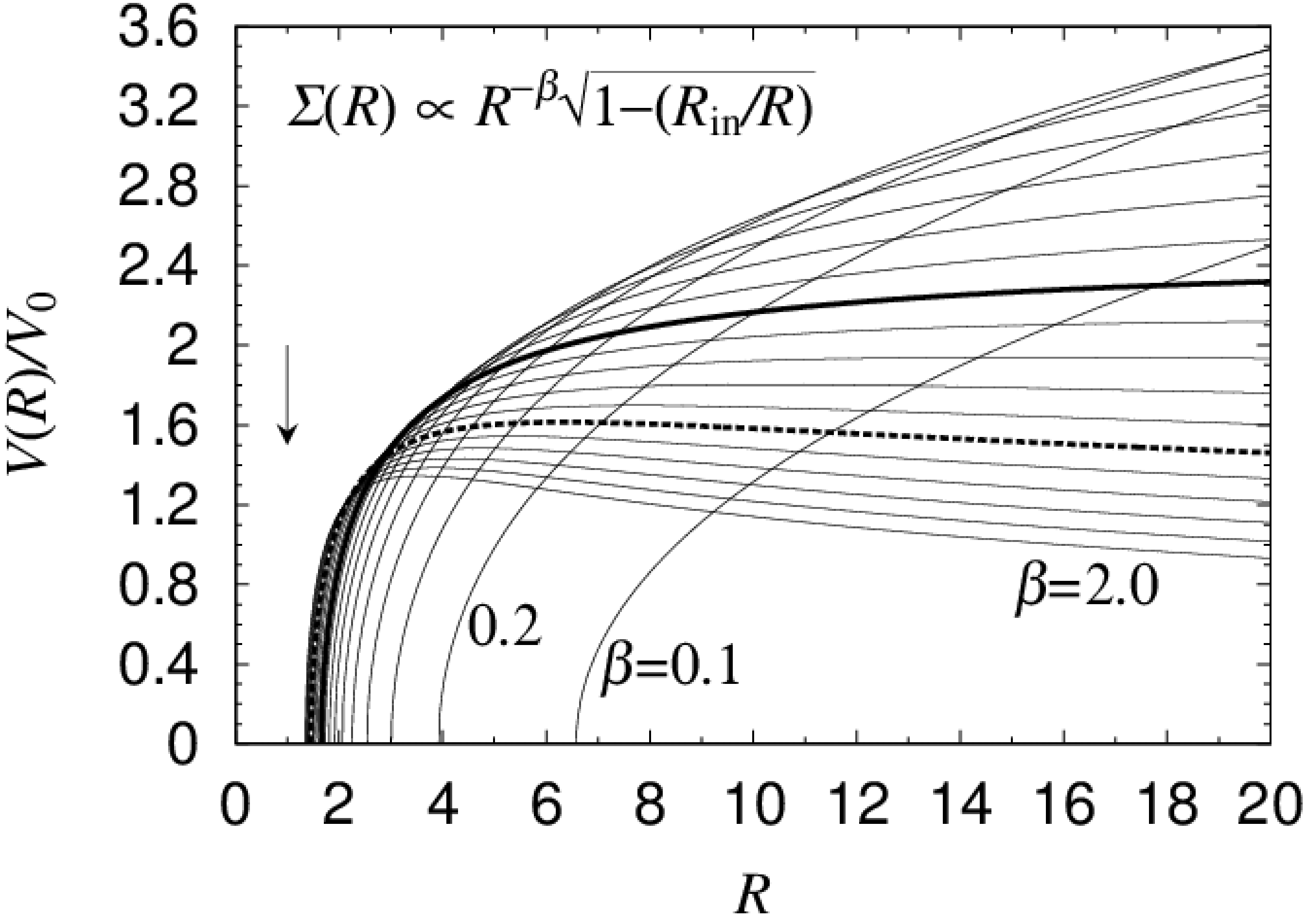


Only Approximate Relation

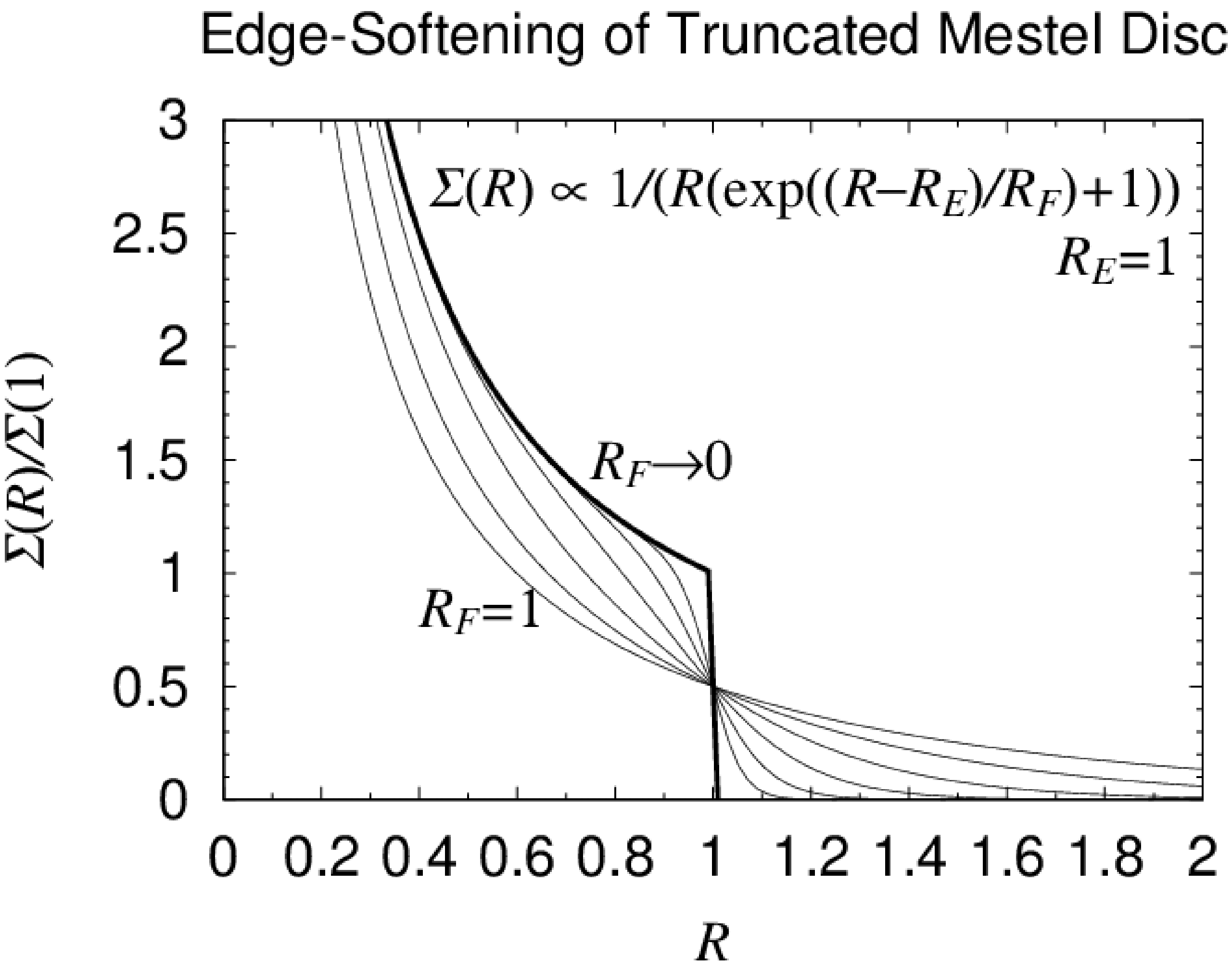


Hole Effect

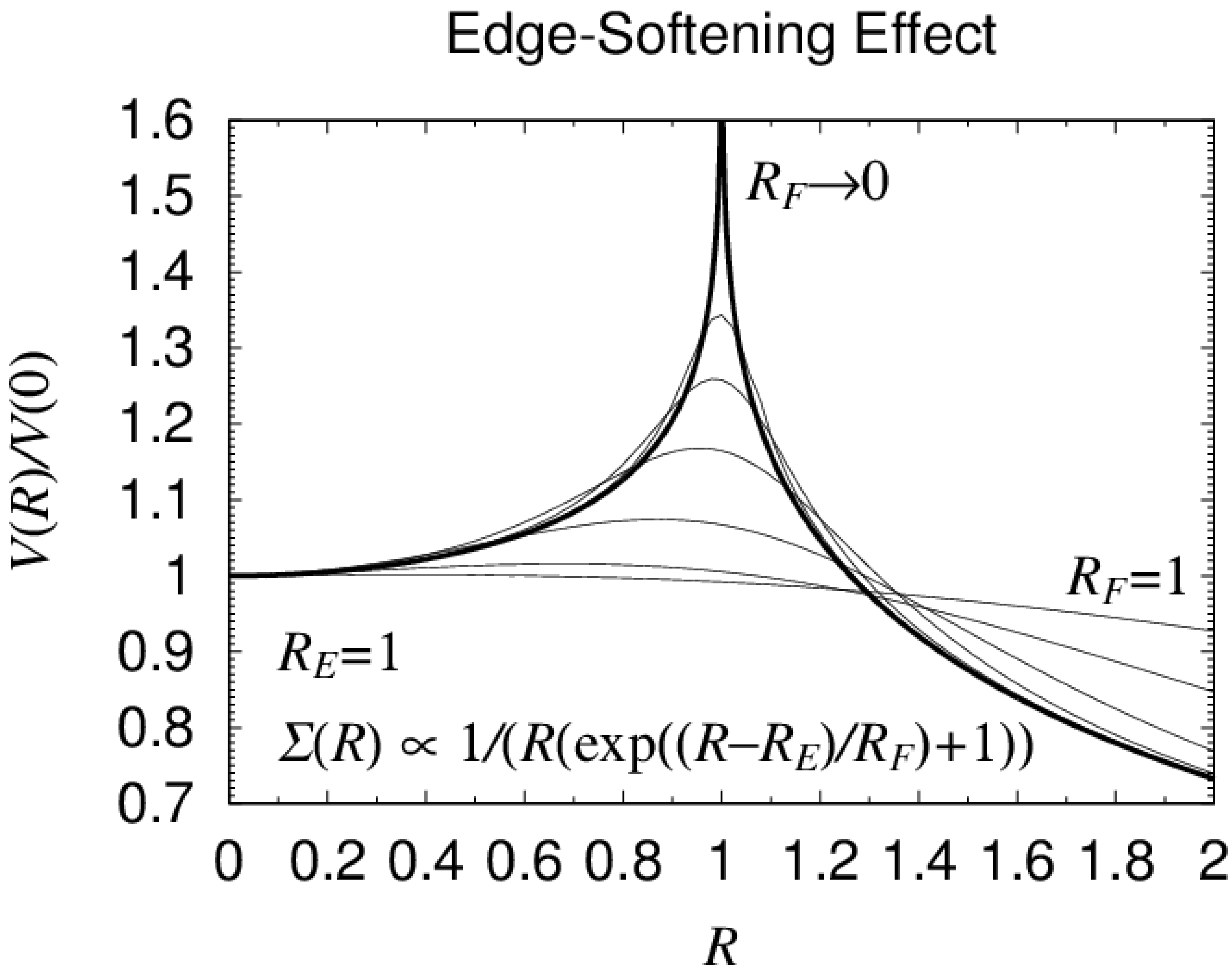
Hole Effect in Accretion Disc



Edge Softening of Density Function



Edge Softened Rotation Curve



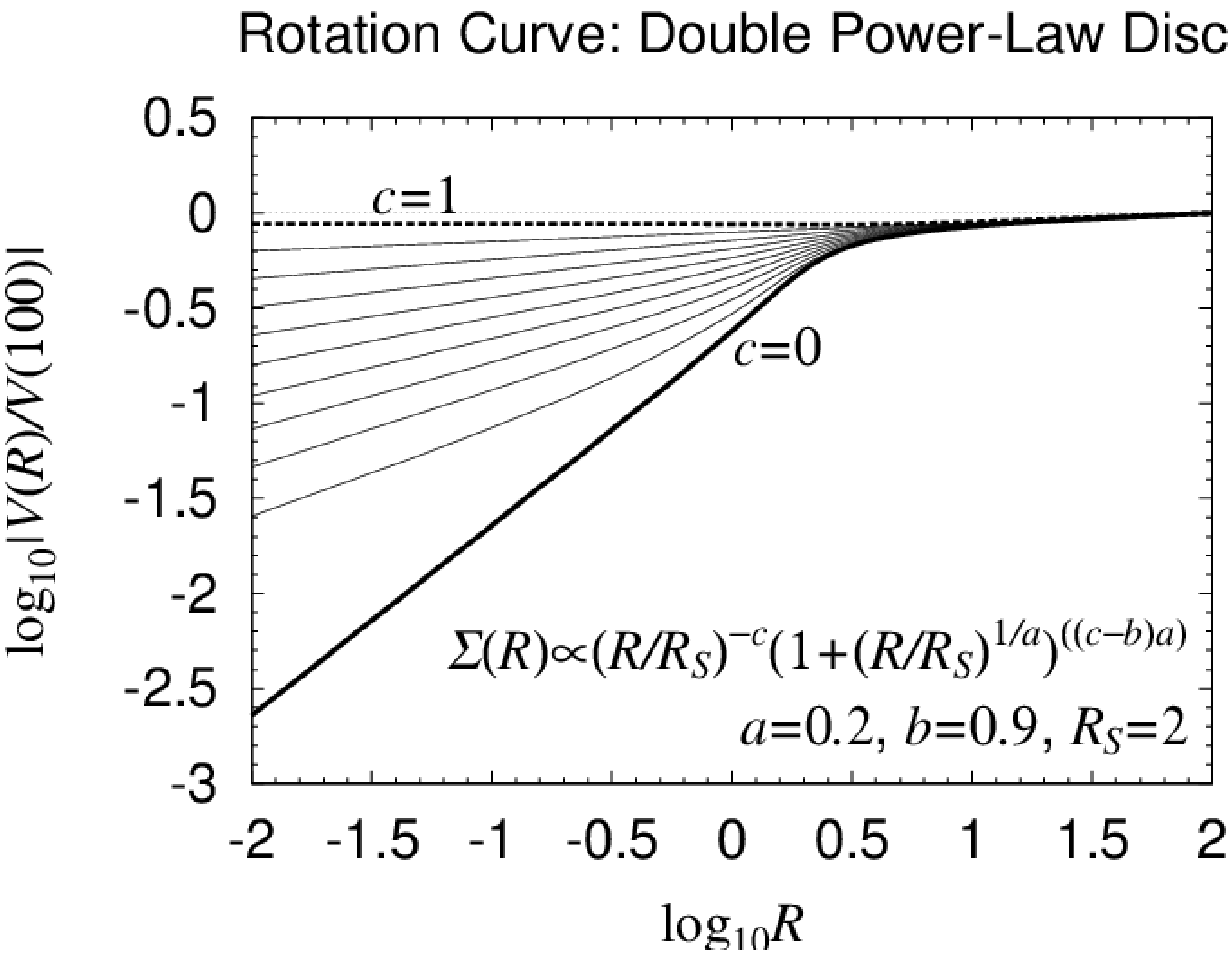
