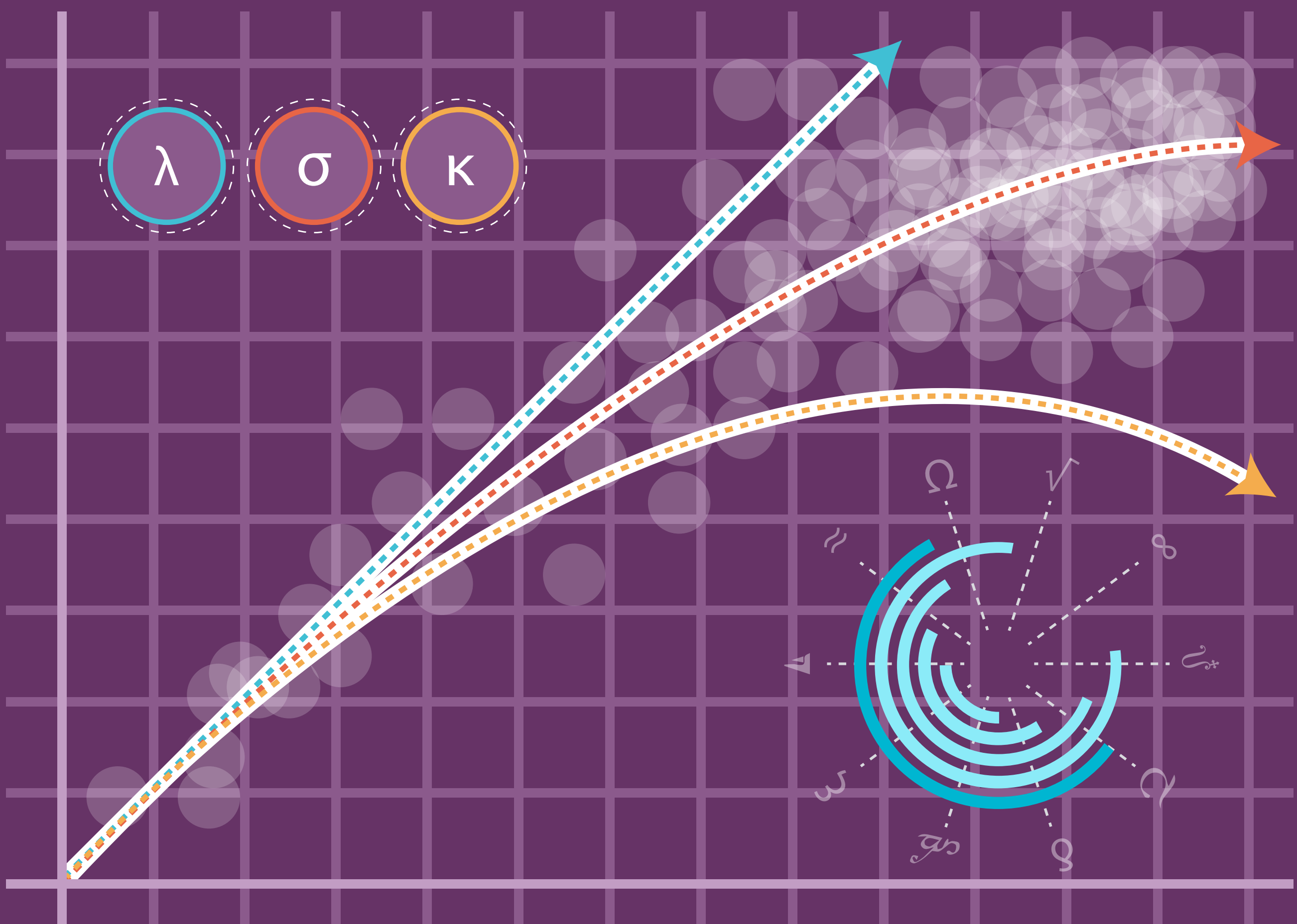
Practical Scalability Analysis With The

Universal Scalability Law



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VividCortex

Meet the Author

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Baron is a performance and scalability expert who participates in various database, opensource, and distributed systems communities. He has helped build and scale many large, high-traffic services for Fortune 1000 clients. He has written several books, including O'Reilly's best-selling High Performance MySQL. Baron has a CS degree from the University of Virginia.

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Introduction

Making systems big, fast, and efficient is one of the most interesting and satisfying things I've done. It's a great feeling when you fix a bottleneck, and get dramatically improved performance at scale. You suddenly realize how wasteful your systems were before the improvement.

I've participated in lots of projects that have produced those kinds of outcomes. It's no coincidence that the best results came from the projects where the most disciplined analysis was performed up front. Like performance optimization, scalability optimization can be a real mystery unless you have an accurate model of how the world works, good measurements of how your systems are performing, and the ability to identify the problem and its likely fix with high certainty.

What's even better than fixing problems with scalability is the ability to design systems (and organizations) that scale in the first place. This is worth its weight in gold.

Scalability is quite a scientific topic, but it doesn't need to be mysterious. It's true that queueing theory is intimately involved, and queueing is complicated and unintuitive. But what's really cool is that scalability, correctly understood, is quite straightforward despite the complexity of what's going on behind the scenes.

I wrote this book to help you understand the simple, but profoundly powerful, truths about scalability. I also wanted to help you understand the connections between scalability and other disciplines, such as performance optimization or the study of queueing. My hope is that this book is as transformational and rewarding for you as the process of learning these concepts has been for me.

What is Scalability?

Scalability is ambiguous for many people—a vague term often bandied about in conference presentations, for example. It can be confusingly similar to performance, efficiency, capacity, availability, and many other terms related to making things big and fast.

Wikipedia's definition of scalability, borrowed from a 2000 paper by André B. Bondi, is "the capability of a system, network, or process to handle a growing amount of work, or its potential to be enlarged in order to accommodate that growth." This isn't wrong, but it's still a bit informal, and this book needs a more formal definition.

Dr. Neil J. Gunther provides one such definition: scalability is a *function*. I read his books and heard him speak, but it was still a year or so before I understood. *Scalability can be defined as a mathematical function*, a relationship between independent and dependent variables (input and output). This is the type of formal definition you need to model and analyze scalability.

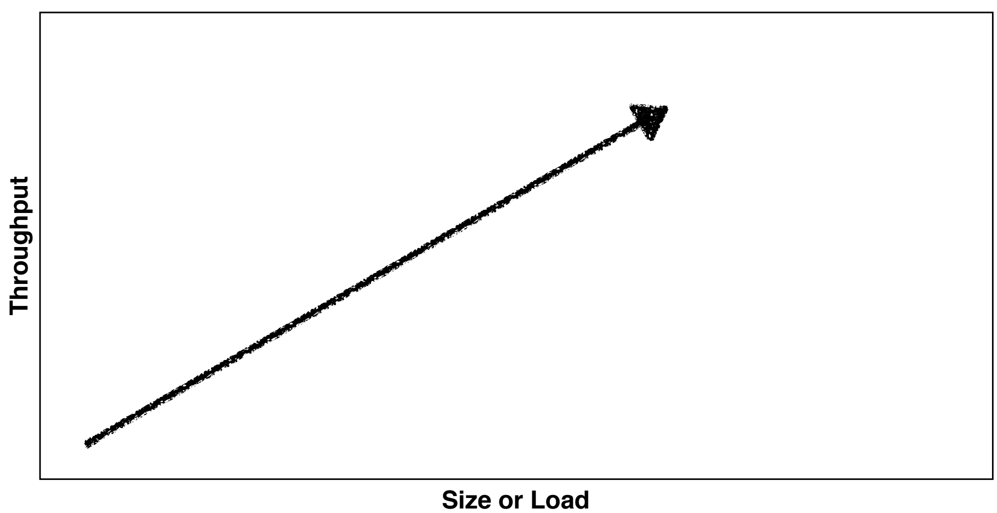
The most important part of understanding such a scalability model is choosing the correct variables to describe the way systems really operate. Bondi's definition provides a good clue: *work* is the driving factor of scalability. Useful ways to think about work include, to mention a few,

- Units of work (requests).
- The rate of requests over time (arrival rate).
- The number of units of work in a system at a time (concurrency).
- The number of customers or users sending requests.

Each of these can play sensible roles in the scalability function, depending on how you view it. For example, it's quite common to configure the number of threads a benchmark uses to send requests to a database. The benchmark usually sends requests as fast as possible with zero think time, so the arrival rate is related to[1] the benchmark configuration. You could say that the amount of work requested is the input to the benchmark's scalability function, and the completion rate is the output.

In another scenario, you might vary the number of CPUs for the system under test (SUT) while holding constant the load per CPU, or if it's a clustered database, vary the cluster size and hold constant the load per node. In this case, the independent variable is the system size and the dependent variable is the completion rate.

In most cases I've analyzed, either size or load is the sensible independent variable for the scalability function, meaning that scalability is *a function of size or load*. The dependent variable will be the rate at which the system can complete work, or *throughput*. The hope is that the system should complete more work as size or load grows, so it should be an increasing function.

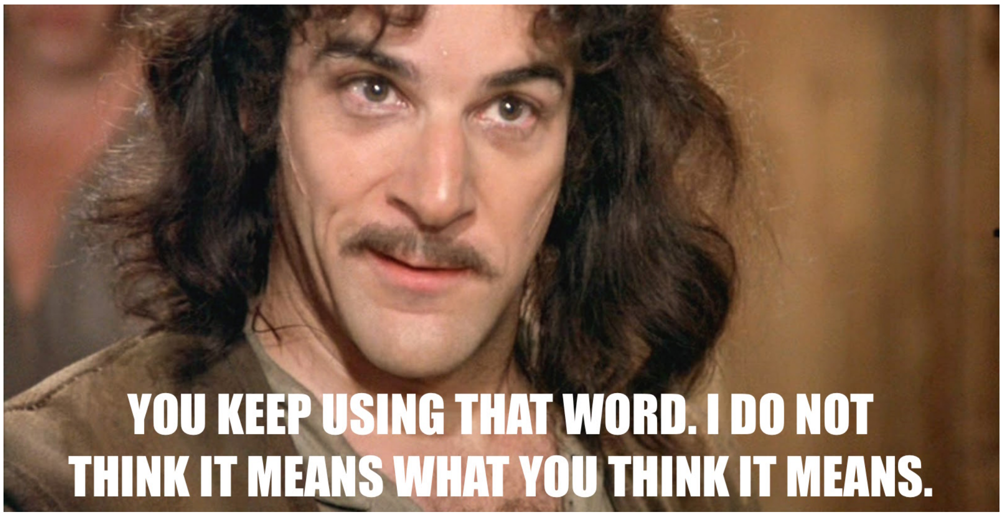


For those who are like me and need extra emphasis, I'll repeat that this is a mathematical function, with size or load on the X axis, and throughput on the Y axis. I'll make this more precise later.

[1] But not strictly controlled by, because it is determined by how quickly the database finishes each request.

Linear Scalability: The Holy Grail

In my experience, never was a marketechture slide deck created that mentions scalability without also including the word "linear." But as you might expect, "linear scalability" is not what people would have you believe.



Hand-waving claims of linear scaling usually coincide with vague definitions of scalability, and people who know a lot about scalability rarely say the word "linear." Here are a few of the misdefinitions of linear scalability I've heard:

- A web architect at a conference said, "I designed our system to be shared-nothing so it would be linearly scalable." He meant there was no single resource or system imposing a hard upper limit on how many servers could be added to the system. But he didn't really know whether his system actually scaled linearly.
- A technical evangelist giving a presentation about a clustered database said, "adding a node to the cluster adds a predictable amount of capacity." Predictable isn't the same as linear.
- A sales presentation for another clustered database said the

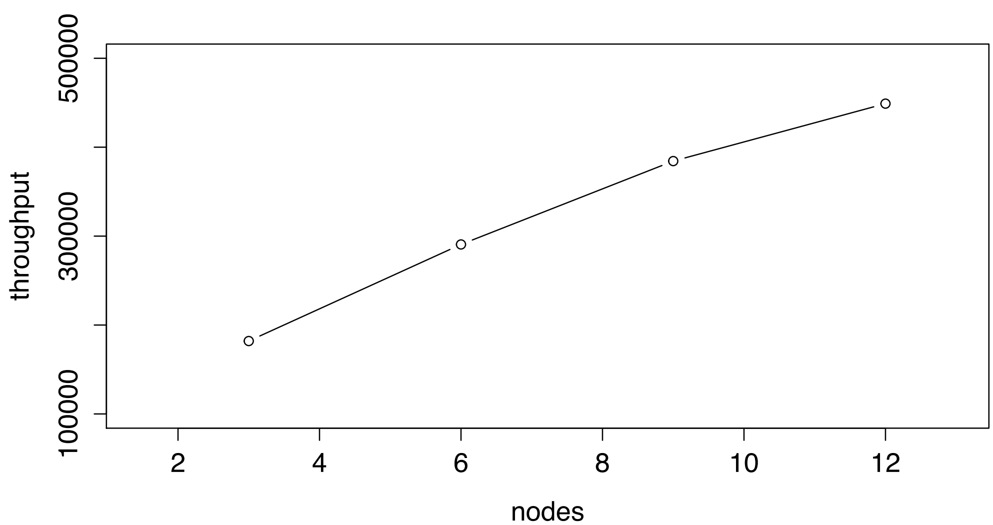
database scaled linearly, with a "linearity factor" of 97%, meaning that each additional node increased the system's capacity by 0.97 times the amount the previous node added. That's a curve, not a line. (Later you'll learn how to instantly determine the asymptotic upper bound on such a system's total capacity.)

