長波長非線形揺らぎから探る初期宇宙モデル

計算科学センター

Yuichi Takamizu

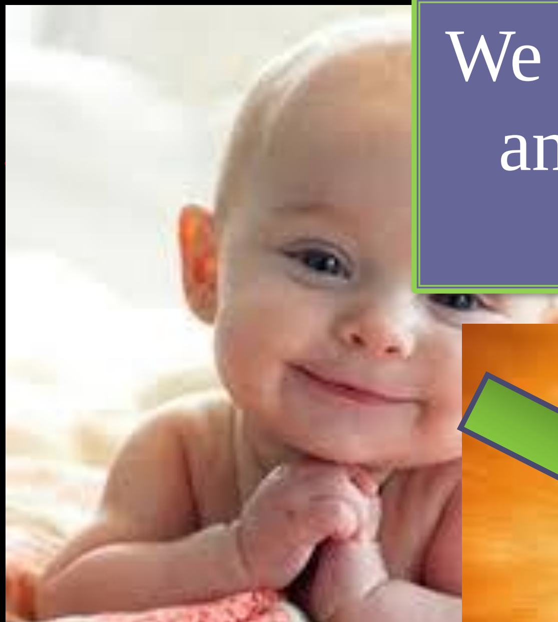
18th JAN,2017@ 筑波宇宙フォーラム

Collaborators: **J.Yokoyama** (U of Tokyo), **M.Sasaki**, **S.Mukohyama**, **A.Naruko** (YITP,Kyoto U)

Ref: **PRD 89 043528 (2014) PRD 83 043504 (2011)**
JCAP 06 019 (2010) & JCAP 01 013 (2009)

Origin: Where did we come from？

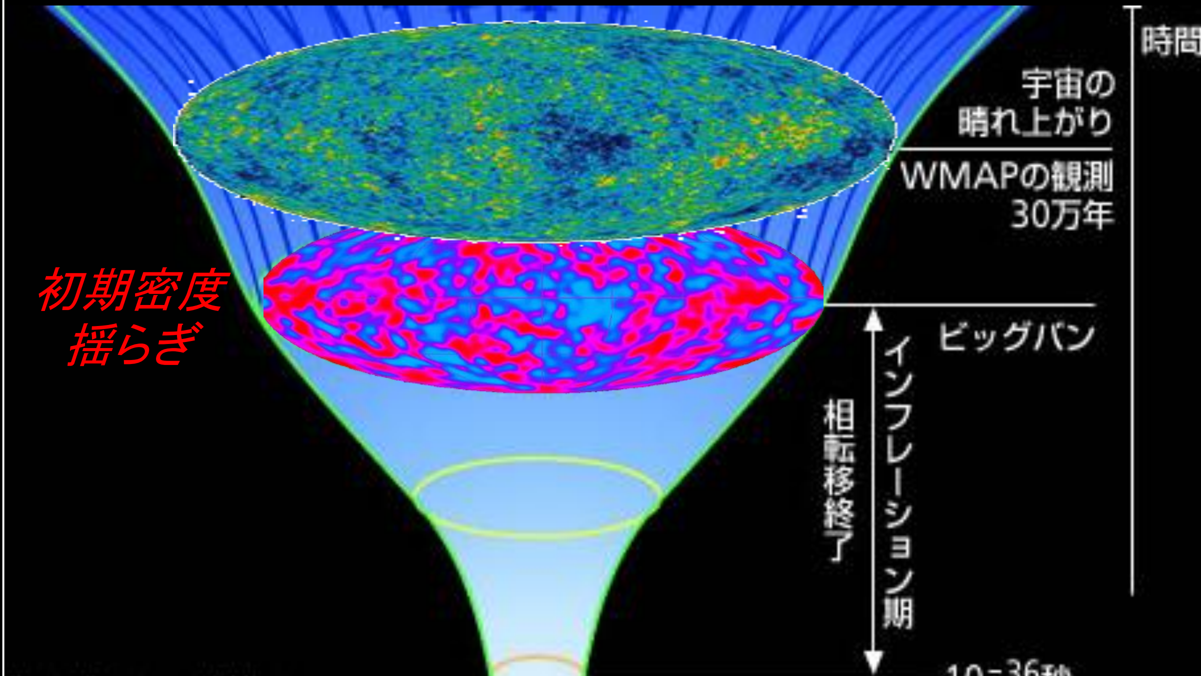
We were made of **Element by supernovae** and were bone on the Earth(Planet) and Sun(Star)



Star bone in galaxy

Galaxy evolves in Cluster

宇宙の進化と宇宙背景輻射



◆ *観測*
（CMB, 銀河分布）

初期密度揺らぎ

◆ *初期宇宙モデル*

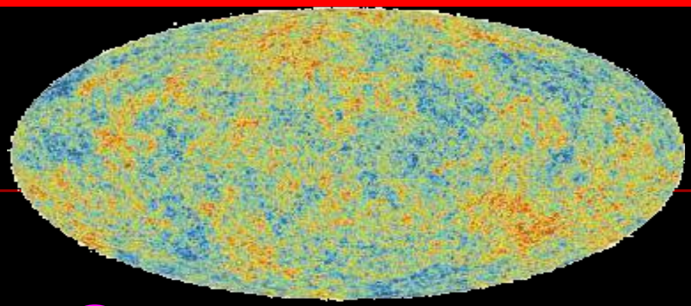
● 初期宇宙の加速膨張（**インフレーションシナリオ**）は
現在宇宙論の標準模型

● これによって'種'となる**初期密度揺らぎ**が与えられる

最古の宇宙地図を与えてくれる。

□銀河団などの大規模構造は、
この*種*（***初期密度揺らぎ***）を元に成長していく。

□ Cosmic Microwave Background (***CMB***)



PLANCK satellite
(2009-)

$$\boxed{T = 2.73K}$$

① Homogeneous and isotropic (FLRW)
→ Evidence for Cosmological principle
＋ the *horizon* problem

$$\boxed{\frac{\delta T}{T} = 10^{-5}}$$

② Tiny fluctuation as a ***Seed***

→ Origin of large scale Structure formation
(Cluster & Galaxy)

◆ ***Key roles: Paradigm of Inflation*** ***(Sato, Guth '81)***

①Accelerated expansion + ②Primordial density perturb.

◆ What is a scalar field named *Inflaton* ?

