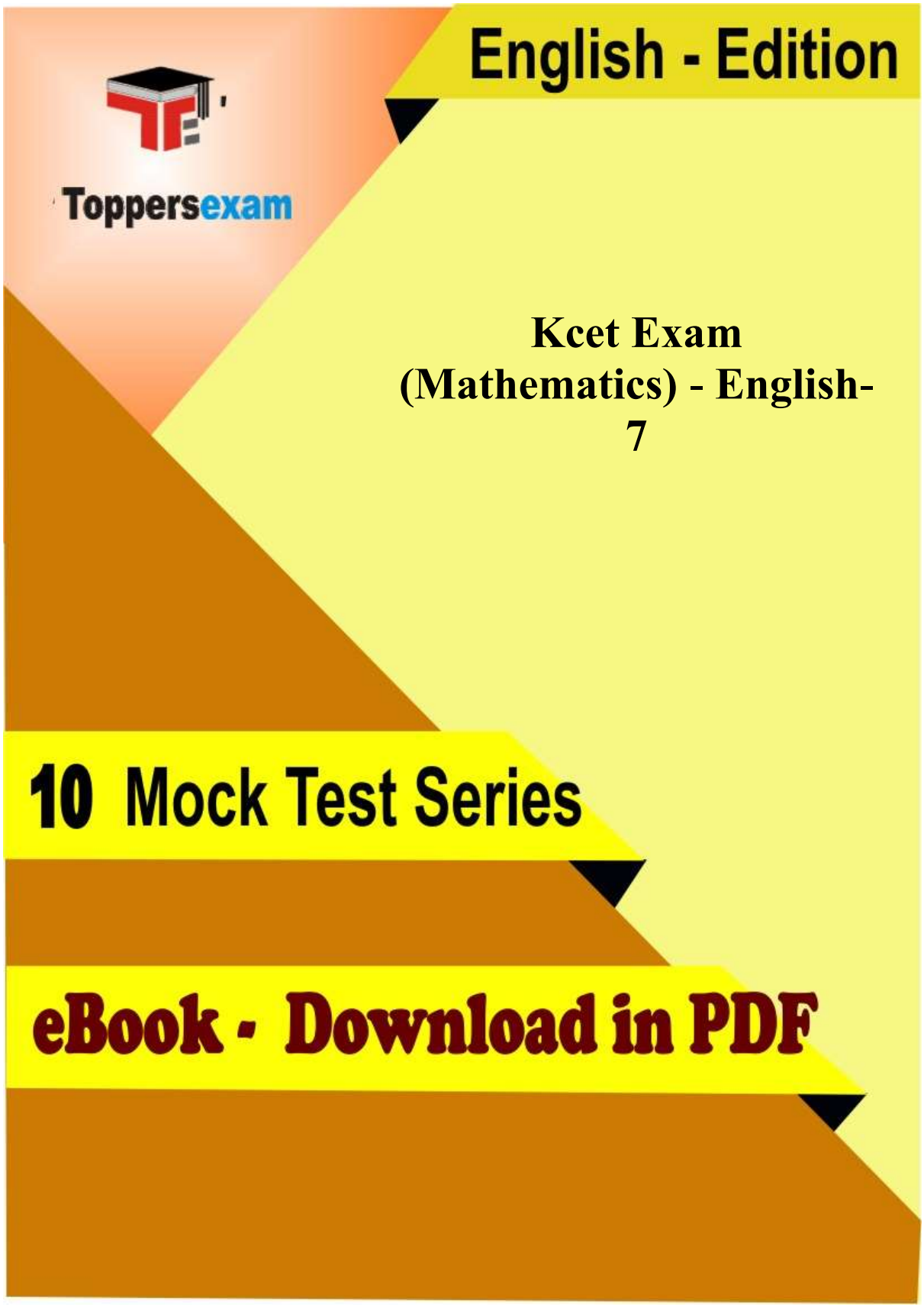


Toppersexam.com

Kcet Exam (Mathematics) - English-7

Paper Questions

SUBJECT: Physics

Question 1 : A cylinder of radius r and length l is placed in an uniform electric field E parallel to the axis of the cylinder. The total flux for the surface of the cylinder is given by:

(A) zero

(B) 2 r^2E

(C) r^2E

(D) (r^2 + l^2)E

Correct Answer: A

Question 2 : Two electric bulbs A and B are related as 60 W and 100 W. they are connected in parallel to the same source. Then:

(A) B draws more current than A

(B) currents drawn are in the ratio of their resistances

(C) both draw the same current

(D) A draws more current than B

Correct Answer: A

Question 3 : A thin plano-convex lens acts like a concave mirror of focal length 0.2 m when silvered from its plane surface. The refractive index of the material of the lens is 1.5. the radius of curvature of the convex surface of the lens will be:

(A) 0.1 m

(B) 0.75 m

(C) 0.4 m

(D) 0.2 m

Correct Answer: D

Question 4 : The physical quantity having the same dimensions as Planck's constant h is:

(A) linear momentum

(B) angular momentum

(C) Boltzmann constant

(D) force

Correct Answer: B

Question 5 : A balloon is rising vertically up with a velocity of 29 ms^{-1}. A stone is dropped from it and it reaches the ground in 10 seconds. The height of the ballon when the stone was dropped from it is (g=9.8 ms^{-1})

(A) 400 m

(B) 150 m

(C) 100 m

(D) 200 m

Correct Answer: D

Question 6 : A wire has a resistance of 6Ω. It is cut into two parts and both half values are connected in parallel. The new resistance is:

(A) 3Ω

(B) 6Ω

(C) 12Ω

(D) 1.5Ω

Correct Answer: D

Question 7 : In a Young's double slit experiment, the separation between the two slits is 0.9 mm and the fringes are observed one metre away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, the wavelength of the monochromatic source of light used is:

(A) 450 nm

(B) 400 nm

(C) 500 nm

(D) 600 nm

Correct Answer: D

Question 8 : When light is incident on a diffraction grating, the zero order principal maximum will be:

(A) spectrum of the colours

(B) white

(C) one of the component colours

(D) absent

Correct Answer: B

Question 9 : H-polaroid is prepared by:

(A) orienting herapathite crystal in the same direction in nitrocellulose

(B) using thin tourmaline crystals

(C) stretching polyvinyl alcohol and then heated with dehydrated agent

(D) stretching polyvinyl alcohol and then impregnating with iodine

Correct Answer: D

Question 10 : SI unit of permittivity is:

(A) $C^2m^2N^2$

(B) $C^2m^{-2}N^{-1}$

(C) $C^2m^2N^{-1}$

(D) $C^{-1}m^2N^{-2}$

Correct Answer: B

Question 11 : A metal wire is subjected to a constant potential difference. When the temperature of the metal wire increases, the drift velocity of the electron in it

(A) increases, thermal velocity of the electron decreases

(B) decreases, thermal velocity of the electron decreases

(C) increases, thermal velocity of the electron increases

(D) decreases, thermal velocity of the electron increases

Correct Answer: D

Question 12 : Two equal forces (P each) act at a point inclined to each other at an angle of 120^{0} C . The magnitude of their resultant is:

(A) P/2

(B) P/4

(C) P

(D) 2P

Correct Answer: C

Question 13 : Threshold wavelength for photoelectric emission from a metal surface is 5200 Å. Photoelectrons will be emitted when this surface is illuminated with monochromatic radiation from:

(A) 1 W IR lamp

(B) 50 W UV lamp

(C) 50 W IR lamp

(D) 10 W IR lamp

Correct Answer: B

Question 14 : In young's double slit experiment if monochromatic light used is replaced by white light, then:

(A) no fringes are observed

(B) only central fringe is white, all other fringes are coloured

(C) all bright fringes become white

(D) all bright fringes have coloures between violet and red

Correct Answer: B

Question 15 : Which state of triply ionized Beryllium (Be^{+++}) has the same orbital radius as that of the ground state of hydrogen?

(A) n = 3

(B) n = 4

(C) n=1

(D) n = 2

Correct Answer: D

Question 16 : If l_1, l_2, l_3 are the lengths of the emitter, base and collector of a transistor, then:

(A) $l_3 < l_2 < l_1$

(B) $l_1 = l_2 = l_3$

(C) $l_3 > l_1 > l_2$

(D) $l_3 < l_1 < l_2$

Correct Answer: D

Question 17 : A count rate meter shows a count of 240 per minute from a given radioactive source. One hour later the meter shows a count rate of 30 per minute. The half-life of the source is:

(A) 80 min

(B) 120 min

(C) 20 min

(D) 30 min

Correct Answer: C

Question 18 : Two conductors of the same material have theirdiameters in the ratio 1:2 and their lengths in the ratio 2:1. If the temperature differences between their ends is the same, then the ratio of amounts of heat conducted per second through them will be:

(A) 4:1

(B) 1:4

(C) 8:1

(D) 1:8

Correct Answer: D

Question 19 : Blowing air with open mouth is an example of:

(A) isobaric process

(B) isochoric process

(C) isothermal process

(D) adiabatic process

Correct Answer: A

Question 20 : Sound waves in air are always longitudinal because:

(A) of the inherent characteristics of sound waves in air

(B) air does not have a modules of rigidity

(C) air is a mixture of several gases

(D) density of air is very small

Correct Answer: B

Question 21 : An uncharged sphere of metal is placed inside a charged parallel plate capacitor. The lines of force will look like:

(A) 

(B) 

(C) 

(D) 

Correct Answer: A

Question 22 : A current flows in a conductor from east to west. The direction of the magnetic field at a point above the conductor is:

(A) towards east

(B) towards west

(C) towards north

(D) towards south

Correct Answer: C

Question 23 : A bar magnet is equivalent to:

(A) torroid carrying current

(B) straight conductor carrying current

(C) solenoid carrying current

(D) circular coil carrying current

Correct Answer: C

Question 24 : Excitation energy of a hydrogen like ion in its first excitation state is 40.8 eV from the ion in ground state is:

(A) 40.8 eV

(B) 27.2 eV

(C) 54.4 eV

(D) 13.6 eV

Correct Answer: C

Question 25 : The refractive index of a particular material is 1.67 for blue light, 1.65 for yellow light power and 1.63 for red light. The dispersive power of the material is:

(A) 0.031

(B) 1.60

(C) 0.0615

(D) 0.024

Correct Answer: C

Question 26 : An ideal gas heat engine operates in a Carnot's cycle between $227^{0}C$ and 127^{0} C . It absorbs $6x10^{4}$ J at high temperature. The amount of heat converted into work is:

(A) $1.6x10^{4}$ J

(B) $1.2x10^{4}$ J

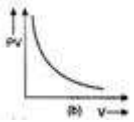
(C) $4.8x10^{4}$ J

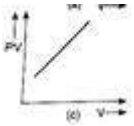
(D) $3.5x10^{4}$ J

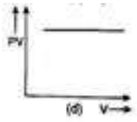
Correct Answer: B

Question 27 : Which one of the following graphs represents the behavior of an ideal gas?