Case 2: Double Power-Law Disc

- Hinted from Generalized Three-Dimensional Volume Mass Density Model (Zhao, 1996, MNRAS)

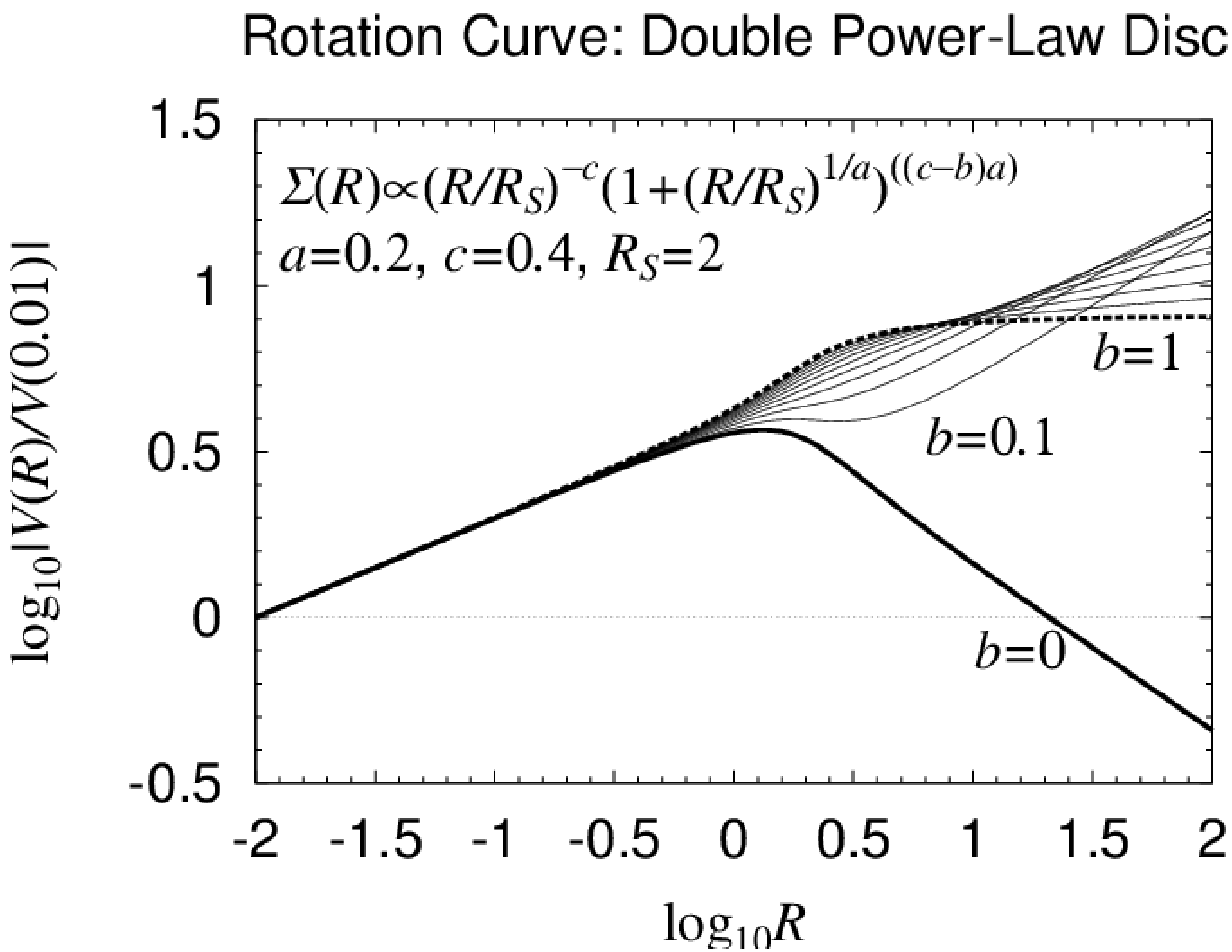
$$\Sigma(R) \equiv \Sigma_0 (R/R_S)^{-c} \left[1 + (R/R_S)^{1/a}\right]^{(c-b)a}$$

- Inner Power-Law Index: c
- Outer Power-Law Index: b
- Curvature of Transition Zone: 1/a