This may seem like a pointless rant, but it's actually important if you want to be able to design and improve highly scalable systems.

Spotting bogus linearity claims is fun. Here are some ways "benchmarketing" makes systems appear linear:

- Show graphs without numbers, so readers can't do the math.
- Show graphs with nonlinear axes.
- Begin the axes, especially the Y axis, at a nonzero value.

Here is a real example, redacted to protect the not-so-innocent, that employs some of these tricks.



Looks pretty linear, doesn't it? Yet if you do the math, it's nowhere near linear. It's an optical illusion, because the X axis begins around 1.45

instead of zero, and the Y axis starts at 100000, so you can't tell that the chart isn't going to intersect the origin if you extend it downwards.

The real test of linearity is whether the transactions per second per node remains constant as the node count increases. The chart's original source mentioned that throughput increased from "182k transactions per second for 3 nodes to 449k for 12 nodes." The math is easy: the system achieves 60700 transactions per second per node at 3 nodes, but only 37400 at 12 nodes, which represents a *39% drop in throughput* versus linear scalability. If it actually scaled linearly, it would achieve 728k transactions per second at 12 nodes.

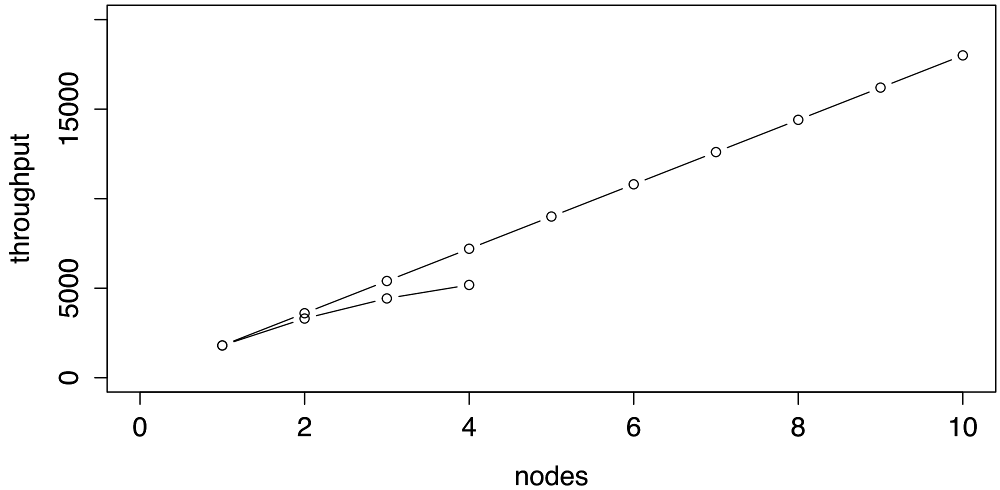
Linear means linear, folks! And seemingly small amounts of nonlinearity really matter, as you'll see later, because small sublinear effects grow very quickly at larger scale.[1]

Why Systems Scale Sublinearly

Linear scalability is the ideal, yet despite the claims, systems that actually scale linearly are rare. It's very useful to understand the reasons for this, because a correct understanding of scalability, and the reasons and sources of sublinear scaling, is the key to building more scalable systems. That's why it's important to be a linearity skeptic. It's not just being pedantic.

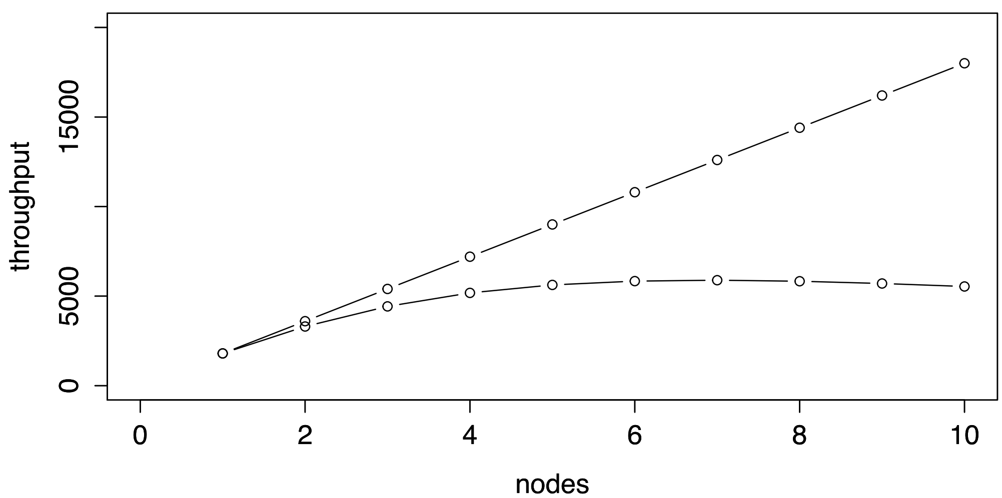
The best way to think about linearity is as a ratio of the system's performance at a size of 1. Neil Gunther calls this the *efficiency*. If a system produces 1800 transactions per second with 1 node, then ideally 4 nodes produce 7200 transactions per second. That would be 100% efficient. If the system loses a bit of efficiency with each node and 4 nodes produce, say, 5180 TPS, then the 4-node system is only 72% efficient:

1 In fact, they grow—wait for it—nonlinearly!



If you do this math, you'll often be surprised at how large the efficiency loss is. Graphs can be deceptive, but the numbers are quite clear.[1]

In the real world there's almost always some efficiency loss, and if you can figure out why, you may be able to fix it. In fact, you've probably noticed that real systems tend not only to fall behind linear scalability a bit, but actually exhibit *retrograde* scalability at some point:



This is quite common in the real world—you scale things up and at some point your system starts going backwards and *losing* performance, instead of just gaining more and more slowly. In the MySQL 5.0 days, for

1 Drawing a linear scaling line on the graph helps, too. Without that line, the eye tends to see the graph as more linear than it really is, and the efficiency loss becomes less obvious.

example, it was common to see people upgrading from 4-core servers to 8-core servers and losing performance.

Why does this happen? Why don't systems scale linearly, and why do they sometimes show retrograde scalability?

According to Neil Gunther, there are two reasons: **contention** and **crosstalk**. Contention degrades scalability because parts of the work can't be parallelized and queue up, so speedup is limited. Crosstalk introduces a coherency penalty as workers (threads, CPUs, etc) communicate to share and synchronize mutable state. I'll explore these effects in the next section.

The Universal Scalability Law

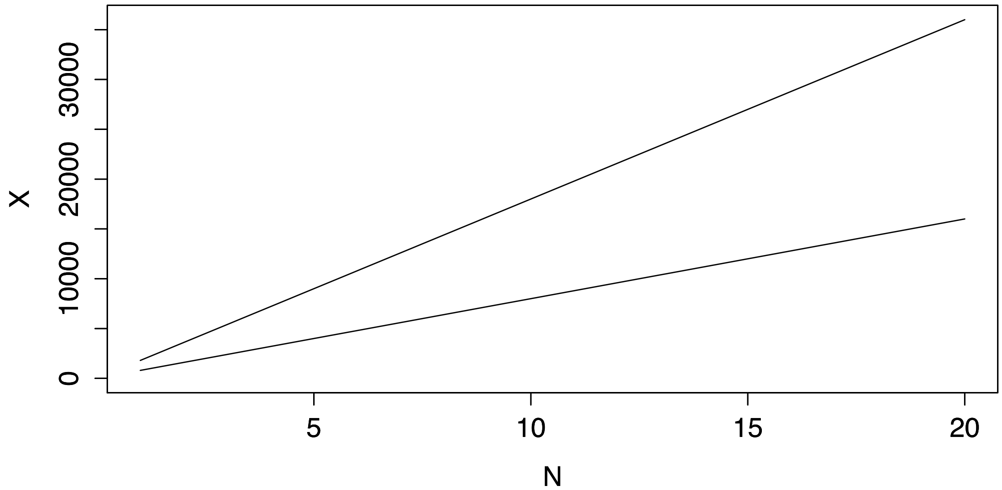
Neil Gunther's Universal Scalability Law (USL) provides a formal definition of scalability,[1] and a conceptual framework for understanding, evaluating, comparing, and improving scalability. It does this by modeling the effects of linear speedup, contention delay, and coherency delay due to crosstalk.

Let's see how this works, piece by piece. An ideal system of size 1 achieves some amount λ of throughput X, in completed requests per second. Because the system is ideal, the throughput doubles at size N=2, and so on. This is perfect linear scaling:

$$X(N) = \frac{\lambda N}{1} \tag{1}$$

The λ parameter defines the slope of the line. I call it the *coefficient of performance*. It's how fast the system performs in the special case when there's no contention or crosstalk penalty. Here are two ideal systems, with λ of 1800 and 800, respectively.

1 Neil Gunther originally called a slightly different form of the USL "superserial," and you may encounter this terminology, especially in older books and papers.



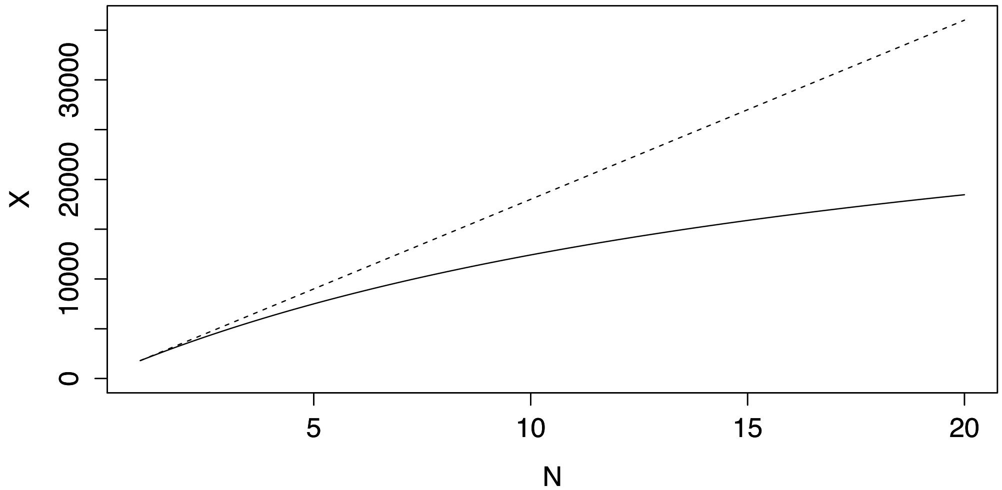
Note that every linearly scalable system is just as scalable as any other, regardless of the slope of the line. They have different performance but identical scalability characteristics: speedup is unlimited.

Contention appears in most systems[1] at some point, for example as a final stage of assembling the multiple outputs generated in parallel into a single final result. As parallelization increases, contention becomes the limiting factor. This is codified in Amdahl's Law, which states that the maximum speedup possible is the reciprocal of the serial fraction. If I add a term to the denominator expressing the serial fraction of the work, multiplied by σ, the coefficient of contention, it becomes Amdahl's Law:

$$X(N) = \frac{\lambda N}{1 + \sigma(N-1)} \tag{2}$$

A system with contention will asymptotically approach a ceiling on speedup. If σ is .05, for example, the speedup approaches 20. Let's see that graphically:

1 Including teams of people. This is a joke but it's also true in the queueing sense.



Remember the system I mentioned earlier, which a salesperson claimed to have 97% scalability with each additional node? That's 3% loss of scalability per node, so this system will never achieve a speedup factor of more than 33, no matter how many nodes it has.

The last bit is the crosstalk penalty, also called the consistency or coherency penalty.[1] Crosstalk potentially happens between each pair of workers in the system (threads, CPUs, servers, etc). You probably remember that the number of edges in a fully connected graph is $n(n-1)$. The USL represents the amount of crosstalk with another term, multiplied by κ, the coefficient of crosstalk:

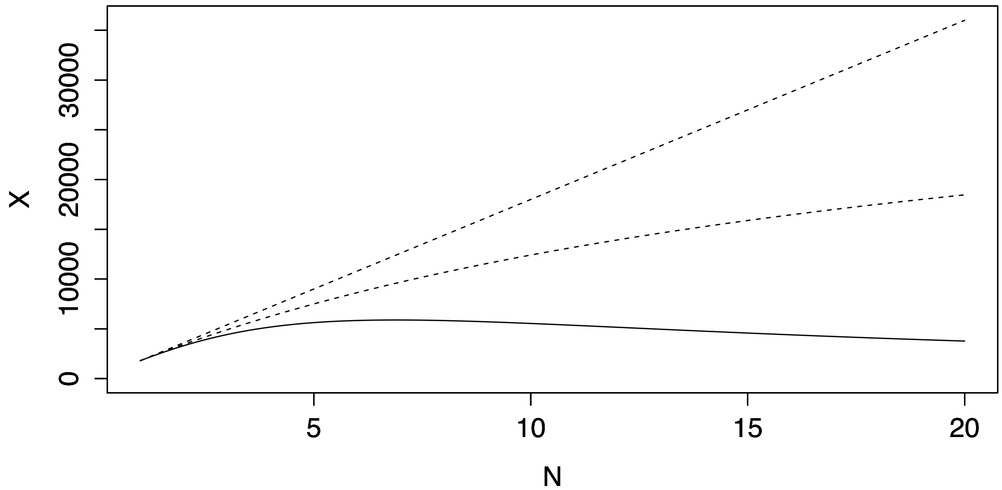
$$X(N) = \frac{\lambda N}{1 + \sigma(N-1) + \kappa N(N-1)} \tag{3}$$

Equation 3 is the Universal Scalability Law.