(A) 

(B) 

(C) 

(D) 

Correct Answer: D

Question 28 : Rainbow is formed due to:

(A) total internal reflection

(B) scattering

(C) refraction

(D) dispersion and total internal reflection

Correct Answer: D

Question 29 : A beam of parallel rays is brought to focus by a plano-convex lens. A thin concave lens of the same focal length is joined to the first lens. The effect of this is:

(A) the focus shifts to infinity

(B) the focal point shifts towards the lens by a small distance

(C) the focal point shifts away from the lens by a small distance

(D) the focus remains undisturbed

Correct Answer: A

Question 30 : When a body is earth connected, electrons from the earth flow into the body. This means the body is:

(A) charged negatively

(B) an insulator

(C) uncharged

(D) charged positively

Correct Answer: D

Question 31 : A direct current I flows along the length of an infinitely long straight thin walled pipe, then the magnetic field is

(A) uniform throughout the pipe not zero

(B) zero only along the axis of the pipe

(C) zero at any point inside the pipe

(D) maximum at the centre and minimum at the edge

Correct Answer: C

Question 32 : If M is the mass of the earth and R its radius, the ratio of the gravitational acceleration and the gravitational constant is :

(A) $\frac{R2}{M}$

(B) $\frac{M}{R2}$

(C) MR^2

(D) $\frac{M}{R}$

Correct Answer: B

Question 33 : A student unable to answer a question on Newton's laws of motion attempts to pull himself up By tugging on his hair. He will not succed :

(A) as the force exerted is small

(B) the frictional force while gripping is small

(C) Newton's law of inertia is not applicable to living beings

(D) as the force applied is internal to the system

Correct Answer: D

Question 34 : From the adjacent figure, the correct observation is:

(A) the pressure on the bottom of tank A is greater than at the bottom of B

(B) the pressure on the bottom of the tank A is smaller than at the bottom of B

(C) the pressure depends on the shape of the container

(D) the pressure on the bottom of A and B is the same

Correct Answer: D

Question 35 : which one of the following is not a unit of young's modulus ?

(A) Nm^{-1}

(B) Nm^{-2}

(C) dyne cm^{-2}

(D) mega pascal

Correct Answer: D

Question 36 : A piece of blue glass heated to a high temperature and a piece of red glass at room temperature, are taken inside a dimly lit room, then :

(A) the blue piece will look blue and red will look as usual

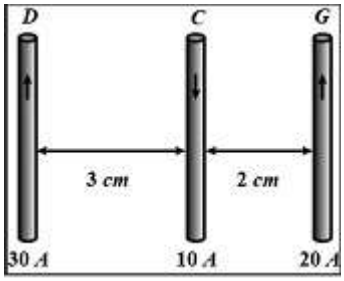
(B) red looks brighter red and blue looks ordinary blue

(C) blue shines like brighter red compared to the red piece

(D) both the pieces will look equally red

Correct Answer: C

Question 37 : Three long, straight parallel wires, carrying current, are arranged as shown in figure. The force experienced by a 25 cm length of wire C is :



(A) 10^{-3}

(B) 2.5×10^{-3} N

(C) zero

(D) 1.5×10^{-3} N

Correct Answer: C

Question 38 : A 5.0 amp.current is setup in an external circuit by a 6.0 volt storage battery for 6.0 minutes. The chemical energy of the battery is reduced by :

(A) 1.08×10^{4}J

(B) 1.08×10^{-4}J

(C) 1.8×10^{-4}J

(D) 1.8×10^{4}J

Correct Answer: A

Question 39 : The current in a simple series circuit is 5.0 amp. When an additional resistance of 2.0 ohms is inserted, the current drops to 4.0 amp. The original resistance of the circuit, in ohms, is;

(A) 1.25

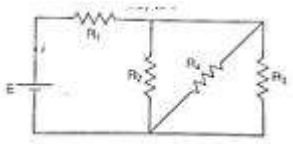
(B) 8

(C) 10

(D) 20

Correct Answer: B

Question 40 : In the circuit given E = 6.0 volt, $R_1 = 100\Omega$ $R_2 = R_3 = 50\Omega$, $R_4 = 75\Omega$. The equivalent resistance of the circuit, in ohms, is



(A) 11.875

(B) 26.31

(C) 118.75

(D) none of these

Correct Answer: C

Question 41 : Two resistance are connected in two gapes of a metre bridge. The balance point is 20 cm from the zero end. A resistance of 15 ohms is connected in series with the smaller of the two. The null point shifts to 40 cm. the value of the smaller resistance in ohms is :

(A) 3

(B) 6

(C) 9

(D) 12

Correct Answer: C

Question 42 : By using only two resistance coils-singly, in series or in parallel one should be able to obtain resistance of 3, 4, 12 and 16 ohms. The separate resistance of the coil are :

(A) 3 and 4

(B) 4 and 12

(C) 12 and 16

(D) 16 and 3

Correct Answer: D

Question 43 : The electrons in the beam of a television tube move horizontally from south to north. The vertical component of the earth's magenetic field points down. The electron is deflected towards :

(A) west

(B) no deflection

(C) east

(D) north to south

Correct Answer: C

Question 44 : A tangent galvanometer has a reduction factor of 1 A and it is placed with the plane of its coil perpendicular to the magnetic meridian. The deflection produced when a current of 1 A is passed through it is :

(A) 60^0

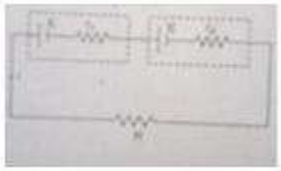
(B) 45^0

(C) 30^0

(D) none of these

Correct Answer: B

Question 45 : If the potential difference across the internal resistance r_1 is equal to the emf E of the battery, then



(A) $R = r_1 + r_2$

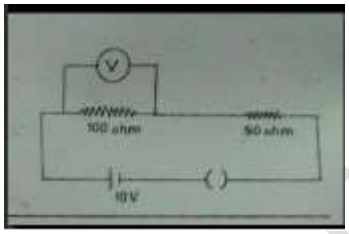
(B) $R = \frac{r_1}{r_2}$

(C) $R = r_1 - r_2$

(D) $R = \frac{r_2}{r_1}$

Correct Answer: C

Question 46 : In the given circuit, the voltmeter records 5 volts.The resistance of the voltmeter in ohms is :



(A) 200

(B) 100

(C) 10

(D) 50

Correct Answer: B

Question 47 : The wavelength of the radiation emitteeby a body depends upon:

(A) the nature of the surface

(B) the area of the surface

(C) the temperature of the surface

(D) all of the above factors

Correct Answer: C

Question 48 : Which mirror is to be used to obtain a parallel beam of light from a small lamp ?

(A) Plane mirror

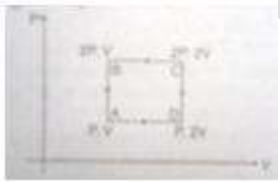
(B) convex mirror

(C) concave mirror

(D) Anyone of the above

Correct Answer: C

Question 49 : An ideal monoatomic gas is taken around the cycle ABCDA as shown in the PV diagram. The work done during the cycle is given bby :



(A) 1
2 PV

(B) PV

(C) 2 PV

(D) 4 PV

Correct Answer: B

Question 50 : Which of the following is a wrong statement ?

(A) D = 1/ *f* where *f* is the focal length and D is called the referactive power of a lens.

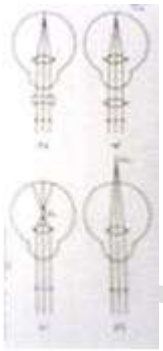
(B) Power is expressed in a diopter when *f is in metres*

(C) power is expressed in diopter and does not depend on the system of unit used to measure *f*

(D) *D is posiive for convergent lens and negative for divergent lens.*

Correct Answer: C

Question 51 : Identify the wrong description of the above figures:



(A) 1 represents far-sightedness

(B) 2 correction for short sightedness

(C) 3 represents far- sightedness

(D) 4 correction for far-sightedness

Correct Answer: A

Question 52 : An electric field of 1500 V/m and a magentic field of 0.40 weber / metre^2 act on a moving electron. The minimum uniform speed along a straight line the electron could have is :

(A) 1.6 x 10^{15}m/s

(B) 6 x 10^{-16}m/s

(C) 3.75 x 10^{3}m/s

(D) 3.75 x 10^{2}m/s

Correct Answer: C

Question 53 : In an ammeter 10% of main current is passing through the galvanometer. If the resistance of the galvanometer is G, then the shunt resistance, in ohms is:

(A) 9G

(B)

(C) 90G

(D)

Correct Answer: B

Question 54 : Among the following properties describing diamagnetism identify the property that is wrongly stated :

(A) Diamagnetic material do not have permanent magnetic moment

(B) Ddiamagnetism is explained in terms of eletromagntic induction

(C) Diamagnetic materials have a small positive susceptibility

(D) the magnetic moment of individual electrons neutralize each other

Correct Answer: C

Question 55 : The induction coil works on the principle of :

(A) self-indution

(B) mutual induction

(C) Ampere's rule

(D) Fleming's right hand rule

Correct Answer: B

Question 56 : The square root of the product of inductance and capacitance has the dimension :

(A) length

(B) mass

(C) time

(D) no dimension

Correct Answer: C

Question 57 : When a body falls in air, the resistance of air depends to a great extent on the shape of the body. 3 different shapes are given. Identify the combination of air resistances which truly represents the physical situation,(The cross sectional areas are the same)

$(20\pi, 22\pi]$

(A) 1< 2< 3

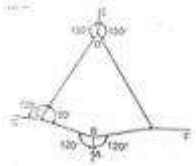
(B) 2<3<1

(C) 3<2<1

(D) 3<1<2

Correct Answer: C

Question 58 : The adjacent figure is the part of a horizontally strected net. Section ABis stretched with a force of 10 N. the tensions in the sections BC and BF are:



(A) 10 N, 11 N

(B) 10 N, 6 N

(C) 10 N, 10 N

(D) can't be calculated due to insufficient data

Correct Answer: C

Question 59 : Out of the following four dimensional quantities, which one qualities to be called a dimensional constant ?

