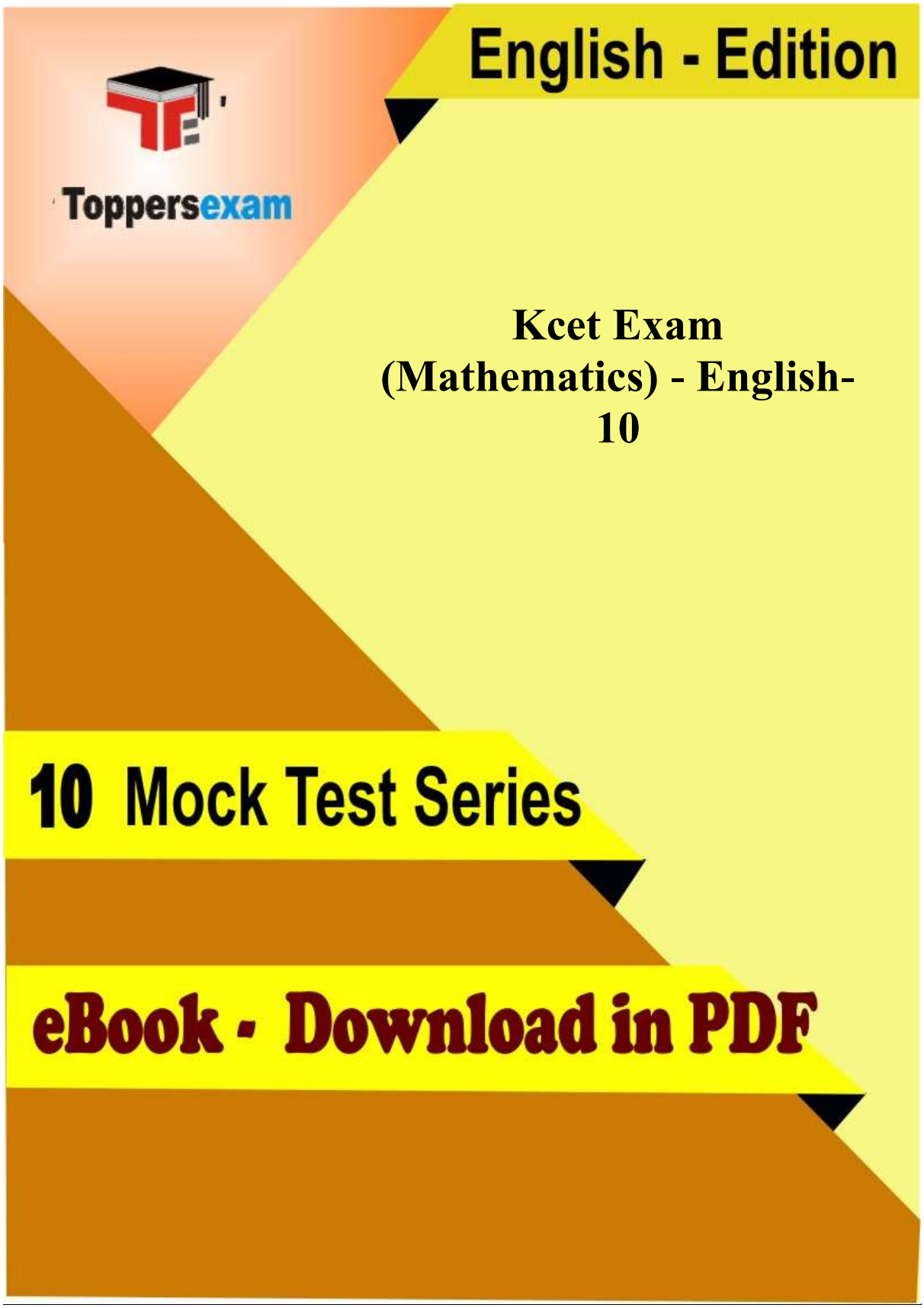


Kcet Exam (Mathematics) - English-10

Paper Questions

SUBJECT: Physics

Question 1 : The equation of state for *n* moles of an ideal gas is *p*V = nRT, where R is a constant. The SI unit for R is

(A) JK^{-1} per molecule

(B) $JK^{-1}\ mol^{-1}$

(C) $JK^{-1}K^{-1}$

(D) $JK^{-1}g^{-1}$

Correct Answer: B

Question 2 : Sound waves transfer

(A) only energy not momentumx

(B) energy

(C) momentum

(D) both (a) and (b)

Correct Answer: D

Question 3 : The number of electron is 2 coulomb of charge is

(A) 5×10^{29}

(B) 12.5 x 10^{18}

(C) 1.6 x 10^{19}

(D) 9 x 10^{11}

Correct Answer: B

Question 4 : The current flowing through a wire depends on time as $I = 3t^2 + 2t + 5$. The charge flowing through the cross-section of the wire in time from t = 0 to t = 2 s is

(A) 22 C

(B) 20 C

(C) 18 C

(D) 5 C

Correct Answer: A

Question 5 : If the charge on a capacitor is increased by 2 coulomb, the energy stored in it increases by 21% . The original charge on the capacitor is

(A) 10C

(B) 20C

(C) 30C

(D) 40C

Correct Answer: B

Question 6 : The work done in carrying a charge Q once around a circle of radius r about a charge q

at the centre is

(A) $\frac{qQ}{4\pi\varepsilon_0 r}$

(B) $\frac{qQ}{4\pi\varepsilon_0}\frac{1}{\pi r}$

(C) $\frac{qQ}{4\pi\varepsilon_0 r}\left(\frac{1}{2\pi r}\right)$

(D) zero

Correct Answer: D

Question 7 : Four capacitors of equal capacitance have an equivalent capacitance C_1 when connected in series and an equivalent capacitance C_2 when connected in parallel. The ratio is

(A) 1/4

(B) 1/16

(C) 1/8

(D) 1/12

Correct Answer: B

Question 8 : Magnetic filed intensity H at the centre of a circular loop of radius r carrying current I emu is

(A) $\frac{r}{I}$ oersted

(B) $\frac{2\pi I}{I}$ oersted

(C) $\frac{I}{2\pi I}$ oersted

(D) $\frac{2\pi I}{I}$ oersted

Correct Answer: B

Question 9 : Which of the following materials is the best conductor of electricity ?

(A) Platinum

(B) Gold

(C) Silicon

(D) Copper

Correct Answer: D

Question 10 : In the Bohr model of the hydrogen atom, let R, V and E represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which of the following quantity is proportional to the quantum number n?

(A) E/V

(B) R/E

(C) VR

(D) RE

Correct Answer: C

Question 11 : Experimental investigations show that the intensity of solar radiation is maximum for a wavelength 480 nm in the visible region. Estimate the surface temperature of sun(Given Wien's constant b = 2.88 x 10^{-3} mK)

(A) 4000 K

(B) 6000 K

(C) 8000 K

(D) 10^6 K

Correct Answer: B

Question 12 : The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K, the root mean square speed of gas molecules is *v*, then at 480 K, it will be

(A) 4 *v*

(B) 2*v*

(C) $\frac{1}{8}$ 6 v

(D) 8v

Correct Answer: B

Question 13 : The radius of the light circle observed by a fish at a depth of 12 m is (refractive index of water = 4/3)

(A) $36\sqrt{7}$

(B) $\frac{36}{\sqrt{7}}$

(C) $36\sqrt{5}$

(D) $4\sqrt{5}$

Correct Answer: B

Question 14 :

G P Thomson experimentally confirmed the existence of matter waves by the phenomena

(A) diffraction

(B) refraction

(C) polarisation

(D) scattering

Correct Answer: A

Question 15 : A plano-convex lens (f = 20 cm) is silvered at plane surface. Now focal length will be

(A) 20 cm

(B) 40 cm

(C) 30 cm

(D) 10 cm

Correct Answer: D

Question 16 : The light beams of intensities in the ratio of 9 : 1 are allowed to interfere. What will be

the ratio of the intensities of maxima and minima ?

(A) 3 : 1

(B) 4 : 1

(C) 25 : 9

(D) 81 : 1

Correct Answer: B

Question 17 : If x_1 be the size of the magnified image and x_2 the size of the diminished image in Lens Displacement method, then the size of the object is

(A) $\sqrt{x_1 x_2}$

(B) x_1x_2

(C) $x_1^2 x^2$

(D) $x_1 x_2^2$

Correct Answer: A

Question 18 : A point charge +q is placed at the centre of a cube of side L. The electric flux emerging from the cube is

(A) $\frac{q}{\varepsilon_0}$

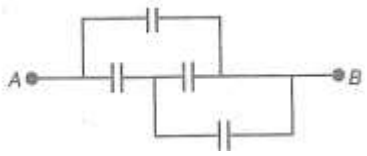
(B) Zero

(C) $\frac{6q L^2}{\varepsilon_0}$

(D) $\frac{q}{6L^2\varepsilon_0}$

Correct Answer: A

Question 19 : In the figure below the capacitance of each capacitor is 3μF. The effective capacitance between A and B is



(A) $\frac{3}{4}$ μF

(B) 3μF

(C) 6μF

(D) 5μF

Correct Answer: D

Question 20 : n identical droplets are charged to V volt each . If they coalesce to from a single drop, then its potential will be

(A) $n^{2/3}$ B

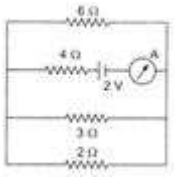
(B) $n^{1/3}$ V

(C) nV

(D) V/n

Correct Answer: A

Question 21 : The reading of the ammeter in the following figure will be



(A) 0.8 A

(B) 0.6 A

(C) 0.4 A

(D) 0.2 A

Correct Answer: C

Question 22 : A wire of resistance R is elongated n-fold to make a new uniform wire. The resistance of new wire

(A) nR

(B) n^2R

(C) 2nR

(D) $2n^2R$

Correct Answer: B

Question 23 : The ratio of magnetic field and magnetic moment at the centre of a current carrying circular loop is *x*. When both the current and radius is doubled the ratio will be

(A) *x*/8

(B) *x*/4

(C) *x/2*

(D) *2x*

Correct Answer: A

Question 24 : The current through a coil of self-inductance L= 2mH is given by $i = t^2e^{-1}$ at time t. How long it will take to make the emf zero

(A) 1 s

(B) 2 s

(C) 3 s

(D) 4 s

Correct Answer: B

Question 25 : The magnetic flux through a loop of resistance 10 Ω is given ($\phi = 5t^2 - 4t + 1$ Wb). How much current is induced in the loop after 0.2 s ?