$\delta\phi$

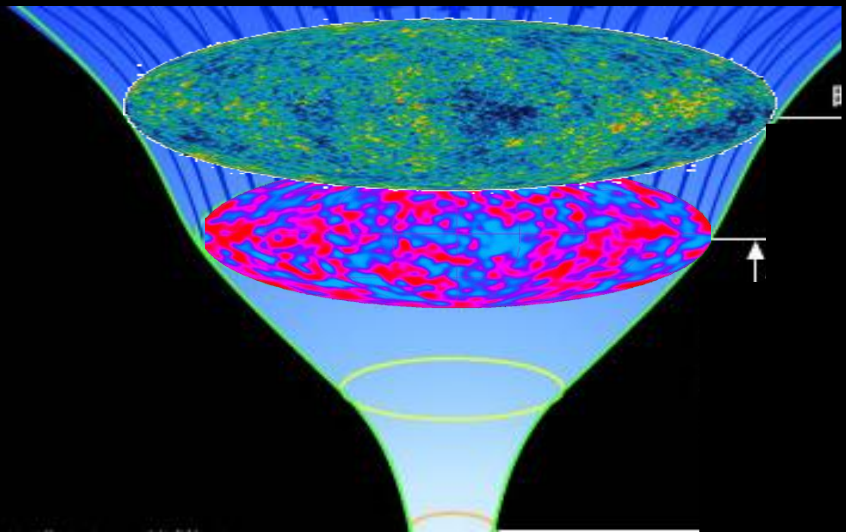
Inflaton

Evolution in Superhorizon scale

$$\frac{\delta T}{T} = 10^{-5}$$

CMB anisotropy

◆ Primordial perturbation as a 「**Window**」 to ‘see’ the high energy physics before the Big Bang universe



◆**Observation**
（CMB, Galaxies）

Primordial perturbation

◆**Model for early universe**

◆ Evolution of primordial curvature perturbation

$\delta\phi$ → Evolution in Superhorizon scale → $\dfrac{\delta T}{T} = 10^{-5}$

Inflaton

CMB anisotropy

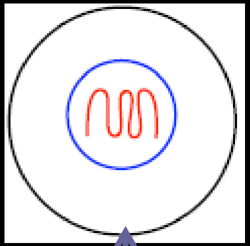
$$\zeta = \psi + H\frac{\delta\phi}{\dot{\phi}}$$

Gravitational Potential

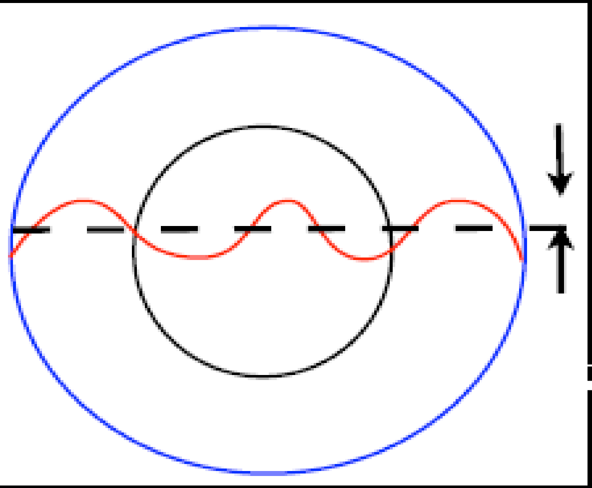
● Conserved : $\boxed{\zeta}$

If adiabatic fluid or single slow-roll scalar

$$\frac{\delta T}{T} = \frac{1}{5}\zeta$$



Horizon



初期密度揺らぎから探る
初期宇宙物理

スカラー場(インフラトン)によって加速膨張を引き起こす（インフレーション）

□ インフラトンの量子揺らぎが初期密度揺らぎを与える。

しかし、スカラー場が解決の糸口を与えることしかわかっていない。。

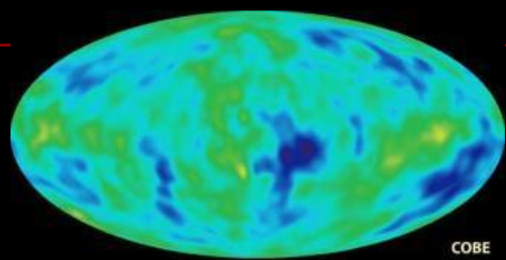
実際インフラトンが何であるかわかっていない。

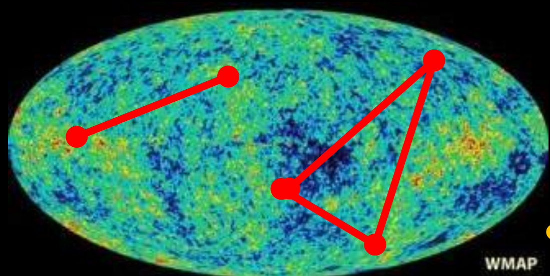
（ヒッグス場？超弦理論などとの関連は？ ）

• この時期の物理は、重力理論と量子論を統一する究極的な基礎理論と深く関係しており、**初期密度揺らぎを探る物理はその重要な手掛かりとなり得る。**

インフラトンの正体解明は、極めて高いエネルギースケールの新たな物理を‘実験’することができるので、現代物理学の発展に大きな鍵となる！

最古の宇宙地図(CMB)の詳細観測



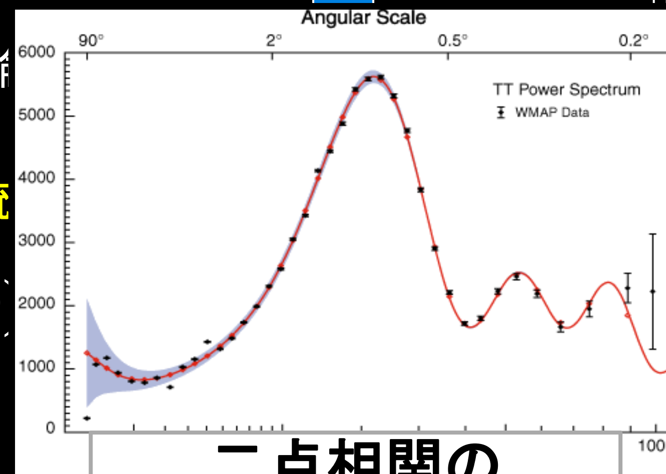


First all-sky image (July,2010)

