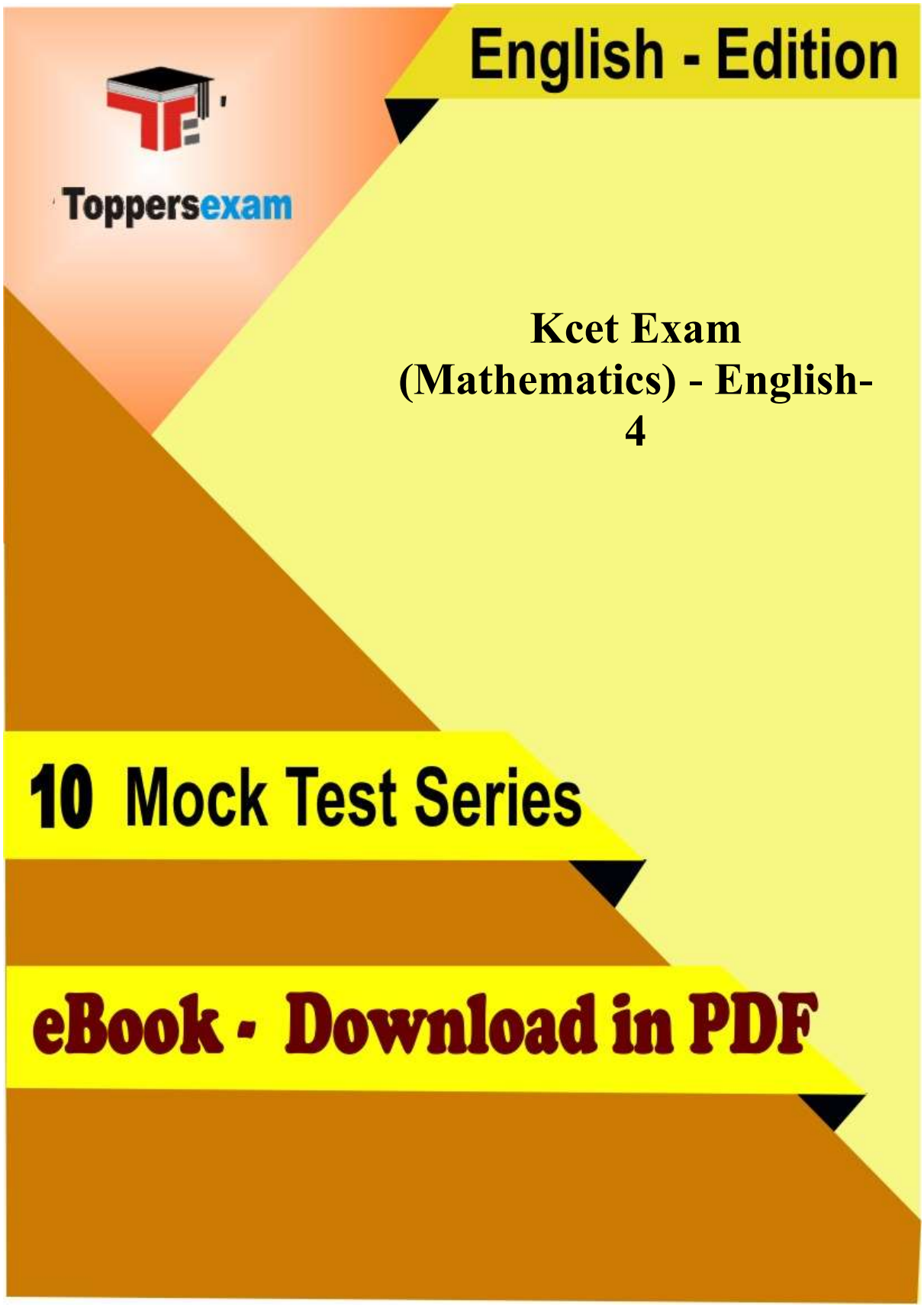

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Kcet Exam (Mathematics) - English-4

Paper Questions

SUBJECT: Physics

Question 1 : An astronaut on a strange planet finds that acceleration due to gravity is twice as that on the surface of earth. Which of the following could explain this?

(A) Both the mass and radius of the planet are half as that of earth

(B) Radius of the planet is half as that of earth, but the mass is the same as that of earth

(C) Both the mass and radius of the planet are twice as that of earth

(D) Mass of the planet is half as that of earth, but radius is same as that of earth

Correct Answer: A

Question 2 : Which of the following substances has the highest elasticity?

(A) Sponge

(B) Steel

(C) Rubber

(D) Copper

Correct Answer: B

Question 3 : Three liquids of equal masses are taken in three identical cubical vessels A,B and C. Their densities are p_A, p_B and p_C respectively but $p_A < p_B < p_C$. The force exerted by the liquid on the base of the cubical vessel is

(A) maximum in vessel C

(B) Minimum in vessel C

(C) the same in all the vessels

(D) maximum in vessel A

Correct Answer: C

Question 4 : Water is in streamline flows along a horizontal pipe with non-uniform cross-section. At a point in the pipe where the area of cross-section is 10 cm^2, the velocity of water is 1 ms^{-1} and the pressure is 2000 Pa. The pressure at another point where the cross-sectional area is 5 cm2 is

(A) 4000 Pa

(B) 2000 Pa

(C) 1000 Pa

(D) 500 Pa

Correct Answer: D

Question 5 : The height of a waterfall is 50 m. IF g = 9.8 ms^{-2} the difference between the temperature at the top and the bottom of the waterfall is

(A) 1.17^0C

(B) 2.17^0C

(C) 0.117^0C

(D) 1.43^0C

Correct Answer: C

Question 6 : There is a uniform magnetic field directed perpendicular and into the plane of the paper. An irregular shaped conducting loop is slowly changing into a circular loop in the plane of the paper. Then

(A) current is induced in the loop in the anticlockwise direction

(B) current is induced in the loop in the clockwise direction

(C) AC is induced in the loop

(D) no cu rrent is induced in the loop

Correct Answer: A

Question 7 : It is difficult to cook rice in an open vessel by boiling it at high altitudes because of

(A) Low boiling point and high pressure

(B) High boiling point and low pressure

(C) Low boiling point and low pressure

(D) High boiling point and high pressure

Correct Answer: C

Question 8 : A train is moving slowly on a straight track with a constant speed of 2 ms^{-1}. A passenger in that train starts walking at a steady speed of 2 ms^{-1} to the back of the train in the opposite direction of the motion of the train. So to an observer standing on the platform directly in front of that passenger, the velocity of the passenger appears to be

(A) 4 ms^{-1}

(B) 2 ms^{-1}

(C) 2 ms^{-1} in the opposite direction of the train

(D) zero

Correct Answer: D

Question 9 : If the linear momentum of a body is increased by 50%, then the kinetic energy of that body increases by

(A) 100%

(B) 125%

(C) 225%

(D) 25%

Correct Answer: B

Question 10 : The temperature of a gas contained in a closed vessel of constant volume increased by $1^{o}C$ when the pressure of the gas is increased by 1%. The initial temperature of the gas is

(A) 100 K

(B) $273^{o}K$

(C) $100^{o}C$

(D) 200 K

Correct Answer: A

Question 11 : A motorboat covers a given distance in 6 h moving downstream on a river. It covers the same distance in 10 h moving upstream. The time it takes to cover the same distance in still water is

(A) 9 h

(B) 7.5 h

(C) 6.5 h

(D) 8 h

Correct Answer: B

Question 12 : 19 g of water of 30^0C and 5 g of at -20^0C are mixed together in a calorimeter. What is the final temperature of the mixture ? (Given specific heat of ice = 0.5 cal g^{-1} $(^0C)^{-1}$ and latent heat of fusion of ice = 80 cal g^{-1})

(A) ^{0}C

(B) -5^0C

(C) 5^0C

(D) 10^0C

Correct Answer: C

Question 13 : Hot water cools from 60^oC to 50^oC in the first 10 min and 10 min and to 42^oC in the next 10 min. Then the temperature of the surrounding is

(A) 20^oC

(B) 30^oC

(C) 15^oC

(D) 10^oC

Correct Answer: D

Question 14 : The efficiency of Carnot's heat engine is 0.5 when the temperature of the source is T_1 and that of sink is T_2. The efficiency of another Carnot's heat engine is also 0.5. The temperatures of source and sink of the second engine are respectively

(A) $2T_1$, $2T_2$

(B) $2T_1$

(C) $T_1 + 5$, $T_2 - 5$

(D) $T_1 + 10$, $T_2 - 10$

Correct Answer: A

Question 15 : Two rods of equal length and diameter have thermal conductivities 3 and 4 units respectively. IF they are joined in series. The thermal conductivity of the combination would be

(A) 3.43

(B) 3.5

(C) 3.4

(D) 3.34

Correct Answer: A

Question 16 : The poisson's ratio of a material is 0.5. if a force is applied to a wire of this material there is a decrease in the cross- sectional area by 4% The percentage increase in the length is

(A) 1%

(B) 2%

(C) 2.5%

(D) 4%

Correct Answer: D

Question 17 : The spectrum of an oil flame is an example for

(A) line emission spectrum

(B) continuous emission spectrum

(C) line absorption spectrum

(D) band emission spectrum

Correct Answer: B

Question 18 : According to Einstein's photoelectric equation, the graph of KE of the photoelectron emitted from the metal versus the frequency of the incident radiation gives a straight line graph, whose slope

(A) depends on the intensity of the incident radiation

(B) depends on the nature of the metal and also on the intensity of incident radiation

(C) is same for all metals and independent of the intensity of the incident radiation

(D) depends on the nature of the metal

Correct Answer: C

Question 19 : An electrical cable having a resistance of 0.2 delivers 10 kW at 200 B DC to a factory. What is the efficiency of transmission

(A) 65%

(B) 75%

(C) 85%

(D) 95%

Correct Answer: D

Question 20 : One kg of copper is drawn into a wire of 1 mm diameter and a wire of 2 mm diameter. The resistance of the two wires will be in the ratio

(A) 2 : 1

(B) 1 : 2

(C)

16 : 1

(D)

4 : 1

Correct Answer: C

Question 21 : Pick out the correct statement from the following.