(A) 0.4 A

(B) 0.2 A

(C) 0.04 A

(D) 0.02 A

Correct Answer: B

Question 26 : The decimal equivalent of the binary number $(11010.101)_2$ is

(A) 9.625

(B) 25.265

(C) 26.625

(D) 26.265

Correct Answer: C

Question 27 : In a common emitter configuration a transistor has β = 50 and input resistance 1 kΩ If the peak value of AC input is 0.01 V, then the peak value of collector current is

(A) 0.01 µA

(B) 0.25 µA

(C) 100 µA

(D) 500 µA

Correct Answer: D

Question 28 : Half –life of radioactive substance is 20 min. The time between 20% and 80% decay will be

(A) 20 min

(B) 30 min

(C) 40 min

(D) 25 min

Correct Answer: C

Question 29 : The energy released by the fission of one uranium atom is 200 MeV. The number of fissions per second required to produce fissions per second required to produce 3.2 W of power is(Take, 1 eV = 1.6 x 10^{-19}J)

(A) 10^{7}

(B) 10^{10}

(C) 10^{15}

(D) 10^{11}

Correct Answer: D

Question 30 :

The resistance of a wire at 300 K is found to be 0.3 Ω. If the temperature coefficient of resistance of wire is 1.5 x 10^{-3} K^{-1}, the temperature at which the resistance becomes 0.6 Ω is

(A) 720 K

(B) 345 K

(C) 993 K

(D) 690 K

Correct Answer: C

Question 31 : A ball is projected horizontally with a velocity of 5 m/s from the top of a building 19.6 m high. How long will the ball take to hit the ground

(A) $\sqrt{2}$ s

(B) 2 s

(C) $\sqrt{3}$ s

(D) 3 s

Correct Answer: B

Question 32 : A stone falls freely from rest and the total distance covered by it in the last second of its motion equals the distance covered by it in the first three seconds of its motion. The stone remains in the air for

(A) 6 s

(B) 5 s

(C) 7 s

(D) 4 s

Correct Answer: B

Question 33 : Two blocks of 2 kg and 1 kg are in contact on a frictionless table. IF a force of 3 N is applied on 2 kg block, then the force of contact between the two blocks will be

(A) Zero

(B) 1 N

(C) 2 N

(D) 3 N

Correct Answer: B

Question 34 : If momentum is increased by 20%, then kinetic energy increases by

(A) 48%

(B) 44%

(C) 40%

(D) 36%

Correct Answer: B

Question 35 : A boy of mass 40 kg is climbing a vertical pole at a constant speed. If the coefficient of friction between his palms and the pole is 0.8 and g = 10 m/s^2 , the horizontal force that he is applying on the pole is

(A) 300 N

(B) 400 N

(C) 500 N

(D) 600 N

Correct Answer: C

Question 36 :

y = 3 sin represents an equation of a progressive wave, where is in second and x is in metre. The distance travelled by the wave in 5 s is

(A)

8 m

(B) 10 m

(C) 5 m

(D) 32 m

Correct Answer: B

Question 37 : The phenomena in which proton flips is

(A) nuclear magnetic resonance

(B) lasers

(C) radioactivity

(D) nuclear fusion

Correct Answer: A

Question 38 : The height vertically above the earth's surface at which the acceleration due to gravity becomes 1% of its value at the surface is (R is the radius of the earth)

(A) 8R

(B) 9R

(C) 10R

(D) 20R

Correct Answer: B

Question 39 : The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the Earth is (Here R is the radius of the earth)

(A) $\left(\frac{n}{n+1}\right)mgr$

(B) $\left(\frac{n}{n-1}\right)mgr$

(C) nmgR

(D) $\frac{mgR}{n}$

Correct Answer: A

Question 40 :

The amount of heat energy radiated by a metal at temperature T is E. When the temperature is increased to 3T, energy radiated is

(A) 81E

(B)

9E

(C) 3E

(D) 27 E

Correct Answer: A

Question 41 : A springs of force constant k is cut into three equal parts. The force constant of each part would be

(A) $\frac{k}{3}$

(B) 3 k

(C) K

(D) 2k

Correct Answer: B

Question 42 : A body floats in water with 40% of its volume outside water. When the same body floats in oil 60% of its volume remains outside oil the relative density of the oil is

(A) 0.9

(B) 1.2

(C) 1.5

(D) 1.8

Correct Answer: C

Question 43 :

Weight of a body of mass decreases by 1% when it is raised to height above the earth's surface. If the body is taken to a depth in a mine, change in its weight is:

(A) 0.5% decrease

(B) 2% decrease

(C) 0.5% increase

(D) 1% increase

Correct Answer: A

Question 44 : Two solid spheres of same metal but of mass M and 8M fall simultaneously on a viscous liquid and their terminal velocities are v and nv, then value of n is

(A) 16

(B) 8

(C) 4

(D) 2

Correct Answer: C

Question 45 : A particle is executing linear simple harmonic motion of amplitude A. At what displacement is the energy of the particle half potential and half knetic ?

(A) A
4

(B) A
2

(C) $\frac{A}{\sqrt{2}}$

(D) $\frac{A}{\sqrt{3}}$

Correct Answer: C

Question 46 : The equation of a progressive wave is y = 4sin (4πt – 0.04*x* + π/3) where *x* is in metre and t is in second. The velocity of the wave is

(A) 100 π m/s

(B) 50 π m/s

(C) 250 π m/s

(D) π m/s

Correct Answer: A

Question 47 :

How many 6 f, 200 V condensers are needed to make a condenser of 18 F, 600 V?

(A) 9

(B) 18

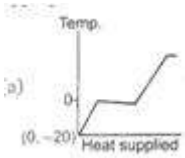
(C) 3

(D) 27

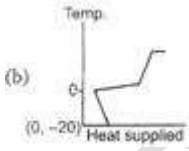
Correct Answer: D

Question 48 : A block of ice at temperature -20^{0}C is slowly heated and converted to steam at 100^{0}C which of the following diagram is most appropriate

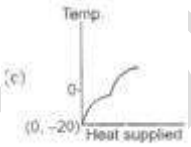
(A)



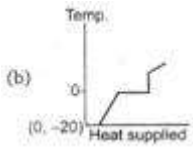
(B)



(C)



(D)



Correct Answer: A

Question 49 : Two black bodies at temperature 327^0C and 427^0C are kept in an evacuated chamber 27^0C. The ratio of their rates of loss of heat are

(A) $\frac{6}{7}$

(B) $(\frac{6}{7})^2$

(C) $(\frac{6}{7})^3$

(D) $\frac{243}{464}$

Correct Answer: D

Question 50 : In a mercury thermometer, the ice point (lower fixed point) is marked as 10^0 and the steam point (upper fixed point) is marked as 130^0 At 40^0C temperature, what will this thermometer read ??

(A) 78^0

(B) 66^0

(C) 62^0

(D) 58^0

Correct Answer: D

Question 51 : Water is following through a very narrow tube. The velocity of water below which the flow remains a streamline flow is known as

(A) Relative velocity

(B) Terminal velocity

(C) Critical velocity

(D) Particle velocity

Correct Answer: C

Question 52 : It the velocity of light in vacuum is 3×10^8 ms^{-1} the time taken (In nanosecond) to travel through a glass plate of thickness 10 cm and refractive index 1.5 is

(A) 0.5

(B) 1.0

(C) 2.0

(D) 3.0

Correct Answer: A

Question 53 : Cavitation is a special application property exhibited only by:

(A) ultrasonics

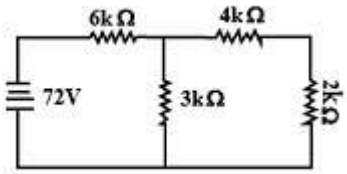
(B) electromagnetic waves

(C) audible sound

(D) infrasonics

Correct Answer: A

Question 54 : What current will flow through the 2 kΩ resistor in the circuit shown in the figure ?



(A) 3 mA

(B) 6 mA

(C) 12 mA

(D) 36 mA

Correct Answer: A

Question 55 : In a region the intensity of an electric field is given by E = 2i + 3j + k in NC^{-1}. The electric flux through a surface S = 10i m^2 in the region is

(A) 5 Nm^2C^{-1}

(B) 10 Nm^2C^{-1}

(C) 15 Nm^2C^{-1}

(D) 20 Nm^2C^{-1}

Correct Answer: D

Question 56 : A train approaching a railway platform with a speed of 20 ms^{-1} starts in air is 340 ms^{-1} . IF the frequency of the emitted sound from the whistle is 640 Hz, The frequency of sound to a person standing on the platform will appear to be

(A) 600 Hz

(B) 640 Hz

(C) 680 Hz

(D) 720 Hz

Correct Answer: C

Question 57 : A straight wire of length 2 m carries a current of 10 A. If this wire Is placed in a uniform magnetic field of 0.15 T making an angle of 45^0 with the magnetic field, The applied force on the wire will be

(A) 1.5 N

(B) 3 N

(C) $3^{\sqrt{2}}$ N

(D) $\frac{3}{\sqrt{2}}$ N

Correct Answer: D

Question 58 : Yellow light is used in single slit diffraction experiment with slit width 0.6 mm. if yellow light is replaced by X-rays, then the pattern will reveal that:

(A) no diffraction pattern

(B) that the central maxima narrower

(C) less number of fringes

(D) more number of fringes

Correct Answer: A

Question 59 : Heat is produced at a rate given by H in a resistor when it is connected across a supply of voltage V . If now the resistance of the resistor is doubled and the supply voltage is made V/3, then

the rate of production of heat in the resistor will be

(A) H/18

(B) H/9

(C) 6H

(D) 18H

Correct Answer: A

Question 60 : Two elements A and B with number Z_A and Z_B are used to produce characteristic X-rays with frequencies v_A and v_B respectively. If $Z_A : Z_B = 1:2$, then $v_A : v_B$ will be

(A) 1 : $\sqrt{2}$

(B) 1 : 8

(C) 4 : 1

(D) 1 :4

Correct Answer: D

SUBJECT: Chemistry

Question 61 :

Which one of the following metallic hydroxides does not dissolve in sodium hydroxide solution?

