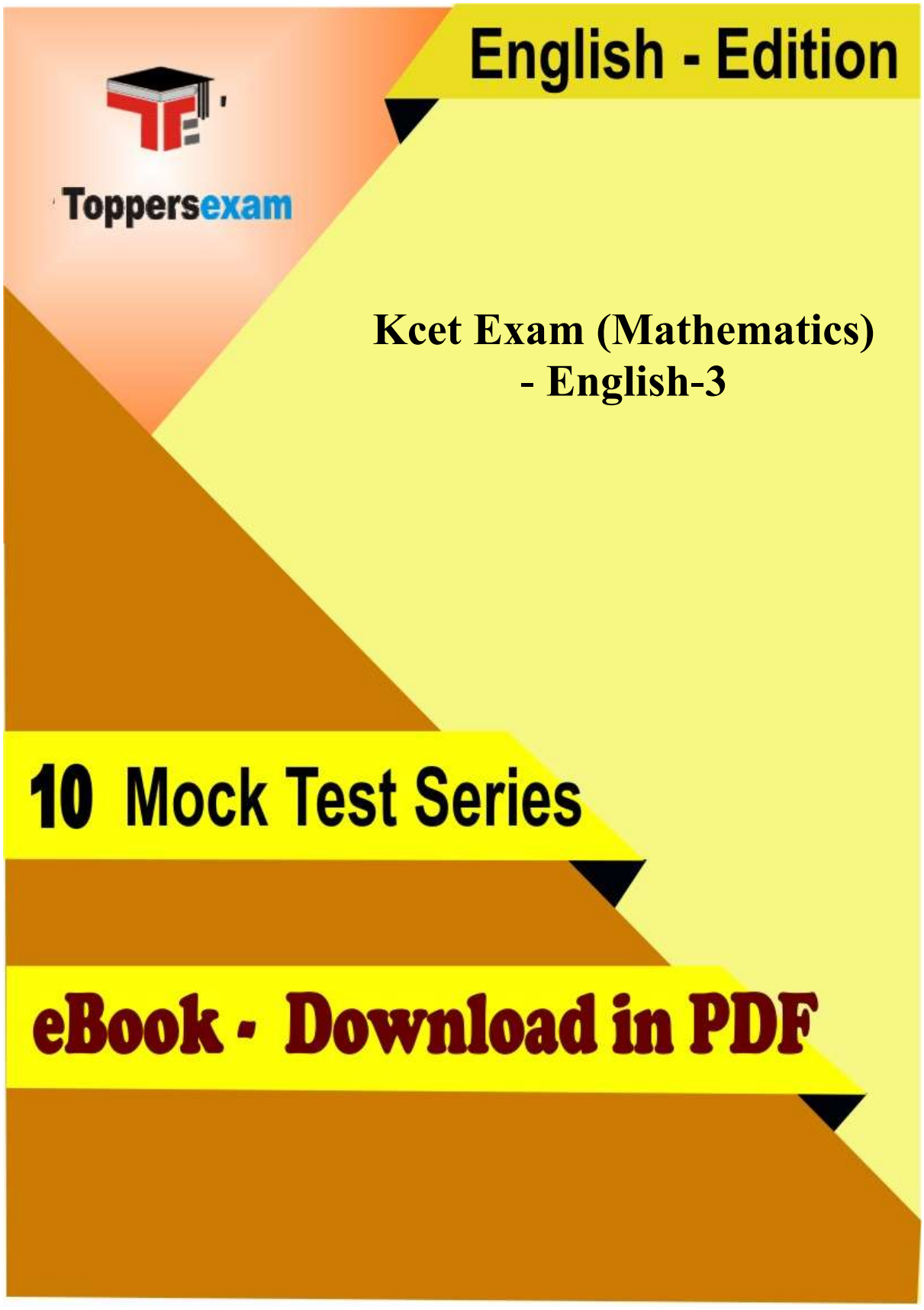

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Kcet Exam (Mathematics) - English-3

Paper Questions

SUBJECT: Physics

Question 1 : In Young's double slit experiment the central bright fringe can be identified:

(A) As it a narrower than other bright fringes

(B) By using white light instead of monochromatic light

(C) As it has a greater intensity than other bright fringes

(D) As it is wider than other bright fringes

Correct Answer: B

Question 2 : *n* identical bulbs, each designated to draw a power P from a certain voltage supply. The total power which they will draw is:

(A) P/n

(B) P/n_2

(C) Np

(D) P

Correct Answer: C

Question 3 : The tension of a stretched string is increased by 69%. In order to keep its frequency of vibration constant its length must be increased by:

(A) 30%

(B) 20%

(C) 69%

(D) 40%

Correct Answer: A

Question 4 : When a neutron is disintegrated to give a β-particle,

(A) a neutrino along is emitted

(B) a proton and neutrino are emitted

(C) a proton alone is emitted

(D) a proton and an antineutrino are emitted

Correct Answer: D

Question 5 : 64 small drops of mercury, each of radius r and charge q coalesce to form a big drop. The ratio of the surface density of charge of each small drop with of the big drop is:

(A) 64 : 1

(B) 1 : 64

(C) 1 : 4

(D) 4 : 1

Correct Answer: C

Question 6 : Two capacitors of capacitances 3 µF and 6 µF are charged to a potential of 12 V each. They are now connected to each other, with the positive plate to each joined to the negative plate to the other. The potential difference across each will be:

(A) 4 V

(B) 6 V

(C) Zero

(D) 3 V

Correct Answer: A

Question 7 : The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is:

(A) 120^0

(B) 60^0

(C) 90^0

(D) 150^0

Correct Answer: A

Question 8 : A body is projected vertically upwards from the surface of a planet of radius R with a velocity equal to half the escape velocity for that planet. The maximum height attained by the body is:

(A) R/2

(B) R/3

(C) R/5

(D) R/4

Correct Answer: B

Question 9 : One kilogram of ice at 0 C is mixed with one kilogram of water at 80° C. the final temperature of the mixture is (take : specific heat of water = 4200 J kg^{-1} K^{-1}. Heat of ice = 336 kJ kg^{-1})

(A) 40° C

(B) 60° C

(C) 0° C

(D) 50°C

Correct Answer: C

Question 10 : A carnot's engine is made to work between 200° C and 0° C first and then between 0° C and - 200° C. the ratio of the engine in the two cases is:

(A) 1 : 1.73

(B) 1.73 : 1

(C) 1 : 2

(D) 1 : 1

Correct Answer: A

Question 11 : An object is a placed 12 cm to the left of a converging lens of local length 8 cm. another converging lens of 6 cm focal length is placed at a distance of 30 cm. to the right of the first lens. The second lens will produce:

(A) A virtual enlarged image

(B) No image

(C) A real inverted image

(D) A real enlarged image

Correct Answer: C

Question 12 : The resistance of an incandescent lamp is:

(A) Smaller when switched on

(B) Greater when switched off

(C) The same whether it is switch off or switch on

(D) Greater when switched on

Correct Answer: D

Question 13 : A superconductor exhibits perfect:

(A) Ferromagnetism

(B) Ferrimagnetism

(C) Diamagnetism

(D) Paramagnetism

Correct Answer: C

Question 14 : A magnet is dropped down an infinitely long vertical copper tube. Then:

(A) The magnet moves with continuously decreasing velocity and ultimately comes to rest

(B) The magnet moves with continuously increasing velocity and ultimately acquires a constant terminal velocity

(C) The magnet moves with continuously increasing velocity and acceleration

(D) The magnet moves with continuously increasing velocity but constant acceleration

Correct Answer: B

Question 15 : Whenever a hydrogen atom emits a photon in the Balmer series, it:

(A) May emit another photon in the paschen series

(B) Need not emit any more photon

(C) May emit another photon in the balmer series

(D) Must emit another photon in the lyman series

Correct Answer: D

Question 16 : The SI unit of radioactivity is:

(A) Rutherford

(B) Roentgen

(C) Becqueral

(D) Curie

Correct Answer: D

Question 17 : An ammeter and a voltmeter are joined in series to a cell. Their reading are A and V respectively. If a resistance is now joined in parallel with the voltmeter, then:

(A) A will decreases, V will increase

(B) A will increase, V will decrease

(C) Both A and V will increase

(D) Both A and V will increase

Correct Answer: B

Question 18 : Two copper balls, each weighing 10 g are kept in air 10 cm apart. If one electron from every 10^6 atoms is transferred from one ball to the other, the coulomb force between them is: (atomic weight of copper is 63.5)

(A) 2.0×10^4 N

(B) 2.0×10^{10} N

(C) 2.0×10^6 N

(D) 2.0×10^8 N

Correct Answer: D

Question 19 : A cell supplies a current of 0.9 A through a 2Ω resistor and a current of 0.3 through a 7Ω resistor. The internal resistance of the cell is:

(A) 1.2Ω

(B) 2.0Ω

(C) 0.5Ω

(D) 1.0Ω

Correct Answer: C

Question 20 : What fraction of the energy drawn from the charging battery is stored in a capacitor?

(A) 75%

(B) 100%

(C) 25%

(D) 50%

Correct Answer: C

Question 21 : A caesium photocell, with a steady potential difference of 60 V across it, is illuminated by a bright point source of light 50 cm away. When the same light is places 1cm. away, the photoelectrons emitted from the cell:

(A) half as numerous

(B)

Are one quarter as numerous

(C) Each carry one quarter of their previous energy

(D)

Each carry one quarter of their previous momentum

Correct Answer: B

Question 22 : To increase both the resolving power and magnifying power of a telescope:

(A) The focal length of the objective has to be increased

(B) Both the focal length and aperture of the objective has to be increased

(C) The wavelength of light has to be decreased

(D) The aperture of the objective has to be increased

Correct Answer: B

Question 23 : Forty one tuning forks are arranged in increasing order of frequencies such that every fork gives 5 beats with the next. The last fork has a frequency that is double the frequency of the first fork is:

(A) 210

(B) 400

(C) 205

(D) 200

Correct Answer: D

Question 24 : In a stationary wave all the particles:

(A) In the region between two nodes vibrate in same phase

(B) On either side of a node vibrate in same phase

(C) Of the medium vibrate in same phase

(D) In the region between two antinodes vibrate in same phase

Correct Answer: A

Question 25 : In a p-n junction diode not connected to any circuit

(A) the potential is the same everywhere

(B) the p-type side has a higher potential than the n-type side

(C) there is an electric field at the junction directed from the n-type side to p-type side

(D) there is an electric field at the junction directed from the p-type side to n-type side

Correct Answer: C

Question 26 :

The forbidden energy gap in Ge is 0.72 eV. Given, hc = 12400 eV-Å. The maximum wavelength of radiation that will generate electron hole pair is

(A)

172220Å

(B)

172.2Å

(C) 17222Å

(D) 1722 Å

Correct Answer: C

Question 27 : When 100 V D.C. is applied across a coil, a current of 1 A flows through it. When 100 V A. C. of 50 Hz is applied to the same coil only 0.5 A flows. The inductance of the coil is:

(A) 5.5 mH

(B) 0.55 mH

(C) 55 mH

(D) 0.55 H

Correct Answer: D

Question 28 : In a sample of radioactive material what percentage of the initial number of active nuclei will decay during one mean life?