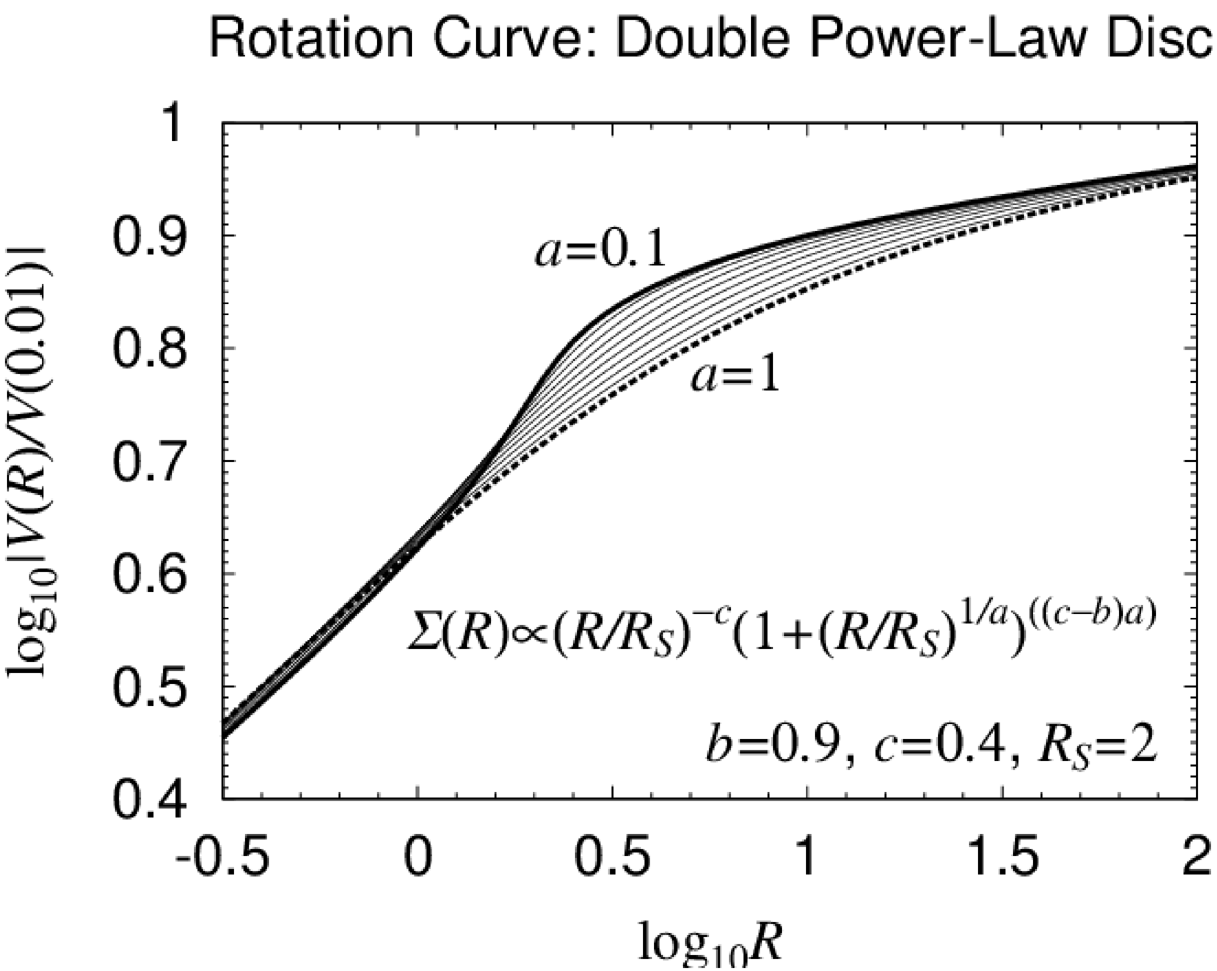
Inner Power-Law Index Dependence



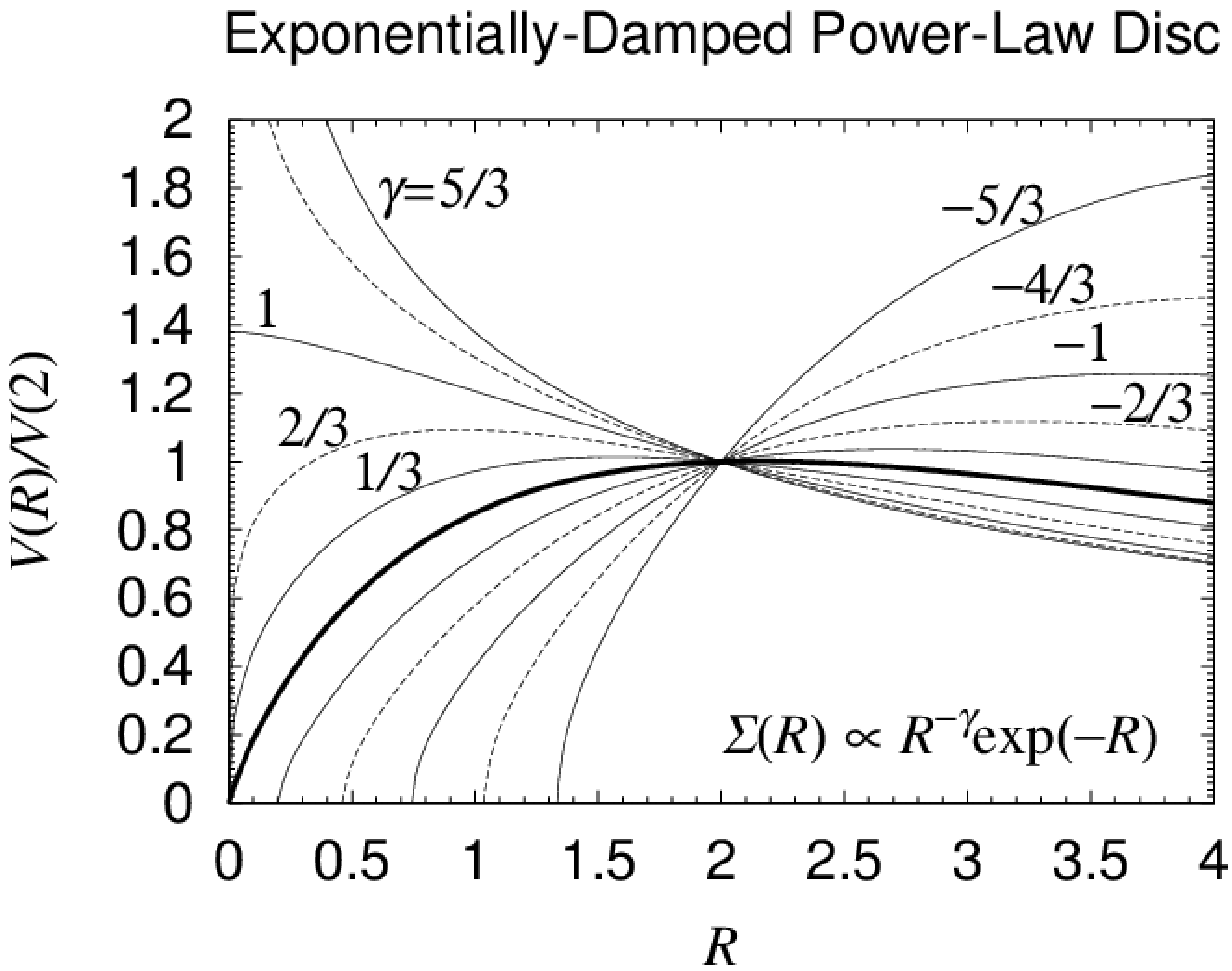
Outer Power-Law Index Dependence



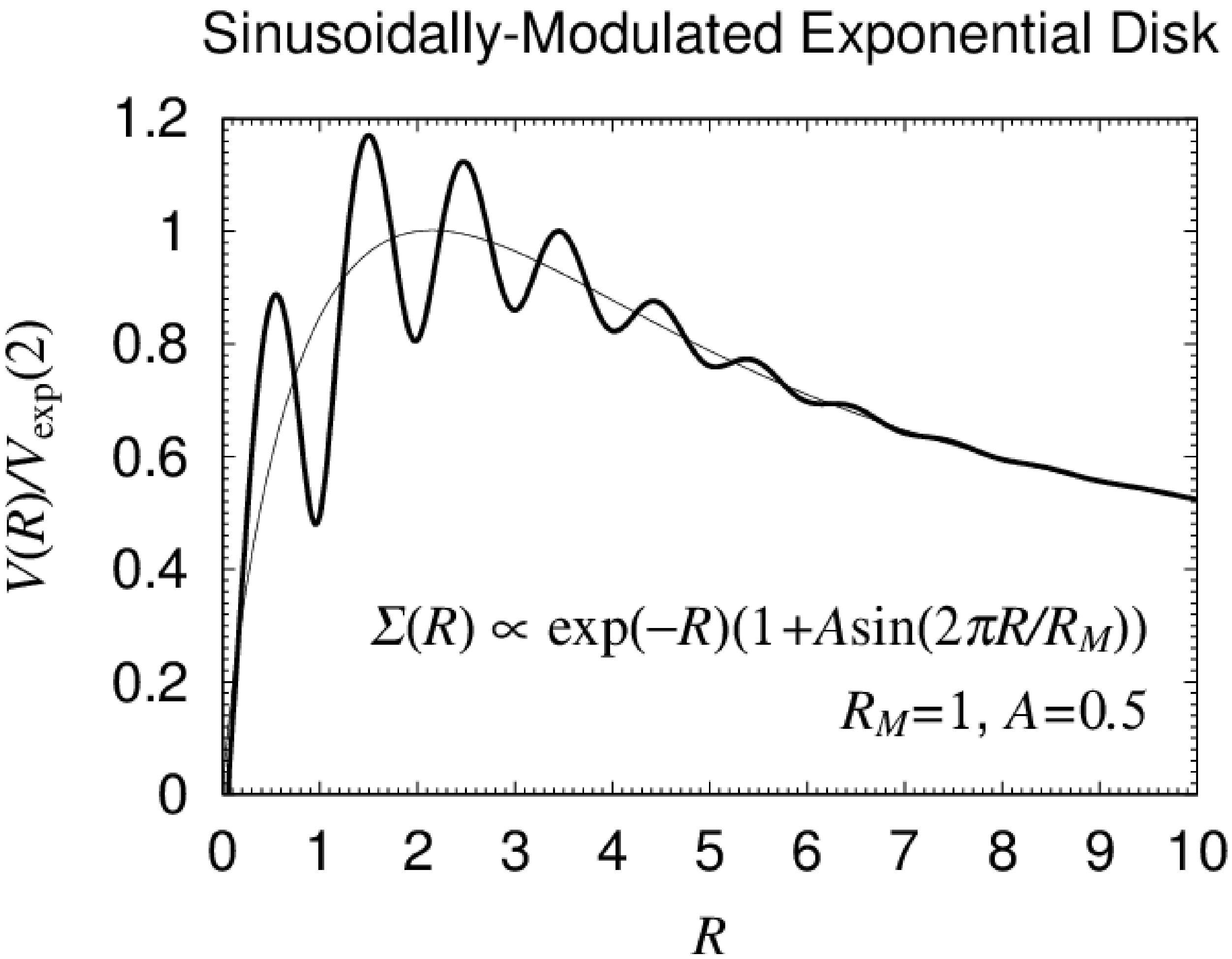
Curvature Index Dependence



Case 3: Exponentially-Damped Power-Law



Case 4: Sine-Modulated Exponential Disc



Empire Strikes Back

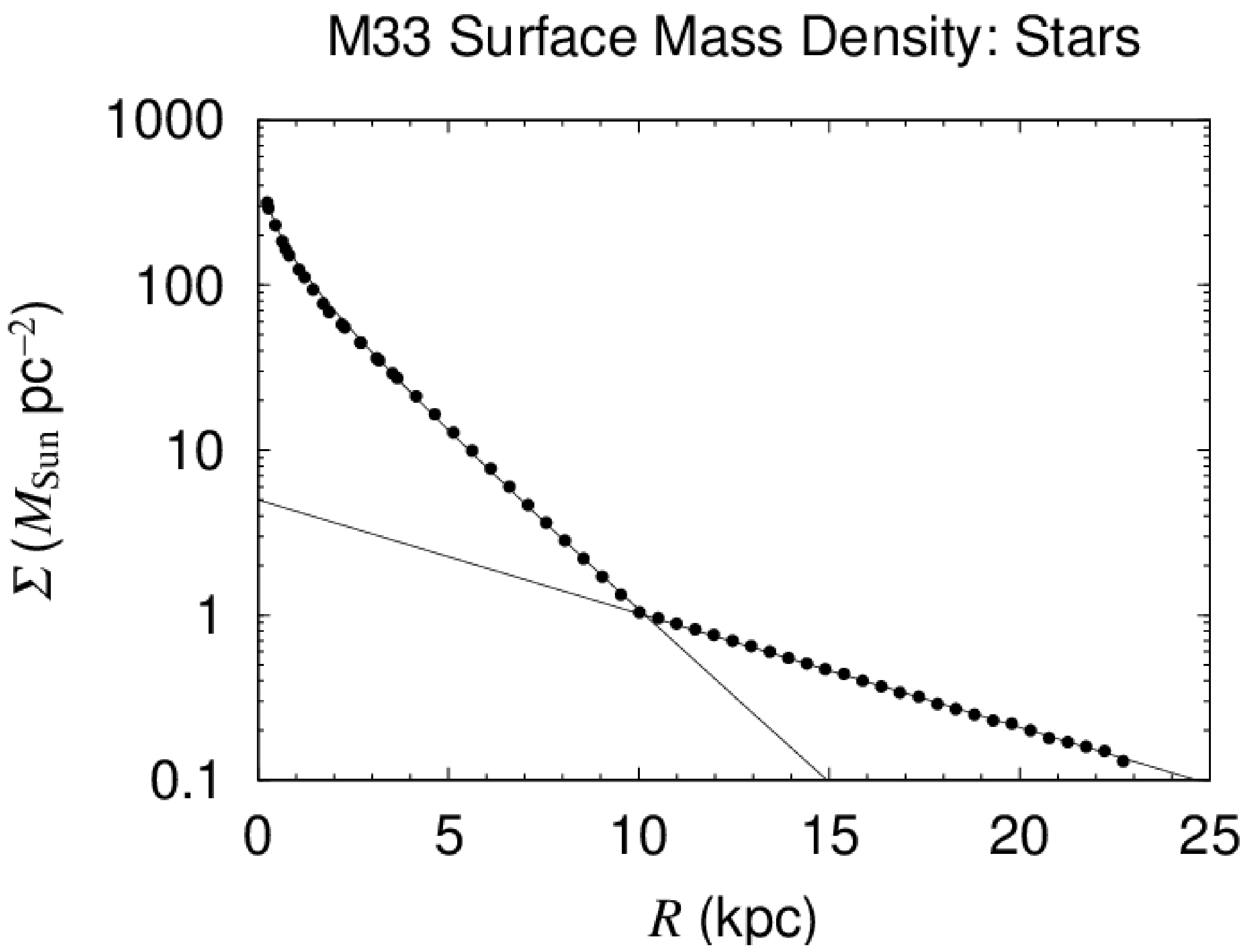
Stars & Gas Density Models: M33

- Two-Piece Models for Stars and Gas
- Stars
  - Inner $$\Sigma(R) = \Sigma_A (R/R_A)^{-1/3} \exp(-R/R_A)$$
  - Outer $$\Sigma(R) = \Sigma_B \exp(-R/R_B)$$