(A) $Zn(OH)_2$

(B) $Al(OH)_3$

(C) $Fe(OH)_3$

(D) $Pb(OH)_2$

Correct Answer: C

Question 62 : The time taken for 10% completion of a first order reaction is 20 min. Then for 19% completion, the reaction will take

(A) 40 min

(B) 60 min

(C) 30 min

(D) 50 min

Correct Answer: D

Question 63 : Which of the following will decrease the pH of a 50 mL solution of 0.01 M HCl ?

(A) Addition of 5 mL of 1 M HCl

(B) Addition of 50 mL of 0.01 M HCl

(C) Addition of 5 mL of 0.002 M HCl

(D) Addition of Mg

Correct Answer: D

Question 64 :

IUPAC name of K3Fe(CN)6 is:

(A) Potassium ferricyanide

(B) Hexacyano ferrate (III)

(C) Potassium hexacyno ferrate (III)

(D) Potassium hexacyno ferrate (II)

Correct Answer: C

Question 65 : Hybridisation of central atom in NF_3 is

(A) sp^3

(B) sp

(C) sp^2

(D) dsp^2

Correct Answer: A

Question 66 : of the following compounds the most a ideic is

(A) As_2O_3

(B) P_2O_5

(C) Sb_2O_3

(D) Bi_2O_3

Correct Answer: B

Question 67 : The half life of a radioactive element is 10 h. How much will be left after 4 h in 1 g atom sample

(A) 45.6×10^{23} atom

(B) 4.56×10^{23} atom

(C) 4.56×10^{21} atom

(D) 4.56×10^{20} atom

Correct Answer: B

Question 68 :

Unpleasant smell of carbylamines is obtained when chloroform and alcoholic KOH are heated with:

(A) Any aliphatic amine

(B) Any amine

(C) Any primary amine

(D) Any aromatic amine

Correct Answer: C

Question 69 : Under which of the following condition is the relation $\Delta H = \Delta E + P\Delta V$ valid for a closed system ?

(A) Constant pressure

(B) Constant temperature

(C) Constant temperature and pressure

(D) Constant temperature, pressure and composition

Correct Answer: A

Question 70 : An organic compound made of C, H and N contains 20% nitrogen. Its molecular weight is

(A) 70

(B) 140

(C) 100

(D) 65

Correct Answer: A

Question 71 : In cu-ammonia complex, the state of hybridization of Cu^{2+} is

(A) sp^3

(B) d^3s

(C) sp^2f

(D) dsp^2

Correct Answer: D

Question 72 : The reaction that takes place when Cl_2 gas is passed through conc. NaOH solution is

(A) Oxidation

(B) Reduction

(C) Displacement

(D) Disproportionation

Correct Answer: D

Question 73 : "Electron" is an alloy of

(A) Mg and Zn

(B) Fe and Mg

(C) Ni and Zn

(D) Al and Zn

Correct Answer: A

Question 74 : Blackened oil painting can be restored into original form by the action of

(A) Cl_2

(B) BaO_2

(C) H_2O_2

(D) MnO_2

Correct Answer: C

Question 75 : Of the following acids the one which has the capability to from complex compound and also possesses oxidizing and reducing properties is

(A) HNO_3

(B) HNO_2

(C) HCOOH

(D) HCN

Correct Answer: B

Question 76 : Atoms in P_4 molecule of white phosphorus are arranged regularly in the following way

(A) At the corners of a cube

(B) At the corners of an octahedron

(C) At the corners of a tetrahedron

(D) At the centre and corners of a tetrahedron

Correct Answer: C

Question 77 : Which of the following statements is not correct ?

(A) Silicon is extensively used as a semiconductor

(B) Carborundum is SiC

(C) Silicon occurs in free state in nature

(D) Mica contains the element silicon

Correct Answer: C

Question 78 : The addition of HBr to 2-pentene gives

(A) 2- bromopentane only

(B) 3- bromopentane only

(C) 2- bromopentane and 3- bromopentane

(D) 1- bromopentane and 3- bromopentane

Correct Answer: C

Question 79 : Ethylene can be separated from acetylene by passing the mixture through

(A) Fuming H_2SO_4

(B) Pyrogallol

(C) Ammoniacal Cu_2Cl_2

(D) Charcoal powder

Correct Answer: C

Question 80 : Reaction of R OH with R' MgX produces

(A) RH

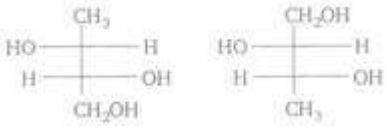
(B) R'H

(C) R – R

(D) R'- R'

Correct Answer: B

Question 81 : The two structures written below represent



(A) Pair of diastereomers

(B) Pair of enantimoers

(C) Same molecule

(D) Both are optically inactive

Correct Answer: C

Question 82 : Which of the following carbocations will be most stable ?

(A) Ph_3C^+

(B) $CH_3—\overset{+}{C}H_2$

(C) $(CH_3)_2$ CH

(D) $CH_2 = CH – CH_2$

Correct Answer: A

Question 83 : Which statement is incorrect ?

(A) Phenol is a weak acid

(B) Phenol is an aromatic compound

(C) Phenol liberates CO_2 from Na_2CO_3 solution

(D) Phenol is soluble in NaOH

Correct Answer: C

Question 84 : In which of the following reactions new carbon-carbon bond is not formed ?

(A) Cannizaro reaction

(B) Wurtz reaction

(C) Aldol condensation

(D) Friedel-Craft reaction

Correct Answer: A

Question 85 : A compound is formed by substitution of two chlorine for two hydrogens in propane. The number of possible isomeric compounds is

(A) 4

(B) 3

(C) 5

(D) 2

Correct Answer: A

Question 86 : Which one of the following is called a carbylamines

(A) RCN

(B) $RCONH_2$

(C) RCH = NH

(D) RNC

Correct Answer: D

Question 87 : For making distinction between 2-pentanone and 3- pentanone the reagent to be employed is

(A) $K_2Cr_2O_7/H_2SO_4$

(B) Zn-Hg/HCl

(C) SeO_2

(D) Iodine/NaOH

Correct Answer: D

Question 88 : Which one of the following formulae does not represent an organic compound ?

(A) $C_4H_{10}O_4$

(B) $C_4H_gO_4$

(C) $C_4H_7ClO_4$

(D) $C_4H_9O_4$

Correct Answer: D

Question 89 : The catalyst used for olefin polymerization is

(A) Ziegler-Natta catalyst

(B) Wilkinson catalyst

(C) Raney nickel catalyst

(D) Merrifield resin

Correct Answer: A

Question 90 : The oxidant which is used as an antiseptic is

(A) $KBrO_3$

(B) $KMnO_4$

(C) CrO_3

(D) KNO_3

Correct Answer: B

Question 91 : Which of the following contributes to the double helical structure of DNA ?

(A) Hydrogen bond

(B) Covalent bond

(C) Disulphide bond

(D) Van der Waals force

Correct Answer: A

Question 92 : The monomer used to produce orlon is

(A) $CH_2 = CHF$

(B) $CH_2 = CCl_2$

(C) $CH_2 = CHCl$

(D) $CH_2 = CH - CN$

Correct Answer: D

Question 93 :

Which of the following is an aldohexose?

(A) Cellulose

(B) Sucrose

(C) Glucose

(D) Raffinose

Correct Answer: C

Question 94 :

When a mixture of calcium acetate and calcium formate is dry distilled, the product formed as:

(A)

Ethanal

(B)

Butanone

(C)

Methanal

(D) Acetophenone

Correct Answer: A

Question 95 : Which of the following thermodynamic relation is correct ?

(A) dG = Vdp – SdT

(B) dE = pdV + Tds

(C) dH = - Vdp + TdS

(D) dG = Vdp + SdT

Correct Answer: A

Question 96 : In the hydrolysis of an organic chloride in presence of large excess of water .RCl + $H_2O \rightarrow ROH + HCl$

(A) Molecularity and order of reaction both are 2

(B) Molecularity is 2 but order of reaction is 1

(C) Molecularity is 1 but order of reaction is 2

(D) Molecularity is 1 and order of reaction is also

Correct Answer: B

Question 97 : The potential of a hydrogen electrode at pH = 10 is

(A) 0.59 V

(B) 0.00 V

(C) - 0.59 V

(D) -0.059

Correct Answer: C

Question 98 : Calculate Kc for the reversible process given below if K_p = 167 T = 800^0C

$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$

(A) 1.95

(B) 1.85

(C) 1.89

(D) 1.60

Correct Answer: C

Question 99 : For a reversible chemical reaction where the forward process is exothermic, which of the following statements is correct ?

(A) The backward reaction has higher activation energy than the forward reaction

(B) The backward and the forward processes have the same activation energy

(C) The backward reaction has lower activation energy

(D) No activation energy is required at all since energy is liberated in the process

Correct Answer: A

Question 100 : In sommerfeld's modification of Bohr's theory, the trajectory of an electron in a hydrogen atom is

(A) A perfect ellipse

(B) A closed ellipse-like curve, narrower at the perihelion position and flatter at the aphelion position

(C) A closed loop on spherical surface

(D) A rosette

Correct Answer: C

Question 101 : In the reaction of sodium thiosulphate with I_2 in aqueous medium, the equivalent weight of sodium thiosulphate is equal to

(A) Molar mass of sodium thiosulphate

(B) The average of molar masses of $Na_2S_2O_3$ and I_2

(C) Half the molar mass of sodium thiosulphate

(D) Molar mass of sodium thiosulphate x 2

Correct Answer: A

Question 102 : 0.1 M HCl and 0.1 M H_2SO_4 each of volume 2 mL are mixed and the volume is made upto 6 mL by adding 2 mL of 0.01 N NaCl soulution. The pH of the resulting mixture is

(A) 1.17

(B) 1.0

(C) 0.3

(D) Log 2 – log 3

Correct Answer: B

Question 103 : The molarity of a NaoH solution by dissolving 4 g of it in 250 ml water is

(A) 0.4 M

(B) 0.8 M

(C) 0.2 M

(D) 0.1 M

Correct Answer: A

Question 104 : If a species has 16 protons, 18 electrons and 16 neutrons, find the species and its charge

(A) S^{1-}

(B) Si^{2-}

(C) P^{3-}

(D) S^{2-}

Correct Answer: D

Question 105 : In periodic table, the basic character oxides

(A) Increases from left to right and decreases from top to bottom

(B) Decreases from right to left and increases from top to bottom

(C) Decreases from left to right and increases from top to bottom

(D) Decreases from left to right and increases from top to Top

Correct Answer: C

Question 106 : Which one of the following contains P — O — P bond?

