

ASSIGNMENT-1

AI1110:Probability and random variables

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12.13.1.10

X_1 and X_2 are independent.

QUESTION: Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

ANSWER: $\frac{1}{15}$

SOLUTION: Let S be the sample space of the event.

$$S = \begin{bmatrix} (1, 1) & (2, 1) & (3, 1) & (4, 1) & (5, 1) & (6, 1) \\ (2, 1) & (2, 2) & (3, 2) & (4, 2) & (5, 2) & (6, 2) \\ (3, 1) & (2, 3) & (3, 3) & (4, 3) & (5, 3) & (6, 3) \\ (4, 1) & (2, 4) & (3, 4) & (4, 4) & (5, 4) & (6, 4) \\ (5, 1) & (2, 5) & (3, 5) & (4, 5) & (5, 5) & (6, 5) \\ (6, 1) & (2, 6) & (3, 6) & (4, 6) & (5, 6) & (6, 6) \end{bmatrix} \quad (1)$$

$n(S)=36$

Let Y be the random variable represents the outcome of a die.

$$\Pr(X_1 = n - k | X_2 = k) = \Pr(X_1 = n - k) = p_{X_1}(n - k) \quad (7)$$

$$p_X(n) = \sum_k p_{X_1}(n - k) p_{X_2}(k) \quad (8)$$

$$p_X(n) = \frac{1}{6} \sum_{k=1}^6 p_{X_1}(n - k) = \frac{1}{6} \sum_{k=n-6}^{n-1} p_{X_1}(k) \quad (9)$$

$$\therefore p_{X_1}(k) = 0, k < 1, k > 6. \quad (10)$$

From (9)

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{1}{6} \sum_{k=1}^{n-1} p_{X_1}(k) & 1 \leq n-1 \leq 6 \\ \frac{1}{6} \sum_{k=n-6}^6 p_{X_1}(k) & 1 < n-6 \leq 6 \\ 0 & n > 12 \end{cases} \quad (11)$$

$$p_Y(n) = \begin{cases} \frac{1}{6} & 1 \leq n \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (2) \quad \text{Substituting (2) in (11)}$$

Let X be a random variable represents the event "the sum of numbers on the dice".

Let X_1, X_2 represents the numbers on each dice respectively.

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \leq n \leq 7 \\ \frac{13-n}{36} & 7 < n \leq 12 \\ 0 & n > 12 \end{cases} \quad (12)$$

$$X = X_1 + X_2 \quad (3)$$

$$X = \{1, 2, 3, \dots, 12\} \quad (4)$$

Here $n=4$,

$$p_X(n) = \Pr(X_1 + X_2 = n) = \Pr(X_1 = n - X_2) \quad (5)$$

$$= \sum_k \Pr(X_1 = n - k | X_2 = k) p_{X_2}(k) \quad (6)$$

$$p_X(4) = \frac{3}{36} \quad (13)$$

Given that $X_1 \neq X_2$

$$(X_1 = X_2) = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\} \quad (14)$$

$$n(X_1 = X_2) = 6 \quad (15)$$

$$\Pr(X_1 \neq X_2) = 1 - \Pr(X_1 = X_2) \quad (16)$$

$$\text{From (15)} \quad (17)$$

$$= 1 - \frac{6}{36} \quad (18)$$

$$= \frac{30}{36} \quad (19)$$

$$(X = 4) \cap (X_1 \neq X_2) = \{(1, 3), (3, 1)\} \quad (20)$$

$$n((X = 4) \cap (X_1 \neq X_2)) = 2 \quad (21)$$

$$\Pr((X = 4) \cap (X_1 \neq X_2)) = \frac{2}{36} \quad (22)$$

Sum	Numbers(X_1, X_2)
4	(1,3)
4	(3,1)

TABLE 0

VALUES SATISFYING GIVEN CONDITION

Hence required probability is,

$$\Pr((X = 4)|(X_1 \neq X_2)) = \frac{\Pr((X = 4) \cap (X_1 \neq X_2))}{\Pr(X_1 \neq X_2)} \quad (23)$$

Substituting (19) and (22) in (23)

$$\Pr((X = 4)|(X_1 \neq X_2)) = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{2}{30} = \frac{1}{15} \quad (24)$$