Indian Institute of Technology, Bombay
Aerospace Engineering Department

Guide: Prof. Arnab Maity
Course: AE 305 - Flight Mechanics

Flight Mechanics Project

Cessna 310 Analysis

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Abstract

This report presents the analysis of a Cessna 310 aircraft model assigned to our group in the AE 305 Flight Mechanics II project. The stability derivatives for the longitudinal and lateral motions of the aircraft are calculated, and the dynamics of both motions are expressed in a state space form. The eigenvalues and eigenvectors in polar form are determined, and the different modes of both longitudinal and lateral motions are distinguished. Transfer functions for longitudinal and lateral states with elevator, throttle, aileron, and rudder as control inputs are obtained, and their responses to various inputs are plotted. The report concludes with a discussion of the results and their implications for the flight dynamics of the Cessna 310 aircraft model.

1 Aircraft

For this project, we have chosen the Cessna 310 Aircraft. To formulate the state space equations, we have gathered the necessary data from various references (see references section). The equations of motion used to form the matrices are provided below. ‘

Figure 1: Cessna 310

1.1 Longitudinal analysis

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u + M_{\dot{w}} Z_u & M_w + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} u_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ Z_{\delta_e} & 0 \\ M^*_{\delta_e} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_T \end{bmatrix}$$

1.2 Lateral anaylsis

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & -(u_0 - Y_r) & -g \\ L^*_v + \frac{I_{xz}}{I_z} N^*_v & L^*_p + \frac{I_{xz}}{I_z} N^*_p & L^*_r + \frac{I_{xz}}{I_z} N^*_r & 0 \\ N^*_v + \frac{I_{xz}}{I_z} L^*_v & N^*_p + \frac{I_{xz}}{I_z} L^*_p & N^*_r + \frac{I_{xz}}{I_z} L^*_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 & Y_{\delta_r} \\ L^*_{\delta_a} + \frac{I_{xz}}{I_z} N^*_{\delta_a} & L^*_{\delta_r} + \frac{I_{xz}}{I_z} N^*_{\delta_r} \\ N^*_{\delta_a} + \frac{I_{xz}}{I_z} L^*_{\delta_a} & N^*_{\delta_r} + \frac{I_{xz}}{I_z} L^*_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

(Note : The force and moment derivatives in the matrices above have been divided by mass of the airplane or the moment of inertia.)

2 Question 1

2.1 Stability derivatives longitudinal and lateral motions in a state space form.

For the aircraft model data assigned to your group, calculate the stability derivatives corresponding to the longitudinal and lateral motions of the aircraft and express the dynamics of both motions in a state space form.

2.2 Eigenvalues and eigenvectors in polar form

From the system matrices of both longitudinal and lateral dynamics, calculate eigenvalues and eigenvectors in polar form and distinguish them into different modes of both longitudinal and lateral motions.

Solution:

1. Longitudinal Dynamics:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.0003 & 0.0200 & 0 & -32.1700 \\ -0.0406 & -0.0103 & 49.2485 & 0 \\ 0.0006 & -0.0033 & -2.3490 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -0.0883 & 0 \\ -2.7978 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_T \end{bmatrix}$$

- Eigen Values:
  e_1 = -2.2812 + 0.0000i (Phugoid Mode)
  e_2 = 0.0256 + 0.1208i (Short-Period Mode)
  e_3 = 0.0256 - 0.1208i (Short-Period Mode)
  e_4 = -0.1295 + 0.0000i (Phugoid Mode)

- Eigen Vectors:
  $\vec{v_1} = \begin{bmatrix} 0.2648 + 0.0000i & 0.9849 + 0.0000i & 0.9849 + 0.0000i & 0.8671 + 0.0000i \end{bmatrix}$
  $\vec{v_2} = \begin{bmatrix} 0.9631 + 0.0000i & -0.1180 + 0.1270i & -0.1180 - 0.1270i & 0.4982 + 0.0000i \end{bmatrix}$
  $\vec{v_3} = \begin{bmatrix} -0.0442 + 0.0000i & 0.0004 - 0.0002iZ & 0.0004 + 0.0002i & -0.0005 + 0.0000i \end{bmatrix}$
  $\vec{v_4} = \begin{bmatrix} 0.0194 + 0.0000i & -0.0009 - 0.0036i & -0.0009 + 0.0036i & 0.0038 + 0.0000i \end{bmatrix}$