- Penzias & Wilson (65)
  2.7Kの黒体輻射として観測
- COBE 衛星(92)
  温度揺らぎの振幅を初めて観測 $\dfrac{\Delta T}{T} \sim 10^{-5}$
  インフレーションの線形理論
  予言と完全に一致
  スケール不変性とガウス統
- WMAP 衛星 (03-10)
  スケール不変性からの差を観測し始めた
- PLANCK 衛星(09-)
  三点相関量が観測できる可能性
  $\left\langle \left(\dfrac{\delta T}{T}\right)^3 \right\rangle$

History

線形理論

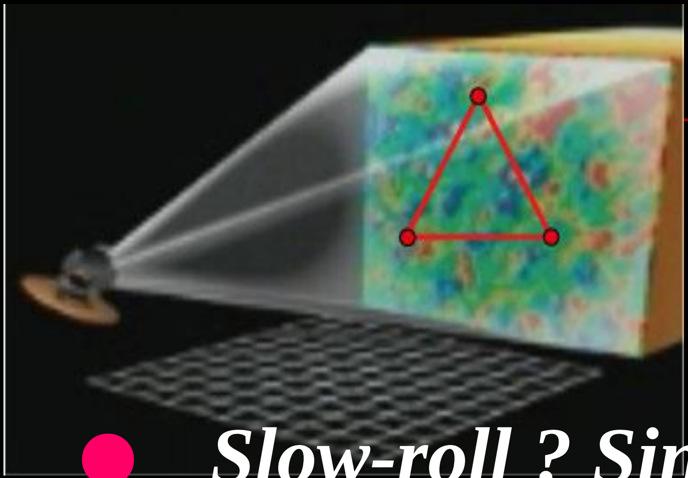


二点相関のスペクトル分解

非線形理論

◆ Non-Gaussianity

$$\zeta(x) = \zeta_G(x) + \frac{3}{5} f_{NL}^{\mathrm{local}} \zeta_G^2(x)$$



● **Current bound**

WMAP 7-year $-10 < f_{\mathrm{NL}}^{\mathrm{local}} < 74$

PLANK 2009- detect within $|f_{\mathrm{NL}}| \gtrsim 5$

● ***Slow-roll ? Single field ? Canonical kinetic ?***

- Standard single slow-roll scalar $f_{\mathrm{NL}} = O(10^{-2})$
- Many models predicting Large Non-Gaussianity

  （Multi-fields, DBI inflation & Curvaton） $f_{\mathrm{NL}} \gg O(1)$

□ *Non-Gaussianity will be one of powerful tool to discriminate many possible inflationary models with the future precision observations*

◆ Non-Gaussianity

To search on Physics behind inflation

- Detecting **Non-Gaussianity** is a big step in order to explore the origin of inflaton.

Linear theory → Nonlinear theory

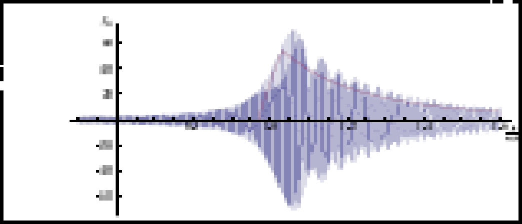
□ PLANCK data result (2015)

$$-8.9 < f_{NL}^{Loc} < 14.3 \qquad 2\sigma(95\%)$$

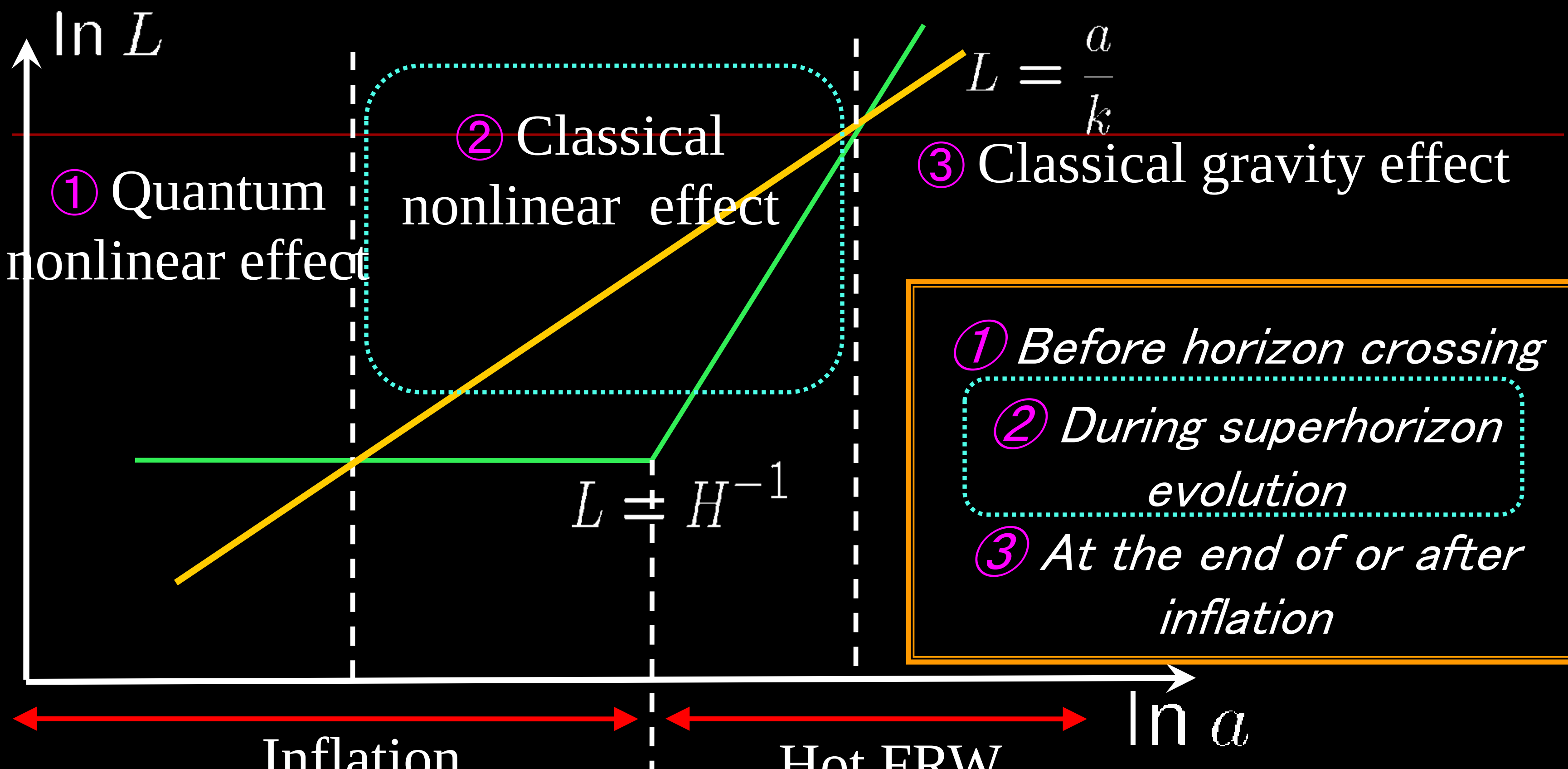
$$-192 < f_{NL}^{Eql} < 108 \qquad 2\sigma(95\%)$$

