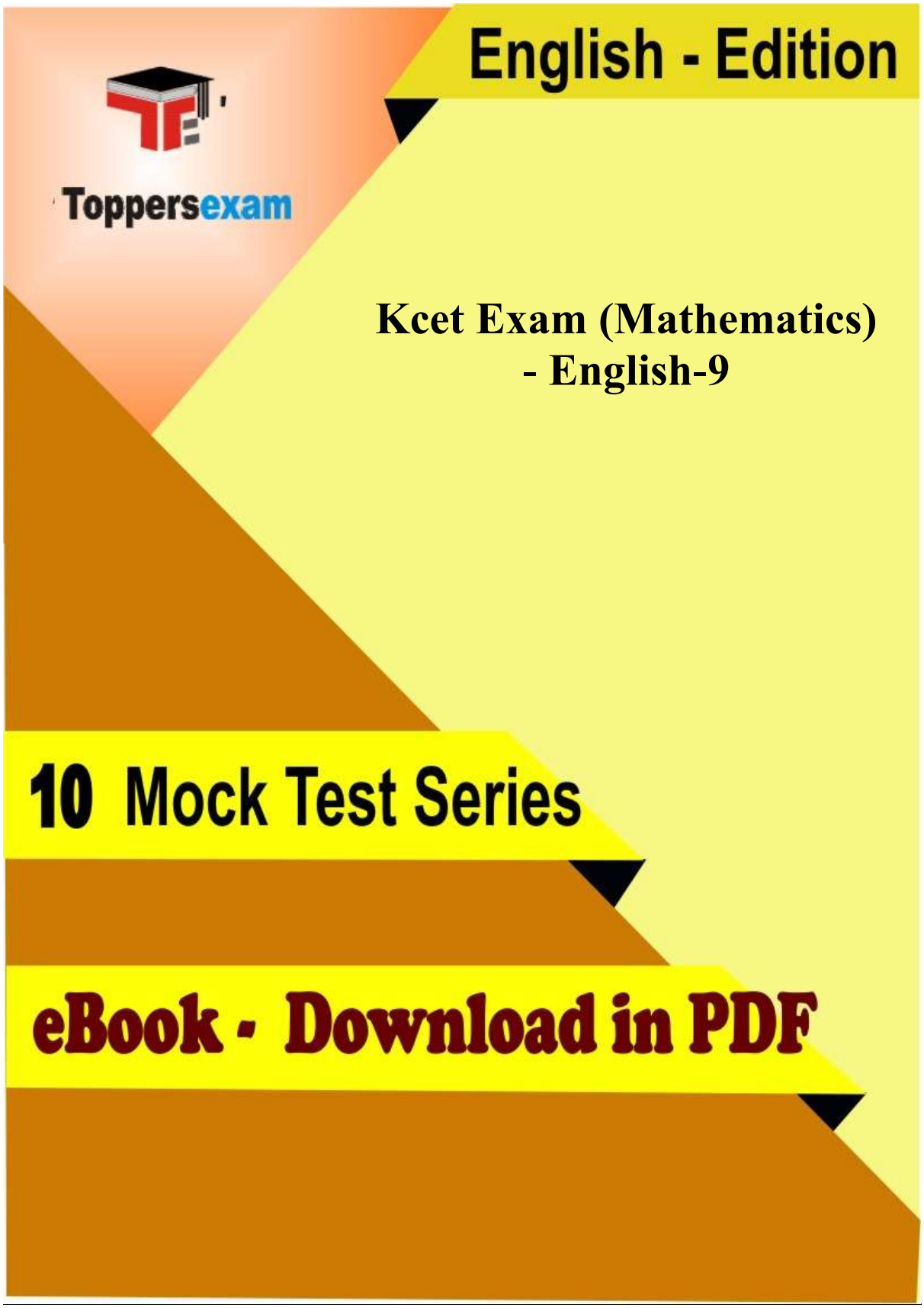

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Kcet Exam (Mathematics) - English-9

Paper Questions

SUBJECT: Physics

Question 1 : The ratio of the nuclear radii of elements with mass numbers 216 an 125 is

(A) 216 : 125

(B) $\sqrt{216}:\sqrt{125}$

(C) 6 : 5

(D) None of these

Correct Answer: C

Question 2 : On bombarding U^{235} by slow neutron, 200 MeV energy is released. If the power output of atomic reactor is 1.6 MW, then the rate of fission will be

(A) $5 \times 10^{22}/s$

(B) $5 \times 10^{16}/s$

(C) $8 \times 10^{16}/s$

(D) $20 \times 10^{16}/s$

Correct Answer: B

Question 3 : A ray of light enters from a rarer to a denser medium. The angle of incidence is i. Then the reflected and refracted rays are mutually perpendicular to each other. The critical angle for the pair of media is

(A) $\sin^{-1}$ (tan i)

(B) tan-1 (sin i)

(C) $\sin^{-1}$ (cot i)

(D) cos-1 (tan i)

Correct Answer: C

Question 4 : A fish in water (refractive index n) looks at a bird vertically above in the air. If y is the height of the bird and x is the depth of the fish from the surface, then the distance of the bird as estimated by the fish is

(A) $x + y\left(1 - \frac{1}{n}\right)$

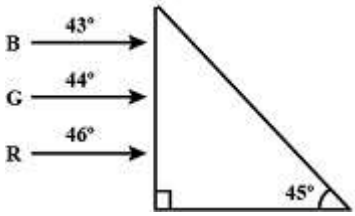
(B) $x + ny$

(C) $x + y\left(1 + \frac{1}{n}\right)$

(D) $y + x\left(1 - \frac{1}{n}\right)$

Correct Answer: B

Question 5 : Figure shows a mixture of blue, green and red coloured rays incident normally on a right angled prism. The critical angles of the material of the prism for red, green and blue are 46", 44° and 43° respectively. The arrangement will separate



(A) Red colour from blue and green

(B) Blue colour from red and green

(C) Green colour from red and blue

(D) all the three colours

Correct Answer: A

Question 6 : A convex and a concave lens separated by distance d are then put in contact. The .focal length of the combination

(A) decreases

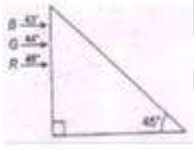
(B) increases

(C) becomes 0

(D) remains the same

Correct Answer: A

Question 7 : A convex lens is made of .3 layers of glass of 3 different materials as in the figures. A point object is placed on its axis. The number of images of the object are



(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: A

Question 8 : An unpolarised beam of intensity /0 falls on a polaroid. The intensity of the emergent light is

(A) $\frac{I_0}{2}$

(B) *lo*

(C) $\frac{I_0}{4}$

(D) Zero

Correct Answer: A

Question 9 : Which of the following as a dichroic crystal?

(A) Quartz

(B) Tourmaline

(C) Mica

(D) Selenite

Correct Answer: B

Question 10 : Two identical metal spheres charged with + 12µF and —8 µF are kept at certain distance in air. They are brought into contact and then kept at the same distance. The ratio of the magnitudes of electrostatic forces between them before and after contact is

(A) 12 : 1

(B) 8 : 1

(C) 24 : 1

(D) 4 : 1

Correct Answer: C

Question 11 : A small conducting sphere of radius r is lying concentrically inside a bigger hollow conducting sphere of radius R. The bigger and smaller spheres are charged with Q and q (Q > q) and are insulated from each other. The potential difference between the spheres will be

(A) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}-\frac{q}{R}\right)$

(B) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}-\frac{Q}{r}\right)$

(C) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}-\frac{Q}{R}\right)$

(D) $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}+\frac{q}{r}\right)$

Correct Answer: A

Question 12 : The charges Q +q and +q are placed at the vertices of an equilateral triangle of side l. If the net electrostatic potential energy of the system is zero, then Q is equal to

(A) $-\frac{q}{2}$

(B) – q

(C) $\frac{+q}{2}$

(D) Zero

Correct Answer: A

Question 13 : Dimensional formula for the universal gravitational constant G is

(A) $[M^{-1}L^{2}T^{-1}]$

(B) $[M^{0}L^{0}T^{0}]$

(C) $[M^{-1}L^{3}T^{-2}]$

(D) $[M^{-1}L^{3}T^{-1}]$

Correct Answer: C

Question 14 : A body is projected vertically upwards. The times corresponding to height h while ascending and while descending are t_1 and t_2 respectively. Then the velocity of projection is (g is acceleration due to gravity)

(A) $g\sqrt{t_1 t_2}$

(B) $\frac{g t_1 t_2}{t_1 + t_2}$

(C) $\frac{g\sqrt{t_1 t_2}}{2}$

(D) $\frac{g(t_1 t_2)}{2}$

Correct Answer: D

Question 15 : A mass of 10 kg is suspended from a spring balance. It is pulled aside by a horizontal string so that it makes an angle of 60° with the vertical. The new reading of the balance is

(A) 20 kg-wt

(B) 10 kg-wt

(C) $10\sqrt{3}$ kg-wt

(D) $20\sqrt{3}$ kg-wt

Correct Answer: A

Question 16 : A body weighs 50 g in air and 40 g in water. How much would it weigh in a liquid of specific gravity 1.5?

(A) 30 g

(B) 35 g

(C) 65 g

(D) 45 g

Correct Answer: B

Question 17 : A body of mass 4 kg is accelerated upon by a constant force, travels a distance of 5 m in the first second and a distance of 2 m in the third second. The force acting on the body is

(A) 2 N

(B) 4 N

(C) 6 N

(D) 8 N

Correct Answer: C

Question 18 : A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest its time period is T. With what acceleration should the lift be accelerated upwards in order to reduce its period to T / 2? (g is acceleration due to gravity).

(A) 2 g

(B) 3 g

(C) 4 g

(D) g

Correct Answer: B

Question 19 :

Three resistors 1 Ω, 2 Ω, 3Ω resistor a 3 V battery is connected. The current through 3 Ω resistor is

(A) 0.75 A

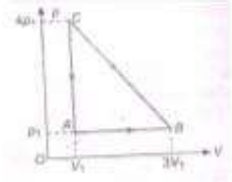
(B) 1 A

(C) 2 A

(D) 1.5 A

Correct Answer: B

Question 20 : An ideal gas is taken via path ABCA as shown in figure. The net work done in the whole cycle is



(A) $3p_1V_1$

(B) $-3p_1V_1$

(C) 6 p_1V_1

(D) Zero

Correct Answer: B

Question 21 : In which of the processes, does the internal energy of the system remain constant?

(A) Adiabatic

(B) Isochoric

(C) Isobaric

(D) Isothermal

Correct Answer: D

Question 22 : The coefficient of thermal conductivity of copper is 9 times that of steel. In the composite cylindrical bar shown in the figure, what will be the temperature at the junction of copper and steel?

