ASSIGNMENT-1
AI1110:Probability and random variables

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QUESTION:Given that the two numbers appearing on throwing two dice are different.Find the probability of the event 'the sum of numbers on the dice is 4'.
ANSWER:$\frac{1}{15}$
SOLUTION:Let S be the sample space of the event.

$$S = \begin{bmatrix} 11 & 21 & 31 & 41 & 51 & 61 \\ 21 & 22 & 32 & 42 & 52 & 62 \\ 31 & 23 & 33 & 43 & 53 & 63 \\ 41 & 24 & 34 & 44 & 54 & 64 \\ 51 & 25 & 35 & 45 & 55 & 65 \\ 61 & 26 & 36 & 46 & 56 & 66 \end{bmatrix}$$

n(S)=36

Let Y be the random variable represents the outcome of a die.

$$p_Y(n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \tag{1.1}$$

Let X be a random variable represents the event "the sum of numbers on the dice".
Let X_1, X_2 represents the numbers on each dice respectively.

$$X = X_1 + X_2$$
$$X = \{1, 2, 3, \dots, 12\}$$
$$p_X(n) = \Pr(X_1 + X_2 = n) = \Pr(X_1 = n - X_2)$$
$$= \sum_k \Pr(X_1 = n - k | X_2 = k)\, p_{X_2}(k)$$

X_1 and X_2 are independent.

$$\Pr(X_1 = n - k | X_2 = k) = \Pr(X_1 = n - k) = p_{X_1}(n - k)$$
$$p_X(n) = \sum_k p_{X_1}(n - k) p_{X_2}(k)$$
$$p_x(n) = \frac{1}{6}\sum_{k=1}^{6} p_{X_1}(n-k) = \frac{1}{6}\sum_{k=n-6}^{n-1} p_{X_1}(k) \tag{1.2}$$

$\because p_{X_1}(k) = 0, \quad k < 1, k > 6.$

From (1.2)

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{1}{6}\sum_{k=1}^{n-1} p_{X_1}(k) & 1 \le n-1 \le 6 \\ \frac{1}{6}\sum_{k=n-6}^{6} p_{X_1}(k) & 1 < n-6 \le 6 \\ 0 & n > 12 \end{cases} \tag{1.3}$$

Substituting (1.1) in (1.3)

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \le n \le 7 \\ \frac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases}$$

Here n=4,

$$p_X(4) = \frac{3}{36}$$

Let B be the event represents "the two numbers appearing on throwing two dice are different".

$$B = \begin{bmatrix} 12 & 21 & 32 & 41 & 51 & 61 \\ 13 & 23 & 33 & 43 & 53 & 63 \\ 14 & 24 & 34 & 44 & 54 & 64 \\ 15 & 25 & 35 & 45 & 55 & 65 \\ 16 & 26 & 36 & 46 & 56 & 66 \end{bmatrix}$$

$$n(B) = 30 \implies \Pr(B) = \frac{30}{36}$$
$$(X = 4) \cap B = \{13, 31\} \implies n((X = 4) \cap B) = 2$$
$$\Pr((X = 4) \cap B) = \frac{2}{36}$$

Hence required probability is,

$$\Pr((X = 4)/B) = \frac{\Pr((X = 4) \cap B)}{\Pr(B)}$$

$$\Pr((X = 4)/B) = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{2}{30} = \frac{1}{15}$$