2. Lateral Dynamics:

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} -0.0131 & -0.0488 & -49.1258 & 32.1700 \\ -0.1496 & -13.8784 & 1.8362 & 0 \\ 0.1592 & -0.5228 & -3.0409 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 & 0.2123 \\ -11.5642 & 1.2909 \\ 0.9122 & -6.2548 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

- Eigen Values:
  e_1 = -13.8472 + 0.0000i (Spiral Mode)
  e_2 = -1.5199 + 2.4553i (Dutch Roll)
  e_3 = -1.5199 - 2.4553i (Dutch Roll)
  e_4 = -0.0453 + 0.0000i (Roll Mode)

- Eigen Vectors:
  $\vec{v_1} = \begin{bmatrix} 0.3091 + 0.0000i & -0.9983 + 0.0000i & -0.9983 + 0.0000i & 0.9953 + 0.0000i \end{bmatrix}$
  $\vec{v_2} = \begin{bmatrix} 0.9477 + 0.0000i & 0.0086 + 0.0054i & 0.0086 - 0.0054i & -0.0037 + 0.0000i \end{bmatrix}$
  $\vec{v_3} = \begin{bmatrix} 0.0413 + 0.0000i & -0.0306 + 0.0476i & -0.0306 - 0.0476i & 0.0535 + 0.0000i \end{bmatrix}$
  $\vec{v_4} = \begin{bmatrix} -0.0684 + 0.0000i & 0.0000 - 0.0035i & 0.0000 + 0.0035i & 0.0807 + 0.0000i \end{bmatrix}$

3 Question 2

3.1 Transfer functions for longitudinal states with elevator and throttle as control inputs

Obtain all the eight transfer functions for longitudinal states (u, w, q and θ) with elevator (δ_e) and throttle (δ_T) as control inputs.

$$\frac{u(s)}{\delta_a(s)} = \frac{-0.001766s^2 + 87.25s + 0.9177}{s^4 + 2.36s^3 + 0.1882s^2 + 0.02067s + 0.004509} \quad \frac{w(s)}{\delta_a(s)} = \frac{-0.0883s^3 - 138s^2 - 0.0414s - 3.656}{s^4 + 2.36s^3 + 0.1882s^2 + 0.02067s + 0.004509}$$

$$\frac{q(s)}{\delta_a(s)} = \frac{-2.798s^3 - 0.02937s^2 - 0.002281s - 2.031e - 20}{s^4 + 2.36s^3 + 0.1882s^2 + 0.02067s + 0.004509} \quad \frac{\theta(s)}{\delta_a(s)} = \frac{-2.798s^2 - 0.02937s - 0.002281}{s^4 + 2.36s^3 + 0.1882s^2 + 0.02067s + 0.004509}$$

$$\frac{u(s)}{\delta_T(s)} = \frac{0}{s^4 + 2.36s^3 + 0.1882s^2 + 0.02067s + 0.004509} \quad \frac{w(s)}{\delta_T(s)} = \frac{0}{s^4 + 2.36s^3 + 0.1882s^2 + 0.02067s + 0.004509}$$

$$\frac{q(s)}{\delta_T(s)} = \frac{0}{s^4 + 2.36s^3 + 0.1882s^2 + 0.02067s + 0.004509} \quad \frac{\theta(s)}{\delta_T(s)} = \frac{0}{s^4 + 2.36s^3 + 0.1882s^2 + 0.02067s + 0.004509}$$

3.2 Responses of flight path angle, speed, and angle of attack to a step input of 1○ of elevator deflection

Plot short-term and long-term responses of flight path angle ($\Delta\gamma$), speed ($\Delta\alpha$), and angle of attack ($\Delta\alpha$) to a step input of 1○ of elevator deflection.