(A) 75°C

(B) 67°C

(C) 25°C

(D) 33°C

Correct Answer: A

Question 23 : The equation of a simple harmonic wave is given by $y = 6 \sin 2\pi (2t - 0.1x)$, where x and y are in mm and t is in seconds. The phase difference between two particles 2 mm apart at any instant is

(A) 18°

(B) 36°

(C) 54°

(D) 72°

Correct Answer: D

Question 24 : With what velocity should an observer approach a stationary sound source, so that the apparent frequency of sound should appear double the actual frequency? (v is velocity of sound).

(A) $\frac{v}{2}$

(B) 3v

(C) 2v

(D) v

Correct Answer: D

Question 25 : If a black body emits 0.5 J of energy per second when it is at 27°C, then the amount of energy emitted by it when it is at 627°C will be

(A) 40.5 J

(B) 162 J

(C) 13.5 J

(D) 135 J

Correct Answer: A

Question 26 : A string vibrates with a frequency of 200 Hz. When its length is doubled and tension is altered, it begins to vibrate with a frequency of 300 Hz. The ratio of the new tension to the original tension is

(A) 9 : 1

(B) 1 : 9

(C) 3 : 1

(D) 1 : 3

Correct Answer: A

Question 27 : How many times more intense is a 60 dB sound than a 30 dB sound?

(A) 1000

(B) 2

(C) 100

(D) 4

Correct Answer: A

Question 28 : The masses of two radioactive substances are same and their half-lives are 1 yr and 2 yr respectively. The ratio of their activities after 4 yr will be

(A) 1 : 4

(B) 1 : 2

(C) 1 : 3

(D) 1 : 6

Correct Answer: A

Question 29 : $_{92}U^{235}$ undergoes successive disintegrations with the end product of $_{82}Pb^{203}$. The number of α and β particles emitted are

(A) α = 6, β = 4

(B) α = 6, β = 0

(C) α = 8, β = 6

(D) α = 3, β = 3

Correct Answer: C

Question 30 : The most stable particle in Baryon group is

(A) neutron

(B) omega-particle

(C) proton

(D) lambda-particle

Correct Answer: C

Question 31 : In an unbiased p-n junction

(A) Potential at p is more than that at n

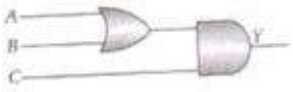
(B) Potential at p is less than that at n

(C) Potential at p is equal to that at n

(D) Potential at p is +ve and that at n is -ve

Correct Answer: B

Question 32 : To get an output Y =1 from the circuit shown, the inputs A, B and C must be respectively A



(A) 0, 1, 0

(B) 1, 0, 0

(C) 1, 0, 1

(D) 1, 1, 0

Correct Answer: C

Question 33 : One kg of copper is drawn into a wire of 1 mm diameter and a wire of 2 mm diameter. The resistance of the two wires will be in the ratio

(A) 2 : 1

(B) 1 : 2

(C) 16 : 1

(D) 4 : 1

Correct Answer: C

Question 34 : An electrical cable having a resistance of 0.2 delivers 10 kW at 200 B DC to a factory. What is the efficiency of transmission

(A) 65%

(B) 75%

(C) 85%

(D) 95%

Correct Answer: D

Question 35 : A wire of resistance 5 Ω is drawn out so that its new length is 3 times its original length. What is the resistance of the new wire ?

(A) 45 Ω

(B) 15 Ω

(C) 5/3 Ω

(D) 5 Ω

Correct Answer: A

Question 36 : Two identical cells each of E and internal resistance r are connected in parallel with an external resistance R. To get maximum power developed across R, the value R is

(A) $R = \frac{r}{2}$

(B) R = r

(C) $R = \frac{r}{3}$

(D) R = 2r

Correct Answer: A

Question 37 : To write the decimal number 37 in binary, how many binary digits are required ?

(A) 5

(B) 6

(C) 7

(D) 4

Correct Answer: B

Question 38 :

In a common emitter amplifier the input signal is applied across

(A) anywhere

(B) emitter-collector

(C) collector-base

(D) base-emitter

Correct Answer: D

Question 39 : Ferromagnetic materials used in a transformer must have

(A) low permeability and high hysterisis loss

(B) high permeability and low hysterisis loss

(C) high permeability and high hysterisis loss

(D) low permeability and low hysterisis loss

Correct Answer: B

Question 40 : What is the particle X in the following nuclear reaction

$${}^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + X$$

(A) Electron

(B) Proton

(C) Photone

(D) Neutron

Correct Answer: D

Question 41 : An alternating current of rms value 10 A is passed through a 12Ω resistor. The maximum potential difference across the resistor is

(A) 20 V

(B) 90 V

(C) 169.68 V

(D) None of the pce

Correct Answer: C

Question 42 : Two solid pieces, one of steel and the other of aluminum when immersed completely in water have equal weights. When the solid pieces are weighed in air

(A) the weight of aluminum is half the weight of steel

(B) steel piece will weigh more

(C) they have the same weight

(D) aluminium piece will weigh more

Correct Answer: A

Question 43 :

Which of the following is not a thermodynamic coordinate?

(A) Gas constant (R)

(B) Pressure (p)

(C) Volume (V)

(D) Temperature (T)

Correct Answer: A

Question 44 : According to Newton's corpuscular theory, the speed of light is

(A) same in all the medium

(B) lesser in rarer medium

(C) lesser in denser medium

(D) independent of the medium

Correct Answer: B

Question 45 : The figure below shows the distance time graph of the motion of a car. It follows the graph that the car is

(A) At rest

(B) In uniform motion

(C) In non-uniform acceleration

(D) Uniformly accelerated

Correct Answer: D

Question 46 : Two particles have mass m and 4m and their kinetic energies are the ratio of their linear momenta ?

(A) $\frac{1}{\sqrt{2}}$

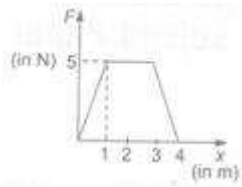
(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{1}{16}$

Correct Answer: A

Question 47 : The force F acting on a particle moving in straight line is shown below. What is the work done by the force on the particle in the 1st metre of the trajectory ?



(A) 5 J

(B) 10 J

(C) 15 J

(D) 2.5 J

Correct Answer: D

Question 48 : The light reflected by a plane mirror may form a real image:

(A) If the rays incident on the mirror are converging

(B) If the rays incident on the mirror are diverging

(C) Under no circumstances

(D) If the object is placed very close to the mirror

Correct Answer: A

Question 49 : A particle is projected at 60^0 to the horizontal with a kinetic energy KE. The kinetic energy at the highest point is

(A) KE

(B) Zero

(C) $\frac{kE}{4}$

(D) $\frac{kE}{2}$

Correct Answer: C

Question 50 : The poisson's ratio of a material is 0.5. if a force is applied to a wire of this material there is a decrease in the cross- sectional area by 4% The percentage increase in the length is

(A) 1%

(B) 2%

(C) 2.5%

(D) 4%

Correct Answer: D

Question 51 : Two massless springs of force constants k_1 and k_2 are joined end to end. The resultant force constant k of the system is

(A) $k = \frac{k_1 + k_2}{k_1 k_2}$

(B) $k = \frac{k_1 - k_2}{k_1 k_2}$

(C) $k = \frac{k_1 k_2}{k_1 + k_2}$

(D) $k = \frac{k_1 k_2}{k_1 - k_2}$

Correct Answer: C

Question 52 : A spring of force constant k is cut into two equal halves. The force constant of each half is

(A) $\frac{k}{\sqrt{2}}$

(B) k

(C) $\frac{k}{2}$

(D) 2k

Correct Answer: D

Question 53 : Two rods of equal length and diameter have thermal conductivities 3 and 4 units respectively. IF they are joined in series. The thermal conductivity of the combination would be

(A) 3.43

(B) 3.5

(C) 3.4

(D) 3.34

Correct Answer: A

Question 54 : 19 g of water of 30^0C and 5 g of at -20^0C are mixed together in a calorimeter. What is the final temperature of the mixture ? (Given specific heat of ice = 0.5 cal g^{-1} $(^0C)^{-1}$ and latent heat of fusion of ice = 80 cal g^{-1})

(A) ^{0}C

(B) -5^0C

(C) 5^0C

(D) 10^0C

Correct Answer: C

Question 55 : It is difficult to cook rice in an open vessel by boiling it at high altitudes because of

(A) Low boiling point and high pressure

(B) High boiling point and low pressure

(C) Low boiling point and low pressure

(D) High boiling point and high pressure

Correct Answer: C

Question 56 : The height of a waterfall is 50 m. IF g = 9.8 ms^{-2} the difference between the temperature at the top and the bottom of the waterfall is

(A) 1.17^0C

(B) 2.17^0C

(C) 0.117^0C

(D) 1.43^0C

Correct Answer: C

Question 57 : The distance between an object and a divergent lens is *m* times the focal length of the lens. The linear magnification produced by the lens is

(A) *m*

(B) $\underline{1}$

m

(C) $m + 1$

(D) $\frac{1}{m+1}$

Correct Answer: D

Question 58 : A 2.0 cm object is placed 15 cm is front of a concave mirror of focal length 10 cm. What is the size and nature of the image ?

(A) 4 cm, real

(B) 4 cm, virtual

(C) 1.0 cm. real

(D) None of these

Correct Answer: A

Question 59 : Three concurrent co-planar forces 1 N, 2 N and 3 N acting along different directions on a body

(A) can keep the body in equilibrium if 1 N and 3 N act at right angle

(B) can keep the body in equilibrium if 1 N and 2 N act at right angle

(C) cannot keep the body in equilibrium

(D) can keep the body in equilibrium in 1 N and 3 N act at an acute angle

Correct Answer: C

Question 60 :

The number of significant figures in the numbers 4.8000 x 104 48000.50 are respectively

(A) 5 and 6

(B) 5 and 7

(C) 2 and 7

(D) 2 and 6

Correct Answer: B

SUBJECT: Chemistry

Question 61 : Which of the following has the maximum number of unpaired `d' electrons?

(A) Zn^{2+}

(B) Fe^{2+}

(C) Ni^{3+}

(D) Cu^{+}

Correct Answer: B

Question 62 : One mole of which of the following has the highest entropy?

(A) Liquid nitrogen

(B) Hydrogen gas

(C) Mercury

(D) Diamond

Correct Answer: B

Question 63 : Which of the following species does not exert a resonance effect?