- Gas
  - Inner $$\Sigma(R) = \Sigma_C (R/R_C)^{-c} \left[1 + (R/R_C)^{1/a}\right]^{(c-b)a}$$
  - Outer $$\Sigma(R) = \Sigma_D (R/R_C)^{-3}$$
- Separation Radius: R_D

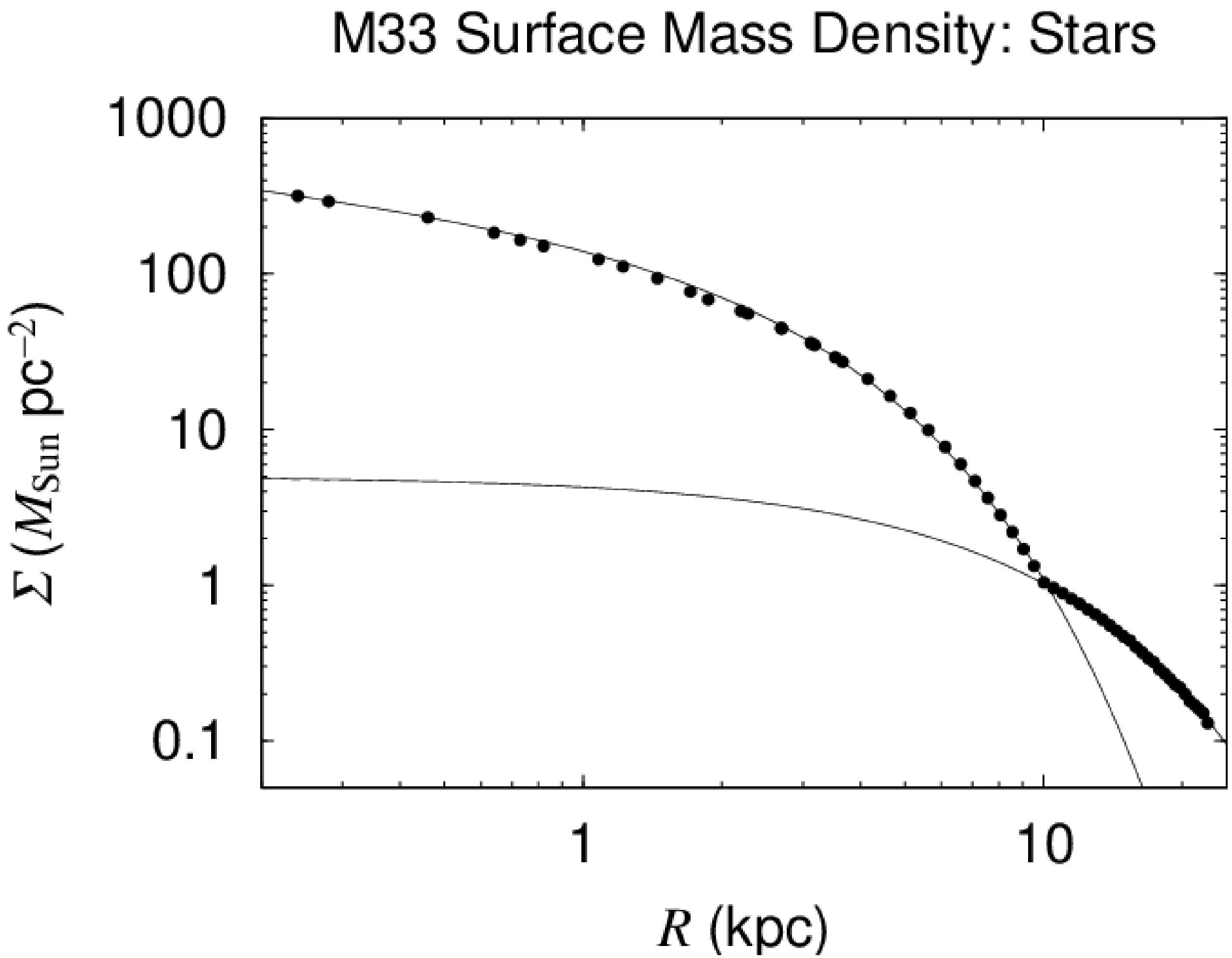
Determined Model Parameters: M33

- Stars Component
  - $\Sigma_A = 169\ M_{sun}pc^{-2}$, $\Sigma_B = 5\ M_{sun}pc^{-2}$
  - $R_A = 2.2$ kpc, $R_B = 6.3$ kpc
- Gas Component
  - $\Sigma_C = 6\ M_{sun}pc^{-2}$, $\Sigma_D = 2.5\ M_{sun}pc^{-2}$
  - $R_C = 7.2$ kpc
  - $a = 0.05$, $b = 5.5$, $c = 0.05$
- Separation Radius
  - $R_D = 10.18$ kpc