(A) Acceleration due to gravity

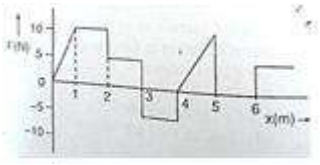
(B) Surface tension of water

(C) weight of a standard kilogram mass

(D) the velocity of light in vacuum

Correct Answer: D

Question 60 : The relationship between the force F and position x of a body is as shown in figure. The work done in displacing the body from x = 1 m to x = 5 m will be :



(A) 30 J

(B) 15 J

(C) 25J

(D) 20J

Correct Answer: B

SUBJECT: Chemistry

Question 61 : A solution contains 1.2046×10^{24} hydrochloric acid molecules In one dm^3 of the solution. The strength of the solution is:

(A) 6 N

(B) 2N

(C) 4 N

(D) 8 N

Correct Answer: B

Question 62 : Nuclear theory of the atom was put forward by:

(A) Rurherford

(B) Aston

(C) Neils Bohr

(D) J.J. Thomson

Correct Answer: A

Question 63 : In acetylene molecule, the two carbon atoms are linked by:

(A) one sigma- bond and two pi-bonds

(B) two sigma- bond and one pi-bonds

(C) three sigma- bonds

(D) three pi-bonds

Correct Answer: A

Question 64 : The enthalpy of reaction, $H_{2(g)} + O_{2(g)} \rightarrow H_2O_{(g)}$ is ΔH_1 and that of $H_{2(g)} + O_{2(g)} \rightarrow H_2O_{(l)}$ is ΔH_2. Then:

(A) $\Delta H_1 < \Delta H_2$

(B) $\Delta H_1 + \Delta H_2 = 0$

(C) $\Delta H_1 > \Delta H_2$

(D) $\Delta H_1 = \Delta H_2$

Correct Answer: A

Question 65 : A radioactive isotope decays at such a rate that offer 192 minutes only 1/16 of the original amount remains:

(A) 32 min

(B) 48 min

(C) 12 min

(D) 24 min

Correct Answer: B

Question 66 : The pressure and temperature of 4 dm^3 of carbon dioxide gas are doubled. Then the volume of carbon dioxide gas would be:

(A) 2 dm^3

(B) 3 dm^3

(C) 4 dm^3

(D) 8 dm^3

Correct Answer: C

Question 67 : 4g of copper was dissolved in concentrated nitric acid. The copper nitrate solution on strong heating gave 5 g of its oxide. The equivalent weight of copper is:

(A) 23

(B) 32

(C) 12

(D) 20

Correct Answer: B

Question 68 : in the manufacture of ammonia by Haber's process, $N_{2(g)}$ + $3H_2$ $P(AB) = 1 - P(\bar{A}\bar{B})$ $2NH_{3(g)}$ + 92.3 kJ Which of the following conditions is unfavourable?

(A) increasing the temperature

(B) increasing the pressure

(C) reducing the temperature

(D) removing ammonia as it is formed

Correct Answer: A

Question 69 : the chemical equilibrium of a reversible reaction is not influenced by:

(A) pressure

(B) catalyst

(C) concentration of the reactants

(D) temperature

Correct Answer: B

Question 70 : Cumene process is the most important commercial method for the manufacture of Phenol. Cumene is:

(A) 1-methyl ethyl benzene

(B) ethyl benzene

(C) vinyl benzene

(D) propyl benzene

Correct Answer: A

Question 71 : The reagent which does not give acid chloride on trating with a carboxylic acid is:

(A) PCl_5

(B) Cl_2

(C) $SOCl_2$

(D) PCl_3

Correct Answer: B

Question 72 : Among the halogens, the one which is oxidized by nitric acid is:

(A) fluorine

(B) iodine

(C) chlorine

(D) bromine

Correct Answer: B

Question 73 : The metal which does not form ammonium nitrate by reaction with dilute nitric acid is:

(A) Al

(B) Fe

(C) Pb

(D) Mg

Correct Answer: C

Question 74 : The elements with atomic numbers 9,17,35,53,85 are all:

(A) noble gases

(B) halogens

(C) heavy metals

(D) light metals

Correct Answer: B

Question 75 : In the electrolytic method of obtaining aluminum from purified bauxite, cryolite is added to the charge in order to:

(A) minimize the heat loss due to radiation

(B) protect aluminium produced from oxygen

(C) dissolve bauxite and render it conductor of electricity

(D) lower the melting point of bauxite

Correct Answer: C

Question 76 : The number of 2p electrons having spin quantum number s= -1/2 are:

(A) 6

(C) 2

(D) 3

Correct Answer: D

Question 77 : Pick out the alkane which differs from the other members of the group:

(A) 2,2-dimethyl propane

(B) pentane

(C) 2-methyl butane

(D) 2,2-dimethyl butane

Correct Answer: D

Question 78 : 56 g of nitrogen and 8g of hydrogen gas are heated in a closed vessel. At equilibrium 34 g of ammonia are present. The equilibrium number of moles of nitrogen, hydrogen and ammonia are respectively:

(A) 1,2,2

(B) 2,2,1

(C) 1,1,2

(D) 2,1,2

Correct Answer: C

Question 79 : A process is taking place at constant temperature and pressure. Then :

(A) $\Delta H = \Delta E$

(B) $\Delta H = T\Delta S$

(C) $\Delta H = 0$

(D) $\Delta S = 0$

Correct Answer: A

Question 80 : In a galvanic cell, the electrons flow from:

(A) anode to cathode through the solution

(B) cathode to anode through the solution

(C) anode to cathode through the external circuit

(D) cathode to anode through the external circuit

Correct Answer: C

$$P\left(\frac{A}{B}\right) - P(B).\ P\left(\frac{B}{A}\right) = P(A)$$

Question 81 : The reaction, $2SO_{2(g)} + O_2 \rightarrow 2SO_{3(g)}$ is carried out in a 1 dm^3 vessel and 2 dm^3 vessel separately. The ratio of the reaction velocities will be:

(A) 1:8

(B) 1:4

(C) 4:1

(D) 8:1

Correct Answer: D

Question 82 : In a mixture of acetic acid and sodium acetate the ratio of concentrations of the salt to the acid is increased ten times. Then the pH of the solution:

(A) increases by one

(B) decreases by one

(C) decreases ten fold

(D) increases ten fold

Correct Answer: A

Question 83 : When a mixture of methane and oxygen is passed through heated molybdenum oxide, the main product formed is:

(A) methanoic acid

(B) ethanal

(C) methanol

(D) methanal

Correct Answer: D

Question 84 : Benzene can be obtained by heating either benzoic acid with X or phenol Y.X and Y are respectively:

(A) zinc dust and soda lime

(B) soda lime and zinc dust

(C) zinc dust and sodium hydroxide

(D) soda lie and copper

Correct Answer: B

Question 85 : An organic compound is boiled with alcoholic potash. The product is cooled and acidified with HCl. A white solid separates out. The starting compound may be:

(A) ethyl benzoate

(B) ethyl formate

(C) ethyl acetate

(D) methyl acetate

Correct Answer: A

Question 86 : A nitrogen containing organic compound gave an oily liquid on heating with bromine and potassium hydroxide solution. On shaking the product with acetic anhydride, an antipyretic drug was obtained. The reactions indicate that the starting compound is:

(A) aniline

(B) benzamide

(C) acetamide

(D) nitrobenzene

Correct Answer: B

Question 87 : The silver salt of a fatty acid on refluxing with an alkyl halide gives an:

(A) acid

(B) ester

(C) ether

(D) amine

Correct Answer: B

Question 88 : Pick out the one which does not belong to the family:

(A) pepsin

(B) cellulose

(C) ptyalin

(D) lipase

Correct Answer: B

Question 89 : Which one of the following is wrongly matched?

(A) Saponification of $CH_3COOC_2H_5$ - Second order reaction

(B) Hydrolusis of CH_3COOCH_3 - Pseudo molecular reaction

(C) Decomposition of H_2O_2 – First order reaction

(D) Combination of H_2 and Br_2 to give HBr – First order reaction

Correct Answer: D

Question 90 : The diameter of colloidal particles rang from:

(A) 10^{-6} m to 10^{-9} m

(B) 10^{-9} m to 10^{-12}m

(C) 10^{3} m to 10^{-3} m

(D) 10^{-3} m to 10^{-6} m

Correct Answer: A

Question 91 : On treating a mixture of two alkyl halides with sodium metal in dry ether, 2-methyl propane was obtained. The alkyl halides are:

(A) 2-chloropropane and chloromethane

(B) 2-chloropropane and chloroethane

(C) chloromethane and chloroethane

(D) chloromethane and1-chloropropane

Correct Answer: A

Question 92 : Which of the following statements about benzyl chloride is incorrect?

(A) it is less reactive than alkyl halides

(B) it can be oxidized to benzaldehyde by boiling with copper nitrate solution.

(C) it is a lachrymatory liquid and answers Beilstein's test

(D) it gives a white precipitate with alcoholic silver nitrate

Correct Answer: A

Question 93 : The main product obtained when a solution of sodium carbonate reacts with mercuric chloride is:

(A) $Hg(OH)_2$

(B) $HgCO_3.HgO$

(C) $HgCO_3$

(D) $HgCO_3.Hg(OH)_2$

Correct Answer: B

Question 94 : In the electrothermal process, the compound displaced by silica from calcium phosphate is:

(A) calcium phosphide

(B) phosphine

(C) phosphorus

(D) phosphorus pentoxide

Correct Answer: D

Question 95 : The enthalpy of combustion of methane at 25^{o} C is 890 kJ. The heat liberated when 3.2 g of methane is burnt is air is:

(A) 445 kJ

(B) 278 kJ

(C) -890 kJ

(D) 178 kJ

Correct Answer: D

Question 96 : The velocity constant of a reaction at 290 K was found to be 3.2 x 10^{-3} s^{-1}. When the temperature is raised to 310 K, it will be about:

(A) 6.4 x 10^{-3}

(B) 3.2 x 10^{-4}

(C) 9.6 x 10^{-3}

(D) 1.28 x 10^{-2}

Correct Answer: D

Question 97 : Select the pK_a value of the strongest acid from the following:

(A) 1.0

(B) 3.0

(C) 2.0

(D) 4.5

Correct Answer: A

Question 98 : Pick out the unsaturated fatty acid from the following:

(A) stearic acid

(B) lauric acid

(C) oleic acid

(D) plamitic acid

Correct Answer: C

Question 99 : Nylon is not a :

(A) condensation polymer

(B) polyamide

(C) copolymer

(D) homopolymer

Correct Answer: D

Question 100 : The coal tar fraction which contains phenol is:

(A) middle oil

(B) green oil

(C) heavy oil

(D) light oil

Correct Answer: A

Question 101 : The compounds A and B are mixed in equimolar proportion to form the products,A + B $P(A \cap B) = \frac{P(A)}{P(B)}$ C + D . At equilibrium, one third of A and B are consumed. The equilibrium constant for the reaction is:

(A) 0.5

(B) 4.0

(C) 2.5

(D) 0.25

Correct Answer: D

Question 102 : In forth floatation process for the purification of ores, the particles of ore float because:

(A) their surface is not easily wetted by water

(B) they are light

(C) they are insoluble

(D) they bear electrostatic charge

Correct Answer: A

Question 103 : Which of the following statements about amorphous solids is incorrect ?