(A) $C_6H_5NH_2$

(B) $C_6H_5NH_3$

(C) C_6H_5OH

(D) C_6H_5Cl

Correct Answer: B

Question 64 : A complex compound in which the oxidation number of a metal is zero is

(A) $K_4[Fe(CN)_6]$

(B) $K_3[Fe(CN)_6]$

(C) $[Ni(CO)_4]$

(D) $[Pt(NH_3)_4]Cl_2$

Correct Answer: C

Question 65 : Catalytic dehydrogenation of a primary alcohol gives a

(A) secondary alcohol

(B) aldehyde

(C) ketone

(D) ester

Correct Answer: B

Question 66 : Excess of PCl_5 reacts with conc H_2SO_4 giving

(A) chlorosulphonic acid

(B) thionyl chloride

(C) sulphuryl chloride

(D) sulphurous acid

Correct Answer: C

Question 67 : If one mole of ammonia and one mole of hydrogen chloride are mixed in a closed container to form ammonium chloride gas, then

(A) $\Delta H > \Delta U$

(B) $\Delta H = \Delta U$

(C) $\Delta H < \Delta U$

(D) there is no relationship

Correct Answer: C

Question 68 : The compound on dehydrogenation gives a ketone. The original compound is

(A) primary alcohol

(B) secondary alcohol

(C) tertiary alcohol

(D) carboxylic acid

Correct Answer: B

Question 69 : Which is the most easily liquifiable rare gas?

(A) Xe

(B) Kr

(C) Ar

(D) Ne

Correct Answer: A

Question 70 : Three moles of PCl_5, three moles of PCl_3 and two moles of Cl_2 are taken in a closed vessel. If at equilibrium the vessel has 1.5 moles of PCl_5, the number of moles of PCI_3 present in it is

(A) 5

(B) 3

(C) 6

(D) 4.5

Correct Answer: D

Question 71 : How many optically active stereomers are possible for butan-2, 3-diol?

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: B

Question 72 : An octahedral complex is formed when hybrid orbitals of the following type are involved

(A) sp^3

(B) dsp^2

(C) d^2sp^3

(D) sp^2d^2

Correct Answer: C

Question 73 : For the reaction the equilibrium constant depends upon

(A) temperature

(B) pressure

(C) catalyst

(D) volume

Correct Answer: A

Question 74 : The angle strain in cyclobutane is

(A) 24°44'

(B) 29°16'

(C) 19° 22'

(D) 9°44'

Correct Answer: D

Question 75 : Methoxy methane and ethanol are

(A) position isomers

(B) chain isomers

(C) functional isomers

(D) optical isomers

Correct Answer: C

Question 76 : When the azimuthal quantum number has the value of 2, the number of orbitals possible are

(A) 7

(B) 5

(C) 3

Correct Answer: B

Question 77 : For the reaction $Fe_2O_3 + 3C0 \rightarrow 2Fe + 3CO_2$, the volume of carbon monoxide required to reduce one mole of ferric oxide is

(A) 22.4 dm^3

(B) 44.8 dm^3

(C) 67.2 dm^3

(D) 11.2 dm^3

Correct Answer: C

Question 78 : The monomers of buna-S rubber are

(A) vinyl chloride and sulphur

(B) butadiene

(C) styrene and butadiene

(D) isoprene and butadiene

Correct Answer: C

Question 79 : An element with atomic number 21 is a

(A) halogen

(B) representative element

(C) transition element

(D) alkali metal

Correct Answer: C

Question 80 : n-propyl bromide on treating with alcoholic KOH produces

(A) propane

(B) propene

(C) propyne

(D) propanol

Correct Answer: B

Question 81 : Mercury is a liquid metal because

(A) it has a completely filled s orbital

(B) it has a small atomic size

(C) it has a completely filled d orbital that prevents d-d overlapping of orbitals

(D) it has a completely filled d orbital that causes d-d overlapping

Correct Answer: C

Question 82 : A compound is formed by elements A and B. This crystallises in the cubic structure where the A atoms are at the corners of the cube and B atoms are at the body centres. The simplest formula of the compound is

(A) AB

(B) A_6B

(C) A_8B_4

(D) AB_6

Correct Answer: A

Question 83 : Anisole can be prepared by the action of methyl iodide on sodium phenate. The reaction is called

(A) Wurtz's reaction

(B) Williamson's reaction

(C) Fittig's reaction

(D) Etard's reaction

Correct Answer: B

Question 84 : Malleability and ductility of metals can be accounted due to

(A) the presence of electrostatic force

(B) the crystalline structure in metal

(C) the capacity of layers of metal ions to slide over the other

(D) the interaction of electrons with metal ions in the lattice

Correct Answer: C

Question 85 : The correct order in which the first ionisation potential increases is

(A) Na, K, Be

(B) K, Na, Be

(C) K, Be, Na

(D) Be, Na, K

Correct Answer: B

Question 86 : 10 cm^3 of 0.1 N monobasic acid requires 15 cm^3 of sodium hydroxide solution whose normality is

(A) 1.5 N

(B) 0.15 N

(C) 0.066 N

(D) 0.66 N

Correct Answer: C

Question 87 : The IUPAC name for tertiary butyl iodide is

(A) 4-iodo butane

(B) 2-iodo butane

(C) 1-iodo-3-methyl propane

(D) 2-iodo-2-methyl propane

Correct Answer: D

Question 88 : When sulphur dioxide is passed in an acidified $K_2Cr_2O_7$ solution, the oxidation state of sulphur is changed from

(A) + 4 to 0

(B) + 4 to +2

(C) + 4 to +6

(D) +6 to +4

Correct Answer: C

Question 89 : Mass of 0.1 mole of methane is

(A) 1 g

(B) 16 g

(C) 1.6 g

(D) 0.1 g

Correct Answer: C

Question 90 : The maximum number of hydrogen bonds that a molecule of water can have is

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: D

Question 91 : A gas deviates from ideal behaviour at a high pressure because its molecules

(A) attract one another

(B) show the Tyndall effect

(C) have kinetic energy

(D) are bound by covalent bonds

Correct Answer: A

Question 92 : The reagent used to convert an alkyne to alkene is

(A) Zn/HCl

(B) Sn/HCl

(C) Zn-Hg/HCl

(D) Pd/H_2

Correct Answer: D

Question 93 : When compared to ΔG° for the formation of Al_2O_3, the ΔG° for the formation of Cr_2O_3 is

(A) higher

(B) lower

(C) same

(D) unpredicted

Correct Answer: A

Question 94 : In order to increase the volume of a gas by 10%, the pressure of the gas should be

(A) increased by 10%

(B) increased by 1%

(C) decreased by 10%

(D) decreased by 1%

Correct Answer: C

Question 95 : Helium is used in balloons in place of hydrogen because it is

(A) incombustible

(B) lighter than hydrogen

(C) radioactive

(D) more abundant than hyrogen

Correct Answer: A

Question 96 : The basic principle of Cottnell's precipitator is

(A) Le-Chatelier's principle

(B) peptisation

(C) neutralisation of charge on colloidal particles

(D) scattering of light

Correct Answer: C

Question 97 : When carbon monoxide is passed over solid caustic soda heated to 200°C, it forms

(A) Na_2C0_3

(B) $NaHCO_3$

(C) HCOONa

(D) CH_3COONa

Correct Answer: C

Question 98 : What is the effect of the increase of temperature on the equilibrium of the reaction?
$N_2 + 3H_2 \rightleftharpoons 2NH_3 + heat.$

(A) Equilibrium is shifted to the left

(B) Equilibrium is shifted to the right

(C) Equilibrium is unaltered

(D) Reaction rate does not change

Correct Answer: A

Question 99 : Hydrogen gas is not liberated when the following metal is added to dil HCl

(A) Ag

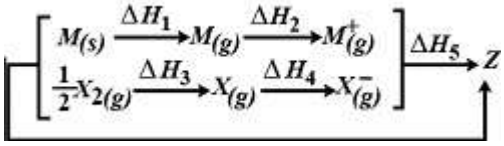
(B) Zn

(C) Mg

(D) Sn

Correct Answer: A

Question 100 : Consider the Born-Haber cycle for the formation of an ionic compound given below and identify the compound (Z) formed.



(A) M^+X^-

(B) $M^+X^-(s)$

(C) MX

(D) $M^+X^-(g)$

Correct Answer: B

Question 101 : In the brown ring test, the brown colour of the ring is due to

(A) ferrous nitrate

(B) ferric nitrate

(C) a mixture of NO and NO_2

(D) nitrosoferrous sulphate

Correct Answer: D

Question 102 : Amines behave as

(A) Lewis acid

(B) Lewis base

(C) aprotic acid

(D) neutral compound

Correct Answer: B

Question 103 : Dalda is prepared from oils by

(A) oxidation

(B) reduction

(C) hydrolysis

(D) distillation

Correct Answer: B

Question 104 : The chemical name of anisole is

(A) ethanoic acid

(B) methoxy benzene

(C) propanone

(D) acetone

Correct Answer: B

Question 105 : The number of disulphide linkages present in insulin are

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: B

Question 106 : 80 g of oxygen contains as many atoms as in

(A) 80 g of hydrogen

(B) 1 g of hydrogen

(C) 10 g of hydrogen

(D) 5 g of hydrogen

Correct Answer: D

Question 107 : Which metal has a greater tendency to form metal oxide?

(A) Cr

(B) Fe

(C) Al

(D) Ca

Correct Answer: D

Question 108 : Identify the reaction that does not take place in a blast furnace.

(A) $CaCO_3 \rightarrow CaO + CO_2$

(B) $CaO + SiO_2 \rightarrow CaSiO_3$

(C) $2Fe_2O_3 + 3C \rightarrow 4Fe + 3CO_2$

(D) $CO_2 + C \rightarrow 2C0$

Correct Answer: C

Question 109 : Waxes are esters of

(A) glycerol

(B) long chain alcohols

(C) glycerol and fatty acid

(D) long chain alcohols and long chain fatty acids

Correct Answer: D

Question 110 : An ionic .compound is expected to have tetrahedral structure if r_+/r_- lies in the range of

(A) 0.414 to 0.732

(B) 0.225 to 0.414

(C) 0.155 to 0.225

(D) 0.732 to 1

Correct Answer: B

Question 111 : Among the following, which is least acidic?

(A) phenol

(B) o-cresol

(C) p-nitrophenol

(D) p-chlorophenol

Correct Answer: B

Question 112 : A ligand can also be regarded as

(A) Lewis acid

(B) Bronsted base

(C) Lewis base

(D) Bronsted acid

Correct Answer: C

Question 113 : The colour of sky is due to

(A) transmission of light

(B) wavelength of scattered light

(C) absorption of light by atomspheric gases

(D) All of the above

Correct Answer: B

Question 114 : Which of the following organic compounds answers to both iodoform test and Fehling's test?