(A) Hypophosphorus acid

(B) Phosphorus acid

(C) Pyrophosphoric acid

(D) Orthophosphoric acid

Correct Answer: C

Question 107 : Which of the following orders regarding ionisation energy is correct ?

(A) N > O > F

(B) N < O <F

(C) N > O < F

(D) N < O > F

Correct Answer: C

Question 108 : Which of the following statements regarding ozone is not correct ?

(A) The ozone molecule is angular in shape

(B) The ozone is a resonance hybrid of two structures

(C) The oxygen-oxygen bond length in ozone I identical with that of molecular oxygen

(D) Ozone is used as germicide and disinfectant for the purification of air

Correct Answer: C

Question 109 : P_4O_{14} is the anhydride of

(A) H_3PO_2

(B) H_3PO_3

(C) H_3PO_4

(D) $H_3P_2O_7$

Correct Answer: C

Question 110 : Which of the following metals has the largest abundance in the earth's crust ?

(A) Aluminium

(B) Calcium

(C) Magnesium

(D) Sodium

Correct Answer: A

Question 111 : Which of the following orbitals will have zero probability of finding the electron in the yz plane ?

(A) P_x

(B) P_y

(C) P_z

(D) d_{yz}

Correct Answer: A

Question 112 : What type of orbital hybridization considered on P in PCl_5 ?

(A) sp^3d

(B) dsp^3

(C) sp^3d^2

(D) d^2sp^3

Correct Answer: A

Question 113 : For which element the inertness of the electron pair will not be observed ?

(A) Sn

(B) Fe

(C) Pb

(D) In

Correct Answer: B

Question 114 : In which of the following molecules is hydrogen bridge bond present

(A) Water

(B) Inorganic benzene

(C) Diborane

(D) Methanol

Correct Answer: C

Question 115 : When a maganous salt is fused with a mixture of KNO_3 and solid NaOH, The oxidation number of Mn changes from +2 to

(A) +4

(B) +3

(C) +6

(D) +7

Correct Answer: C

Question 116 : In haemoglobin, the metal ion present is

(A) Fe^{2+}

(B) Zn^{2+}

(C) Co^{2+}

(D) Cu^{2+}

Correct Answer: A

Question 117 : Ortho and para hydrogens have

(A) Identical chemical properties but different physical properties

(B) Identical physical and chemical properties

(C) Identical physical properties but different chemical properties

(D) Different physical and chemical properties

Correct Answer: A

Question 118 : The bond order of CO molecule is

(A) 2

(B) 2.5

(C) 3

(D) 3.5

Correct Answer: C

Question 119 : Vitamin C is

(A) Citric acid

(B) Lactic acid

(C) Paracetamol

(D) Ascorbic acid

Correct Answer: D

Question 120 : On mixing alkane with chlorine and irradiating with ultraviolet light, It forms only one monochloroalkane. The alkane is

(A) Propane

(B) Pentane

(C) Iso-pentane

(D) neo-pentane

Correct Answer: D

SUBJECT: Mathematics

Question 122 : Consider the sets

A = {x : x is a real number and $x^2 + 1 = 0$}

B = {x : x is a real number and $x^2 - 16 = 0$}

C = {x : x is a real number and $x^2 = x + 2$}

Which of the following is are null set?

(A) A

(B) A and B

(C) A and C

(D) A, B and C

Solution :

Since, $x^2 + 1 = 0 \Rightarrow x^2 = -1$ and we know there is no real number whose square is negative.

A is an empty set.

Now, $x^2 - 16 = 0 \Rightarrow x^2 = 16 \Rightarrow x = -4, 4$

So, B is not an empty set.

Again, $x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x = -1, 2$

So, C is not an empty set.

Correct Answer: A

Question 123 : Which of the following functions is inverse of itself?

(A) $f(x) = \frac{1-x}{1+x}$

(B) $f(x) = 5^{\log x}$

(C) $f(x) = 2^{x(x-1)}$

(D) None of the above

$$fof(x) = f(f(x))$$

$$= f\left(\frac{1-x}{1+x}\right) = \frac{1-\left(\frac{1-x}{1+x}\right)}{1+\left(\frac{1-x}{1+x}\right)}$$

$$= \frac{1+x-1+x}{1+x+1-x} = \frac{2x}{2} = x = I(x)$$

Solution : $\Rightarrow$ Inverse of f is f.

Correct Answer: A

Question 124 :

If $f(x) = e^x$ and $g(x) = \log ex$, then which of the following is true?

(A) $f\{g(x)\} \neq g\{f(x)\}$

(B) $f\{g(x)\} = g\{f(x)\}$

(C) $f\{g(x)\} + g\{f(x)\} = 0$

(D) $f\{g(x)\} - g\{f(x)\} = 1$

Solution :

Given, $f(x) = e^x$ and $g(x) = \log_e x$

Now, $f\{g(x)\} = {}_e\log_e x$

and $g\{f(x)\} = \log_e ex = x$
$f\{g(x)\} = g\{f(x)\}$

Correct Answer: B

If $z = x - iy$ and $z^{1/3} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right)\Big/(p^2 + q^2)$

Question 125 : is equal to

(A) 1

(B) –1

(C) 2

(D) –2

Solution :

$z^{1/3} - p + iq$

$\therefore\ z = (p + iq)^3 = p^3 - iq^3 + 3ip^2q - 3pq^2$

Given that,

$x - iy = (p^3 - 3pq^2) + i(3p^2q - q^3)$

$\Rightarrow x = p^3 - 3pq^2$

and $-y = 3p^2q - q^3 \Rightarrow \frac{x}{p} = p^2 - 3q^2$

and $\frac{y}{q} = -3p^2 + q^2 \Rightarrow \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2$

$\therefore\ \frac{1}{(p^2 + q^2)}\left(\frac{x}{p} + \frac{y}{q}\right) = -2$

Correct Answer: D

Question 126 : $\left(\frac{1+2i}{2+i}\right)^2$ What is the conjugate of

(A) $\frac{7}{25} + i\frac{24}{25}$

(B) $-\frac{7}{25} - i\frac{24}{25}$

(C) $-\frac{7}{25}+i\frac{24}{25}$

(D) $\frac{7}{25}-i\frac{24}{25}$

$$\left(\frac{1+2i}{2+i}\right)^2=\left(\frac{1+2i}{2+i}\times\frac{2-i}{2-i}\right)^2$$

$$=\left(\frac{2-i+4i-2i^2}{4-i^2}\right)=\left(\frac{4+3i}{5}\right)^2$$

$$=\frac{16+9i^2+24i}{25}=\frac{7}{25}+\frac{24}{25}i$$

$$\therefore \text{ Conjugate of } \left(\frac{1+2i}{2+i}\right)^2=\frac{7}{24}-\frac{24}{25}i$$

Solution :

Correct Answer: D

Question 127 : If a^2, b^2, c^2 are in AP, then which of the following is also in AP?

(A)

sinA, sinB, sinC

(B) tanA, tanB, tanC

(C) cotA, cotB, cotC

(D) None of the above

Solution :

a2, b2, c2 are inAP

⇒ sin2B – sin2A = sin2C – sin2B

[By using sine rule]

⇒ sin(B + A)sin(B – A) = sin(C + B)sin(C – B)

⇒ sinC(sinB cosA – cosB sinA)

= sinA (sinC cosB – cosC sinB)

On dividing by sin A sinB sinC, we get

2 cotB = cotA + cotC

⇒cotA, cotB, cotC are in AP.

Correct Answer: C

Question 128 : If $a_1, a_2, a_3, \ldots, a_{4001}$ are terms of an AP such that $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \ldots + \frac{1}{a_{4000}a_{4001}} = 10$ and $a_2 + a_{4000} = 50$, then $|a_1 - a_{4001}|$ is equal to

(A) 20

(B) 30

(C) 40

(D) None of the above

Solution :

$$\text{Consider, } \frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots.+\frac{1}{a_{4000}a_{4001}}$$

$$=\frac{1}{d}\left(\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_2}{a_2a_3}+\ldots..\frac{a_{4001}-a_{4000}}{a_{4000}a_{4001}}\right)$$

$$=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\ldots.\frac{1}{a_{4000}}-\frac{1}{a_{4001}}\right)$$

$$=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{4001}}\right)=\frac{4000}{a_1a_{4001}}=10(\text{given})$$

$\Rightarrow a_1a_{4001} = 400$ .....(i)

$a_1 + a_{4001} = a_2 + a_{400} = 50$ .....(ii)

$\therefore (a_1 - a_{4001})^2 = (a_1 + a_{4001})^2 - 4a_1a_{4001}$

$= (50)^2 - 1600$

$\Rightarrow |a_1 - a_{4001}| = 30$

Correct Answer: B

Question 129 : If p is a prime, then 2(p – 3)! + 1 is a multiple of

(A) p – 1

(B) p

(C) p + 1

(D) None of the above

Solution :

Since, p is prime by Wilson's theorem,

we have, $(p - 1)! + 1 = 0 \pmod p$

$\Rightarrow (p - 1)(p - 2)(p - 3)! + 1 = 0 \pmod p$

$\Rightarrow (p^2 - 3p + 2)(p - 3)! + 1 = 0 \pmod p$

$\Rightarrow 2(p - 3)! + 1 + (p^2 - 3p)(3p)! = 0 \pmod p$

$\Rightarrow 2(p - 3)! + 1 = 0 \pmod p$

$[Q\ (p^2 - 3p)(p - 3) = 0] \pmod p$

Correct Answer: B

Question 130 : If (n, 7) = 1 i.e. if n is prime to 7, then $n^{12} - 1$ is divisible by

(A) 9

(B) 3

(C) 5

(D) 7

Solution : Since, 7 is prime (n, 7) = 1, therefore, by Fermet, theorem, We have, n7 – 1 = 1(mod 7)
⇒ n6 = 1 (mod 7)
⇒ (n6)2 = 12(mod 7)
⇒ n12 = 1(mod 7)
⇒ 7| (n2 – 1)

Correct Answer: D

Question 131 : Number of divisors of the form (4n + 2),n > 0 of the integer 240 is

(A) 4

(B) 8

(C) 10

(D) 3

Solution : Since, $240 = 2^4.3.5$
Total number of divisors
= (4 + 1) (1 + 1) (1 + 1) = 20
Out of these 2, 6, 10 and 30 are of the form 4n + 2.