The crosstalk penalty grows fast. Because it's quadratic,[2] eventually it grows faster than the linear speedup of the ideal system we started with, no matter how small κ is. That's what makes retrograde scalability happen, as you can see in the following chart:

1 I call it crosstalk because in my opinion it's the best description of the pairwise communication that must occur to make distributed data or other shared resources consistent or coherent.

2 The cost of $n(n-1)$ is $\mathcal{O}(n^2)$. If you're not familiar with it, this blog post introduces Big-O notation.



That's the Universal Scalability Law in all its glory. This plot has the same parameters as the ones I showed before, where a system of size 4 produced only 72% of its ideal output. That system has 5% contention and 2% crosstalk, and now that I've plotted it out to size 20 you can see it's embarrassingly inefficient. In fact, I should have given up trying to scale this system after size 6 or so.

This shows visually how much harm a "small amount" of nonlinearity can do in the long run. Even very small amounts of these damaging coefficients will create this effect sooner or later (mostly sooner). This is why it's rare to find clustered systems that scale well beyond a couple dozen nodes or so. If you'd like to experiment with this interactively, I've made a graph of it at Desmos.

The USL's Relationship to Queueing Theory

The USL is closely related to queueing theory. Neil Gunther proved that it's equivalent to synchronous repairman queueing. If you're not familiar with queueing theory, I wrote an approachable introduction called Everything You Need To Know About Queueing Theory.

The two causes of sublinearity have important relationships to queueing theory. Contention, the first term I added to the denominator to obtain Amdahl's Law, expresses the penalty from queueing delay that occurs when there is competition for shared resources—the servers (in the queueing theory sense) that process work from queues.

The queue length is *nonlinear* with respect to utilization and therefore to offered load. Queueing theory is confusing and counterintuitive! As the queues lengthen, the queueing delay lengthens in direct proportion.

As you probably know, queueing theory treats service time—the amount of time it takes to complete a job after it leaves the queue and enters service—as independent of utilization or queueing. The job takes as long as needed to execute all the instructions, whether the server is busy or idle. The customer's total wait time at a busy server is longer only because of queueing delay.

Coherency penalty, which comes from crosstalk, actually expresses an *increase in service time* that is not due to queueing delay. As the system has to do more crosstalk to synchronize mutable shared state, the jobs take longer and longer.[1] This is *not* due to queueing—the job is already out of the queue and in service.

These effects are clearly visible in the USL when it enters the region of retrograde scalability. An increase in service time is the only thing that can explain retrograde scalability. If the service time remained constant, throughput would approach a flat line. Queue length and queue wait time cannot explain retrograde scalability; queueing can only cap throughput, not decrease it.

1 An alternative explanation is that the cost of managing the queues actually increases as they grow. My personal experience leads me to believe it's typically crosstalk in multi-node clusters, and it's more often queue management in single-server systems under high load. For examples, see John D.C. Little's paper *Little's Law as Viewed on Its 50th Anniversary*, http://on.fb.me/1NmVeGM, http://bit.ly/1MtQdqM, and http://bit.ly/1NNE0k4.

Measuring Scalability

To recap, at this point we've figured out the right dimensions for a formal model of scalability that seems to behave as we know real systems behave, and examined Neil Gunther's USL, which fits that framework well and gives us an equation for scalability. (Are you excited yet?)

Now what can you do with it?

Great question! It turns out you can do a lot of extremely useful things with the USL. Unlike a lot of models of system behavior, this one is actually practical to apply in the real world. That's the real genius of it, in fact. Not only is the equation uncomplicated, but the variables it describes are easy to get most of the time.

I use the USL mostly for modeling system scalability, by working backwards from observed system behavior and estimating the likely coefficients. To accomplish this, you need a set of measurements of the system's load or size (usually concurrency or node count) and the corresponding throughput. Then you *fit* the USL to this dataset, using nonlinear least squares regression. This is a statistical technique that finds the optimal coefficient values in order to calculate a best-fit line through the measurements. The result is values for λ, σ, and κ.

If you're reading about the USL in Neil Gunther's books, he takes a different approach. First, he doesn't use regression to determine λ, he assumes that you can measure it in a controlled way at $N = 1$. (I've often found that's not true for me.) Secondly, there are a couple of different forms of the USL—one for hardware scaling and one for software scaling—which are the same equation, but with different parameters. For simplicity I'm treating them as interchangeable. I will write a bit more about this later.

Examples of systems I've analyzed with the USL include:

- Black-box analysis of networked software simply by observing and correlating packet arrivals and departures, looking at the IP addresses, port numbers, and timestamps. From this I computed the concurrency by averaging the amount of time the system was busy servicing requests over periods of time. The throughput was straightforward to get by counting packet departures.
- MySQL database servers. Some of its `SHOW STATUS` counters are essentially equivalent to throughput and concurrency.
- Linux block devices (disks) by looking at `/proc/diskstats`, from which you can get both instantaneous and average concurrency over time deltas, as well as throughput (number of I/Os completed).
- Lots and lots—and lots—of benchmark results.

I've built a variety of tools to help clean, resample, and analyze the data before arriving at satisfactory results. Most of them were commandline, though these days I use R more than anything else. This is an important topic: you will get dirty data, and that will make your results less useful. You need to visualize both in scatterplot form as well as in time-series form and ensure you're working with a relatively consistent set of data. You can remove individual points or trim the time range you use, and you may need to experiment with averaging the data over time to get good results.

As for the R code, I'll give a little bit of a quickstart to show the soup-to-nuts approach. You'll save the data into a delimited file, with column headers `size` and `tput`. Then you'll load this into a variable in R and regress it against the USL.

Here's a complete sample, based on a benchmark that Vadim Tkachenko ran on a Cisco server:

```
size tput
1 955.16
2 1878.91
3 2688.01
4 3548.68
5 4315.54
6 5130.43
7 5931.37
8 6531.08
9 7219.8
10 7867.61
11 8278.71
12 8646.7
13 9047.84
14 9426.55
15 9645.37
16 9897.24
17 10097.6
18 10240.5
19 10532.39
20 10798.52
21 11151.43
22 11518.63
23 11806
24 12089.37
25 12075.41
26 12177.29
27 12211.41
28 12158.93
29 12155.27
30 12118.04
31 12140.4
32 12074.39
```

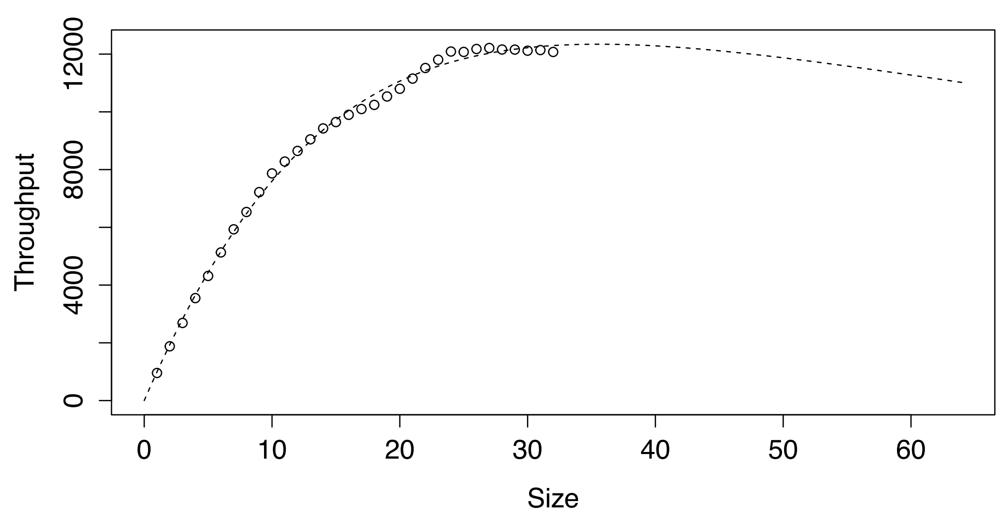
Save that data into a file, say, `benchmark.txt`. Then load it and run the following commands:

```
benchmark <- read.csv("/path/to/benchmark.txt", sep="")
usl <- nls(tput ~ lambda*size/(1 + sigma * (size-1) + kappa * size *
   (size-1)), benchmark, start=c(sigma=0.1, kappa=0.01, lambda=1000))
summary(usl)
sigma <- coef(usl)['sigma']
kappa <- coef(usl)['kappa']
lambda <- coef(usl)['lambda']
u=function(x){y=x*lambda/(1+sigma*(x-1)+kappa*x*(x-1))}
plot(u, 0, max(benchmark$size)*2, xlab="Size", ylab="Throughput", lty="dashed")
points(benchmark$size, benchmark$tput)
```

The results are as follows:

$$\begin{array}{ll} \lambda & 995.6486 \\ \sigma & 0.02671591 \\ \kappa & 0.0007690945 \end{array}$$

Note the extremely small value for κ which nonetheless degrades scalability before N becomes very large. Here's the resulting plot:



If you're an R user, that's probably all you need to get going. You really should do more diligence, such as checking the R^2 value of the fit. But instead of doing all this work manually (which you can certainly do if you want), I suggest trying the USL package from CRAN. It has many features built in, although it does have some limitations.

One final thing: if the κ coefficient has a nonzero value, the function has a maximum. You can find the size of the system at that maximum as follows:

$$N_{max} = \left\lfloor \sqrt{\frac{1-\sigma}{\kappa}} \right\rfloor \tag{4}$$

You can find the maximum predicted throughput by plugging N_{max} into Equation 3. Doing so with the coefficients in this example predicts the system's throughput will increase until $N = 35$, which in this case means 35 threads, and the peak throughput will be 12341 queries per second. It also found λ, the throughput at $N = 1$, to be 995 QPS, which is close to the actual value of 955.

It's always interesting to use the USL on a subset of the performance data, such as the first third or so, to see how well it predicts the higher N values. This can be quite educational.

Note that you should have at least half a dozen or so data points in order to get good results in most circumstances. In practice I usually try to capture at least a dozen for benchmarks, and more—often thousands—when analyzing systems that aren't in a controlled laboratory setting.

Modeling Response Time

What is the relationship between scalability and performance? Throughput and latency are two common ways to describe and measure performance. Most benchmarks measure overall system throughput, and claims of performance are almost always in throughput terms: “a million transactions per second,” and so on. Benchmarks usually define performance as how much work the system can do.

On the other hand, users care mostly about the performance of individual requests. For example, a user’s opinion of website performance is based entirely on how quickly pages load and render. From this viewpoint, as Cary Millsap says, *performance is response time*.

Which view is right? Both. System performance is measured in throughput, and request performance is measured in latency.[1] And because throughput and latency are related, scalability also has dual meanings: the system’s ability to complete more work at larger sizes, and its response time characteristics.

I’ve shown that the USL can model and forecast how size affects throughput. Can it also model latency? Yes, it can when the independent variable is concurrency, because of a relationship called Little’s Law:

$$N = XR \tag{5}$$

Little’s Law says that the mean number of requests resident in a system is equal to the throughput times the mean response time. This relationship is valid for stable systems, in which all requests eventually complete.

If you use Little’s Law to solve the USL for response time as a function of concurrency, the result is a quadratic function:

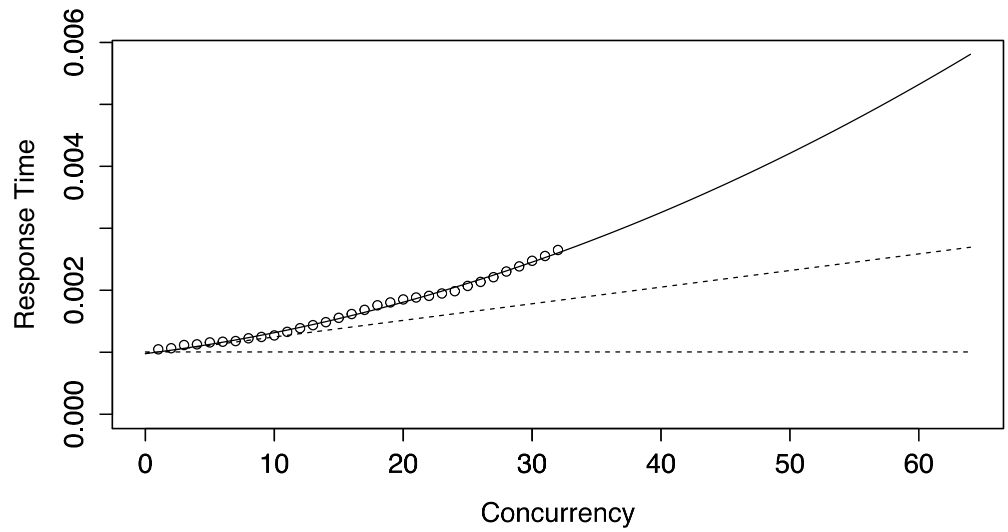
$$R(N) = \frac{1 + \sigma(N-1) + \kappa N(N-1)}{\lambda} \tag{6}$$

This means that response time[2] is related to the square of concurrency.