- **Now Observation shows Perfect Gaussian statistic**
- **However it may be detected as some tiny localized feature**

● *Featured bispectrum*, especially on the equilateral type



◆ Three stages for generating Non-Gaussianity



◆ Generation of NG in superhorizon needs violation of the condition for ①*slow-roll* **or(and)** ②*single*

Nonlinear perturbations on superhorizon scales

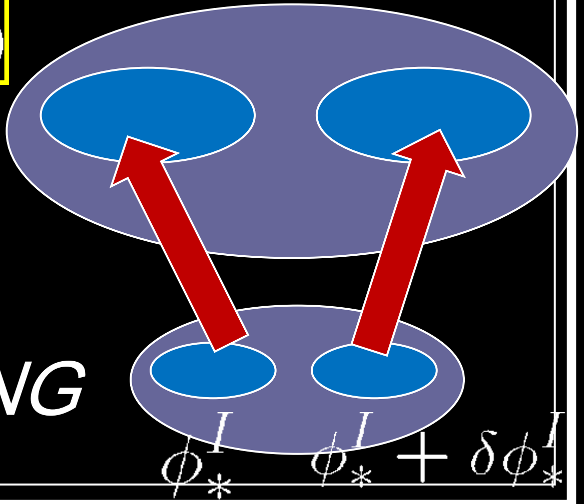
- *Spatial gradient approach* : $\boxed{\epsilon = 1/(HL)}$ Salopek & Bond (90)
  - Spatial derivatives are small compared to time derivative
  - Expand Einstein eqs in terms of small parameter ε, and can solve them for nonlinear perturbations iteratively

δ N formalism *(Separated universe)*

(Starobinsky 85, Nambu & Taruya 96 , Sasaki & Stewart 96)

$$\boxed{\zeta(t,\mathrm{x}) = \delta N \equiv N(t,\mathrm{x}) - N_0(t)} \qquad \boxed{O(\epsilon^0)}$$

Curvature perturbation = Fluctuations of the local e-folding number



◇ *Powerful tool for the estimation of NG*

δN formalism (Separated universe approach)

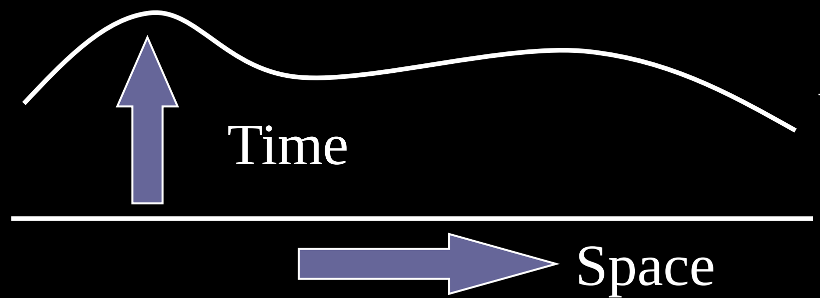
$\det\gamma = 1$

Spatial metric: $g_{ij} = a^2(1+2\zeta)\gamma_{ij} = e^{2N}(1+2\delta N)\gamma_{ij}$

Curvature perturbation   *e*-folding perturbation

$(N = \log a)$

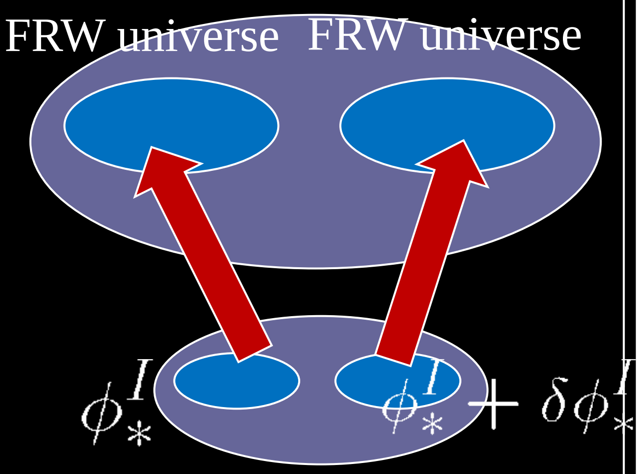
Two hypersurfaces:



$$\boxed{\zeta(t,\mathbf{x}) = \delta N \equiv N(t,\mathbf{x}) - N_0(t)}$$

● Local expansion
= Expansion of the **unperturbed** Universe

$$N(t_c,\vec{x}) = N(t_c,\phi^I(t_c,\vec{x}))$$



◆ δ N formalism

[Ex] Single slow-roll scalar: $\dot{\phi} \approx -\frac{V'(\phi)}{3H}, \; H^2 \approx \frac{V(\phi)}{3}$

***e*-folding number**

$$N = \int H dt = \int \frac{d\phi}{\dot{\phi}} H \quad \Rightarrow \quad \delta N = N_{,\phi}\delta\phi = \frac{H}{\dot{\phi}}\delta\phi$$

◆Calculation of Power spectrum

well known result

$$< \zeta\zeta > = N_{,\phi}^2 < \delta\phi\delta\phi > = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2$$

➢Also *Easy* to calculate **Non-Gaussianity** by only using **background equation**

➢ **Applied to Multi-scalar system**

[①Violation of single]

◆ *Temporary violating of slow-roll condition*

[②Violation of slow-roll]

For Single inflaton-field (this talk)

● **Multi-field** inflation always shows (With Naruko, Sasaki PTEP 2013)

◆ *δ N formalism*

$\boxed{O(\epsilon^0)}$ $\boxed{\zeta(t, \mathbf{x}) = \mathrm{const}}$

➢ Ignore the **decaying mode** of curvature perturbation

Not conserved !

