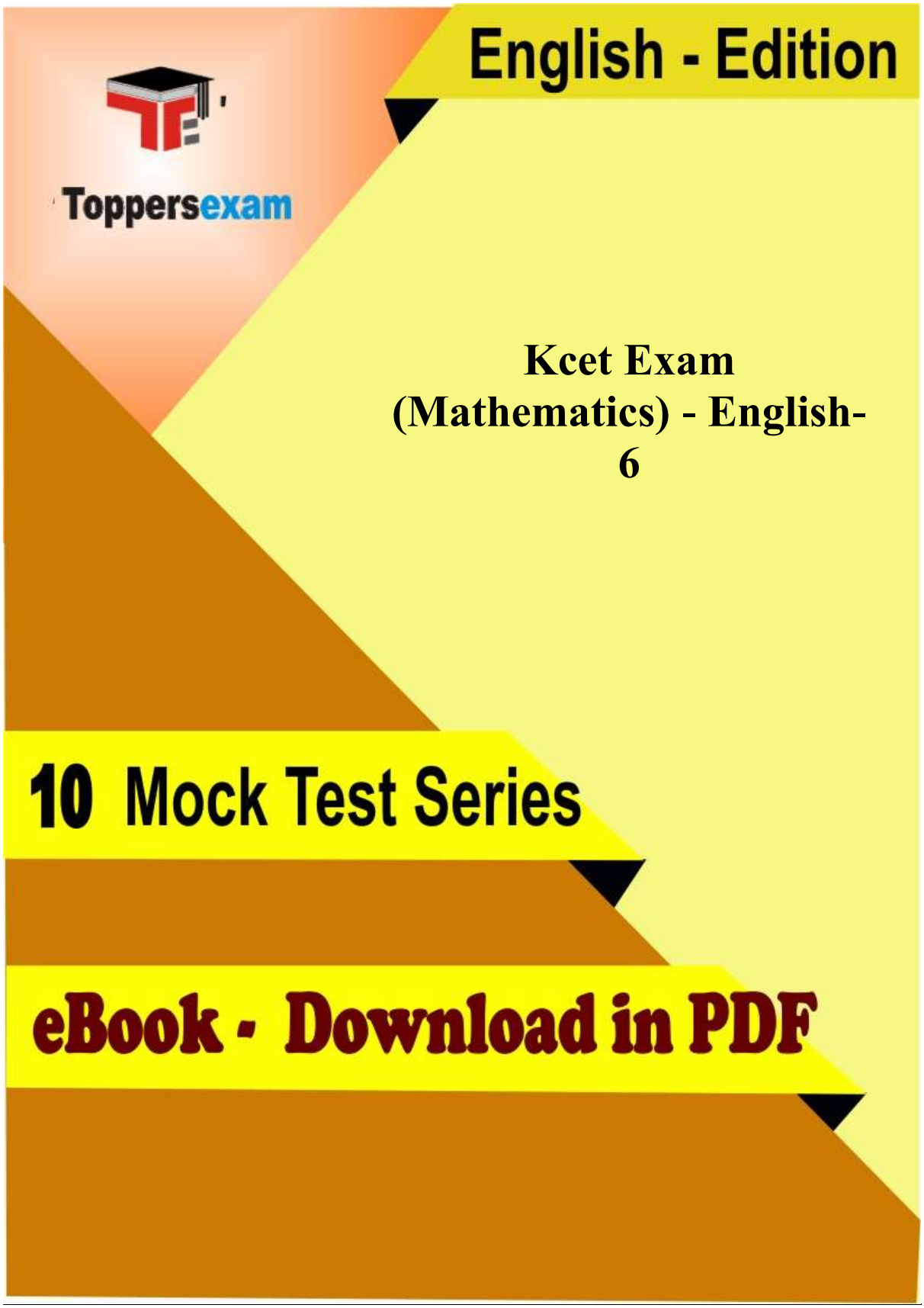

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Kcet Exam (Mathematics) - English-6

Paper Questions

SUBJECT: Physics

Question 1 : As a result of interference of two coherent sources of light energy is:

(A) redistributed and the distribution does not very with time

(B) increased

(C) redistributed and the distribution changes with time

(D) decreased

Correct Answer: A

Question 2 : Two bodies of masses 1 Kg and 2 Kg have equal momentum. Then, the ratio of their kinetic energy is:

(A) 2:1

(B) 3:1

(C) 1:3

(D) 1:1

Correct Answer: A

Question 3 : A charge q is placed at the center of the line joining two equal point charges each equal to Q . The system of 3 charges will be in equilibrium if q is equal to:

(A) +Q / 4

(B) – Q /2

(C) + Q / 2

(D) –Q / 4

Correct Answer: D

Question 4 : The nuclear reactor at kaiga is a:

(A) research reactor

(B) Fusion reactor

(C) breeder reactor

(D) power reactor

Correct Answer: D

Question 5 : The spectrum obtained from the chromospheres of the sun at the time of total solar eclipse is :

(A) line emission spectrum

(B) band emission spectrum

(C) continuous emission spectrum

(D) line absorption spectrum

Correct Answer: D

Question 6 : The maximum current that can be measured by a galvanometer of resistance 40 , is 10 mA. It is converted into a voltmeter that can read upto 50 V. the resistance to be connected in series with the galvanometer (in ohms) is:

(A) 2010

(B) 4050

(C) 5040

(D) 4960

Correct Answer: D

Question 7 : Curie-Weiss law is obeyed by iron:

(A) at Curie temperature only

(B) at all temperature

(C) below curie temperature

(D) above Curie temperature

Correct Answer: D

Question 8 : Excitation energy of a hydrogen like ion in its first excitation state is 40.8 eV from the ion in ground state is:

(A) 40.8 eV

(B) 27.2 eV

(C) 54.4 eV

(D) 13.6 eV

Correct Answer: C

Question 9 : If a 30 V, 90W bulb is to be worked in 120V line, the resistance to be connected in series with the bulb is:

(A) 20Ω

(B) 10Ω

(C) 40Ω

(D) 30Ω

Correct Answer: D

Question 10 : A count rate meter shows a count of 240 per minute from a given radioactive source. One hour later the meter shows a count rate of 30 per minute. The half-life of the source is:

(A) 80 min

(B) 120 min

(C) 20 min

(D) 30 min

Correct Answer: C

Question 11 : A bar magnet is equivalent to:

(A) torroid carrying current

(B) straight conductor carrying current

(C) solenoid carrying current

(D) circular coil carrying current

Correct Answer: C

Question 12 : Blowing air with open mouth is an example of:

(A) isobaric process

(B) isochoric process

(C) isothermal process

(D) adiabatic process

Correct Answer: A

Question 13 : The temperature coefficient of resistance of a wire is 0.00125/° C . Its resistance is 1 Ohm at 300K. its resistance will be 2 Ohm at:

(A) 1127 K

(B) 1400 K

(C) 1154 K

(D) 1100 K

Correct Answer: A

Question 14 : A potentiometer has uniform potential gradient. The specific resistance of the potentiometer wire is 10^{-7} ohm-meter and the current passing through it is 0.1 ampere, cross –section of the wire is 10^{-6} m^2. The potential gradient along the potentiometer wire is:

(A) 10^{-6} V/m

(B) 10^{-4} V/m

(C) 10^{-8} V/m

(D) 10^{-2} V/m

Correct Answer: D

Question 15 : A fuse wire with radius 1 mm blows at 1.5 ampere. The radius of the fuse wire of the same material to blow at 3A will be:

(A) $3^{1/4}$ mm

(B) $4^{1/3}$ mm

(C) $3^{1/2}$ mm

(D) $2^{1/3}$ mm

Correct Answer: B

Question 16 : A wire in the form of a circular loop of one turn carrying a current produces a magnetic field B at the centre. If the same wire is looped into a coil of two turns and carries the same current, the new value of magnetic induction at the centre is:

(A) 3B

(B) 5B

(C) 4B

(D) 2B

Correct Answer: C

Question 17 : To send 10% of the main current through a moving coil galvanometer of resistance 99 ohm, the shunt required is:

(A) 10 ohm

(B) 9.9 ohm

(C) 9 ohm

(D) 11 ohms

Correct Answer: D

Question 18 : The magnetic flux linked with a coil at any instant 't' is given by Ø $5t^3 - 100t + 300$, the emf induced in the coil at t = 2 second is:

(A) 40 V

(B) -40 V

(C) 300 V

(D) 140 V

Correct Answer: A

Question 19 : Sound waves in air are always longitudinal because:

(A) of the inherent characteristics of sound waves in air

(B) air does not have a modules of rigidity

(C) air is a mixture of several gases

(D) density of air is very small

Correct Answer: B

Question 20 : The time taken by A.C. of 50 Hz in reaching from zero to the maximum value is:

(A) 50×10^{-3} sec

(B) $5 \times 10^{-3 \text{ sec}}$

(C) 1×10^{-3} sec

(D) 2×10^{-3} sec

Correct Answer: B

Question 21 : The ratio of the secondary to the primary turns in a transformer is 3:2 and the output power is . Neglecting all power losses, the input power must be:

(A) P/2

(B) P

(C) 2P/3

(D) 3P/2

Correct Answer: B

Question 22 : The material used for permanent ,magnet has:

(A) low retentivity , high coercivity

(B) high retentivity , low coercivity

(C) high retentivity , high coercivity

(D) low retentivity , low coercivity

Correct Answer: C

Question 23 : Rainbow is formed due to:

(A) total internal reflection

(B) scatterin

(C) refraction

(D) dispersion and total internal reflection

Correct Answer: D

Question 24 : A current flows in a conductor from east to west. The direction of the magnetic field at a point above the conductor is:

(A) towards east

(B) towards west

(C) towards north

(D) towards south

Correct Answer: C

Question 25 : If elements with principal quantum number n >4 were not allowed in nature, have number of possible elements would have been:

(A) 32

(B) 60

(C) 64

(D) 4

Correct Answer: B

Question 26 : In photoelectric effect, the number of electrons ejected per second is:

(A) proportional to the wavelength of light

(B) proportional to the intensity of light

(C) proportional to the work function of the metal

(D) proportional to thefrquency of light

Correct Answer: B

Question 27 : Half life of a radioactive substance is 20 minutes. The time between 20% and 80% decay will be:

(A) 40 minutes

(B) 20 minutes

(C) 25 minutes

(D) 30 minutes

Correct Answer: A

Question 28 : When a body is earth connected, electrons from the earth flow into the body. This

means the body is:

(A) charged negatively

(B) an insulator

(C) uncharged

(D) charged positively

Correct Answer: D

Question 29 : A beam of parallel rays is brought to focus by a plano-convex lens. A thin concave lens of the same focal length is joined to the first lens. The effect of this is:

(A) the focus shifts to infinity

(B) the focal point shifts towards the lens by a small distance

(C) the focal point shifts away from the lens by a small distance

(D) the focus remains undisturbed

Correct Answer: A

Question 30 : If m,m_n and m_p are the masses of ${}_zX^A$ nucleus, neutron and proton respectively:

(A) $m = (A - Z)m_n + Zm_p$

(B) $m < (A - Z)m_n + Zm_p$

(C) $m > (A - Z)m_n + Zm_p$

(D) $m = (A - Z)m_p + Zm_n$

Correct Answer: B

Question 31 : The electrical used to get smooth D.C. output from a rectifier circuit is called:

(A) filter

(B) oscillator

(C) logic gates

(D) amplifier

Correct Answer: A

Question 32 : In the case of constants **α** and β of a transistor:

(A) $\alpha = \beta$

(B) $\beta < 1, \alpha > 1$

(C) $\alpha\beta = 1$

(D) $\beta > 1, \alpha < 1$

Correct Answer: D

Question 33 : The difference between the apparent frequency of a source of sound as perceived by the observer during its approach and recession is 2% of the frequency of the source. If the speed of sound in air is 300 ms^{-1}, the velocity of the source is:

(A) 1.5 ms^{-1}

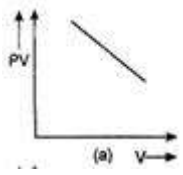
(B) 12 ms^{-1}

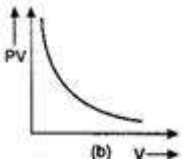
(C) 6 ms^{-1}

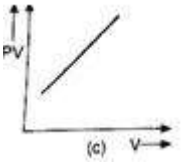
(D) 3 ms^{-1}

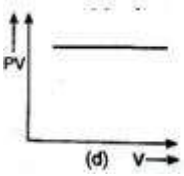
Correct Answer: D

Question 34 : Which one of the following graphs represents the behavior of an ideal gas?

(A) 

(B) 

(C) 

(D) 

Correct Answer: D

Question 35 : Curie-Weiss law is obeyed by iron:

(A) at Curie temperature only

(B) at all temperature

(C) below curie temperature

(D) above Curie temperature

Correct Answer: D

Question 36 : The refractive index of a particular material is 1.67 for blue light, 1.65 for yellow light power and 1.63 for red light. The dispersive power of the material is:

(A) 0.031

(B) 1.60

(C) 0.0615

(D) 0.024

Correct Answer: C

Question 37 : The dimensional formula for inductance is:

(A) $[ML^2T^{-2}A^{-2}]$

(B) $[ML^2TA^{-2}]$

(C) $[ML^2T^{-1}A^{-2}]$

(D) $[ML^2T^{-2}A^{-1}]$

Correct Answer: A

Question 38 : The maximum current that can be measured by a galvanometer of resistance 40Ω, is 10 mA. It is converted into a voltmeter that can read upto 50 V. the resistance to be connected in series with the galvanometer (in ohms) is:

(A) 2010

(B) 4050

(C) 5040

(D) 4960

Correct Answer: D

Question 39 : The spectrum obtained from the chromospheres of the sun at the time of total solar eclipse is :

(A) line emission spectrum

(B) band emission spectrum

(C) continuous emission spectrum

(D) line absorption spectrum

Correct Answer: D

Question 40 : Heavy water is:

(A) compound of deuterium and oxygen

(B) water at 4° C

(C) water, in which soap does not lather

(D) compound of heavy oxygen and heavy hydrogen

Correct Answer: A

Question 41 : The nuclear reactor at kaiga is a:

(A) research reactor

(B) Fusion reactor

(C) breeder reactor

(D) power reactor

Correct Answer: D

Question 42 : A bullet moving with a speed of 100 ms^{-1} can just penetrate two planks of equal thickness. Then, the number of such planks penetrated by the same bullet when the speed is double will be:

(A) 6

(B) 10

(C) 4

(D) 8

Correct Answer: D

Question 43 : Two bodies of masses 1 Kg and 2 Kg have equal momentum. Then, the ratio of their kinetic energy is:

(A) 2:1

(B) 3:1

(C) 1:3

(D) 1:1

Correct Answer: A

Question 44 : Absorption coefficient of an open window is:

(A) 1

(B) 0.25

(C) Zero

(D) 0.5

Correct Answer: C

Question 45 : The loudness and pitch of a sound note depend on :

(A) intensity and velocity

(B) frequency and velocity

(C) intensity and frequency

(D) frequency and number of harmonics

Correct Answer: C

Question 46 : In Melde's experiment in the transverse mode, the frequency of the tuning fork and the frequency of the waves in the string are in the ratio:

(A) 2:1

(B) 4:1

(C) 1:1

(D) 1:2

Correct Answer: C

Question 47 : An electron is accelerated through a potential difference of 45.5 volt. The velocity

acquired by it is (in ms^{-1}):

(A) 10^6

(B) Zero

(C) 4×10^6

(D) 4×10^4

Correct Answer: C

Question 48 : An unknown resistance R_1 is connected in series with a resistance of 10Ω This combination is connected to one gap of a metre bridge while a resistance R_2 is connected in the other gap. The balance point is at 50 cm. Now, when the 10 Ω resistance is removed the balance point shifts to 40 cm. the value of R_1 is (in ohms):

(A) 20

(B) 10

(C) 60

(D) 40

Correct Answer: A

Question 49 : The physical quantity having the same dimensions as Planck's constant h is

(A) linear momentum

(B) angular momentum

(C) Boltzmann constant

(D) force

Correct Answer: B

Question 50 : In young's double slit experiment if monochromatic light used is replaced by white light, then:

(A) no fringes are observed

(B) only central fringe is white, all other fringes are coloured

(C) all bright fringes become white

(D) all bright fringes have coloures between violet and red

Correct Answer: B

Question 51 : In a LCR circuit the potential difference between the terminals of the inductance is 60 V, between the terminals of the capacitor is 30 V and that between the terminal of resistance is 40 V. the supply voltage will be equal to:

(A) 130 V

(B) 10 V

(C) 50 V

(D) 0 V

Correct Answer: C

Question 52 : A vertical circular coil of radius 0.1 m and having 10 turns carries a steady current. When the plane of the coil is normal to the magnetic meridian, a neutral point is observed at the centre of the coil. If $B_H = 0.314 \times 10^{-4}$T, the current in the coil is:

(A) 0.5 A

(B) 0.25 A

(C) 2 A

(D) 1 A

Correct Answer: A

Question 53 : When light is incident on a diffraction grating, the zero order principal maximum will be:

(A) spectrum of the colours

(B) white

(C) one of the component colours

(D) absent

Correct Answer: B

Question 54 : thin plano-convex lens acts like a concave mirror of focal length 0.2 m when silvered from its plane surface. The refractive index of the material of the lens is 1.5. the radius of curvature of the convex surface of the lens will be:

(A) 0.1 m

(B) 0.75 m

(C) 0.4 m

(D) 0.2 m

Correct Answer: D

Question 55 : A balloon is rising vertically up with a velocity of 29 ms^{-1}. A stone is dropped from it and it reaches the ground in 10 seconds. The height of the ballon when the stone was dropped from it is (g=9.8 ms^{-1})

(A) 400 m

(B) 150 m

(C) 100 m

(D) 200 m

Correct Answer: D

Question 56 : A man, standing between two cliffs, claps his hands and starts hearing a series of echoes at intervals of one second. If the speed of sound in air is 340 ms^{-1}, the distance between the cliffs is:

(A) 680 m

(B) 1700 m

(C) 340 m

(D) 1620 m

Correct Answer: C

Question 57 : A beam of light of wavelength 600 nm from a source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is:

(A) 2.4 cm

(B) 2.4 mm

(C) 1.2 mm

(D) 1.2 cm

Correct Answer: B

Question 58 : Specific rotation of sugar solution is 0.01 SI units. 200 kgm^{-3} of impure sugar solution is taken in a polarimeter tube of length 0.25 m and an optical rotation of 0.4 rad is observed. The percentage of purity of sugar in the sample as:

(A) 11%

(B) 20%

(C) 80%

(D) 89%

Correct Answer: C

Question 59 : The emitter-base junction of a transistor is …….biased while the collector-base junction is……. biased.

(A) forward, forward

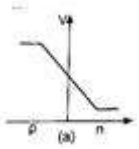
(B) forward, reverse

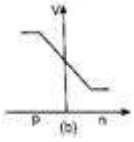
(C) reverse, forward

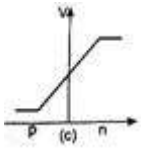
(D) reverse ,reverse

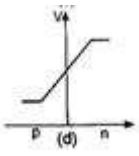
Correct Answer: B

Question 60 : In a forward biased p-n junction diode, the potential barrier in the depletion region is of the form:

(A) 

(B) 

(C) 

(D) 

Correct Answer: D

SUBJECT: Chemistry

Question 61 : Heat liberated with 100 ml of 1 N Naoh is neutralized by 300 ml of 1N HCl:

(A) 11.56 kJ

(B) 5.73 kJ

(C) 22.92 kJ

(D) 17.19 kJ

Correct Answer: B

Question 62 : 5 moles of SO_2 and 5 moles of O_2 are allowed to react. At equilibrium, it was found that 60% of SO_2 is used up. If the partial pressure of the equilibrium mixture is one atmosphere, the partial pressure of O_2 is

(A) 0.82 atm

(B) 0.52 atm

(C) 0.21 atm

(D) 0.41 atm

Correct Answer: D

Question 63 : In Goldshmidt aluminothermic process, thermite contains:

(A) 3 parts of Al_2O_3 and 4 parts of Al

(B) 3 parts of Fe_2O_3 and 2 parts of Al

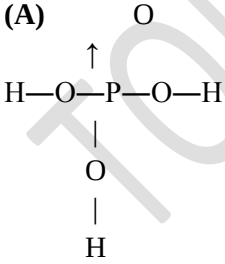
(C) 3 parts of Fe_2O_3 and 1 part of Al

(D) 1 part of Fe_2O_3 and 1 part of Al

Correct Answer: C

Question 64 : The structure of orthophosphoric acid is:

(A)



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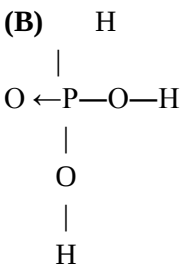