(A) Energy released per unit mass of the reactant is less in case of fusion reaction

(B) Packing fraction may be positive or may be negative

(C) Pu^{239} is not suitable for a fission reaction

(D) For stable nucleus, the specific binding energy is low

Correct Answer: B

Question 22 : A radioactive sample S_1 having the activity A_1 has twice the number of nuclei as another sample S_2 of activity A_2. If $A_2 = 2A_1$, then the ratio of half-life of S_1 to the half-life of S_2 is

(A) 4

(B) 2

(C) 0.25

(D) 0.75

Correct Answer: A

Question 23 : When a neutron is disintegrated to give a β-particle,

(A) a neutrino along is emitted

(B) a proton and neutrino are emitted

(C) a proton alone is emitted

(D) a proton and an antineutrino are emitted

Correct Answer: D

Question 24 : The forbidden energy gap in Ge is 0.72 eV. Given, hc = 12400 eV-Å. The maximum wavelength of radiation that will generate electron hole pair is

(A) 172220Å

(B) 172.2Å

(C) 17222Å

(D) 1722 Å

Correct Answer: C

Question 25 : In a p-n junction diode not connected to any circuit

(A) the potential is the same everywhere

(B) the p-type side has a higher potential than the n-type side

(C) there is an electric field at the junction directed from the n-type side to p-type side

(D) there is an electric field at the junction directed from the p-type side to n-type side

Correct Answer: C

Question 26 : In a given direction, the intensities of the scattered light by a scattering substance for two beams of light are in the ratio of 256 : 81. The ratio of the frequency of the first beam to the frequency of the second beams is

(A) 64 : 127

(B) 1 : 2

(C) 64 : 27

(D) None of these

Correct Answer: D

Question 27 : The de-Broglie wavelength of the electron in the ground state of the hydrogen atom is ……..(radius of the first orbit of hydrogen atom = 0.53Å).

(A) 1.67 Å

(B) 3.33Å

(C) 1.06Å

(D) 0.53Å

Correct Answer: B

Question 28 : Intensity level of sound whose intensity is 10-8 Wm-2 is :

(A) 80 dB

(B) 8 dB

(C) 4 dB

(D) 40 dB

Correct Answer: D

Question 29 : In a series resonant *R-L-C* circuit, the voltage across R is 100 v and the value of R = 1000Ω. The capacitance of the capacitor is $2x10^{-6}$F; angular frequency of AC is 200 rad $^{s-1}$. Then the potential difference across the inductance coil is

(A) 100 V

(B) 40 V

(C) 250 V

(D) 400 V

Correct Answer: C

Question 30 : A capacitor and an inductance coil are connected in separate AC circuits with a bulb glowing in both the circuits. The bulb glows more brightly when

(A) an iron rod is introduced into the inductance coil

(B) the number of turns in the inductance coil is increased

(C) separation between the plates of the capacitor is increased

(D) a dielectric is introduced into the gap between the plates of the capacitor

Correct Answer: D

Question 31 : A charge + Q is moving upwards vertically. It enters a magnetic field direction to north.The force on the charge will be towards

(A) north

(B) south

(C) east

(D) west

Correct Answer: D

Question 32 : An α-particle if mass 6.4×10^{-27} kg and charge 3.2×10^{-19} C is situated in a uniform electric field of 1.6×10^{5} Vm^{-1}. The velocity of the particle at the end of 2×10^{-2} m path when it starts from rest is

(A) $\times 10^{5}$ ms^{-1}

(B) 8×10^{5} ms^{-1}

(C) 16×10^{5} ms^{-1}

(D) $\times 10^{5}$ ms^{-1}

Correct Answer: D

Question 33 : A cylindrical tube open at both the ends has a fundamental frequency of 390 Hz in air.

If th of the tube is immersed vertically in water the fundamental frequency of air column is

(A) 260 Hz

(B) 130 Hz

(C) 390 Hz

(D) 520 Hz

Correct Answer: A

Question 34 : In ruby laser, the stimulated emission is due to transition from

(A) metastable state to any lower state

(B) any higher state of lower state

(C) metastable state to ground state

(D) any higher state to ground state

Correct Answer: C

Question 35 : Which of the following is incorrect?

(A) 'Diffraction' helps us to distinguish between sound wave and light Wave

(B) If the wave is longitudinal, it must be a mechanical wave

(C) If the wave is mechanical, it may or may not be a transverse wave

(D) Mechanical waves cannot propagate in vacuum

Correct Answer: B

Question 36 : The equation of state for *n* moles of an ideal gas is *p*V = nRT, where R is a constant. The SI unit for R is

(A) JK^{-1} per molecule

(B) $JK^{-1}\ mol^{-1}$

(C) $JK^{-1}K^{-1}$

(D) $JK^{-1}g^{-1}$

Correct Answer: B

Question 37 : A chimpanzee swinging on a swing in a sitting position, stands up suddenly, the time period will :

(A) become infinite

(B) remain same

(C) increase

(D) decrease

Correct Answer: D

Question 38 : The number of electron is 2 coulomb of charge is

(A) 5×10^{29}

(B) 12.5×10^{18}

(C) 1.6×10^{19}

(D) 9×10^{11}

Correct Answer: B

Question 39 : Transverse waves can propagate in :

(A) liquids

(B) solids

(C) gases

(D) none of these

Correct Answer: B

Question 40 : A box is lying on an inclined plane what is the coefficient of static friction if the box starts sliding when an angle of inclination is 60^{o} :

(A) 1.173

(B) 1.732

(C) 2.732

(D) 1.677

Correct Answer: B

Question 41 : For which of the following combination of working tempratures, the efficiency of Carnot's engines is highest ?

(A) 100 K, 80 K

(B) 80 K, 60 K

(C) 60 K, 40 K

(D) 40 K, 20 K

Correct Answer: D

Question 42 : Myopia is corrected by :

(A) cylindrical lens

(B) bifocal lens

(C) concave lens

(D) convex lens

Correct Answer: C

Question 43 : On increasing the temperature of a substance gradyually, Which of the following colours will be noticed by you :

(A) white

(B) yellow

(C) green

(D) red

Correct Answer: D

Question 44 : The current flowing through a wire depends on time as $I = 3t^2 + 2t + 5$. The charge flowing through the cross-section of the wire in time from t = 0 to t = 2 s is

(A) 22 C

(B) 20 C

(C) 18 C

(D) 5 C

Correct Answer: A

Question 45 : An aeroplane is flying with a uniform speed of 100 m/ s along a Circular path of radius 100 m. The angular speed of the aeroplane will be :

(A) 1rad / sec

(B) 2 rad / sec

(C) 3 rad / sec

(D) 4 rad / sec

Correct Answer: B

Question 46 : If the charge on a capacitor is increased by 2 coulomb, the energy stored in it increases by 21% .The original charge on the capacitor is

(A)

10C

(B) 20C

(C) 30C

(D) 40C

Correct Answer: B

Question 47 : v_1 and v_2 are the velocities of sound at the same temperature in two monoatomic gases of densities p1 and p2 respectively. If p1/p2 = ,then the ratio of velocities v_1 and v_2 will be :

(A) 1 : 2

(B) 4 : 1

(C) 2 : 1

(D) 1 : 4

Correct Answer: C

Question 48 : The resistance of a conductor is 5 ohm at 50°C and 6 ohm at 100°C. Its resistahce at 0°C is :

(A) 1 ohm

(B) 2 ohm

(C) 3 ohm

(D) 4 ohm

Correct Answer: D

Question 49 : Which of the following materials is the best conductor of electricity ?

(A) Platinum

(B) Gold

(C) Silicon

(D) Copper

Correct Answer: D

Question 50 : If in a Wire of Young's modulus Y, longitudinal strain X is produced, then the value of potential energy stored in its unit volume will be :

(A) $0.5YX^2$

(B) $0.5Y^2X$

(C) $2YX^2$

(D) YX^2

Correct Answer: A

Question 51 : Light is incident on a glass plate at an angle 600, the reflected and refracted rays are mutually perpendicular to each other, then the refractive index of plate will be :

(A) 1.732

(B) 1.5

(C) 1.4

(D) 1.6

Correct Answer: A

Question 52 : A closed organ pipe an open organ pipe of same length produce 2 beats/second while vibrating in their fundamental modes. The length of the open organ pipe is halved and that of closed pipe is doubled. Then the number of beats produced per second while vibrating in the fundamental mode is

(A) 2

(B) 6

(C) 8

(D) 7

Correct Answer: D

Question 53 : The Younge's double slit experiment, the separation between the slits is halved and the distance between slits and screen is doubled. The fringes width will :

(A) remain the same

(B) be quadrupled

(C) be doubled

(D) be halved

Correct Answer: B

Question 54 : A source X of unknown frequency produces 8 beats per second, with a source of 250 Hz and 12 beats per second with a source of 270 Hz. Then the frequency of the source X will be :

(A) 284 Hz

(B) 265 Hz

(C) 258 Hz

(D) 252 Hz

Correct Answer: C

Question 55 : A plano-convex lens is made of refractive index of 1.6. The radius of curvature of

the curved surface is 60 cm. The focal length of the lens is

(A) 400

(B) 200 cm

(C) 100 cm

(D) 50 cm

Correct Answer: C

Question 56 : A diffraction is obtained by using a beam of red light. What will happen if the redlight is replaced by the blue light ?

(A) bands will narrower and crowd full together

(B) bands become broader and further apart

(C) no change will take place

(D) bands disappear

Correct Answer: A

Question 57 : After an interval of one day 1 / 6th of the initial amount of a radioactive material remains in a sample. Its half life will be :

(A) 2 hour

(B) 3 hour

(C) 6 hour

(D) 12 hour

Correct Answer: C

Question 58 : Two coherent light beams of intensity I and 4I are superimposed. The maximum and minimum possible intensities in the resulting beam are :-

(A) 9I and 3I

(B) 9I and I

(C) 5I and 3I

(D) 5I and I

Correct Answer: B

Question 59 : If the energy released in the fission of one nucleus is 200 MeV. Then the number of nuclei required per second in a power plant of 16 kW will be :

(A) 0.5 $x10^{14}$

(B) 0.5 x 10^{12}

(C) 5 x 10^{12}

(D) 5 x 10^{14}

Correct Answer: D

Question 60 : Diffraction effects are easier to notice in the case of sound waves than in case of light waves, because the sound waves are :

(A) of longer wavelengths

(B) of shorter wavelengths

(C) longitudinal waves

(D) mechanical Waves

Correct Answer: A

SUBJECT: Chemistry

Question 61 : Benzaldehyde and acetone can be best distinguished using

(A) Fehling's solution

(B) sodium hydroxide solution

(C) 2, 4-DNA

(D) Tollen's reagent

Correct Answer: D

Question 62 : Which one of the following statement is true?