◆ *Beyond δ N formalism*

$\boxed{O(\epsilon^2)}$ $\boxed{\zeta(t, \mathbf{x}) \neq \mathrm{const}}$

➢ **Decaying modes** cannot be neglected in this case

➢ **Enhancement** of curvature perturbation in the **linear theory**

[Seto et al (01), Leach et al (01)]

◆ *Linear theory for a single scalar field*

● **Mukhanov-Sasaki equation：Master equation**

$$\boxed{\mathcal{R}_c^{\mathrm{Lin}\prime\prime} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{Lin}\prime} + k^2 c_s^2 \mathcal{R}_c^{\mathrm{Lin}} = 0}$$

□ If we focus on a scalar type perturbation, One can derive the equation for One basic variable as one degree of freedom（Density）

Solution in Long wavelength expansion (gradient expansion)

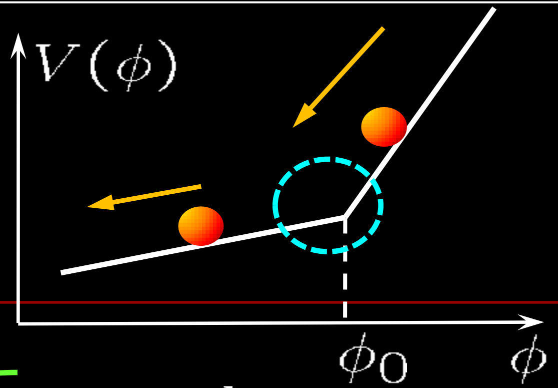
$\boxed{O(k^0)}$ $\mathcal{R} = \mathrm{const}$ $\boxed{O(k^2)}$ $\boxed{\propto k^2 \int \frac{d\eta}{z^2}}$

Growing mode

Decaying mode

◆ Example

- Starobinsky's model (92):
  - There is a stage at which slow-roll conditions are violated



- Leach, Sasaki, Wands & Liddle (01)
  - Linear theory
  - The $\boxed{O(\epsilon^2)}$ in the expansion

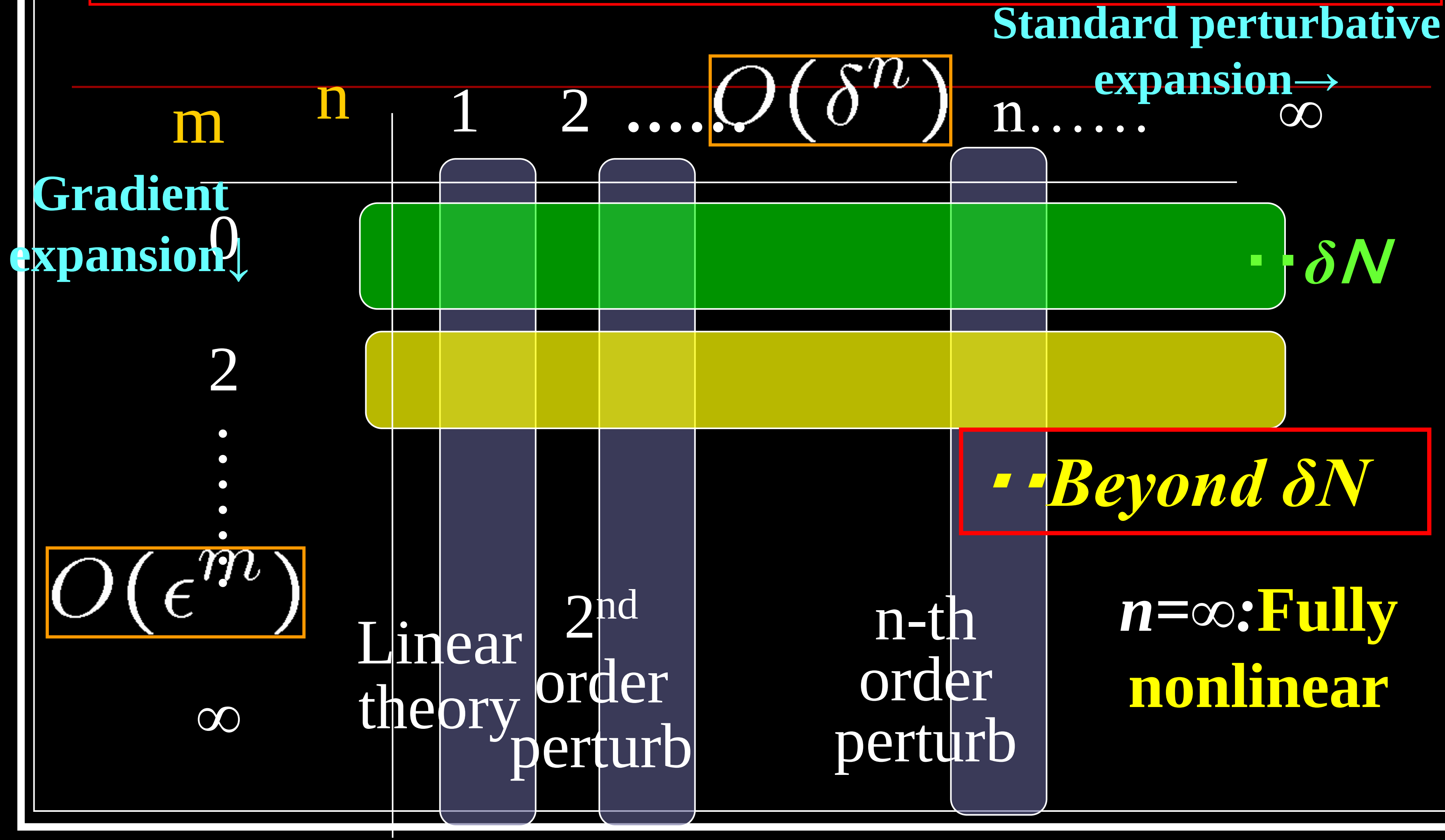
Violating of Slow-Roll
Decaying mode
⇒ $\mathcal{R}'(\eta) \neq 0$

⇒ $$\boxed{\mathcal{R}(0) = \alpha^{\mathrm{Lin}} \mathcal{R}(\eta_*)}$$

$\eta = 0$ (Final) η_* (initial)

◆ *Enhancement of curvature perturbation near* $\boxed{\phi = \phi_0}$

● *Nonlinear perturbations on superhorizon scales up to Next-leading order in the expansion*



◆ *Beyond δN-formalism for single scalar-field*

YT, S.Mukohyama, M.Sasaki & Y.Tanaka JCAP(2010)

Simple result !