(A) 63%

(B) 69.3%

(C)

37%

(D) 50%

Correct Answer: B

Question 29 : In the Bohr model of the hydrogen atom, let R, V and E represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which of the following quantity is proportional to the quantum number n?

(A) E/V

(B) R/E

(C) VR

(D) RE

Correct Answer: C

Question 30 : The Instrument _____ is used for detecting electric current is

(A) Galvanometer

(B) Tube tester

(C) Altimeter

(D) Fathometer

Correct Answer: B

Question 31 : A caesium photocell, with a steady potential difference of 60 V across it, is illuminated by a bright point source of light 50 cm away. When the same light is places 1cm. away, the photoelectrons emitted from the cell:

(A) Are half as numerous

(B) Are one quarter as numerous

(C) Each carry one quarter of their previous energy

(D) Each carry one quarter of their previous momentum

Correct Answer: B

Question 32 : In the Bohr model of the hydrogen atom, let R, V and E represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which of the following quantity is proportional to the quantum number n?

(A) E/V

(B) R/E

(C) VR

(D) RE

Correct Answer: C

Question 33 : A light points fixed to one prong of a tuning fork touches a vertical plate. The fork is set vibrating and the plate is allowed to fall freely. If eight oscillations are counted when the plate falls through 10 cm, the frequency of the tuning fork is:

(A) 280 Hz

(B) 360 Hz

(C) 56 Hz

(D) 560 Hz

Correct Answer: C

Question 34 : Three point charges are placed at the corners of an equilateral triangle. Assuming only electrostatic forces are acting. Then the system:

(A) Will be equilibrium if the charges rotate about the centre of the triangle

(B) Can never be in equilibrium

(C) Will be in equilibrium if the charges have the same magnitudes but different signs

(D) Will be in equilibrium if the charges have different magnitude and different signs

Correct Answer: B

Question 35 : Two copper balls, each weighing 10 g are kept in air 10 cm apart. If one electron from every 10^6 atoms is transferred from one ball to the other, the coulomb force between them is:(atomic weight of copper is 63.5)

(A) 2.0×10^4 N

(B) 2.0×10^{10} N

(C) 2.0×10^6 N

(D) 2.0×10^8 N

Correct Answer: D

Question 36 : What fraction of the energy drawn from the charging battery is stored in a capacitor?

(A) 75%

(B) 100%

(C) 25%

(D) 50%

Correct Answer: C

Question 37 : In a stationary wave all the particles:

(A) In the region between two nodes vibrate in same phase

(B) On either side of a node vibrate in same phase

(C) Of the medium vibrate in same phase

(D) In the region between two antinodes vibrate in same phase

Correct Answer: A

Question 38 : A body is projected vertically upwards from the surface of a planet of radius R with a velocity equal to half the escape velocity for that planet. The maximum height attained by the body is:

(A) R/2

(B) R/3

(C) R/5

(D) R/4

Correct Answer: B

Question 39 :

In a given direction, the intensities of the scattered light by a scattering substance for two beams of light are in the ratio of 256 : 81. The ratio of the frequency of the first beam to the frequency of the second

beams is

(A)

64 : 127

(B)

1 : 2

(C)

64 : 27

(D) None of these

Correct Answer: D

Question 40 : Forty one tuning forks are arranged in increasing order of frequencies such that every fork gives 5 beats with the next. The last fork has a frequency that is double the frequency of the first fork is:

(A) 210

(B) 400

(C) 205

(D) 200

Correct Answer: D

Question 41 : The energy spectrum of a black body exhibits a maximum around a wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum around a wavelength $3\lambda_0/4$. The power radiated by the black body will now increase by a factor of:

(A) 64/27

(B) 256/81

(C) 4/3

(D) 16/9

Correct Answer: B

Question 42 : *n* identical bulbs, each designated to draw a power P from a certain voltage supply. The total power which they will draw is:

(A) P/n

(B) P/n_2

(C) Np

(D) P

Correct Answer: C

Question 43 : An ammeter and a voltmeter are joined in series to a cell. Their reading are A and V respectively. If a resistance is now joined in parallel with the voltmeter, then:

(A) A will decreases, V will increase

(B) A will increase, V will decrease

(C) Both A and V will increase

(D) Both A and V will increase

Correct Answer: B

Question 44 : Whenever a hydrogen atom emits a photon in the Balmer series, it:

(A) May emit another photon in the paschen series

(B)

Need not emit any more photon

(C)

May emit another photon in the balmer series

(D)

Must emit another photon in the lyman series

Correct Answer: D

Question 45 : Two capacitors of capacitances 3 F and 6 F are charged to a potential of 12 V each. They are now connected to each other, with the positive plate to each joined to the negative plate to the other. The potential difference across each will be:

(A) 4 V

(B) 6 V

(C) Zero

(D) 3 V

Correct Answer: A

Question 46 : 64 small drops of mercury, each of radius r and charge q coalesce to form a big drop. The ratio of the surface density of charge of each small drop with of the big drop is:

(A) 64 : 1

(B) 1 : 64

(C) 1 : 4

(D) 4 : 1

Correct Answer: C

Question 47 : Two identical conducting balls A and B have positive charges q_1 and q_2 respectively but $q_1 \neq q_2$. The ball are brought together so that they touch each other and then kept in their original positions. The force between them is

(A) less than that before the balls touched

(B) greater than that before the balls touched

(C) same as that before the balls touched

(D) zero

Correct Answer: B

Question 48 : Red light of wavelength 625 nm is incident normally on an optical diffraction grating with 2 $x10^5$ lines/m. including central principal maxima, how many maxima may be observed on a screen which is far from the grating?

(A) 15

(B) 17

(C) 8

(D) 16

Correct Answer: B

Question 49 : The light reflected by a plane mirror may form a real image:

(A) If the rays incident on the mirror are converging

(B) If the rays incident on the mirror are diverging

(C) Under no circumstances

(D) If the object is placed very close to the mirror

Correct Answer: A

Question 50 : In nuclear fission the percentage of mass converted into energy is about:

(A)

0.01%

(B) 10%

(C) 1%

(D) 0.1%

Correct Answer: D

Question 51 : Two beams of red and violet colours are made to pass separately through a prism of A=60^o. In the minimum inside the prism will be

(A) greater for red colour

(B) equal but not 30^o for both the colours

(C) greater for violet colour

(D) 30^o for both the colours

Correct Answer: D

Question 52 : A plano-convex lens is made of refractive index of 1.6. The radius of curvature of the curved surface is 60 cm. The focal length of the lens is

(A) 400 cm

(B) 200 cm

(C) 100 cm

(D) 50 cm

Correct Answer: C

Question 53 : In nuclear fission the percentage of mass converted into energy is about:

(A)

0.01%

(B) 10%

(C) 1%

(D) 0.1%

Correct Answer: D

Question 54 : A bat flies at a steady speed of 4 ms^{-1} emitting a sound of f=90x10^3 Hz. It is flying horizontally of the reflected sound as detected by the bat will be (take velocity of sound un air is 330ms^{-1})

(A) 88x10^{-3} Hz

(B) 87.1x10^3 Hz

(C) 92.1 x10^3 Hz

(D) 89.1x10^3 Hz

Correct Answer: C

Question 55 : A closed organ pipe an open organ pipe of same length produce 2 beats/second while vibrating in their fundamental modes. The length of the open organ pipe is halved and that of closed pipe is doubled. Then the number of beats produced per second while vibrating in the fundamental mode is

(A) 2

(B) 6

(C) 8

(D) 7

Correct Answer: D

Question 56 : When one of the slits of Young's experiment is covered with a transparent sheet of thickness 4.8 mm, the central shifts to a position originally occupies by the 30th bright fringe. What should be the thickness of the sheet if the central fringe has to shift to the position occupied by 20th bright fringe?

(A) 1.6 mm

(B) 3.8 mm

(C) 3.2 mm

(D) 7.6 mm

Correct Answer: C

Question 57 :

In a series resonant R-L-C circuit, the voltage across R is 100 v and the value of R = 1000Ω. The capacitance of the capacitor is $2x10^{-6}F$; angular frequency of AC is 200 rad s^{-1}. Then the potential difference across the inductance coil is

(A)

100 V

(B) 40 V

(C) 250 V

(D) 400 V

Correct Answer: C

Question 58 : What is the minimum thickness of a thin film required for constructive interference in the reflected light from it? Given, the refractive index of the film = 1.5, wavelength of the light incident on the film =600 nm.