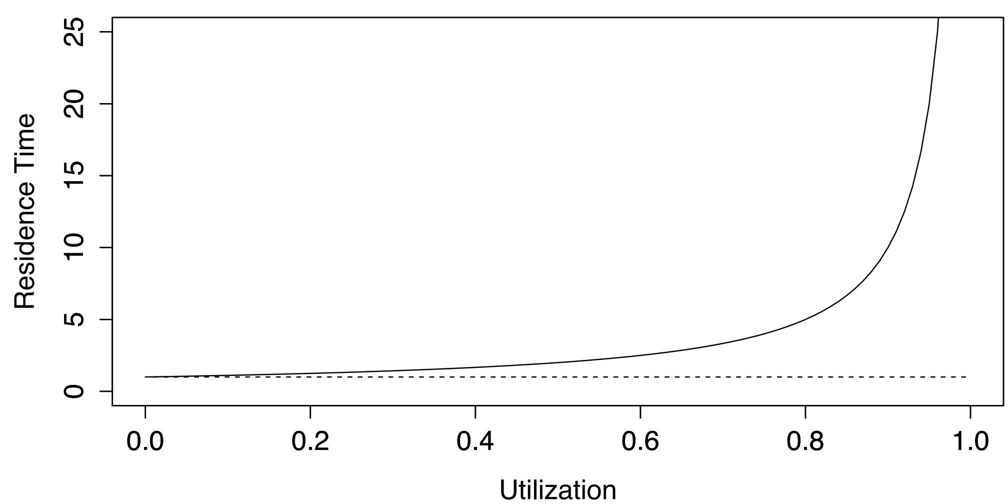
1 Response time, latency, and residence time are synonymous.

2 *Mean* response time. You could use queueing theory to add nuance, such as quantiles or the probability that any given request needed to wait more than a set amount of time.

Of course, just as with the USL, if the σ or κ coefficients are zero, the equation is simplified, removing the causes of nonlinear behavior. One of the nice things about Equation 6 is that you can now see the effects of contention (linearly increasing queueing delay) and coherency (quadratically increasing service time) on residence time. Here's the same data I used previously, rearranged to predict response time. The dashed lines show linear and Amdahl response time scaling for the same λ:



Be careful not to confuse this chart with another famous "hockey stick" chart, that of response time versus utilization, which is familiar from queueing theory:



The difference is that one chart uses utilization as the independent variable, which ranges only from 0 to 1, whereas the other uses concurrency, which has no fixed upper limit.[1]

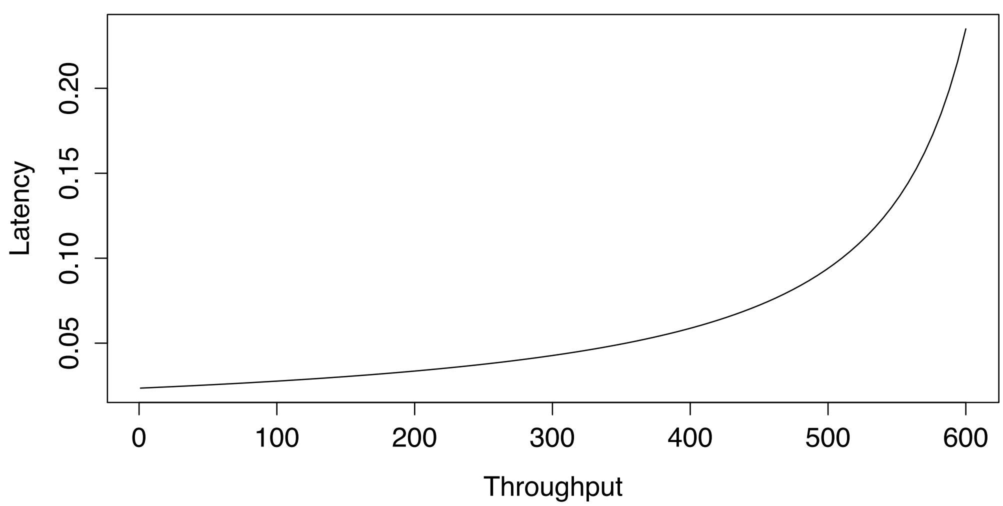
Little's Law also lets you rearrange the USL in terms of the relationship between throughput and latency, which is useful for a few reasons. It lets you model scalability and performance in these dimensions, it helps build intuition about what happens when systems don't scale linearly, and it relates the USL to queueing theory visually in an important way. I'll begin by showing response time as a function of throughput, which is a common way I've seen people (and vendors) plot it. Beginning with a linearly scalable system,

$$R(X) = \frac{1}{\lambda}$$

Response time is constant. Adding a positive σ coefficient[2] makes it an exponential function:

$$R(X) = \frac{\sigma - 1}{\sigma X - \lambda} \tag{7}$$

Response time goes to infinity as throughput approaches λ/σ. The following plots are generated with the parameters $\sigma = \kappa = .06, \lambda = 40$:

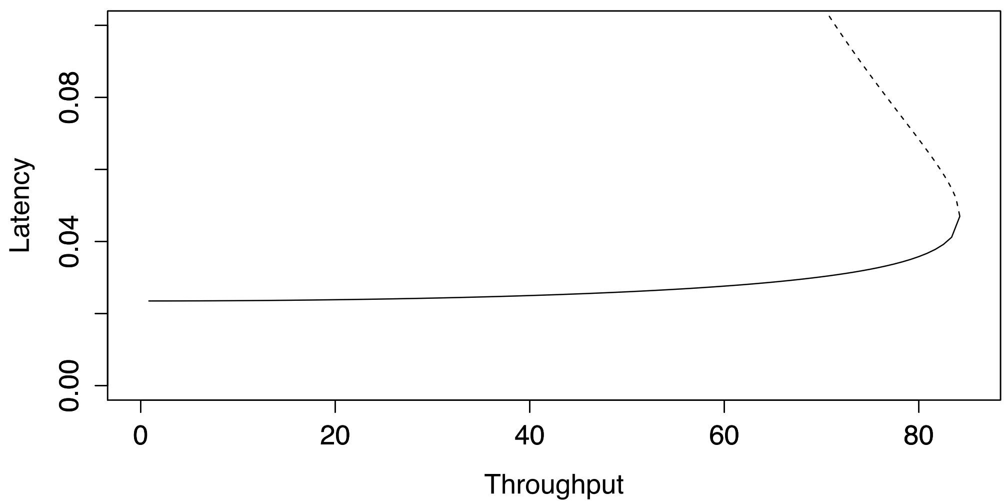


1 The relationship between concurrency and utilization is nonlinear; according to the USL, it is quadratic.
2 I am omitting step-by-step solutions for brevity, but Wolfram Alpha can show this if you're curious. The easiest way is to solve the USL for R as a function of X and simplify.

The response time curve's relationship to the queueing theory response time curve (which is in terms of ρ) is more obvious now. What happens when you add coherency to the equation? The equation has multiple solutions. Instead of merely lifting away from the response-time line, it folds back on itself like a "nose." The solution for the lower portion of the curve is as follows:

$$R(X) = \frac{-\sqrt{X^2(\kappa^2 + 2\kappa(\sigma - 2) + \sigma^2) + 2\lambda X(\kappa - \sigma) + \lambda^2} + \kappa X + \lambda - \sigma X}{2\kappa X^2} \tag{8}$$

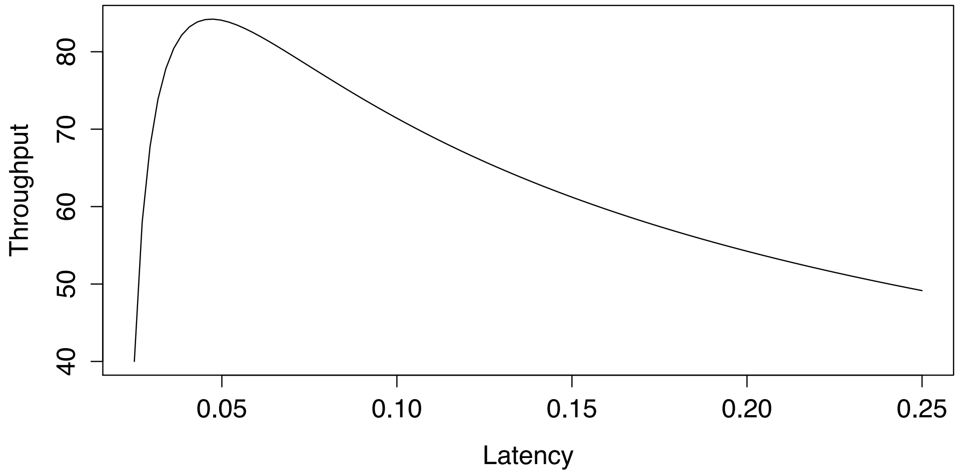
The other solution is simply the negative. Here's a plot of both:



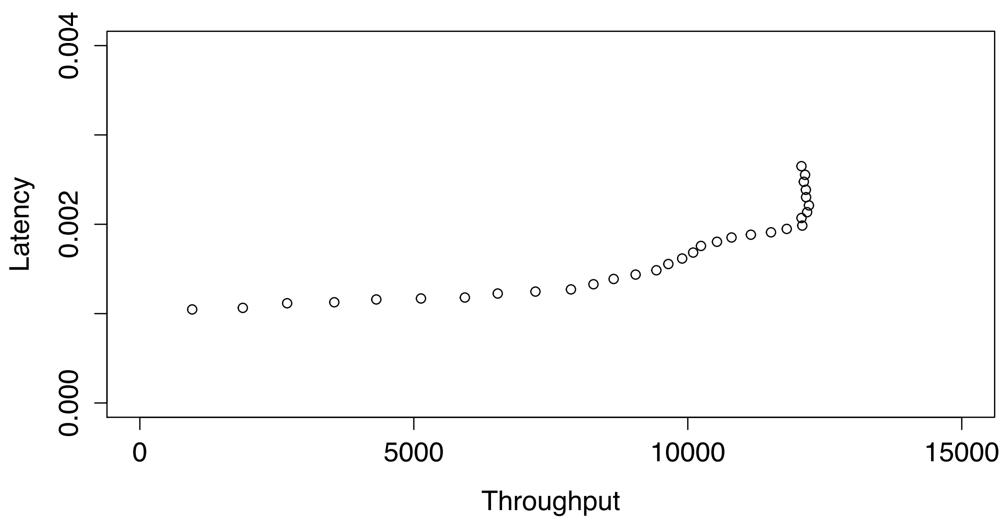
As you can see, this is not a simple function of X, since a single X value can map to two values of R due to the multiple solutions. *Response time is not a simple function of throughput if κ is nonzero.* The inverse is actually true—given a target latency, you can uniquely identify the throughput at which it will occur:

$$X(R) = \frac{\sqrt{\sigma^2 + \kappa^2 + 2\kappa(2\lambda R + \sigma - 2)} - \kappa + \sigma}{2\kappa R} \tag{9}$$

Here's a plot of that:

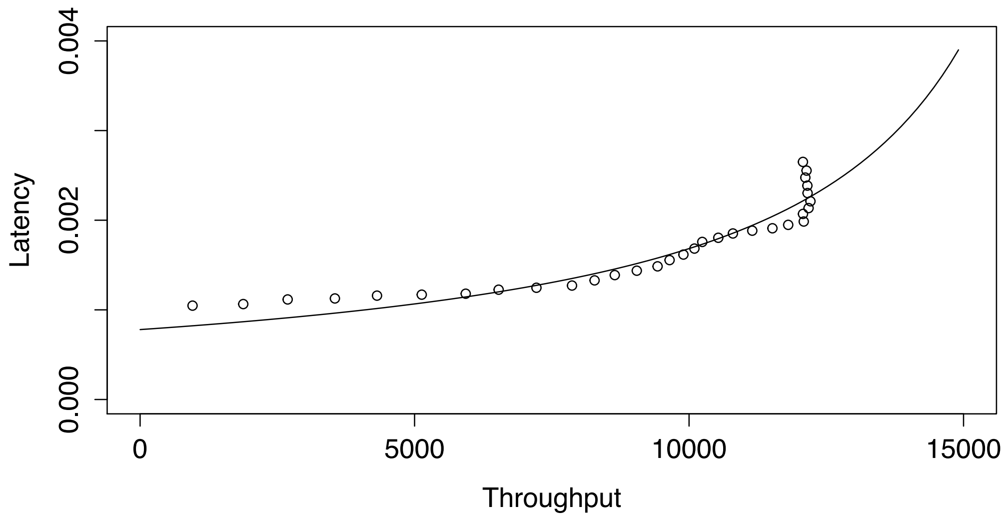


The maximum throughput is the same as in Equation 3 with the same parameters; it's equivalent to N_{max} from Equation 4, but in latency instead of concurrency. You can use these equations to model scalability and performance in terms of latency, just as the USL models it in terms of concurrency or size.[1] Referencing the Cisco benchmark again, a scatterplot shows that the system has exceeded the point of diminishing returns and is climbing up the nose:

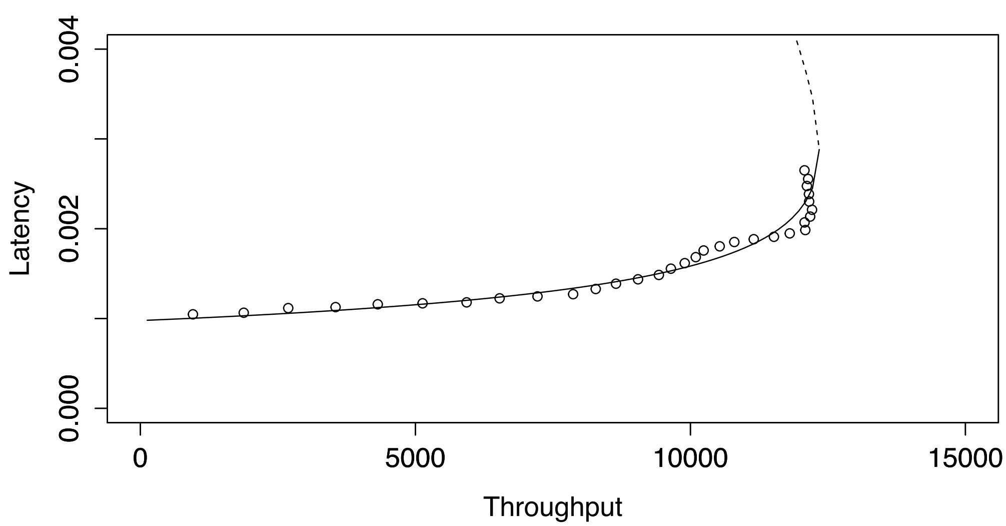


1 Because of the folded-back nature of the nose curve, it's difficult to use regression to estimate the USL parameters in this form. It's easy to use Little's Law to compute concurrency, though, and just perform the regression against that instead.