(A) Saponification of oil yields a diol

(B) Drying of oil involves hydrolysis

(C) Addition of antioxidant to oil minimizes rancidity

(D) Refining of oil involves hydrogenation

Correct Answer: C

Question 63 : The following data is obtained during the first order thermal decomposition of 2A(g) -> B(g) + C(s) at constant volume and temperature.

S.NO.	Time	Total pressure in Pascal
1.	At the end of 10 min	300
2.	After completion	200

The rate constant in min-1 is

(A) 0.0693

(B) 6.93

(C) 0.00693

(D) 69.3

Correct Answer: A

Question 64 : Saccharin, an artificial sweetener, is manufactured from :

(A) toluene

(B) cyclohexane

(C) starch

(D) cellulose

Correct Answer: A

Question 65 : The correct sequence of steps involved in the mechanism of Cannizaro's reaction is

(A) nuccleophilic attack, transfer of H^- and transfer of H^+

(B) transfer of H^-, transfer of H^+ and nucleophilic attack

(C) transfer of H^+, nucleophilic attack and transfer of H^-

(D) electrophilic attack by OH^-, transfer of H^+ and transfer of H^-

Correct Answer: A

Question 66 : Which one of the following is an example for homogeneous catalysis?

(A) Manufacture of sulphuric acid by Contact process

(B) manufacture of ammonia by Haber's process

(C) Hydrolysis of sucrose in presence of dilute hydrochloric acid

(D) Hydrogenation of oil

Correct Answer: C

Question 67 : The empirical formula of a non-electrolyte is CH_2O. A solution containing 6g of the compound exerts the same osmotic pressure as that of 0.05 M glucose solution at the same temperature. The molecular formula of the compound is

(A) $C_2H_4O_2$

(B) $C_2H_6O_3$

(C) $C_5H_{10}O_5$

(D) $C_4H_8O_4$

Correct Answer: D

Question 68 : A white crystaline salt A reacts with dilute HCl to liberate a suffocationg gas B and also forms a yellow precipitate. The gas B turns potassium dichromate acidified with dilute H_2SO_4 to a green coloured solution C. A, B and C are respectively

(A) Na_2SO_3, SO_2,$Cr_2(SO_4)_3$

(B) NaS_2O_3,SO_2,$Cr_2(SO_4)_3$

(C) Na_2S,SO_2,$Cr_2(SO_4)_3$

(D) Na_2SO_4,SO_2,$Cr_2(SO_4)_3$

Correct Answer: B

Question 69 : Molecules of a noble gas do not possess vibrational energy because a noble gas

(A) is monoatomic

(B) is chemically inert

(C) has completely filled shells

(D) is diamagnetic

Correct Answer: A

Question 70 : 1 dm3 solution containing 10^{-5} moles each of ions and Cr ions is treated with 10^{-4} moles of silver nitrate. Which one of the following observations is made?
[$K_{sp}Ag_2CrO_4=4x10^{-12}$]
[$K_{sp}AgCl=1x10^{-10}$]

(A) Precipitation does not occur

(B) Silver chromate gets precipitated first

(C) Silver chloride gets precipitated first

(D) Both silver chromate and silver chloride start precipitating simultaneously

Correct Answer: C

Question 71 : E_1,E_2,E_3 are the emf values of the three galvanic cells respectively Which one of the following is true?

(A) $E_2>E_31$

(B) $E_3>E_2>E_1$

(C) $E_1>E_2>E_3$

(D) $E_1>E_3>E_2$

Correct Answer: B

Question 72 : Which one of the following forms propane nitrile as the major product?

(A) Ethyl bromide + alcoholic KCN

(B) Propyl bromide + alcoholic KCN

(C) Propyl bromide + alcoholic AgCN

(D) Ethyl bromide + alcoholic AgCN

Correct Answer: A

Question 73 : Oil of winter green is :

(A) a carboxylic acid

(B) an alcohol

(C) a ketone

(D) an ester

Correct Answer: D

Question 74 : Which of the following is not true for S_N1 reaction ?

(A) 3°-alkyl halides generally react through S_N 1 reaction

(B) The rate of the reaction does not depend upon the molar concentration of the nucleophile

(C) 1°-alkyl halides generally react through S_N1 reaction

(D) It is favoured by polar solvents

Correct Answer: C

Question 75 : 2 g of a radioactive sample having half-life of 15 days was synthesised on 1st Jan 2009. The amount (including both the days) is

(A) 0 g

(B) 0.125 g

(C) 1 g

(D) 0.5 g

Correct Answer: B

Question 76 : For a chemical reaction A → B, the rate of the reaction is 2×10^{-3} mol dm-3 s^{-3}, when the initial concentration is 0.05 mol dm-3. The rate of the same initial reaction 1.6×10^{-2} mol dm^{-3} s^{-1} when the initial concentration is 0.1 mol dm^{-3}. The order of the reaction is

(A) 2

(C) 3

(D) 1

Correct Answer: C

Question 77 : For the decomposition of a compound AB at 600 K, the following data were obtained

[AB] mol dm^{-3}	**Rate of decomposition of AB in mol dm^{-3} s^{-1}**
0.20	2.75×10^{-8}
0.40	11.0×10^{-8}
0.60	24.75×10^{-8}

The order for the decomposition of AB is

(A) 1.5

(C) 1

(D) 2

Correct Answer: D

Question 78 : The reaction that takes place when Cl_2 gas is passed through conc. NaOH solution is

(A) Oxidation

(B) Reduction

(C) Displacement

(D) Disproportionation

Correct Answer: D

Question 79 : In cu-ammonia complex, the state of hybridization of Cu^{2+} is

(A) sp^3

(B) d^3s

(C) sp^2f

(D) dsp^2

Correct Answer: D

Question 80 : An aqueous solution containing 6.5 g of NaCl of 90% purity was subjected to electrolysis. After the complete electrolysis, the solution was evaporated to get solid NaOH. The volume of 1 M acetic acid required to neutralise NaOH obtained above is

(A) 1000 cm^3

(B) 2000 cm^3

(C) 100 cm^3

(D) 200 cm^3

Correct Answer: B

Question 81 : The standard electrode potential for the half-cell reaction are
Zn^{2+} $2e^- \rightarrow Zn$; $E^o = -0.76$ V
$Fe^{2+} + 2e^- \rightarrow Fe$; $E^o = -0.44$ V
The emf of the cell reaction,
$Fe^{2+} + Zn \rightarrow Zn^{2+} + Fe$ is

(A) -0.32 V

(B) -0.20 V

(C) + 1.20 V

(D) + 0.32 V

Correct Answer: B

Question 82 : An organic compound made of C, H and N contains 20% nitrogen. Its molecular weight is

(A) 70

(B) 140

(C)

100

(D)

65

Correct Answer: A

Question 83 : The half life of a radioactive element is 10 h. How much will be left after 4 h in 1 g atom sample

(A) 45.6×10^{23} atom

(B) 4.56×10^{23} atom

(C)

4.56×10^{21} atom

(D) 4.56×10^{20} atom

Correct Answer: B

Question 84 : of the following compounds the most a ideic is

(A) As_2O_3

(B) P_2O_5

(C) Sb_2O_3

(D) Bi_2O_3

Correct Answer: B

Question 85 : Hybridisation of central atom in NF_3 is

(A) sp^3

(B) sp

(C) sp^2

(D) dsp^2

Correct Answer: A

Question 86 : The compound which forms acetaldehyde when heated with dilute NaOH is

(A) 1,1 dichloroethane

(B) 1,1,1-trichloroetane

(C) 1-chloroethane

(D) 1,2-dichloroethane

Correct Answer: A

Question 87 : Cooking is fast in a pressure cooker, because

(A) food particles are effectively smashed

(B) water boils at higher temperature inside the pressure cooker

(C) food is cooked at constant volume

(D) loss of heat due to radiation is minimum

Correct Answer: B

Question 88 : The one that is concentrated by froth flotation process is

(A) zincite

(B) cinnabar

(C) bauxite

(D) malachite

Correct Answer: B

Question 89 : Which of the following will decrease the pH of a 50 mL solution of 0.01 M HCl ?

(A) Addition of 5 mL of 1 M HCl

(B) Addition of 50 mL of 0.01 M HCl

(C) Addition of 5 mL of 0.002 M HCl

(D) Addition of Mg

Correct Answer: D

Question 90 : A body of mass x kg is moving with a velocity of 100 ms^{-1}. Its de-Broglie wavelength is 6.62 $x10^{-35}$m. Hence is ($h=6.62x10^{-34}Js$)

(A) 0.1 kg

(B) 0.25 kg

(C) 0.15 kg

(D) 0.2 kg

Correct Answer: A

Question 91 : The correct order of ionisation energy of C,N,O, F is

(A) F<O<N<C

(B) F<N<C<O

(C) C<N<O<F

(D) C<O<N<F

Correct Answer: D

Question 92 : The oxide of an element whose electronic configuration is $1s^2,2s^2,2p^6,3s^1$ is

(A) neutral

(B) amphoteric

(C) basic

(D) acidic

Correct Answer: C

Question 93 : The characteristic not related to alkali metal is

(A) high ionisation energy

(B) their ions are isoelectronic with noble gases

(C) low melting point

(D) low electronegativity

Correct Answer: A

Question 94 : Among the following, the compound that contains ionic, covalent and coordinate linkage is

(A) NH_3

(B) NH_3Cl

(C) NaCl

(D) CaO

Correct Answer: B

Question 95 : A covalent molecule AB3 has pyramidal structure. The number lone pair and bond pair of electrons in the molecule are respectively

(A) 2 and 2

(B) 0 and 4

(C) 3 and 1

(D) 1 and 3

Correct Answer: D

Question 96 : Excess of carbon dioxide is passed through 50 ml, of 0.5 M calcium hydroxide

solution. After the completion of the reaction, the solution was evaporated to dryness. The solid calcium carbonate was completely neutralised with 0.1 N hydrochloric acid. The volume of bydrochloric acid required is (Atomic mass of calcium =40)