- Nonlinear theory in $\boxed{O(\epsilon^2)}$
  - **Nonlinear variable** (including δN)
  - **Nonlinear Source term**

$$\mathcal{R}_c^{\mathrm{NL}\,\prime\prime} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{NL}\,\prime} + \frac{c_s^2}{4}K^{(2)}[\,\mathcal{R}_c^{\mathrm{NL}}\,] = O(\epsilon^4)$$

Ricci scalar of spatial metric

- Linear theory

$$\mathcal{R}_c^{\mathrm{Lin}\,\prime\prime} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{Lin}\,\prime} + k^2 c_s^2 \mathcal{R}_c^{\mathrm{Lin}} = 0$$

◆ *Beyond δN-formalism*

Single scalar case

System : $$\boxed{I = \int d^4x \sqrt{-g}\left[\frac{R}{16\pi G_N} + P(X,\phi)\right]},\quad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- *Single scalar field with general potential & kinetic term*

including K- inflation & DBI etc

ADM decomposition & Gradient expansion

Small parameter: $\boxed{\epsilon = 1/(HL)}$ $\quad \partial_i\psi = \psi\times O(\epsilon)$

◆ **Background is the flat FLRW universe**

Basic assumption:

$$\boxed{\beta^i = O(\epsilon),\quad v^i = O(\epsilon),\quad \partial_t\tilde{\gamma}_{ij} = O(\epsilon)} \Rightarrow \boxed{\partial_t\tilde{\gamma}_{ij} = O(\epsilon^2)}$$

- Absence of any decaying at leading order
- Can be justified in taking the background as FLRW

◆ Scalar field in a perfect fluid form

- This formulation can be developed for couple ways
- We can recognize the difference between them

$$I = \int d^4x\sqrt{-g}P(X,\phi),\ X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$T_{\mu\nu} = 2P_X\partial_\mu\phi\partial_\nu\phi + Pg_{\mu\nu} = (\rho + P)u_\mu u_\nu + Pg_{\mu\nu},$$

$$\boxed{\rho(X,\phi) = 2P_X X - P, \quad u_\mu = -\frac{\partial_\mu\phi}{\sqrt{X}}.}$$

● *The difference between a perfect fluid and a scalsr field*

$$\delta P = c_s^2\delta\rho + \rho\Gamma\delta\phi,$$

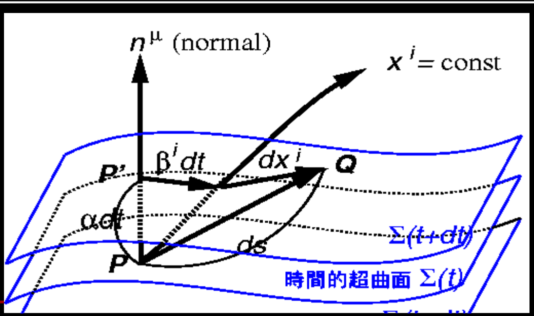
$$\boxed{c_s^2 = \frac{P_X}{2P_{XX}X + P_X}, \quad \Gamma = \frac{1}{\rho}\left(P_\phi - c_s^2\rho_\phi\right).}$$

For a perfect fluid（adiabatic）

$$\delta P = c_s^2\delta\rho \quad c_s^2 = \dot{P}/\dot{\rho}$$

The propagation speed of perturbation（Sound speed）

ADM decomposition



Lapse　　　　Shift　$n_\mu = (-\alpha, 0, 0, 0)$

$$ds^2 = (-\alpha^2 + \beta_k \beta^k)dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$$

- *Gauge degree of freedom*

EOM: Two constraint equations　①Hamiltonian & Momentum constraint eqs

- *EOM for* γ_{ij} *: dynamical equations*

In order to express as a set of first-order diff eqs,

Introduce the extrinsic curvature

$$K_{ij} = -\frac{1}{2\alpha}(\partial_t \gamma_{ij} - D_i \beta_j - D_j \beta_i)$$

② Evolution eqs for K_{ij}, γ_{ij}

- *Conservation law* $T^{\mu\nu}{}_{;\mu} = 0$

③ Energy & Momentum conservation eqs (Hydrodynamic eqs)

Further decompose,

② Four Evolution eqs

- **Spatial metric** $\gamma_{ij} = a^2 \psi^4 \tilde{\gamma}_{ij}$; $\det(\tilde{\gamma}_{ij}) = 1$
- **Extrinsic curvature** $K_{ij} = \frac{\gamma_{ij}}{3} K + \psi^4 a^2 \tilde{A}_{ij}$ ← Traceless part

Solve the Einstein equation after ADM decomposition

General solution in Uniform Hubble + Time-orthogonal slicing

valid up to $\boxed{O(\epsilon^2)}$

YT& Mukohyama, JCAP 01(2009)

Curvature perturbation

Spatial metric $\gamma_{ij} = a^2 \boxed{e^{2\zeta}} \tilde{\gamma}_{ij};\ \det(\tilde{\gamma}_{ij}) = 1$

$$\zeta \simeq (\mathrm{const}) + \underbrace{(\mathrm{time}-\mathrm{dep})}_{O(\epsilon^2)} + O(\epsilon^4),$$

Constant (δ N)

$\delta P = c_s^2 \delta\rho + \rho\Gamma\delta\phi,$

- Variation of Pressure (speed of sound etc)
- Scalar decaying mode

◆ Applications of our formula

(Temporary violating of slow-roll condition)

$$z = \frac{a}{H}\left(\frac{\rho + P}{c_s^2}\right)^{\frac{1}{2}}$$

Calculate the integrals;

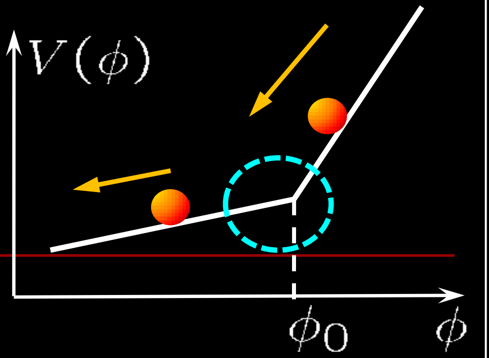
$$D_k = 3\mathcal{H}(\eta_k)\int_{\eta_k}^{0} d\eta' \frac{z^2(\eta_k)}{z^2(\eta')} \quad F_k = \int_{\eta_k}^{0} \frac{d\eta'}{z^2} \int_{\eta_k}^{\eta'} z^2 c_s^2 d\eta''$$