(A) 100 nm

(B) 300 nm

(C) 50 nm

(D) 200 nm

Correct Answer: A

Question 59 : Critical angle for certain medium is $\sin^{-1}(0.6)$. The polarizing angle of that medium is

(A) $\tan^{-1}[1.5]$

(B) $\sin^{-1}[0.8]$

(C) $\tan^{-1}[1.6667]$

(D) $\tan^{-1}[0.6667]$

Correct Answer: C

Question 60 : The speed of electromagnetic wave in vacuum depends upon the source of radiation

(A) increases as we move from y –rays to radio waves

(B) decreases as we move from y-rays to radio waves

(C) is same for all of them

(D) None of the above

Correct Answer: C

SUBJECT: Chemistry

Question 61 : Ammonium ion is

(A) Neither an acid nor base

(B) Both an acid and a base

(C) A conjugate acid

(D) A conjugate base

Correct Answer: C

Question 62 : Water is a

(A) Amphoteric acid

(B) Aprotic solvent

(C) Protophobic solvent

(D) Protophilic solvent

Correct Answer: A

Question 63 : A chemical reaction was carried out at 300 K and 280 K. the rate constants were found to be K_1 and K_2 respectively, then:

(A) $K_2 = 0.25\ K_1$

(B) $K_2 = 0.5\ K_1$

(C) $K_2 = 4\ K_1$

(D) $K_2 = 2K_1$

Correct Answer: A

Question 64 : A smuggler could not carry gold by depositing iron and gold surface since:

(A) Gold has higher standard reduction potential than iron

(B) Gold has lower standard reduction potential than iron

(C) Gold is denser

(D) Iron rusts

Correct Answer: A

Question 65 : A is an aqueous acid; B is an aqueous base. They are diluted separately, then:

(A) pH of A decreases and pH of B increases

(B) pH of A increases and pH of B decreases till pH in each case is 7

(C) pH of A and B increase

(D) pH of B and A decreases

Correct Answer: A

Question 66 : kinetic theory of gases presumes that collision between the molecules to be perfectly elastic because:

(A) collisions will not split the molecules

(B) the molecules are tiny

(C) the molecules are rigid

(D) the temperature remains constant irrespective collisions

Correct Answer: D

Question 67 : ${}_{z}X^{M} + {}_{2}He^{4} \rightarrow {}_{15}P^{30} + {}_{0}n^{1}$. Then:

(A) Z = 12, M = 27

(B) Z = 13, M = 27

(C) Z = 12, M = 17

(D) Z = 13, M = 28

Correct Answer: B

Question 68 : Bio- gas production is more useful when compared to the direct use of dung because

(A) Both fuel value and fertilizer value are effectively utilized

(B) Production of bio-gas involves less labour

(C) Fuel is quickly produced

(D) The fertilizer produced is a fluid

Correct Answer: A

Question 69 : Compounds with high heat of formation are less stable because

(A) High temperature is required to synthesis them

(B) Molecules of such compounds are distorted

(C) It is difficult to synthesis them

(D) Energy rich state leads to instability

Correct Answer: D

Question 70 :

The volume of water to be added to 100 cm^3 of 0.5 N H_2SO_4 to get decinormal concentration is:

(A) 400 cm^3

(B) 500 cm^3

(C) 450 cm^3

(D) 100 cm^3

Correct Answer: A

Question 71 : Heat of neutralization of weak acid and strong base is less than the heat of neutralization of strong acid an strong base due to

(A) Energy has to be spent for the total dissociation of weak acid

(B) Salt of weak acid and strong base is not stable

(C) Incomplete dissociation of weak acid

(D) Incomplete neutralization of weak acid

Correct Answer: A

Question 72 : A radio isotope will not emit

(A) Gamma and alpha rays simultaneously

(B) Gamma rays only

(C) Alpha and beta rays simultaneously

(D) Beta and gamma rays simultaneously

Correct Answer: C

Question 73 : Heat treatment alters the properties of steel due to:

(A) Chemical reaction on heating

(B) Partial rusting

(C) Change in the residual energy

(D) Change in the lattice structure due to differential rate of cooling

Correct Answer: D

Question 74 : Chemical bond implies

(A) Attraction and impulses

(B) Attraction and repulsion balanced at a particular distance

(C) Attraction

(D) Repulsion

Correct Answer: B

Question 75 : Grignard reagent is not prepared in aqueous medium but prepared in either medium. Because

(A) The reagent reacts with water

(B) The reagent becomes inactive in water

(C) It is insoluble in water

(D) The reagent is highly reactive in water

Correct Answer: A

Question 76 : A lone pair of electrons in an atom implies:

(A) A pair of valence electrons not involved in bonding

(B) A pair of electrons involved in bonding

(C) A pair of electrons

(D) A pair of valence electrons

Correct Answer: A

Question 77 : In the electrolytic refining of zinc

(A) graphite is at the anode

(B) the impure metal is at the cathode

(C) the metal ion get reduced at the anode

(D) acidified zinc sulphate is the electrolyte

Correct Answer: D

Question 78 :

In which of the following equilibrium systems is the rate of the backward reaction favoured by increase of pressure?

(A)

PCl_5 PCl_3 $+Cl_2$

(B)

$2SO_2 + O_2 \rightleftharpoons 2SO_3$

(C) $N_2 + 3H_2 \rightleftharpoons 2NH_3$

(D) $N_2 + O_2 \rightarrow 2NO$

Correct Answer: A

Question 79 : Halo alkane in the presence of alcoholic KOH undergoes:

(A) Elimination

(B) Polymerisation

(C) Dimerization

(D) Substitution

Correct Answer: A

Question 80 : Enthalpy of vaporization of benzene is +35.3kJ mol-1 at its boiling point, 80^oC. The entropy change in the transition of the vapour to liquid at its boiling point in$[JK^{-1}mol^{-1}]$ is

(A) -441

(B) -100

(C) +441

(D) +100

Correct Answer: B

Question 81 : Reaction of aniline with benzaldehyde is:

(A) Polymerisation

(B) Condensation

(C) Addition

(D) Substitution

Correct Answer: B

Question 82 : In chromite ore, the oxidation number of iron, and chromium are respectively

(A) +3,+2

(B) +3,+6

(C) +2,+6

(D) +2,+3

Correct Answer: D

Question 83 :

In order to decompose 9g water 142.5 kJ heat is required. Hence the enthalpy of formation of water is:

(A) – 142.5 Kj

(B) + 142.5 kJ

(C) – 285 Kj

(D) + 285 kJ

Correct Answer: C

Question 84 : The amount of heat evolved when 500 cm^3 of 0.1 M HCl is mixed with 200 with 200 cm^3 of 0.2 M NaOH is

(A) 2.292 kJ

(B) 1.292 kJ

(C) 0.292 kJ

(D) 3.392 kJ

Correct Answer: A

Question 85 :

A quantity of PCl_5 was heated in a 10 dm^3 vessel at 250 C PCl_5 (g) PCl_3 (g) + Cl_2 (g) At equilibrium the vessel contains 0.1 mole of PCl_5 and 0.2 mole of Cl_2. The equilibrium constant of the reaction is:

(A) 0.05

(B) 0.02

(C) 0.025

(D) 0.04

Correct Answer: D

Question 86 : An ester used as medicine is:

(A) Ethyl benzoate

(B) Methyl salicylate

(C)

Methyl acetate

(D) Ethyl acetate

Correct Answer: B

Question 87 : Peroxide ion……
(i) has five completely filled antibonding molecular orbitals
(ii) is diamagnetic
(iii) has bond order one
(iv) is isoelectronic with neon
Which one of these is correct?

(A) (ii) and (iii)

(B) (i), (ii) and (iv)

(C) (i), (ii) and (iii)

(D) (i) and (iv)

Correct Answer: A

Question 88 : Which one of these is not true for benzene?

(A) It forms only one type of monosubstituted product

(B) There are three carbon-carbon single bonds and three carbon-carbon double bonds

(C) The heat of hydrogenation of benzene is less than the theoretical value

(D) The bond angle between the carbon-carbon bonds is 120^{o}

Correct Answer: B

Question 89 : A mixture of $CaCl_2$ and NaCl weighing 4.44 g is treated with sodium carbonate solution to precipitate all the Ca^{2+} ions as calcium carbonate. The calcium carbonate so obtained is heated strongly to get 0.56 g of CaO. The percentage of NaCl in the mixture (atomic mass of Ca=40)is

(A) 75

(B) 30.6

(C) 25

(D) 69.4

Correct Answer: A

Question 90 : For one mole of an ideal gas, increasing the temperature from 10^oC to 20^oC

(A) increases the average kinetic energy by two times

(B) increases the rms velocity by times

(C) increases the rms velocity by two times

(D) increases both the average kinetic energy and rms velocity, but not significantly

Correct Answer: D

Question 91 : Generally, the first ionization energy increases along a period. But there are some exceptions. One which is not an exception is

(A) N and O

(B) Na and Mg

(C) Mg and Al

(D) Be and B

Correct Answer: B

Question 92 : 50 cm^3 of 0.2 N HCI is titrated against 0.1 N NaOH solution. The titration is

discontinued after adding 50 cm^3 of NaOH. The remaining titration is completed by adding 0.5 N KOH. The volume of KOH required for completing the titration is

(A) 12 cm^3

(B) 10 cm^3

(C) 25 cm^3

(D) 10.5 cm^3

Correct Answer: B

Question 93 : In which one of the following, does the given amount of chlorine exert the least pressure in a vessel of capacity 1 dm^3 at 273 K?

(A) 0.0355g

(B) 0.071 g

(C) $6.023x10^{21}$ molecules

(D) 0.02 mol

Correct Answer: A

Question 94 : Based on the first law of thermodynamics, which one of the following is correct?

(A) For an isochoric process = $\Delta E=-q$

(B) For an adiabatic process =$\Delta E=-w$

(C) For an isothermal process $+q=+w$

(D) For a cyclic process $q = -w$

Correct Answer: D

Question 95 : For alkali metals, which one of the following trends is incorrect?

(A) Hydration energy : Li > Na > K > Rb

(B) Ionization energy : Li >Na>K>Rb

(C) Density : Li < Na < K < Rb

(D) Atomic size : Li < Na < K < Rb

Correct Answer: C

Question 96 : 1 g of silver gets distributed between 10 cm^3 of molten zinc and 100 cm^3 of molten lead at 800°C. The percentage of silver in the zinc layer is approximately

(A) 89

(B) 91

(C) 97

(D) 94

Correct Answer: C

Question 97 : One mole of an organic compound A with the formula C_3H_8O reacts completely with two moles of HI to form X and Y. when Y is boiled with aqueous alkali it form Z. Z answers the iodoform test. The compound A is

(A) propan-2ol

(B) propan-l-ol

(C) ethoxyethane

(D) methoxyethane

Correct Answer: D

Question 98 : The IUPAC name of $K_2[Ni(CN)_4]$ is

(A) potassium tetracyanonickelate (II)

(B) potassium tetracyanatonickelate (III)

(C) potassium tetracyanatonickel(II)

(D) potassium tetracyanonickel (III)

Correct Answer: A

Question 99 : The spin only magnetic moment of Mn^{4+} ion is nearly

(A) 3 BM

(B) 6 BM

(C) 4 BM

(D) 5 BM

Correct Answer: C

Question 100 : In Kjeldahl's method, ammonia from 5 g of food neutralizes 30 cm^3 of 0.1 N acid. The percentage of nitrogen in the food is

(A) 0.84

(B) 8.4

(C) 16.8

(D) 1.68

Correct Answer: A

Question 101 : Carbon can reduce ferric oxide to iron at a temperature above 983 K because

(A) carbon monoxide formed is thermodynamically less stable than ferric oxide

(B) carbon has a higher affinity towards oxidation than iron

(C) free energy change for the formation of carbon dioxide is less negative than that for ferric oxide

(D) iron has a higher affinity towards oxygen than carbon

Correct Answer: B

Question 102 : An Oxygen containing organic compound upon oxidation forms a carboxylic acid as the only organic product with its molecular mass higher by 14 units. The organic compound is

(A) an aldehyde

(B) a pri8mary alcohol

(C) a secondary alcohol

(D) a ketone

Correct Answer: B

Question 103 : The compound obtained when acetaldehyde reacts with dilute aqueous sodium hydroxide exhibits

(A) geometrical isomerism

(B) optical isomerism

(C) neither optical nor geometrical isomerism

(D) both optical and geometrical isomerism

Correct Answer: B

Question 104 : The activation energy for a reaction at the temperature T K was found to be 2.303 RT J mol^{-1}. The ratio of the rate constant to Arrhenius factor is

(A) 10^{-1}

(B) 10^{-2}

(C) $2x10^{-3}$

(D) $2x10^{-2}$

Correct Answer: A

Question 105 : A dibromo derivative of an alkane reacts with sodium metal to form an alicyclic hydrocarbon. The derivative is

(A) 1, 1-dibromopropane

(B) 2,2-dibromobutane

(C) 1,2-dibromoethane

(D) 1,4-dibromobutane

Correct Answer: D

Question 106 : An organic acid without a carboxylic acid group is:

(A) Picric acid

(B) Oxalic acid

(C)

Vinegar

(D) Ascorbic acid

Correct Answer: A

Question 107 : 0.023 g of sodium metal is reacted with 100 cm^3 of water. The pH of the resulting solution is

(A) 10

(B) 11

(C) 9

(D) 12

Correct Answer: D

Question 108 : Which one of the following is wrongly matched ?