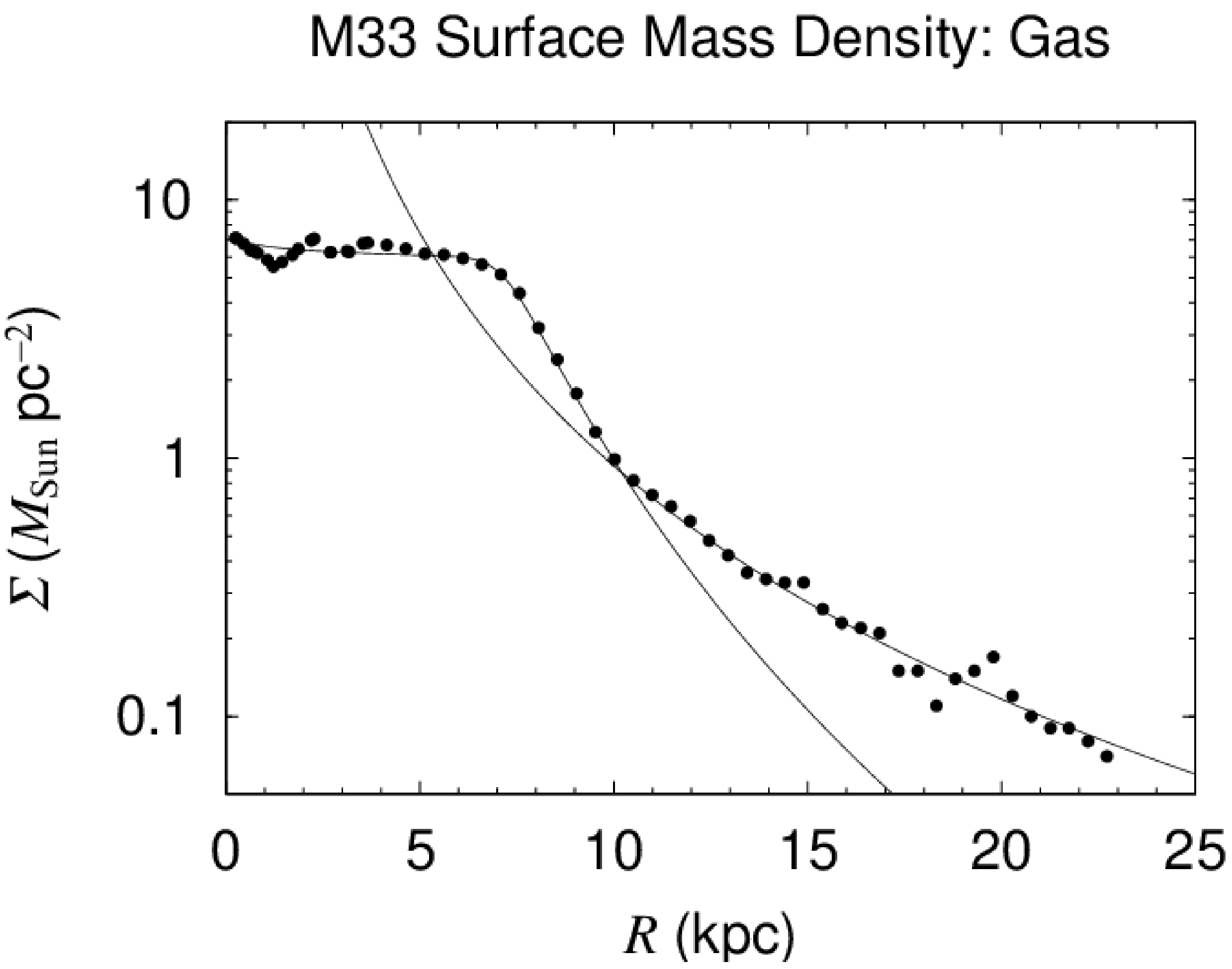
Determined Stars Disc Model of M33



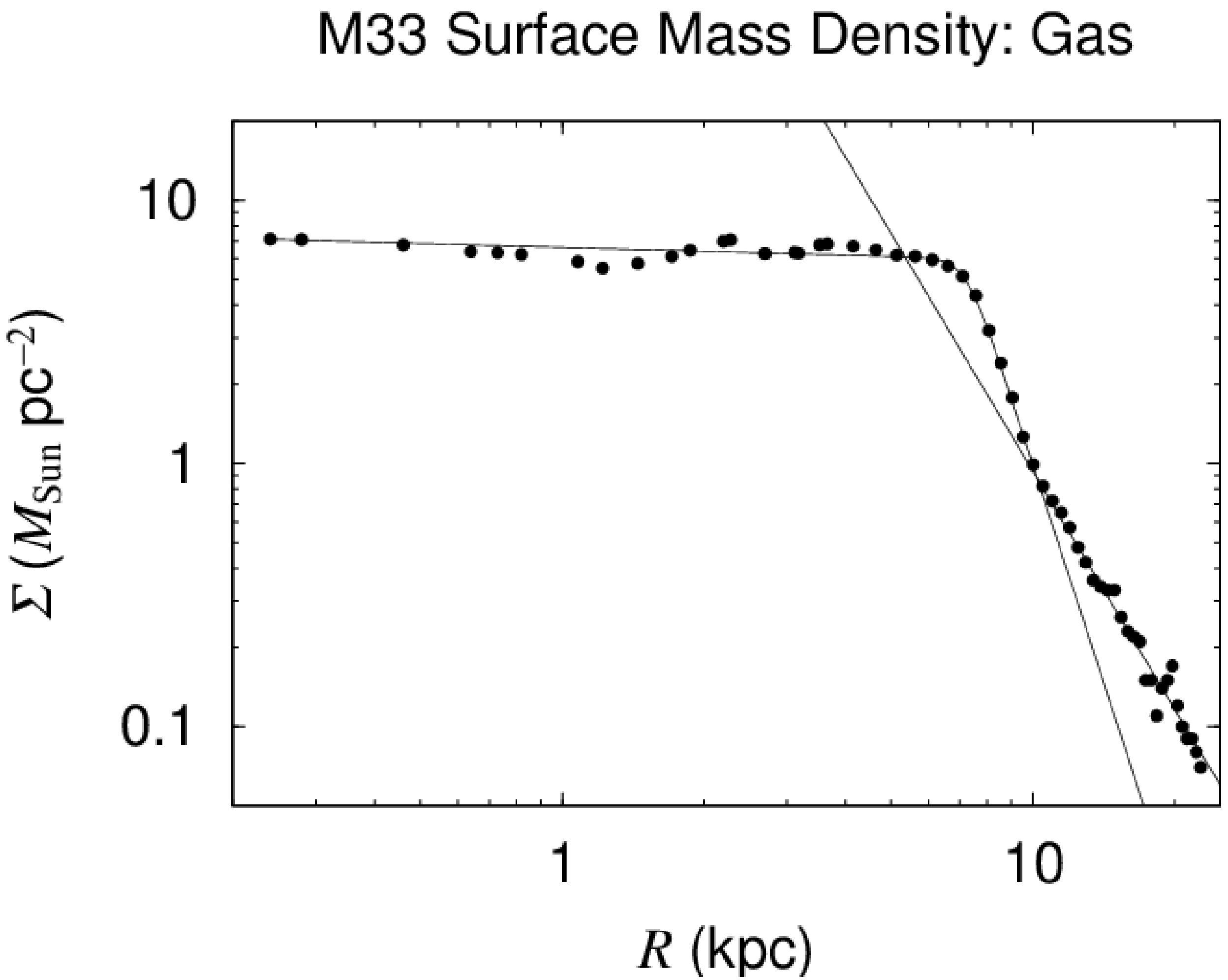
Determined Stars Disc Model of M33



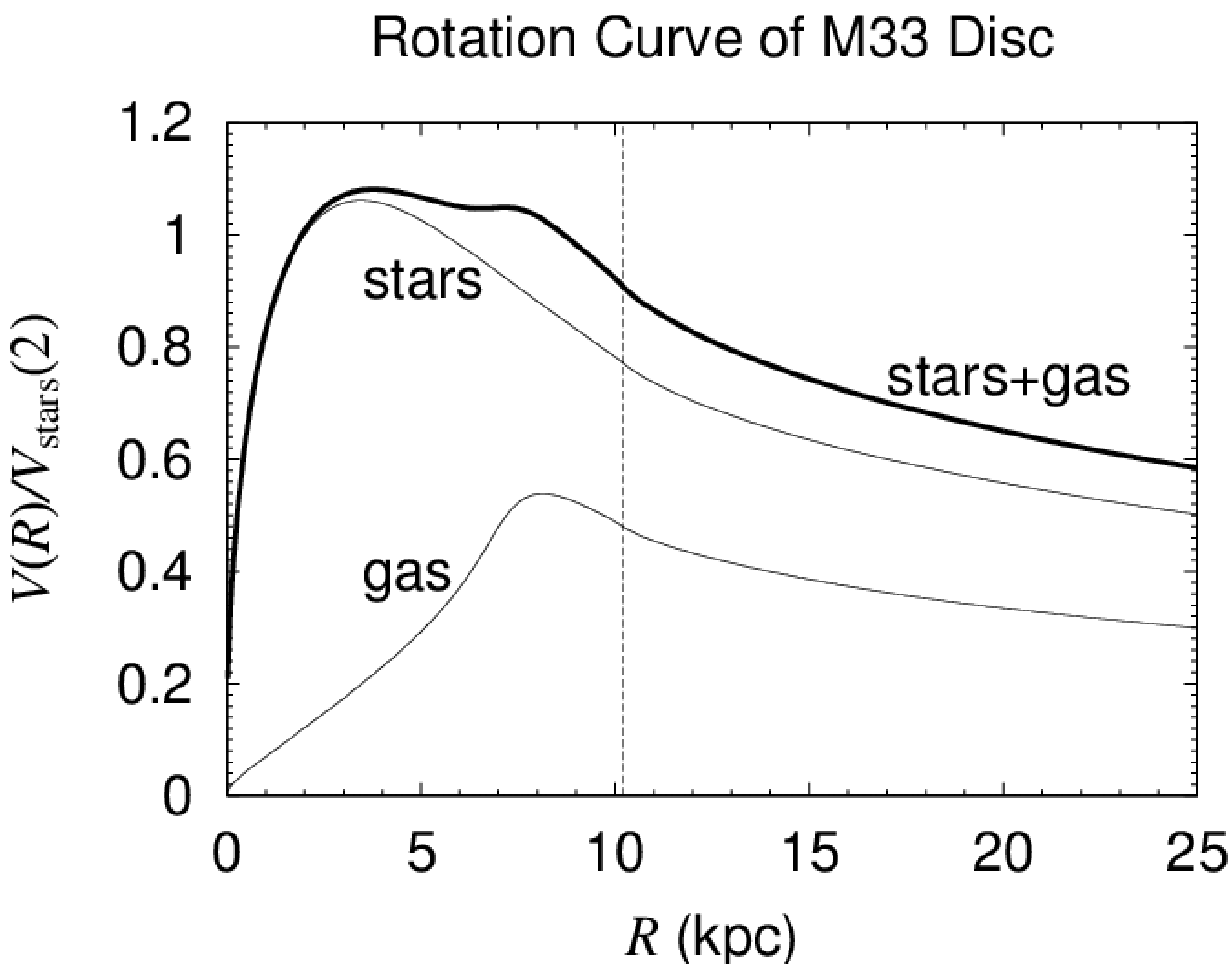