- Non-Gaussianity in this formula matched to DBI inflation
- Apply to varying sound velocity
- Trispectrum of the feature models
- Extension to nonlinear Gravitational wave
- Extension to the models of Multi-scalar field

(naturally gives temporary violating of slow-roll cond)

◆ Application to Starobinsky model

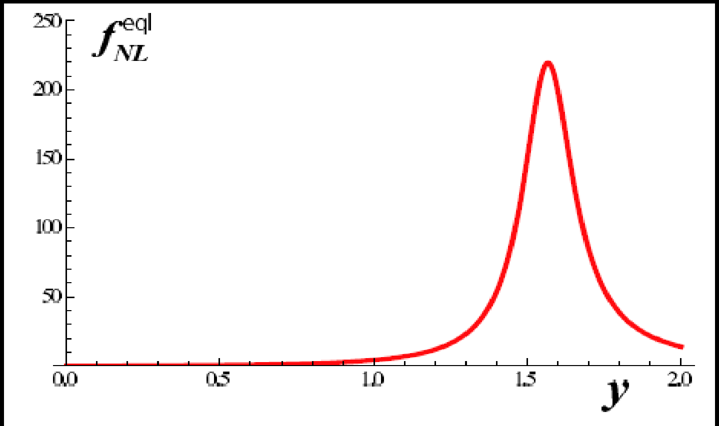
➢ There is a stage at which slow-roll conditions are violated



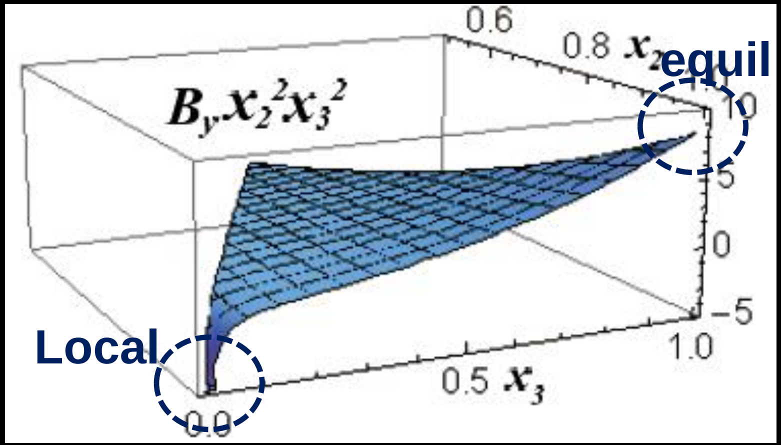
● In Fourier space, calculate Bispectrum

$$T = \left(\frac{A_+}{A_-} - 1\right) = 10^2$$

◆ Equilateral $k = k_1 = k_2 = k_3$



◆ Shape of bispectrum



$\boxed{f_{NL}^{eql} \simeq 2T}$ $y = \sqrt{T}k/k_0 \simeq 1.5$

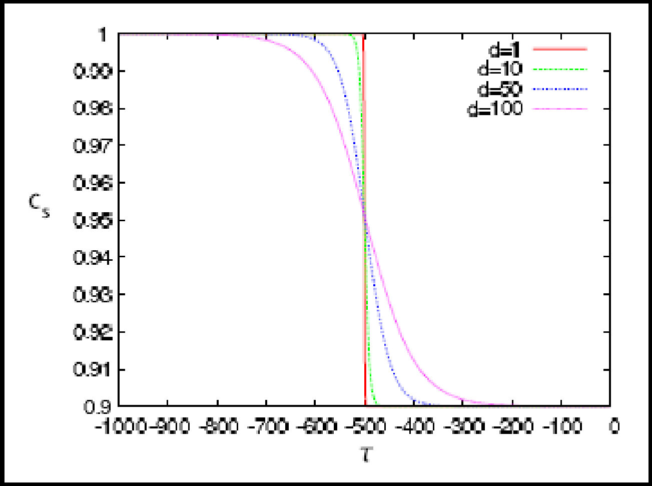
$y = 10$

◆ Application to varying sound speed

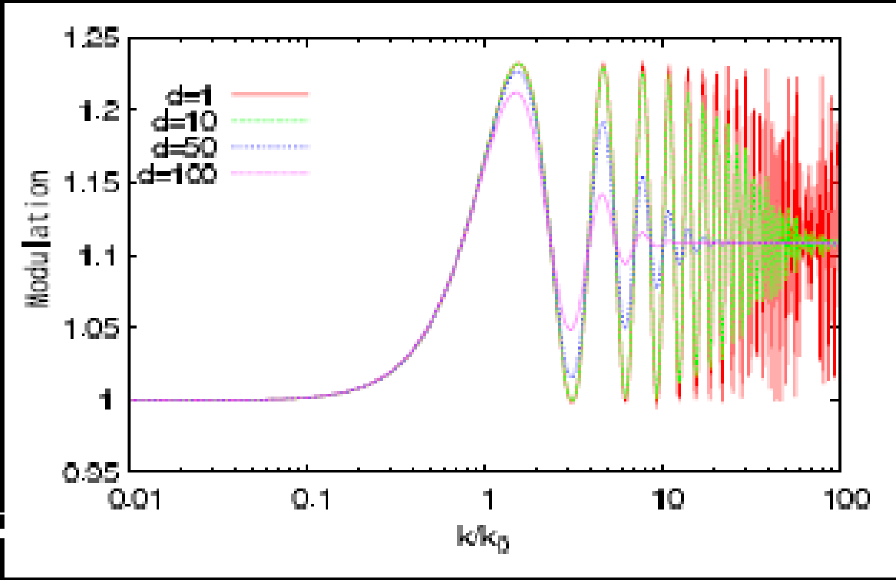
● In order to create localized feature in power and bispectrum