(A) Ethanol

(B) Methanal

(C) Ethanal

(D) Propanone

Correct Answer: C

Question 115 : 1 mole of photon, each of frequency 2500 s^{-1}, would have approximately a total energy of

(A) 1 erg

(B) 1 J

(C) 1 eV

(D) 1 MeV

Correct Answer: A

Question 116 : If n_t, number of ratio atoms are present at time t, the following expression will be a constant

(A) $\frac{n_t}{t}$

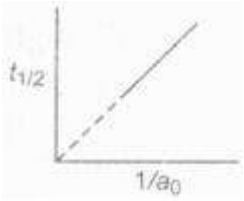
(B) In $\frac{n_t}{t}$

(C) $\frac{d}{dt}$ (In nt)

(D) t . n_t

Correct Answer: C

Question 118 : The second law of thermodynamics says that in a cyclic process



(A) Work cannot be converted into heat

(B) Heat cannot be converted into work

(C) Work cannot be completely converted into heat

(D) Heat cannot be completely converted into work

Correct Answer: D

Question 119 : The equilibrium constant (K) of a reaction may be written as

(A) $K = e^{-\Delta G/RT}$

(B) $K = e^{-\Delta G0/RT}$

(C) $K = e^{-\Delta H/RT}$

(D) $K = e^{-\Delta H0/RT}$

Correct Answer: B

Question 120 :

Which among the following species has the highest ionization potential?

(A) B

(B) Li

(C) Ne

(D) F

Correct Answer: C

SUBJECT: Mathematics

Question 121 :

The set $(A \cup B \cup C) \cap (A \cap B' \cap C)' \cap C'$ is equal to

(A) $B \cap C'$

(B) $A \cap C$

(C) $B' \cap C'$

(D) None of these

Solution :

Correct Answer: A

Question 123 :

If N be the set of natural numbers and define the relation R on N by R = {(1, 2)(2, 5), (3, 10)(4, 17)(5, 26)}. Then, the set builder form of R is

(A) $\{(x, x^2 + 1) | x \in N \text{ and } x < 6\}$

(B) $\{(x, x^2 - 1) | x \in N \text{ and } x < 6\}$

(C) $\{(x, 2x + 1) | x \in N \text{ and } x < 6\}$

(D) None of the above

Solution :

Here, R = {(1, 2), (2, 5), (3, 10), (4, 17), (5, 26)}

Now, consider the ordered pairs

(1, 2), (2, 5), (3, 10), (4, 17), (5, 26)

(1, 2) → 2 can be written as $2 = 1^2 + 1$

(2, 5) → 5 can be written as $5 = 2^2 + 1$

(3, 10) → 10 can be written as $10 = 32 + 1$

Similarly, for (4, 17) and (5, 26)

Thus, the set builder form of R is

$R = \{(x, x^2 + 1) | x \in N \text{ and } x < 6\}$

Correct Answer: A

Question 124 :

Let R be a relation in the set {1, 2, 3, 4} given by

R = {(1, 1), (1, 2), (2, 2), (4, 4), (1, 3), (3, 3), (3, 2)}.

Choose the correct option

(A) R is reflexive

(B) R is transitive

(C) R is symmetric

(D) R is reflexive and transitive but not symmetric

Correct Answer: D

Question 125 : The real value of a for which the expression $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely real is, when n € N

(A) $(n+1)\frac{\pi}{2}$

(B) $(2n+1)\frac{\pi}{2}$

(C) $n\pi$

(D) None of these

$$\text{Let } z = \frac{1-i\sin\alpha}{1+2i\sin\alpha}\times\frac{(1-2i\sin\alpha)}{(1-2i\sin\alpha)}$$

$$= \frac{1-2i\sin\alpha-i\sin\alpha+(-1)2\sin^2\alpha}{1^2-(2i\sin\alpha)^2}$$

$$= \frac{1-2\sin^2\alpha-3i\sin\alpha}{1+4\sin^2\alpha}$$

Since, z is purely real, therefore Im(z) = 0

$$\frac{-3\sin\alpha}{1+4\sin^2\alpha}=0$$

Solution : $\Rightarrow \sin\alpha = 0 \Rightarrow \alpha = n\pi$

Correct Answer: C

Question 126 : What is the argument of $(1-\sin\theta)+i\cos\theta$? $(i=\sqrt{-1})$

(A) $\frac{\pi}{2}-\frac{\theta}{2}$

(B) $\frac{\pi}{2}+\frac{\theta}{2}$

(C) $\frac{\pi}{4}-\frac{\theta}{2}$

(D) $\frac{\pi}{4}+\frac{\theta}{2}$

Solution :

Let $z = (1 - \sin\theta) + i\cos\theta$, then

$$\arg(z) = \tan^{-1}\left(\frac{Im(z)}{Re(z)}\right) = \tan^{-1}\left(\frac{\cos\theta}{1-\sin\theta}\right)$$

$$= \tan^{-1}\left\{\frac{(\cos^2\theta/2 - \sin^2\theta/2)}{\sin^2\theta/2 + \cos^2\theta/2 - 2\sin\theta/2.\cos\theta/2}\right\}$$

$$= \tan^{-1}\left\{\frac{(\cos\theta/2 - \sin\theta/2)(\cos\theta/2 + \sin\theta/2)}{(\cos\theta/2 - \sin\theta/2)^2}\right\}$$

$$= \tan^{-1}\left\{\frac{(\cos\theta/2 + \sin\theta/2)}{\cos\theta/2 - \sin\theta/2}\right\}$$

$$= \tan^{-1}\left(\frac{1+\tan\theta/2}{1-\tan\theta/2}\right)$$

$$= \tan^{-1}\tan\left[\frac{\pi}{4} + \frac{\theta}{2}\right] = \frac{\pi}{4} + \frac{\theta}{2}$$

Correct Answer: D

Question 127 : If $a_1, a_2, a_3, \ldots\ldots, a_{24}$ are in arithmetic progression and $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then $a_1 + a_2 + a_3 + \ldots\ldots + a_{23} + a_{24}$ is equal to

(A) 909

(B) 75

(C) 750

(D) 900

Given that,

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow (a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 225$

$\Rightarrow 3(a_1 + a_{24}) = 225$

$\Rightarrow a_1 + a_{24} = 75$ .....(i)

[$\because$ In an AP, the sum of the terms equidistant from the beginning and the end is same and is equal to the sum of the first and last term]

$\therefore\ a_1 + a_2 + ... a_{24} = \frac{n}{2}(a + l)$

$= \frac{24}{2}(a_1 + a_{24})$

Solution : $= 12 \times 75 = 900$ [$\because$ from eq. (i)]

Correct Answer: D

Question 128 :

The sum of integers from 1 to 100 that are divisible by 2 or 5 is

(A) 3000

(B) 3050

(C) 4050

(D) None of these

Solution :

The sum of integers from 1 to 100 that are divisible by 2 or 5.

= Sum of series divisible by 2 + Sum of series divisible by 5 – Sum of series divisible by 2 and 5 = (2 + 4 + 6 +.....+ 100) + (5 + 10 + 15 +.....+ 100) – (10 + 20 + 30 + ... + 100)

= 2(1 + 2 + 1 .... + 50) + 5(1 + 2 + ... + 20) – 10(1 + 2 + ...10)

$$= 2\left(\frac{50\times 51}{2}\right)+5\left(\frac{20\times 21}{2}\right)-10\left(\frac{10\times 11}{2}\right)$$

$$= \frac{1}{2}[5100+2100-1100]$$

$$= \frac{1}{2}6100 = 3050$$

Correct Answer: B

Question 129 : The product of $\sqrt[3]{2}\times\sqrt[4]{2}\times\sqrt[12]{32}$, is equal to

(A) $\sqrt{2}$

(B) 2

(C) $\sqrt[12]{2}$

(D) $\sqrt[12]{32}$

LCM of 3, 4 and 12 = 12

Now, $\sqrt[3]{2} = \sqrt[12]{2^4}$

$\sqrt[4]{2} = \sqrt[12]{2^3}$

and $\sqrt[12]{32} = \sqrt[12]{2^5}$

Product of $\sqrt[3]{2}$ $\sqrt[4]{2}$ $\sqrt[12]{32}$

$= \sqrt[12]{2^4}.\sqrt[12]{2^3}.\sqrt[12]{2^5} = \sqrt[12]{2^4.2^3.2^5}$

Solution : $= \sqrt[12]{2^{12}} = 2^{12\times\frac{1}{12}} = 2$

Correct Answer: B

Question 130 : The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively, is

(A) 1651

(B) 2032

(C) 127

(D) None of these

Solution :

Required number

= HCF of (1657 – 6) and (2037 – 5)

= HCF of 1651 and 2032

By using Euclid's division Lemma, we get

2032 = (1651× 1) + 381

Here, divisor is 1651 and remainder is 381.

Again, by using Euclid's division lemma, we get

1651 = (381 × 4) = 127

Here, divisor is 381 and remainder is 127.

Again, by using Euclid's division lemma, we get

$381 = (127 \times 3) + 0$

Here, the remainder is zero.

So, HCF of 1651 and 2032 is 127.

Correct Answer: C

Question 131 : The number of mappings (functions) from the set A = {1, 2, 3} into the set B = {1,2,3 4.5 6,7} such that f(i) £ f(j), whenever i < j, is

(A) 84

(B) 90

(C) 88

(D) None of these

Solution :

If the function is one-one, then select any three from the set B in $^7C_3$ ways i.e. 35 ways.
If the function in many-one, then there are two possibilities. All three corresponds to same element number of such functions = $^7C_1$ = 7 ways. Two corresponds to same element.
Select any two from the set B. The larger one corresponds to the larger and the smaller one corresponds to the smaller the third may corresponds to any two.
Number of such functions
$= {}^7C_2 \times 2 = 42$
So, the required number of mappings
$= 35 + 7 + 42 = 84$

Correct Answer: A

Question 132 : If a polygon of n sides has 275 diagonals, then n is equal to

(A) 25

(B) 35

(C) 20

(D) 15

Solution :

A polygon of n sides has number of diagonals

$= \frac{n(n-3)}{2} = 275$ [given]

$\Rightarrow n^2 - 3n - 550 = 0$

$\Rightarrow (n - 25)(n + 22) = 0$

$\therefore n = 25$ [Q $n \neq -25$]

Correct Answer: A

Question 133 : ${}^{15}C_3 + {}^{15}C_5 + ... + {}^{15}C_{15}$ is equal to

(A) 2^{14}

(B) $2^{14} - 15$

(C) $2^{14} + 15$

(D) $2^{14} - 1$

Solution :

Consider, ${}^{15}C_3 + {}^{15}C_5 + ... + {}^{15}C_{15}$

$= ({}^{15}C_1 + ... + {}^{15}C_{15}) - {}^{15}C_1$

$= \left(\frac{2^{15}}{2}\right) - 15 = 2^{14} - 15$

Correct Answer: B

If the value of x is so small that x^2 and greater powers can be neglected, then $\frac{\sqrt{1+x}+\sqrt[3]{(1-x)^2}}{1+x+\sqrt{1+x}}$ is equal to

Question 134 :

(A) $1+\frac{5}{6}x$

(B) $1-\frac{5}{6}x$

(C) $1+\frac{2}{3}x$

(D) $1-\frac{2}{3}x$

Solution :

Given expression $\frac{\sqrt{1+x}+\sqrt[3]{(1-x)^2}}{1+x+\sqrt{1+x}}$

can be written as $\frac{(1+x)^{1/2}+(1-x)^{2/3}}{1+x+(1+x)^{1/2}}$

$$= \frac{\left[1+\frac{1}{2}x-\frac{1}{8}x^2+....\right]+\left[1-\frac{2}{3}x-\frac{1}{9}x^2-.....\right]}{1+x+\left[1+\frac{1}{2}x-\frac{1}{8}x^2+....\right]}$$

$$= \frac{2-\frac{1}{6}x-\frac{17}{72}x^2+....}{2+\frac{3}{2}x-\frac{1}{8}x^2+....}$$

$$= \frac{\left[1-\frac{1}{12}x-\frac{17}{144}x^2+....\right]}{\left[1+\frac{3}{4}x-\frac{1}{16}x^2+....\right]}$$

$$= \left[1-\frac{1}{12}x-\frac{17}{144}x^2+....\right]\left[1+\frac{3}{4}x-\frac{1}{16}x^2+....\right]^{-1}$$

$$= 1-\frac{5}{6}x+.... = 1-\frac{5}{6}x$$

[on neglecting the higher power of x]

Correct Answer: B

Question 135 : For all x, $x^2 + 2ax + (10 – 3a) > 0$, then the interval in which a lies, is

(A) $a < –5$

(B) $–5 < a < 2$

(C) $a > 5$

(D) $2 < a < 5$

Solution :

As we know, $ax^2 + bx + c > 0$ for all $x \in R$, iff $a > 0$ and $D < 0$.