Determined Gas Disc Model of M33



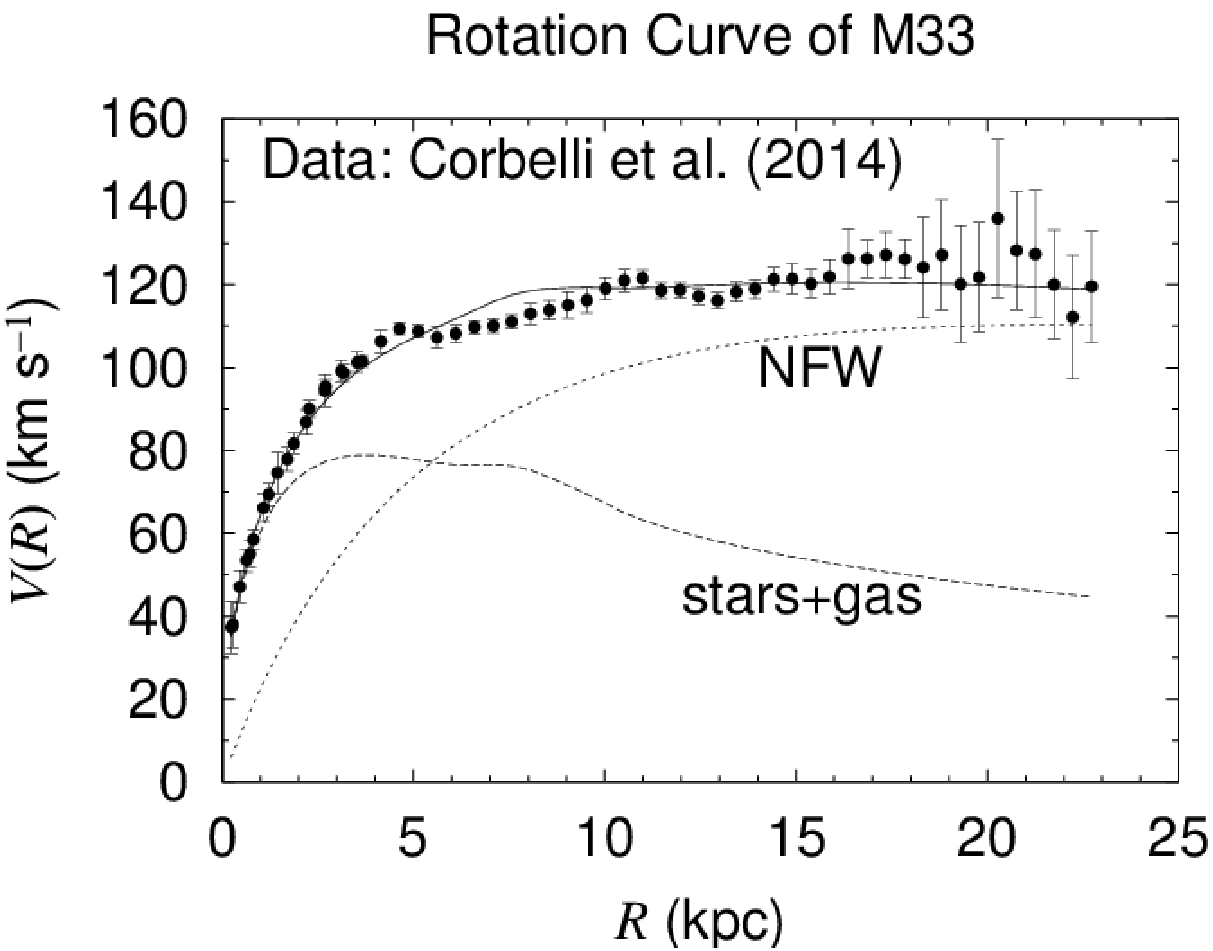
Determined Gas Disc Model of M33



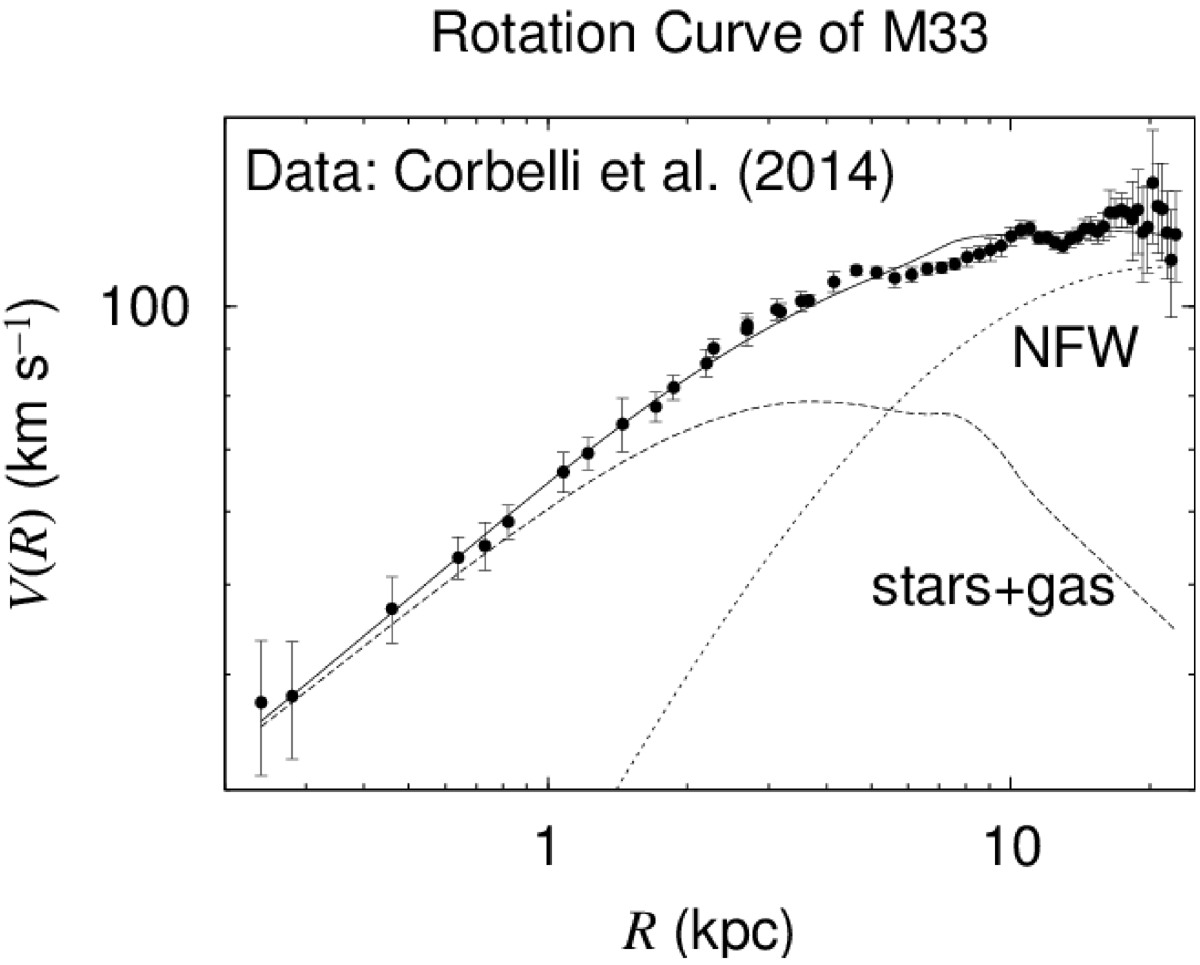
Determined Rotation Curve of Stars and Gas