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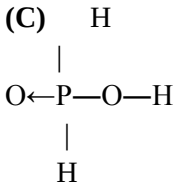

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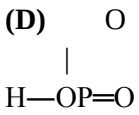


Correct Answer: A

Question 65 : A bivalent metal has an equivalent mass 32. The molecular mass of the metal nitrate is

(A) 182

(B) 168

(C) 192

(D) 188

Correct Answer: D

Question 66 : Same amount of electric current is passed through solution of $AgNO_3$ and HCl. If 1.08 g of silver is obtained in the first case, the amount of hydrogen liberated at S.T.P. in the second case is:

(A) 224 cm^3

(B) 1.008 g

(C) 112 cm^3

(D) 22400 cm^3

Correct Answer: C

Question 67 : The flame colours of metal ions are due to:

(A) Frankel defect

(B) Schottky defect

(C) Metal deficiency defect

(D) Metal excess defect

Correct Answer: D

Question 68 : The order of reactivities of methyl halides in the formation of Grignard reagent is:

(A) $CH_3I > CH_3Br > CH_3Cl$

(B) $CH_3Cl > CH_3Br > CH_3I$

(C) $CH_3Br > CH_3Cl > CH_3I$

(D) $CH_3Br > CH_3I > CH_3Cl$

Correct Answer: A

Question 69 : The reaction of an organic compound with ammonia followed by nitration of the product gives a powerful explosive, called RDX. The organic compound is:

(A) phenol

(B) toluene

(C) glycerin

(D) formaldehyde

Correct Answer: A

Question 70 : A signature written in carbon pencil weights 1 mg. what is the number of carbon atoms present in the signature?

(A) 5.02×10^{23}

(B) 5.02×10^{20}

(C) 6.02×10^{20}

(D) 0.502×10^{20}

Correct Answer: D

Question 71 : When 50 cm^3 of 0.2 N H_2SO_4 is mixed with 50 cm^3 of 1 N KOH, the heat liberated is:

(A) 11.46 KJ

(B) 57.3 KJ

(C) 573 KJ

(D) 573 J

Correct Answer: D

Question 72 : NH_3 and HCl gas are introduced simultaneously from the two ends of a long tube. A white ring of NH_4Cl appears first:

(A) nearer to the HCl end

(B) at the centre of the tube

(C) throughout the tube

(D) nearer to the NH_3 end

Correct Answer: A

Question 73 : A gas formed by the action of alcoholic KOH on ethyl iodide, decolourises alkaline $KMnO_4$. The gas is:

(A) C_2H_6

(B) CH_4

(C) C_2H_2

(D) C_2H_4

Correct Answer: D

Question 74 : Which of the following is not a characteristic of chemisorptions?

(A) is of the order of 400 kJ

(B) Adsorption is irreversible

(C) Adsorption may be multimolecular layer

(D) Adsorption is specific

Correct Answer: C

Question 75 : The concentration of electrolyte required to coagulate a given amount of As_2S_3 sol. Is

minimum in the case of:

(A) magnesium nitrate

(B) potassium nitrate

(C) potassium sulphate

(D) aluminum nitrate

Correct Answer: D

Question 76 : Identify the organic compound which on heating with strong solution of NaOH, partly covered into an acid salt and partly into alcohol:

(A) Benzyl alcohol

(B) Acetaldehyde

(C) Acetone

(D) Benzaldehyde

Correct Answer: D

Question 77 : The process of which synthesis of protein takes place based on the genetic information present in m-RNA is called:

(A) translation

(B) transcription

(C) replication

(D) messenger hypothesis

Correct Answer: A

Question 78 : The enthalpies of formation of Al_2O_3 and Cr_2O_3 are -1596 kJ and -1134 kJ respectively. ΔH for the reaction,$2Al + Cr_2O_3 \rightarrow 2Cr + Al_2O_3$ is:

(A) -2730 kJ

(B) -462 kJ

(C) -1365 kJ

(D) +2730 kJ

Correct Answer: B

Question 79 : Stainless steel does not rust because:

(A) chromium and nickel combine with iron

(B) chromium forms an oxide layer and protects iron from rusting

(C) nickel present in it, does not rust

(D) iron forms a hard chemical compound with chromium presemt in it

Correct Answer: B

Question 80 : Temperature coefficient of a reaction is 2. When temperature is increased from 30^{o} C to 100^{o} C , rate of the reaction increases by:

(A) 128 times

(B) 100 times

(C) 500 times

(D) 250 times

Correct Answer: A

Question 81 : Which of the following combinations can be used to synthesis ethanol?

(A) CH_3MgI and CH_3COCH_3

(B) CH_3MgI and C_2H_5OH

(C) CH_3MgI and $CHH_3COOC_2H_5$

(D) CH_3MgI and $HCOOC_2H_5$

Correct Answer: C

Question 82 : The equivalent weight of a certain trivalent is 20. Molecular weight of its oxide is:

(A) 152

(B) 56

(C) 168

(D) 68

Correct Answer: C

Question 83 : Identify the reaction that doesn't take place during the smelting process of copper extraction:

(A) $2FeS+3O_2 \rightarrow 2FeO + 2SO_2 \uparrow$

(B) $Cu_2O+FeS \rightarrow Cu_2S + FeO$

(C) $2Cu_2S+3O_2 \rightarrow 2CU_2O + 2SO_2 \uparrow$

(D) $FeO+SiO_2 \rightarrow FeSiO_3$

Correct Answer: A

Question 84 : Pick out the complex compound in which the central metal atom obeys EAN rule strictly:

(A) $K_4[Fe(CN)_6]$

(B) $K_3[Fe(CN)_6]$

(C) $[Cr(H_2O)_6]Cl_3$

(D) $[Cu(NH_3)_4]SO_4$

Correct Answer: A

Question 85 : In a reversible reaction, the catalyst:

(A) increases the activation energy of the backward reaction

(B) increases the activation energy of the forward reaction

(C) decreases the activation energy of both, forward and backward reaction

(D) decreases the activation energy of forward reaction

Correct Answer: C

Question 86 : Solubility product of a salt AB is $1x10^{-8}$ in a solution in which concentration of A is 10^{-3} M. the salt will precipitate when the concentration of B becomes more than:

(A) 10^{-4} M

(B) 10^{-7} M

(C) 10^{-6} M

(D) 10^{-5} M

Correct Answer: D

Question 87 : A solution contains 1.2046×10^{24} hydrochloric acid molecules In one dm^3 of the solution. The strength of the solution is:

(A) 6 N

(B) 2N

(C) 4 N

(D) 8 N

Correct Answer: B

Question 88 : The ratio of cationic radius to anionic radius in an ionic crystal is greater than 0.732. its co-ordination number is:

(A) 6

(B) 8

(C) 1

(D) 4

Correct Answer: B

Question 89 : Dacron is obtained by the condensation polymerization of:

(A) Dimethyl terephthalate and ethylene glycol

(B) Terephthalic acid and formaldehyde

(C) Phenol and phthalic acid

(D) Phenol and formaldehyde

Correct Answer: A

Question 90 : 4-chloro-3, 5-dimethyl phenol is called:

(A) Chloramphenicol

(B) Paracetamol

(C) Barbital

(D) Dettol

Correct Answer: D

Question 91 : The percentage s-character of the hybrid orbitals In methane, ethane and ethyne are respectively:

(A) 25,33,50

(B) 25,50,75

(C) 50,75,100

(D) 10,20,40

Correct Answer: A

Question 92 : In the manufacture of sulfuric acid by contact process,Tyndall box is used to:

(A) filter dust particles

(B) remove impurities

(C) convert SO_2 to SO_3

(D) test the presence of dust particles

Correct Answer: D

Question 93 : The pH value of gastric juice in human stomach is about 1.8 and in the small intestine it is about 7.8. the pK_a value of aspirin is 3.5. Aspirin will be:

(A) completely ionized in the small intestine and in the stomach

(B) unionized in the small intestine and in the stomach

(C) ionized in the small intestine and almost unionized in the stomach

(D) ionized in the stomach and almost unionized in the small intestine

Correct Answer: D

Question 94 : The number of **α** and β particles emitted during the transformation of ${}_{90}Th^{232}$ to ${}_{82}Pb^{208}$ are respectively:

(A) 4,2

(B) 2,2

(C) 8,6

(D) 6,4

Correct Answer: D

Question 95 : When chlorine is passed through warm benzene in presence of the sunlight, the product obtained is:

(A) Benzotrichloride

(B) Chlorobenzene

(C) Gammexane

(D) D D T

Correct Answer: C

Question 96 : Ethyl benzoate reacts with PCl_5 to give:

(A) $C_2H_5Cl + C_6H_5COCl + POCl_3 + HCl$

(B) $C_2H_5Cl + C_6H_5COCl + POCl_3$

(C) $CH_3COCl + C_6H_5COCl + POCl_3$

(D) $C_2H_5Cl + C_6H_5COOH + POCl_3$

Correct Answer: B

Question 97 : Pick out the statement which is not relevant in the discussion of colloids:

(A) Sodium aluminum silicate is used in the softening of hard water

(B) Potash alum is used in shaving rounds and as a styptic in medicine

(C) Artificial rain is caused by throwing electrified sand on the clouds from an aeroplane

(D) Deltas are formed at a place where the river pours its water into the sea

Correct Answer: A

Question 98 : A wooden box excavated from Indus valley had an activity of $1.18x10^{13}$ disintegration per minute per gm. of carbon. What is the approximate age of this civilizations?

(A) 4000 years

(B) 5700 years

(C) 8100 years

(D) 6000 years

Correct Answer: C

Question 99 : For a reaction if $K_p > K_c$, the forward reaction is favoured by:

(A) low pressure

(B) high pressure

(C) high temperature

(D) low temperature

Correct Answer: A

Question 100 : In a time kiln, to get higher yield of CO_2, the measure that can be taken is:

(A) to remove CaO

(B) to add more $CaCO_3$

(C) to maintain high temperature

(D) to pump out CO_2

Correct Answer: D

Question 101 : What is the volume of ''20 volume H_2O_2" required to get 5000 cm^3 of oxygen at S.T.P.?

(A) 250 cm^3

(B) 50 cm^3

(C) 100 cm^3

(D) 125 cm^3

Correct Answer: A

Question 102 : The IUPAC name of $(CH_3)_3C—CH═CH_2$ is:

(A) 1,1,1-trimethyl-2-propane

(B) 3,3,3-trimethyl-2-propane

(C) 2,2-dimethyl-3-butene

(D) 3,3-dimethyl-1-butene

Correct Answer: D

Question 103 : Railways wagon axles are made by heating iron rods embedded in charcoal powder. This process is known as:

(A) Tempering

(B) Case hardening

(C) Sherardising

(D) Annealing

Correct Answer: D

Question 104 : Thomas Slag is . . .

