ASSIGNMENT-1
AI1110:Probability and random variables

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QUESTION:Given that the two numbers appearing on throwing two dice are different.Find the probability of the event 'the sum of numbers on the dice is 4'.
ANSWER:$\frac{1}{15}$
SOLUTION:Let S be the sample space of the event.

$$S = \begin{bmatrix} (1,1) & (2,1) & (3,1) & (4,1) & (5,1) & (6,1) \\ (2,1) & (2,2) & (3,2) & (4,2) & (5,2) & (6,2) \\ (3,1) & (2,3) & (3,3) & (4,3) & (5,3) & (6,3) \\ (4,1) & (2,4) & (3,4) & (4,4) & (5,4) & (6,4) \\ (5,1) & (2,5) & (3,5) & (4,5) & (5,5) & (6,5) \\ (6,1) & (2,6) & (3,6) & (4,6) & (5,6) & (6,6) \end{bmatrix}$$

n(S)=36

Let Y be the random variable represents the outcome of a die.

$$p_Y(n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \tag{1.1}$$

Let X be a random variable represents the event "the sum of numbers on the dice".
Let X_1, X_2 represents the numbers on each dice respectively.

$$X = X_1 + X_2$$
$$X = \{1, 2, 3, \ldots, 12\}$$
$$p_X(n) = \Pr(X_1 + X_2 = n) = \Pr(X_1 = n - X_2)$$
$$= \sum_k \Pr(X_1 = n - k | X_2 = k)\, p_{X_2}(k)$$

X_1 and X_2 are independent.

$$\Pr(X_1 = n - k | X_2 = k) = \Pr(X_1 = n - k) = p_{X_1}(n - k)$$
$$p_X(n) = \sum_k p_{X_1}(n - k) p_{X_2}(k)$$
$$p_x(n) = \frac{1}{6} \sum_{k=1}^{6} p_{X_1}(n - k) = \frac{1}{6} \sum_{k=n-6}^{n-1} p_{X_1}(k) \tag{1.2}$$
$$\because p_{X_1}(k) = 0, \quad k < 1, k > 6.$$

From (1.2)

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{1}{6}\sum_{k=1}^{n-1} p_{X_1}(k) & 1 \le n - 1 \le 6 \\ \frac{1}{6}\sum_{k=n-6}^{6} p_{X_1}(k) & 1 < n - 6 \le 6 \\ 0 & n > 12 \end{cases} \tag{1.3}$$

Substituting (1.1) in (1.3)

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \le n \le 7 \\ \frac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases} \tag{1.4}$$

Here n=4,

$$p_X(4) = \frac{3}{36} \tag{1.5}$$

Given that $X_1 \neq X_2$

$$(X_1 = X_2) = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$
$$n(X_1 = X_2) = 6 \tag{1.6}$$
$$\Pr(X_1 \neq X_2) = 1 - \Pr(X_1 = X_2)$$
$$= 1 - \frac{6}{36} \qquad (From 1.6)$$
$$= \frac{30}{36} \tag{1.7}$$
$$(X = 4) \cap (X_1 \neq X_2) = \{(1,3), (3,1)\}$$
$$n((X = 4) \cap (X_1 \neq X_2)) = 2$$
$$\Pr((X = 4) \cap (X_1 \neq X_2)) = \frac{2}{36} \tag{1.8}$$

Sum	Numbers($X_1 \neq X_2$)
4	(1,3)
4	(3,1)

Hence required probability is,

$$\Pr((X = 4) | (X_1 \neq X_2)) = \frac{\Pr((X = 4) \cap (X_1 \neq X_2))}{\Pr(X_1 \neq X_2)} \tag{1.9}$$

Substituting (1.7) and (1.8) in (1.9)

$$\Pr\left((X = 4) | (X_1 \neq X_2)\right) = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{2}{30} = \frac{1}{15}$$