$\therefore\ x^2 + 2ax + (10 - 3a) > 0, \forall x \in R$

$\Rightarrow D < 0$

$\Rightarrow 4a^2 - 4(10 - 3a) < 0$

$\Rightarrow (a + 5)(a \quad 2) < 0$

Using number line rule

$a \in (-5, 2)$

Correct Answer: B

Question 136 :

If $\sin\alpha$, $\sin\beta$ and $\cos\alpha$ are in GP, then roots of $x^2 + 2x\cot\beta + 1 = 0$ are always

(A) real

(B) real and negative

(C) greater than one

(D) non-real

Solution :

Since, $\sin\alpha$, $\sin\beta$ and $\cos\alpha$ are in GP, then

$\sin^2\beta = \sin\alpha\cos\alpha$ .....(i)

Given equation is $x^2 + 2x\cot\beta + 1 = 0$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= (2\cot\beta)^2 - 4 = 4(\text{cosec}^2\beta - 2)$

$= 4(\text{cosec}\alpha\sec\alpha - 2)$ [From eq. (i)]

$= 4(2\ \text{cosec}\ 2\alpha - 2) \geq 0$

$\therefore$ Roots are real.

Correct Answer: A

Question 137 : If |x| < 3 and x is a real number, then

(A) $x \geq 3$

(B) $-3 < x < 3$

(C) $x < -3$

(D) $-3 < x < 3$

Solution :

We have, $|x| < 3$

$\Rightarrow -3 < x < 3$ $\quad [\because |x| < a \Rightarrow [-a < x < a]$

Correct Answer: B

Question 138 : If |3 – 4x| > 9, then

(A) $x \in \left(-\infty, -\frac{3}{2}\right)$

(B) $x \in [3, \infty)$

(C) $x \in \left(-\infty, \frac{-3}{2}\right] \cup [3, \infty)$

(D)

Solution :

We have, $|3 - 4x| \geq 9$

$\Rightarrow 3 - 4x \leq -9$ or $3 - 4x \geq 9$

$[\square\ |x| \geq a \Rightarrow x \leq x - a$ or $x \geq a]$

Subtracting 3 from both sides of each inequality.

$3 - 4x - 3 \leq -9 - 3$

or $3 - 4x - 3 \geq 9 - 3$

$-4x \leq -12$

or $-4x \geq 6$

On dividing by (–4) both sides of each inequality,

$$\frac{-4x}{-4} \geq \frac{-12}{-4} \text{ or } \frac{-4x}{-4} \leq \frac{6}{-4}$$

$$x \geq 3 \text{ or } x \leq -\frac{3}{2}$$

$$\therefore\ x \in \left(-\infty, -\frac{3}{2}\right) \cup [3, \infty)$$

Correct Answer: C

Question 139 :

If $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$, if $Q_1 = \frac{1}{2}(A + A')$ and $Q_2 = \frac{1}{2}(A - A')$, then Q_1, Q_2 is equal to

(A) $\begin{bmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

Solution :

$$\text{Since } P=\begin{bmatrix}i & 0 & -i\\0 & -i & i\\-i & i & 0\end{bmatrix} \text{ and } Q=\begin{bmatrix}-i & i\\0 & 0\\i & -i\end{bmatrix}$$

$$\therefore PQ=\begin{bmatrix}i & 0 & -i\\0 & -i & i\\-i & i & 0\end{bmatrix}\begin{bmatrix}-i & i\\0 & 0\\i & -i\end{bmatrix}$$

$$=\begin{bmatrix}-i^2-i^2 & i^2+i^2\\i^2 & -i^2\\i^2 & -i^2\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & -1-1\\-1 & 1\\-1 & 1\end{bmatrix}=\begin{bmatrix}2 & -2\\-1 & 1\\-1 & 1\end{bmatrix}$$

Correct Answer: B

Question 140 : A square matrix P satisfies P2 = I – P, where I is the identity matrix. If $P^n = 5I – 8P$, then n is equal to

(A) 4

(B) 5

(C) 6

(D) 7

$Q\,P^3 = P(1 - P)$ $[Q\,P^2 = I - P]$

$= PI - P^2 = PI - (I - P)$

$= P - I + P = 2P - I$

Now, $P^4 = P.P^3$

$\Rightarrow P^4 = P(2P - I)$

$\Rightarrow P^4 = 2P^2 - P$

$\Rightarrow P^4 = 2I - 2P - P$

$\Rightarrow P^4 = 2I - 3P$

and $P^5 = P(2I - 3P)$

$\Rightarrow P^5 = 2P - 3(I - P)$

$\Rightarrow P^5 = 5P - 3I$

Also, $P^6 = P(5P - 3I)$

$\Rightarrow P^6 = 5P^2 - 3P$

$\Rightarrow P^6 = 5(I - P) - 3P$

$\Rightarrow P^6 = 5I - 8P$

Solution : So, n = 6

Correct Answer: C

Question 141 : $\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix}$ is equal to

(A) $(x + p)(x^2 + px - 2q^2)$

(B) $(x - p)(x^2 + px - 2q^2)$

(C) $(x - p)(x^2 - px + 2q^2)$

(D) None of the above

Solution :

Applying $C_1 \to C_1 - C_2$, we have

$$\Delta = \begin{vmatrix} x-p & p & q \\ p-x & x & q \\ 0 & q & x \end{vmatrix} = (x-p)\begin{vmatrix} 1 & p & q \\ -1 & x & q \\ 0 & q & x \end{vmatrix}$$

$$= (x-p)\begin{vmatrix} 0 & p+x & 2q \\ -1 & x & q \\ 0 & q & x \end{vmatrix}$$

[applying $R_1 \to R_1 + R_2$]

Expanding along C_1, we have

$\Delta = (x-p)(px + x^2 - 2q^2)$

$= (x-p)(x^2 + px - 2q^2)$

Correct Answer: B

Question 142 : If $\Delta = \begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix}$, then Δ is equal to

(A) a + b + c

(B) abc

(C) $\frac{1}{2}(a+b+c)$

(D) ZERO

Interchanging rows and columns, we get

$$\Delta = \begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$$

$$\Delta = (-1)^3 \begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix} = -\Delta$$

Solution : $\Rightarrow 2\Delta = 0$ or $\Delta = 0$

Correct Answer: D

Question 143 : If the angles of a triangle are in AP and the ratio of the number of degrees in the least is to the number of radians in the greatest is 60 : π, then the angles in radian are.

(A) $\frac{\pi}{8}, \frac{\pi}{4}, \frac{\pi}{2}$

(B) $\frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{5}$

(C) $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$

(D) $\frac{2\pi}{9}, \frac{\pi}{3}, \frac{4\pi}{9}$

Solution : Let the angles of a triangle be
$(a - d)°$, $a°$ and $(a + d)°$
So, $a - d + a + a + d = 180°$
$\Rightarrow a = 60°$
$\therefore$ Angles in degree are
$\Rightarrow (60 - d)°$ is the least angle and $(60 + d)°$ is the greatest angle.
Now, the greatest angle in radian

$$= \left[(60 + d) \times \frac{\pi}{180}\right]^c$$

Thus, we get $\frac{(60-d)}{(60+d)\frac{\pi}{180}} = \frac{60}{\pi}$

$\Rightarrow 60\pi - d\pi$

$= 60 \times 60 \times \frac{\pi}{180} + 60 \times d \times \frac{\pi}{180}$

$\Rightarrow 60\pi - d\pi = 20\pi + \frac{d\pi}{3}$

$\Rightarrow d\left(\frac{\pi}{3} + \pi\right) = 40\pi$

$\Rightarrow d = 40\,\pi \times \frac{3}{4\pi} = 30°$

Hence,the angles are
$(60 - 30)°, 60°, (60 + 30)°$

i.e. $30°, 60°, 90°$ or $\frac{\pi^c}{9}, \frac{\pi^c}{3}, \frac{\pi^c}{2}$.