(A) $CaSiO_3$

(B) $Ca_3(PO_4)_2$

(C) $MnsiO_3$

(D) $CaCO_3$

Correct Answer: B

Question 105 : Urea is preferred to ammonium sulphate as a nitrogenous fertilizer because:

(A) it is more soluble in water

(B) it is cheaper than ammonium sulphate

(C) it is quite stable

(D) it does not cause acidity in the soil

Correct Answer: D

Question 106 : Two gas cylinders having same capacity have been filled with 44 g of H_2 and 44 g of CO_2 respectively. If the pressure in CO_2 cylinder is 1 atmosphere at a particular temperature, the pressure in the hydrogen cylinder at the same temperature is:

(A) 2 atmosphere

(B) 1 2 atmosphere

(C) 22 2 atmosphere

(D) 44 2 atmosphere

Correct Answer: C

Question 107 : Nuclear theory of the atom was put forward by:

(A) Rurherford

(B) Aston

(C) Neils Bohr

(D) J.J. Thomson

Correct Answer: A

Question 108 : The element with atomic number 36 belongs to …… block in the periodic table:

(A) p

(B) s

(C) f

(D) d

Correct Answer: A

Question 109 : The function of $AlCl_3$ in Friedel-Craft's reaction is:

(A) to absorb HCl

(B) to absorb water

(C) to produce nucleophile

(D) to produce electrophile

Correct Answer: D

Question 110 : An important reaction of acetone is autocondensation in presence of concentrated sulphuric acid to give the aromatic compound…..

(A) mesitylene

(B) mesityl oxide

(C) trioxan

(D) phorone

Correct Answer: A

Question 111 : Kinetic energy of one mole of an ideal gas at 300 K in is:

(A) 3.74

(B) 348

(C) 34.8

(D) 3.48

Correct Answer: A

Question 112 : The tripeptide hormone present in most living cell is:

(A) glulathione

(B) glutamine

(C) oxytocin

(D) ptyalin

Correct Answer: A

Question 113 : In acetylene molecule, the two carbon atoms are linked by:

(A) one sigma- bond and two pi-bonds

(B) two sigma- bond and one pi-bonds

(C) three sigma- bonds

(D) three pi-bonds

Correct Answer: A

Question 114 : Energy is stored in our body in the form of:

(A) ATP

(B) protein

(C) Fats

(D) Carbohydrates

Correct Answer: A

Question 115 : An organic compound answers Molisch's test as well as Benedict's test. But it does not answer Scliwanoff's test. Most probably, it is:

(A) sucrose

(B) protein

(C) fructose

(D) maltose

Correct Answer: D

Question 116 : Which one of the following is not an amphoteric substance?

(A) HNO_3

(B) HCO^-_3

(C) H_2O

(D) NH_3

Correct Answer: A

Question 117 : When 50 cm^3 of 0.2 N H_2SO_4 is mixed with 50 cm^3 of 1 N KOH, the heat liberated is:

(A) 11.46 KJ

(B) 57.3 KJ

(C) 573 KJ

(D) 573 J

Correct Answer: D

Question 118 : the chemical equilibrium of a reversible reaction is not influenced by:

(A) pressure

(B)

catalyst

(C) concentration of the reactants

(D) temperature

Correct Answer: B

Question 119 : Stainless steel does not rust because:

(A) chromium and nickel combine with iron

(B) chromium forms an oxide layer and protects iron from rusting

(C) nickel present in it, does not rust

(D) iron forms a hard chemical compound with chromium presemt in it

Correct Answer: B

Question 120 : Which of the following combinations can be used to synthesis ethanol?

(A) CH_3MgI and CH_3COCH_3

(B) CH_3MgI and C_2H_5OH

(C) CH_3MgI and $CHH_3COOC_2H_5$

(D) CH_3MgI and $HCOOC_2H_5$

Correct Answer: C

SUBJECT: Mathematics

Question 121 :

Which of the following is/are empty set?
A = {x : x2 + x + 1 = 0 and x Î Z} B = {x : x is a prime number and 19 < x < 23}

(A) A

(B) B

(C) Both A and B

(D) None of the above

Solution :

The equation x2 + x + 1 = 0 is not satisfied by any integral value of x. So, A is an empty set.

Clearly, there is no prime number which lie between 19 and 23.

So, B is also an empty set

Correct Answer: C

Question 122 :

The set {x : $x^4 = 1$ and x ∈ R} is .....X.... set. Here, X refers to

(A) empty

(B)

finite

(C) infinite

(D) singleton

Solution :

Consider $x^4 = 1$

$\Rightarrow x4 - 1 = 0 \Rightarrow (x^2 - 1)(x^2 + 1) = 0$

$\Rightarrow (x - 1)(x + 1) = 0$ $\because [Q\ x^2 + 1 \neq 0\ \forall, x \in R]$

$x = -1, 1$

Thus, the given set is {–1, 1}.

Correct Answer: B

Question 123 : If R is a relation on a finite set A having (n – 1) elements, then the number of relations on A is

(A) 2^{n-1}

(B) $2^{(n-1)^2}$

(C) $(n-1)^2$

(D) $(n-1)^{n-1}$

Correct Answer: B

Question 124 : If $R = \{(x, y): x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation, then domain of R is

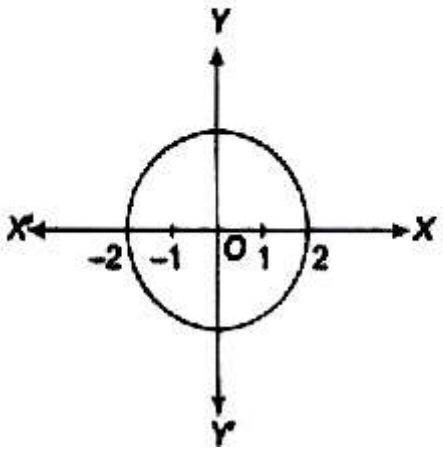
(A) { 0, 1, 2, }

(B) { 0, -1 , -2 }

(C) { -2 , -1 ,0 , 1 , 2 , }

(D) None of the above

We know, $x^2 + y^2 = a^2$ represent circle, with centre (0, 0) and radius a. Given equation is $x^2 + y^2 \leq 4^2$. Hence, we can also take interior integral points of the circle $x^2 + y^2 = 4$.



Solution :

Correct Answer: C

Question 125 : If 1, a_1, a_2,....., a_{n-1} are n roots of unity, then the value of $(1 - a_1)(1 - a_2)(1 - a_3).....(1 - a_{n-1})$ is

(A) $\sqrt{3}$

(B)

1/2

(C) n

(D) Zero

Solution :

Q 1, $a_1, a_2, \ldots, a_{n-1}$ are n roots of unity.

$\therefore \frac{x^n-1}{x-1} = (x-a_1)(x-a_2)\ldots(x-a_{n-1})$

$\Rightarrow x^{n-1} + x^{n-2} + \ldots + x^2 + x + 1$

$= (x - a_1)(x - a_2)\ldots(x - a_{n-1})$

Put $x = 1$

$\therefore (1 - a_1)(1 - a_2) \ldots (1 - a_{n-1})$

$= 1 + 1 + \ldots$ n times $= n$

Correct Answer: C

Question 126 :

In the argand plane the complex number $z = 4 - 3i$ is turned in the clockwise sense through 180° and stretched three times. The complex number represented by the new number is

(A) $12 + 9i$

(B) $12 - 9i$

(C) -12 - 9i

(D) -12 + 9i

Solution :

We have, $z = 4 - 3i$

$\therefore |z| = \sqrt{4^2 + (-3)^2} = 5$

Let z_1 be the new complex number obtained by rotating z in the clockwise sense through 180°, therefore, $z_1 = -4 + 3i$

Therefore, required complex number is

$3(-4 + 3i) = -12 + 9i$

Correct Answer: D

Let S_n denote the sum of the first n terms of an AP.

Question 127 : If $S_{2n} = 3S_n$, then $S_{3n} = S_n$ is equal to

(A) 4

(B)

6

(C) 8

(D) 10

Solution :

Given, $S_{2n} = 3S_n$

$$\Rightarrow \frac{2n}{2}[2a + (2n-1)d]$$

$$= 3\left\{\frac{n}{2}[2a + (n-1)d]\right\} 4a + (4n-2)d$$

$$= 6a + (3n-3)d$$

$$\Rightarrow 2a = (n+1)d \quad \ldots..(i)$$

$$\text{Now, } \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a + (3n-1)d]}{\frac{n}{2}[2a + (n-1)d]} +$$

$$= \frac{3\{(n+1)d + (3n-1)d\}}{(n+1)d + (n-1)d} \quad [\square \text{ from eq. (i)}]$$

$$= \frac{3(4nd)}{2nd} = 6$$

Correct Answer: B

Question 128 :

The number which should be added to the numbers 2, 14, 62, so that the resulting numbers may be in GP, is

(A) 1

(B) 2

(C) 3

(D) 4

Solution :

Suppose that the added number be x,
then x + 2, x + 14, x + 62 are in GP
$\therefore (x + 14)^2 = (x + 2)(x + 62)$
$\Rightarrow x^2 + 196 + 28x = x^2 + 64x + 124$
$\Rightarrow 36x = 72$
$\therefore x = 2$

Correct Answer: B

Question 129 : $3^{2n} - 8n - 1$ is divisible by

(A) 65

(B) 67

(C) 71

(D) 64

Solution :

We use the principle of induction to prove the statement.

Let f(x) = 32n – 8n – 1

Step I: f(1) = 9 – 8 – 1 = 0, f(1) is divisible by 64.

Therefore, the statement is true for n = 1.

Step II: f(k + 1) – f(k)

= {32k + 2 – 8(k + 1) – 1} – {32k – 8k – 1}

= 8(32k – 1) = 8(9k – 1)

= 8.8(9k – 1 + 9k – 2 + ... + 1)

= 64p, where p is an integer.

f(k + 1) is divisible by 64, if f(k) is true.

Thus, proves the statement.

Correct Answer: D

Question 130 : If n > 2, then n5 – 5n3 + 4n is divisible by

(A)

120

(B) 110

(C) 100

(D) 90

Solution :

We have
$n^5 - 5n^3 + 4n = n(n^4 - 5n^2 + 4)$

$= n(n^2 - 1)(n^2 - 4)$

$= n(n - 1)(n + 1)(n - 2)(n + 2)$

$= (n - 2)(n - 1)n(n + 1)(n + 2)$

Thus, if n > 2, then $n5 - 5n^3 + 4n$ has been expressed as a product of ifve consecutive natural numbers and so it is divisible by 5! i.e. 120.