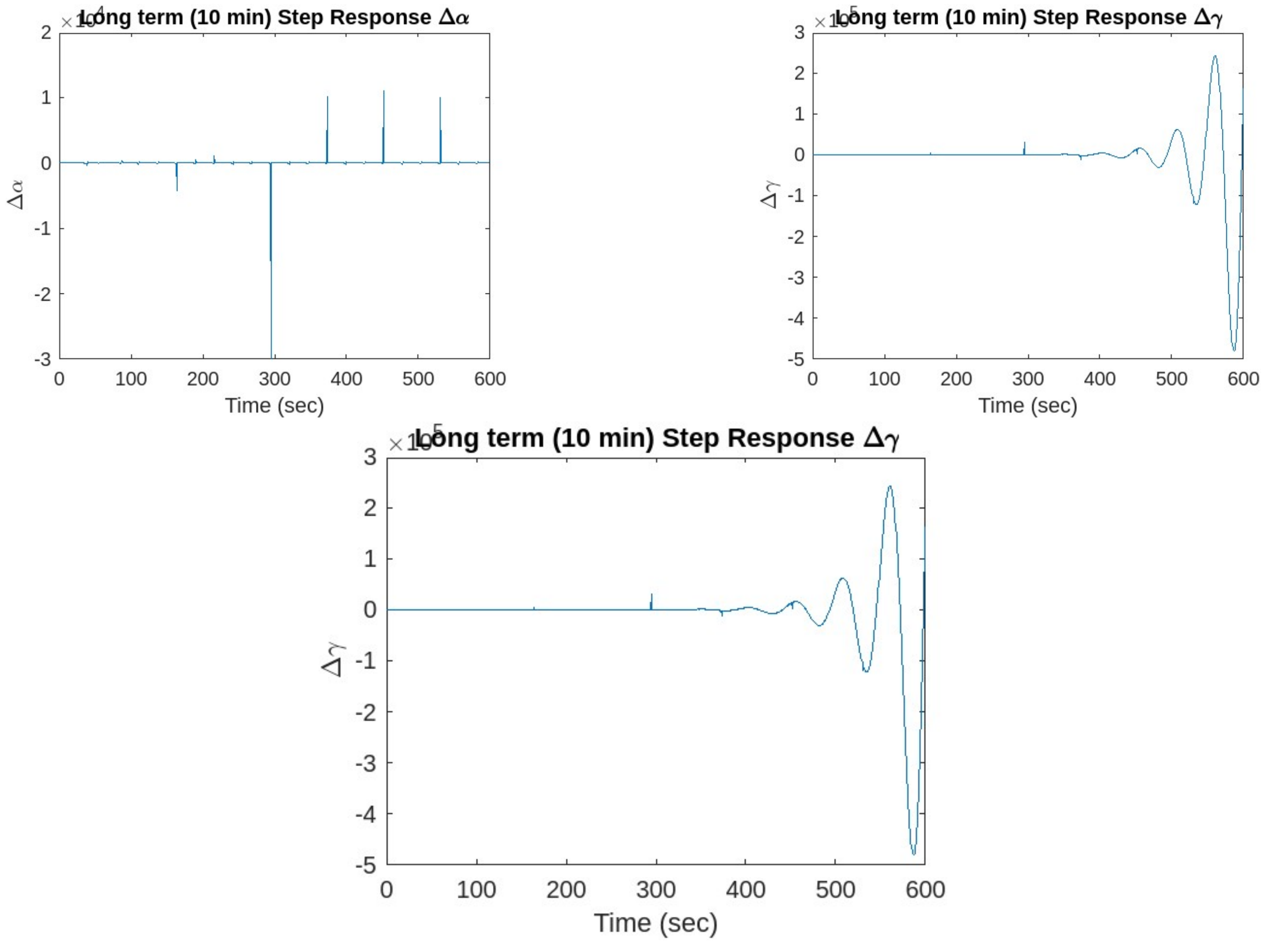
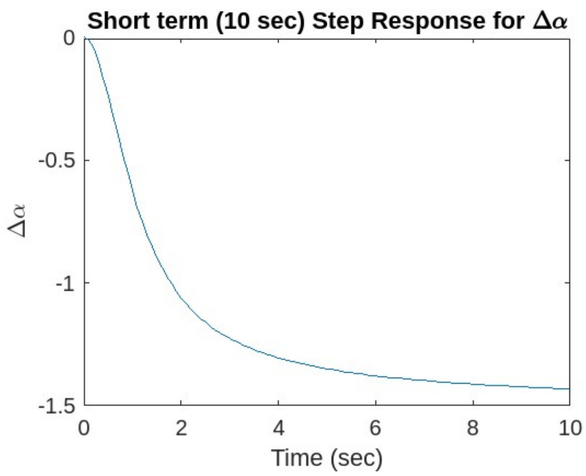
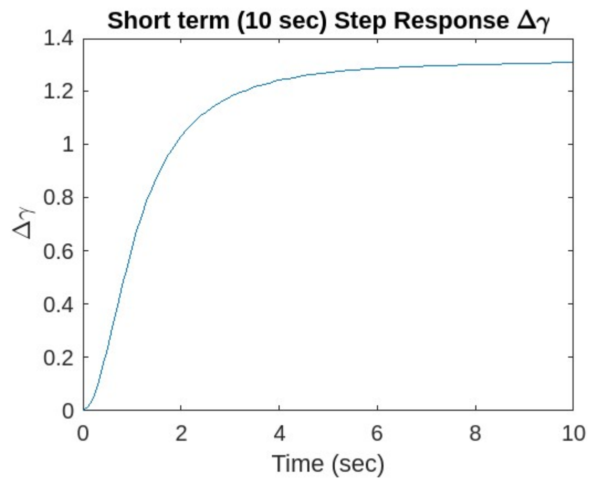


