

# ASSIGNMENT-1

## AI1110:Probability and random variables

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### 12.13.1.10

**QUESTION:** Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

**ANSWER:**  $\frac{1}{15}$

**SOLUTION:** Let S be the sample space of the event.

$$S = \begin{bmatrix} 11 & 21 & 31 & 41 & 51 & 61 \\ 21 & 22 & 32 & 42 & 52 & 62 \\ 31 & 23 & 33 & 43 & 53 & 63 \\ 41 & 24 & 34 & 44 & 54 & 64 \\ 51 & 25 & 35 & 45 & 55 & 65 \\ 61 & 26 & 36 & 46 & 56 & 66 \end{bmatrix}$$

$$n(S) = 36$$

Let Y be the random variable represents the outcome of a die.

$$p_Y(n) = \begin{cases} \frac{1}{6} & 1 \leq n \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (1.1)$$

Let X be a random variable represents the event "the sum of numbers on the dice".

Let  $X_1, X_2$  represents the numbers on each dice respectively.

$$X = X_1 + X_2$$

$$X = \{1, 2, 3, \dots, 12\}$$

$$\begin{aligned} p_X(n) &= \Pr(X_1 + X_2 = n) = \Pr(X_1 = n - X_2) \\ &= \sum_k \Pr(X_1 = n - k | X_2 = k) p_{X_2}(k) \end{aligned}$$

$X_1$  and  $X_2$  are independent.

$$\Pr(X_1 = n - k | X_2 = k) = \Pr(X_1 = n - k) = p_{X_1}(n - k)$$

$$p_X(n) = \sum_k p_{X_1}(n - k) p_{X_2}(k)$$

$$p_X(n) = \frac{1}{6} \sum_{k=1}^6 p_{X_1}(n - k) = \frac{1}{6} \sum_{k=n-6}^{n-1} p_{X_1}(k) \quad (1.2)$$

$$\therefore p_{X_1}(k) = 0, \quad k < 1, k > 6.$$

From (1.2)

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{1}{6} \sum_{k=1}^{n-1} p_{X_1}(k) & 1 \leq n-1 \leq 6 \\ \frac{1}{6} \sum_{k=n-6}^6 p_{X_1}(k) & 1 < n-6 \leq 6 \\ 0 & n > 12 \end{cases} \quad (1.3)$$

Substituting (1.1) in (1.3)

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \leq n \leq 7 \\ \frac{13-n}{36} & 7 < n \leq 12 \\ 0 & n > 12 \end{cases} \quad (1.4)$$

Here  $n=4$ ,

$$p_X(4) = \frac{3}{36} \quad (1.5)$$

Given that  $X_1 \neq X_2$

$$(X_1 = X_2) = \{11, 22, 33, 44, 55, 66\}$$

$$n(X_1 = X_2) = 6 \quad (1.6)$$

$$\Pr(X_1 \neq X_2) = 1 - \Pr(X_1 = X_2)$$

$$= 1 - \frac{6}{36} \quad (\text{From 1.6})$$

$$= \frac{30}{36} \quad (1.7)$$

$$(X = 4) \cap (X_1 \neq X_2) = \{13, 31\}$$

$$n((X = 4) \cap (X_1 \neq X_2)) = 2$$

$$\Pr((X = 4) \cap (X_1 \neq X_2)) = \frac{2}{36} \quad (1.8)$$

Hence required probability is,

$$\Pr((X = 4) / (X_1 \neq X_2)) = \frac{\Pr((X = 4) \cap (X_1 \neq X_2))}{\Pr(X_1 \neq X_2)} \quad (1.9)$$

Substituting (1.7) and (1.8) in (1.9)

$$\Pr((X = 4) / (X_1 \neq X_2)) = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{2}{30} = \frac{1}{15}$$