Correct Answer: A

Question 131 : The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time is

(A) 432

(B) 108

(C) 36

(D) 18

Solution : The sum of the digit in unit place of all the numbers formed =
$3!(3 + 4 + 5 + 6)$
$= 6 \times 18 = 108$

Correct Answer: B

Question 132 : The sum of all five-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 when repetition of digits is not allowed, is

(A) 366000

(B) 660000

(C) 360000

(D) 3999960

Solution : Required sum = (Sum of the digits)

$$(n-1)!\left(\frac{10^n-1}{10-1}\right)$$

$$=(1+2+3+4+5)(5-1)!\left(\frac{10^5-1}{10-1}\right)$$

$$=360\left(\frac{100000-1}{9}\right)$$

$= 40 \times 99999 = 3999960$

Correct Answer: D

Question 133 : If $n > (8+3\sqrt{7})^{10}, n \in N$, then the least value of n is

(A) $(8+3\sqrt{7})^{10}-(8-3\sqrt{7})^{10}$

(B) $(8+3\sqrt{7})^{10}+(8-3\sqrt{7})^{10}$

(C) $(8+3\sqrt{7})^{10}-(8-3\sqrt{7})^{10}+1$

(D) $(8+3\sqrt{7})^{10}-(8-3\sqrt{7})^{10}-1$

Let $f=(8-3\sqrt{7})^{10}$, here $0 < f < 1$

$\therefore$ $(8+3\sqrt{7})^{10}+(8-3\sqrt{7})^{10}$ is an integer, hence

Solution : this is the value of n.

Correct Answer: B

Question 134 : The two successive terms in the expansion of $(1 + x)^{24}$ whose coefficients are in the ratio 1 : 4 are

(A) 3rd and 4th

(B) 4th and 5th

(C) 5th and 6th

(D) 6th and 7th

Solution :

Given expression is $(1 + x)^{24}$.

Let two successive terms are $(r + 1)$th and $(r + 2)$th terms.

$\Rightarrow T_{r+1} = {}^{24}C_r . x^r$

and $T_{r+2} = {}^{24}C_{r+1} x^{r+1}$

Now, ratio of coefficients $= \frac{1}{4}$

$\Rightarrow \frac{{}^{24}C_r}{{}^{24}C_{r+1}} = \frac{1}{4} \Rightarrow \frac{r+1}{24-r} = \frac{1}{4}$

$\Rightarrow 4r + 4 = 24 - r \Rightarrow r = 4$

$\Rightarrow$ Required terms are 5th and 6th.

Correct Answer: C

Question 135 : The number of real roots of is

(A) ZERO

(B) 2

(C) 1

(D) 4

Solution :

Correct Answer: B

Question 137 : The solution of the inequality $3x + 2y > 6$ in the graph is represented by shading the

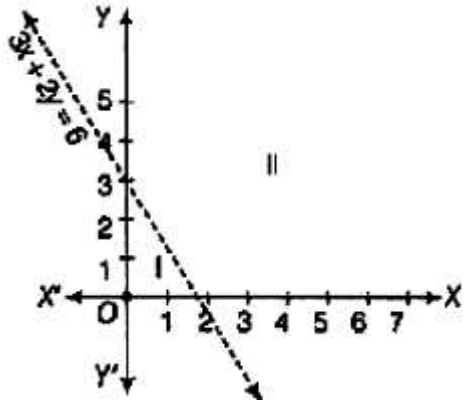
(A) half plane, which contains origin including the points on the line

(B) half plane, which does not contain origin excluding the points on the line

(C) half plane, which does not contain origin including the points on the line

(D) half plane, which contain origin excluding the points on the line

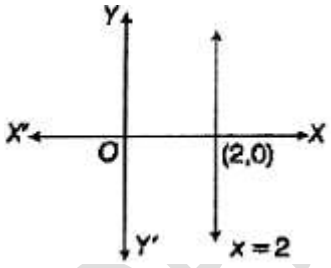
Solution : Graph of ax+ 2y = 6 is given as dotted line in the figure. This line divides the xy-plane in two half planes I and II. We select a point (not on the line), say (0, 0), which lies in one of the half planes (see figure). Now, determine whether this point satisfies the given inequality or not. We note that 3(0) + 2(0) > 6 or 0 > 6 which is false. Hence, half plane I is not the solution region of the given inequality. Clearly, any point on the line does not satisfy the given strict inequality. In other words, the shaded half plane II excluding the points on the line is the solution region of the inequality.



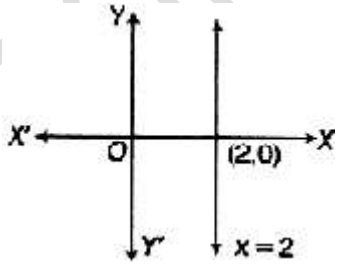
Correct Answer: B

Question 138 : The graphical solution of 3x – 6 > 0 is

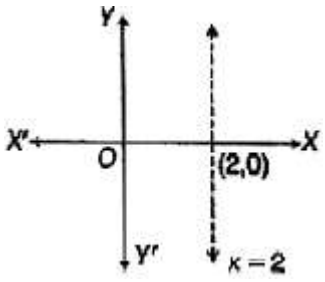
(A)



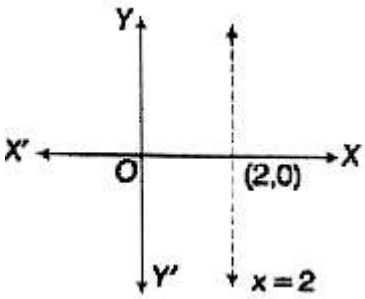
(B)



(C)

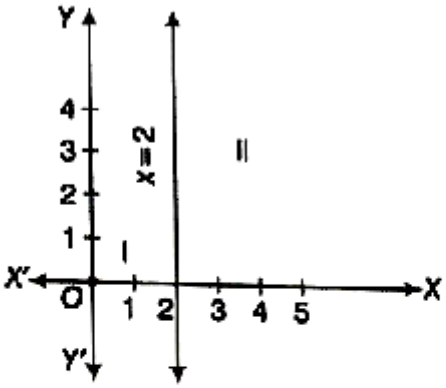


(D)



Solution :

Graph of 3x – 6 = 0is given in the figure.



We select a point say (0, 0) and substituting it in given inequality, we see that

3(0) – 6 ≥ 0 or –6 ≥ 0 which is false.

Thus, the solution region is the shaded region on the right hand side of the line x = 2.

Also, all the points on the line 3x – 6 = 0 will be included in the solution.

Hence, a dark line is drawn in the solution region.

Correct Answer: A

Question 139 : If the system of equations x + ky – z = 0,

$3x - ky - z = 0$ and $x - 3y + z = 0$, has non-zero solution, then k is equal to

(A) –1

(B) ZERO

(C) 1

(D)

2

Solution :

The system has non-zero solution, if

$$\begin{bmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{bmatrix} = 0$$

$\Rightarrow 1(-k - 3) - k(3 + 1) - 1(-9 + k) = 0$

$\Rightarrow -6k + 6 = 0$

$\therefore k = 1$

Correct Answer: C

Question 140 :

The existence of the unique solution of the system of equations $x + y + z = \beta$; $5x - y + \alpha z = 10$ and $2x + 3y - z = 6$ depends on

(A) α only

(B) β only

(C) both α and β

(D) Neither β nor α

Solution :

Given, system of equation is

$x + y + z = \beta$

$5x - y + \alpha z = 10$ and $2x + 3y - z = 6$

For unique solution, $\begin{vmatrix} 1 & 1 & 1 \\ 5 & -1 & \alpha \\ 2 & 3 & -1 \end{vmatrix} \neq 0$

$\Rightarrow 1(1 - 3\alpha) - 1(-5 - 2\alpha) + 1(15 + 2) \neq 0$

$\Rightarrow 1 - 3\alpha + 5 + 2\alpha + 17 \neq 0$

$\Rightarrow -\alpha + 23 \neq 0, a \neq 23$

Hence, for the existence of the unique solution system of equations depends on α only.

Correct Answer: A

Question 141 : $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & c^2 \end{vmatrix}$ is equal to

(A) $a^2b^2c^2$

(B) $2a^2b^2c^2$

(C) $4a^2b^2c^2$

(D) $8a^2b^2c^2$

Solution : (i) Take out factors a, b, c from R^1 R^2 and R^3 respectively.
(ii) Take out factors a, b, c from $C^{1,}$ C^2 and C^3 respectively.
It will change the determinant into easiest form which you will be able to solve.

$$= \begin{vmatrix} -a^2 & b & ac \\ ba & -b^2 & c \\ ca & cb & -c^2 \end{vmatrix} = abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

[taking out factors a from R_1, b from R_2 and c from R_3]

$$= (abc)\,(abc) \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

[taking out factors a from C_1, b from C_2 and c from C_3]

$$= a^2b^2c^2 \begin{vmatrix} 0 & 0 & 2 \\ 0 & -2 & 2 \\ 1 & 1 & -1 \end{vmatrix}$$

[using $R_1 \to R_1 + R_2$ and $R_2 \to R_2 - R_3$]
Expanding correspoding to first row R_1 we get

$$a^2b^2c^2 \left[0 \begin{vmatrix} -2 & 2 \\ 1 & -1 \end{vmatrix} - 0 \begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} + 2 \begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix} \right]$$

$= a^2b^2c^2\,[0 - 0 + 2\,(0 + 2)] = 4a^2b^2c^2$

Correct Answer: C

Question 142 : $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$ is equal to

(A) $k(y + k)$

(B) $k^2(y + k)$

(C) $k^2(3y + k)$

(D) $k^2(y + 3k)$

$$= \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

$$= \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix}$$

[using $C_1 \to C_1 + C_2 + C_3$]

$$= (3y + k) \begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$$

[take out $(3y + k)$ common from C_1]

$$= (3y + k) \begin{vmatrix} 1 & y & y \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix}$$

[using $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]
Expanding along C_3, we get

Solution : $(3y + k)\ (1 \times k \cdot k) = k^2(3y + k)$

Correct Answer: C

Question 143 : If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$ then $\tan x$ is equal to