(A) 300 m^3

(B) 200 cm^3

(C) 500 cm^3

(D) 400 cm^3

Correct Answer: C

Question 97 : A bivalent metal has an equivalent mass 32. The molecular mass of the metal nitrate is

(A) 182

(B) 168

(C) 192

(D) 188

Correct Answer: D

Question 98 : The rms velocity of molecules of a gas density 4 kg m^{-3} and pressure $1.2x10^5$ Nm^{-2} is

(A) 300 ms^{-1}

(B) 900 ms^{-1}

(C) 120 ms^{-1}

(D) 600 ms^{-1}

Correct Answer: A

Question 99 : 0.5 mole of each of H_2,SO_2 and CH_4 are kept in a container. A hole was made in the container. After 3 h, the order of partial pressures in the container will be

(A) $pSO_2 > pH_2 > pCH_4$

(B) $pSO_2 > pCH_4 > pH_2$

(C) $pH_2 > pSO_2 > pCH_4$

(D) $pH_2 > pCH_4 > pSO_2$

Correct Answer: B

Question 100 : The enthalpy of formation of NH_3 is -46 kJ mol^{-1}. The enthalpy change for the reaction. $2NH_3)g) \rightarrow N_2(g) + 3H_2(g)$ is

(A) +184 kJ

(B) +23 kJ

(C) +92 kJ

(D) +46 kJ

Correct Answer: C

Question 101 : 5 moles of SO_2 and 5 moles of O_2 are allowed to react. At equilibrium, it was found that 60% of SO_2 is used up. If the partial pressure of the equilibrium mixture is one atmosphere, the partial pressure of O_2 is

(A) 0.82 atm

(B) 0.52 atm

(C) 0.21 atm

(D) 0.41 atm

Correct Answer: D

Question 102 : The time taken for 10% completion of a first order reaction is 20 min. Then for 19% completion, the reaction will take

(A) 40 min

(B) 60 min

(C) 30 min

(D) 50 min

Correct Answer: D

Question 103 : Rate of physical adsorption increases with

(A) decrease in surface area

(B) decrease in temperature

(C) decrease in pressure

(D) increase in temperature

Correct Answer: B

Question 104 : IUPAC name of $(CH_3)_3CCl$ is

(A) n-butyl chloride

(B) 3-chloro butane

(C) 2-chloro 2—methyl propane

(D) t-butyl chloride

Correct Answer: C

Question 105 : Lucas test is associated with

(A) aldehydes

(B) phenols

(C) carboxylic acids

(D) alcohols

Correct Answer: D

Question 106 : An organic compound on heating with CuO produces CO_2 but no water. The organic compound may be

(A) carbon tetrachloride

(B) chloroform

(C) methane

(D) ethyl iodide

Correct Answer: A

Question 107 : The condensation polymer among the following is

(A) rubber

(B) protein

(C) PVC

(D) polyethene

Correct Answer: B

Question 108 : The order of stability of metal oxides is

(A) $Al_2O_3<MgO<Fe_2O_3<Cr_2O_3$

(B) $Cr_2O_3<MgO<Al_2O_3<Fe_2O_3$

(C) $Fe_2O_3<Cr_2O_3<Al_2O_3<MgO$

(D) $Fe_2O_3<Al_2O_3<Cr_2O_3<MgO$

Correct Answer: C

Question 109 : The temperature of the slag zone in the metallurgy of iron using blast furnace is

(A) 1200-1500^oC

(B) 1500-1600^oC

(C) 400-700^oC

(D) 800-1000^oC

Correct Answer: D

Question 110 : The function of $Fe(OH)_3$ in the contact process is

(A) to remove arsenic impurity

(B) to detect colloidal impurity

(C) to remove moisture

(D) to remove dust particles

Correct Answer: A

Question 111 : In which of the following, NH_3 is not used?

(A) Tollen's reagent

(B) Nessler's reagent

(C) Group reagent for the analysis of IV group basic radicals

(D) Group reagent for the analysis of III group basic radicals

Correct Answer: B

Question 112 : Argon is used

(A) in filling airships

(B) to obtain low temperature

(C) in high temperature welding

(D) in radiotherapy for treatment of cancer

Correct Answer: C

Question 113 : The incorrect statement in respect of chromyl chloride test is

(A) formation of red vapours

(B) formation of lead chromate

(C) formation of chromyl chloride

(D) liberation of chlorine

Correct Answer: D

Question 114 : A 2.0 cm object is placed 15 cm is front of a concave mirror of focal length 10 cm. What is the size and nature of the image ?

(A) 4 cm, real

(B) 4 cm, virtual

(C) 1.0 cm. real

(D) None of these

Correct Answer: A

Question 115 : The IUPAC name of $[Co(NH_3)_5ONO]^{2+}$ ion is

(A) penta ammine nitrito cobalt (IV) ion

(B) penta ammine nitrito cobalt (III) ion

(C) penta ammine nitro cobalt (III) ion

(D) penta ammine nitro cobalt (IV) ion

Correct Answer: B

Question 116 : The oxidation state of Fe in the brown ring complex: $[Fe(H_2O)_5NO]SO_4$ is

(A) +3

(C) +2

(D) +1

Correct Answer: D

Question 117 : Which one of the following defects in the crystals lowers its density ?

(A) F centres

(B) Interstitial defect

(C) Frenkel defect

(D) Schottky

Correct Answer: D

Question 118 : A emulsifier is a substance which :

(A) Stabilises the emulsion

(B) Homogenizes the emulsion

(C) Coagulates the emulsion

(D) Accelerates the dispersion of liquid in liquid

Correct Answer: A

Question 119 : The fatty acid that shows reducing property is :

(A) acetic acid

(B) ethanoic acid

(C) oxalic acid

(D) formic acid

Correct Answer: D

Question 120 : By adding 20 ml 0.1 N HCl to 20 ml 0.001 N KOH, the pH of the obtained solution will be :

(A) 2

(B) 1.3

(C) 0

(D) 7

Correct Answer: B

SUBJECT: Mathematics

Question 121 :

The number of elements of the power set of a set containing m elements is

(A) $2^m - 1$

(B) m

(C) m^2

(D) 2^m

Solution :

Let A is a set with n(A) = m

Number of elements of the power set of a given set is equal to number of subsets of the set.
$n[P(A)] = 2^m$

Correct Answer: D

Question 122 : If $A = \{x : x = n^2, n = 1, 2, 3\}$, then number of proper subsets is

(A) 3

(B) 8

(C) 7

(D) None of these

Solution :

Given that, $A = \{x : x = n^2, n = 1,2, 3\}$

Number of elements in A is 3.

So, number of proper subsets = $2^3 - 1 = 7$

Correct Answer: C

Question 123 : Let A = {1, 2, 3}. The total number of distinct relations that can be defined over A, is

(A) 2^9

(B) 6

(C) 8

(D) None of these

Solution : $n(A \times A) = (n(A))^2 = 3^2 = 9$. So, the total number of subsets of $A \times A$ is and hence total

number of relations over the set A is 2^9.

Correct Answer: A

Question 124 : Let A = {x, y, z} and B = {1, 2}. Then, the number of relations from A to B is

(A) 32

(B) 64

(C) 128

(D) 8

Correct Answer: B

Question 125 : Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg(zw) = π. Then, arg(z) is equal to

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$

(D) $\frac{5\pi}{4}$

Solution :

Given that, $z = iw = 0$

$\Rightarrow \bar{z} = -i\bar{w} \Rightarrow z = iw \Rightarrow w = iz$

And $\arg(zw) = \pi$

$\Rightarrow \arg(-iz^2) = \pi$

$\Rightarrow \arg(-i) + 2\arg(z) = \pi$

$\Rightarrow -\frac{\pi}{2} + 2\arg(z) = \pi \quad \left[Q\ \arg(-i) = -\frac{\pi}{2}\right]$

$\arg(z) = \frac{3\pi}{4}$

Correct Answer: C

Question 126 : The conjugate of $\frac{2-i}{(1-2i)^2}$ is

(A) $\frac{2}{25} - \frac{i11}{25}$

(B) $\frac{-2}{25} - \frac{i11}{25}$

(C) $\frac{-2}{25} + \frac{i11}{25}$

(D) $\frac{2}{25} + \frac{i11}{25}$

Solution :

Let $z = \frac{2-i}{(1-2i)^2}$, then conjugate of

$z = \bar{z} = \frac{2+i}{(1+2i)^2} \qquad \left[Q\ \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}\right]$

Now, $\bar{z} = \frac{2+i}{1^2 - 4 + 4i} = \frac{2+i}{4i-3} \times \frac{4i+3}{4i+3}$

$= \frac{11i+2}{(4i)^2 - (3)^2} = \frac{11i+2}{16-9} = \frac{-2}{25} - i\frac{11}{25}$

Correct Answer: B

Question 127 : nth term of the series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots\ldots$ will be

(A) $\frac{3n+1}{5^{n-1}}$

(B) $\frac{3n-1}{5^n}$

(C) $\frac{3n-2}{5^{n-1}}$

(D) $\frac{3n+2}{5^{n-1}}$

Solution

The given series is clearly an AGS and the corresponding AP is 1, 4, 7, 10..... having nth term = 3n – 2 and corresponding GP is 1, $\frac{1}{5}, \frac{1}{5^2}, \cdots$

: having nth term = $\frac{1}{5^{n-1}}$.