Correct Answer: A

Question 132 : What is the total number of non-negative integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 7$

(A) 120

(B) 150

(C) 180

(D) None of these

Solution :

By using the formula ${}^{n+r-1}C_{r-1}$ required number non-negative integral solution

$= {}^{7+4-1}C_{4-1}$ [here, n = 7, r = 4]

$= {}^{10}C_3$

$= \frac{10!}{(10-3)!3!} = 150$

Correct Answer: B

Question 133 : The expression ${}^nC_0 + 2^n C_1 + 3^n C_2 + ... + (n+1)^nC_n$ is equal to

(A) $(n+1)2^n$

(B) $2^n(n+2)$

(C) $(n+2)2^{n-1}$

(D) $(n+2)2^{n+1}$

Solution :

We know, $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... {}^nC_nx^n$

$\Rightarrow x(1 + x)^n = {}^nC_0x + {}^nC_1x^2 + {}^nC_2x^3 + ... + {}^nC_nx^n + 1$

On differentiating both sides w.r.t. x, we get

$(1 + x)^n + nx(1 + x)^{n-1} = {}^nC_0 + 2^n\ C_1x +$

$.... + (n + 1)^nC_n\ x^n$

Put x = 1, we get

$2^n + n.2^{n-1} = {}^nC_0 + 2\ {}^nC_1 + ... + (n + 1)^nC_n$

$\Rightarrow 2^{n-1}(n + 2) = {}^nC_0 + 2^n\ C_1 + ... + (n + 1)^nC_n$

Correct Answer: C

Question 134 : If $C_0, C_1, ..., C_n$ denote the binomial coefficients in the expansion of $(1 + x)^n$. Then, the value of $C_1 - 2C_2 + 3C_3 - 4C_4 + ...$ (upto n terms) is

(A) 2^n

(B) 2^{-n}

(C) ZERO

(D) 1

Solution :

Since, $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$

On differentiating both sides w.r.t. x, we get

$n(1 + x)^{n-1} = C_1 + 2C_2x + ... + {}^nC_n\ x^{n-1}$

Or putting x = –1, we get

$0 = C_1 - 2C_2 + ... + n(-1)^{n-1}C_n$

Correct Answer: C

Question 135 : The solution set of the equation

pqx2 – (p + q)2 + (p + q)2 = 0 is

(A) $\left\{\frac{p}{q}, \frac{q}{p}\right\}$

(B) $\{pq, \frac{p}{q}\}$

(C) $\left\{\frac{q}{p}, pq\right\}$

(D) $\left\{\frac{p+q}{p}, \frac{p+q}{q}\right\}$

Solution :

The given equation is

$pqx^2 - (p+q)^2x + (p+q)^2 = 0.$

$$x = \frac{(p+q)^2 \pm \sqrt{(p+q)^4 - 4pq(p+q)^2}}{2pq}$$

$$x = \frac{(p+q)^2 \pm (p-q)^2}{2pq}$$

Now, taking (+ve) sign

$$x = \frac{p+q}{q}$$

and taking (–ve) sign

$$x = \frac{p+q}{q}$$

$\therefore$ Solution set is $\left\{\frac{p+q}{q}, \frac{p+q}{q}\right\}$.

Correct Answer: B

Question 136 : If a root of the equation $ax^2 + bx + c = 0$ is reciprocal of a root of the equation $a'x^2 + b'x + c' = 0$, then

(A) $(cc' - aa')^2 = (ba' - cb')(ab' - bc')$

(B) $(bb' - aa')^2 = (ca' - bc')(ab' - bc')$

(C) $(cc' - aa')^2 = (ba' - cb')(ab' + bc')$

(D) None of the above

Solution : Let α is a root of equation $\alpha x^2 + bx + c = 0$, then $\frac{1}{\alpha}$ is a root of second equation, therefore

$a\alpha^2 + b\alpha + c = 0$ .....(i)

and $a'\frac{1}{\alpha^2} + b'\frac{1}{\alpha} + c' = 0$

$\Rightarrow c'\alpha^2 + b'\alpha + a' = 0$ .....(ii)

On solving eqs. (i) and (ii), we get

$$\frac{\alpha^2}{ba' - b'c} = \frac{\alpha}{cc' - aa'} = \frac{1}{ab' - bc'}$$

$\Rightarrow (cc' - aa')^2 = (ba' - cb')(ab' - bc')$

Correct Answer: A

Question 137 : If $\frac{|x-2|}{x-2} \geq 0$, then

(A) $x \in [2, \infty)$

(B) $x \in (2, \infty)$

(C) $x \in (-\infty, 2)$

(D) $x \in (-\infty, 2]$

We have, $\frac{|x-2|}{x-2} \geq 0$

Case I: If $x < 2$; $\frac{|x-2|}{x-2} \geq 0$

$\Rightarrow \frac{-(x-2)}{x-2} \geq 0$ [$\because |x-2| = -(x-2)$, if $x < 2$]

$\Rightarrow -1 \geq 0$

which is not true.

Case II: If $x > 2$

$\frac{|x-2|}{x-2} \geq 0$

$\Rightarrow \frac{x-2}{x-2} \geq 0$ [$\because |x-2| = -x-2$, if $x > 2$]

$\Rightarrow 1 \geq 0$

which is true.

Also, given expression is not defined at $x = 2$, since denominator at $x = 2$ is 0.

Solution : There is no solution at $x = 2$

Hence, $x > 2$ is the required solution. i.e. $x \in (2, \infty)$.

Correct Answer: B

Question 138 : If $x, y, z > 0$, then $\frac{x^8 + y^8 + z^8}{x^3y^3z^3}$

(A) $\geq \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

(B) $\leq \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

(C) $\geq x + y + z$

(D) None of these

Solution :

We have, $x, y, z > 0$

From Tchebychef's inequality

$3(x^8 + y^8 + z^8) \geq (x^6 + y^6 + z^6)(x^2 + y^2 + z^2)$

$\geq 3x^2y^2z^2(x^2 + y^2 + z^2)$

$\geq 3x^2y^2z^2(xy + yz + zx)$

$$\therefore \quad \frac{x^8 + y^8 + z^8}{x^3y^3z^3} \geq \frac{3x^2y^2z^2(xy + yz + zx)}{3x^3y^3z^3}$$

$$\Rightarrow \frac{x^8 + y^8 + z^8}{x^3y^3z^3} \geq \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

Correct Answer: A

Question 139 : If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which on eof the following holds for all $n \geq 1$, by the principle of mathematical induction

(A) $A^n = nA + (n - 1)I$

(B) $A^n = 2^{n-1}A + (n - 1)I$

(C) $A^n = nA - (n - 1)I$

(D) $A^n = 2^{n-1}A - (n - 1)I$

$$A^2 = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$$

$$A^3 = \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$$

$$\therefore\ A^n = \begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}$$

$$nA = \begin{bmatrix}n & 0\\ n & n\end{bmatrix},\ (n-1)I = \begin{bmatrix}n-1 & 0\\ 0 & n-1\end{bmatrix}$$

Solution : $$nA - (n-1)I = \begin{bmatrix}1 & 0\\ n & 1\end{bmatrix} = A^n$$

Correct Answer: C

If $A = \begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$ and $B = \begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$, then value of α for

Question 140 : which $A^2 = B$ is

(A) 1

(B) –1

(C) 4

(D) No real values

Given that, $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} \alpha^2+0 & 0+0 \\ \alpha+1 & 0+1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

Also, $B = A^2$ [Given]

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

Solution : Clearly, this is not satisfied by any real value of α

Correct Answer: D

If $x = -4$ is root of

Question 141 : $\Delta = \begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the other roots are

(A) 1, 3

(B) 0, 2

(C) – 1, 1

(D) 2, 4

Applying $R_1 \to (R_1 + R_2 + R_3)$, we get

$$\Delta = \begin{vmatrix} x+4 & x+4 & x+4 \\ 1 & x & 1 \\ 8 & 2 & x \end{vmatrix}$$

Taking $(x + 4)$ common from R_1, we get

$$\Delta = (x+4)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we get

$$\Delta = (x+4)\begin{vmatrix} 1 & 0 & 0 \\ 1 & x-1 & 0 \\ 3 & -1 & x-3 \end{vmatrix}$$

Expanding along R_1

$\Delta = (x + 4)[(x - 1)(x - 3) - 0]$

Solution : Thus, $\Delta = 0$ implies $\Rightarrow x = -4, 1, 3$

Correct Answer: A

If the determinant $\Delta \begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$, then

Question 142 : the value of $\sin\theta$ is

(A) $\frac{1}{3}$ or 1

(B) $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{3}}{2}$

(C) 0 or $\frac{1}{2}$

(D) None of these

Solution :

Applying $R_2 \rightarrow R_2 + 4R_1$ and $R_3 \rightarrow R_3 + 7R_1$, we get

$$\begin{vmatrix} 3 & -2 & \sin 3\theta \\ 5 & 0 & \cos 3\theta + 4\sin 3\theta \\ 10 & 0 & 2 + 7\sin 3\theta \end{vmatrix} = 0$$

$\Rightarrow 2[5(2 + 7\sin 3\theta) - 10(\cos 2\theta) + 4\sin 3\theta)] = 0$

$\Rightarrow 2 - 2\cos 2\theta - \sin 3\theta = 0$

$\Rightarrow \sin\theta\,(4\sin^2\theta + 4\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = 0$ or $(2\sin\theta - 1) = 0$

or $(2\sin\theta + 3) = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = \frac{1}{2}$

Correct Answer: C

Question 143 :

$\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true, if and only if

(A) $x + y \neq 0$

(B) $x = y,\ x \neq 0,\ y \neq 0$

(C) $x = y$

(D) $x \neq 0,\ y \neq 0$

Solution :

We know that, $\sec^2\theta \geq 1$

$\Rightarrow \frac{4xy}{(x+y)^2} \geq 1 \Rightarrow 4xy \geq (x + y)^2$

$\Rightarrow (x - y)^2 \leq 0$

$\Rightarrow x - y = 0$

$\Rightarrow y = x$ and $x \neq 0,\ y \neq 0$

Correct Answer: B

Question 144 : The value of the expression $\sin^6\theta + \cos^6\theta + 3\sin^2\theta \cdot \cos^2\theta$ is