(A) They melt over a range of temperature

(B) They are anisotropic

(C) They is no orderly arrangement of particles

(D) They are rigid and incompressible

Correct Answer: B

Question 104 : Hydrogen diffuse six times faster than gas A. the molar mass of gas A is:

(A) 72

(B) 6

(C) 24

(D) 36

Correct Answer: A

Question 105 : Dulong and Petit's law is valid only for:

(A) metals

(B) non-metals

(C) gaseous elements

(D) solid elements

Correct Answer: D

Question 106 : Identify the gas which is readily adsorbed by activated charcoal:

(A) N_2

(B) SO_2

(C) H_2

(D) O_2

Correct Answer: B

Question 107 : If the distance between Na^+ and Cl^- ions in sodium chloride crystal is X pm, the

length of the edge of the unit cell is:

(A) 4 X pm

(B) X/4 pm

(C) X/2 pm

(D) 2X pm

Correct Answer: D

Question 108 : Which of the following statement is incorrect?

(A) In $[K_3Fe(CN)_6]$ the ligand has satisfied only the secondary valency of ferric ion

(B) In $[K_3Fe(CN)_6]$ the ligand has satisfied both primary and secondary valency of ferric ion

(C) In $[K_3Fe(CN)_6]$ the ligand has satisfied both primary and secondary valency of ferrous ion

(D) In $[K_3Fe(CN)_6]$ the ligand has satisfied only the secondary valency of coper

Correct Answer: A

Question 109 : 2-acetoxy benzoic acid is used as an:

(A) antimalarial

(B) antidepressant

(C) antiseptic

(D) antipyretic

Correct Answer: D

Question 110 : A nucleoside on hydrolysis gives:

(A) a heterocyclic base and orthophosphoric acid

(B) an aldopentose, a heterocyclic base and orthophosphoric acid

(C) an aldopentose, a heterocyclic base

(D) an aldopentose and orthophosphoric acid

Correct Answer: C

Question 111 : In qualitative analysis, in order to detect second group basic radical, H_2S gas is passed in the presence of dilute HCl to :

(A) increase the dissociation of H_2S

(B) decrease the dissociation of salt solution

(C) decrease the dissociation of H_2S

(D) increase the dissociation of of salt solution

Correct Answer: C

Question 112 : Aluminium displaces hydrogen from dilute HCl whereas silver does not. The e.m.f. of a cell prepared by combining Al/Al^{3+} and Ag/Ag^{+} is 2.46 V. the reduction potential of silver electrode is +0.80 V. The reduction potential of aluminium electrode is:

(A) +1.66 V

(B) -3.26 V

(C) 3.26 V

(D) -1.66 V

Correct Answer: D

Question 113 : The first reaction obtained during the fractionation of petroleum is:

(A) hydrocarbon gases

(B) kerosene oil

(C) gasoline

(D) diesel oil

Correct Answer: A

Question 114 : Which of the following compounds gives trichloromethane on distilling or bleaching powder?

(A) Methanal

(B) Phenol

(C) Ethanol

(D) Methanol

Correct Answer: C

Question 115 : Benzoin is:

(A) compound containing an aldehyde and a ketonic group

(B) **α**, β-unsaturated acid

(C) **α**-hydroxy aldehyde

(D) **α**- hydroxyl ketone

Correct Answer: D

Question 116 : 15 Moles of H_2 and 5.2 moles of I_2 are mixed and allowed tyo attain equilibrium at 500^0C. At equilibrium, the concentration of HI is found to be 10 moles. The equilibrium constant for the formation of HI is :

(A) 50

(B) 15

(C) 100

(D) 25

Correct Answer: A

Question 117 : If, in the reaction N_2O_4 $2NO_2$, x is that part of N_2O_4 which dissociates, then the number of molecules at equilibrium will be :

(A) 1

(B) 3

(C) (1+x)

(D) $(1+x)^2$

Correct Answer: C

Question 118 : Which of these dose not influence the rate of reaction ?

(A) Nature of the reactants

(B) Concentration of the reactants

(C) Temperature of the reaction

(D) Molecularity of the reaction

Correct Answer: D

Question 119 : For the reaction A + B → C, it is found that doubling the concentration of A increases the rate by 4 times, and doubling the concentration of B doubles the reaction rate. What is the overall order of the reaction:

(A) 4

(B) 3/2

(C) 3

(D) 1

Correct Answer: C

Question 120 : The rate at which a substance reacts depends on its:

(A) atomic weight

(B) atomic number

(C) molecular weight

(D) active mass

Correct Answer: D

SUBJECT: Mathematics

Question 121 :

The universal set for the sets
A = {x : x^2 – 5x + 6 = 0} and
B = {x : x^2 – 3x + 2 = 0} having least number of elements is

(A) {1, 2}

(B) {1, 3}

(C) {1, 2, 3}

(D) {0, 1, 2, 3}

Solution :

A = {x : x^2 – 5x + 6 = 0} = {2, 3}

B = {x : x^2 – 3x + 2 = 0} = {1, 2}

Clearly, the universal set for A and B with least elements is {1, 2, 3}.

Correct Answer: C

Question 122 :

If A = {1, 2, 3}, B = {x € R : x2 – 2x + 1 = 0},

C = {1, 2, 2, 3} and

D = {x € R : x3 – 6x2 + 11x – 6 = 0}, then the equal sets are

(A) A and B

(B) A and C

(C) A, B and C

(D) A, C and D

Solution :

Here, B = {x€R : (x – 1)2 = 0} = {1}

C = {1, 2, 2, 3} = {1, 2, 3}

[repetition of elements in a set is immaterial]

and D = {x € R : x3 – 6x2 + 11x – 6 = 0}

= {x € R : (x – 1)(x – 1 – 5x + 6) = 0}

= {1, 2, 3}

Correct Answer: D

Question 123 : Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A ={3, 6, 9, 12}
The relation is

(A) an equivalence reation

(B) reflexive symmetric

(C) reflexive and trasitive

(D) Only reflexive

Correct Answer: C

Question 124 : Let A = {5, 6, 7} be a set and define a relation R = {(5, 6), (6, 5)}. Then, R is

(A) reflexive and transitive

(B) symmetric only

(C) reflexive and symmetric

(D) equivalence relation

Since, (5, 5), (6, 6), (7, 7) ∉ R
Therefore, R is not reflexive.
Now, (5, 6) ∈ R
⇒ (6, 5) ∈ R
So, R is symmetric.
Also, (5, 6), (6, 5) ∈ R
but (5, 5) ∉ R
Thus, R is not transitive.

Solution : Hence, R is symmetric only.

Correct Answer: B

Question 125 : The locus of the poins z which satisfy the condition $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$, is

(A) a straight line

(B) a circle

(C) a parabola

(D) none of these

Let $z = x + iy$

$$\therefore \frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x^2+y^2-1)+2iy}{(x+1)^2+y^2}$$

$$\therefore \arg\left(\frac{z-1}{z+1}\right) = \tan^{-1}\frac{2y}{x^2+y^2-1}$$

$$\Rightarrow \tan^{-1}\frac{2y}{x^2+y^2-1} = \frac{\pi}{3} \quad \text{(given)}$$

$$\Rightarrow \frac{2y}{x^2+y^2-1} = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\Rightarrow x^2 + y^2 - 1 = \frac{2}{\sqrt{3}}y$$

$$\Rightarrow x^2 + y^2 - \frac{2}{\sqrt{3}}y - 1 = 0$$

Solution : which is an equation of a circle.

Correct Answer: B

Question 126 : If $x^{2n} + 1 = 0$, $x \in R$, then root of the equation is/are

(A) x = 1

(B) x = –1

(C) x = ±1

(D) No real roots

Solution :

$x^{2n} + 1 = 0$, $x \in R$, $x^{2n} = -1$ but $x^{2n} > 0$.

So, above equation has no real roots.

Correct Answer: D

Question 127 : The sum of the geometric progression 0.15, 0.015, 0.0015, ... 20 terms is

(A) $\frac{1}{6}[1-(0.1)^{20}]$

(B) $[1-(0.1)^{20}]$

(C) $\frac{1}{6}[1+(0.1)^{20}]$

(D) None of these

Solution :

Here, $a = 0.15$,

$$r = \frac{0.015}{0.15} = \frac{15}{1000} \times \frac{100}{15}$$

$$r = \frac{1}{10} < 1 \text{ and } n = 20$$

$$\text{Now, } S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\Rightarrow S_{20} = \frac{0.15\left[1-\left(\frac{1}{10}\right)^{20}\right]}{1-\frac{1}{10}}$$

$$= \frac{15 \times 10}{900}\left[1-\left(\frac{1}{10}\right)^{20}\right]$$

$$= \frac{1}{6}[1-(0.1)^{20}]$$

Correct Answer: A

Question 128 : The value of 0.234 is

(A) $\frac{232}{990}$

(B) $\frac{232}{9990}$

(C) $\frac{232}{900}$

(D) $\frac{232}{9909}$

Solution :

$$0.234 = 0.2343434.....$$
$$= 0.2 + 0.034 + 0.00034 + ...$$
$$= \frac{2}{10} + 34\left(\frac{1}{10^3} + \frac{1}{10^5} + ...\right)$$
$$= \frac{2}{10} + 34 \times \frac{1}{1000} \times \frac{100}{99}$$
$$= \frac{2}{10} + \frac{34}{990} = \frac{232}{990}$$

Correct Answer: A

Question 129 : n(n + 1)(n + 5) is a multiple of

(A) 5

(B) 6

(C) 7

(D) None of these

Solution :

We have, n(n + 1)(n + 5) = n(n + 1) [(n + 2) + 3]

= n(n + 1)(n + 2) + 3n(n + 1)

Now, n(n + 1)(n + 2) is a product of three consecutive natural numbers and so it is divisible by 3! i.e.