Deconvolved Rotation Curve of M33



Deconvolved Rotation Curve of M33



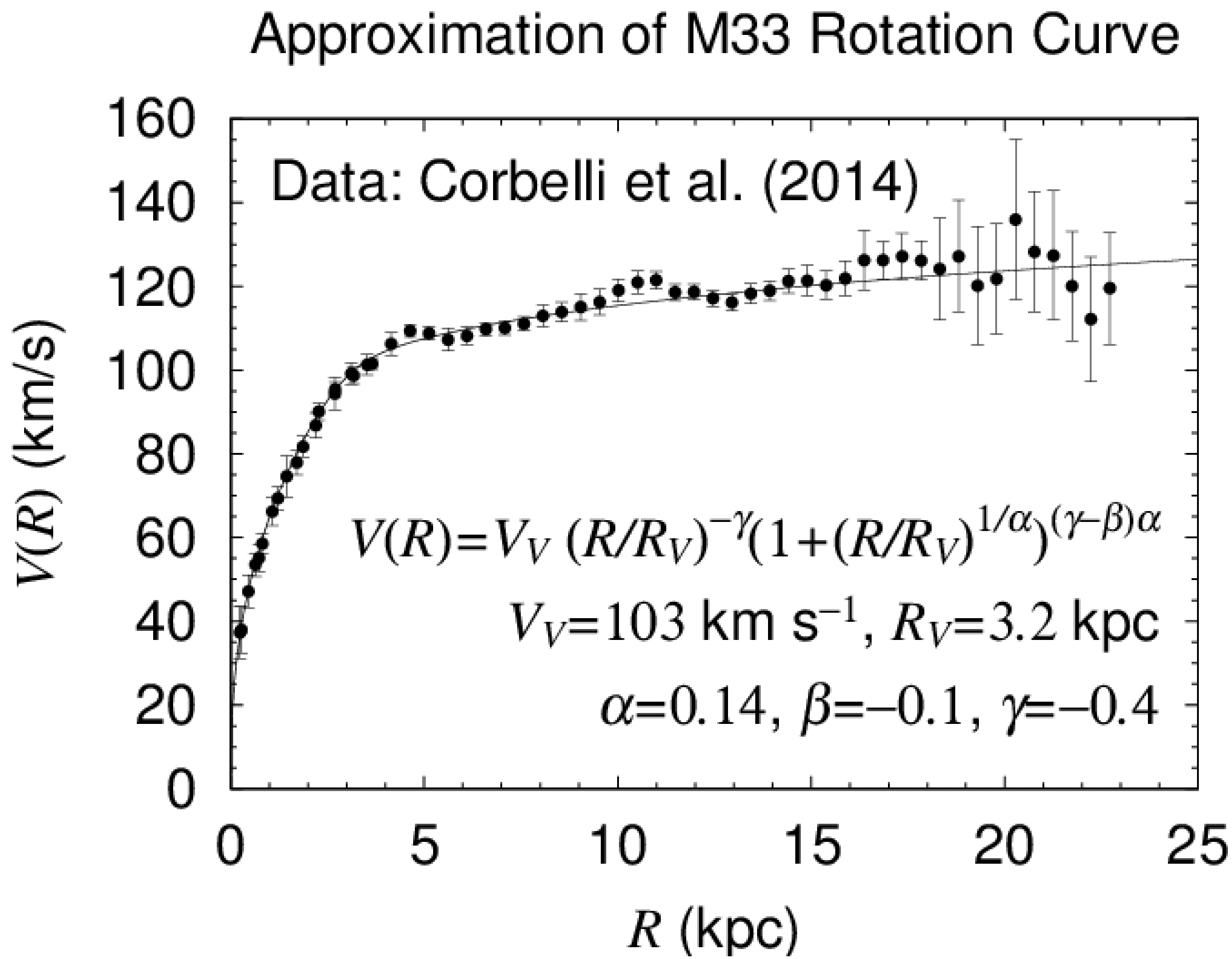
The Force Awakens

Trial Explanation by Disc Mass Model

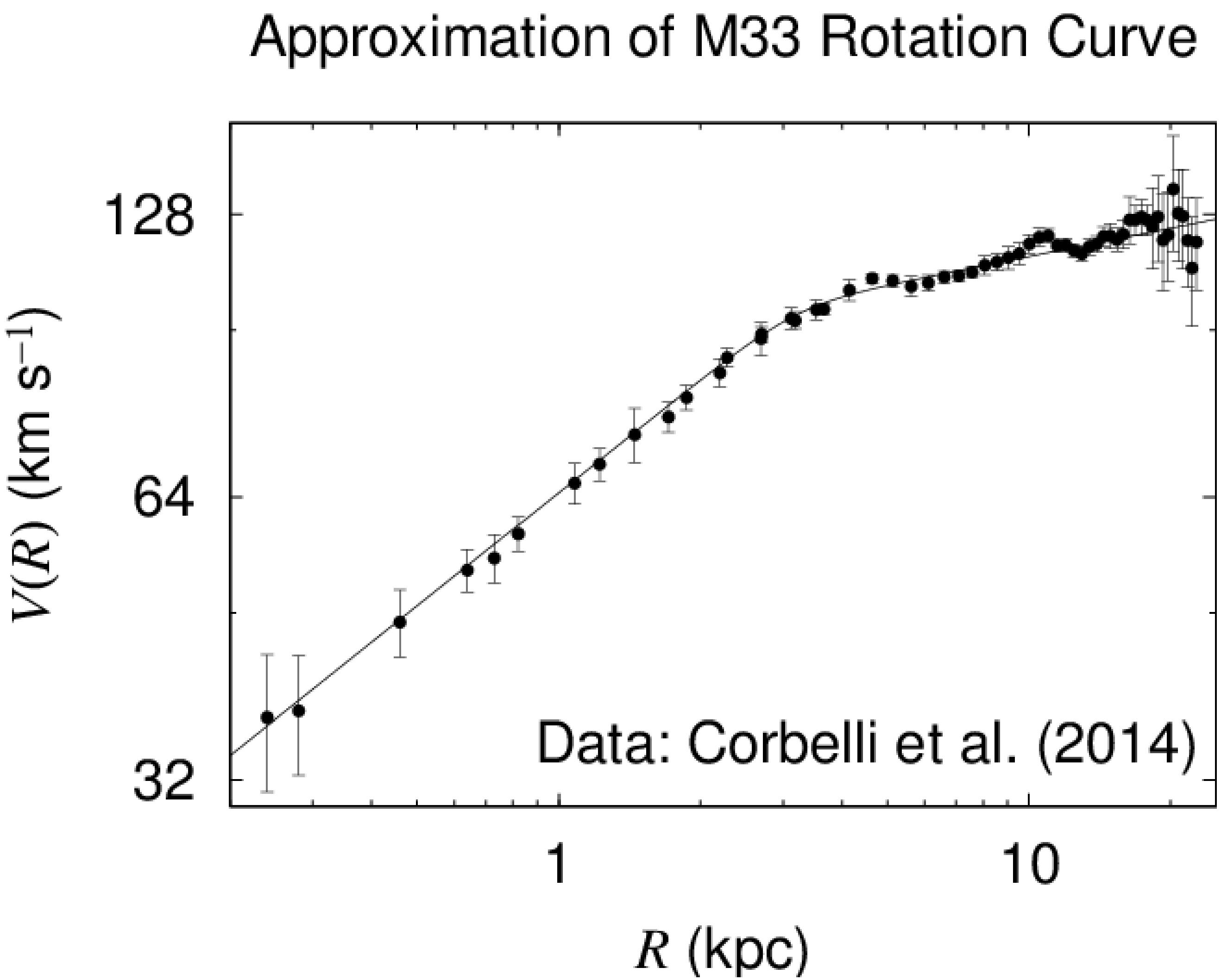
- Unsatisfactory Result of Deconvolution
  - Hump near R = 3-8 kpc
- Assumption: Disc Mass Only
  - Unknown Surface Mass Density Profile
- Hints from Rotation Curve Itself
  - Double-Power-Law-like Feature

$$V(R) = V_0 (R/R_V)^{-\gamma} \left[1 + (R/R_V)^{1/\alpha} \right]^{(\gamma-\beta)\alpha}$$

Rotation Curve Model



Rotation Curve Model



Double Power-Law Disc Mass Model

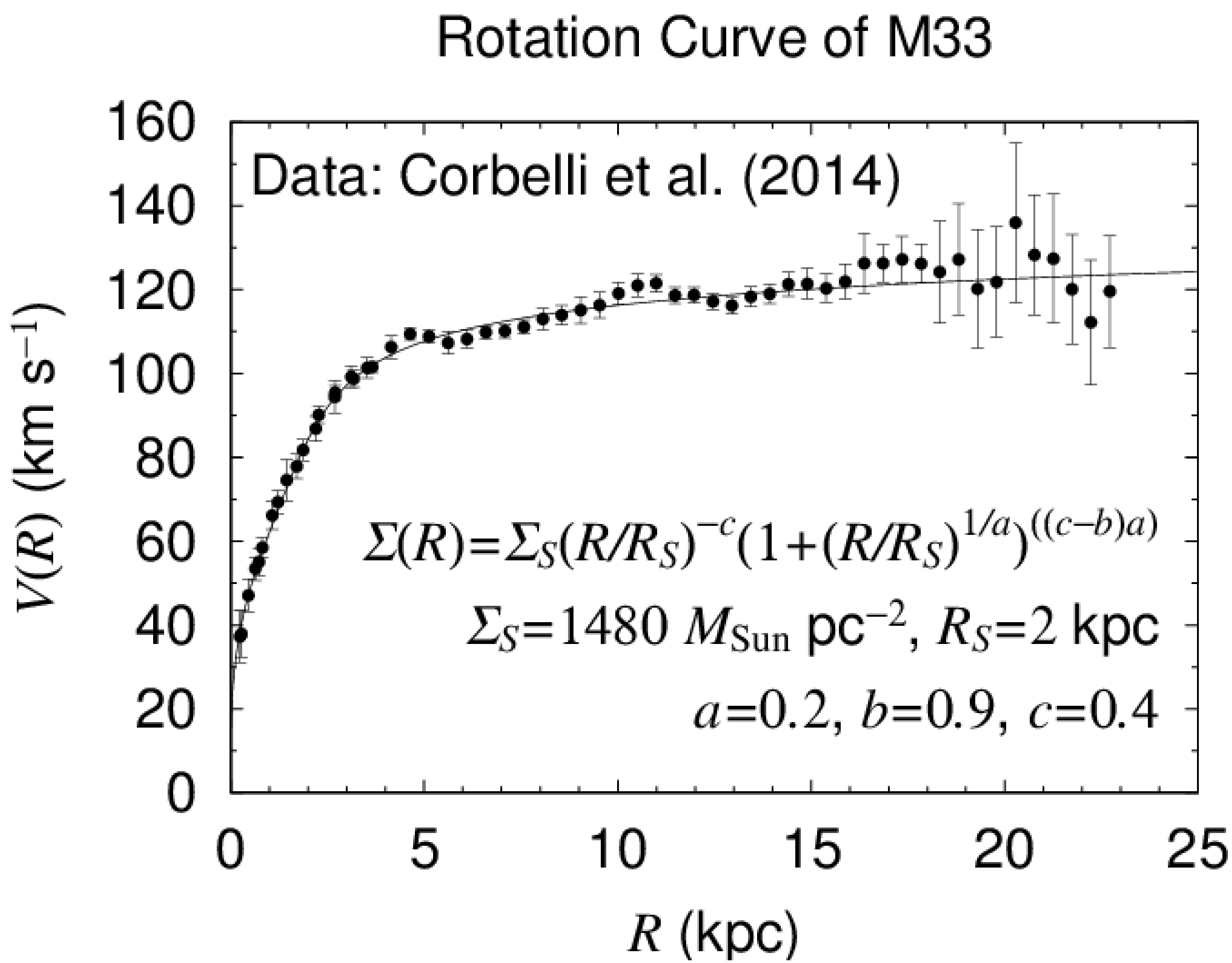
- Natural Expectation
- Double Power-Law Rotation Curve from