Figure 2: Long Term Responses For Deflection of 1 Degree





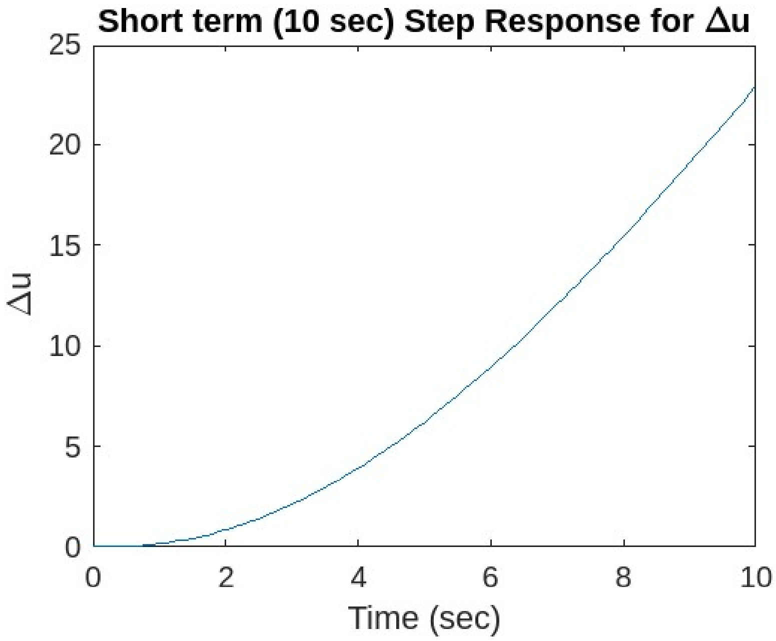


Figure 3: Short Term Responses For Deflection of 1 Degree.

3.3 Long-term response to throttle magnitude

Plot a long-term response of flight path angle ($\Delta\gamma$), speed (Δu) and angle of attack ($\Delta\alpha$) to the throttle magnitude of 1/6.

Solution:

Since we couldn't find any data for throttle magnitude derivatives , we couldn't compute the responses for the same. The same has been mentioned in the email sent to the TA .

4 Question 3

4.1 Transfer functions for lateral states with aileron and rudder as control inputs

Obtain all the eight transfer functions for lateral states (v, p, r and ϕ) with aileron (δ_e) and rudder (δ_T) as control inputs.

$$\frac{v(s)}{\delta_e(s)} = \frac{-44.25s^2 - 1289s - 1077}{s^4 + 16.93s^3 + 51.2s^2 + 117.8s + 5.231} \qquad \frac{p(s)}{\delta_e(s)} = \frac{-11.56s^3 - 33.64s^2 - 84.18s + 8.756e - 17}{s^4 + 16.93s^3 + 51.2s^2 + 117.8s + 5.231}$$