6.

Again, n(n + 1) is a product of two consecutive natural numbers and so it is divisible by 2! i.e. consequently 3n(n + 1) is divisible by 6.

Thus, n(n + 1)(n + 5) is divisible by 6.

Correct Answer: B

Question 130 :

(A) $\frac{17}{26}$

(B) $\frac{25}{26}$

(C) $\frac{13}{16}$

(D) $\frac{15}{16}$

Solution : $\text{HCF}(1095, 1168) = 73$

$\therefore \frac{1095}{1168} = \frac{1095+73}{1168+73} = \frac{15}{16}$

Correct Answer: D

Question 131 : Out of 7 consonants and 4 vowels, the humber of words (not necesarily meaningful) that can be made, each consisting of 3 consonants and 2 vowels, is

(A) 24800

(B) 25100

(C) 25200

(D) 25400

Solution :

3 consonants can be selected from 7 consonants
$= {}^7C_3$ ways
2 vowels can be selected from 4 vowels
$= {}^4C_2$ ways
$\therefore$ Required number of words
$= {}^7C_3 \times {}^4C_2 \times 5!$
[selected 5 letters can be arranged in 5! ways, to get a different word]

$35 \times 6 \times 120 = 25200$

Correct Answer: C

Question 132 : A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if atleast one black ball is to be included in the draw?

(A) 64

(B) 24

(C) 3

(D) 12

Solution :

Case I: When 3 drawn balls contain 1 black ball

$= {}^6C_2 \times {}^3C_1 = \frac{6 \times 5}{2 \times 1} \times 3 = 45$

Case II: When 3 drawn balls contain 2 black balls

$= {}^6C_1 \times {}^3C_2 = 6 \ \frac{3 \times 2}{2 \times 1} = 18$

Case III: When 3 drawn balls contain all black balls
$= {}^3C_3 = 1$
On adding all three cases,we get required number of ways = 45 + 18 + 1 = 64

Correct Answer: A

Question 133 : If the coefficients of 2^{nd}, 3^{rd} and the 4^{th} terms in the expansion of (1 + x)n are in AP, then value of n is

(A) 2

(B) 7

(C) 11

(D) 14

Solution :

Given expression is $(1 + x)^n$.

Coefficients of 2nd, 3rd and 4th terms are ${}^nC_1$, ${}^nC_2$ and ${}^nC_3$, respectively.

Since, ${}^nC_1$, ${}^nC_2$, ${}^nC_3$ are in AP.

$\Rightarrow 2{}^nC_2 = {}^nC_1 + {}^nC_3$

$$\Rightarrow 2=\frac{{}^nC_1}{{}^nC_2}+\frac{{}^nC_3}{{}^nC_2} \Rightarrow 2=\frac{2}{n-1}+\frac{n-2}{3}$$

$\Rightarrow n^2 - 9n + 14 = 0 \Rightarrow n = 2, 7$

But $n \neq 2$

$\therefore n = 7$

Correct Answer: B

If $\frac{e^x}{1-x}=B_0+B_1x+B_2x^2+.....+B_nx^n+....$, then

Question 134 : $B_n - B_{n-1}$ equals

(A) $\frac{1}{n!}$

(B) $\frac{1}{(n-1)!}$

(C) $\frac{1}{n!}-\frac{1}{(n-1)!}$

(D) 1

Solution :

We have, $\frac{e^x}{1-x} = B_0 + B_1x + B_2x^2 + ... + B_nx^n + ...$

$\Rightarrow e^x(1-x)^{-1} = B_0 + B_1x + B_2x^2 + ... + B_nx^n + ...$

$\Rightarrow \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + ...\right)(1 + x + x^2 + ...)$

$= B_0 + B_1x + B_2x^2 + ...$

$\Rightarrow (1 + x + x^2 + ...)$

$\left(1 + \frac{x}{1!} + \frac{x^2}{1!} + ...\right) + \left(\frac{x^2}{2!} + \frac{x^3}{2!} + ...\right) + ...$

$= B_0 + B_1x + B_2x^2 + ...$

$\Rightarrow 1 + \left(1 + \frac{1}{1!}\right)x + \left(1 + \frac{1}{1!} + \frac{1}{2!}\right)x^2 + ...$

$= B_0 + B_1x + B_2x^2 + ...$

On comparing, we get $B_0 = 1$,

$B_1 = 1 + \frac{1}{1!}$, $B_2 = 1 + \frac{1}{1!} + \frac{1}{2!}$ .....

$\therefore B_n = 1 + \frac{1}{1!} + \frac{1}{2!} ..... + \frac{1}{n!}$

Now, $B_n - B_{n-1} = \left(1 + \frac{1}{1!} + ... \frac{1}{n!}\right)$

$- \left(1 + \frac{1}{1!} + ... \frac{1}{(n-1)!}\right) = \frac{1}{n!}$

Correct Answer: A

Question 137 : The region represented by 2x + y > 6 is

(A) bounded

(B) unbounded

(C) does not exist

(D) None of these

Solution :

We have the given inequality,

$2x + y \geq 6$ ....(i)

Step I: Consider the inequation as a strict equation,

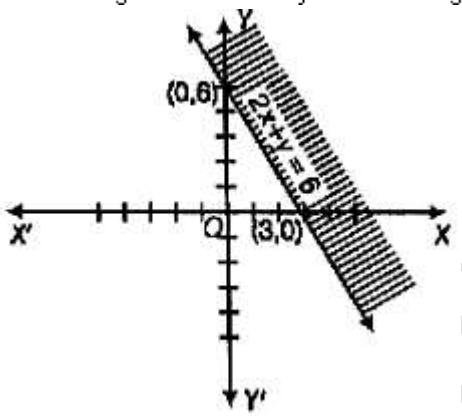
i.e. $2x + y = 6$

Step II: Find the points on X-axis and Y-axis, i.e.

x	3	0
y	0	6

Step III: Plot the graph using the above table.

Step IV: Take a point (0, 0) and put it in the given inequation (i), we get $0 + 0 \geq 6$ which is false, so shaded region will be away from the origin.



Here, shaded region shows the inequality

$2x + y > 6$.

Note: The region is said to be bounded, if it is enclosed. otherwise it is unbounded.

Correct Answer: B

Question 138 : The graphical solution $-3x + 2y > -6$ is represented by shade the

(A) half plane II including the points on the line $-3x + 2y = -6$

(B) half plane II excluding the points on the line $-3x + 2y = -6$

(C) half plane I including the points on the line –3x + 2y = –6

(D) half plane I excluding the points on the line –3x + 2y = –6

Solution :

The given inequation is

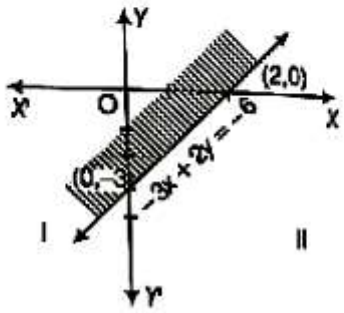
–3x + 2y ≥ –6 .....(i)

Step I: Consider the inequation as a strict equation

i.e. 3x + 2y = –6

Step II: Find the points on the X-axis and Y-axis i.e.

x	0	2
y	–3	0



Step III: Plot the graph using the above table.

Step IV: Take a point (0, 0) and put it in the given inequation (i), we get 0 + 0 ≥ –6 which is true, so the shaded region will be towards the origin.

Hence, the shaded half plane I including the points on the line is the solution of given inequality.

Correct Answer: C

Question 139 : If A is matrix of order m × n and B is a matrix such that AB' and B'A are both defined, then order of matrix B is

(A) m × m

(B) n × n

(C) n × m

(D) m × n

Given, that the order of A = m × n
Since AB′ and B′A are both defined.
Then, for AB, B must be of the order like as
n × p .....(i)
But for B′A, B′ must be the order like as
q × m ....(ii)
∴ From Eqs. (i) and (ii), we get the required order of B′ is n × m.

Solution : ⇒ Order of matrix B is m × n.

Correct Answer: D

If $A=\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix}$ and $kA=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$ then the values of k, a, b are respectively

Question 140 :

(A) –6, –12, –18

(B) –6, 4, 9

(C) –6, –4, –9

(D)

–6, 12, 18

$$Q\ kA=\begin{bmatrix}0 & 3\alpha\\ 2b & 24\end{bmatrix}$$

$$\Rightarrow k\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix}=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0 & 2k\\ 3k & -4k\end{bmatrix}=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$$

$$\Rightarrow 2k=3a,\ 3k=2b,\ -4k=24$$

$$\Rightarrow a=\frac{2k}{3},\ b=\frac{3k}{2},\ k=-6$$

Solution : ∴ a = –4, b = –9, k = –6

Correct Answer: C

Question 141 : $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$ is equal to

(A) $(5x - 4)(4 + x)^2$

(B) $(5x + 4)(4 - x)^2$

(C) $(5x - 4)^2 (4 + x)$

(D) $(5x + 4)^2 (4 - x)$

Solution : $(5x + 4)(4 - x)^2$

Correct Answer: B

Question 142 : The roots ot the equatoin $\begin{vmatrix} x & \alpha & 1 \\ \beta & x & 1 \\ \beta & \gamma & 1 \end{vmatrix} = 0$ are independent of

(A) α

(B) β

(C) λ

(D) α, β and λ

Given $\begin{vmatrix} x & \alpha & 1 \\ \beta & x & 1 \\ \beta & \gamma & 1 \end{vmatrix} = 0$

Use operations $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$,

$$\begin{vmatrix} x & \alpha & 1 \\ \beta - x & x - \alpha & 0 \\ \beta - x & \gamma - \alpha & 0 \end{vmatrix} = 0$$

Expand along C_3,

$(\beta - x)(\gamma - \alpha) - (x - \alpha)(\beta - x) = 0$

$\Rightarrow (\beta - x)\{(-\alpha + \gamma) - (x - \alpha)\} = 0$

$\Rightarrow (\beta - x)\{-\alpha + \gamma - x + \alpha\} = 0$

$\Rightarrow (\beta - x)(\gamma - x) = 0$

$x = \beta \, . \, \gamma$

Solution : So, roots of the given equation is independent ot α

Correct Answer: A

Question 143 : The general solution of sin

$\sin 3x + \sin x - 3 \sin 2x = \cos 3x + \cos x - 3 \cos 2x$ is

(A) $\frac{n\pi}{2} + \frac{\pi}{8}$ for n integer

(B) $\frac{n\pi}{2} - \frac{\pi}{8}$ for n integer

(C) $\frac{n\pi}{2} + \frac{\pi}{6}$ for n integer

(D) $\frac{n\pi}{2} - \frac{\pi}{6}$ for n integer

Solution :

Given that,

$(\sin 3x + \sin x) - 3 \sin 2x = (\cos 3x + \cos x) - 3 \cos 2x$

$\Rightarrow 2. \sin 2x. \cos x - 3 \sin 2x = 2 \cos 2x. \cos x - 3 \cos 2x$

$\Rightarrow \sin 2x(2 \cos x - 3) = \cos 2x(2 \cos x - 3)$

$\Rightarrow (2 \cos x - 3) (\sin 2x - \cos 2x) = 0$

$\therefore \cos x = \frac{3}{2}$, which is not possible.