  Double Power-Law Surface Mass Density

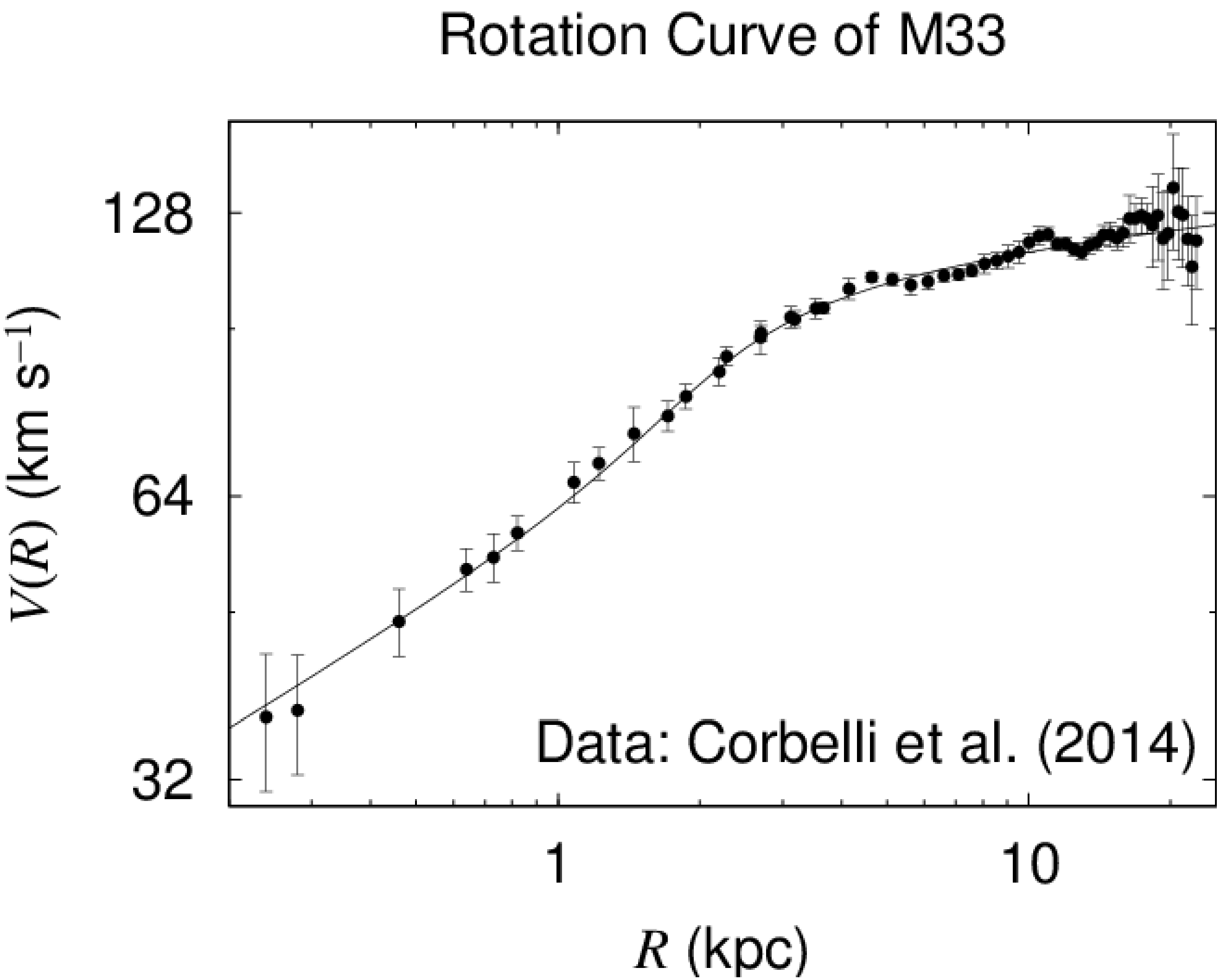
$$\Sigma(R) = \Sigma_S \left(R / R_S\right)^{-c} \left[1 + \left(R / R_S\right)^{1/a}\right]^{(c-b)a}$$

- Determined Model Parameters
  - Σ_S = 1480 $M_{sun}pc^{-2}$, R_S = 2 kpc
  - a=0.2, b=0.9, c=0.4

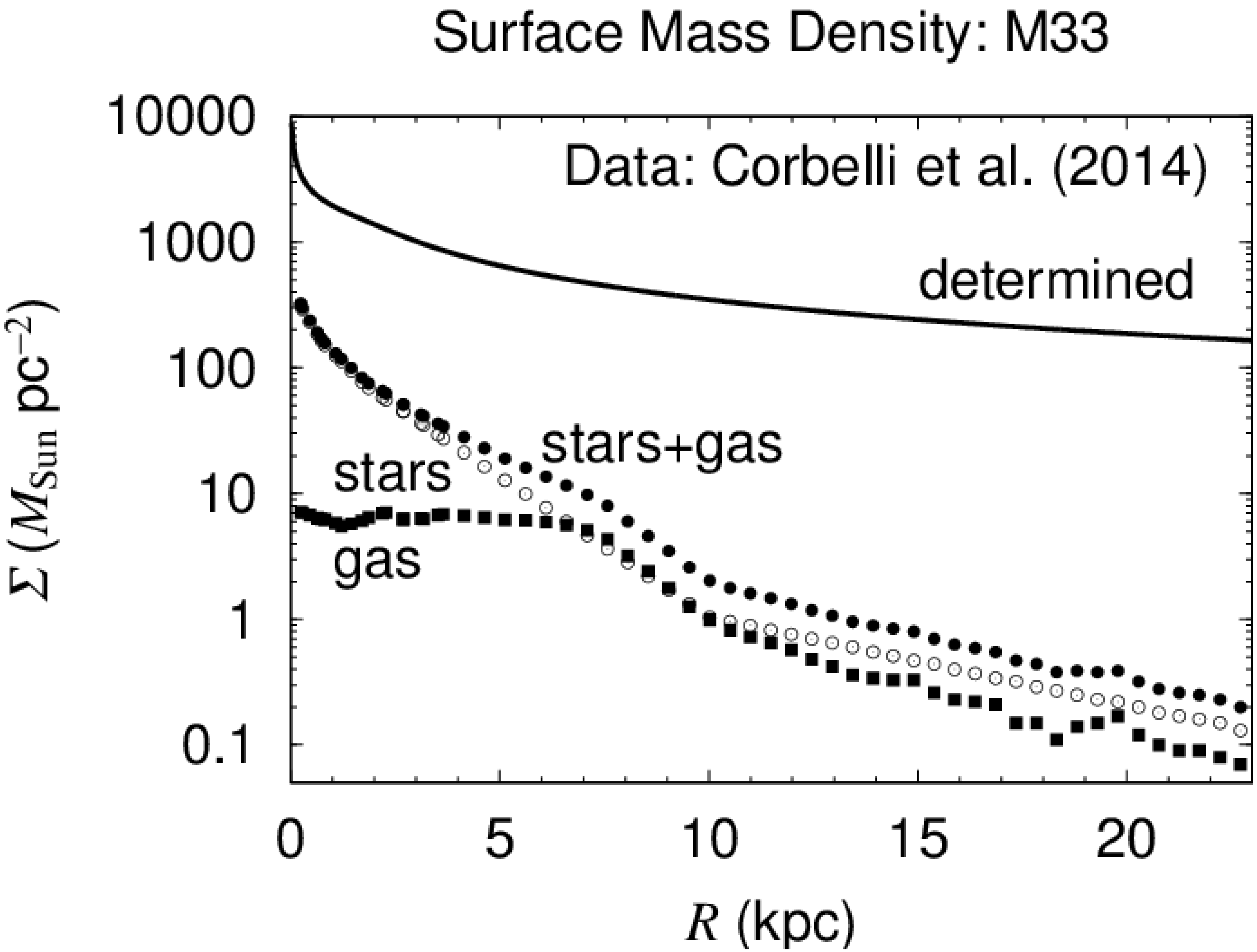
Model Rotation Curve



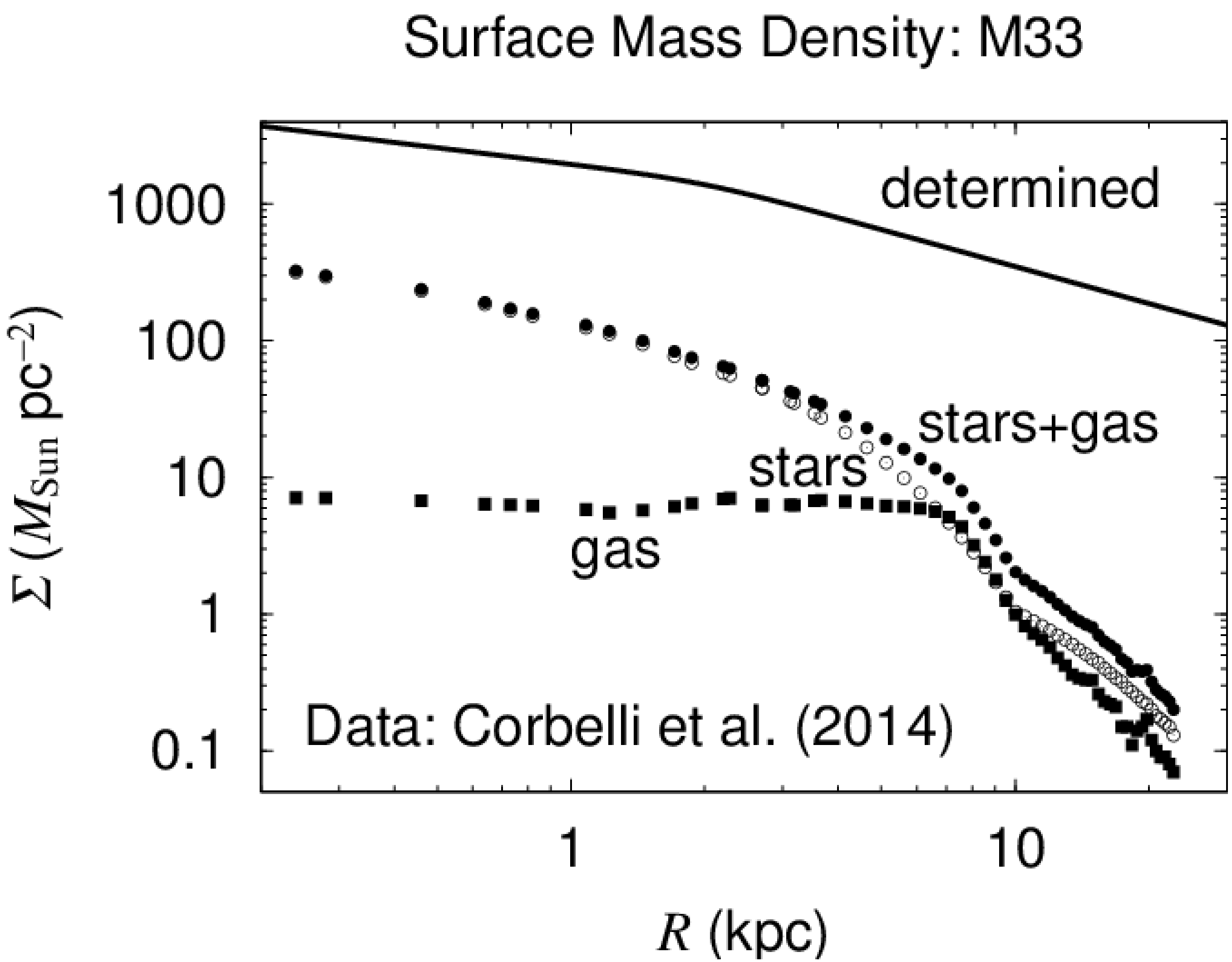
Model Rotation Curve



Determined Disc Mass



Determined Disc Mass



Conclusion

- New Method to Compute Gravitational Field of Infinitely-Thin Disc
- Split Quadrature + Numerical Diff.
- Precise and Fast
- Test Computation of Various Discs
- Application to M33 Rotation Curve
  - Better Fit by Disc Dark Matter

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Best is Yet to Come