(A) $\frac{(4-\sqrt{7})}{3}$

(B) $-\frac{(4+\sqrt{7})}{3}$

(C) $\frac{(1+\sqrt{7})}{4}$

(D) $\frac{(1-\sqrt{7})}{4}$

Solution :

$$\text{Given, } \cos x + \sin x = \frac{1}{2}$$

$$\Rightarrow \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1}{2}$$

$$\text{Let } \tan\frac{x}{2} = t, \text{ then}$$

$$\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = \frac{1}{2}$$

$$\Rightarrow 3t^2 - 4t - 1 = 0$$

$$\Rightarrow t = \frac{2 \pm \sqrt{7}}{3}$$

$$\Rightarrow t = \tan\frac{x}{2} = \frac{2+\sqrt{7}}{3}$$

$$\left[\because 0 < \frac{x}{2} < \frac{\pi}{2}, \tan\frac{x}{2} \text{ is positive}\right]$$

$$\text{Now, } \tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

$$= \frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2} = -\frac{3(2+\sqrt{7})}{2+2\sqrt{7}} \times \frac{2-2\sqrt{7}}{2-2\sqrt{7}}$$

$$\therefore \tan x = -\left(\frac{(4+\sqrt{7})}{3}\right)$$

Correct Answer: B

Question 144 : The number of solutions of the equation $(1 - \cos 2x)(\cos 2x + \cot^2 x) = 0, 0 \le x \le 2\pi$ is

(A) 3

(B) 2

(C) 1

(D) ZERO

Solution :
$(1 - \cos 2x)(\cos 2x + \cos^2 x) = 0$
$\Rightarrow \cos 2x = 1 \Rightarrow x = 0, \pi, 2\pi$
But for all values of x, cot x is not defined.
Hence, no solution exist.

Correct Answer: D

Question 145 : The value of $\sin(2 \sin^{-1}0.8)$ is

(A) 0.48

(B) $\sin 1.2^o$

(C) $\sin 1.6^o$

(D) 0.96

Solution :
Let $E = \sin(2 \sin^{-1}0.8)$
Put $\sin^{-1}0.8 = \theta$
$\Rightarrow \sin\theta = 0.8$
$\therefore \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - 0.64}$
$= \sqrt{0.36} = 0.6$
$\therefore E = \sin(2\theta) = 2 \sin\theta \cos\theta$
$= 2 \times 0.8 \times 0.6 = 0.96$

Correct Answer: D

Question 146 : If x takes negative permissible value, then $\sin^{-1} x$ is equal to

(A) $-\cos^{-1}\sqrt{1-x^2}$

(B) $\cos^{-1}\sqrt{x^2-1}$

(C) $\pi-\cos^{-1}\sqrt{1-x^2}$

(D) $\cos^{-1}\sqrt{1-x^2}$

Solution :

Let $x = -y, y > 0$

$\therefore \sin^{-1} x = \sin^{-1}(-y) = -\sin^{-1} y$

$= -\cos^{-1}\sqrt{1-y^2}$

$= -\cos^{-1}\sqrt{1-x^2}$

Correct Answer: A

Question 147 : Let f be differentiable for all x, If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then

(A) $f(6) = 5$

(B) $f(6) < 5$

(C) $f(6) < 8$

(D) $f(6) \geq 8$

Since, $\frac{f(6)-f(1)}{6-1} \geq 2$

$$\left[\text{Q } f'(x) \frac{y_2 - y_1}{x_2 - x_1}\right]$$

$\Rightarrow f(6) - f(1) \geq 10$

$\Rightarrow f(6) + 2 \geq 10$

Solution : $\Rightarrow f(6) \geq 8$

Correct Answer: D

Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| \geq 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$. If $f(x)$ is

Question 148 : continuous and differentiable everywhere, then

(A) $a = \frac{1}{2}, b = -\frac{3}{2}$

(B) $a = -\frac{1}{2}, b = \frac{3}{2}$

(C) a = 1, b = – 1

(D) a =b = 1

The given function is clearly continuous at all points except possibly at $x = \pm 1$

As $f(x)$ is an even function, so we need to check its continuity only at $x = 1$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} f(x) = f(1)$$

$$\Rightarrow \lim_{x \to 1} (ax^2 + b) = \lim_{x \to 1} \frac{1}{|x|}$$

$$\Rightarrow a + b = 1 \quad \text{.....(i)}$$

Clearly, $f(x)$ is differentiable for all x, except possible at $x = \pm 1$. As $f(x)$ is an even function, so we need to check its differentiability at $x = 1$ only.

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow \lim_{x \to 1} \frac{ax^2 + b - 1}{x - 1} = \lim_{x \to 1} \frac{\frac{1}{|x|} - 1}{x - 1}$$

$$\Rightarrow \lim_{x \to 1} \frac{ax^2 - b}{x - 1} = \lim_{x \to 1} \frac{-1}{x}$$

$$\Rightarrow 2a = -1$$

$$\Rightarrow a = -\frac{1}{2}$$

Solution :

On putting $a = -\frac{1}{2}$ in Eq. (i),

we get $b = \frac{3}{2}$

Hence, (2) is the correct answer.

Correct Answer: B

Question 149 : If $y = e^{\frac{1}{2}\log(1+\tan^2 x)}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{2}\sec^2 x$

(B) $\sec^2 x$

(C) secx tanx

(D) $e^{\frac{1}{2}\log(1+\tan^2 x)}$

Solution :

$$y = e^{\frac{1}{2}\log(1+\tan^2 x)}$$

$$y = (\sec^2 x)^{1/2} = \sec x$$

$$\therefore \frac{dy}{dx} = \sec x \tan x$$

Correct Answer: C

Question 150 : Derivative of x6 + 6x with respect to x is

(A) 12x

(B) x + 4

(C) $6x^5 + 6^x \log^6$

(D) $6x^5 + x6x^{-1}$

Solution :

Let $y = x^6 + 6^x$

On differentiating w.r.t x, we get

$$\frac{dy}{dx} = 6x^5 + 6^x \log 6$$

Correct Answer: C

Question 151 : The maximum value of $f(x) = 2x^3 - 24x + 107$ in the interval [1, 3] is

(A) 89

(B) 85

(C) 75

(D) 40

Solution :

$f(x) = 2x^3 - 24x + 107 \Rightarrow f'(x) = 6x^2 - 24$

For maximum or minimum, put $f'(x) = 0$

$\Rightarrow 6x^2 - 24 = 0$

$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

But $x = -2 \notin [1, 3]$

$\therefore$ $x = 2$ is the stationary point.

Now, $f(1) = 2 - 24 + 107 = 85$

$f(2) = 2(2)^3 - 24(2) + 107 = 75$

$f(3) = 2(3)^2 - 24 \times 3 + 107 = 89$

Hence, the maximum value of $f(x)$ is 89 at $x = 3$.

Correct Answer: A

Question 152 :

It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ on $[1, 3]$, Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$.

The values of a and b, if $f(1) = f(3) = 0$ are respectively, is

(A) 11, –6

(B) 2, 4

(C) 3, 2

(D) None of the above

Solution :

Given that, $f(1) = f(3) = 0$

$\Rightarrow (1)^3 - 6(1)^2 + a + b = 3^3$

$\Rightarrow -6(3)^2 + 3a + b = 0$

$\Rightarrow a + b = 5$

and $3a + b = 27$

$a = 11$

and $b = -6$

We have, $f(x) = x^3 - 6x^2 + ax + b$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 - 12x + a$

$= 3x^2 - 12x + 11 \qquad [\because a = 11]$

At $x = c$, $f'(c) = 3c^2 - 12c + 11$

$$= 3\left(2+\frac{1}{\sqrt{3}}\right)^2 - 12\left(2+\frac{1}{\sqrt{3}}\right)+11$$

$$= 12+\frac{12}{\sqrt{3}}+1-24-\frac{12}{\sqrt{3}}+11=0$$

$\therefore a = 11$

and $b = -6$

Correct Answer: A

Question 153 : $\int_0^{\pi/2} \frac{\cos x}{1+\sin x}\,dx$ equals to

(A) $\log 2$

(B) $2\log 2$

(C) $(\log 2)^2$

(D) $\frac{1}{2}\log 2$

$$I = \int_0^{\pi/2} \frac{\cos x}{1+\sin x}\,dx$$

$$= [\log(1+\sin x)]_0^{\pi/2}$$

Solution : $= \log 2 - \log 1 = \log 2$

Correct Answer: A

Question 154 : $\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{4-\sin^2\theta}}\,d\theta$ is equal to

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{5}$

$$\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{4-\sin^2\theta}}\,d\theta \left[\sin^{-1}\left(\frac{\sin\theta}{2}\right)\right]_0^{\pi/2}$$

Solution : $= \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Correct Answer: B

Question 155 : What is the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$?

(A) 3

(B) $\frac{2}{3}$

(C) 2

(D) 6

We have, $\left(\frac{d^3y}{dx^3}\right)^{2/3} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2 = \left(\frac{3d^2y}{dx^2} - \frac{5dy}{dx} - 4\right)^3$

Solution : Clearly, it is differential equatoin of degree 2.

Correct Answer: C

Question 156 : The second order differential equation of a parabola with its principal axis along the X-axis is

(A) $y'' + 2(y')^2 = 0$

(B) $3yy'' + 2(y')^3 = 0$

(C) $yy'' + (y') = 0$

(D) $y'' + 2(y')^4 = 0$

Solution : Equation of parabola is $y^2 = 4ax$.
On differentiating w.r.t. x, we get
$2yy' = 4a$
Again differentiating, we get
$2yy'' + 2(y')^2 = 0 \Rightarrow yy'' + (y')^2 = 0$

Correct Answer: C

Question 157 : The distance of the point (–2, 3) from the line x – y = 5 is

(A) $5\sqrt{2}$

(B) $2\sqrt{5}$

(C) $3\sqrt{5}$

(D) $5\sqrt{3}$

Solution : The distance of the point (–2, 3) from the line x – y = 5

$$p = \left|\frac{-2-3-5}{\sqrt{(1)^2+(-1)^2}}\right| = \left|\frac{-10}{\sqrt{2}}\right| = \frac{10}{\sqrt{2}} = 5\sqrt{2}$$

Correct Answer: A

Question 158 : Equation of the bisector of the acute angle between lines 3x + 4y + 5 = 0 and 12x – 5y – 7= 0 is

(A) 21x + 77y + 100 = 0

(B) 99x – 27y + 30 =0

(C) 99x + 27y + 30 = 0

(D) 21 – 77y – 100 =0

Solution :