Correct Answer: C

Question 128 : The sum of the series $\frac{1}{1.2.3}+\frac{1}{3.4.5}+\frac{1}{5.6.7}+\ldots\ldots$ is

(A) $\log_e 2-\frac{1}{2}$

(B) $\log_e 2$

(C) $\log_e 2 + \frac{1}{2}$

(D) $\log_e 2 + 1$

Solution :

Let $S = \frac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + \ldots$

$$T_n = \frac{1}{(2n-1)(2n)(2n+1)}$$

$$= \frac{1}{(2n-1)} - \frac{1}{2n} + \frac{1}{2(2n+1)}$$

$$= \frac{1}{2}\left[\frac{1}{2n-1} - \frac{1}{2n}\right] - \frac{1}{2}\left[\frac{1}{2n} - \frac{1}{2n+1}\right]$$

On putting n = 1, 2, 3,....., we get

$$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{2}\right] - \frac{1}{2}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{4}\right] - \frac{1}{2}\left[\frac{1}{4} - \frac{1}{5}\right]$$

Now, $S = T_1 + T_2 + T_3 + \ldots\ldots + T_n + \ldots$

$$= \frac{1}{2}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} + \ldots\right]$$

On putting n = 1, 2, 3,....., we get

$$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{2}\right] - \frac{1}{2}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{4}\right] - \frac{1}{2}\left[\frac{1}{4} - \frac{1}{5}\right]$$

Now, $S = T_1 + T_2 + T_3 + ... + T_n + ...$

$$= \frac{1}{2}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} + ...\right]$$

$$- \frac{1}{2}\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + ...\right]$$

$$= \frac{1}{2}\log_e(1+1) + \frac{1}{2}\left[-1 + \left\{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\right\}\right]$$

$$= \frac{1}{2} log_e\ 2 - \frac{1}{2} + (1+1)$$

Correct Answer: A

Question 129 : What is the highest power of 3 contained in 100!?

(A) 49

(B) 48

(C) 47

(D) 46

Solution : Highest power of 3 continued in 100!

$$= \left[\frac{100}{3}\right] + \left[\frac{100}{3^2}\right] + \left[\frac{100}{3^3}\right] + ...$$

$= 33 + 11 + 3 + 1 + 0 + 0 + ... = 48$

Correct Answer: B

Question 130 : The least positive integer having remainders 2, 3, 2 when divided by 3, 5, 7 respectively
is

(A) 27

(B) 25

(C) 24

(D) 23

Solution :

$N = 2(mod 3)$
$N = 2(mod\ 7)$ $\Rightarrow N = 2\ (mod 21)$

$\Rightarrow N = 21p + 2 = (5 \times 4p + p) + 2$
$= (p + 2)\ (mod\ 5)$
But $N \equiv 3\ (mod\ 5)$
$\therefore\ p + 2 \equiv 3(mod\ 5)$
$\Rightarrow p = 1$
$\therefore$ Least number satisfying the given criteria = 23

[Q N = 21 p + 2]

Correct Answer: D

Question 131 : Let f: {1, 2, 3, 4, 5} → {1, 2, 3, 4, 5} that are onto and f(x) ≠ i is equal to

(A) 9

(B) 44

(C) 16

(D) None of these

Solution : Total number of functions
= Number of dearrangement of 5 objects

$= 5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 44$

Correct Answer: B

Question 132 : There are 5 letters and 5 different envelopes. The number of ways in which all the letters can be put in wrong envelope, is

(A) 119

(B) 44

(C) 59

(D) 40

Solution : Required numbers

$$= 5!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right]=44$$

Correct Answer: B

Question 133 : The greatest term in the expansin of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ is

(A) $\frac{26840}{9}$

(B) $\frac{24840}{9}$

(C) $\frac{25840}{9}$

(D) None of these

Solution :

Correct Answer: C

Question 137 : Ravi obtained 70 and 75 marks in first two unit tests. Then, the minimum marks he should get in the third test to have an average of atleast 60 marks, are

(A) 45

(B) 35

(C) 25

(D) None of these

Solution :

Let Ravi got x marks in third unit test.

∴ Average marks obtained by Ravi

$$= \frac{\text{Sum of marks in all tests}}{\text{Number of tests}}$$

$$= \frac{70+75+x}{3} = \frac{145+x}{3}$$

Now, it is given that he wants to obtain an average of atleast 60 marks.

Atleast 60 marks means that the marks should be greater than or equal to 60.

i.e. $\frac{145+x}{3} \geq 60$

$\Rightarrow 145 + x \geq 60 \times 3$

$\Rightarrow 145 + x \geq 180$

Now, transferring the term 145 to RHS,

$x \geq 180 - 145$

$\Rightarrow x \geq 35$

i.e. Ravi should get greater than or equal to 35 marks in third unit test to get an average of atleast 60 marks.

∴ Minimum marks Ravi should get = 35.

Correct Answer: B

Question 138 : If a point satisfying the line ax + by = c, then

(A) it will lie on the line

(B) It will lie in upper half plane

(C) It will lie in lower half plane

(D) None of the above

Solution : All points (x, y) satisfying ax + by = c, lies on the line it represent.

Correct Answer: A

Question 139 : Matrix $A = \begin{bmatrix} 1 & 0 & k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ is invertible for

(A) k = 1

(B) k = –1

(C) k = R – {1, –1}

(D) None of these

Solution : Since A is invertible.

$$|A| \neq 0 \Rightarrow \begin{bmatrix} 1 & 0 & k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix} \neq 0$$

$$\Rightarrow 1(1 - 0) + k(0 - k) \neq 0$$

$$\Rightarrow 1 - k^2 \neq = 0 \Rightarrow k \neq \pm 1$$

Correct Answer: C

Question 140 : For any 2 × 2 matrix A, if $A(\text{adj } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then |A| is equal to

(A) ZERO

(B) 10

(C) 20

(D) 100

Since A[adj(A)] = |A|.I

$$\therefore \begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix} = |A| \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}|A| & 0\\ 0 & |A|\end{bmatrix}$$

Solution : $\therefore |A| = 10$

Correct Answer: B

Question 141 : The minors of – 4, 9 and the cofactors og – 4, 9 in matrix $\begin{bmatrix}-1 & -2 & 3\\ -4 & -5 & -6\\ -7 & 8 & 9\end{bmatrix}$ are respectively

(A) 42, 3, – 42, 3

(B) – 42, – 3, 42, – 3

(C) 42, 3, – 42, – 3

(D) 42, 3, 42, 3

Solution : – 42, – 3, 42, – 3

Correct Answer: B

Question 142 : The area of a triangle with vertices (– 3, 0), (3, 0) and (0, k) is 9 sq units. The value of k will be

(A) 9

(B) 3

(C) – 9

(D) 6

Solution :

Required area = 9 sq units

$$\Rightarrow \frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix} = 9 \Rightarrow -k(-3-3) = 18$$

$$-k(-6) = 18$$

$$\Rightarrow 6k = 18 \Rightarrow k = 3$$

Correct Answer: B

Question 143 : If A + B = 45°, then (cot A – 1) (cot B – 1) is equal to

(A) 3

(B) $\frac{1}{2}$

(C) – 1

(D) 2

Solution :

Given, A + B = 45°

$\cot(A + B) = 1$

$$\Rightarrow \frac{\cot A \cot B - 1}{\cot A + \cot B} = 1$$

$$\Rightarrow \cot A \cot B - (\cot A + \cot B) = 1 \quad \ldots\ldots(i)$$

Now, (cot A – 1) (cot B – 1)

= cot A cot B – (cot A + cot B) + 1

= 1 + 1 = 2 [from Eq. (i)]

Correct Answer: B

Question 144 : If $\frac{\cos x}{\cos(x-2y)} = \lambda$, then $\tan(x-y)\tan y$ is equal to

(A) $\frac{1+\lambda}{1-\lambda}$

(B) $\frac{1-\lambda}{1+\lambda}$

(C) $\frac{\lambda}{1+\lambda}$

(D) $\frac{\lambda}{1-\lambda}$

Solution :

$$\tan(x-y)\tan y = \frac{\sin(x-y)\sin y}{\cos(x-y)\cos y} \times \frac{2}{2}$$

$$= \frac{\cos(x-2y)-\cos(y)}{\cos(x-2y)+\cos(y)}$$

$$= \frac{1-\frac{\cos x}{\cos(x-2y)}}{1+\frac{\cos x}{\cos(x-2y)}}$$

$$= \frac{1-\lambda}{1+\lambda} \qquad \left[Q\ \lambda = \frac{\cos x}{\cos(x-2y)}\right]$$

Correct Answer: B

Question 145 : The principal value of $\sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right]$ is

(A) $\frac{-2\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{2}$

(D) None of these

Solution :

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Correct Answer: D

Question 146 : Let $x \in (0, 1)$, The set of all x such that $\sin^{-1}x > \cos^{-1}x$ is the interval

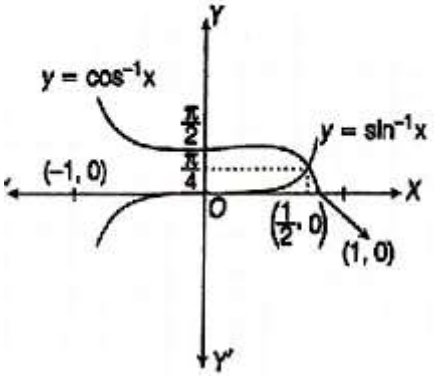
(A) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

(B) $\left(\frac{1}{\sqrt{2}}, 1\right)$

(C) (0, 1)

(D) $\left(0, \frac{\sqrt{3}}{2}\right)$

$\therefore \sin^{-1}x > \cos^{-1}x, \forall x \in \left(\frac{1}{\sqrt{2}}, 1\right)$



Solution :

Correct Answer: B

Let $f(x) = \begin{cases} 1+x, 0 \le x \le 2 \\ 3-x, 2 < x \le 3 \end{cases}$ then the points of

Question 147 : discontinuity of g(x) = f[f(x)] is/are

(A) only at x = 1

(B) x = 2, 3

(C) only at x = 3

(D) x = 1, 2

$$g(x) = f[f(x)] = \begin{cases} 1+x, & 0 \le x \le 2 \\ 3-x, & 2 < x \le 3 \end{cases}$$

$$g(x) = \begin{cases} 2+x, & 0 \le x \le 1 \\ 2-x, & 1 < x \le 1 \\ 4-x, & 2 < x \le 3 \end{cases}$$

Here, g(x) changes the inequality sign at $x = 1$ and $x = 2$. Thus, to check the continuity of g(x) at $x = 1$ and $x = 2$

At $x = 1$,

$$\text{LHL} = \lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} (2 + x) = 3$$

$$\text{RHL} = \lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} (2 - x) = 1$$

$\therefore$ LHL $\neq$ RHL $\Rightarrow$ g(x) = is discontinuous at $x = 1$

At $x = 2$

$$\text{LHL} = \lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} (2 - x) = 0$$

$$\text{RHL} = \lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} (4 - x) = 2$$

$\therefore$ LHL $\neq$ RHL $\Rightarrow$ g(x) is discontinuous at $x = 2$

Solution :