I'll repeat that this isn't a simple function of throughput alone. That is why you can't model it with a curve such as a parabola or an exponential function. The lower portion of the nose might *look like* an exponential curve, but it isn't. It's quite different. Here's what happens if you try to fit an exponential function, such as Equation 7, through the benchmark's results.



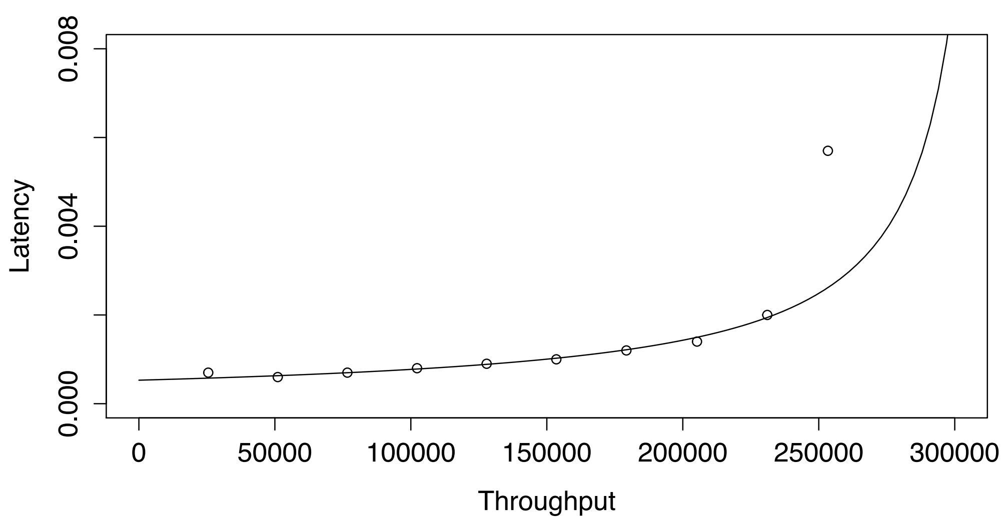
That dramatically underestimates how fast latency grows. Using Equation 9 is a better fit:



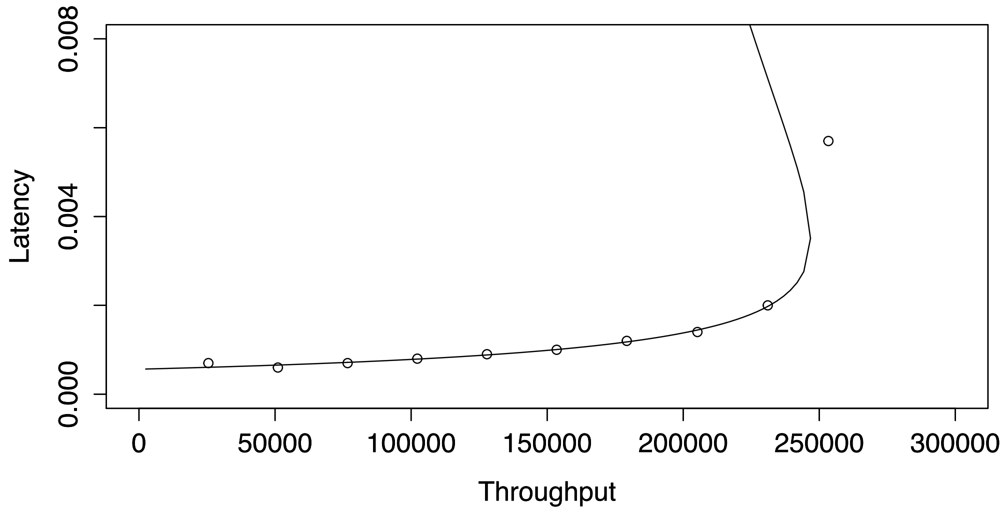
Another example of this apparently beginning to happen is the SPEC benchmark for NFS on the Isilon S210, which requires reporting latency as a function of throughput. Here are the results:

Throughput (ops/sec)	**Response (msec)**
25504	0.7
51054	0.6
76667	0.7
102288	0.8
127879	0.9
153497	1.0
179261	1.2
205226	1.4
231069	2.0
253357	5.7

Let's pretend that we didn't see the last row in that table. If you thought response time was an exponential function of throughput, your curve would appear to "fit" very well. What would you predict would happen at 253,357 ops/sec? You'd expect latency to be much lower than it actually is. Here's the curve, fit to all but the last point, overlaid with all the measurements:



Using the USL will model the effect more closely, but note that the USL appears to be a bit pessimistic, a topic we'll return to later:



Most people probably never will see a system curve around the tip of the nose and start climbing backwards “in the wild,” because the system’s performance becomes extremely bad. Often this doesn’t happen in production because the load-generating systems (and users) are finite, experience back pressure, and can’t continue adding work to the system you’re measuring. The only way to drive a system past the nose is to increase concurrency beyond the system’s saturation point. Another way to look at this is that the request *arrival rate* is a free parameter you can choose at will, but *completion rate* (throughput) isn’t; the system can’t produce more than its capacity, no matter how many requests arrive. If arrivals are constrained by back pressure you won’t see the nose.

If you won’t see it in production, then where will you? Easy—in benchmarks, where you can fire up lots of driver threads. I’ve seen it in many benchmarks. So has John Little; his paper Little’s Law as Viewed on Its 50th Anniversary shows a plot of this phenomenon on page 544.

To sum up: even though production systems often don’t climb around the tip of the nose, the nose’s precise equation matters a lot. That’s because if you want to model how response time behaves as it *approaches* the nose, you need the right model. With the wrong model you’ll be far off when doing things like capacity planning, which coincidentally is the next topic I want to cover.

Capacity Planning with the USL

"How much load can this system sustain?" is a common question in capacity planning. The practical purpose is usually something like the following:

- How soon will the system begin to perform badly as load increases?
- How many servers will I need for the expected holiday load?
- Is this system close to a point of failure?
- Are we overprovisioned? By how much?

Capacity planning is often a difficult problem because it's hard to tell what a system's true capacity is. The USL can help you estimate this.

Conventional ways to determine system capacity are often difficult, expensive, and don't give results you can really believe in. For example, you can set up load tests, but it takes a lot of work and time, and the results are suspect because the workload is always artificial in some way.

You can also run benchmarks, but most benchmarks are pretty useless for predicting a system's usable capacity. In addition to being an artificial workload, they push a system to its maximum throughput and beyond. As I mentioned, it's rare for benchmarks to be run by people who understand the importance of latency. But when I do see benchmarks that measure latency percentiles, the systems almost always perform very badly at their peak throughput.[1]

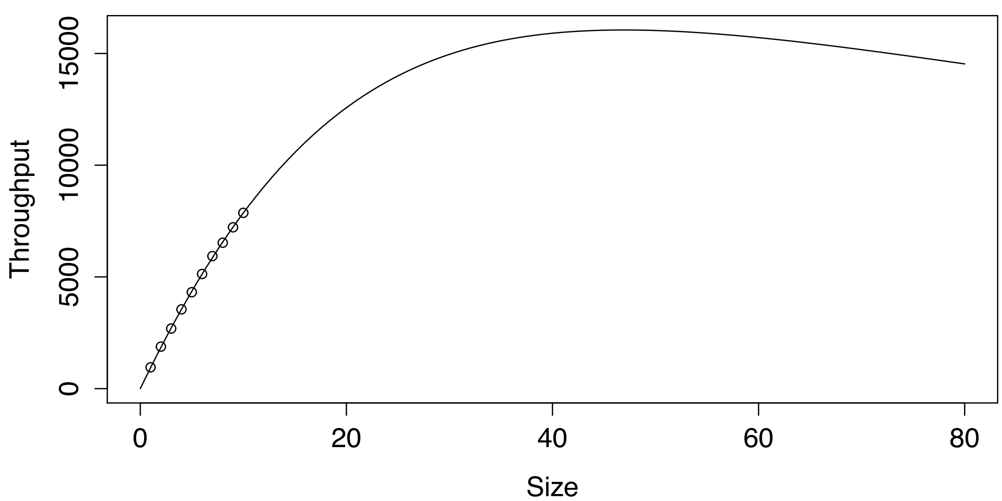
Another way I've tried to predict system capacity in the past is with queueing theory, using the Erlang C formula to predict response time at a

1 This is a problem with the way benchmarks are usually designed, in my opinion. I'd really prefer for the benchmark system to be intelligent enough to back off and reduce pressure on the system under test if it violates a service level objective (SLO). Ideally, this is defined as a quantile, such as 99th percentile latency less than 10ms. A smart benchmark would throttle load, eventually finding a longterm stable arrival rate at which the SUT can consistently perform well.

given utilization. Unfortunately, this requires that you know service times, which are often impossible to obtain. You can measure total response time, but that includes waiting time in the queue, so it's not the same thing as the service time. The utilization is also often deceptive, because the real utilization of the resources you're trying to model can be difficult to measure correctly too. Most people I know consider the Erlang approach to be difficult to apply.

If load tests, benchmarks, and queueing theory are difficult to use, can the USL help? Yes, it can. Because the USL is a *model*, it can help you predict how a system will perform under load beyond what you can observe. The USL's point of maximum predicts the system's maximum throughput, so it's a way to assess a system's capacity. It can help you get a better idea of how close you are to the system's maximum capacity.

Here's an example. Imagine that I had measured the first 10 data points in the Cisco benchmark, in a live production environment serving real users, not a lab. Here's the result of fitting the USL to the data:



Using the formula $N_{max} = \sqrt{(1-\sigma)/\kappa}$ from Equation 4, the USL predicts a max of 16,049 queries per second at a concurrency of 46 threads.

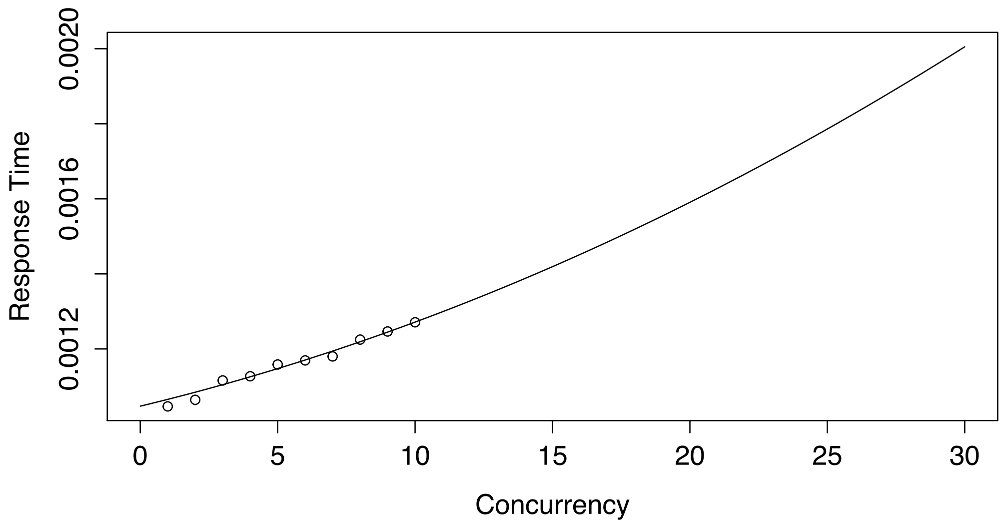
I have a rule of thumb for using the USL to project out into the unknown. I've seen so many systems that appear to be scaling beautifully—fitting the USL cleanly, just as this one does—and then they hit rough waters, that I don't trust anything farther out than twice the measured throughput or twice the measured size, whichever comes first.[1] And that's if I'm not seeing telltale signs of leveling off or retrograde throughput. If I see those signs, I lower my expectations accordingly. I will also include other information such as CPU utilization to guide my estimates, if I have it, but in the absence of more data this is a good way to keep expectations capped.

Back to the model: I have measurements only to $N = 10$, where observed throughput is 7,867, so I'm going to compute the predicted throughput at $N = 20$. The result is a throughput forecast of 12,572. This is less than twice my maximum observed throughput, so I'll allow it. In my experience, it's an optimistic but not unrealistic guess that I won't get more than about 12,500 queries per second from this server. (As you may remember, this system topped out at 12,211 QPS.)