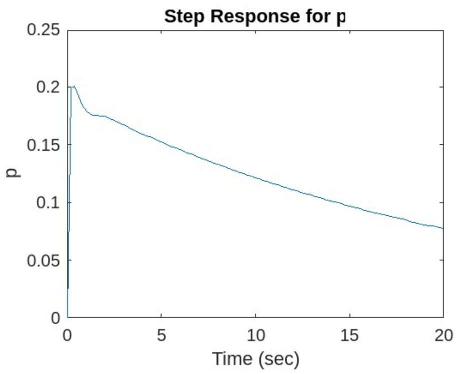
$$\frac{r(s)}{\delta_e(s)} = \frac{0.9122s^3 + 18.72s^2 + 0.3282s - 54.84}{s^4 + 16.93s^3 + 51.2s^2 + 117.8s + 5.231} \qquad \frac{\phi(s)}{\delta_e(s)} = \frac{-11.56s^2 - 33.64s - 84.18}{s^4 + 16.93s^3 + 51.2s^2 + 117}$$

$$\frac{v(s)}{\delta_T(s)} = \frac{0.2123s^3 + 310.8s^2 + 4349s - 243.2}{s^4 + 16.93s^3 + 51.2s^2 + 117.8s + 5.231} \qquad \frac{p(s)}{\delta_T(s)} = \frac{1.291s^3 - 7.574s^2 - 36.01s + 5.797e - 16}{s^4 + 16.93s^3 + 51.2s^2 + 117.8s + 5.231}$$

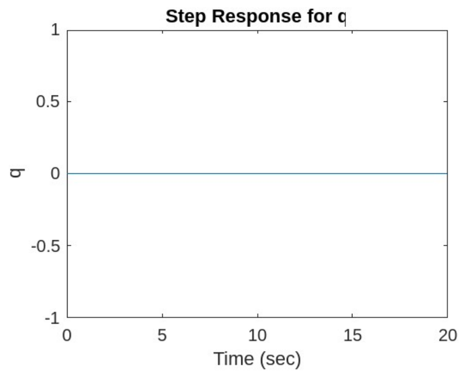
$$\frac{r(s)}{\delta_T(s)} = \frac{-6.255s^3 - 87.53s^2 - 0.6247s - 23.49}{s^4 + 16.93s^3 + 51.2s^2 + 117.8s + 5.231} \qquad \frac{\phi(s)}{\delta_T(s)} = \frac{1.291s^2 - 7.574s - 36.01}{s^4 + 16.93s^3 + 51.2s^2 + 117.8s + 5.231}$$

4.2 Plot response for velocity and attitude to aileron step input

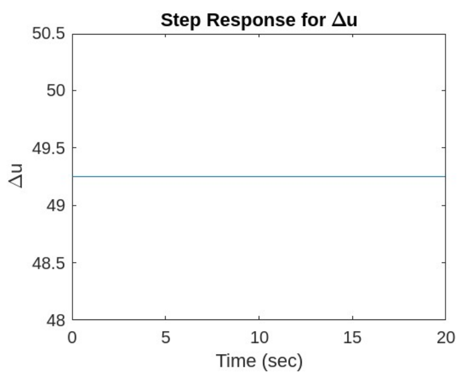
Plot response for velocity components (u,v,w) angular velocity (p, q, r) and attitude angles (ϕ, θ, ψ) to a step input of 15° of aileron deflection.