$[Q - 1 \leq \cos x \leq 1]$

$\therefore \sin 2x = \cos 2x$

$\Rightarrow \tan 2x = 1 = \tan \frac{\pi}{4}$

$\Rightarrow 2x = n\pi + \frac{\pi}{4}$

$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}$, for n integer

Correct Answer: A

Question 144 : If $\sin 2x = 4 \cos x$, then is equal to

(A) $\frac{n\pi}{2} \pm \frac{\pi}{4}, n \in Z$

(B) no value

(C) $n\pi + (-1)^n \frac{\pi}{4}, n \in Z$

(D) $2n\pi \pm \frac{\pi}{2}, n \in Z$

Solution :

We have, $\sin 2x = 4 \cos x$

$\Rightarrow 2 \sin x \cos x = 4 \cos x$

$\Rightarrow \cos x (\sin x - 2) = 0$

$\Rightarrow \cos x = 0 \qquad [Q \sin x \neq 2]$

$\Rightarrow \cos x = 0 = \cos \frac{\pi}{2}$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{2}, n \in Z$

Correct Answer: D

Question 145 : $\cot^{-1}(2.1^2) + \cot^{-1}(2.2^2) + \cot^{-1}(2.3^2) + \ldots$ upto ∞ is equal to

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{5}$

$\cot^{-1}(2.1^2) + \cot^{-1}(2.2^2) + \cot^{-1}(2.3^2) + \ldots\ldots \infty$

$= \sum_{r=1}^{\infty} cot^{-1}(2.r^2)$

$\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{2r^2}\right)$

$= \sum_{r=1}^{\infty} \tan^{-1}\left[\frac{(1+2r)+(1-2r)}{1-(1+2r)(1-2r)}\right]$

$= \sum_{r=1}^{\infty}\left[\tan^{-1}(1+2r) + \tan^{-1}(1-2r)\right]$

$= \tan^{-1}3 - \tan^{-1}1 + \tan^{-1}5 - \tan^{-1}3 + \tan^{-1}7$
$- \tan^{-1}5 + \ldots\ldots + \tan^{-1}\infty$

$= -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$

Solution :

Correct Answer: A

If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, then $\cos^{-1}x + \cos^{-1}y$ is equal to

Question 146 :

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) $\frac{3\pi}{4}$

Solution :

Given, $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$

$\therefore \frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} y = \frac{\pi}{2}$

$\Rightarrow \cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$

Correct Answer: A

Question 147 : $\lim_{m \to \infty} \left(\cos \frac{x}{m}\right)^m$ is equal to

(A) ZERO

(B) e

(C) $\frac{1}{e}$

(D) 1

$$\lim_{m \to \infty} \left(\cos \frac{x}{m}\right)^m$$

$$= \lim_{m \to \infty} \left[1 + \left(\cos \frac{x}{m} - 1\right)\right]^m$$

$$= \lim_{m \to \infty} \left[1 - \left(1 - \cos \frac{x}{m}\right)\right]^m$$

$$= \lim_{m \to \infty} \left[1 - 2 \sin^2 \frac{x}{2m}\right]^m$$

$$= e^{\lim_{m \to \infty} m\left[-2 \sin^2 \frac{x}{2m}\right]}$$

$$= e^{\lim_{m \to \infty} -2\left(\sin \frac{\frac{x}{2m}}{\frac{x}{2m}}\right)^2 \left(\frac{x^2}{4m^2}\right) m}$$

Solution : $= e^{-2 \lim_{m \to \infty} \frac{x^2}{4m}} = e^0 = 1$

Correct Answer: D

Question 148 : If $f(x) = (x5 – 1)(x3 + 1)$, $g(x) = (x2 – 1)(x2 – x + 1)$ and h(x) be such that $f(x) = g(x)\, h(x)$, then lim h(x) is equal to $x \to 1$

(A) 5

(B) 1

(C) 3

(D) 4

Given, $f(x) = (x^5 - 1)\ (x^3 + 1)$

$g(x) = (x^2 - 1)\ (x^2 - x + 1)$

Q $f(x) = g(x)\ h(x)$

$$\therefore\ h(x) = \frac{f(x)}{g(x)}$$

$$\lim_{x \to 1} h(x) = \lim_{x \to 1} \frac{(x^5 - 1)(x^3 + 1)}{(x^2 - 1)(x^2 - x + 1)}$$

$$= \lim_{x \to 1} \frac{(x^5 - 1)(x + 1)(x^2 - x + 1)}{(x - 1)(x + 1)(x^2 - x + 1)}$$

$$= \lim_{x \to 1} \frac{(x^5 - 1)}{(x - 1)} \qquad \left[\frac{0}{0} \text{ form}\right]$$

Solution :

$$= \lim_{x \to 1} \frac{5x^4}{1}$$

[using L'Hospital's rule]

$$= \frac{5(1)^4}{1} = 5$$

Correct Answer: A

Question 149 : If f(x) = sin(cosx), then f ' (x) is

(A) cos(cosx)

(B) sin(–sinx)

(C) –sin(cosx)

(D) –sinx cos(cosx)

Solution :

We have, $f(x) = \sin(\cos x)$

On differentiating w.r.t x, we get

$$f'(x) = \cos(\cos x)\frac{d}{dx}(\cos x)$$

$$\therefore f'(x) = \cos(\cos x).(-\sin x)$$

$$= -\sin x.\cos(\cos x)$$

Correct Answer: D

Question 150 : If $f(x) = \sin^2 x^2$, then what is the value of f '(x)?

(A) $4x \sin(x^2) \cos(x^2)$

(B) $2 \sin(x^2) \cos(x^2)$

(C) $4 \sin(x^2) \sin^2 x$

(D) $2x \cos^2 x^2$

Solution :

$$Q\ f(x) = \sin^2 x^2$$

$$\Rightarrow f'(x) = 2\sin x^2.\cos x^2.\frac{d}{dx}(x^2)$$

$$\therefore f'(x) = 2\sin x^2.\cos x^2.2x$$

$$= 4x\ \sin x^2\ \cos x^2$$

Correct Answer: A

Question 151 : Let f be the differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for all $x \in [1, 6]$, then the minimum value of

(A) 4

(B) 2

(C) 8

(D) None of these

Solution :

By Lagrange's mean value theorem there exists c Ï

(1, 6), such that

$$f'(c) = \frac{f(6) - f(1)}{6 - 1}$$

$$\Rightarrow \frac{f(6) + 2)}{5} = f'(c) \quad [\text{Q } f'(x) \geq 2 \text{ for all} \in [1, 6]]$$

$$\Rightarrow f(6) + 2 \geq 10$$

$$\Rightarrow f(6) \geq 8$$

Correct Answer: C

Question 152 : The speed v of a particle moving along a straight line is given by $a + bv^2 = x^2$ (where x is its distance from the origin). The acceleration of the particle is

(A) bx

(B) x/a

(C) x/b

(D) x/ab

Solution :

Given equatoin is $a + bv^2 = x^2$

On differnetiating, we get

$$\Rightarrow 0 + b\left(2v\frac{dv}{dt}\right) = 2v\frac{dx}{dt}$$

$$\Rightarrow vb\frac{dv}{dt} = x\frac{dx}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{x}{vb}.\frac{dx}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{x}{b} \qquad \left(\text{Q } \frac{dx}{dt} = v\right)$$

Correct Answer: C

Question 153 : $\int_{-2}^{2} |x|\,dx$ is equal to

(A) ZERO

(B) 1

(C) 2

(D) 4

Solution :

$$\text{Let } I = \int_{-2}^{2} |x|\,dx = -\int_{-2}^{0} x\,dx + \int_{0}^{2} x\,dx$$

$$= -\left[\frac{x^2}{2}\right]_{0}^{2} + \left[\frac{x^2}{2}\right]_{0}^{2} = -(0-2)$$

+ (2 - 0) = 4

Correct Answer: D

Question 154 : The area enclosed between the curves $y = x^3$ and $y = \sqrt{x}$ is [in sq. unit]

(A) 5/3

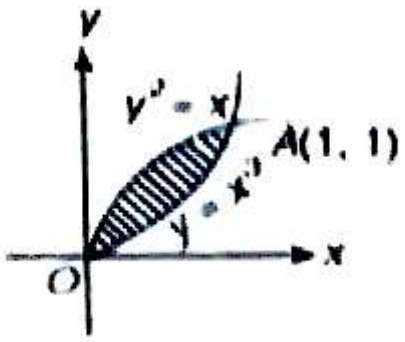
(B) 5/4

(C) 5/12

(D) 12/5

Solving $y = \sqrt{x}$ or $y^3 = -x(y = 0)$ and $y = x^3$

We get points of intersection (0, 0) and (1, 1)



Solution : $\therefore$ Required area $= \int_0^1 (\sqrt{x} - x^3)dx$

$= \left[\frac{x^{3/2}}{3/2} - \frac{x^4}{4}\right]_0^1 = \frac{5}{12}$ sq. units

Correct Answer: C

Question 155 : The differential equation of all non-vertical lines in a plane is

(A) $\frac{d^2y}{dx^2} = 0$

(B) $\frac{d^2x}{dy^2} = 0$

(C) $\frac{dy}{dx} = 0$

(D) $\frac{dx}{dy} = 0$

Solution :

The general equation of all non-vertical ines in a plane is $ax + by = 1$,
where $b \neq 0$.
On differentiating both sides w.r.t. x, are

get $a + b\frac{dy}{dx} = 0$

Again, differentiating w.r.t. x, we get

$b\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$

Correct Answer: A

Question 156 : The solution of the differential equation $\frac{dy}{dx} = \frac{x-2y+1}{2x-4y}$ is

(A) $(x - 2y)^2 + 2x = C$

(B) $(x - 2y)^2 + x = C$

(C) $(x - 2y)^2 + 2x^2 = C$

(D) $(x - 2y)^2 + x^2 = C$

Solution :

Given, $\frac{dy}{dx} = \frac{x-2y+1}{2x-4y}$

Put $x - 2y = z$

$\Rightarrow 1 - 2\frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \frac{1}{2}\left[-\frac{dz}{dx} + 1\right] = \frac{z+1}{2z} \Rightarrow 2dz = -dx$

$\Rightarrow \frac{z^2}{2} = -x + C$

$\Rightarrow (x - 2y)^2 + 2x = C$ [Q $z = x - 2y$]

Correct Answer: A

Question 157 : The values of λ, for which the equation $x^2 - y^2 - x + \lambda y - 2 = 0$ represents a pair of straight lines, is

(A) –3, 1

(B) –1, 1

(C) 3, –3

(D) 3, 1

Solution :

Given equation is

$x^2 - y^2 - x + \lambda y - 2 = 0$

On comparing given equation with standard equation, we get

$a = 1, b = -1, c = -2, h = 0,$

$\therefore\ g = -1/2$ and $f = \lambda/2$

Given equation represents a pair of straight line.