Correct Answer: C

Question 144 : If θ lies in the first quadrant and $5\tan\theta = 4$, then $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta}$ is equal to

(A) $\frac{5}{14}$

(B) $\frac{3}{14}$

(C) $\frac{1}{14}$

(D) ZERO

Solution :

Given, $\tan\theta = \frac{4}{5}$

$\therefore \sin\theta = \frac{4}{\sqrt{41}}$ and $\cos\theta = \frac{5}{\sqrt{41}}$

Now, $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta} = \frac{5 \times \frac{4}{41} - 3 \times \frac{5}{\sqrt{41}}}{\frac{4}{41} + 2 \times \frac{5}{\sqrt{41}}} = \frac{5}{14}$

Correct Answer: A

Question 145 :

The value of $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$ is

(A) 1

(B) ZERO

(C) -1

(D) $\frac{\pi}{2}$

Solution :

Consider,

$$\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

Now, substitute $x = \tan\theta$, we get

$$\sin\left[\tan^{-1}\left(\frac{1-\tan^2}{2\tan\theta}\right)+\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right]$$

$$= \sin\left[\tan^{-1}(\cot 2\theta)+\cos^{-1}(\cos 2\theta)\right]$$

$$= \sin\left[\tan^{-1}\left\{\tan\left(\frac{\pi}{2}-2\theta\right)\right\}+2\theta\right]$$

$$= \sin\left(\frac{\pi}{2}-2\theta+2\theta\right) = 1$$

Correct Answer: A

Question 146 : Given, $0 \le x \le \frac{1}{2}$, then the value of

$$\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right\}-\sin^{-1}x\right]$$ is

(A) 1

(B) $\sqrt{3}$

(C) -1

(D) $\frac{1}{\sqrt{3}}$

Solution :

Given, for $0 \le x \le \frac{1}{2}$,

$$\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right\}-\sin^{-1}x\right]$$

$$=\tan\left[\sin^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}-\sin^{-1}x\right]$$

$$=\tan\left[\sin^{-1}\left\{\frac{\sin\theta+\sqrt{1-\sin^2\theta}}{\sqrt{2}}\right\}-\theta\right]$$

$$[\text{put } \sin^{-1}x=\theta \Rightarrow x=\sin\theta]$$

$$=\tan\left[\sin^{-1}\left\{\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right\}-\theta\right]$$

$$=\tan\left[\sin^{-1}\left\{\sin\left(\theta+\frac{\pi}{2}\right)\right\}-\theta\right]$$

$$=\tan\left[\theta+\frac{\pi}{4}-\theta\right]=\tan\frac{\pi}{4}=1$$

Correct Answer: A

Question 147 : $\lim_{x\to\frac{\pi}{2}}\frac{\frac{\pi}{2}-\theta}{\cot\theta}$ is equal to

(A) ZERO

(B) -1

(C) 1

(D) ∞

Solution :

We have, $\lim_{\theta \to \frac{\pi}{2}} \frac{\frac{\pi}{2} - \theta}{\cot \theta} = \lim_{\theta \to \frac{\pi}{2}} \frac{-1}{-\operatorname{cosec}^2 \theta}$

$= \lim_{\theta \to \frac{\pi}{2}} \sin^2 \theta = 1$

Correct Answer: C

Question 148 : If $f(x) = \begin{cases} \frac{\sin [x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$ where, [x] denotes the greatest integer less than or equal to x, then $\lim_{x \to 0} f(x)$ is equal to

(A) 1

(B) ZERO

(C) -1

(D) None of these

Solution :

As, $f(x) = \begin{cases} \frac{\sin [x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$

$\therefore$ RHL at $x = 0$, $\lim_{x \to 0} 0 = 0$

LHL at $x = 0$, $\lim_{h \to 0} \frac{\sin [0 - h]}{[0 - h]}$

$= \lim_{h \to 0} \frac{\sin (-1)}{-1} = \sin 1$

RHL $\neq$ LHL

Hence, limit does not exist.

Correct Answer: D

Question 149 : If $xy = e^{(x-y)}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{y(x-1)}{x(1+y)}$

(B) $\frac{x(x-1)}{y(1+y)}$

(C) $\frac{x(1+x)}{y(1-y)}$

(D) $\frac{y(1+x)}{x(1-y)}$

Solution :

Given, $xy = e^{(x-y)}$

On differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}(xy) = \frac{d}{dx}(e^{x-y})$$

$$\Rightarrow x\frac{dy}{dx} + y.1 = e^{x-y}\frac{d}{dx}(x-y)$$

[using product rule in LHS and chain rule in RHS]

$$\Rightarrow x\frac{dy}{dx} + y = e^{x-y}\left(1-\frac{dy}{dx}\right)$$

$$\Rightarrow x\frac{dy}{dx} + e^{x-y}\frac{dy}{dx} = e^{x-y} - y$$

$$\Rightarrow (x+e^{x-y})\frac{dy}{dx} = e^{x-y} - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{x-y}-y}{x+e^{x-y}} = \frac{xy-y}{x+xy} \quad [\text{Q } e^{x-y} = xy \text{ is given}]$$

$= \frac{y(x-1)}{x(1+y)}$

Correct Answer: A

Question 150 : What is the derivative of $x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)$?

(A) $\sqrt{a^2 - x^2}$

(B) $2\sqrt{a^2 - x^2}$

(C) $\sqrt{x^2 - a^2}$

(D) $2\sqrt{x^2 - a^2}$

Solution :

Let $y = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = x.\frac{(-2x)}{2\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2} + \frac{a^2}{a\sqrt{1 - \frac{x^2}{a^2}}}$$

$$= \frac{-x^2}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2} + \frac{a^2}{\sqrt{a^2 - x^2}}$$

$$= \sqrt{a^2 - x^2} + \sqrt{a^2 - x^2}$$

$$= 2\sqrt{a^2 - x^2}$$

Correct Answer: B

Question 151 : The equation of tangent to the curve y = be–x/a at the point, where it crosses Y-axis, is

(A) ax + by = 1

(B) ax – by = 1

(C) $\frac{x}{a} - \frac{y}{b} = 1$

(D) $\frac{x}{a}+\frac{y}{b}=1$

Solution :

Equation of the curve $x^2y^2 = a^4$

we get $x^2 2y\frac{dy}{dx}+y^2 2x=0$

$\Rightarrow \frac{dy}{dx}=\frac{-y}{x}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(-a,a)}=-\left(\frac{a}{-a}\right)=1$

Therefore, subtangent at the point (–a, a).

$=\frac{y}{\left(\frac{dy}{dx}\right)}=\frac{a}{1}=a$

Correct Answer: D

Question 152 : The angle of intersection of the curves $y = x^2$ and $x = y^2$ at (1, 1) is

(A) $\tan^{-1}\left(\frac{4}{3}\right)$

(B) tan–1(1)

(C) 90º

(D) $\tan^{-1}\left(\frac{3}{4}\right)$

The slope of the curve,

$y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 2 = m_1$ (say)

and $x = y_2 \Rightarrow 1 = 2y\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{1}{2} = m_2$ (say)

$\therefore$ Angle of intersection at the point of (1, 1).

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}} = \frac{3}{4}$

Solution :

$\Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$

Correct Answer: D

Question 153 : $\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ equals

(A) $-\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$

(B) $\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$

(C) $\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$

(D) $-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$

$$\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$$

$$= \int \frac{dx}{\sin x \frac{\sqrt{2}}{\sqrt{2}} - \cos \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2}}$$

$$= \int \frac{dx}{\sqrt{2}(\sin x \sin \frac{\pi}{4} - \cos x \cos \frac{\pi}{4} + 1)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos\left(x + \frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos 2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{2\sin^2\left(\frac{x}{2} + \frac{\pi}{8}\right)} = \frac{1}{2\sqrt{2}} \int \operatorname{cosec}^2\left(\frac{x}{2} + \frac{\pi}{8}\right) dx$$

Solution :

$$= -\frac{1}{2\sqrt{2}} \cdot \frac{-\cot\left(\frac{x}{2} + \frac{\pi}{8}\right)}{\frac{1}{2}} + C = \frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$$

Correct Answer: C

Question 154 : $\int \frac{dx}{x(x^7 + 1)}$ is equal to

(A) $\log\left(\frac{x^7}{x^7 + 1}\right) + C$

(B) $\frac{1}{7}\log\left(\frac{x^7}{x^7 + 1}\right) + C$

(C) $\log\left(\frac{x^7 + 1}{x^7}\right) + C$

(D) $\frac{1}{7}\log\left(\frac{x^7 + 1}{x^7}\right) + C$

Solution :

Given that, $I = \int \frac{dx}{x(x^7+1)}$

Put $x^7 = t \Rightarrow 7x^6 dx = dt$

$\Rightarrow dx = \frac{1}{7x^6} dt$

$\therefore I = \int \frac{1}{7x^7} \cdot \frac{dt}{(t+1)} = \int \frac{dt}{7t(t+1)} = \frac{1}{7}\int \left[\frac{1}{t} - \frac{1}{t+1}\right] dt$

$= \frac{1}{7}[\log t - \log(t+1)] + C$

$= \frac{1}{7}\log\frac{t}{t+1} + C = \frac{1}{7}\log\frac{x^7}{x^7+1} + C$

Correct Answer: B

Question 155 : The solution of the differential equation $(x^2 + y^2)dx = 2xy\ dy$ is

(A) $y = e^{-x}(x - 1)$

(B) $y = xe^x$

(C) $y = xe^{-x} + 1$

(D) $y = xe^{-x}$

Solution :

Given equation, $\frac{dy}{dx} + y = e^{-x}$

It is a linear differential equation, comparing with the standard equation

$\frac{dy}{dx} + Py = Q$

$\Rightarrow P = 1,\ Q = e^{-x}$

$IF = e^{\int P\,dx} = e^{x}$

$\therefore$ Required solution is

$y^{x} = \int e^{-x} e^{x} dx + C = \int 1 dx + C$

$\Rightarrow ye^{x} = x + C$

At $x = 0 \Rightarrow y = 0$

$\therefore C = 0$

Hence, the required solution is

$ye^{x} = x \Rightarrow y = xe^{-x}$

Correct Answer: D

Question 156 : Solve $\frac{dy}{dx} + \frac{y}{x} = x^{3}$

(A) $y = \frac{x^{4}}{5} + C / x$

(B) $y = \frac{x^{3}}{3} + C / x$

(C) $y = \frac{x^{3}}{3} + C$

(D) $y = \frac{x^{4}}{4} + x$

We have, $\frac{dy}{dx}+\frac{y}{x}=x^3$

On comparing with $\frac{dy}{dx}+Py=Q$, we get

$P=\frac{1}{x}, Q=x^3$

$\therefore$ $IF=e^{\int\frac{1}{x}dx}=e^{\log_e x}=x$

Hence, required solutionof given differential equation is

$y.IF=\int Q.IF\,dx+C$

$\Rightarrow yx=\int x^3.x\,dx+C$

$\Rightarrow yx=\int x^4\,dx+C$

Solution :

$\Rightarrow yx=\frac{x^5}{5}+C \Rightarrow x=\frac{x^4}{5}+\frac{C}{x}$

Correct Answer: A

Question 157 : Let A(1, k), B(1,1) and C(2,1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by

(A) (1, 3)

(B) (0, 2)

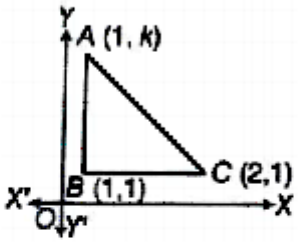
(C) (–1, 3)

(D) (–3, –2)

Solution :

Since A(1 , k), B(1 , 1) and C(2, 1) are the vertices of a right angled DABC.

$\begin{bmatrix} Q\ h = 1 \\ \angle B = 90° \end{bmatrix}$



Now, given that area of the triangle is 1.

Then, area $(\Delta ABC) = \frac{1}{2} \times AB \times BC$

$\Rightarrow 1 = \frac{1}{2} \times \sqrt{(1-1)^2 + (1-k)^2} \times 1$

$\Rightarrow 2 = \sqrt{(1-k)^2} \Rightarrow 2 = \sqrt{(k-1)^2}$

On squaring both sides, we get

$4 = k^2 + 1 - 2k$

$\Rightarrow k^2 - 2k - 3 = 0$

$\Rightarrow k = -1, 3$

Thus, the set of values of k is (–1, 3).