(A) $[Cu(NH_3)_4]^{2+}$ - Square planar

(B) $[Ni(CO)_4]$ - Neutral ligand

(C) $[Fe(CN_6)^{-3}]$ - sp^3d^2

(D) $[Co(en)_3]^{3+}$ - Follows EAN rule

Correct Answer: C

Question 109 : Which one of the following conformations of cyclohexane is the least stable?

(A) Half-chair

(B) Boat

(C) Twisted-boat

(D) Diamond

Correct Answer: A

Question 110 : Which one of the following is a molecular crystal?

(A) Rock salt

(B) Boat

(C) Dry ice

(D) Diamond

Correct Answer: C

Question 111 : A buffer solution contains 0.1 mole of sodium acetate in 1000 cm^3 of 0.1 M acetic acid. To the above buffer solution, 0.1 mole of sodium acetate is further added and dissolved. The pH of the resulting buffer is equal to

(A) pK_a – log 2

(B) pK_a

(C) pK_a + 2

(D) pK_a + log 2

Correct Answer: D

Question 112 : Chloracetic acid is a stronger acid than acetic acid. This can be explained using

(A) –M effect

(B) –I effect

(C) + M effect

(D) + I effect

Correct Answer: B

Question 113 : The correct sequence of reactions to convert p-nitrophenol into quinol involves

(A) reduction, diazotization and hydrolysis

(B) hydrolysis, diazotization and reduction

(C) hydrolysis, reduction and diazotization

(D) diazotization, reduction and hydrolysis

Correct Answer: A

Question 114 : Glacial acetic acid is obtained by:

(A)

Chemically separating acetic acid

(B) Treating vinegar with dehydrating agents

(C) Crystallising, separating and melting acetic acid

(D) Distilling vinegar

Correct Answer: D

Question 115 : The letter 'D' in d-glucose signifies

(A) configuration at all chiral carbons

(B) dextrorotatory

(C) that it is a monosaccharide

(D) Configuration at a particular chiral carbon

Correct Answer: D

Question 116 : Reaction of methyl bromide with aqueous sodium hydroxide involves

(A) racemisation

(B) S_N1 mechanism

(C) retention of configuration

(D) S_N2 mechanism

Correct Answer: D

Question 117 : 9.65 C of electric current is passed through fused anhydrous magnesium chloride. The magnesium metal thus, obtained is completely converted into a Grignard reagent. The number of moles of the Grignard reagent obtained is

(A) $5x10^{-4}$

(B) $1x10^{-4}$

(C) $5x10^{-5}$

(D) $1x10^{-5}$

Correct Answer: C

Question 118 : Which one of the following does not involve coagulation?

(A) Formation of delta regions

(B) Peptization

(C) Treatment of drinking water by potash alum

(D) Clotting of blood by the use of ferric chloride

Correct Answer: B

Question 119 : In alkaline medium, alanine exists predominantly as

(A) anion

(B) Zwitter ion

(C) cation

(D) covalent form

Correct Answer: A

Question 120 : The standard emf of galvanic cell involving 3 moles of electrons in its redox reaction

is 0.59 V. The equilibrium constant for the reaction of the cell is

(A) 10^{25}

(B) 10^{20}

(C) 10^{15}

(D) 10^{30}

Correct Answer: D

SUBJECT: Mathematics

Question 121 :

If A $\subset$ B and A $\neq$ B, then

(A) A is called a proper subset of B

(B) A is called a super set of B

(C) A is not a subset of B

(D) B is a subset of A

Solution : If A $\subset$ B and A $\neq$ B, then A is called a proper subset of B and B is called a super set of A.

Correct Answer: A

Question 122 :

The number of non-empty subsets of the set{1, 2, 3, 4} is

(A) 15

(B) 14

(C) 16

(D) 17

Solution : Number of non-empty subsets = $2^4 - 1$.

Correct Answer: A

Question 123 :

If A = {1, 2, 3}, B = {3, 8}, then (A $\cup$ B) × (A $\cap$ B) is equal to

(A) {(8, 3), (8, 2), (8, 1), (8, 8)}

(B) {(1, 2), (2, 2), (3, 3), (8, 8)}

(C) {(3, 1), (3, 2), (3, 3), (3, 8)}

(D) {(1, 3), (2, 3), (3, 3), (8, 3)}

Solution :

$A \cup B = \{1, 2, 3\} \cup \{3, 8\} = \{1, 2, 3, 8\}$ and
$A \cap B = \{1, 2, 3\} \cap \{3, 8\} = \{3\}$
$\therefore (A \cup B) \times (A \cap B) = \{1, 2, 3, 8\} \times \{3\}$
$= \{(1, 3), (2, 3), (3, 3), (8, 3)\}$

Correct Answer: D

Question 124 : The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Then, the remaining elements of A × A are

(A) (0, 0), (0, 1), (0, –1), (–1, 1), (–1, 0), (1, 0), (1, –1)

(B) (–1, 1), (–1, 0), (0, 0), (0, –1), (0, 1), (1, 0), (1, –1)

(C) (0, –1), (1, 0), (1, –1)

(D) None of the above

Solution :

If A × A has 9 elements, then
$n(A \times A) = 9$
$\Rightarrow n(A) \times n(A) = 9$
$\{n(A)\}^2 = 9$
$n(A) = 3$
$\therefore$ A contains 3 elements.
Since, $(-1, 0), (0, 1) \in A \times A$
$\therefore A = \{-1, 0, 1\}$
$A \times A = \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)\}$
Hence, the remaining elements of
$A \times A$ are $(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)$

Correct Answer: D

Question 125 : If square root of $-7 + 24i$ is $x + iy$, then x is equal to

(A) ± 1

(B) ± 2

(C) ± 3

(D) ± 4

Solution :

Given, $x + iy = \sqrt{-7 + 24i}$

$$\pm\left[\sqrt{\frac{1}{2}\left(\sqrt{(-7)^2 + (24)^2} + (-7)\right)}\right.$$

$$\left. + i\sqrt{\frac{1}{2}\left(\sqrt{(-7)^2 + (24)^2} - (-7)\right)}\right]$$

$$\therefore x = \pm\sqrt{\frac{1}{2}\left[\sqrt{(-7)^2 + (24)^2} - 7\right]} = \pm 3$$

Correct Answer: C

Question 126 : What is the modulus of $\frac{1}{1+3i}-\frac{1}{1-3i}$?

(A) $\frac{3}{5}$

(B) $\frac{9}{25}$

(C) $\frac{3}{25}$

(D) $\frac{5}{3}$

Correct Answer: A

Question 127 : If A is the arithmetic mean and G1, G2 be two geometric means between any two numbers, then $\frac{G_1^2}{G_2}+\frac{G_2^2}{G_1}$ is equal to

(A) 2A

(B)A

(C) 3A

(D) None of these

Solution :

Let the two numbers are a and b.

Then, $A = \frac{a+b}{2}$ and $G_1 = ar$, $G_2 = ar^2$,

where, $r = \left(\frac{b}{a}\right)^{\frac{1}{2+1}} = \left(\frac{b}{a}\right)^{1/3}$ .....(i)

Now, $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{a^2r^2}{ar^2} + \frac{a^2r^4}{ar}$

$= a + ar^3 = a + a\left(\frac{b}{a}\right)$ [From eq. (i)]

$= a + b = 2A \left[Q\ A = \frac{a+b}{2}\right]$

Correct Answer: A

The sum of the series $\frac{1}{3\times7} + \frac{1}{7\times11} + \frac{1}{11\times15} + ...$

Question 128 : is

(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) $\frac{1}{9}$

(D) $\frac{1}{12}$

Solution :

$$\text{Let } S = \frac{1}{3\times7}+\frac{1}{7\times11}+\frac{1}{11\times15}+\ldots\infty$$

$$= \frac{1}{4}\left[\left\{\frac{1}{3}-\frac{1}{7}\right\}+\left\{\frac{1}{7}-\frac{1}{11}\right\}+\ldots\right] = \frac{1}{4}\left(\frac{1}{3}+0\right) = \frac{1}{12}$$

Correct Answer: D

Question 129 : The sum of the divisors of 630 is

(A)809

(B) 1439

(C) 1872

(D) 1579

Solution :

$630 \times 2^1 \times 3^2 \times 5^1 \times 7^1$

Sum of all the divisors

$$\frac{2^2-1}{2-1}\times\frac{3^3-1}{3-}\times\frac{5^2-1}{5-1}\times\frac{7^2-1}{7-1} = 1872$$

Correct Answer: C

Question 130 : The number of zeroes at the end, if 100! is fully expanded and written out is

(A) 23

(B) 26

(C) 25

(D) 24

Solution : Highest power of 2 in 100! is 97. Similarly, power of 5 in 100! is 24. Each pair of 5 and 2 will give rise to 10 or a zero at the end.

Hence, the number of zero in 100! is equal to 24. (least in 24 and 97).

Correct Answer: D

Question 131 : There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated, is

(A) 210

(B) 10!