(A) ZERO

(B) 2

(C) 3

(D) 1

Solution :

$$\sin^6\theta + \cos^6\theta + 3\sin^2\theta \cdot \cos^2\theta$$
$$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta \cos^2\theta$$
$$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta \cos^2\theta) + 3\sin^2\theta \cos^2\theta$$
$$= 1[(\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta \cos^2\theta] + 3\sin^2\theta \cos^2\theta$$
$$= 1 - 3\sin^2\theta \cos^2\theta + 3\sin^2\theta \cos^2\theta = 1$$

Correct Answer: D

Question 145 : $\cos^{-1}\left(\frac{-1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right) + 3\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) - 4\tan^{-1}(-1)$ is equal to

(A) $\frac{19\pi}{12}$

(B) $\frac{35\pi}{12}$

(C) $\frac{47\pi}{12}$

(D) $\frac{43\pi}{12}$

Solution :

$$\text{Given, } \sin^{-1}\left(\frac{3}{x}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{x}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \cos^{-1}\left(\frac{4}{x}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \sin^{-1}\left(\frac{\sqrt{x^2-16}}{x}\right)$$

$$\Rightarrow \frac{3}{x} = \left(\frac{\sqrt{x^2-16}}{x}\right) \Rightarrow x = \pm 5 \qquad \therefore\ x = 5$$

Correct Answer: D

Question 146 : $\sin\left(2\sin^{-1}\sqrt{\frac{63}{65}}\right)$ is equal to

(A) $\frac{2\sqrt{126}}{65}$

(B) $\frac{4\sqrt{65}}{65}$

(C) $\frac{8\sqrt{63}}{65}$

(D) $\frac{\sqrt{63}}{65}$

Solution :

$$\sin\left(2\sin^{-1}\sqrt{\frac{63}{65}}\right)$$

$$= \sin\left(\pi - \sin^{-1} 2\sqrt{\frac{63}{65}}\sqrt{1-\frac{63}{65}}\right)$$

$$= \sin\left(\sin^{-1}\frac{2\sqrt{126}}{65}\right) = \frac{2\sqrt{126}}{65}$$

Correct Answer: A

Question 147 : $\lim_{x \to 0} \left[\frac{e^x - e^{\sin x}}{x - \sin x} \right]$ is equal to

(A) – 1

(B) ZERO

(C) 1

(D) None of these

Solution :

$$\lim_{x \to 0} \left[\frac{e^{\sin x} - e^x}{\sin x - x} \right]$$

$$= \lim_{x \to 0} e^x \lim_{x \to 0} \left[\frac{e^{\sin x - x} - e^x}{\sin x - x} \right] = e^0 \times 1 = 1$$

Correct Answer: C

Question 148 : $\lim_{x \to 0} \frac{x2^x - x}{1 - \cos x}$ is equal to

(A) 2 log 2

(B) log 2

(C) $\frac{1}{2} \log 2$

(D) $\frac{1}{2}$

Solution :

$$\text{Consider, } \lim_{x\to 0}\frac{x2^x - x}{1-\cos x} \qquad \left[\frac{0}{0}\text{ form}\right]$$

$$= \lim_{x\to 0}\frac{2^x + x2^x \log 2 - 1}{\sin x} \qquad \left[\frac{0}{0}\text{ form}\right]$$

$$= \lim_{x\to 0}\frac{2^x \log 2 + 2^x \log 2 + x2^x \log 2 . \log 2}{\cos x}$$

$$= \log 2 + \log 2 = 2 \log 2$$

Correct Answer: A

Question 149 : The derivative of $y = x^{\ln} x$ is

(A) $x^{\ln x} \ln x$

(B) $x^{\ln x} - 1 \ln x$

(C) $2x^{\ln x} - 1 \ln x$

(D) $x^{\ln x} - 2$

Solution :

Q $y = x^{\ln x}$

On taking log both sides, we get

$\ln y = (\ln x)^2$

On differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\ln x}{x}$$

$$\frac{dy}{dx} = y\frac{2\ln x}{x} = \frac{2(x^{\ln x})\ln x}{x}$$

$$\frac{dy}{dx} = 2x^{\ln x - 1}\ln x$$

Correct Answer: C

Question 150 : The differential of e^{x^3} w.r.t. logx is

(A) e^{x^3}

(B) $3x^2e^{x^3}$

(C) $3x^2 2e^{x3}$

(D) $3x^2e^{x^3} + 3x^2$

Solution :

$y = e^{x^3}$, $z = \log x$

On differentiating the equation,

$\frac{dy}{dx} = e^{x^3}(3x^2) = 3x^2e^{x^3}$ and $\frac{dz}{dx} = \frac{1}{x}$

$\therefore \quad \frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{3x^2e^{x^3}}{\left(\frac{1}{x}\right)} = 3x^3e^{x^3}$

Correct Answer: C

Question 151 : The function xx is increasing, when

(A) $x > \frac{1}{e}$

(B) $x < \frac{1}{e}$

(C) $x < 0$

(D) for all real x

Solution :

Let $y = x^x$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = x^x(1+\log x)$$

For increasing function, $\frac{dy}{dx} > 0$

$\therefore\ x^x(1+\log x) > 0 \Rightarrow 1+\log x > 0$

$\Rightarrow \log_e x > \log_e \frac{1}{e} \Rightarrow x > \frac{1}{e}$

Function is increasing when $x > \frac{1}{e}$.

Correct Answer: A

Question 152 : Maximum slope of the curve
y = –x3 + 3x2 + 9x – 27 is

(A) ZERO

(B) 12

(C) 16

(D) 32

Solution :

Let $y = f(x) = -x^3 + 3x^2 + 9x - 27$

The slope of this curve

$f'(x) = -3x^2 + 6x + 9$

Let $g(x) = f'(x) = -3x^2 + 6x + 9$

On differentiating w.r.t. x, we get

$g'(x) = -6x + 6$

For maxima or minima, put

$g'(x) = 0 \Rightarrow x = 1$

Now, $g''(x) = -6 < 0$ and hence at $x = 1$, g(x) slope will have maximum value.

$\therefore\ [g(1)]max = -3 \times 1 + 6(1) + 9 = 12$

Correct Answer: B

Question 153 : The value of $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x}dx$ is equal to

(A) x + C

(B) log(x + e) + C

(C) $\log(e^x + x^e) + C$

(D) $\log (x^e + e^x)^{1/e} + C$

Solution :

Given that, $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x}dx$

Put, $x^e + x^e = t$

$\Rightarrow (ex^{e-1} + e^x)dx = dt$

$\therefore I = \int \frac{1}{t}.\frac{dt}{e} = \frac{1}{e}\log t + C$

$= \frac{1}{e}\log(x^e + e^x) + C$

Correct Answer: D

Question 154 : What is the value of $\int (e^x + 1)^{-1}dx$?

(A) $\log(e^x + 1) + C$

(B) $\log(e^{-x} + 1) + C$

(C) $-\log(e^{-x} + 1) + C$

(D) $-(e^x + 1) + C$

Solution :

Let $I = \int (e^x + 1)^{-1} dx$

$= \int \frac{1}{e^x + 1} dx = \int \frac{e^{-x}}{1 + e^{-x}} dx$

Let $1 + e^{-x} = t$

$\Rightarrow -e^{-x} dx = dt$

$= -\log t + C$

$= -\log(1 + e^{-x}) + C$

Correct Answer: C

Question 155 : What is the solution of the differential equation x dy – y dx = xy^2 dx?

(A) $yx^2 + 2x = 2Cy$

(B) $y^2x + 2y = 2Cx$

(C) $y^2x^2 + 2x = 2Cy$

(D) None of the above

Solution :

$xdy - y\,dx = xy2\,dx$ [given]

$\Rightarrow \frac{xdy - ydx}{y^2} = xdx$

$\Rightarrow \frac{ydx - xdy}{y^2} = -xdx$

$\Rightarrow \int d\left(\frac{x}{y}\right) = -\int x\,dx$

$\Rightarrow \frac{x}{y} = -\frac{x^2}{2} + C$

$\Rightarrow 2x + x^2y = 2Cy$

Correct Answer: A

Question 156 : The order and degree of the differenital equation $\sqrt{\sin x}(dx+dy)=\sqrt{\cos x}(dx-dy)$ are

(A) (1, 2)

(B) (2, 2)

(C) (1, 1)

(D) (2, 1)

Solution :

Given, $\sqrt{\sin x}\left(1+\frac{dy}{dx}\right)=\sqrt{\cos x}\left(1-\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{\cos x}-\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$

Hence, order = 1
and degree = 1.

Correct Answer: C

Question 157 : ABC is a triangle with vertices A(–1, 4), B (6, –2) and C (–2, 4). D, E and F are the points which divide each AB, BC and CA, respectively in the ratio 3:1 internally. Then, the centroid of the DDEF is

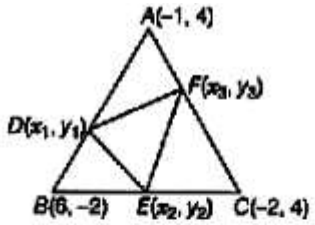
(A) (3, 6)

(B) (1, 2)

(C) (4, 8)

(D) (–3, 6)

Solution : Let (x^1, y^1), (x^2, y^2) and (x^3, y^3) be the coordinates of the points D, E and F which divide each AB, BC and CA, respectively in the ratio 3 : 1internally.



$\therefore\ x_1 = \frac{3\times6-1\times1}{4} = \frac{17}{4}$

and $y_1 = \frac{-2\times3+4\times1}{4} = -\frac{2}{4} = -\frac{1}{2}$

Similarly, $x_2 = 0,\ y_2 = \frac{5}{2}$

and $x_3 = -\frac{5}{4},\ y_3 = 4$

For centroid,

$x = \frac{x_1+x_2+x_3}{3} = \frac{\frac{17}{4}+0-\frac{5}{4}}{3} = \frac{12}{12} = 1$

and $y = \frac{y_1+y_2+y_3}{3} = \frac{-\frac{1}{2}+\frac{5}{2}+4}{3} = \frac{6}{3} = 2$

Hence, centroid of the ΔDEF is (1, 2).

Correct Answer: B

Question 158 : The incentre of the triangle with vertices , (0, 0) and (2, 0) is

(A) $\left(1, \frac{\sqrt{3}}{2}\right)$

(B) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$

(D) $\left(1, \frac{1}{\sqrt{3}}\right)$

Solution :

Let $A(1,\sqrt{3})$, $B(0, 0)$ and $C(2, 0)$ be the given points.