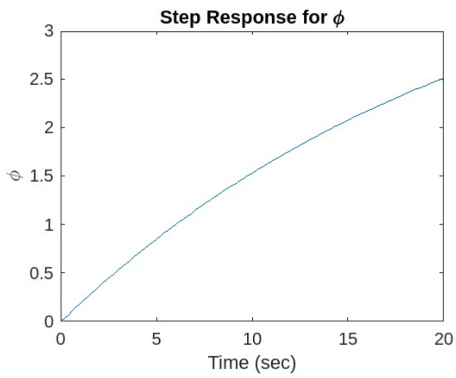
(a) Step response of p



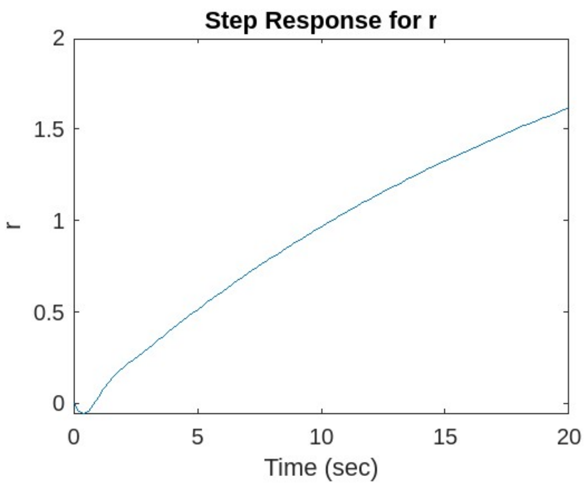
(b) Step response of q



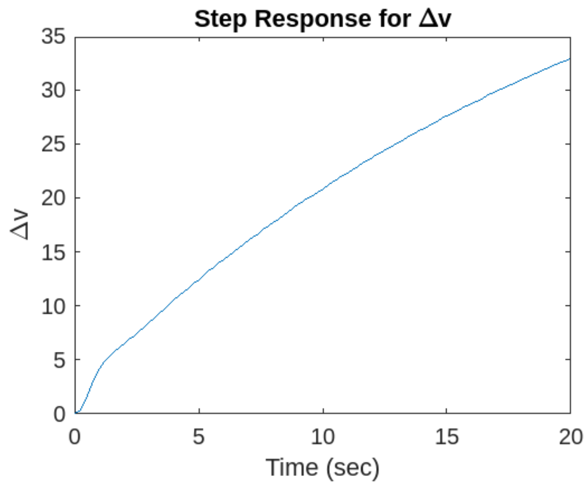
(c) Step response of u



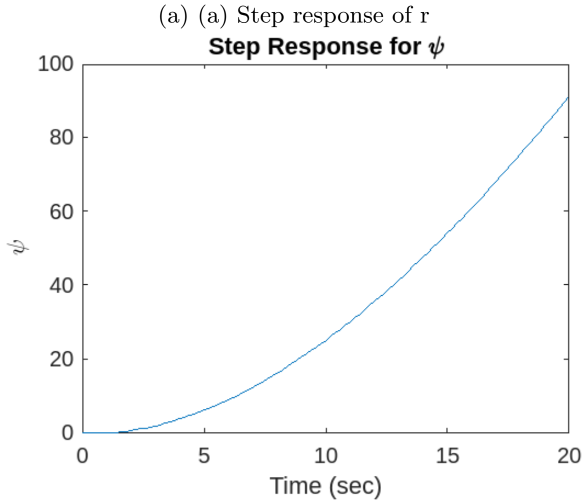
(d) Step response of phi



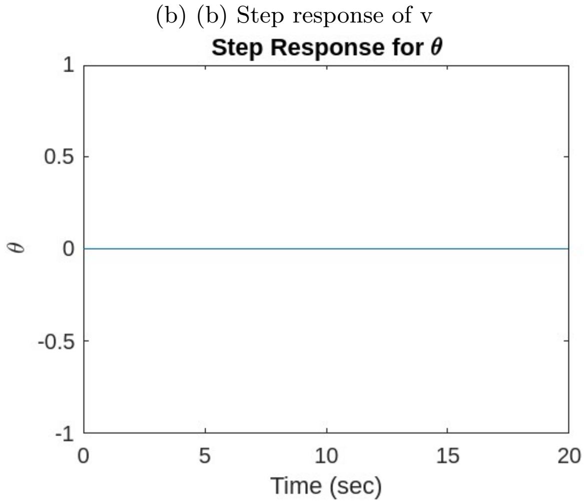
(a) (a) Step response of r



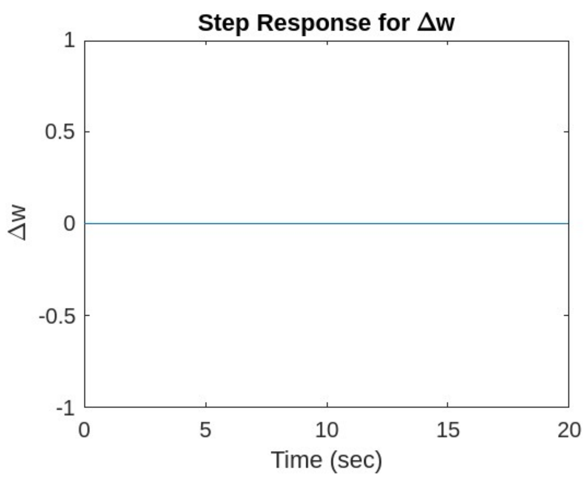
(b) (b) Step response of v



(c) (c) Step response of psi



(d) (d) Step response of theta



(e) (e) Step response of w

Figure 4: Step response of the system for different variables.

5 References

1. Flight Stability and Automatic Control
2. Jan Roskam - Airplane Flight Dynamics and Automatic Flight Controls. partI-DARcorporation (2001)
3. Aircraft: Cessna 310 Cruise configuration