The outcome is that my system appears to be operating at about half of its maximum capacity. However, as discussed previously, maximum throughput isn't maximum *usable* capacity. Again, when the system is at its maximum throughput, response time is probably terrible, and will be extremely inconsistent. That's why it's more important to focus on the system's maximum throughput within the constraints of a service level objective.

As a first step towards this, I can use average latency to help understand the potential QoS end-users will get from this server. Again using the rearranged form of the USL, I obtain the following response time forecast:

1 When I explain superlinear scaling, you'll see why I don't use throughput alone as a guide.



Using the estimated coefficients and the formula for response time from Equation 6, I can predict a mean response time of 0.00159 seconds at 20 threads. Let's imagine that this is unacceptable; I need mean response times to be 1.5ms or less. Solving the response time equation as a function of latency lets me use it to compute the maximum usable concurrency. The resulting equation has two roots; I'm only interested in the positive one:

$$N(R) = \frac{\kappa - \sigma + \sqrt{\sigma^2 + \kappa^2 + 2\kappa(2\lambda R + \sigma - 2)}}{2\kappa} \tag{10}$$

Plugging in an R target of 0.0015 yields N = 17, so if I want to avoid violating my SLO I can't drive my server higher than approximately 11,450 QPS. I'm actually at about two-thirds of my usable capacity, not half. I can grow traffic about 150% before I get into trouble, if I'm lucky.

This process is something like what I might use if I were encountering this server in the wild. It's not perfect; as Niels Bohr said, "It's hard to make predictions, especially about the future." Despite the uncertainty that remains, this approach is much better than staring at a chart and thinking, "I don't know, it looks like it's scaling linearly and CPU utilization is only 10%, so I guess we have a lot of headroom?" You usually have less headroom than you think, because of how nonlinearly throughput and latency degrade.

The USL has a few nice properties that make it suitable for this type of capacity planning:

- It's a "black box" technique, which uses data that's usually easy to get.
- Gathering data and using regression to analyze it is also easy.
- The USL is a relatively simple model, so people like me can understand the math.
- The USL is highly intuitive in comparison to most other approaches.

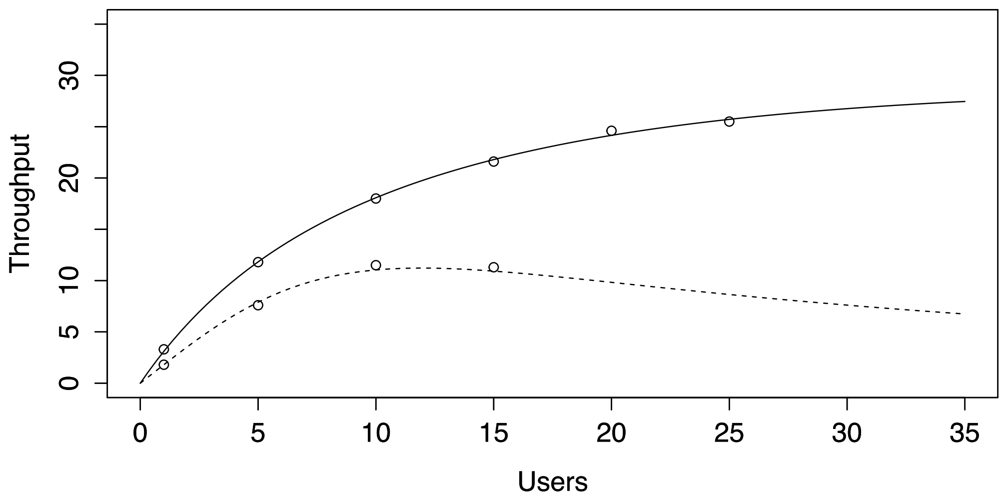
I would just repeat my caution that a lot of systems perform worse than the USL predicts they will, because their degradation in scalability at larger sizes is more severe than predicted. This is why I suggest viewing the USL's prediction as optimistic: "I won't count on being able to scale this system as high as the USL predicts I can."

You can also combine the USL with selected techniques from queueing theory, such as the Square Root Staffing Rule, to forecast how much capacity is needed and what quality of service it will provide. See the aforementioned queueing theory book for more on this topic.

Using the USL to Improve Scalability

One of the best uses of the USL is to explain *why* a system doesn't scale as well as it might. Armed with this knowledge, you can get clues about where to look for bottlenecks, so you might be able to alleviate them and improve the system's scalability. With practice, you'll also develop a mindset of scalability, building intuition about which design decisions can cause serious sublinearity.

An example will help illustrate. A few years ago PayPal published benchmark results of a Java application they rewrote in NodeJS. I analyzed their benchmark results and wrote about it on the VividCortex blog. Here are the plots and the key scalability parameters:



	σ	κ
Java	0.000011	0.006323
NodeJS	0.080319	0.000222

These systems scale very differently,[1] and for very different reasons. In a nutshell, the Java benchmark shows much higher crosstalk penalty, whereas the NodeJS benchmark exhibits more contention from queueing and serialization. Examining the architectures of the two systems reveals why: the Java app is multi-threaded and NodeJS is single-threaded with an event loop, and the PayPal blog post even mentions that they used “a single core for the NodeJS application compared to five cores in Java.”

This is a great real-life example of key scalability tenets:

- Avoid serialization and queueing; make things as parallel as possible.
- Avoid crosstalk and synchronization.

If you’re using the USL to model and analyze system scalability, another valuable practice is to approach the USL as a pessimistic scenario.

[1] Both of them scale pretty poorly, in fact.

Synchronous repairman queueing, the basis of the USL, is actually a worst-case in terms of the amount of queueing delay that occurs in a system. This is another way of saying that well-built systems theoretically ought to scale *at least* as well as the USL predicts. This should prompt you to ask the question, “why is this system degrading more than it should?” The answer is to look at whether the system degrades because of contention (queueing, serialization) or crosstalk (synchronization, communication, pairwise data interchange). If you can identify the likely cause, you might suspect that you need to look at mutex contention, for example.

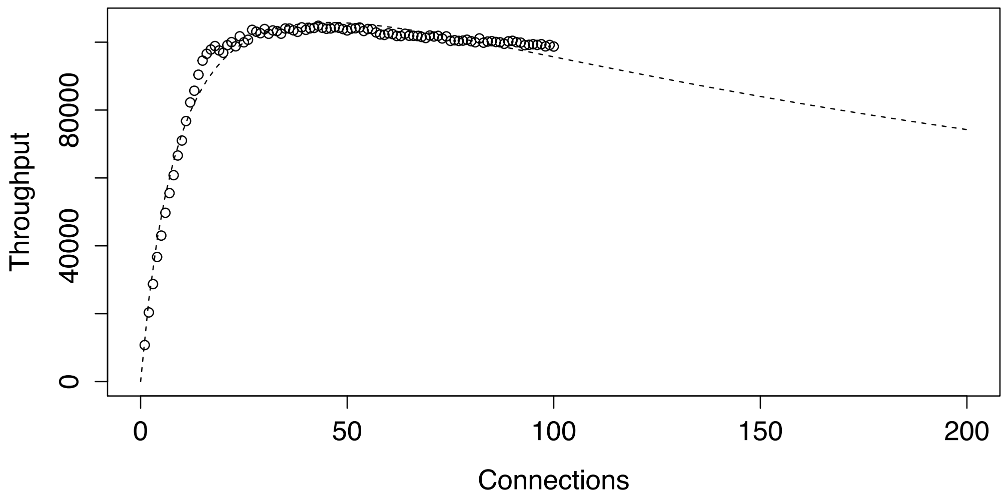
One of the biggest changes in my mindset since learning to use the USL is that it’s possible, and important, to explain why systems behave as they do. As Neil Gunther tweeted, it’s no longer enough just to benchmark, measure, and present the results on a chart. You can and should explain the results. As I tweeted myself, “Benchmarking is good, publishing results is better, explaining results is best. Publishing charts only (pictures without numbers) is sad.”

Thinking Critically About The USL

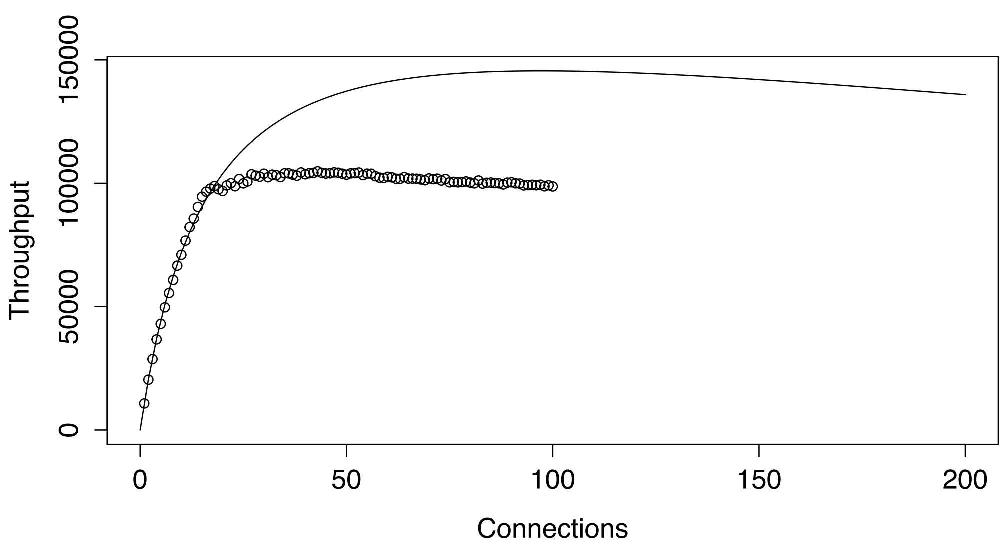
For many people, the USL is a huge shift in mindset. I know it was for me. But is it the be-all and end-all? Of course not.

Not only is the USL not the answer for every problem related to scalability or capacity planning, sometimes it’s hard to apply for the problems it *can* solve. The examples I’ve shown thus far are remarkable in that they’re extremely clean data. Real-world systems often have noisy data that doesn’t “look like” the USL much at all. It may look more like something your cat threw up on the screen. Even when there is a nice-looking shape to the data, regression can produce unphysical results, or refuse to produce results.

And then there are the systems for which you have nice, highly reproducible, clean-room measurements, but they just don't seem to fit the USL very well. They might appear to scale nicely, following the USL, but suddenly flatline instead of continuing a graceful curve. Or they abruptly degrade faster than predicted—or the reverse, appearing to degrade more *slowly* than predicted once you enter the region where the higher-order terms prevail. Here's an example:



Maybe this is queue saturation or resource saturation, or maybe it's something else, but the USL can't model or predict it. If you'd tried to predict this behavior by fitting the USL to the first dozen or so data points, you'd get something like this:



The USL doesn't give you the tools to forecast those kinds of behavioral shifts. There's no way to tell it how many servers are servicing queues, for example. I've seen many benchmarks that fit the USL nicely with increasing numbers of threads until there is one thread per physical CPU core; then there's an abrupt change such as those in the diagram. I've seen others where a resource such as network bandwidth becomes a limiting factor.

I say this not to criticize the USL, but to point out that even a Swiss Army Knife is not always the tool you need.

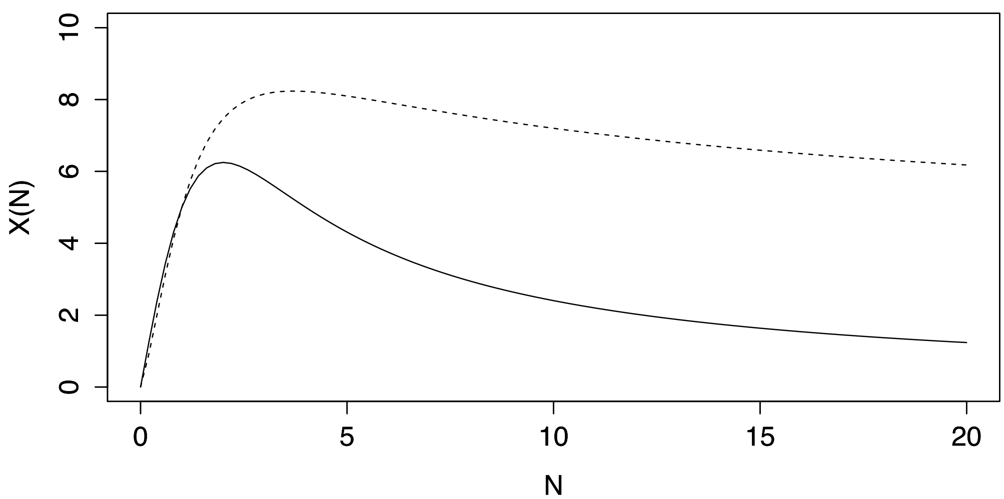
If the USL is incomplete, what is it good for? The answer is lots. Any model is better than none. In the absence of a model explaining the workings of system scalability, there isn't even a point of comparison to assess your expectations and results. There's no frame of reference to say, "this system should scale better than it does," or "these measurements look wrong." Whether the USL is applicable to a given problem or not, it still provides a framework. Without it, why not just draw lines at will? You could get a set of French curves and follow your muse.

As George E. P. Box famously said, "all models are wrong, but some are useful." And as Richard Feynman said in a 1964 lecture at Cornell University, "If it disagrees with experiment, it's wrong. That simple statement is the key to science." If the USL were able to model the physical world completely and correctly, for example including knowledge about capped resources such as number of CPUs (which is a vitally important parameter for queueing theory problems) or network bandwidth, it would certainly describe more systems and scenarios than it does in my experience.