Given lines are $3x + 4y + 5 = 0$ and

$12x - 5y - 7 = 0$

Here, $a_1 = 3$, $b_1 = 4$, $a_2 = 12$ and $b_2 = -5$

$\therefore\ a_1a_2 + b_1b_2 = 3 \times 12 + 4 \times (-5) = 16 > 0$

For acute angle bisector,

$$\frac{3x+4y+5}{\sqrt{9+16}} = -\frac{(12x-5y-7)}{\sqrt{12^2+(-5)^2}}$$

$$\Rightarrow \frac{3x+4y+5}{5} = -\frac{(12x-5y-7)}{13}$$

$\Rightarrow 39x + 52y + 65 = -60x + 25y + 35$

$\Rightarrow 99x + 27y + 30 = 0$

Correct Answer: C

Question 159 : The equation of the circle passing through (1, 1) and the points of intersection of the circles

$x^2 + y^2 + 13x - 3y = 0$ and

$2x^2 + 2y^2 + 4x - 7y - 25 = 0$

(A) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

(B) $x^2 - y^2 + 30x + 13y + 25 = 0$

(C) $x^2 + 3y^2 - 30x + 13y - 25 = 0$

(D) None of the above

Solution :

The given circles are

$x^2 + y^2 + 13x - 3y = 0$ .....(i)

and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$

$\Rightarrow x^2 + y^2 + 2x - \frac{7}{2}y - \frac{25}{2} = 0$ .....(ii)

Equation of any circle passing through the point of intersection of circles (i) and (ii) is

$(x^2 + y^2 + 13x - 3y)$

$$+ \lambda\left(x^2 + y^2 + 2x - \frac{7}{2}x - \frac{25}{2}\right) = 0 \quad \text{.....(iii)}$$

Its passes through (1, 1) then

$$(1 + 1 + 13 - 3) + \lambda\left(1 + 1 + 2 - \frac{7}{2} - \frac{25}{2}\right) = 0$$

$\Rightarrow 12 + \lambda(-12) = 0$

$\lambda = 1$

On substituting the value of λ in eq. (iii), the required equation is

$$x^2 + y^2 + 13x - 3y + x^2 + y^2 + 2x - \frac{7}{2}y - \frac{25}{2} = 0$$

$$\Rightarrow 2x^2 + 2y^2 + 15x - \frac{13}{2} - \frac{25}{2} = 0$$

$\Rightarrow 4x^2 + 4y^2 + 30x - 13y - 25 = 0$

Correct Answer: A

Question 160 : The equation of the circle through points of intersection of the circle $x^2 + y^2 - 2x - 4y + 4 = 0$ and the line $x + 2y = 4$ which touches the line $x + 2y = 0$, is

(A) $x^2 + y^2 + x + 2y = 0$

(B) $x^2 + y^2 - x - 2y = 0$

(C) $x^2 + y^2 - x + 2y = 0$

(D) None of these

Solution :

Equation of any circle through points of intersection of the given circle and the line is

$(x^2 + y^2 - 2x - 4y + 4) + \lambda(x + 2y - 4) = 0$

$\Rightarrow x^2 + y^2 + (\lambda - 2)x + (2\lambda - 4)y + 4(1 - \lambda) = 0$ .....(i)

It will touch the line $x + 2y = 0$, if the solution of eq. (i) and $x = -2y$ be unique.

Hence, the roots of the equation

$(-2y)^2 + y^2 + (\lambda - 2)(-2y) + (2\lambda - 4)y + 4(1 - \lambda) = 0$

or $5y^2 + 4(1 - \lambda) = 0$ must be equal.

Then, $0 = 4.5.4.(1 - \lambda)$

$\Rightarrow \lambda = 1$

$\therefore$ From eq. (i), the required circle is

$x^2 + y^2 - x - 2y = 0$

Correct Answer: B

Question 161 : One of the points on the parabola $y^2 = 12x$ with focal distance 12 is

(A) (3, 6)

(B) $(9, 6\sqrt{3})$

(C) $(7, 2\sqrt{21})$

(D) $(8, 4\sqrt{6})$

Solution :

Let the point be $(3t^2, 6t)$.

$\therefore$ Focal distance $= 3t^2 + 3$

$\Rightarrow 3t^2 + 3 = 12$

$\Rightarrow t^2 = 3 \Rightarrow t = \sqrt{3}$

Hence, the required point is $(9, 6\sqrt{3})$.

Correct Answer: B

Question 162 : The straight lines $y = \pm x$ intersect the parabola $y^2 = 8x$ in points P and Q, then length

of PQ is

(A) 4

(B) $4\sqrt{2}$

(C) 8

(D) 16

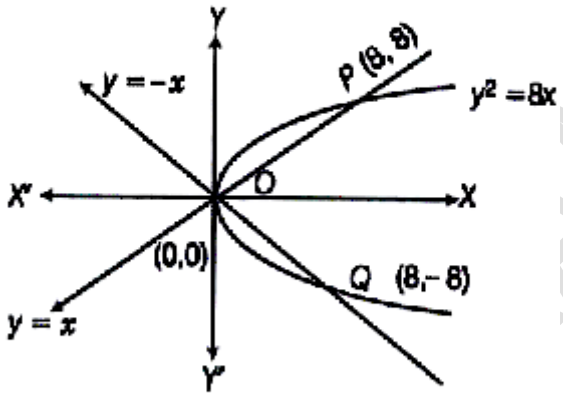
Solution :

. We have, $y^2 = 8x$

$\therefore\ y^2 = \pm 8y \qquad [Q\ x = \pm y]$

$\Rightarrow\ y^2 \mp 8y = 0$

$\Rightarrow\ y(y \mp 8) = 0$



$\Rightarrow\ y = 0, \pm 8$

$\therefore\ P(8, 8),\ Q(8, -8)$

$\therefore$ Length of PQ = $\sqrt{(16)^2 + (0)^2} = 16$

Correct Answer: D

Question 163 : If a, b and c are the three vectors mutually perpendicular to each other to form a right handed system and |a| = 1, |b| = 3 and |c| = 5, then [a – 2b b – 3c c – 4a] is equal to

(A) ZERO

(B) –24

(C) 3600

(D) –215

Solution :

Given that, $|\mathbf{a}| = 1$, $|\mathbf{b}| = 3$ and $|\mathbf{c}| = 5$

$\therefore\ [\mathbf{a} - 2\mathbf{b}\ \mathbf{b} - 3\mathbf{c}\ \mathbf{c} - 4\mathbf{a}]$

$= (\mathbf{a} - 2\mathbf{b}).\{(\mathbf{b} - 3\mathbf{c}) \times (\mathbf{c} - 4\mathbf{a})\}$

$= (\mathbf{a} - 2\mathbf{b}).\{\mathbf{b} \times \mathbf{c} - 4\mathbf{b} \times \mathbf{a} + 12\mathbf{c} \times \mathbf{a}\}$

$= (\mathbf{a} - 2\mathbf{b}).(\mathbf{a} + 4\mathbf{c} + 12\mathbf{b})$

$= \mathbf{a}.\mathbf{a} - 24\mathbf{b}.\mathbf{b} = 1 - 24 \times 9 = -215$

Correct Answer: D

Question 164 : $a \times [a \times (a \times b)]$ is equal to

(A) $(a \times a).(b \times a)$

(B) $a.(b \times a) - b.(a \times b)$

(C) $[a.(a \times b)]a$

(D) $(a.a)(b \times a)$

Solution :

$\mathbf{a} \times [\mathbf{a} \times (\mathbf{a} \times \mathbf{b})]$

$= \mathbf{a} \times \{(\mathbf{a}.\mathbf{b})\mathbf{a} - (\mathbf{a}.\mathbf{b})\mathbf{b}\}$

[Expanding by vector triple product]

$= (\mathbf{a}.\mathbf{b})(\mathbf{a} \times \mathbf{a}) - (\mathbf{a}.\mathbf{a})(\mathbf{a} \times \mathbf{b})$

$= (\mathbf{a}.\mathbf{a})(\mathbf{b} \times \mathbf{a})$ [$\because (\mathbf{a} \times \mathbf{a}) = 0$]

Correct Answer: D

Question 165 : If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are non-coplanar vectors and if $\mathbf{d}$ is such that $\mathbf{d} = \frac{1}{x}(\mathbf{a} + \mathbf{b} + \mathbf{c})$ and $\mathbf{d} = \frac{1}{y}(\mathbf{b} + \mathbf{c} + \mathbf{d})$, where x and y are non-zero real numbers, then $\frac{1}{xy}(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d})$ equals

(A) 3c

(B) –a

(C) ZERO

(D) 2a

Solution :

Given, $\mathbf{d} = \frac{1}{x}(\mathbf{a}+\mathbf{b}+\mathbf{c})$ and $\mathbf{d} = \frac{1}{x}(\mathbf{b}+\mathbf{c}+\mathbf{d})$

$\therefore \mathbf{a} + \mathbf{b} + \mathbf{c} - x\mathbf{d} = 0$

and $\mathbf{b} + \mathbf{c} + \mathbf{d} - y\mathbf{d} = 0$

$\Rightarrow \mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = 0$

$\therefore \frac{1}{xy}(\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}) = \frac{1}{xy}(0) = 0$

Correct Answer: C

Question 166 : The angle between the line $\frac{3x-1}{3} = \frac{y+3}{-1} = \frac{5-2z}{4}$ and the plane $3x - 3y - 6z = 0$ is equal to

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Solution :

Given line and plane can be rewritten as

$$\frac{x-\frac{1}{3}}{1}=\frac{y+3}{-1}=\frac{\left(z-\frac{5}{2}\right)}{-2}$$

and $x - y - 2z = 0$

Here, $a_1 = 1, b_1 = -1, c_1 = -2$

and $a_2 = 1, b_2 = -1, c_2 = -2$

$$\therefore \sin\theta = \frac{1\times1+(-1)\times(-1)+(-2)\times(-2)}{\sqrt{1+1+4}\sqrt{1+1+4}}$$

$$= \frac{6}{\sqrt{6}\sqrt{6}} = 1 \Rightarrow \theta = \frac{\pi}{2}$$

Correct Answer: D

Question 167 : What is the diameter of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z - 7 = 0$?

(A) 4 units

(B) 5 units

(C) 6 units

(D) 12 units

Solution :

On comparing the given equation with

$ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$, we get

$u = -2, v = 3, w = -4$ and $d = -7$

$$\therefore \text{Radius of sphere} = \sqrt{u^2+v^2+w^2-d}$$

$$= \sqrt{(-2)^2+(3)^2+(-4)^2+7} = 6$$

$\therefore$ Diameter $= 2 \times 6 = 12$ units

Correct Answer: D

Question 168 : Find the intersection of the spheres
$x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and
$x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$.