$\therefore\ abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\Rightarrow 2 + 0 - \frac{\lambda^2}{4} + \frac{1}{4} = 0$$

$$\Rightarrow \frac{\lambda^2}{4} = \frac{9}{4} \Rightarrow \lambda = \pm 3$$

Correct Answer: C

Question 158 : Fhe equation $12x^2 + 7xy + ay2 + 13x - y + 3 = 0$ represents a pair of perpendicular lines. Then, the value of a is

(A) $\frac{7}{2}$

(B) –19

(C) –12

(D) 12

Solution :

Given equation is

$12x^2 + 7xy + ay^2 + 13x - y + 3 = 0$

On comparing the given equation with standard equation, we get a = 12 and b = a

For perpendicular lines, coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore 12 + a = 0 \Rightarrow a = -12$

Correct Answer: C

Question 159 : The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other at two distinct points, if

(A) r < 2

(B) r > 8

(C) 2 < r < 8

(D) $2 \le r \le 8$

Solution :

The centre and radii of two circles are $C_1(1, -3)$, $C_2(0, 0)$

and $r_1 = \sqrt{25 + 0 - 16} = 3, r_2 = r$

Now, $C_1C_2 = 5$

For intersection of two circles

$r_2 - r_1 < C_1C_2 < r_1 + r_2$

$\Rightarrow r - 3 < 5 < 3 + r$

$\Rightarrow r < 8$ and $r > 2$

$\Rightarrow 2 < r < 8$

Correct Answer: C

Question 160 : The angle between the circles
S: $x^2 + y^2 – 4x + 6y + 11 = 0$ and
S‘: $x^2 + y^2 – 2x + 8y + 13 = 0$ is

(A) 45º

(B) 90º

(C) 60º

(D) None of these

Solution :

Centres and radii of circles S and S′ are $C_1(2, –3)$,

$r_1 = \sqrt{2}$; $C_2(1, – 4), r_2 = 2$

Distance between centres,

$d = |C_1C_2| = \sqrt{(2-1)^2 + (-3+4)^2} = \sqrt{2}$

If an angle between the circles is θ, then

$$\cos\theta = \left|\frac{2+4-2}{2\sqrt{2}.2}\right| = \frac{1}{\sqrt{2}}$$

$\therefore \theta = 45°$

Correct Answer: A

Question 161 : The coordinates of the focus of the parabola described parametrically by x = 5t + 2, y = 10t + 4 are

(A) (7, 4)

(B) (3, 4)

(C) (3, –4)

(D) (–7, 4)

Solution : Given, parametric curves are x
$= 5t^2 + 2$, y = 10t + 4

$\Rightarrow \frac{x-2}{5} = t^2, \frac{y-4}{10} = t$

$\Rightarrow \frac{x-2}{5} = \left(\frac{y-4}{10}\right)^2$

$\Rightarrow (y-4)^2 = 20(x-2)$

$\Rightarrow Y^2 = 20X$

where $Y = y-4, X = x-2$

So, the coordinates of focus are(5, 0).

i.e. $x-2=5, y-4=0$

$\Rightarrow x = 7$ and $y = 4$

Hence, the required coordinates are (7, 4)

Correct Answer: A

Question 162 : The locus of the mid-point of the line joining the focus and any point on the parabola $y^2 = 4ax$ is a parabola with the equation of directrix as

(A) $x + a = 0$

(B) $2x + a = 0$

(C) $x = 0$

(D) $x = \frac{a}{2}$

Let the coordinates of mid-points of forms S(a, 0) and points P(at^2, 2at) on the parabola be (x_1, y_1)

$\therefore \quad x_1 = \frac{a + at^2}{2}, y_1 = \frac{0 + 2at}{2}$

$\Rightarrow at^2 = 2x_1 - a,\ y_1 = at$

$\Rightarrow a\left(\frac{y_1}{a}\right)^2 = 2x_1 - a$

$\Rightarrow y_1^2 = 2x_1 a - a^2$

Hence, the locus of the mid-point is

$y^2 = 2a\left(x - \frac{a}{2}\right)$.

$\therefore$ Equation of directrix is

$x - \frac{a}{2} = -\frac{a}{2}$

Solution : $\Rightarrow x = 0$

Correct Answer: C

Question 163 : If a = i + j + pk and b = i + j + k, then |a + b| = |a| + |b|, holds for

(A) all real p

(B) no real p

(C) p = –1

(D) p = 1

$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$

$\therefore \sqrt{\{4 + 4 + (p+1)^2\}} = \sqrt{(p^2 + 2)} + \sqrt{3}$

$\Rightarrow 2(p+2) = 2\sqrt{3}\sqrt{(p^2+2)}$

Solution : p = 1

Correct Answer: D

Question 164 : For what value of m are the point with position vectors $10\hat{i}+3\hat{j}$, $12\hat{i}-5\hat{j}$ and $m\hat{i}+11\hat{j}$ collinear?

(A) –8

(B) 4

(C) 8

(D) 12

Solution :

Let OA = $10\hat{i}+3\hat{j}$, OB = $12\hat{i}-5\hat{j}$ and

OC = $m\hat{i}+11\hat{j}$

Since, A, B and C are collinear vectors.
Then, AB = λBC

$\Rightarrow 2\hat{i}-8\hat{j}=\lambda\{(m-12)\hat{i}+16\hat{j}\}$

$\lambda(m-12)=2$ and $16\lambda=-8$ .....(i)

$\lambda=-\frac{1}{2}$

From eq. (i) $\Rightarrow$ m = 8

Correct Answer: C

Question 165 : ABCDEF is a regular hexagon with centre at the origin such that AD = EB + FC = lED. Then, l is equal to

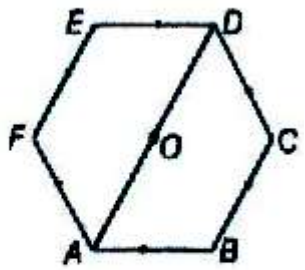
(A) 2

(B) 4

(C) 6

(D) 3

Given, AD + EB + FC = λED



Now, AD + EB + FC
= 2(OD + EO + ED)
= 2(ED + ED) = 4ED

Solution : ∴ λ = 4

Correct Answer: B

Question 166 : The shortest distance from the plane
$12x + 4y + 3z = 327$ to the sphere
$x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

(A) 26

(B) $11\frac{4}{13}$

(C) 13

(D) 39

The distance from the centre of sphere to the plane

$$= \left|\frac{-2\times12+4\times1+3\times3-327}{\sqrt{144+16+9}}\right| = 26$$

Solution : ∴ Shortest distance

$= 26 - \sqrt{4+1+9+155} = 13$

Correct Answer: C

Question 167 : The ratio in which the line joining (2, 4,5), (3, 5, –4) is divided by the YZ-plane is

(A) 2 : 3

(B) 3 : 2

(C) –2 : 3

(D) 4 : –3

Solution :

Let the required ratio be $\lambda : 1$. Then, the point is

$$\left(\frac{3\lambda+2}{\lambda+1}, \frac{5\lambda+4}{\lambda+1}, \frac{-4\lambda+5}{\lambda+1}\right)$$

It lies on YZ-plane, so its X-coordinate of $X = 0$

i.e., $\frac{3\lambda+2}{\lambda+1} = 0 \Rightarrow \lambda = -\frac{2}{3}$

So, the ratio is 2 : 3 externally.

Correct Answer: C

Question 168 : if the ine joining A (1, 3, 4) and B is divided by the point (–2, 3, 5) in the ratio 1 : 3 then B is equal to

(A) (–11, 3, 8)

(B) (–11, 3, –8)

(C) (–8, 12, 20)

(D) (13, 6, –13)

Solution :

Using internally ratio formula,

$$\Rightarrow (-2, 3, 5) = \left(\frac{x+3}{4}, \frac{y+9}{4}, \frac{z+12}{4}\right)$$

$\Rightarrow x = -11, y = 3, z = 8$

$\therefore$ Required point = B(–11, 3, 8)

Correct Answer: A

Question 169 : If the variance of $1, 2, 3, 5, \ldots\ldots, 10$ is $\frac{99}{12}$, then the standard deviation of $3, 6, 9, 12, \ldots\ldots, 30$ is

(A) $\frac{297}{4}$

(B) $\frac{3}{2}\sqrt{33}$

(C) $\frac{3}{2}\sqrt{99}$

(D) $\sqrt{\frac{99}{12}}$

Solution : Given, $\sigma_{10}^2 = \frac{99}{12} = \frac{33}{4} \Rightarrow \sigma_{10} = \frac{\sqrt{33}}{2}$

$\therefore$ SD of required series $= 3\sigma_{10} = \frac{3\sqrt{33}}{2}$

Correct Answer: B

Question 170 : The variance of first n natural numbers is

(A) $\frac{n(n+1)}{2}$

(B) $\frac{(n+1)(n+5)}{12}$

(C) $\frac{(n+1)(n-5)}{12}$

(D) $\frac{(n^2-1)}{12}$

Solution :

$$\text{Variance of first n natural numbers} = \frac{\Sigma n^2}{n} - \left(\frac{\Sigma n}{n}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

$$= (n+1)\left[\frac{2n+1}{6} - \frac{(n+1)}{4}\right] = \frac{n^2-1}{12}$$

Correct Answer: D

Question 171 : Coefficient of variation of two distributions are 50% and 60% and their arithmetic means are 30 and 25, respectively. Difference of their standard deviation is

(A) 1

(B) 1.5

(C) 2.5

(D) ZERO

Solution :

$$\square\ CV_1 = \frac{\sigma_1}{\bar{x}} \times 100 \Rightarrow \sigma_1 = 50 \times \frac{30}{100}$$

$$\text{and } \sigma_2 = 60 \times \frac{25}{100}$$

$$\therefore \sigma_1 - \sigma_2 = 0$$

Correct Answer: D

Question 172 : Out of 15 persons 10 can speak Hindi and 8 can speak English. If two persons are chosen at random, then the probability that one person speaks Hindi only and the other speaks both Hindi and English is

(A) $\frac{3}{5}$

(B) $\frac{7}{12}$

(C) $\frac{1}{5}$

(D) $\frac{2}{5}$

Solution :

Total number of persons = 15

and number of persons, who can speak Hindi and English both = 10 + 8 – 15 = 3

$\therefore$ Required probability

$$= \frac{{}^{7}C_1 \times {}^{3}C_1}{{}^{15}C_2} = \frac{7 \times 3}{\frac{15 \times 14}{2}} = \frac{1}{5}$$

Correct Answer: C

Question 173 : For a party 8 guests are invited by a husband and his wife. They sit for a dinner around a round table. The probability that the husband and his wife sit together is

(A) $\frac{2}{7}$

(B) $\frac{2}{9}$

(C) $\frac{1}{9}$

(D) $\frac{4}{9}$

Solution :

Total number of ways of sitting = 9!

and number of favourable ways of sitting = 2 × 8!