Correct Answer: C

Question 158 : The vertices P, Q and R of a triangle are (2, 1), (5, 2) and (3, 4), respectively. Then, the circumcentre is

(A) $\left(\frac{13}{4}, -\frac{9}{4}\right)$

(B) $\left(-\frac{13}{4}, \frac{9}{4}\right)$

(C) $\left(-\frac{13}{4}, -\frac{9}{4}\right)$

(D) $\left(\frac{13}{4}, \frac{9}{4}\right)$

Solution :

Let the vertices of a triangle be P(2, 1), Q(5, 2) and R(3, 4) and A(x, y) be the circumcentre of ΔPQR.

$\therefore AP^2 = AQ^2$

$\Rightarrow (2 - x)^2 + (1 - y)^2 = (5 - x)^2 + (2 - y)^2$

$\Rightarrow 4 + x^2 - 4x + 1 + y^2 - 2y$

$= 25 + x^2 - 10x + 4 + y^2 - 4y$

$\Rightarrow 6x + 2y = 24$

$\Rightarrow 3x + y = 12 \quad \ldots\ldots(i)$

and $AP^2 = AR^2$

$(2 - x)^2 + (1 - y)^2 = (3 - x)^2 + (4 - y)^2$

$\Rightarrow 4 + x^2 - 4x + 1 + y^2 - 2y$

$= 9 + x^2 - 6x + 16 + y^2 - 8y$

$\Rightarrow 2x + 6y = 20$

$\Rightarrow x + 3y = 10 \quad \ldots\ldots(ii)$

On solving eqs. (i) and (ii), we get

$x = \frac{13}{4}$ and $y = \frac{9}{4}$

So, the circumcentre is $\left(\frac{13}{4}, \frac{9}{4}\right)$.

Correct Answer: D

Question 159 : If the equation $\lambda x^2 + (2\lambda - 3)y^2 - 4x - 1 = 0$ represents a circle, then its radius is

(A) $\frac{\sqrt{11}}{3}$

(B) $\frac{\sqrt{13}}{3}$

(C) $\frac{\sqrt{7}}{3}$

(D) $\frac{1}{3}$

Given equation is

$\lambda x^2 + (2\lambda - 3)y^2 - 4x - 1 = 0$

Here, $a = \lambda$, $b = (2\lambda - 3)$

It represents a circle, if $a = b$

$\Rightarrow \lambda = 2\lambda - 3$

$\therefore \lambda = 3$

Solution : Then, equation becomes

$3x^2 + 3y^2 - 4x - 1 = 0 \Rightarrow x^2 + y^2 - \frac{4}{3}x - \frac{1}{3} = 0$

Here, $g = -\frac{2}{3}$, $c = -\frac{1}{3}$, $f = 0$

$\therefore$ Radius $= \sqrt{\left(-\frac{2}{3}\right)^2 + 0 - \left(-\frac{1}{3}\right)}$

$= \sqrt{\frac{4}{9} + \frac{1}{3}} = \frac{\sqrt{7}}{3}$

Correct Answer: C

Question 160 : The equation of circle which touches Xand Y-axes at the points (1, 0) and (0,1) respectively is

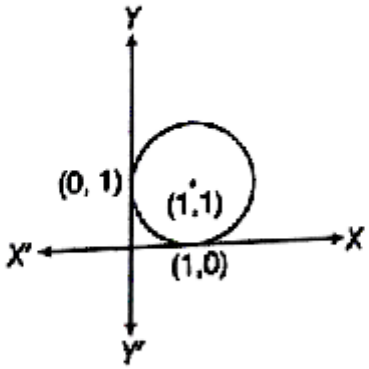
(A) $x^2 + y^2 - 4y + 3 = 0$

(B) $x^2 + y^2 - 2y - 2 = 0$

(C) $x^2 + y^2 - 2x - 2y + 2 = 0$

(D) $x^2 + y^2 - 2x - 2y + 1 = 0$

Since circle touches X-axis and Y-axis at points (1,0) and (0, 1), respectively.
So, centre of circle is (1, 1) and radius is 1.
Hence, equation of circle is



Solution : $(x-1)^2 + (y-1)^2 = 12$
$\Rightarrow x^2 + y^2 - 2x - 2y + 1 = 0$

Correct Answer: D

Question 161 : Eccentricity of the ellipse $x^2 + 2y^2 - 2x + 3y + 2 = 0$ is

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) $\frac{1}{2\sqrt{2}}$

(D) $\frac{1}{\sqrt{3}}$

Solution :

Given, equation of ellipse can be rewritten as

$$\frac{(x-1)^2}{1/8}+\frac{\left(y+\frac{3}{4}\right)^2}{1/16}=1$$

$$\therefore \text{ Eccentricity } = \sqrt{1-\frac{b^2}{a^2}}$$

$$= \sqrt{1-\frac{8}{16}} = \frac{1}{\sqrt{2}}$$

Correct Answer: A

Question 162 : If the foci of the ellipse $\frac{x^2}{9}+y^2=1$ subtend a right angle at a point P. Then, the locus of P is

(A) $x^2 + y^2 = 1$

(B) $x^2 + y^2 = 2$

(C) $x^2 + y^2 = 4$

(D) $x^2 + y^2 = 8$

Solution :

Given, $\frac{x^2}{9}+\frac{y^2}{1}=1$ .....(i)

$$\Rightarrow e = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

Two foci are $(\pm ae, 0)$ i.e. $(\pm 2\sqrt{2}, 0)$.

Let P(h,k) be any point on the ellipse.

$$\therefore \frac{k-0}{h-2\sqrt{2}} \times \frac{k-0}{h+2\sqrt{2}} = -1 \text{ [From given condition]}$$

$$\Rightarrow h^2 - 8 = -k^2$$

$$\Rightarrow x^2 + y^2 = 8$$

Correct Answer: D

Question 163 : The length of longer diagonal of the parallelogram constructed on $5\mathbf{a} + 2\mathbf{b}$ and $\mathbf{a} - 3\mathbf{b}$, if it is given that $|\mathbf{a}| = 2\sqrt{2}$, $|\mathbf{b}| = 3$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{4}$, is

(A) 15

(B) $\sqrt{113}$

(C) $\sqrt{593}$

(D) $\sqrt{369}$

Solution :

Given that, $|\mathbf{a}| = 2\sqrt{2}$ $|\mathbf{b}| = 3$

The longer diagonal = $6\mathbf{a} - \mathbf{b}$

Its length = $|6\mathbf{a} - \mathbf{b}|$

$= \sqrt{36\mathbf{a}^2 + \mathbf{b}^2 - 2 \times 6 |\mathbf{a}| . |\mathbf{b}| . \cos 45°} = 15$

Other diagonal = $4\mathbf{a} + 5\mathbf{b}$.

Its length = $\sqrt{16 \times 8 + 25 \times 9 + 40 \times 6} = \sqrt{593}$

Correct Answer: C

Question 164 : If |a| = 3, |b| = 4 and |c| = 5 such that each is perpendicular to sum of the other two, then |a + b + c| is

(A) $5\sqrt{2}$

(B) $\frac{5}{\sqrt{2}}$

(C) $10\sqrt{2}$

(D) $5\sqrt{3}$

$\mathbf{a}.(\mathbf{b}+\mathbf{c})=0,\ \mathbf{b}.(\mathbf{c}+\mathbf{a})=0$

$\mathbf{c}.(\mathbf{a}+\mathbf{b})=0$

$\therefore\ 2\Sigma\mathbf{a}.\mathbf{b}=0$

Now, $(\mathbf{a}+\mathbf{b}+\mathbf{c})^2=\Sigma\mathbf{a}^2+2\Sigma\mathbf{a}.\mathbf{b}$

$=9+16+25+0=50$

Solution : $\Rightarrow |\mathbf{a}+\mathbf{b}+\mathbf{c}|=5\sqrt{2}$

Correct Answer: A

A vector which is a linear combination of the vectors $3\hat{i}+4\hat{j}+5\hat{k}$ and $6\hat{i}-7\hat{j}-3\hat{k}$ and is perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is

Question 165 :

(A) $3\hat{i}-11\hat{j}-8\hat{k}$

(B) $-3\hat{i}+11\hat{j}+8\hat{k}$

(C) $-9\hat{i}+3\hat{j}-2\hat{k}$

(D) $9\hat{i}-3\hat{j}+2\hat{k}$

$r=(3\hat{i}+4\hat{j}+5\hat{k})+b(6\hat{i}-7\hat{j}-3\hat{k})$

$=(3+6b)\hat{i}+(4-7b)\hat{j}+(5-3b)\hat{k}$

Since, $r.(\hat{i}+\hat{j}-\hat{k})=0$

$\Rightarrow (3+6b)1+(4-7b)1-(5-3b)=0$

Solution : $\Rightarrow b=-1$

$r=-3\hat{i}+11\hat{j}+8\hat{k}$

Correct Answer: B

Question 166 : If the straight line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ intersect the curve $ax^2 + by^2 = 1,\ z = 0$, then the value of $a(\alpha n - \gamma l)^2 + b(\beta n - \gamma m)^2$ is

(A) n^2

(B) m^2

(C) l^2

(D) ZERO

Solution :

Given, $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}=k$

$\therefore$ Any point on the given line

$$(kl+\alpha,\ km+\beta,\ kn+\gamma)$$

Since, it lies on the curve $ax^2 + by^2 = 1,\ z = 0$ therefore

$a(\alpha + kl)^2 + b(\beta + km)^2 = 1$ .....(i)

and $kn + \gamma = 0 \Rightarrow k = \frac{-\gamma}{n}$

On putting value of k in eq. (i), we get

$$a\left(\alpha-\frac{l\gamma^2}{n}\right)+b\left(\beta-\frac{m\gamma^2}{n}\right)=1$$

$\Rightarrow a(n\alpha - l\gamma)^2 + b(n\beta - m\gamma)^2 = n^2$

Correct Answer: A

Question 167 : If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{3}$ intersect at a point, then the integer k is equal to

(A) –2

(B) –5

(C) 5

(D) 2

Solution :

Since, two lines intersect at a point.

Then, shortest distance between them is zero.

$$\therefore \begin{vmatrix} k & 2 & 3 \\ 3 & k & 2 \\ 1 & 1 & -2 \end{vmatrix} = 0$$

$$\Rightarrow k(-2k-2) - 2(-6-2) + 3(3-k) = 0$$

$$\Rightarrow k = \frac{5}{2}, -5$$

Hence, integer value of k is –5.