(A) $x - y - z = 1$

(B) $x - 2y - z = 1$

(C) $x - y - 2z = 1$

(D) $2x - y - z = 1$

Solution : We have two spheres
$S_1 = x^2 + y^2 + z^2 + 7x - 2y - z = 13$
and $S_2 = x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$
So, the intersection of S_1 and S_2 is given by
$S_1 - S_2 = 0$.
$\therefore 7x + 3x - 2y - 3y - z - 4z = 13 - 8$
$\Rightarrow 2x - y - z = 1$

Correct Answer: D

Question 169 : The SD of 15 items of 6 and if each item is decreased by 1, then standard deviation will be

(A) 5

(B) 7

(C) 9

(D) 6

Solution : If each item of a data is increased or decreased by the same constant, then standard deviation of the data remains unchanged, i.e., SD is 6.

Correct Answer: D

Question 170 : The mean of four observations is 3. If the sum of the squares of these observations is

48, then their standard deviation is

(A) $\sqrt{7}$

(B) $\sqrt{2}$

(C) $\sqrt{3}$

(D) $\sqrt{5}$

Solution : Let four observations be x_1, x_2, x_3 and x_4.

Given mean $(\bar{x}) = 3$ and $\Sigma x_i^2 = 48$

$\therefore \text{SD} = \sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2} = \sqrt{\frac{48}{4} - (3)^2} = \sqrt{12 - 9} = \sqrt{3}$

Correct Answer: C

Question 171 : The standard deviation of the following frequency distribution

Class	Frequency
1 – 10	11
11 – 20	29
21 – 30	18
31 – 40	4
41 – 50	5
51 – 60	3

(A) 12.7

(B) 13.6

(C) 15.3

(D) None of these

Let A, the assumed mean, be 25.5. Here, h = 10

Class	x_i	$y_i = \frac{x_i - 25.5}{10}$	f_i	$f_i y_i$	$f_i y_i^2$
1-10	5.5	–2	11	-22	44
11-20	15.5	–1	29	–29	29
21-30	25.5	0	18	0	0
31-40	35.5	1	4	4	4
41-50	45.5	2	5	10	20
51-60	55.5	3	3	9	27
			70	–28	124

$$\frac{\Sigma f_i y_i}{\Sigma f_i} = \frac{-28}{70} = -0.4$$

$$\bar{x} = A + \frac{\Sigma f_i y_i}{\Sigma f_i} \times h$$

Mean = $\bar{x}$ = 25.5 + (10)(–0.4) = 21.5

Variance $(\sigma^2) = \left[\frac{h}{N}\sqrt{N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2}\right]^2$

Solution :

$$= \frac{10 \times 10}{70 \times 70}[70(124) - (-28)^2] = 161.14$$

$$S.D(\sigma) = \sqrt{161.14} = 12.69 = 12.7$$

Correct Answer: A

Question 172 : A fair six-faced die is rolled 12 times The probability that each face turns up twice is equal to

(A) $\frac{12!}{6!6!6^{12}}$

(B) $\frac{2^{12}}{2^6 6^{12}}$

(C) $\frac{12!}{2^6 6^{12}}$

(D) $\frac{12!}{6^2 6^{12}}$

Solution :

Required probability

$$= {}^{12}C_2 \times {}^{10}C_2 \times {}^{8}C_2 \times {}^{6}C_2 \times {}^{4}C_2 \times {}^{2}C_2 \times \left(\frac{1}{6}\right)^{12}$$

$$= \frac{12!}{10!\times 2!} \times \frac{10!}{8!\times 2!} \times \frac{8!}{6!\times 2!} \times \frac{6!}{4!\times 2!}$$

$$\times \frac{4!}{2!\times 2!} \times \frac{2!}{2!\times 02!} \times \left(\frac{1}{6}\right)^{12}$$

$$= \frac{12!}{2^6 \times 6^{12}}$$

Correct Answer: C

Question 173 : 4 boys and 2 girls occupy seats in a row at random. Then, the probability that the two girls occupy seats side by side is

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{6}$

Solution :

Girls sit side by side, it means both girls are seated together in 2! ways.

Let E = 4 boys and 2 girls occupy seats side by side.

$\therefore$ n(E) = 5! 2!

$\therefore$ Required probability = $\frac{5!2!}{6!} = \frac{2}{6} = \frac{1}{3}$

Correct Answer: C

Question 174 : Three letters, to each of which corresponds an envelope, are placed in the envelopes at random. The probability that all the letters are not placed in the right envelopes, is

(A) $\frac{1}{6}$

(B) $\frac{5}{6}$

(C) $\frac{1}{3}$

(D) $\frac{2}{3}$

Solution :

Three letters can be placed in 3 envelopes in 3! ways, whereas there is only one way of placing them in their right envelopes.

So, that all the letters are placed in the right envelopes = $\frac{1}{3!}$

$\therefore$ Required probability $= 1 - \frac{1}{3!} = \frac{5}{6}$

Correct Answer: B

Question 175 : Given the mapping f: V3(F) → V2(F) defined by f(a1, a2, a3) = (a1, a2)

(A) f is homomorphism of $V_3(F)$ onto $V_2(F)$

(B) f is not a homomorphism of $V_3(F)$ onto $V_2(F)$

(C) {0, 0, a) : a ∈ F} is a kernel of f

(D) {0, 0, 0) : a ∈ F} is not a kernel of f

Solution :

Let $a = (a_1, a_2, a_3)$ and $\beta = (b_1, b_2, b_3)$ be any two elements of $V_3(V)$.
Let a, b be any two elements of F, then
$f(a\alpha + b\beta) = f[a(a_1, a_2, a_3) + b(b_1, b_2, b_3)]$
$= f[aa_1 + bb_1, aa_2 + bb_2, aa_3 + bb_3)]$
$= af(a_1, a_2, a_3) + bf(b_1, b_2, b_3)$
$= af(a) + bf(\beta)$
Hence, f is a linear transformation.
Let (a_1, a_2) be an element of V(F),
then $(a_1, a_2, 0) \in V_3(F)$
and $f(a_1, a_2, 0) = (a_1, a_2)$
Hence, f is onto.
Thus, f is a homomorphism of $V_3(F)$ onto $V_2(F)$.
If W is the kernel of this homomorphism, then
$W = \{(0, 0, a) : a \in F\}$
Thus, $f(0, 0, a) = (0, 0)$ = the zero vector of $V_2(f)$, $\forall$ a $\in$ F
Also, if $f(a_1, a_2, a_3) = (0, 0)$
Then, $f(a_1, a_2, a_3) = (a_1, a_2) = (0, 0)$
i.e. $a_1 = 0 = a_2$
Hence $(a_1, a_2, a_3) \in W$
Thus, W is the kernel of f.

Correct Answer: A

Question 176 :

Let $T : R^3 \rightarrow R^2$ be the linear transformation given by $T(x, y, z) = (x, y)$, $\forall (x, y, z) \in R^3$ with respect to standard basis of R^3 and the basic $\{(1, 0), (1, 1)\}$ of R^2.
What is the matrix representation of T?

(A) $\begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

Solution :

Here, it is given that

T: $R^3 \to R^2$ be the linear transformation given by

$T(x, y, z) = (x, y)\ \forall\ (x, y, z) \in R^3$

basis $\{(1, 0), (1, 1)\}$ of R2

$\therefore$ Standard basis of

$R_3 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

$= \{e_1, e_2, e_3\}$

$\therefore\ T(1, 0, 0) = (1, 0) = 1(1,0) + 0(1, 1)$

$T(0, 1, 0) = (0, 1) = -1(1, 0) + 1(1, 1)$

$T(0, 0, 1) = (0, 0) = 0(1, 0) + 0(1, 1)$

Hence, matrix of $T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Correct Answer: C

Question 177 : The eigen values of the matrix $\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$ are

(A) 1, 4

(B) –1, 2

(C) 0, 5

(D) cannot be determined

Solution :

The characteristic equation is

$|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & -2 \\ -2 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow (4 - \lambda)(1 - \lambda) - [(-2) \times (-2)] = 0$

$\Rightarrow \lambda^2 - 5\lambda = 0$

$\Rightarrow \lambda(\lambda - 5) = 0$

$\Rightarrow \lambda = 0, 5$ are the eigen value.

Correct Answer: C

Question 178 : The function $f(z) = \frac{z - ib}{z^2 + b^2}$ is continuous at

(A) ib

(B) –ib

(C) i3b

(D) None of these

Solution : We have $\lim_{z \to ib} f(z) = \lim_{z \to b} \frac{z - ib}{z^2 + b^2} = -\frac{i}{2b}$

and $f(ib) = -\frac{i}{2b}$

Hence, f is continuous at z = ib

Correct Answer: A

Question 179 : The nature of the singularities of $f(z) = \sin\frac{1}{z}$ at z = 0 is

(A) isolated

(B) isolated essential

(C) removable

(D) non-isolated essential

Solution :

Let $f(z) = \sin\frac{1}{z'}$ hence zeroes of f(z) are given by

$\sin\frac{1}{z} = 0$ or $\frac{1}{z} = n\pi$

or $z = \frac{1}{n\pi}, n \in I$

Obviously, z = 0 is a limit point of zeroes of f(z), hence z = 0 is an isolated essential singularity of f(z).

Correct Answer: B

Question 180 : $\frac{d}{dx}\left(\int_0^y e^{-t^2}dt + \int_0^{x^2}\sin^2 t\,dt\right) = 0$, then $\frac{dy}{dx}$ is equal to

(A) $-2xe^{y^2}\sin^2 x^2$

(B) $2xw^{y^2}\sin^2 x^2$

(C) $2xe^{y^2}\cos^2 x^2$

(D) $3x^2e^{y^2}\sin^2 x^2$

Solution :

We have, $\frac{d}{dx}\left(\int_0^y e^{-t^2}dt + \int_0^{x^2}\sin^2 t\,dt\right) = 0$

$\Rightarrow e^{-y^2}.\left\{\frac{d}{dx}(y)\right\} - e^{-0}\left\{\frac{d}{dx}(0)\right\}$

$+\sin^2(x^2)\left\{\frac{d}{dx}(x^2)\right\} - \sin^2 0\left\{\frac{d}{dx}(0)\right\} = 0$

$\Rightarrow e^{-y^2}\frac{dy}{dx} + 2x\sin^2 x^2 = 0$

$\Rightarrow e^{-y^2}\frac{dy}{dx} + 2x\sin^2 x^2 = 0$

Correct Answer: A