I'd also like to note that you could easily conjecture and analyze other USL-like models. For example, many computer algorithms, such as those that might perform pairwise interchange to cause the coherency penalty,[1] can be shown by analysis to be $\mathcal{O}(n \log n)$ instead of $\mathcal{O}(n^2)$. You might intuit that the quadratic κ term is too pessimistic, and could be replaced by a logarithmic term, like so:

$$X(N) = \frac{\lambda N}{1 + \sigma(N-1) + \kappa \log(N)(N-1)} \tag{11}$$

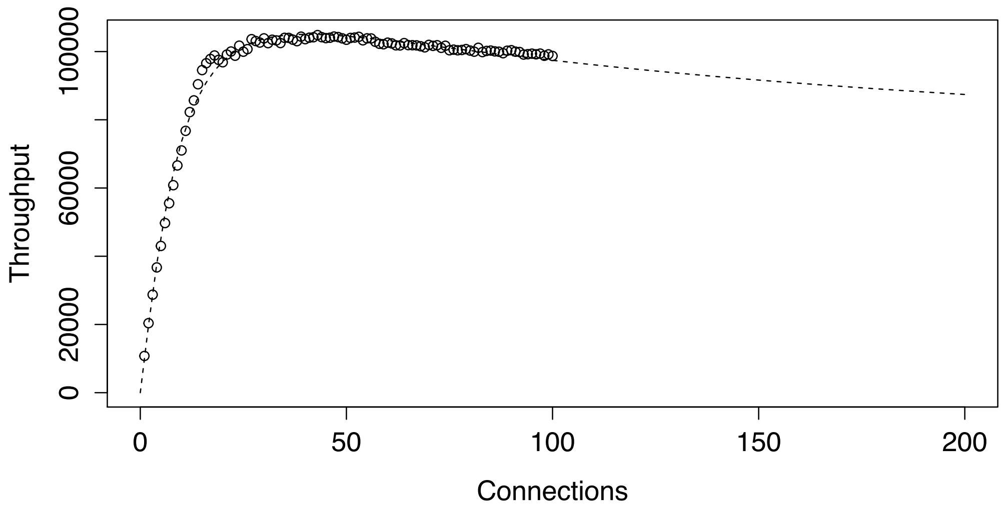
To give some visual intuition of how this differs from the accepted form of the USL, here they are together on a single plot. The standard USL is the solid line and the logarithmic variant is the dashed line.



You might think that you could choose parameters to the standard USL to make it follow the dashed line better, but it doesn't work. The functions are of different order and type, and won't behave the same.

Would this be a better model for the data I showed previously? Visually, it does appear to model the observed data somewhat more closely:

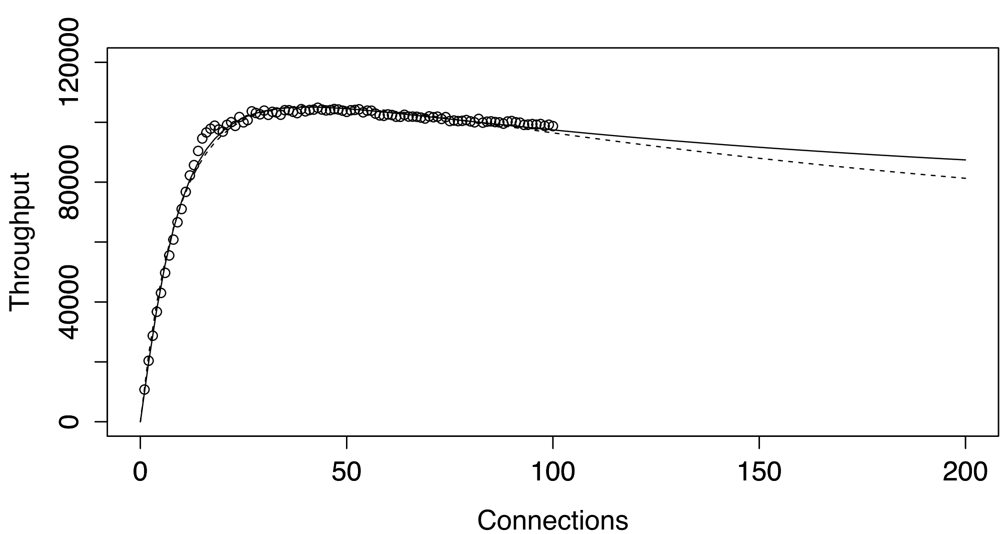
1 Or queue management, if you believe that's the explanation of service time bloat; see John Little's paper.



Dr. Jayanta Choudhury has suggested changes to the USL to model cases such as this, where resources apparently become saturated. His Asymptotically Improved Super-Serial Law[1] is slightly more complex:

$$X(N) = \frac{\lambda N}{1 + \sigma(N-1) + \sigma\kappa N^{\beta}(N-1)} \tag{12}$$

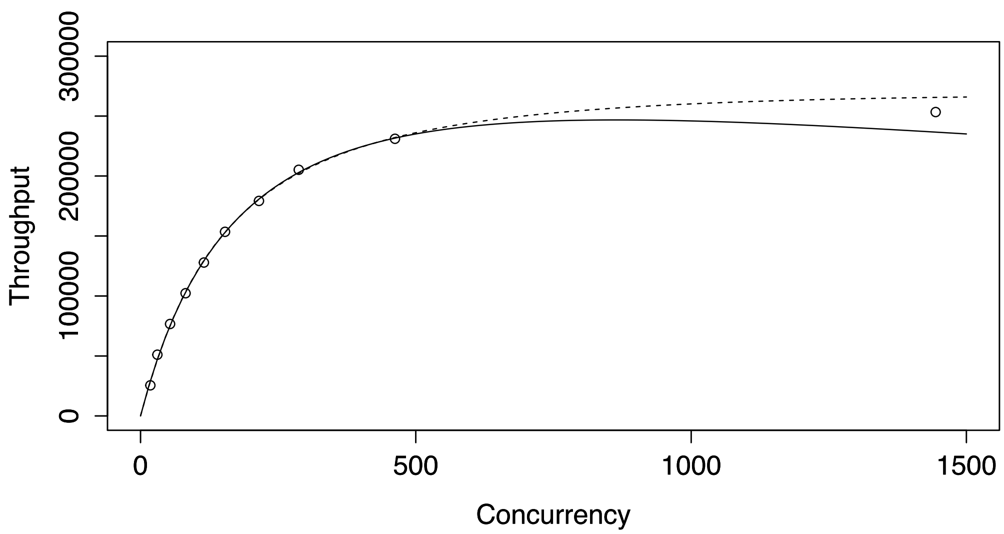
The β parameter ranges from 0 to 1, inclusive. The following plot shows the logarithmic variant I proposed in a solid line, and the AISSL in a dashed line. As you can see, the logarithmic variation fits the data better.



1 See *Parameter Estimation of Asymptotically Improved Super-serial Scalability Law* by Dr. Jayanta Choudhury, of TeamQuest Corporation.

Unfortunately it's a bit difficult to estimate parameters for the AISSL, making it harder to use than the USL.

If you recall the Isilon example from before, it seemed that the USL was too pessimistic in predicting the maximum achievable throughput. This is another way of saying that retrograde scalability was less severe than the USL predicted. On the other hand, that system clearly didn't scale exactly like Amdahl's Law, either. Maybe the logarithmic variant of the USL is more realistic? I've fit the USL as a solid line and the logarithmic variant as a dashed line to see:



As I'm sure you can imagine, you could play games like this all day long. Dreaming up an idea is much easier than proving that it's correct or showing how it might arise analytically from the underlying mechanisms as we understand them to operate in the system.

One way to look at some of the examples I've given in this section is to stop trying to predict what happens after retrograde scalability or resource saturation kicks in. There are at least two good reasons for taking this pragmatic approach.

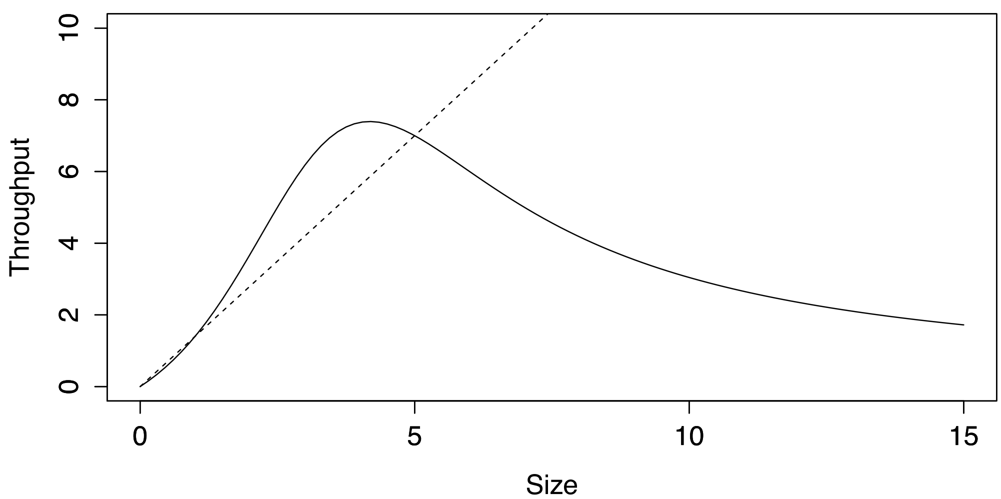
1. **It's a different model.** The system's behavior is being influenced by factors that weren't present at a smaller size. It's not just different

parameters, the entire model has changed, and I doubt that a single model could explain those wildly different behaviors.

2. **It's pointless.** When you see retrograde scaling, you know the system has gone past the point where it's in trouble. Nothing good can come of pushing it further, so why should you try to model how bad it is? It's a lost cause with no practical purpose.

Superlinear Scaling

I've spent some time analyzing the possible causes of sublinear scaling. Is superlinear scaling possible? As I've worked with the USL over the years, I've found a number of cases where systems apparently do scale superlinearly. It manifests as a negative σ coefficient and a USL curve that has a more complex shape, rising above linear and then below again:



At first I dismissed this result as unphysical (how can there be less than zero contention?), but after repeatedly seeing this happen and having many conversations with Neil Gunther about it, I started to wonder. So did he, and eventually he reproduced and explained the effect on a large-scale Hadoop TeraSort benchmark.

The TeraSort case is quite a specific one. In the more general case, I would explain superlinear scalability as a disproportionate scaling of some resource relative to the load placed upon it, creating an economy of scale. For example, adding more nodes to a clustered database system adds more memory; if the dataset size is not scaled proportionately, then more of the data fits into memory on each node, and access times improve relative to disk reads. Any resource that is more efficient when shared than when used singly may cause this effect.

It's worth noting that this initial boost, depending upon its cause, may be countered by a correspondingly disproportionate "payback" later when performance falls quickly below linearity again.

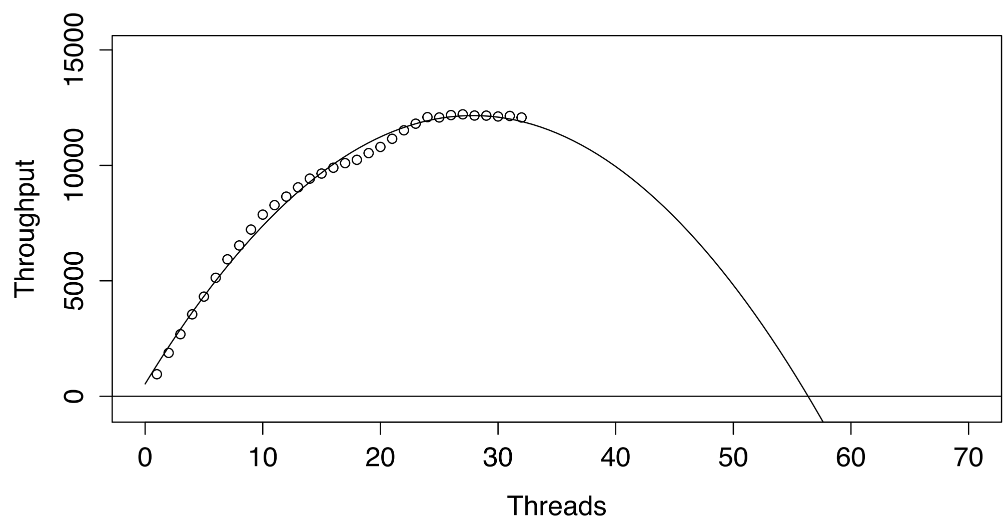
Another special case to be aware of is that some clustered systems behave differently at sizes 1 and 2 than they do at 3 and above. For example, at size 1 there is no crosstalk or contention—it isn't a distributed system. At size 2 special-cases may be in play. At size 3 and above, usually generic algorithms and techniques suited for any size n are in use. Some clustered systems have to be benchmarked or measured at larger sizes in order to avoid skew from these effects.

Finally, negative σ coefficients can arise from data that includes some type of skew. For example, if you are measuring throughput at various levels of concurrency and something is inflating the apparent concurrency measurements (such as idling threads), all of your points will be shifted to the right on the scatterplot. This can cause apparently superlinear scaling (though it's just bad data).

Other Scalability Models

In addition to the USL and the variants I've already discussed, there are many other potential models of scalability you could consider. In my opinion some are good, some are useful, some are not. I also believe that there is much more work to be done on this topic.