(C) 1023

(D) 102

Solution : The hall can be illuminated by switching on atleast one of the 10 bulbs. Therefore, the required number of ways is

$= ({}^{10}C_1 + {}^{10}C_2 + \ldots\ldots + {}^{10}C_{10})$

$= ({}^{10}C_0 + {}^{10}C_1 + \ldots\ldots + {}^{10}C_{10}) - {}^{10}C_0$

$= 2^{10} - 1 = 1023$

Correct Answer: C

Question 132 : Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.

(A) 111

(B) 112

(C) 113

(D) None of these

Any number divisible by 5, if either 0 or 5 in unit place

6			0or5
Th	H	T	U

In unit place, the number of ways = $^2C_1 = 2$
In thousand place, number 6 is fixed. In ten and hundred place the number of ways of selection
$= 8 \times 7.$

Solution : $\therefore$ Required number of ways $= 2 \times 8 \times 7 = 112$

Correct Answer: B

Question 133 : If n is even,then the middle term in the expansion

(A) 10

(B) 12

(C)14

(D) None of these

Since, n is even, therefore $\left(\frac{n}{2}+1\right)^{th}$ term is the middle term.

$\therefore \quad T_{\frac{n}{2}+1} = {}^nC_{n'2}(x^2)^{n'2}\left(\frac{1}{x}\right)^{n'2}$

$= 924x^6$ [Given]

Solution : $\Rightarrow x^{n/2} = x^6 \Rightarrow n = 12$

Correct Answer: B

Question 134 : In the expansion of , then coefficient of x0 is

(A) 20

(B) –20

(C)30

(D) –30

Solution :

Let (r + 1)th term be the coefficient of x0 in the expansion of $\left(x-\frac{1}{x}\right)^6$.

$\therefore T_{r+1} = {}^6C_r x^{6-r}\left(-\frac{1}{x}\right)^r$

$= (-1)^r\ {}^6C_r\ x^{6-2r}$

Since, this term is a constant term.

$\therefore 6 - 2r = 0 \Rightarrow r = 3$

Now, $T_4 = (-1)^3\ {}^6C_3 = -20$

Correct Answer: B

Question 137 : The set of real x satisfying the inequality $\frac{5-2x}{3} \le \frac{x}{6} - 5$ is

(A) $(-\infty, 8)$

(B) $(8, \infty)$

(C) $[8, \infty)$

(D) $(-\infty, 8]$

Solution :

We have, $\frac{5-2x}{3} \le \frac{x}{6} - 5$

or $2(5-2x) \le x - 30$

or $10 - 4x \le x - 30$

or $-5x \le -40$, i.e. $x > 8$

Thus, all real numbers x which are greater than or equal to 8 are the solutions of the given inequality, i.e. $x \in [8, \infty)$.

Correct Answer: C

Question 138 : Which of the following is the solution set of the inequality $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$?

(A) $(4, \infty)$

(B) $(-\infty, 4)$

(C) $[4, \infty)$

(D) $(-\infty, 4)$

Solution :

We have, $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

$\frac{x}{4} < \frac{5(5x-2) - 3(7x-3)}{15}$

$\Rightarrow 15x < 4[(25x - 10) - (21x - 9)]$

$\Rightarrow 15x < 4[(25x - 10 - 21x + 9]$

$\Rightarrow 15x < 4[4x - 1]$

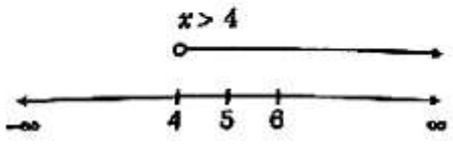
$\Rightarrow 15x < 16x - 4$

Transferring the term 16x to LHS.

$15x - 16x < -4$

$\Rightarrow -x < -4$

On multiplying by –1 both sides, we get $x > 4$



Correct Answer: C

Question 139 : If a square matrix A is such that $AA^T = I = A^TA$, then |A| is equal to

(A) ZERO

(B) ±1

(C) ±2

(D)

None of these

Solution :

Given matrix A is a square matrix and

$AA^T = I = A^TA$

$\Rightarrow |AA^T| = |I| = |A^TA|$

$\Rightarrow |A|\,|A^T| = 1 = |A^T|A|$

$\Rightarrow |A|^2 = 1$ $\quad [Q\ A.AT = |A|^2]$

$\Rightarrow |A| = \pm 1$

Correct Answer: B

Question 140 : If $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ is the sum of a symmetric matrix B and skew-symmetric matrix C, then B is

(A) $\begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 5 & -2 \\ 2 & 2 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 6 & 6 & 7 \\ -6 & 2 & -5 \\ -7 & 5 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 6 & -2 \\ 2 & 0 & -2 \\ -2 & -2 & 0 \end{bmatrix}$

Solution :

$$\text{Given, } A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} \text{ and symmetric matrix,}$$

$$B = \frac{A + A'}{2}$$

$$\therefore B = \frac{1}{2}\left\{\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}\right\} = \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$$

Correct Answer: A

Question 141 :

$$\text{Determinate } \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} \text{ is}$$

(A) independent of θ

(B) dependent of θ

(C) dependent of θ and x

(D) None of these above

Solution :

$$\text{Let } A = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

Expanding to corresponding first row, we get

$$A = x \begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin\theta \begin{vmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{vmatrix} + \cos\theta \begin{vmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{vmatrix}$$

$$= x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$$

$$= -x^3 - x + x(\sin^2\theta + \cos^2\theta)$$

$$= -x^3 - x + x \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= -x^3$$

Hence, A is independent of θ.

Correct Answer: A

Question 142 : $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ is equal to

(A) a + b + c

(B) abc

(C) ZERO

(D) None of these

$$\text{Let } A = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & bc & ab+ac+bc \\ 1 & ca & bc+ba+ca \\ 1 & ab & ca+cb+ab \end{vmatrix}$$

[using $C_3 \to C_3 + C_2$]

Taking ab + bc + ca common form C_3, we get

$$= (ab + bc + ca)\begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$$

= (ab + bc + ca) × 0

Solution : (since the two columns C_1 and C_3 are identical)

Correct Answer: C

If $\tan\theta = \frac{1}{\sqrt{7}}$, then

Question 143 : $\frac{(\text{cosec}^2\theta - \sec^2\theta)}{(\text{cosec}^2\theta + \sec^2\theta)}$ is equal to

(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) $\frac{5}{4}$

(D) 2

Given, $\tan\theta = \frac{1}{\sqrt{7}}$

$\cot\theta = \sqrt{7}$

$\therefore \frac{(\text{cosec}^2\theta - \sec^2\theta)}{(\text{cosec}^2\theta + \sec^2\theta)}$

$= \frac{(1+\cot^2\theta - 1 - \tan^2\theta)}{(1+\cot^2\theta + 1 + \tan^2\theta)}$

$= \frac{\cot^2\theta - \tan^2\theta}{2 + \cot^2\theta + \tan^2\theta}$

$= \frac{(\sqrt{7})^2 - (1/\sqrt{7})^2}{2 + (\sqrt{7})^2 + (1/\sqrt{7})^2}$

Solution : $= \frac{49-1}{7} \times \frac{7}{63+1}$

$= \frac{48}{64} = \frac{3}{4}$

Correct Answer: B

Question 144 : If $\sin A + \cos A = \sqrt{2}$, then the value of $\cos^2 A$ is

(A) $\sqrt{2}$

(B) 1/2

(C) 4

(D) –1

Solution :

Given, $\sin A + \cos A = \sqrt{2}$

$\therefore \frac{1}{\sqrt{2}} \sin A + \frac{1}{\sqrt{2}} \cos A = 1$

$\Rightarrow \sin \frac{\pi}{4} \sin A + \cos A \cos \frac{\pi}{4} = 1$

$\Rightarrow \cos\left(A - \frac{\pi}{4}\right) = 1$

$\Rightarrow A - \frac{\pi}{4} = 0 \Rightarrow A = \frac{\pi}{4}$

Now, $\cos^2 A = \cos^2 \frac{\pi}{4}$

$= \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

Correct Answer: B

Question 145 : In which symmetrical line, there is a relation between Trigonometric and inverse trigonometric function

(A) y = x

(B) y = – x

(C) $y = x^2$

(D) None of these

Solution : Line y = x, inverse trigonometric function is symmetrical to trigonometric function

Correct Answer: A

Question 146 : The value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ is

(A) $-\frac{\pi}{4}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) None of these

Solution :

$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$= \tan^{-1}\left[2\cos\left\{2\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right\}\right] \quad \left[\text{Q } \sin\frac{\pi}{6}=\frac{1}{2}\right]$

$= \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right] \quad \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$

$= \tan^{-1}\left[2\times\frac{1}{2}\right] = \tan^{-1}(1) \quad \left[\text{Q } \cos\frac{\pi}{3}=\frac{1}{2}\right]$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)=\frac{\pi}{4} \quad \left[\text{Q } \tan\frac{\pi}{4}=1\right]$

Correct Answer: B

Question 147 : The values of a and b. Such that the function defined by $f(x)=\begin{cases}5, & \text{if } x\le 2\\ ax+b, & \text{if } 2<x<10\\ 21, & \text{if } x\ge 10\end{cases}$ is a continuous function is

(A) a = 1 and b = 2

(B) a = 2 and b = 1

(C) $a, b \in R$

(D) None of the above

Solution :

Here, $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$

At x = 2,

$LHL = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (5) = 5$

$RHL = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (ax + b)$

Putting $x = 2 + h$ as $x \to 2^+$ when $h \to 0$

$\therefore \lim_{h \to 0} [a(2 + h) + b] = \lim_{h \to 0} (2a + ah + b)$

$= 2a + b$

Also, f(2) = 5

Since f(x) is continuous at x = 2

$\therefore$ LHL = RHL = f(2)

$\Rightarrow 2a + b = 5$ .....(i)

At x = 10, LHL

$= \lim_{x \to 10^-} f(x) = \lim_{x \to 10^-} (ax + b)$

Putting $x = 10 - h$ as $x \to 10^-$ when $h \to 0$

$\therefore \lim_{h \to 0} [a(10 - h) + b] = \lim_{h \to 0} (10a - ah + b)$

$= 10a + b$

$RHL = \lim_{x \to 10^+} f(x) = \lim_{x \to 10^+} (21) = 21$

Also, f(10) = 21

Since, f(x) is continuous at x = 10

LHL = RHL = f(10)

$\Rightarrow 10a + b = 21$ .....(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$8a = 16 \Rightarrow a = 2$

Put a = 2 in Eq. (i), we get

$2 \times 2 + b = 5 \Rightarrow b = 1$

Correct Answer: B

Question 148 : $f(x) = x + |x|$ is continuous for

(A) $x \in (-\infty, \infty)$

(B) $x \in (-\infty, \infty) - \{0\}$

(C) only $x > 0$

(D) no value of x

Solution : Since x and |x| is continuous for every value of x, so their sum is also continuous for every value of x.