YT PRD 2014

$$c_s^2 = c_{s1}^2 + (c_{s2}^2 - c_{s1}^2)\frac{\tanh[(\tau-\tau_0)/d]+1}{2}$$



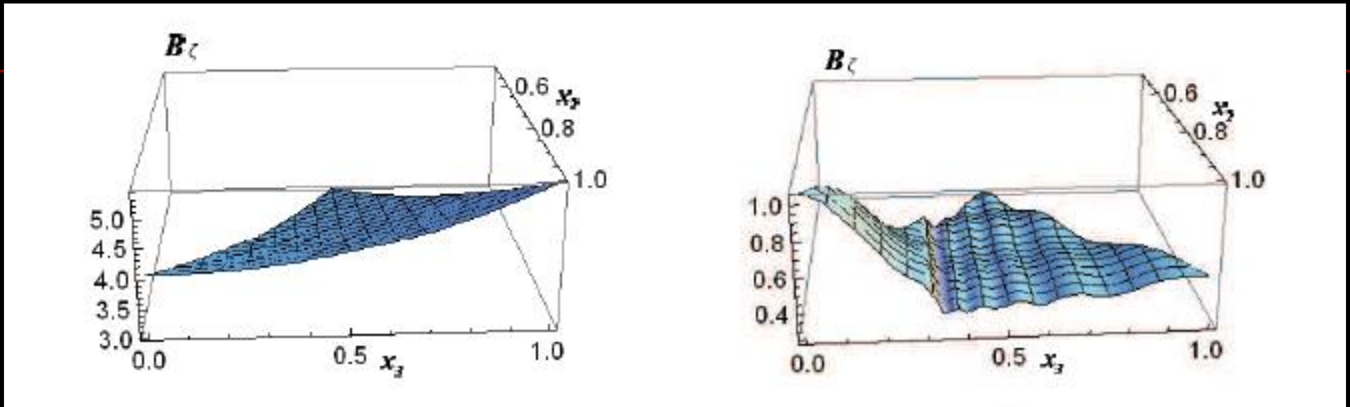
➢ Power spectrum



See also previous study with Saito and Yokoyama PTP 125 (2011)

Application to varying sound speed

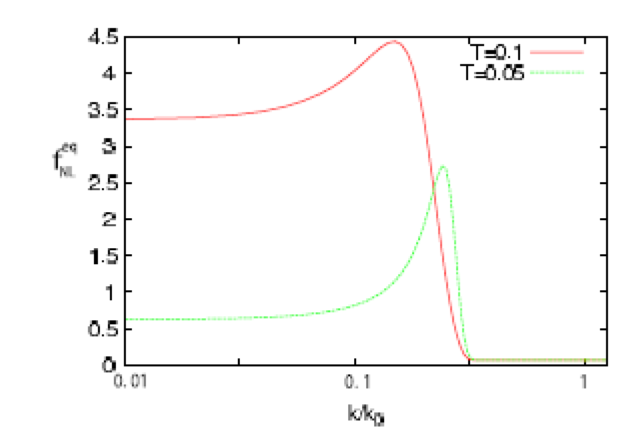
Shape of bispectrum



- Equilateral $k_1 = k_2 = k_3$
- Local $k_3 \ll k1 = k2$

- Equilateral

(Localized Featured bispectrum)



See also recent paper with Saito JCAP06(2013)

Heavy field also shows similar localized feature

◆ Application to Inflaton stopping

$\dot{\phi} \approx 0$

Full nonlinear growing and decaying modes of superhorizon curvature perturbations

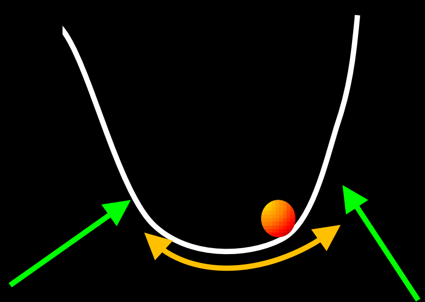
YT & Yokoyama
PRD 83 043504 (2011)

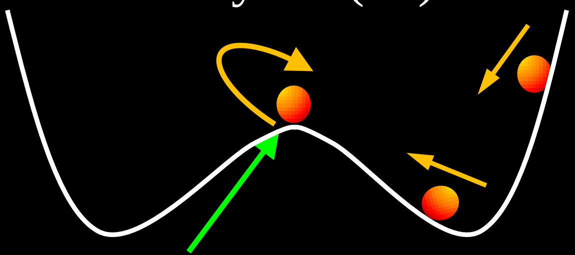
● What happens when the inflaton stops during inflation ?

◆ ***Oscillating inflation*** or ***Chaotic New inflation***

Damour & Mukhanov (98)

Yokoyama(98)





$$\mathcal{R}_c \approx \frac{H^2}{|\dot{\phi}|}$$ **Divergent ?** ➡ $$\mathcal{R}_c \propto \frac{H^3}{V'(\phi)}$$ **Finite !**

Seto Yokoyama & Kodama (00)

◆ Application to Inflaton stopping

◆ Linear theory

$$\mathcal{R}_c^{\mathrm{Lin}''} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{Lin}'} + k^2 c_s^2 \mathcal{R}_c^{\mathrm{Lin}} = 0$$

Solution: $$\mathcal{R}_c^{\mathrm{Lin}} = u^{(0)}\left[C_1^{(2)} + C_2^{(2)} D(\eta) + k^2 F(\eta)\right] + O(k^4)$$

$$\boxed{D(\eta) = \int \frac{d\eta'}{z^2(\eta')}} \qquad F(\eta) = \int \frac{d\eta'}{z^2}\int^{\eta'} z^2 c_s^2 d\eta''$$

Decaying modes **diverge** at $\boxed{z = a\dot{\phi}/H \to 0}$

ただしdecaying modeなので最終的に消えてしまうため、
後には影響が残らない（**問題なし**）
しかし一般に非線形摂動論ではモードが<u>coupleする可能性</u>
がありgrowing modeに影響が残る場合もある

◆ *Summary*

- We develop a theory of nonlinear cosmological perturbations on superhorizon scales for a scalar field with a general potential & kinetic terms
- We employ the **ADM formalism** and the spatial **gradient expansion** approach to obtain general solutions valid up through second-order $O(\epsilon^2)$
- We formulate a general method to match n-th order perturbative solution
- Can applied to Non-Gaussianity *in* ***temporary violating of slow-rolling***
- **Beyond δN-formalism:** *Two nonlinear effects*
- ① **Nonlinear variable : including δN (fully nonlinear)**
- ② **Nonlinear source term : Simple 2nd order diff equation**
- **Applications: Bispectrum** for **Starobinsky** model & **Inflaton stopping**

Thank you for your attention ！