Some alternative theories, however, are just garbage. Chief amongst them is "quadratic scalability." Observing that systems under increasing amounts of load will first increase in throughput, then level off and begin to decline, some people inevitably decide to"fit a curve to it." The curve is always a quadratic polynomial and the fitted curve ends up being a parabola opening downwards. Let's see how this looks on the Cisco benchmark data once again:



Look, Ma, it's a great fit! I don't even need to compute the R^2 value to know that. There's just one problem: it predicts that at some point you'll achieve negative throughput.

You should ignore this model because by definition it doesn't work. Rather than use it, I'd suggest that you get out that French curve set again and get in touch with your inner artist.

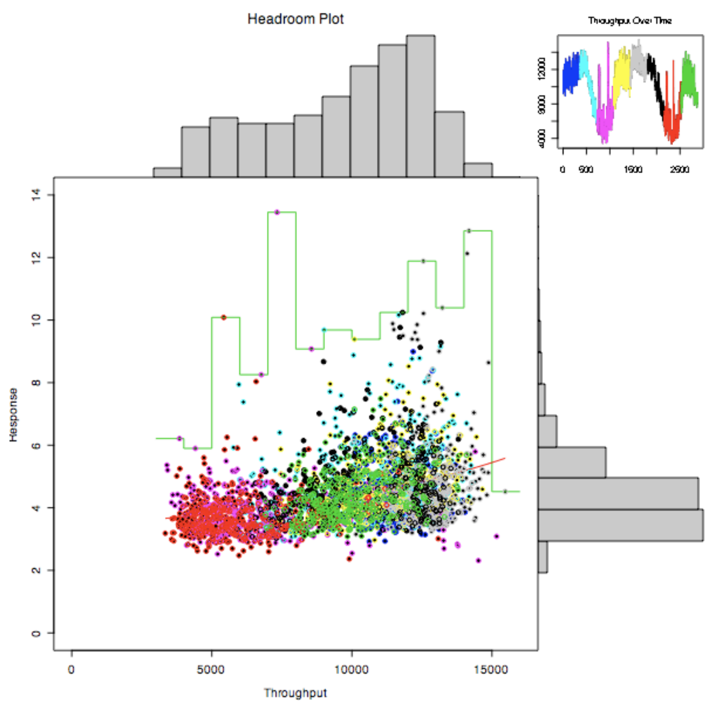
Another model that I've seen is latency as a function of throughput. Here are three examples:

- New Relic's scalability chart, which plots latency as a function of throughput and renders a smoothed line through the points. The line has no predetermined form and doesn't express any particular model.
- AppDynamics's scalability analysis feature does the same thing, but

fits a parabola through the lines instead of a polynomial of arbitrarily high degree.

- Cockcroft headroom plots, by Adrian Cockcroft. These are also latency-versus-throughput charts, but add some histograms and other useful visual cues around the edges.

These three express essentially the same beliefs about the relationship between variables–that throughput is a free parameter that can be set at will, and determines latency. AppDynamics models latency as quadratic, but otherwise the differences among the three are relatively minor. Here's an example of a Cockcroft Headroom Plot:



As I have shown, the relationship between throughput and response time is never quadratic according to the USL model, and is neither exponential nor even a simple function of throughput unless κ is zero. In other words, you really can't model the relationship in this way unless you ignore the possibility of a nonzero κ, which is far from rare. It is better to use a more capable model such as the USL.

Modeling Hardware, Software, and MPP Systems

I mentioned that Neil Gunther actually defines two forms of the USL, one for hardware scaling and one for software. They're essentially the same equation, with different Greek letters.

For most purposes it's not important to care about the distinction. However, it is *very* important, in general, to have a firm grasp on the meaning of the X axis in the USL chart. I've been somewhat casual about it, essentially treating it as a generic metric of size by which you scale the system or workload, but now I will be more precise.

In fact there are at least three important dimensions of the work a system performs, and how it scales, and these three interact with each other. A correct understanding of the concepts is important to get sensible results:

1. **Drivers.** The number of things producing work requests for the system. In a benchmark, for example, this is typically the configured concurrency of the benchmark—that is, the number of driver threads. It could also be the number of connections to the database, the number of users on a web application, and so on.
2. **Servers.** The number of servers as defined in queueing theory. It could be the number of CPUs in a server, the number of servers in a cluster, or the like.
3. **Data.** The size of the dataset. This will most typically be in the usual units—megabytes or gigabytes, number of rows—but will occasionally be the number of logical partitions in the dataset ("shards"). A VoltDB benchmark that I analyzed once needed to be couched in terms of partitions, because of the configured per-partition redundancy.

The key point in scalability analysis with the USL is that everything needs to be held constant relative to the unit of scale you're using, so you are changing only one variable at a time. If you're measuring scalability at different cluster sizes, for example, you need to grow the number of driver threads and the data size proportionately to the number of nodes in the cluster, so each node receives the same amount and rate of work to perform upon the same amount of data no matter the cluster size. (If you hold the dataset constant and increase the number of nodes, you'll get superlinear scalability.)

I've shown that the USL can be solved for a variety of variables, such as response time (latency). Yet another way to think about the USL is in terms of response time *reduction*, or speedup. I mentioned this briefly while motivating the introduction of Amdahl's Law, but a system that literally behaves like Amdahl's Law is probably less familiar to most readers. However, it's a perfect model to use for massively parallel processing (MPP) systems, which subdivide work and execute fractions of it in parallel, then combine the intermediate results into a final answer. In such a system, you can vary the degree of parallelism, measure how latency reduces, and produce a model of the system's scalability in terms of response time reduction.[1]

To accomplish this, I need to make explicit what's been a bit bundled together into the USL when applied to hardware scalability. The assumption, not shown in Equation 3 directly, is that as you scale the number of servers N, you scale the drivers proportionately, and each request is served by a single server. In this way you hold the concurrency constant at each server. Again, I'm using the term "server" in the queueing theory sense of the word, e.g. nodes in the cluster, CPUs in the motherboard.

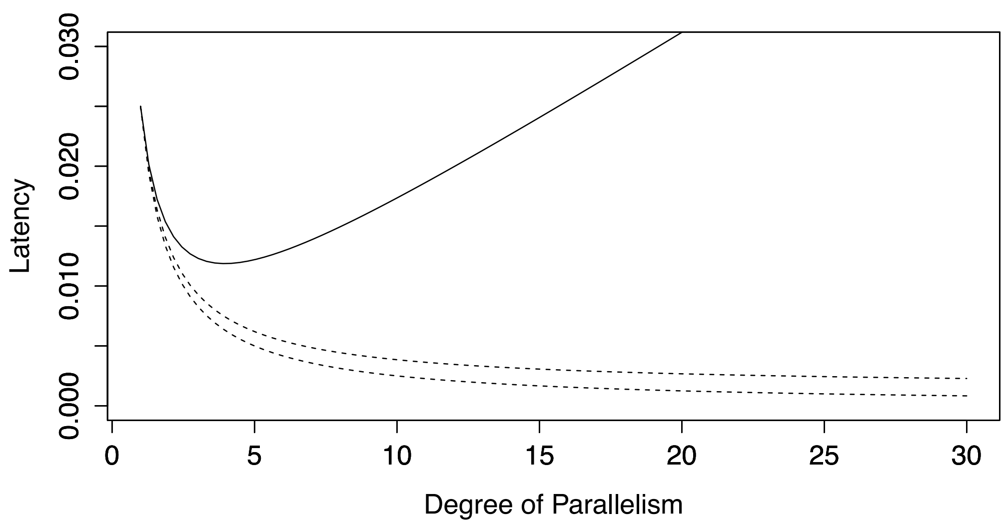
If you solve the USL to represent response time as a function of the

[1] Throughput and concurrency do not enter into the equation; you are not measuring aggregates of number of requests completed per time, but merely measuring the response time of individual requests at a concurrency of 1, by definition.

degree of parallelism in an MPP job, of course you'll find that it's just the inverse of the speedup. And the standard form of the USL is equivalent to speedup in the MPP sense, so the response time of individual requests becomes:

$$R(N) = \frac{1 + \sigma(N-1) + \kappa N(N-1)}{\lambda N} \tag{13}$$

Characteristic response time curves in an MPP system, then, are as follows. A perfectly linear system will result in speedup towards an asymptote of zero. A system with serialization will speedup towards an asymptote of the serial fraction. And a system with retrograde scalability due to service time bloat will actually exhibit, well, *bloat*. It looks like this:



You can use that model just like any other form of the USL: to evaluate scalability, predict performance beyond what you can observe, figure out whether a system is scaling well enough and why, and so on.

Conclusions

The Universal Scalability Law is a wonderfully versatile, easy-to-apply, yet formal framework for modeling, thinking about, and analyzing scalability and performance. It applies to many different situations, as long as you can define the variables correctly. It even explains why human organizations (companies and teams) struggle as they grow. As anyone who's managed a growing company knows, the chief scaling problem in a big company is a communications problem.

In this book I've given you a whirlwind tour of modeling scalability and performance with the USL. A few of the key takeaways are as follows:

- Scalability is a formal concept that is best defined as a mathematical function.
- Linear scalability means equal return on investment. Double down on workers and you'll get twice as much work done; add twice as many nodes and you'll increase the maximum capacity twofold. Linear scalability is oft claimed but seldom delivered.
- Systems scale sublinearly because of contention, which adds queueing delay, and crosstalk, which inflates service times. The penalty for contention grows linearly and the crosstalk penalty grows quadratically. (An alternative to the crosstalk theory is that longer queues are more costly to manage.)
- Contention causes throughput to asymptotically approach the reciprocal of the serialized fraction of the workload. If your workload is 5% serialized you'll never grow the effective speedup by more than 20-fold.
- Crosstalk causes the system to regress. The harder you try to push systems with crosstalk, the more time they spend fighting amongst themselves.
- To build scalable systems, avoid contention (serialization) and crosstalk (synchronization). The contention and crosstalk penalties degrade system scalability and performance *much* faster than you'd think. Even tiny amounts of serialization or pairwise data synchronization cause big losses in efficiency.

- If you can't avoid crosstalk, partition (shard) into smaller systems that will lose less efficiency by avoiding the explosion of service times at larger sizes.
- To model systems with the USL, obtain measurements of throughput at various levels of load or size, and use regression to estimate the parameters to Equation 3.
- To forecast scalability beyond what's observable, be pessimistic and treat the USL as a best-case scenario that won't really happen. Use Equation 4 to forecast the maximum possible throughput, but don't forecast too far out. Use Equation 6 to forecast response time.
- Use your judgment to predict limitations that USL can't see, such as saturation of network bandwidth or changes in the system's model when all of the CPUs become busy.
- Use the USL to explain why systems aren't scaling well. Too much queueing? Too much crosstalk? Treat the USL as a pessimistic model and demand that your systems scale at least as well as it does.
- If you see superlinear scaling, check your measurements and how you've set up the system under test. In most cases σ should be positive, not negative. Make sure you're not varying the system's dimensions relative to each other and creating apparent superlinear efficiencies that don't really exist.
- It's fun to fantasize about models that might match observed system behavior more closely than the USL, but the USL arises analytically from how we know queueing systems work. Invented models might not have any basis in reality. Besides, the USL usually models systems extremely well up to the point of inflection, and modeling what happens beyond that isn't as interesting as knowing *why* it happens.
- Never trust a scatterplot with an arbitrary curve fit through it unless you know why that's the right curve. Don't confuse the USL, hockey stick charts from queueing theory, or other charts that just happen to

have similar shapes. Know what shape various plots should exhibit, and suspect bad measurements or other mistakes if you don't see them.

The USL succeeds because it encapsulates many different effects implicitly, and doesn't require you to capture arcane measurements that may be impossible to obtain. It's simpler to use and often gives better results than many other types of forecasting tools, such as Erlang C formulas. For the same reasons, it's limited in its predictive ability. But remember: as Neil Gunther says, "Models don't get confused, only modelers do."

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Further Reading

The Universal Scalability Law is only a couple of decades old, and there is not a great deal of literature about it. Interested readers should purchase the canonical books from Dr. Neil J. Gunther. I include a variety of other excellent reading in the list below, but Neil Gunther's books are the definitive sources.

- *Guerrilla Capacity Planning* by Neil Gunther. This book is more or less the canonical text.
- *Analyzing Computer System Performance With Perl::PDQ* by Neil Gunther, which introduces the background to the USL.
- *Practical Performance Analyst* by Neil Gunther.

- *Forecasting MySQL Scalability with the Universal Scalability Law* by Baron Schwartz and Ewen Fortune.
- *Fundamentals of Queueing Theory* by Gross and Harris.
- *Practical Queueing Analysis* by Mike Tanner.
- *Probability, Statistics, and Queueing Theory* by Allen.
- *Capacity Planning for Web Performance* by Menascé and Almeida.
- *The Art of Computer Systems Performance Analysis* by Jain.

About VividCortex

VividCortex is a SaaS database performance monitoring platform. The database is the heart of most applications, but it's also the part that's hardest to scale, manage, and optimize even as it's growing 50% year over year. VividCortex has developed a suite of unique technologies that significantly eases this pain for the entire IT department. Unlike traditional monitoring, we measure and analyze the system's work and resource consumption. This leads directly to better performance for IT as a whole, at reduced cost and effort.

Related Resources From VividCortex

Everything You Need To Know About Queueing Theory

This highly accessible introduction demystifies queueing theory without using pages full of equations, helping you build intuition about it.

Case Study: SendGrid

VividCortex is the go-to solution for seeing what's happening in SendGrid's production systems. It has saved months of effort and made query performance data available instantly to the entire team.