Correct Answer: A

Question 149 : If $x^2 + 2xy + y^3 = 42$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{-2(x+y)}{2x+3y^2}$

(B) $\frac{-(2x+3y^2)}{2(x+y)}$

(C) $\frac{-2(x+y)}{2y+3x^2}$

(D) $\frac{2(x^2+y^2)}{2x+3y^2}$

$Q\ x^2 + 2xy + y^2 = 42$

$\therefore \frac{d}{dx}(x^2 + 2xy + y^3) = \frac{d}{dx}(42)$

$\Rightarrow 2x + 2\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) + 3y^2\frac{dy}{dx} = 0$

$\Rightarrow 2x + 2x\frac{dy}{dx} + 2y + 3y^2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(2x + 3y^2) = -2(x + y) \Rightarrow \frac{dy}{dx} = \frac{-2(x+y)}{2x+3y^2}$

Solution :

Correct Answer: A

Question 150 : Find the derivative of $\tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$ with respect to x.

(A) ZERO

(B) 1

(C) –1

(D) $\cos^2 x$

Let $y = \tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right) = \tan^{-1}\left(\frac{1 + \tan x}{1 - \tan x}\right)$

$= \tan^{-1}\left\{\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}\right\} = \tan^{-1}\left\{\tan\left(\frac{\pi}{4} + x\right)\right\}$

Solution : $\Rightarrow y = \frac{\pi}{4} + x \Rightarrow \frac{dy}{dx} = 0 + 1 = 1$

Correct Answer: B

Question 151 : The equation of normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is

(A) $y + 2at^2 = xt - at^2$

(B) $y - 2at = xt - at^2$

(C) $y - 2at = -xt + at^3$

(D) None of the above

Solution :

The equation of given curve is

$y^2 = 4ax$ .....(i)

On differentiating eq.(i) w.r.t x, we get

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(at^2, 2at)} = \frac{4a}{4at} = \frac{1}{t} \quad .....(ii)$$

So, the equation of normal is

$$(y - 2at) = -\frac{1}{1/t}(x - at)^2$$

$\Rightarrow y - 2at = -t(x - at^2)$

$\Rightarrow y - 2at = -xt + at^3$

Correct Answer: C

Question 152 : Length of normal to curve $y^2 = 4ax$ at $(at^2, 2at)$ is

(A) $2a\sqrt{t+1}$

(B) 2a

(C) $2a\sqrt{t^2+1}$

(D) None of these

Solution :

We have, $y^2 = 4ax$ .....(i)

On differentiating eq. (i) w.r.t. 'x', we get

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{4a}{2y}$$

$$\therefore \ 2y\left(\frac{dy}{dx}\right)_{(at^2,2at)} = \left(\frac{4a}{4at}\right)^2 = \left(\frac{1}{t}\right)^2$$

Now, length of normal at $(at^2, 2at)$ is

$$= \left|y\sqrt{1+\left(\frac{dy}{dx}\right)^2_{(x,y)}}\right|$$

$$= \left|2at\sqrt{1+\frac{1}{t^2}}\right| = 2a\sqrt{t^2+1}$$

Correct Answer: C

Question 153 : $\int \frac{x\,dx}{x^2+4x+5}$ is equal to

(A) $\frac{1}{2}\log(x^2+4x+5)+2\tan^{-1}(x)+C$

(B) $\frac{1}{2}\log(x^2+4x+5)-2\tan^{-1}(x+2)+C$

(C) $\frac{1}{2}\log(x^2+4x+5)+\tan^{-1}(x+2)+C$

(D) $\frac{1}{2}\log(x^2+4x+5)-2\tan^{-1}(x+2)+C$

$$\int \frac{x\,dx}{x^2+4x+5} = \int \frac{x}{x^2+4x+4+1}\,dx$$

$$= \int \frac{x+2-2}{(x+2)^2+1}\,dx$$

$$= \frac{1}{2}\int \frac{2(x+2)dx}{(x+2)^2+1} - 2\int \frac{dx}{1+(x+2)^2}$$

$$= \frac{1}{2}\log[(x+2)^2+1] - 2\tan^{-1}(x+2) + C$$

Solution : $$= \frac{1}{2}\log(x^2+4x+5) - 2\tan^{-1}(x+2) + C$$

Correct Answer: D

Question 154 : $\int \frac{dx}{\sqrt[4]{(x+1)^5(x+2)^3}}$ is equal to

(A) $4\left(\frac{x+1}{x+2}\right)^{1/4} + C$

(B) $-4\left(\frac{x+1}{x+2}\right)^{-1/4} + C$

(C) $-4\left(\frac{x+2}{x+2}\right)^{1/4} + C$

(D) None of these

Solution :

$$\text{Let } I = \int \frac{dx}{\sqrt[4]{(x+1)^5(x+2)^3}}$$

$$\text{Put } \frac{x+1}{x+2} = t \Rightarrow \frac{1}{(x+2)^2}dx = dt$$

$$\therefore I = \int \frac{dt}{t^{5/4}} \Rightarrow I = \frac{t^{-1/4}}{\left(-\frac{1}{4}\right)} + C$$

$$\Rightarrow I = -4\left(\frac{x+2}{x+1}\right)^{1/4} + C$$

Correct Answer: C

Question 155 : The solution of $\frac{dy}{dx} = \cos(x+y)$ is

(A) $\sin(x + y) = C$

(B) $\cos(x^2 + y^2) = 2C$

(C) $\tan\frac{(x+y)}{2} = x + C$

(D) None of these

Solution :

Given, $\frac{dy}{dx} = \cos(x + y)$

Let $x + y = t$

$\Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\therefore \frac{dt}{dx} - 1 = \cos t \Rightarrow \frac{dt}{dx} = 1 + \cos t$

Separating the variables, we get

$\Rightarrow \frac{1}{2}\sec^2\frac{t}{2}dt = dx$

On integrating both sides, we get

$\tan\frac{t}{2} = x + C \Rightarrow \tan\frac{(x+y)}{2} = x + C$

Correct Answer: C

Question 156 : The solution of homogeneous equation
$y^2dx + (x^2 - xy)dy = 0$

(A) $Ky = e^{y/x}$

(B) $Ky = e^{x/y}$

(C) $Ky = e^{-y/x}$

(D) $Ky2 = e^{x/y}$

Solution : Given, $y^2dx + (x^2 - xy)dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{xy - x^2}$$

On putting, $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$\int \frac{v-1}{v}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \int\left(1 - \frac{1}{v}\right)dv = \int \frac{dx}{x}$$

$$\Rightarrow v - \log v = \log x + \log K$$

$$\Rightarrow Ky = e^{y/x}$$

Correct Answer: A

Question 157 : If A = (0, 4), B =(0, –4) and | AP – BP| = 6, then locus of the point P is

(A) $7x^2 - 9y^2 + 63 = 0$

(B) $7x^2 - 9y^2 = 63$

(C) $9x^2 - 7y^2 + 63 = 0$

(D) $9x^2 - 7y^2 = 63$

Solution :

Let the point be P(x,y).

Given, points are A = (0, 4), B = (0, –4)

We have

$|AP - BP| = 6$

$\Rightarrow (AP - BP) = \pm 6$

$\Rightarrow (\sqrt{(x-0)^2 + (y-4^2)^2}$

$= (\pm 6 + \sqrt{(x-0)^2 + (y+4^2)^2}$

$\Rightarrow x^2 + y^2 + 16 - 8y$

$= 36 + x^2 + y^2 + 16 + 8y \pm 12\sqrt{x^2 + y^2 + 8y + 16}$

$\Rightarrow -16y - 36 = \pm 12\sqrt{x^2 + y^2 + 8y + 16}$

$\Rightarrow -4y - 9 = \pm 3\sqrt{x^2 + y^2 + 8y + 16}$

$\Rightarrow 16y^2 + 81 + 72y = 9(x^2 + y^2 + 8y + 16)$

$\Rightarrow 9x^2 - 7y^2 + 63 = 0$

Correct Answer: C

Question 158 : A line passes through the point of intersection of the lines 3x + y + 1 = 0 and 2x – y + 3 = 0 and makes equal intercepts with axes. Then, equation of the line is

(A) 5x + 5y – 3 = 0

(B) x + 5y – 3 = 0

(C) 5x – y – 3 = 0

(D) 5x + 5y + 3 = 0

Solution : The point of intersection of the lines

$3x + y - 1 = 0$

and $2x - y + 3 = 0$ is $\left(-\frac{4}{5}, \frac{7}{5}\right)$.

The equation of line, which make equal intercepts with axes, is $x + y = a$

$\therefore -\frac{4}{5} + \frac{7}{5} = a \Rightarrow a = \frac{3}{5}$

Now, equation of line is

$x + y - \frac{3}{5} = 0$

$\Rightarrow 5x + 5y - 3 = 0$

Correct Answer: A

Question 159 : The equation of the circle which passes through the origin and cuts off intercepts 3 and 4 from the positive parts of the axes, respectively is

(A) $4x^2 + 12xy + 4y^2 - 16y = 0$

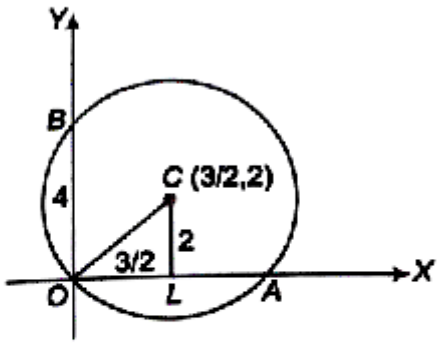
(B) $x^2 + y^2 - 3x - 4y = 0$

(C) $x^2 - 12x + 4y^2 + 16y = 0$

(D) None of the above

We have, OA = 3, OB = 4

$\therefore$ OL = $\frac{3}{2}$ and CL = 2



Thus, centre of circle is $\left(\frac{3}{2}, 2\right)$.

The required equation of circle is

$$\left(x-\frac{3}{2}\right)^2+(y-2)^2=\left(\frac{3}{2}\right)^2+(2)^2$$

Solution : $\Rightarrow x^2 + y^2 - 3x - 4y = 0$

Correct Answer: B

Question 160 : The point (–2.5, 3.5) lie with respect to the circle $x^2 + y^2 = 25$

(A) inside

(B) outside

(C) on the circle

(D) None of these

Given equation of the circle is $x^2 + y^2 = 25$
Let $S = x^2 + y^2 - 25$ .....(i)
Put (x, y) = (–2.5, 3.5) in eq. (i), we get
$S = (-2.5)^2 + (3.5)^2 - 25$
$= 6.5 + 12.25 - 25 = 18.5 - 25 = -6.50 < 0$

Solution : $\Rightarrow$ Points (–2.5, 3.5) lies inside the circle.

