

ASSIGNMENT-1

AI1110:Probability and random variables

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QUESTION: Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

ANSWER: $\frac{1}{15}$

SOLUTION: Let S be the sample space of the event.

$$S = \begin{bmatrix} (1, 1) & (2, 1) & (3, 1) & (4, 1) & (5, 1) & (6, 1) \\ (2, 1) & (2, 2) & (3, 2) & (4, 2) & (5, 2) & (6, 2) \\ (3, 1) & (2, 3) & (3, 3) & (4, 3) & (5, 3) & (6, 3) \\ (4, 1) & (2, 4) & (3, 4) & (4, 4) & (5, 4) & (6, 4) \\ (5, 1) & (2, 5) & (3, 5) & (4, 5) & (5, 5) & (6, 5) \\ (6, 1) & (2, 6) & (3, 6) & (4, 6) & (5, 6) & (6, 6) \end{bmatrix} \quad (1)$$

$$n(S) = 36$$

Let X be a random variable represents the event "the sum of numbers on the dice".

Let X_1, X_2 represents the numbers on each dice respectively.

Parameter	Value	Description
X	$2 \leq X \leq 12$	Sum of the numbers on two dice
X_1	$1 \leq X_1 \leq 6$	Number on die 1
X_2	$1 \leq X_2 \leq 6$	Number on die 2

TABLE 0
DEFINITIONS

$$X = X_1 + X_2 \quad (2)$$

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \leq n \leq 7 \\ \frac{13-n}{36} & 7 < n \leq 12 \\ 0 & n > 12 \end{cases} \quad (3)$$

Here $n=4$,

$$p_X(4) = \frac{3}{36} \quad (4)$$

Given that $X_1 \neq X_2$

$$(X_1 = X_2) = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\} \quad (5)$$

$$n(X_1 = X_2) = 6 \quad (6)$$

$$\Pr(X_1 \neq X_2) = 1 - \Pr(X_1 = X_2) \quad (7)$$

$$\text{From (6)} \quad (8)$$

$$= 1 - \frac{6}{36} \quad (9)$$

$$= \frac{30}{36} \quad (10)$$

$$(X = 4) \cap (X_1 \neq X_2) = \{(1, 3), (3, 1)\} \quad (11)$$

$$n((X = 4) \cap (X_1 \neq X_2)) = 2 \quad (12)$$

$$\Pr((X = 4) \cap (X_1 \neq X_2)) = \frac{2}{36} \quad (13)$$

Sum	Numbers(X_1, X_2)
4	(1,3)
4	(3,1)

TABLE 0

VALUES SATISFYING GIVEN CONDITION

Hence required probability is,

$$\Pr((X = 4)|(X_1 \neq X_2)) = \frac{\Pr((X = 4) \cap (X_1 \neq X_2))}{\Pr(X_1 \neq X_2)} \quad (14)$$

Substituting (10) and (13) in (14)

$$\Pr((X = 4)|(X_1 \neq X_2)) = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{2}{30} = \frac{1}{15} \quad (15)$$