Correct Answer: D

Question 148 : If $f(x) = x \sin \frac{1}{x}$

(A) is continuous but not differentiable at x = 0

(B) is discontinuous but differentiable at x = 0

(C) is differentiable at x = 0

(D) cannot be determined

$$LHL = \lim_{x \to 0^-} \left\{ -h \cdot \sin\left(-\frac{1}{h}\right) \right\} = 0$$

$$RHL = \lim_{x \to 0^+} \left\{ -h \cdot \sin\left(\frac{1}{h}\right) \right\} = 0$$

Also, $f(0) = 0 \times [t \in R, -1 \le t \le 1] = 0$
So, $f(x)$ is continuous for all $x \in R$.
To check differentiability at $x = 0$,

$$LHD = \lim_{x \to 0} \frac{f(-h) - f(0)}{-h}$$

$$= \lim_{x \to 0} \frac{+h \cdot \sin\left(-\frac{1}{h}\right) - 0}{+h}$$

$$= -\sin\left(\frac{1}{h}\right) = -K \; [K \in (-1, 1)]$$

$$RHD = \lim_{x \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{x \to 0} \frac{h \cdot \sin\left(\frac{1}{h}\right) - 0}{h} = \sin\frac{1}{h} = K = LHD \neq RHD$$

Solution : So, $f(x)$ is not differentiable at $x = 0$

Correct Answer: A

Question 149 : If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{\log_e x}{1+\log_e x}$

(B) $\frac{\log_e x}{(1+\log_e x)^2}$

(C) $\frac{\log_e x}{1-\log_e x}$

(D) $\frac{\log_e x}{(1-\log_e x)^2}$

Solution :

Given that, $x^y = e^{x-y}$, $\log_e xy = \log_e x - y$

$\Rightarrow y\log_e x = (x - y)\log_e e \Rightarrow y\log_e x = x - y$

$\Rightarrow y(1 + \log_e x) = x \Rightarrow y = \frac{x}{1+\log_e x}$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\log_e x)\frac{d}{dx}x - x\frac{d}{dx}(1+\log_e x)}{(1+\log_e x)^2}$$

$$= \frac{(1+\log_e x).1 - x.\left(0+\frac{1}{x}\right)}{(1+\log_e x)^2}$$

$$= \frac{1+\log_e x - 1}{(1+\log_e x)^2} = \frac{\log_e x}{(1+\log_e x)^2}$$

Correct Answer: B

Question 150 : What is the derivative of $\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$ w.r.t $\cos^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$?

(A) 1

(B) –1

(C) 2

(D) –2

Solution :

Let $u = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$ and $v = \cos^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

Again, let $t = \tan\theta$

$\therefore u = \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right) = \sin^{-1}(\sin\theta) = \tan^{-1}t$

$\Rightarrow \frac{du}{dt} = \frac{1}{1+t^2}$

and $v = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+\tan^2\theta}}\right)$

$= \cos^{-1}(\cos\theta) = \tan^{-1}t$

$\Rightarrow \frac{dv}{dt} = \frac{1}{1+t^2}$

$\therefore \frac{du}{dv} = \frac{du}{dt}.\frac{dt}{dv} = 1$

Correct Answer: A

Question 151 : What is the slope of a curve which is perpendicular to a curve whose slope is secx + tanx?

(A) tan x – secx

(B) tan x + secx

(C) –(tanx + secx)

(D) None of the above

Let slope of given curve be m_1 and m_2.

Q both the curve are perpendicular to each other.

$\therefore\ m_1m_2 = -1$

$\Rightarrow (\sec x + \tan x)m^2 = -1 \quad [Q\ m_1 = \sec x + \tan x]$

$\Rightarrow m_2 = \dfrac{-1}{\sec x + \tan x}$

$= \dfrac{-(\sec^2 x - \tan^2 x)}{\sec x + \tan x} \quad [Q\ \sec^2 x - \tan^2 x = 1]$

Solution : $= -(\sec x - \tan x) = \tan x - \sec x$

Correct Answer: A

Question 152 : The acute angle of intersection of the curves $y = 2\sin^2 x$ and $y = \cos 2x$ at $x = \dfrac{\pi}{6}$ is

(A) $\dfrac{\pi}{3}$

(B) $\dfrac{\pi}{6}$

(C) $\dfrac{\pi}{4}$

(D) None of these

Solution :

Given, $y = 2\sin^2 x$ ....(i)

and $y = \cos 2x$ .....(ii)

On differentiating eq.(i) w.r.t. x, we get

$$\frac{dy}{dx} = 4\sin x \cos x$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=-\frac{\pi}{6}} = \sqrt{3} = m_1 \text{ [say]}$$

On differentiating eq.(ii) w.r.t. x, we get

$$\frac{dy}{dx} = -2\sin 2x$$

$$\left(\frac{dy}{dx}\right)_{x=-\frac{\pi}{6}} = -\sqrt{3} = m_2 \text{ [say]}$$

Let θ be the angle between them.

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\sqrt{3} + \sqrt{3}}{1 - (\sqrt{3})(\sqrt{3})}\right| = |-\sqrt{3}|$$

$$\Rightarrow \theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

Correct Answer: A

Question 153 : $\int \frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$ is equal to

(A) $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right) + C$

(B) $\frac{a}{b}\tan^{-1}\left(\frac{a\tan x}{b}\right) + C$

(C) $\frac{b}{a}\tan^{-1}\left(\frac{b\tan x}{a}\right) + C$

(D) None of the above

$$\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$$

$$= \int \frac{x^2\, dx}{a^2 \tan^2 x + b^2}$$

$$= \frac{1}{a^2} \int \frac{dt}{t^2 + \left(\frac{b}{a}\right)^2}$$

where t = tanx

$$= \frac{1}{a^2} \cdot \frac{a}{b} \tan^{-1} \frac{t}{b/a} + C$$

Solution : $$= \frac{1}{ab} \tan^{-1} \frac{a(\tan x)}{b} + C$$

Correct Answer: A

Question 154 : $\int \cos(\log_e x)\, dx$ is equal to

(A) $\frac{1}{2} x[\cos(\log_e x) + \sin(\log_e x)] + C$

(B) $x[\cos(\log_e x) + \sin(\log_e x)] + C$

(C) $\frac{1}{2} x[\cos(\log_e x) - \sin(\log_e x)] + C$

(D) $x[\cos(\log_e x) - \sin(\log_e x)] + C$

Solution :

Let

$$I = \int \cos(\log_e x)dx = \int \cos(\log_e x).1\,dx$$

$$= \cos(\log_e x)x - \int \frac{-\sin(\log_e x)}{x}.x\,dx$$

$$= x\cos(\log_e x) + \int \sin(\log_e x)dx$$

$$= x\cos(\log_e x) + \int \sin(\log_e x).1dx$$

$$= x\cos(\log_e x) + \sin(\log_e x)x - \int \frac{\cos(\log_e x)}{x}x\,dx$$

$$= x\cos(\log_e x) + x\sin(\log_e x) - I$$

$$\Rightarrow 2I = x[\cos(\log_e x) + \sin(\log_e x)]$$

$$\Rightarrow I = \frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$$

Correct Answer: A

Question 155 :

The solution of $\frac{dy}{dx} + \frac{y}{x} = \log x$ is

(A) yx = logx + C

(B) $yx = \frac{x^2}{2}(\log x) - \frac{x^2}{4} + C$

(C) $x^2y^2 = \log x + C$

(D) None of the above

Solution :

It is a linear differential equation of the form

$$\frac{dy}{dx}+Py=Q$$

Here, $P=\frac{1}{x}, Q=\log x$

$\therefore$ $IF=e^{\int P\,dx}=e^{\int 1/x\,dx}=e^{\log x}=x$

Hence, the general solution is

$$y.(IF)=\int Q(IF)dx+C$$

$$\Rightarrow yx=\int(\log x)\,x\,dx+C$$

$$\Rightarrow yx=(\log x)\frac{x^2}{2}-\int\frac{1}{x}.\frac{x^2}{2}dx+C$$

$$\Rightarrow yx=\frac{x^2}{2}(\log x)-\frac{x^2}{4}+C$$

Correct Answer: A

Question 156 : The solution of $\frac{dy}{dx}+xy=xy^2$ is

(A) $y=1+ce^{x^2/2}$

(B) $\frac{1}{y}=1+ce^{x^2/2}$

(C) $\frac{1}{y}=1+ce^{x/2}$

(D) None of the above

On dividing by y^2 on both sides of given equation we get

$$y^{-2}\frac{dy}{dx}+\frac{1}{y}x = x$$

$$\text{Let } \frac{1}{y} = z \Rightarrow -\frac{1}{y^2}\frac{dy}{dz} = \frac{dz}{dx}$$

Then, given equation reduces to

$$\frac{dz}{dx} - xz = -x$$

$$\therefore \text{ IF } = e^{\int -x\,dx} = e^{-x^2/2}$$

The solution is

$$ze^{-x^2/2} = \int(-x).e^{-x^2/2}dx = e^{-x^2/2} + c$$

$$\text{i.e. } \frac{1}{y} = 1 + ce^{x^2/2}$$

Solution :

Correct Answer: A

Question 157 : If a line with y-intercept 2, is perpendicular to the line 3x – 2y = 6, then its r-intercept is

(A) 1

(B) 2

(C) –4

(D) 3

Let the equation of perpendicular line to the line
$3x - 2y = 6$ be $3y + 2x = c$ .....(i)
Since it passes through (0, 2).
$\therefore\ c = 6$
On putting the value of c in eq. (i), we get
$3y + 2x = 6$

$$\Rightarrow \frac{x}{3} + \frac{y}{2} = 1$$

Solution : Hence, x-intercept is 3.