Correct Answer: A

Question 161 : The position of the point (2, 3) with respect to parabola $y^2 = 3x$ is

(A) outside the parabola

(B) inside the parabola

(C) on the parabola

(D) None of these

Solution :

Given, $y^2 = 3x \Rightarrow y^2 - 3x = 0$ .....(i)

On putting the point (2, 3) in eq. (i), we get

$(3)^2 - 3(1) = 9 - 3 = 6 > 0$

$\therefore$ Point (2, 3) lies outside the parabola $y^2 = 3x$

Correct Answer: A

Question 162 : Side of an equilateral triangle inscribed in $y^2 = 4x$ such that one vertex of the triangle coincide with the vertex of parabola is

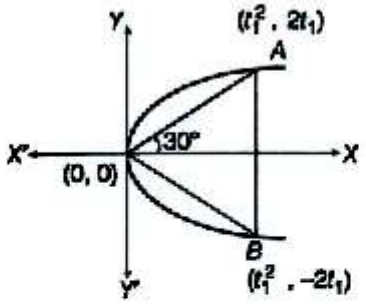
(A) 2

(B) $2\sqrt{3}$

(C) $8\sqrt{3}$

(D) $4\sqrt{3}$

Q $4a = 4 \Rightarrow a = 1$



Coordiantes of the vertices A and B $(t_1^2, 2t_1)$ and $(t_1^2, -2t_1)$ respectively.

$\therefore \tan 30° = \frac{2t_1}{t_1^2}$

Solution : $\Rightarrow \frac{1}{\sqrt{3}} = \frac{2}{t_1} \Rightarrow t_1 = 2\sqrt{3}$

And length of the chord = $4t_1 = 4(2\sqrt{3}) = 8\sqrt{3}$

Correct Answer: C

Question 163 : If **a** and **b** are unit vectors and θ is the angle between them, then $\sin\frac{\theta}{2}$ is equal to

(A) $\frac{|\mathbf{a}-\mathbf{b}|}{2}$

(B) $\frac{|\mathbf{a}+\mathbf{b}|}{2}$

(C) $\frac{|\mathbf{a}|-|\mathbf{b}|}{2}$

(D) $\frac{|\mathbf{a}|+|\mathbf{b}|}{2}$

Solution :

$|\mathbf{a}|^2 = 1, |\mathbf{a}|^2 = 1, \mathbf{a}.\mathbf{b} = \cos\theta$

$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2} = \frac{2-2\cos\theta}{4}$

$= \frac{|\mathbf{a}|^2 + |\mathbf{b}|^2 - 2(\mathbf{a}.\mathbf{b})}{4} = \left|\frac{\mathbf{a}-\mathbf{b}}{2}\right|^2$

$\therefore \sin\frac{\theta}{2} = \frac{|\mathbf{a}-\mathbf{b}|}{2}$

Correct Answer: A

Question 164 : The vectors $\hat{i} - 2x\hat{j} - 2y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other. Then, the locus of the point (x, y) is

(A) hyperbola

(B) ellipse

(C) parabola

(D) circle

Solution :

Let $\mathbf{a} = \hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\mathbf{b} = \hat{i} + 3x\hat{j} + 2y\hat{k}$

Since, $\mathbf{a} + \mathbf{b}$ are orthogonal to each other.

$\therefore \mathbf{a}.\mathbf{b} = 0$

$\Rightarrow (\hat{i} - 2x\hat{j} - 3y\hat{k}).(\hat{i} + 3x\hat{j} + 2y\hat{k}) = 0$

$\Rightarrow 1 - 6x^2 - 6y^2 = 0$

$\Rightarrow x^2 + y^2 = \frac{1}{6}$

So, the locus of the point (x, y) is a circle.

Correct Answer: D

Question 165 : Let $a = \hat{i}+\hat{j}+\hat{k}, b = \hat{i}-\hat{j}+\hat{k}$ and $c = \hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector **v** in the plane of a and b, whose projection on c is $\frac{1}{\sqrt{3}}$, is given

(A) $\hat{i}-3\hat{j}+3\hat{k}$

(B) $-3\hat{i}-3\hat{j}-\hat{k}$

(C) $3\hat{i}-\hat{j}+3\hat{k}$

(D) $\hat{i}+3\hat{j}-3\hat{k}$

Solution :

Let $\mathbf{v} = \mathbf{a} + \lambda\mathbf{b}$

$v = (1+\lambda)\hat{i}+(1-\lambda)\hat{j}+(1+\lambda)\hat{k}$

Projection of **v** on $\mathbf{c} = \frac{\mathbf{v}.\mathbf{c}}{|\mathbf{c}|} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{(1+\lambda)-(1-\lambda)-(1+\lambda)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \lambda = 2$

$\therefore \mathbf{v} = 3\hat{i}-\hat{j}+3\hat{k}$

Correct Answer: C

Question 166 : If a plane meets the coordinate axes at A, B and C such that the centroid of the triangle is (1, 2, 4), then the equation of the plane is

(A) x + 2y + 4z = 12

(B) 4x + 2y + z = 12

(C) x + 2y + 4z = 3

(D) 4x + 2y + z =3

Solution : Equation of a plane through (–2, –2, 2) is given by
$a(x + 2) + b(y + 2) + c(z - 2) = 0$
It contains the line joining the points (1, 1, 1) and B(1, –1, 2), so these points also lie in the plane.
$\therefore\ 3a + 3b - c = 0$ and $3a + b + 0.c = 0$

$$\Rightarrow \frac{a}{1} = \frac{b}{-3} = \frac{c}{-6} = r \quad \text{[say]}$$

$c = -6r$
So, equation of the plane is
$x - 3y - 6z + 8 = 0.$

Correct Answer: B

Question 167 : If a plane meets the coordinate axes at A, B and C such that the centroid of the triangle is (1, 2, 4), then the equation of the plane is

(A) $x + 2y + 4z = 12$

(B) $4x + 2y + z = 12$

(C) $x + 2y + 4z = 3$

(D) $4x + 2y + z = 3$

Solution : Let the equation of plane be

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$$

Then, $A(\alpha, 0, 0)$, $B(0, \beta, 0)$ and $C(0, 0, \gamma)$ are the points on the coordinate axes.
Since, the centroid of triangle is (1, 2, 4).

$$\therefore\ \frac{\alpha}{3} = 1, \frac{\beta}{3} = 2, \frac{\gamma}{2} = 4$$

$\Rightarrow \alpha = 3, \beta = 6, \gamma = 12$
Hence, the equation of the plane is

$$\frac{x}{3} + \frac{y}{6} + \frac{z}{12} = 1$$

$\Rightarrow 4x + 2y + z = 12$

Correct Answer: B

Question 168 : What is the equation to the plane through (1, 2, 3) parallel to 3x + 4y – 5z = 0?

(A) 3x + 4y + 5z + 4 = 0

(B) 3x + 4y – 5z + 14 = 0

(C) 3x + 4y – 5z + 4 = 0

(D) 3x + 4y – 5z – 4 = 0

Solution :

The equation of any plane parallel to the plane
3x + 4y – 5z = 0 may be taken as
3x + 4y – 5z + k = 0 .....(i)
If plane Eq. (i) passes through the point (1, 2, 3)
we get
3(1) + 4(2) – 5(3) + k = 0
k = 4 .....(ii)
On putting k = 4 in Eq. (i),
we get required equation
i.e. 3x + 4y – 5z + 4 = 0

Correct Answer: C

Question 169 : The average of the squares of the numbers 0, 1, 2, 3, 4,., n is

(A) $\frac{1}{2}n(n+1)$

(B) $\frac{1}{6}n(2n+1)$

(C) $\frac{1}{6}(n+1)(2n+1)$

(D) $\frac{1}{6}n(n+1)$

Solution :

$$\text{Mean} = \frac{0^2+1^2+2^2+3^2+.....+n^2}{(n+1)}$$

$$= \frac{n(n+1)(2n+1)}{6(n+1)} = \frac{1}{6}n(2n+1)$$

Correct Answer: B

Question 170 : The mean of the values of 1, 2, 3,..,n with respectively frequencies x, 2x, 3x,..., nx is

(A) $\frac{n}{2}$

(B) $\frac{1}{3}(2n+1)$

(C) $\frac{1}{6}(2n+1)$

(D) $\frac{n}{2}$

Solution :

$$\therefore \text{Mean} = \frac{1\times x+2\times 2x+3\times 3x+.....+n\times nx}{x+2x+3x+.....+nx}$$

$$= \frac{x(1^2+2^2+3^2+.....+n^2)}{x(1+2+3+.....+n)}$$

$$= \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{2n+1}{3}$$

Correct Answer: B

If $u_i = \frac{x_i - 20}{10}, \Sigma f_i u_i = 30$ and $\Sigma f_i = 40$ then find the

Question 171 : value of $\bar{x}$.

(A) 27.5

(B) 28

(C) 29

(D) 30

Solution :

On comparing the given u_i with

$u_i = \frac{x_i - a}{h}$, we get $a = 20, h = 10$

$\therefore \ \bar{x} = a + \left\{\frac{\Sigma f_i u_i}{\Sigma f_i}\right\} \times h$

$= 20 + \left[\frac{30}{40}\right] \times 10 = 27.5$

Correct Answer: A

Question 172 : Three machines E_1, E_2 and E_3 in a certain factory produce 50%, 25% and 25%, respectively, of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines E_1 and E_2 are defective and that 5% of those produced on E_3 are defective. If one tube is picked up at random from a day's production, the probability that it is defective, is

(A) 0.025

(B) 0.125

(C) 0.325

(D) 0.0425

Let D be the event that the picked up tube is defective.

Let A_1, A_2 and A_3 be the events that the tube is produced on machines E_1, E_2 and E_3 respectively.