Correct Answer: B

Question 168 : The point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ is

(A) (0, 0, 0)

(B) (1, 1, 1)

(C) (–1, –1, –1)

(D) (1, 2, 3)

Solution :

Given lines are,

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=r\ [say] \qquad .....(i)$$

and $$\frac{x-4}{5}=\frac{y-1}{2}=z \qquad .....(ii)$$

Any point on the line (i) is
$(2r+1, 3r+2, 4r+3)$.
If they intersect, then this point satisfies the second line, we get

$$\frac{2r+1-4}{5}=\frac{3r+2-1}{2}=4r+3$$

$$\Rightarrow \frac{2r-3}{5}=\frac{3r+1}{2}\Rightarrow r=-1$$

Hence, the required point is $(-1, -1, -1)$.

Correct Answer: C

Question 169 : For a certain frequency table which has been partly reproduced here, the arithmetic mean was found to be Rs. 28.07.

Income (in ₹)	Number of workers
15	8
20	12
25	?
30	16
35	?
40	10

If the total number of workers is 75, then missing frequencies are respectively

(A) 14, 15

(B) 15, 14

(C) 13, 16

(D) 12, 17

$\square\ 8 + 12 + f_1 = 16 + f_2 + 10 = 75$

$\Rightarrow f_1 + f_2 = 29$ .....(i)

and $120 + 240 + 25f_1 + 480 + 35f_2 + 400 = 28.07 \times 75$

$\Rightarrow 5f_1 + 7f_2 = 173.25$ .....(ii)

On solving eqs. (i) and (ii), we get

Solution : $f_1 = 15$ and $f_2 = 14$

Correct Answer: B

Question 170 : If the mean of n observations $1^2, 2^2, 3^2, \ldots n^2$ is $\frac{46n}{11}$, then n is equal to

(A) 11

(B) 12

(C) 23

(D) 22

Solution :

According to the given condition,

$$\frac{1^2 + 2^2 + 3^2 + \ldots\ldots + n^2}{n} = \frac{46n}{11}$$

$$\Rightarrow \frac{n(n+1)(2n+1)}{6n} = \frac{46n}{11}$$

$$\Rightarrow n = 11 \qquad \left[\text{Q } n \neq \frac{1}{22}\right]$$

Correct Answer: A

Question 171 : The AM of the series 1, 2, 4, 8, 16, ..., 2^n is

(A) $\frac{2^n-1}{n}$

(B) $\frac{2^{n+1}-1}{n+1}$

(C) $\frac{2^n+1}{n}$

(D) $\frac{2^n-1}{n+1}$

Solution :

The required AM is

$$\bar{X}=\frac{1+2+2^2+2^3+.....+2^n}{n+1}$$

$$=\frac{1(2^{n-1}-1)}{(2-1)}.\frac{1}{(n+1)}=\frac{2^{n+1}-1}{n+1}$$

Correct Answer: B

Question 172 : The probability of simultaneous occurrence of atleast one of two events A and B is p. If the probrbility that exactly one of A, B occurs is q, then P(A`) + P(B`) is equal to

(A) 2 – 2p + q

(B) 2 + 2p – q

(C) 3 – 3p + q

(D) 2 – 4p + q

Solution :

Since, P(exactly one of A, B occurs) = q(given), we get

$P(A \cup B) - P(A \cap B) = q$

$\Rightarrow p - P(A \cap B) = q$

$\Rightarrow 1 - P(A' \cap B') = p - q$

$\Rightarrow P(A') + P(B') - P(A' \cap B') = 1 + q - p$

$\Rightarrow P(A') + P(B')$

$= (1 - p + q) + [1 - p(A \cup B)]$

$= (1 - p + q) + (1 - p)$

$= 2 - 2p + q$

Correct Answer: A

Question 173 : Out of 50 tickets numbered 00, 01, 02, . 49, one ticket is drawn randomly, the probability of the ticket having the product of its digits 7, given that the sum of the digits is 8, is

(A) $\frac{1}{14}$

(B) $\frac{3}{14}$

(C) $\frac{1}{5}$

(D) None of these

Solution :

Total number of casses = ${}^{50}C_1 = 50$

Let A be the event of selecting ticket with sum of digits '8'.

Favourable cases to A are {08, 17, 26, 35, 44}.

Let B be the event of selecting ticket with product of its digits 7'.

Favourable cases to B is only {17}.

Now, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/50}{5/50} = \frac{1}{5}$

Correct Answer: C

Question 174 : If four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}$ and $\frac{1}{8}$. Then the probability that the problem is solved correctly by atleast one of them, is

(A) $\frac{235}{256}$

(B) $\frac{21}{256}$

(C) $\frac{3}{256}$

(D) $\frac{235}{256}$

Solution :

Probability that the problem is solved correctly by atleast one of them = 1 – P(problem is not solved by all)

$\therefore$ P(problem is solved)

= 1 – P(problem is not solved)

$= 1 - P(\bar{A}).P(\bar{B}).P(\bar{C}).P(\bar{D})$

$= 1 - \left(\frac{1}{2}.\frac{1}{4}.\frac{3}{4}.\frac{7}{8}\right) = \frac{235}{256}$

Correct Answer: D

Question 175 : Consider the real vector space $V = R_3$ and following of its subsets

I. $S = \{(x, y, z) \in V: x = y = 0\}$

II. $T = \{(x, y, z) \in V: x = 0\}$

III. $W = \{(x, y, z) \in V: z \neq 0\}$

Which one of the following statement is correct?

(A) S, T and W are subspaces

(B) Only S and W are subspaces

(C) Only T and W are subspaces

(D) Only S and T are subspaces

Solution : Here, it is given that vector space $V = R^3$ and its subset are

$S = \{(x, y, z) \in V: x = y = 0\}$

$T = \{(x, y, z)\} \in V: x = 0\}$

$W_1 = \{(x, y, z) \in V: z \neq 0\}$

Let $W_1 = (2, 1, -3)$, $W_2 = (1, 2, 3) \in W$

Hence, $W_1 + W_2 = (3, 3, 0) \notin W$

So, it is not a subspace of R^3 but S and T both are subspaces.

Since, $\alpha S_1 + \beta S_2 \in S$ and $\alpha T_1 + \beta T_2 \in T$

where S_1 and $S_2 \in S$ and $T_1, T_2 \in T$, a, $\beta \in F$(field)

Correct Answer: D

Question 176 : Let V be a vector space over the field F of dimension n. Consider the following statements.

I. Every subset of V containing n elements is a basis of V.

II. No linearly independent subset of V contains more than n elements.

Which of the above statement is/are correct?

(A) I only

(B) II only

(C) Both I and II

(D) Neither I nor II

Solution :

I. Every subset of V containing n elements is a basis of V is not true.
Let R^3 be vector space of dimension 3.
But {(2, 2, 4), (1, 1, 2), (1, 0, 1)} is not a basis for R^3 because these are linearly dependent.
II. But we know that, no linearly independent subset of V contains more than n elements. So, it is true because we have not any linearly dependent set which has n elements.

Correct Answer: B

Question 177 : Let X = (3, 2, –1), Y = (2, 4, 1), Z = (4, 0, –3) and W = (10, 4, –5) be vector in R3, a real vector space. Which one of the following is correct?

(A) 2X + Z = W, Y + Z = W

(B) 2X – Y = Z, Y + 2Z = W

(C) X + Z = W, 2X + Y = Z

(D) Y + 2Z = W, X – Y = Z

Solution :

Let X = (3, 2, –1)
Y = (2, 4, 1)
Z = (4, 0, –3)
W = (10, 4, –5)
(i) 2X + Z = 2(3, 2, –1) + (4, 0, –3)
= (10, 4, –5) = W
and Y + Z = (2, 4, 1) + (4, 0, –3)
= (6, 4, –2) ≠ W
Thus, first option is wrong.
(ii) 2X – Y = 2(3, 2, –1) – (2, 4, 1)
= (4, 0, –3) = Z
and Y + 2Z = (2, 4, 1) + 2(4, 0, –3)
⇒ (10, 4, –5) = W
Thus, 2X – Y = Z and Y + 2Z = W

Correct Answer: B

Question 178 : The radius of convergence of the power series $f(z)=\sum\frac{n+1}{(n+2)(n+3)}z^n$ is

(A) 1

(B) 2

(C) 3

(D) 4

Solution :

$$a_n=\frac{n+1}{(n+2)(n+3)}$$

$$\Rightarrow a_n+1=\frac{(n+2)}{(n+3)(n+4)}$$

$$\therefore R=\lim_{n\to\infty}\left|\frac{a_n}{a_n+1}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(n+1)}{(n+2)(n+3)}\cdot\frac{(n+3)(n+4)}{(n+2)}\right|=1$$

Correct Answer: A

Question 179 : Let $f(x)=\begin{cases}0, & x \text{ irrational}\\ 1, & x \text{ rational}\end{cases}$, then

(A) $R\int_a^b f(x)\,dx = a-b$

(B) $R\int_a^b f(x)\,dx = 0$

(C) $R\int_a^b f(x)\,dx = 0$

(D) $R\int_a^b f(x)\,dx = b - a$

Solution :

Let $a = x_1 < x_2 < \ldots\ldots < x_n = b$ be a sub-division of [a, b]. Then, we have

$$R\int_a^b f(x)dx = \text{Inf}\sum_{i=1}^{n}(x_i - x_{i-1})M_i$$

where $M_i = \underset{x_{i-1} < x < x_{i-1}}{\text{Sup}}\ f(x) = 1$

$$\text{Inf}\sum_{i=1}^{n}(x_i - x_{i-1}) = (x_n - x_i) = b - a$$

$$\text{Also, } R\int_a^b f(x)dx = \text{Sup}\sum_{i=1}^{n}(x_i - x_{i-1})m_i$$

where $m_i = \underset{x_{i-1} < x < x_{i-1}}{\text{Inf}}\ f(x) = 0$

$$= \sum_{i=1}^{n}(x_i - x_{i-1}).0 = 0$$

Correct Answer: B

Question 180 :

The integral $\iiint\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}}\,dx\,dy\,dz$ taken over the region $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1$ is equal to

(A) $\frac{\pi^2}{4}abc$

(B) $\frac{\pi^2}{3}abc$

(C) $\frac{\pi^2}{5}abc$

(D) $\frac{\pi^2}{7}abc$

Solution :

Consider the spherical polar coordiantes $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, the Jacobian

$$J = \frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$$

Therefore,

$$I = \iiint\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}}\,dx\,dy\,dz$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{1}\sqrt{1-r^2}(abcr^2\sin\theta)\,dr\,d\theta\,d\phi$$

$$abc\int_0^{2\pi}d\phi\int_0^{\pi}\sin\theta\,d\theta\int_0^{1}r^2\sqrt{1-r^2}\,dr$$

$$= 2\pi abc(-\cos\theta)_0^{\pi/2}\int_0^{\pi/2}\sin^2 t\cos^2 t\,dt$$

(putting $r = \sin t$)

$$= \frac{\pi^2}{4}abc$$

Correct Answer: A