$\therefore BC = \sqrt{(2-0)^2+(0-0)^2} = 2$

$CA = \sqrt{(2-1)^2+(0-\sqrt{3})^2} = 2$

and $AB = \sqrt{1+3} = 2$

So, the triangle is equilateral.

We know that, in equilateral triangle, incentre is the same as centroid of the triangle.

$\therefore$ Incentre is $\left(\frac{1+0+2}{3}, \frac{\sqrt{3}+0+0}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$

Correct Answer: D

Question 159 : If one end of a diameter of the circle x2 + y2 – 4x – 11= 0 is (3, 4), then find the coordinates of the other end of the diameter.

(A) (2, 1)

(B) (1, 2)

(C) (1, 1)

(D) None of these

Solution :

The centre of the given circle $x^2 + y^2 – 4x – 6y + 11 = 0$ is (2, 3).

Let the other end of diameter be (x_1, y_1)

Since centre $= \left(\frac{x_1+3}{2}, \frac{y_1+4}{2}\right)$

$\therefore (2, 3) = \left(\frac{x_1+3}{2}, \frac{y_1+4}{2}\right)$

$\Rightarrow 2 = \frac{x_1+3}{2}, 3 = \frac{y_1+4}{2}$

$\therefore x_1 = 1, y_1 = 2$

Hence, coordinates of other end of the diameter is (1, 2).

Correct Answer: B

Question 160 : The equations of the circle which pass through the origin and make intercepts of lengths 4 and 8 on the X and Y-axes respectively, are

(A) $x^2 + y^2 \pm 4x \pm 8y = 0$

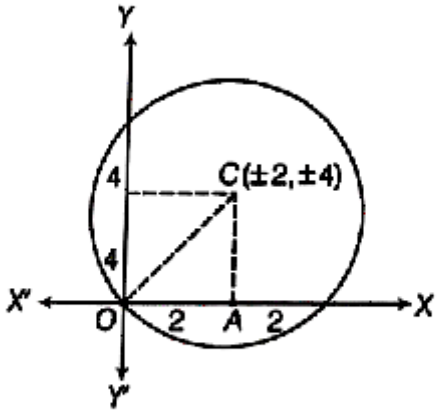
(B) $x^2 + y^2 \pm 2x \pm 4y = 0$

(C) $x^2 + y^2 \pm 8x \pm 16y = 0$

(D) $x^2 + y^2 \pm x \pm y = 0$

Solution :

In ΔOAC, $OC^2 = 2^2 + 4^2 = 20$



$\therefore$ Required equation of circle is

$(x \pm 2)^2 + (y \pm 4)^2 = 20 \Rightarrow x^2 + y^2 \pm 4x \pm 8y = 0$

Correct Answer: A

Question 161 : The parametric form of the ellipse $4(x + 1)^2 + (y - 1)^2 = 4$ is

(A) $x = \cos q - 1, y = 2\sin q - 1$

(B) x = 2cosq – 1, y = sinq + 1

(C) x = cosq – 1, y = 2sinq + 1

(D) x = cosq + 1, y = 2sinq + 1

Solution : Given, equation of ellipse can be rewritten as

$$\frac{(x+1)^2}{1}+\frac{(y-1)^2}{4}=1$$

$\therefore$ Parametric equations of ellipse is

$x + 1 = \cos\theta$ and $y - 1 = 2\sin\theta$

$\Rightarrow x = \cos\theta - 1$ and $y = 2\sin\theta + 1$

Correct Answer: C

Question 162 : If the area of the auxiliary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ is twice the area of the ellipse, then the eccentricity of the ellipse is

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{2}$

Solution : Given, ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose area is π ab.

The auxiliary circle to the given ellipse is

$x^2 + y^2 = a^2$ whose area is πa^2.

Given that, $\pi a^2 = 2\pi$ ab $\Rightarrow$ a = 2b

Now, eccentricity of ellipse

$$\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{b^2}{4b^2}}=\frac{\sqrt{3}}{2}$$

Correct Answer: B

Question 163 : If P, Q, R are the mid-points of the sides AB, BC, CA respectively of DABC and a, p, q are the position vector of A, P, Q respectively, then what is the position vector of R?

(A) 2a – (p – q)

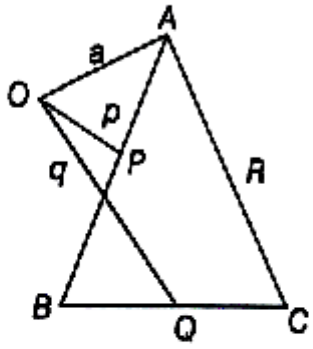
(B) (p – q) – 2a

(C) a – (p – q)

(D) $\frac{a}{2}-\frac{(p-q)}{2}$

Solution :

Let the position vectors of B, C and R are b, c and r respectively.



$\therefore\ p = \frac{\mathbf{a}+\mathbf{b}}{2} \Rightarrow \mathbf{b} = 2\mathbf{p} - \mathbf{a}$ .....(i)

$q = \frac{\mathbf{b}+\mathbf{c}}{2}$

$\Rightarrow \mathbf{c} = 2\mathbf{q} - 2\mathbf{p} + \mathbf{a}$ [From eq. (i)]

and $r = \frac{\mathbf{a}+\mathbf{c}}{2} = \mathbf{a} - (\mathbf{p} - \mathbf{q})$

Correct Answer: C

Question 164 : If a + b + c = 0 and |a| = 3, |b| = 5, |c| = 7, then angle between a and b is

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) π

Solution :

Given, $\mathbf{a} + \mathbf{b} = -\mathbf{c}$

$(\mathbf{a} + \mathbf{b})^2 = (-\mathbf{c})^2$

$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2|\mathbf{a}|.|\mathbf{b}| \cos\theta = |\mathbf{c}|^2$

$\Rightarrow (3)^2 + (5)^2 + 2 \times 3 \times 5\cos\theta = 49$

$\Rightarrow \theta = \frac{\pi}{3}$

Correct Answer: A

Question 165 : If $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are perpendicular to $\mathbf{q} + \mathbf{r}$, $\mathbf{r} + \mathbf{p}$ and $\mathbf{p} + \mathbf{q}$, respectively and if $|\mathbf{p} + \mathbf{q}| = 6$, $|\mathbf{q} + \mathbf{r}| = 4\sqrt{3}$ and $|\mathbf{r} + \mathbf{p}| = 4$, then $|\mathbf{p} + \mathbf{q} + \mathbf{r}|$ is

(A) $5\sqrt{2}$

(B) 10

(C) 15

(D) 5

Solution :

$|\mathbf{p} + \mathbf{q}| = 6$

$\Rightarrow |\mathbf{p} + \mathbf{q}|^2 = 36$

$\Rightarrow \mathbf{p}^2 + \mathbf{q}^2 + 2\mathbf{p}.\mathbf{q} = 36$

Similarly, $\mathbf{q}^2 + \mathbf{r}^2 + 2\mathbf{q}.\mathbf{r} = 48$

and $\mathbf{r}^2 + \mathbf{p}^2 + 2\mathbf{r}.\mathbf{p} = 16$

On adding all, we get

$2(\mathbf{p}^2 + \mathbf{q}^2 + \mathbf{r}^2) = 100$ $[\because \mathbf{p}.\mathbf{q} + \mathbf{q}.\mathbf{r} + \mathbf{r}.\mathbf{p} = 0]$

$\Rightarrow \mathbf{p}^2 + \mathbf{q}^2 + \mathbf{r}^2 = 50$

$\Rightarrow |\mathbf{p} + \mathbf{q} + \mathbf{r}|^2 = 50$

$\Rightarrow |\mathbf{p} + \mathbf{q} + \mathbf{r}| = 5\sqrt{2}$

Correct Answer: A

The equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and

Question 166 : $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to X-axis is

(A) x + y – 3z + 6 = 0

(B) y – 3z + 6 = 0

(C) 3y – z + 6 = 0

(D) y – z + 6 = 0

The intersection equation of planes
$\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$ is
$(\vec{r}.\vec{n}_1 - d_1) + \lambda(\vec{r}.\vec{n}_2 - d_2) = 0$

Given, planes are $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$

and $\vec{r}.(2\hat{i}+3\hat{j}-4\hat{k})+4=0$

The equation of any plane passing through the line of intersection of these planes is

Solution : $[\vec{r}.(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4]=0$

$\vec{r}.[(2\lambda+1)\hat{i}+(3\lambda+1)\hat{j}+(1-\lambda)\hat{k}]+(4\lambda-1)=0$ ....(i)

Its direction ratios are $(2\lambda + 1)$, $(3\lambda + 1)$ and $(1 - \lambda)$.
The required plane is parallel to X-axis. Therefore, its normal is perpendicular to X-axis. The direction ratios of X-axis are 1, 0 and 0.
$\therefore\ 1(2\lambda + 1) + 0(3\lambda + 1) + 0(1 - \lambda) = 0$

$\Rightarrow 2\lambda + 1 = 0 \Rightarrow \lambda = -\frac{1}{2}$

On putting the value of λ in Eq. (i), we obtain

$\vec{r}.(\hat{j}-3\hat{k})+6=0$

Therefore, its cartesian equation is

$y - 3z + 6 = 0$ (put $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$)

Correct Answer: C

Question 167 : Equation of the plane through the mid-point of the line segment joining the points

P(4, 5, –10), Q(–1, 2, 1) and perpendicular to PQ is

(A) $r.\left(\frac{3}{2}\hat{i}+\frac{7}{2}\hat{j}-\frac{9}{2}\hat{k}\right)=45$

(B) $r.\left(-\hat{i}+2\hat{j}+\hat{k}\right)=\frac{135}{2}$

(C) $r.\left(5\hat{i}+3\hat{j}-11\hat{k}\right)+\frac{135}{2}=0$

(D) $r.\left(5\hat{i}+3\hat{j}-11\hat{k}\right)=\frac{135}{2}$

Solution :

Coordinates of mid-point of

P(4, 5, –10) and Q(–1, 2, 1) are $\left(\frac{8}{2},\frac{7}{2},\frac{-9}{2}\right)$

Now, DR's of PQ are (5, 3, –11).