Correct Answer: D

Question 158 : The equation of the line bisecting the join of (3, –4) and (5, 2) and having its intercepts on the X-axis and the Y-axis in the ratio 2 : 1is

(A) $x + y - 3 = 0$

(B) $2x - y = 9$

(C) $x + 2y = 2$

(D) $2x + y = 7$

Solution :

Let the points be A(3, –4) and B(5, 2).
Mid-point of AB = (4, –1).
It is given that the bisecting line intersect the coordinate axes in the ratio 2 : 1.
So, point of coordinate axes are (2k, 0) and (0, k).
The equation of line passing through the above point is

$$y-0=\frac{k-0}{0-2k}(x-2k)$$

$$\Rightarrow y=-\frac{1}{2}(x-2k) \quad .....(i)$$

Since it is passing through the mid-point of AB i.e. (4, –1).

$$\therefore -1=-\frac{1}{2}(4-2k)\Rightarrow k=1$$

On putting the value of k in eq. (i), we get

$$y=-\frac{1}{2}(x-2)\Rightarrow x+2y=2$$

Correct Answer: C

Question 159 : For what value of λ will line $y = 2x + \lambda$ be a tangent to the circle $x^2 + y^2 = 5$

(A) ± 4

(B) ± 5

(C) $\pm 2\sqrt{6}$

(D) $\sqrt{10}$

Solution :

Comparing the given line with $y = mx + c$, we get
$m = 2$, $c = \lambda$ and given circle with $x^2 + y^2 = a^2$
then $a^2 = 5$
Q Condition for tangency is $c^2 = a^2(1 + m^2)$
$\Rightarrow \lambda^2 = 5(1 + m^2)$
$\Rightarrow \lambda^2 = 5(1 + 4)$
$\Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5$

Correct Answer: B

Question 160 : Tangent to circle x2 + y2 = 5 at (1, –2) also touches the circle x2 + y2 – 8x + 6y + 20 = 0. Then, coordinate of the corresponding point of contact is

(A) (3, –4)

(B) (3, –1)

(C) (5, 3)

(D) (4, –1)

Solution :

Equation of tangent to $x^2 + y^2 = 5$ at (1, –2) is
$x - 2y - 5 = 0$.
On putting $x = 2y + 5$ in second circle, we get
$(2y + 5)^2 + y^2 - 8(2y + 5) + 6y + 20 = 0$
$\Rightarrow 5y^2 + 10y + 5 = 0$
$\Rightarrow y = -1 \Rightarrow x = -2 + 5 = 0$
Thus, point of contact is (3, –1).

Correct Answer: B

Question 161 : The equation of the line touching two parabolas $y^2 = 4x$ and $x^2 = -32y$, is

(A) 2y – x = 4

(B) 2x – y = 4

(C) 2y + x = 4

(D) x + 2y = 4

Solution :

Let equation of tangent line for the parabola $y^2 = 4x$ be

$$y = mx + \frac{1}{m} \qquad .....(i)$$

Q $y = mx + \frac{1}{m}$ is tangent to the parabola

$x^2 + 32y = 0$

$$\therefore \; x^2 + 32\left(mx + \frac{1}{m}\right) = 0 \Rightarrow x^2 + 32mx + \frac{32}{m} = 0$$

$\Rightarrow x^2m + 32m^2x + 32 = 0$

Since, parabola touch the line.

$\therefore$ D = 0

i.e. $b^2 - 4ac = 0$

$\Rightarrow (32m^2)^2 - 4.m.(+32) = 0$

$\Rightarrow (32m^2)^2 - 32 \times 4m = 0$

$\Rightarrow 32m(32m^3 - 4) = 0$

$\Rightarrow 32m^3 - 4 = 0 \qquad [Q\ m \neq 0]$

$\Rightarrow 8m^3 - 1 = 0$

$\Rightarrow m = \pm 1/2$

On putting the value of m in eq. (i), we get

$$y = +\frac{1}{2}x + \frac{1}{(+1/2)} \Rightarrow y = +\frac{1}{2}x + 2$$

$\therefore 2y = x + 4 = 0$

$\Rightarrow 2y - x = 4$

Correct Answer: A

Question 162 : The normal meet the parabola $y^2 = 4ax$ at that point where the abscissae of the poit is equal to the ordinate of the point is

(A) (6a, –9a)

(B) (–9a, 6a)

(C) (–6a, 9a)

(D) (9a, –6a)

Solution : If a normal drawn from the point $(at_2^2, 2at_2)$ of the $y^2 = 4ax$ meets at point $(at_2^2, 2at_2)$ of same parabola, then $t_2 = -t_1 - \frac{2}{t_1}$.

Given condition is $x = y$ .....(i)
Q $y^2 = 4ax$ [given]
$\therefore\ y^2 = 4ay$ [Q from eq. (i)]
$\Rightarrow y(y - 4a) = 0 \Rightarrow y = 0$ or $y = 4a$
$\therefore$ Point (0, 0) and (4a, 4a)

$$\Rightarrow 2at_1 = 4a \Rightarrow t_1 = \frac{4a}{2a} = 2$$

$$t_2 = -2 - \frac{2}{2} = -3$$

$\therefore$ Point $(at_2^2, 2at_2) = [a \times (-3)^2, 2a(-3)] = (9a, -6a)$

Correct Answer: D

Question 163 : Constant forces $P_1 = \hat{i} - \hat{j} - \hat{k}$, $P_2 = -\hat{i} + 2\hat{j} - \hat{k}$ and $P_3 = \hat{j} - \hat{k}$ act on a particle at a point A. The work done when the particle is displaced from the point A to B, where $A = 4\hat{i} - 3\hat{j} - 2\hat{k}$ and $B = 6\hat{i} + \hat{j} - 3\hat{k}$ is

(A) 3

(B) 9

(C) 20

(D) None of these

Solution :

Q Total force, $\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3$

$= \hat{i} - \hat{j} + \hat{k} - \hat{i} + 2\hat{j} - \hat{k} + \hat{j} - \hat{k} = 2\hat{j}$

and displacement,

$\mathbf{AB} = 6\hat{i} + \hat{j} - 3\hat{k} - (4\hat{i} - 3\hat{j} - 2\hat{k}) = 2\hat{i} + 4\hat{j} - \hat{k}$

$\therefore$ Work done $= \mathbf{P}.\mathbf{AB}$

$= 2\hat{j}.(2\hat{i} + 4\hat{j} - \hat{k}) = 8$

Correct Answer: D

Question 164 : If q is the angle between vecrots a and b and $|a \times b| = |a.b|$, then q is equal to

(A) ZERO

(B) 180º

(C) 135º

(D) 60º

Solution :

$|ab\sin\theta\,\hat{n}| = |ab\cos\theta|$

$\therefore\ ab\sin\theta = ab\,|\cos\theta|$

$\Rightarrow \sin\theta = \pm\cos\theta \Rightarrow \tan\theta = \pm 1$

$\Rightarrow \theta = 45°, 135°$

Correct Answer: C

Question 165 : If u = a – b, v = a + b and |a| = |b| = 2, then $|u \times v|$ is equal to

(A) $2\sqrt{16 - (\mathbf{a}.\mathbf{b})^2}$

(B) $2\sqrt{4 - (\mathbf{a}.\mathbf{b})^2}$

(C) $\sqrt{16 - (\mathbf{a}.\mathbf{b})^2}$

(D) $\sqrt{4-(\mathbf{a}.\mathbf{b})^2}$

Solution :

$u \times v = 2(a \times b)$

$\therefore |u \times v| = 2\sqrt{a^2b^2 \sin^2\theta}$

$= 2\sqrt{a^2b^2(1-\cos^2\theta)}$

$= 2\sqrt{16-(a.b)^2}$

Correct Answer: A

Question 166 : The equation of the plane through the intersection of the planes $3x - y + 2 - 4 = 0$, $x + y + z - 2 = 0$ and the point (2, 2, 1) is

(A) $7x + 5y + 4z + 8 = 0$

(B) $7x + 5y + 4z - 8 = 0$

(C) $7x - 5y + 4z - 8 = 0$

(D) None of the above

Solution :

The equation of any plane through the intersection of the planes

$3x - y + 2z - 4 = 0$

and $x + y + z - 2 = 0$ is

$(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0$ .....(i)

Since, the plane passes through the point (2, 2, 1).

Therefore, this point will satisfy Eq. (i)

$(3 \times 2 - 2 + 2 \times 1 - 4) + \lambda(2 + 2 + 1 - 2) = 0$

$\Rightarrow \lambda = \frac{-2}{3}$

$(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$

[From Eq. (i)]

$\Rightarrow 7x - 5y + 4z - 8 = 0$

Correct Answer: C

Question 167 : The image of the point with position vector $\hat{i}+3\hat{k}$ in the plane $r.(\hat{i}+\hat{j}+\hat{k})=1$ is

(A) $\hat{i}+2\hat{j}+\hat{k}$

(B) $\hat{i}+2\hat{j}-\hat{k}$

(C) $-\hat{i}-2\hat{j}+\hat{k}$

(D) $\hat{i}-2\hat{j}+\hat{k}$

Solution :

Here, the point is (1, 0, 3) and equation of plane is
$x + y + z = 1$.
Let α, β and γ be the image of plane.

$$\therefore \frac{\alpha-1}{1}=\frac{\beta-0}{1}=\frac{\gamma-3}{1}$$

$$=\frac{-2(1+0+3-1)}{1^2+1^2+1^2}$$

$$\Rightarrow \alpha=-1,\ \beta=-2,\ \gamma=1$$

Hence, the image is $(-\hat{i}-2\hat{j}+\hat{k})$.

Correct Answer: C

Question 168 : The points A(4, 5, 1), B(0, –1, –1), C(3, 9, 4) and D(–4, 4, 4) are

(A) collinear

(B) coplanar

(C) non-coplanar

(D) non-collinear

Solution :

DR's of AB = (–4, –6, –2), AC = (–1, 4, 3).
and AD = (–8, –1, 3)

Now, $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -4(15) + 6(21) - 2(33) = 0$

Hence, points A, B, C and D are coplanar.

Correct Answer: B

Question 169 : The mean height of 25 male workers in a factory is 61 cm and the mean height of 36 female workers in the same factory is 58 cm. The combined mean height of 60 workers in the factory is

(A) 59.25

(B) 59.5

(C) 59.75

(D) 58.75

Solution :

Since, $n_1 = 25, n_2 = 35,$

$\bar{x}_1 = 61, \bar{x}_2 = 58$

$\therefore \bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$\therefore \bar{x} = \dfrac{(25)(61) + (35)(58)}{25 + 35} = 59.25$

Correct Answer: A

Let G_1 and G_2 be the .....geometric means of two.... series $x_1, x_2, \ldots, x_n$; $y_1, y_2, \ldots, y_n$. If G is the geometric mean of $\frac{x_i}{y_i}$, i = 1, 2,....,n. Then, G is

Question 170 : equal to

(A) $G^1 - G^2$

(B) $\frac{\log G_1}{\log G_2}$

(C) $\frac{G_1}{G_2}$

(D) $\log\left(\frac{G_1}{G_2}\right)$

Solution :

Given, $G_1 = (x_1.x_2.....x_n)^{1/n}$
and $G_2 = (y_1.y_2.....y_n)^{1/n}$

$$\therefore \quad G = \left(\frac{x_1}{y_1}.\frac{x_2}{y2}.....\frac{x_n}{y_n}\right)^{1/n}$$

$$= \frac{(x_1x_2....x_n)^{1/n}}{(y_1y_2....y_n)^{1/n}} = \frac{G_1}{G_2}$$

Correct Answer: C

Question 171 : The median of 27 observations of variable is 18. Three more observations are made and the values of these observation are 16, 18 and 50. What is the median of these 30 observations?