$\therefore$ Required probability = $\frac{2 \times 8!}{9!} = \frac{2}{9}$

Correct Answer: B

Question 174 : If A and B are two mutually exclusive events, then

(A) $P(A) < P(\bar{B})$

(B) $P(A) > P(\bar{B})$

(C) P(A) < P(B)

(D) None of these

Solution :

Since, A and B are mutually exclusive events therefore,

$A \cap B = \phi \Rightarrow A \subseteq \bar{B}$

or $B \subseteq \bar{A}$

$\Rightarrow P(A) \leq P(\bar{B})$

or $\Rightarrow P(B) \leq P(\bar{A})$

Correct Answer: A

Question 175 : The matrix P which transform the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$ to diagonal form is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Solution :

We have, $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\Rightarrow (\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$

$\Rightarrow \lambda = 1, 2, 3$

Let X_1 be eigen vector corresponding to the eigen value $\lambda = 1$, we have $(A - \lambda I)X_1 = 0$

$$\therefore \begin{bmatrix} 1-1 & 0 & -1 \\ 1 & 2-1 & 1 \\ 2 & 2 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 0x_1 + 0x_2 - x_3 = 0$ .....(i)

$x_1 + x_2 + x_3 = 0$ .....(ii)

and $2x_1 + 2x_2 + 2x_3 = 0$ .....(iii)

From eqs. (i) and (ii), we get

$$\frac{x}{0+1} = \frac{x_2}{-1+0} = \frac{x_3}{0}$$

$\Rightarrow x_1 = 1,\ x_2 = -1$ and $x_3 = 0$

$$\therefore X_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

For $\lambda = 2$, let X_2 be the eigen vector corresponding to the eigen value $\lambda = 2$, we have $(A - \lambda I)X_2 = 0$

$$\therefore \begin{bmatrix} 1-2 & 0 & -1 \\ 1 & 2-2 & 1 \\ 2 & 2 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} R_1 \to R_1 + R_2$$

$\Rightarrow 0x_1 + 0x_2 + 0x_3 = 0$ .....(iv)
$x_1 + 0x_2 + x_3 = 0$ .....(v)
and $2x_1 + 2x_2 + x_3 = 0$ .....(vi)
From eqs.(v) and (vi), we have

$-2x_1 + 0x_2 - x_3 = 0$
and $x_1 - x_2 + x_3 = 0$
On solving these we get

$$\frac{x}{0-1} = \frac{x_2}{-1+2} = \frac{x_3}{2-0}$$

$\Rightarrow x_1 = -1, x_2 = 1, x_3 = 2$

$$\therefore X_2 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$$\therefore \text{Modal matrix, } P = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\text{and } P^{-1} = \frac{-1}{2}\begin{bmatrix} 0 & 2 & -1 \\ 2 & 2 & 0 \\ -2 & -2 & -1 \end{bmatrix}$$

$$\Rightarrow P^{-1}AP = -\frac{1}{2}\begin{bmatrix} 0 & 2 & -1 \\ 2 & 2 & 0 \\ -2 & -2 & -1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 & -1 \\ -1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = P$$

Correct Answer: A

Question 176 :

The rank of the matrix $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$ is

(A) 1

(B) 2

(C) 5

(D) 4

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & -2 & 14 & -4 \\ 0 & -2 & 14 & -4 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + 2R_1 \\ R_4 \to R_4 + 5R_1 \end{matrix}$$

$$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 2R_2 \end{matrix}$$

Here the 4th order and 3rd order minors are zero
But a minor of second order

$$\begin{vmatrix} 3 & -2 \\ 7 & -2 \end{vmatrix} = -6 + 14 = 8 \neq 0$$

Solution : Rank = Number of non-zero rows = 2

Correct Answer: B

Question 177 : The value class of the real quadratic form $-x_1^2 - 2x_2^2 - 2x_3^2 + 2x_1x_2 + 2x_2x_3$ is

(A) negative semi-definite

(B) negative definite

(C) positive definite

(D) positive semi-definite

Solution :

The given quadratic form is

$$q(x_1, x_2, x_3) = -x_1^2 - 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_2x_3$$

$$= -x_1x_1 + x_1x_2 + 0x_1x_3 + x_2x_1 - 2x_2x_2 + x_2x_3 + 0x_3x_1 + x_3x_2 - 2x_3x_3$$

$$= X^TAX, \text{ where } A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

is a symmetric matrix.

The leading principal miniors of A are

$A_1 = -1$ = negative.

$$A_2 = \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} = 2 - 1 = 1 = \text{positive}$$

$$A_3 = \begin{vmatrix} -1 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{vmatrix} C_2 \to C_2 + C_1$$

$$= \begin{vmatrix} -1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -2 \end{vmatrix} = -1(2-1) = -1 = \text{negative}$$

We see that, $A_1 < 0, A_2 > 0, A_3 < 0$

$\therefore$ The given quadratic form is negative definite.

Correct Answer: B

Question 178 :

The value of $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ is equal to the integral being taken throughout the volume bounded by the planes $x = 0, y = 0, z = 0, x + y + z = 1$

(A) $\frac{1}{2}\log 2 - \frac{5}{16}$

(B) $\frac{1}{3}log^3 + \frac{5}{11}$

(C) $\frac{1}{5}log^3 + \frac{6}{11}$

(D) None of these

Solution :

By Liouville's theorem, when $0 < x + y + z < 1$

$$\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3} = \iiint \frac{x^{l-1}y^{l-1}z^{l-1}dx\,dy\,dz}{(x+y+z+1)^3}$$

$$(0 \le x + y + z \le 1)$$

$$= \frac{\overline{l}\,\overline{l}\,\overline{l}}{\overline{l+1+1}} \int_0^1 \frac{1}{(u+1)^3} u^{3-1} du = \frac{1}{2}\int_0^1 \frac{u^2}{(u+1)^3} du$$

$$= \frac{1}{2}\int_0^1 \left[\frac{1}{u+1} - \frac{2}{(u+1)^2} + \frac{1}{(u+1)^3}\right] du \text{ (partial fractions)}$$

$$= \frac{1}{2}\left[\log 2 + 2\left(\frac{1}{2} - 1\right) - \left(\frac{1}{8} - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\log 2 - \frac{5}{16}$$

Correct Answer: A

Question 179 : Let f be continuous on [0, 1] and let f(x) be in [0, 1] $\forall x \in [0, 1]$. Then,

(A) $f(x) = x$ for some $x \in [0, 1]$

(B) $f(x) = x^2, \forall x \in [0, 1]$

(C) $f(x) = x - 1, \forall x \in [0, 1]$

(D) None of the above

Solution :

If $f(0) = 0$ or $f(1) = 1$, then we have done. Let us consider the case when $f(0) \neq 0$ and $f(1) \neq 1$.

Consider $g(x) = f(x) - x$

Now, since f is continuous on [0, 1].

Therefore, g is also continuous on [0, 1].

Also, $g(0) = f(0)\,0$. Since, $f(0) \in [0, 1]$ and $f(0) \neq 0$.

Again, $g(1) = f(1) - 1 < 0$.
Since, $f(1) \in [0, 1]$, $f(1) \neq 1$.
Since, $g(0)$ and $g(1)$ are opposite signs, therefore by the intermediate value theorem $g(x) = 0$ for some $x \in [0, 1]$.
Hence, $f(x) - x = 0$ for some $x \in [0, 1]$
$\Rightarrow f(x) = x$ for some $x \in [0, 1]$

Correct Answer: A

Question 180 : Which of the following is correct?

(A) $\frac{2}{\pi} > \frac{\sin x}{x} < 1, 0 < x < \frac{\pi}{2}$

(B) $\frac{2}{\pi} > \frac{\sin x}{x} > 1, 0 < x < \frac{\pi}{2}$

(C) $\frac{2}{\pi} < \frac{\sin x}{x} < 1, 0 < x < \frac{\pi}{2}$

(D) $\frac{2}{\pi} < \frac{\sin x}{x} > 1, 0 < x < \frac{\pi}{2}$

Let f be defined as follows

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Now, since $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin x}{x} = 1 = f(0)$.

Therefore, f is continuous at $x = 0$.

Also, f is continuous in $\left[0, \frac{\pi}{2}\right]$.

Therefore, f is continuous in $\left[0, \frac{\pi}{2}\right]$.

When $x \neq 0$, $f'(x) = \frac{(x \cos x - \sin x)}{x^2}$

Solution : $= \frac{(x - \tan x)\cos x}{x^2}$

If $0 < x < \frac{\pi}{2}$, then $\tan x > x$ and $\cos x > 0$.

Therefore, $f'(x) < 0$, whenever $0 < x < \frac{\pi}{2}$.

Consequently, f is strictly decreasing in $\left[0, \frac{\pi}{2}\right]$

So, that whenever $0 < x < \frac{\pi}{2}$,

we have $f(0) > f(x) > f\left(\frac{\pi}{2}\right)$.

$\Rightarrow 1 > \frac{\sin x}{x} > \frac{2}{\pi}$

Correct Answer: C