$\therefore$ Equation of plane passing through $\left(\frac{3}{2},\frac{7}{2},\frac{-9}{2}\right)$

and having DR's (5, 3, –11) is

$$5\left(x-\frac{3}{2}\right)+3\left(y-\frac{7}{2}\right)=11\left(z+\frac{9}{2}\right)=0$$

$$\Rightarrow 5x+3y-11z=\frac{135}{2}$$

$\therefore$ It is written in vector form as

$$r.\left(5\hat{i}+3\hat{j}-11\hat{k}\right)=\frac{135}{2}$$

Correct Answer: D

Question 168 : The plane ax + by + cz = 1 meets the coordinate axes at A, B and C. The centroid of DABC is

(A) (3a, 3b, 3c)

(B) $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

(C) $\left(\frac{1}{3a},\frac{1}{3b},\frac{1}{3c}\right)$

(D) (a, b, c)

Solution :

The coordinates of A, B and C are

$\left(\frac{1}{a},0,0\right),\left(0,\frac{1}{b},0\right)$ and $\left(0,0,\frac{1}{c}\right)$, respectively

Hence, the centroid of ΔABC is

$\left(\frac{1}{3a},\frac{1}{3b},\frac{1}{3c}\right)$

Correct Answer: C

Question 169 : Mean of 100 observations is 45. If it was later found that two observations 19 and 31 were incorrectly recorded as 91 and 13. The correct mean is

(A) 44

(B) 45

(C) 44.46

(D) 45.54

Solution :

Total of corrected observations

$= 4500 - (91 + 13) + (19 + 31)$

$= 4446$

$\therefore$ Mean $= \frac{4446}{100} = 44.46$

Correct Answer: C

Question 170 : If $\bar{x}_1$ and $\bar{x}_2$ are the means of two distributions such that $\bar{x}_1 < \bar{x}_2$ and $\bar{x}$ is the mean of the combined distribution, then

(A) $\bar{x} < \bar{x}_1$

(B) $\bar{x} > \bar{x}_2$

(C) $\bar{x} = \frac{\bar{x}_1 + \bar{x}_2}{2}$

(D) $\bar{x}_1 < \bar{x} < \bar{x}_2$

Solution : Let n_1 and n_2 be the number of observations in two groups having means $\bar{x}_1$ and $\bar{x}_2$, respectively. Then,

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Now, $\bar{x} - \bar{x}_1 = \frac{n_2(\bar{x}_2 - \bar{x}_1)}{n_1 + n_2} > 0$ [$\because \bar{x}_2 > \bar{x}_1$ (given)]

$\Rightarrow \bar{x} > \bar{x}_1$ .....(i)

Also, $\bar{x} - \bar{x}_2 = \frac{n_1(\bar{x}_1 > \bar{x}_2)}{n_1 + n_2} < 0$ [$\because \bar{x}_2 > \bar{x}_1$ (given)]

$\Rightarrow \bar{x} < \bar{x}_2$ .....(ii)

From Eqs. (i) and (ii), we have

$\bar{x}_1 > \bar{x} < \bar{x}_2$

Correct Answer: D

Question 171 : In a class of 100 students, the average amount of pocket money is Rs. 35 per student. If the aveage is Rs. 25 for girls and Rs. 50 for boys, then the number of girls in the clas is

(A) 20

(B) 40

(C) 60

(D) 80

Solution :

Let the number of girls in the class = y

$\therefore$ Number of boys in the class = 100 – y

Now, $\bar{x}_1 = 25$, $n_1 = y$, $\bar{x}_2 = 50$, $n_2 = 100 - y$

and $X_2 = 35$, $n_1 + n_2 = 100$

$$\therefore 35 = \frac{25 \times y + 50 \times (100 - y)}{100}$$

$\Rightarrow 25y = 1500$

$\Rightarrow y = 60$

Correct Answer: C

Question 172 : The probability that in the toss of two dice, we obtain an even sum or a sum less than 5 is

(A) $\frac{1}{2}$

(B) $\frac{1}{6}$

(C) $\frac{2}{3}$

(D) $\frac{5}{9}$

Solution :

Let A be the event of obtaining an even sum and B be the event of obtaining a sum leass than 5.

Then, we have to find $P(A \cup B)$. Since, A, B are not mutually exclusive, we have

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{6}{36} - \frac{4}{36} = \frac{5}{9}$

[Since, there are 18 ways to get an even sum and 6 ways to get a sum less than 5, i.e (1, 3), (3, 1), (2, 2), (1, 2), (2, 1), (1, 1) and 4 ways to get an even sum less than 5 i.e. (1, 3), (3, 1), (2, 2), (1, 1)]

Correct Answer: D

Question 173 :

For k = 1, 2, 3 the box Bk contains k red balls and (k + 1) white balls. Let $P(B_1) = \frac{1}{2}$, $P(B_2) = \frac{1}{3}$ and $P(B_3) = \frac{1}{6}$. A box is seleted at random and a ball is drawn from it. If a red ball is drawn, then the probability that it has come from box B_2, is

(A) $\frac{35}{78}$

(B) $\frac{14}{39}$

(C) $\frac{10}{13}$

(D) $\frac{12}{13}$

$$\frac{\frac{1}{3}\times\frac{2}{5}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{5}+\frac{1}{6}\times\frac{3}{7}}$$

Solution : $= \frac{\frac{2}{15}}{\frac{1}{6}+\frac{2}{15}+\frac{1}{14}} = \frac{14}{39}$

Correct Answer: B

Question 174 : X speaks truth in 60% and Y in 50% of the cases. The probability that they contradict each other narrating the same incident is

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

Solution :

Given, $P(X) = 60\% = \frac{3}{5}$, $P(Y) = 50\% = \frac{1}{2}$

$\Rightarrow P(X') = 1 - \frac{3}{5} = \frac{2}{5}$,

$P(Y') = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ Required probability that they contradict each other narrating the same incident.

$= [P(X) \times P(Y') + P(X') + P(Y)]$

$= \left[\frac{3}{5}\times\frac{1}{2}+\frac{2}{5}\times\frac{1}{2}\right] = \frac{1}{2}$

Correct Answer: C

Question 175 : Given $\bar{a} = (3,1,-5), \bar{b} = (2,2,-3)$

(A) $\bar{a}$ and $\bar{b}$ are linaerly independent vectors

(B) $\bar{a}$ and $\bar{b}$ are linaerly dependent vectors

(C) $c_1\bar{a} + c_2\bar{b} = 0$ only when $c_1 = c_2 \neq 0$

(D) $c_1\bar{a} + c_2\bar{b} = 0$ for some $c_1 \neq 0$ or $c_2 \neq 0$

Solution :

$$c_1\bar{a}_1 - c_2\bar{a}_2 = 0$$
$$\Rightarrow c_1(3, 1, -5) + c_2(2, 2, -3) = 0$$
$$\Rightarrow 3c_1 + 2c_2 = 0$$
$$\Rightarrow c_1 + 2c_2 = 0$$
$$-5c_1 - 3c_2 = 0$$
$$\Rightarrow c_1 = 0, c_2 = 0$$

Hence, $\bar{a}$ and $\bar{b}$ are linearly independent vectors

Correct Answer: A

Question 176 : If V is the vector space of all functions from R to R and W = {f : f(4) = 3 + f(2)},

(A) W is a subspace of V

(B) W is not a subspace of V

(C) W is not closed under scalar multiplication

(D) W is closed under scalar multiplication

Solution :

We have, f, g ∈ W

f(4) = 3 + f(2) and g(4) = 3 + g(2) .....(i)

If a, b ∈ R, then

(af + bg)(4) = af(4) + bg(4)

= a[3 + f(2)] + b[3 + g(2)] [From eq. (i)]

= af(2) + bg(2) + 3a + 3b

= (af + bg)(2) + 3(a + b)

≠ 3 + (af + bg)(2)

⇒ (af + bg)(4) ≠ W

Hence, W is not a subspace of V.

Correct Answer: B

Question 177 : Let T : R2 → R2 be a map defined by
T(x, y) = (x + y, x – y).
Which of the following statements is correct?

(A) T is linear and its kernel has infinite number of element of R^2

(B) T is not linear

(C) The kernel of T consists of only two elements of R^2

(D) Nulity of T is zero

Solution :

If (x, y) ∈ Ker(T)

⇒ T(x, y) = (0, 0)

⇒ (x + y, x – y) = (0, 0)

⇒ x = 0, y = 0

i.e. dimension of Ker(T) is 0 or Nullity of T = 0

Correct Answer: D

$$\text{If } f(x,y) = \begin{cases} xy \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Question 178 : Then,

(A) $f_x(0, 0) = 0 = f_y(0, 0)$

(B) $f_x(0, 0) = 1 = f_y(0, 0)$

(C) $f_x(0, 0)$ and $f_y(0, 0)$ does not exists

(D) None of the above

Solution :

We have,

$$fx(0, 0) = \lim_{h\to 0}\frac{f(0+h,0)-f(0,0)}{h}$$

$$= \lim_{h\to 0}\frac{f(h, 0)}{h} = \lim_{h\to 0} 0 = 0$$

$$\text{and } f_y(0, 0) = \lim_{k\to 0}\frac{f(0, 0+k) - f(0,0)}{k}$$

$$= \lim_{k\to 0}\frac{f(0, k)}{k} = \lim_{k\to 0} 0 = 0$$

$$\therefore f_x(0, 0) = f_y(0, 0) = 0$$

Correct Answer: A

Question 179 : For what value of k, the function

$$f(x,y) = \begin{cases} \dfrac{\sin^{-1}(xy-2)}{\tan^{-1}(3xy-6)}, & (x,y)\neq(1,2) \\ k, & (x,y) = (1,2) \end{cases}$$

is continuous?

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

Solution :

$$\lim_{(x,y)\to(1,2)} f(x,y) = \lim_{(x,y)\to(1,2)} \frac{\sin^{-1}(xy-2)}{\tan^{-1}(3xy-6)}$$

Putting $xy - 2 = t$, we get

$$\lim_{t\to 0} \frac{\sin^{-1} t}{\tan^{-1} 3t} = \lim_{t\to 0} \frac{1/\sqrt{1-t^2}}{3/(1+3t^2)} = \frac{1}{3}$$

$\therefore\ k = \frac{1}{3}$

Correct Answer: B

Question 180 : The function f(z) = secz is

(A) analytic for all z

(B) analytic for $z = \pi$

(C) not analytic at $z = \frac{\pi}{2}$

(D) None of the above

Solution :

$f(z) = \sec z = \frac{1}{\cos z}$ which is not defined when

$\cos z = 0$

$\Rightarrow$ f(z) is not analytic at

$z = (2n+1)\frac{\pi}{2}, n \in I$

Correct Answer: C