(A) 18

(B) 19

(C) 25.5

(D) Cannot be determined

Median of 27 observation = 16

Then, $\left(\frac{27+1}{2}\right)$th observation = 18

and 14th observation = 18

Now, number of observation is 30 (even)

$$\text{Medium} = \frac{\left(\frac{30}{2}\right)\text{th} + \left(\frac{30}{2}+1\right)\text{th}}{2}$$

$$= \frac{15\text{th observation} + 14\text{th observation}}{2}$$

Solution : $= \frac{18+18}{2} = 18$

Correct Answer: A

Question 172 : In an examination, 20 questions of true-false type are asked. Suppose a students tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers true, if it falls tails, he answers false. The probability that he answers atleast 12 questions correctly is

(A) $\left(\frac{1}{2}\right)^{20}\left({}^{20}C_{12} + {}^{20}C_{13} + \ldots\ldots + {}^{20}C_{20}\right)$

(B) $\left(\frac{1}{2}\right)^{10}\left({}^{20}C_{11} + {}^{20}C_{12} + \ldots\ldots + {}^{20}C_{20}\right)$

(C) $\left(\frac{1}{2}\right)^{20}\left({}^{20}C_{11} + {}^{20}C_{12} + \ldots\ldots + {}^{20}C_{20}\right)$

(D) None of the above

Solution :

Let X denote the number of correct answers given by the student. The repeated tosses of a coin are Bernoulli trials. Since, head on a coin represents the true answer and tail represents the false answer, the correctly answered of the question are Bernoulli trials.

$\therefore$ p = P (a success)

= P(coin show up a head) = $\frac{1}{2}$

$\therefore$ q = 1 – p = 1 – $\frac{1}{2}$ = $\frac{1}{2}$

$\therefore$ X has a binomial distribution with

n = 20, p = $\frac{1}{2}$ and q = $\frac{1}{2}$

$\therefore$ $P(X = r) = {}^{20}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{20-r}$

Hence, P (atleast 12 questions are answered as true)

$= P(X \geq 12) = P(12) + P(13) + P(14) + P(15) + P(16) + P(17) + P(18) + P(19) + P(20)$

$= {}^{20}C_{12}p^{12}q^8 + {}^{20}C_{13}p^{13}q^7 + {}^{20}C_{14}p^{14}q^6 + {}^{20}C_{15}p^{15}q^5 + {}^{20}C_{16}p^{16}q^4 + {}^{20}C_{17}p^{17}q^3 + {}^{20}C_{18}p^{18}q^2 + {}^{20}C_{19}p^{19}q^1 + {}^{20}C_{20}p^{20}$

$= \left(\frac{1}{2}\right)^{20} [{}^{20}C_{12} + {}^{20}C_{13} + ... + {}^{20}C_{20}]$

Correct Answer: A

Question 173 : In a trial, the probability of success is twice the probability of failure. In six trials, the probability of atleast four successes will be

(A) $\frac{496}{729}$

(B) $\frac{400}{729}$

(C) $\frac{500}{729}$

(D) $\frac{600}{729}$

Solution :

Let the probability of success and failure be p and q, respectively.

Then, $p = 2q$ and $p + q = 1 \Rightarrow 3q = 1 \Rightarrow q = \frac{1}{3}$

$\therefore\ p = \frac{2}{3}$

Requried probability

$$= {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + {}^6C_6\left(\frac{2}{3}\right)^6 = \frac{496}{729}$$

Correct Answer: A

Question 174 : If the mean and variance of a Binomial variate X are 8 and 4 respectively, then P(X < 3) equals

(A) $\frac{265}{2^{15}}$

(B) $\frac{137}{2^{14}}$

(C) $\frac{137}{2^{16}}$

(D) $\frac{265}{2^{16}}$

Solution :

Given, mean of binomial variable, np = 8

and variance of Binomial variable, npq = 4

$\therefore\ q = \frac{1}{2}$ and $p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$

and $n\left(\frac{1}{2}\right) = 8 \Rightarrow n = 16$

$\therefore\ P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= {}^{16}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{16-0} + {}^{16}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{16-1}$

$+ {}^{16}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{16-2}$

$= 1\left(\frac{1}{2}\right)^{16} + 16\left(\frac{1}{2}\right)^{16} + 120\left(\frac{1}{2}\right)^{16} = \frac{137}{2^{16}}$

Correct Answer: C

Question 175 : Let V is a finite dimensional vector space, with direct sum of its subspaces, U and W

(A) $\dim(U + W) \neq \dim U + \dim V$

(B) U and W are dependent

(C) $U \cap W \neq \{0\}$

(D) $\dim(U \cap W) = 0$

Solution :

If U and W are the subspace of V

$\Rightarrow \dim(U + W) = \dim U + \dim W - \dim(U \cap W)$

Since, U and W are direct

$\Rightarrow$ U and W are independent

$\Rightarrow U \cap W = \{0\}$

$\Rightarrow \dim(U \cap W) = \{0\}$

Hence, $\dim(U + W) = \dim U + \dim V$

Correct Answer: D

Question 176 : Let A be an m × n matrix, b ∈ R^m, then the system of equations Ax = b has a solution if and only if

(A) b ∈ column space of A

(B) b ∈ rows space of A

(C) b = 0

(D) rank [A, b] > rank A

Solution : A system of linear equations Ax – b is consistent if and only if b is in the column space of A.

Correct Answer: C

Question 177 : If V is an inner product space, then the norm fucntion || || : V → R has the property, for all u, v ∈ V and a ∈ R.

(A) $||\alpha u|| = |\alpha| \, ||u||$

(B) $||u|| - ||v|| > ||u - v||$

(C) $||u|| + ||v|| > ||u + v||$

(D) None of the above

Solution : Q $||\alpha u||^2 = (\alpha u, \alpha u)$
$= \alpha(u, \alpha u) = \alpha(\alpha u, u)$
$= \alpha^2(u, u) = \alpha^2 ||u||^2$
$\therefore \; ||\alpha u|| = |\alpha| \, ||u||$

Correct Answer: A

Question 178 : Evaluate $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dz\,dx\,dy$

(A) 2π

(B) 4π

(C) 8π

(D) 16π

Solution :

We have, $\int_0^4 \int_0^{2\sqrt{z}} \left[\int_0^{\sqrt{4z-x^2}} dy \right] dz\,dx$

$= \int_0^4 \int_0^{2\sqrt{z}} \sqrt{4z - x^2}\, dz\, dx$

$= \int_0^4 \left[\int_0^{2\sqrt{z}} \sqrt{4z - x^2} dx \right] dz$

$= \int_0^4 \left[\frac{x}{2}\sqrt{4z - x^2} + \frac{4z}{2} \sin^{-1} \frac{x}{2\sqrt{z}} \right]_0^{2\sqrt{z}} dz$

$= \pi \int_0^4 z\, dz = \pi \left[\frac{z^2}{2} \right]_0^4 = 8\pi$

Correct Answer: C

Question 179 : The integral $\int_5^\infty \frac{2x^2}{x^4 - 1} dx$ is

(A) convergent and equal to $\frac{\pi}{2}$

(B) convergent and equal to $\frac{\pi}{2} - \tan^{-1}2$

(C) convergent and equal to $\frac{\pi}{2} - \tan^{-1}2 + \frac{1}{2}\log 3$

(D) divergent

Solution :

We have, $\int_2^\infty \frac{2x^2}{x^4-1}dx = \lim_{\varepsilon\to\infty}\int_2^\varepsilon \frac{2x^2}{x^4-1}dx$

$= \lim_{\varepsilon\to\infty}\left[\tan^{-1}\varepsilon - \tan^{-1}2 + \frac{1}{2}\log\frac{\varepsilon-1}{\varepsilon+1} + \frac{1}{2}\log 3\right]$

$= \frac{\pi}{2} - \tan^{-1}2 + \frac{1}{2}\log 3$

Since, the limit exists. Hence, given integeral is convergent and equal to $\frac{\pi}{2} - \tan^{-1}2 + \frac{1}{2}\log 3$

Correct Answer: C

Question 180 :

The given integral $\int_0^\infty e^{-a^2x^2}\cos bx\,dx$ then,

(A) integral is converges but not absolutely

(B) integral is not converges

(C) integral also converges absolutely

(D) None of the above

Solution :

We have, $\int_0^\infty |e^{-a^2x^2}\cos bx\,dx| \le \int_0^\infty |e^{-a^2x^2}|dx$

$$= \int_0^{\infty} e^{-a^2x^2}\,dx + \int_0^{\infty} e^{-a^2x^2}\,dx$$

But the integral $\int_0^{\infty} e^{-a^2x^2}\,dx$ is a proper integral and

$\int_{\infty}^{\infty} e^{-a^2x^2}\,dx$ is convergent by μ-test, we have

$$\lim_{x\to\infty} x^{\mu} e^{-a^2x^2} = \lim_{x\to\infty} \frac{x^{\mu}}{1 + a^2x^2 + \frac{\alpha^{\downarrow}x^{\downarrow}}{2!} + \ldots..} = 0, \forall \mu$$

Choose $\mu > 1$, we observe that $\int_{\infty}^{\infty} e^{-a^2x^2}\,dx$ is convergent.

Therefore, $\int_0^{\infty} |e^{-a^2x^2} \cos bx|\,dx$ is also convergent, by comparison test.

Hence, the given integral $\int_0^{\infty} e^{-a^2x^2} \cos bx\,dx$ is absolutely convergent.

Correct Answer: C