$P(D) = P(A_1)\,P(D|A_1) + P(A_2)\,P(D|A_2) + P(A_3)\,P(D|A_3)$.....(i)

$P(A_1) = \frac{50}{100} = \frac{1}{2}$,

$P(A_2) = \frac{1}{4}$, $P(A_3) = \frac{1}{4}$

Also, $P(D|A_1) = P(D|A_2)$

$= \frac{4}{100} = \frac{1}{25}$

$P(D|A_3) = \frac{5}{100} = \frac{1}{20}$

Solution :

On putting these values in eq. (i), we get

$$P(D) = \frac{1}{2}\times\frac{1}{25} + \frac{1}{4}\times\frac{1}{25} + \frac{1}{4}\times\frac{1}{20}$$

$$= \frac{1}{50} + \frac{1}{100} + \frac{1}{80} = \frac{17}{400} = 0.0425$$

Correct Answer: D

Question 173 : A man is known to speak the truth a out of 4 times. He throws a die and reports that it is six. The probability that it is actually a six, is

(A) $\frac{3}{8}$

(B) $\frac{1}{5}$

(C) $\frac{8}{4}$

(D) None of these

Let E denotes the event that a six occurs and A the event that the man reprots that it is a 6

Then, $P(E) = \frac{1}{6}$, $P(E') = \frac{5}{6}$

$P\left(\frac{A}{E}\right) = \frac{3}{4}$ and $P\left(\frac{A}{E'}\right) = \frac{1}{4}$

From Baye's theorem,

$$P\left(\frac{A}{E}\right) = \frac{P(E).P\left(\frac{A}{E}\right)}{P(E).P\left(\frac{A}{E}\right) + P(E').P\left(\frac{A}{E'}\right)}$$

Solution : $= \frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}} = \frac{3}{8}$

Correct Answer: A

A discrete random variable X has the following probability distribution

X	1	2	3	4	5	6	7
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2 + C$

Question 174 : The value of C and the mean of the distribution are

(A) $\frac{1}{10}$ and 3.66

(B) $\frac{1}{20}$ and 2.66

(C) $\frac{1}{15}$ and 1.33

(D) None of these

Solution :

Since, $\Sigma p_i = 1$, we have

$C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$

$\Rightarrow C = \frac{1}{10}, C = -1$

Therefore, the permissible value of

$C = \frac{1}{10}$

$$\text{Mean} = \sum_{i=1}^{n} x_i p_i = \sum_{i=1}^{7} x_i p_i$$

$$= 1\times\frac{1}{10}+2\times\frac{2}{10}+3\times\frac{2}{10}+4\times\frac{3}{10}+5\left(\frac{1}{10}\right)^2 = 3.66$$

Correct Answer: A

Question 175 : Given $\bar{a}_1 = (3,1,-4), \bar{a}_2 = (2,2,-3)$ and $\bar{a}_3 = (0,-4,1)$

(A) $\bar{a}_1, \bar{a}_2$ and $\bar{a}_3$ are linearly dependent

(B) $\bar{a}_1, \bar{a}_2$ and $\bar{a}_3$ are linearly independent

(C) for some $c_1 = 2, c_2 = -3, c_3 = -1$, $\Sigma c_i \bar{a}_i = \bar{0}$

(D) for some $c_1 = c_2 = c_3 \neq 0$, $\Sigma c_i \bar{a}_i = \bar{0}$

$c_1\bar{a}_1 + c_2\bar{a}_2 + c_3\bar{a}_3 = c_1(3,1,-4)$

$+c_2(2,2,-3) + c_3(0,-4,1)$

$= (3c_1 + 2c_2, c_1 + 2c_2 - 4c_3,$

$-4c_1 - 3c_2 + c_3) = (0, 0, 0)$

$\Rightarrow 3c_1 + 2c_2 = 0$

$c_1 + 2c_2 - 4c_3 = 0$

$-4c_1 - 3c_2 + c_3 = 0$

$\Rightarrow c_1 = 2, c_2 = -3, c_3 = -1$

Since, c_1, c_2 and c_3 are not zero.

Solution : Hence, $\bar{a}_1$, $\bar{a}_2$ and $\bar{a}_3$ linearly dependent

Correct Answer: C

Question 176 : Given $\{\alpha + i\beta, a + ib\}$, $\alpha, \beta, a, b \in R$

$a\beta = b\alpha$, then this set from a basis for vector space

(A) C over R

(B) $\frac{a}{\beta} = \frac{b}{\alpha}$, then this set form a basis for vector

(C) Both (1) and (2) are true

(D) Neither (1) nor (2) is true

$(\alpha + i\beta)$ and $(a + ib)$ to the linearly independent.

$\Rightarrow \theta_1(a + ib) + \theta_2(\alpha + i\beta) = 0$

$\Rightarrow \theta_1 a + \theta_2 a = 0$

$\theta_1 b + \theta_1 \beta = 0 \Rightarrow \theta_1(a\beta - b\alpha) = 0$

$\theta_2(a\beta - b\alpha) = 0$

$\Rightarrow \theta_1 = 0 = \theta_2 \Leftrightarrow a\beta - b\alpha = 0$

Solution : Hence, if $a\beta = b\alpha$, then $\{a + i\beta, a + ib\}$ is a basis.

Correct Answer: A

Question 177 : Given a subspace of R^3

$V = \{a_0 + a_1x + a_2x^2 + a_3x_1^2 , x \in R\}$.

(A) S = {1, x, x^2, x^3} is a basis

(B) V has dimension 2

(C) V has dimension 3

(D) V has dimension 5

Solution : S = {1, x, x^2, x^3} is a basis of V, therefore, dimension four.

Correct Answer: A

Question 178 : The integral $\int_0^3 [x]dx$ where [x] is greatest integer function) is

(A) ZERO

(B) 1

(C) 2

(D) 3

Solution :

Since, the function f(x) = [x] is bounded and has only three points of discontinuity at x = 1, 2, 3.

Let $\varepsilon > 0$ be given number and

$P = \{0 = x, x_1, \ldots x_i, y_0, y_1, \ldots, y_m, z_0, z_1, \ldots, z_n = 3\}$ be a partition of [0, 3] where y0 = 1, z0 = 2.

Hence, $U(P, f) = \Sigma 0.\Delta x_i + 1.(y_0 - x_i) + \Sigma 1.\Delta y_i + 2(z_0 - y_m) + \Sigma 2.\Delta z_i + 3(z_n - z_{n-1})$

$= 1 + 2 + \{(y_0 - x_i) + (z_0 - y_m) + (z_n - z_{n-1})\}$

Now, select P such that

$(y_0 - x_i) + (z_0 - y_m) + (z_n - z_{n-1}) < e$

$\Rightarrow U(P, f) < 3 + \varepsilon$

Similarly, $L(P, f) = \Sigma 0.\Delta x_i + S1.\Delta y_i + (z_0 - y_m) + \Sigma 2.\Delta Z_i = 3$

Hence, $U(P, f) - L(P, f) < \varepsilon$

Hence, f(x) = [x] is integrable in [0, 3]

and $\int_0^3 [x]dx = \int_0^3 [x]dx = 3$

Correct Answer: D

Question 179 : Which of the following(s) is/are correct?

(A) Every continuous function is not Riemann integrable

(B) Every continuous function is not Riemann integrable

(C) If function is continuous in an interval it is bounded and uniformly continuous on that interval

(D) If function is continuous in an interval it is not neither bounded nor uniformly continuous on that interval

Solution :

Let f be a continuous function on [a, b], then evidently f is bounded and uniformly continuous on [a, b].

Hence, for a given positive number ε, there exists a partition $P = \{a = x_0, x_1, x_2, \ldots, x_n = b\}$ of [a, b] such that the oscillation $(M_r - m_r)$ of the function f in the sub-interval (x_{r-1}, x_r) is less than

$$\frac{\varepsilon}{b-a}, r = 1, 2, \ldots, n$$

Now, $U(P, f) - L(P, f)$

$$= \sum_{r=1}^{n} M_r (x_r - x_{r-1}) - \sum_{r=1}^{n} m_r (x_r - x_{r-1})$$

$$= \sum_{r=1}^{n} (M_r - m_r)(x_r - x_{r-1})$$

$$= < \sum_{r=1}^{n} \frac{\varepsilon}{b-a} (x_r - x_{r-1}) \quad \left[Q\ M_r - m_r = \frac{\varepsilon}{b-a} \right]$$

$\Rightarrow U(P, f) - L(P, f) < \frac{\varepsilon}{b-a} (x_1 - x_0) + (x_2 - x_1)$

$+ \ldots\ldots + (x_n - x_{n-1})$

$\Rightarrow U(P, f) - L(P, f) < \frac{\varepsilon}{b-a} (x_n - x_0)$

$\Rightarrow U(P, f) - L(P, f) < \frac{\varepsilon}{b-a} (b - a) [Q = x_0 = a, x_n = b]$

$\Rightarrow U(P, f) - L(P, f) < \varepsilon$

Hence, the function f is Reimann integeral on (a, b).

Correct Answer: C

Question 180 : Find the value of integral $\iint_A xy\,dx\,dy$ where A is given by $x^2 + y^2 - 2x = 0$, $y^2 = 2x$ and $y = x$.

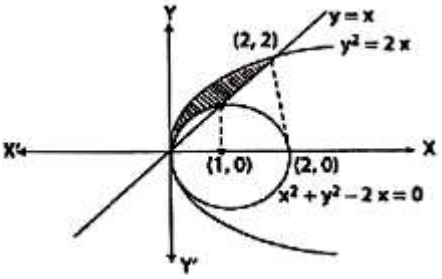
(A) $\frac{6}{13}$

(B) $\frac{7}{12}$

(C) $\frac{7}{13}$

(D) None of these

Solution : We have to evaluate the given integral along the shaded area.



$$\therefore \iint_A xy\,dx\,dy = \int_0^2 \int_x^{\sqrt{2x}} xy\,dx\,dy - \int_0^1 \int_x^{\sqrt{2x-x^2}} xy\,dx\,dy$$

$$= \int_0^2 x\,dy \int_x^{\sqrt{2x}} y\,dy - \int_0^1 x\,dx \int_x^{\sqrt{2x-x^2}} y\,dy$$

$$= \int_0^2 x \left[\frac{y^2}{2}\right]_x^{\sqrt{2x}} dx - \int_0^1 x \left[\frac{y^2}{2}\right]_x^{\sqrt{2x-x^2}} dx$$

$$= \int_0^2 x \left(x - \frac{x^2}{2}\right) dx - \int_0^1 x \left(\frac{2x - 2x^2}{2}\right) dx$$

$$= \int_0^2 x \left(x - \frac{x^2}{2}\right) dx - \int_0^1 x \left(\frac{2x - 2x^2}{2}\right) dx$$

$$= \left(\frac{8}{3} - \frac{16}{8}\right) - \left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= \frac{16}{24} - \frac{1}{12} = \frac{7}{12}$$

Correct Answer: D