APR/MAY-2022

GOVERNMENT OF KARNATAKA
DEPARTMENT OF PRE - UNIVERSITY EDUCATION
II YEAR P.U.C. ANNUAL EXAMINATION

MAY/JUNE - 2021

MAIN ANSWER BOOK

Please read the instructions overleaf before filling in

Answer Book Sl. No.: 1272388

Register No. of the Candidate: 4 0 7 1 5 6

Subject Code: 3 5

Subject: MATHEMATICS

Enter the Serial Number of Map / Graph sheet: 320715

No. of pages used in Main Answer Book	No. of pages used in Additional Answer Book	Total No. of pages used
31	–	31

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23/04/22

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Marks awarded

Part / Question No.											Total Marks
	1	2	3	4	5	6	7	8	9	10	
	1	1		1	0	1	1	1	1	1	08
	11	12	13	14	15	16	17	18	19	20	
	1	1	1 E		1 E		2	0	1 E	1 E	06
	21	22	23	24	25	26	27	28	29	30	
	2	2			2		2	2	2		12
	31	32	33	34	35	36	37	38	39	40	
				3	3	3	3	0	3	3	18
	41	42	43	44	45	46	47	48	49	50	
	3	3	3		2 E			3	3 E		12
	51	52	53	54	55	56	57	58	59	60	
		3 E	5	5		5	5	5			25
	61	62	63	64	65	66	67	68	69	70	
	5			6+4							15
	71	72	73	74	75	76	77	78	79	80	
				R-96							

Grand Total in words: Nity Six only

Grand Total in Figures: 96

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INSTRUCTIONS TO CANDIDATES

1. Write your register number Correctly on the space provided on the Facing Sheet of the Answer book and the top left side of Additional Answer book if any. Over writing should be attested by the Room Invigilator.
2. Write answers in both sides of the sheet using BLUE/BLACK ink or ball point pen.
3. Obtain Additional Booklets, Graph sheets, Mathematical table from the Invigilator if required. Enter the serial numbers of all the Additional Booklets used.
4. Intimate disorders if any, in the Main Answer book/ Additional Booklet to the invigilator.
5. Indicate the Correct question number in the margin.
6. Obtain the permission of the Invigilator for change of PEN / INK.
7. All rough work should be made on a particular page with the heading ROUGH WORK and cross it.
8. Do not write in the margin and leave any page UNUSED except at the end of answers.
9. No Candidate is permitted to leave the examination hall within 30 minutes from the commencement of the examination. Any candidate who leaves after 30 minutes will not be allowed again to the examination hall.
10. If you want to make any request to the Room Invigilator, just stand up to attract his / her attention. Do not shout or leave your place. The invigilator will come to you.
11. During the examination if the candidate wants to go out, for urination etc., same may be informed to the invigilator. While going out, the Answer paper, Question paper etc., should be handed over to the room invigilator for safe custody.
12. After completion, just stand up & inform the same to the Room Invigilator who in turn will collect the papers and gets your signature on the diary maintained by the invigilator.
13. The following misdeeds will attract disciplinary action and criminal prosecution.
    a) Breach of silence.
    b) Use of books, notes, manuscripts, etc., pertaining to the subject in the examination hall.
    c) Talking or signalling to other Candidate.
    d) Candidates copying from the answer books of the other candidates or from other source.
    e) Sending of answer books or additional Booklets or question paper out of the examination hall.
    f) Impersonation.
    g) Taking the answer books or additional Booklet received for writing the answers out of the examination hall during or after the examination.
    h) Tearing or insertion to the answer books and the additional answer book if any.
    i) Writing an appeal or request to the valuator in the answer book.
    j) Mobile Phones, pagers are strictly prohibited in the Examination Hall.
    k) Simple calculators can be used, Scientific calculators allowed only for Statistics paper.
14. After completion of writing, Count the No. of pages used and fill the columns provided on the facing sheet of the main answer book.
15. Candidates suffering from infectious diseases are not allowed to sit in the examination hall.
16. Candidate should strike off the subject which is not applicable.
17. Invigilator should put an END SEAL with his/her signature on the next page of the answer booklet where a student ends his/her writing.

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4	0	7	1	5	6

PART-D

5a] given that $f: R \to R$
Ans defined by $f(x) = 1+x^2$

* Let $x_1, x_2 \in R$ be any elements such that

$f(x_1) = 1+x_1^2$

$f(x_2) = 1+x_2^2$

Let $f(x_1) = f(x_2)$

$\Rightarrow 1+x_1^2 = 1+x_2^2$

$\boxed{x_1^2 = x_2^2}$

$\Rightarrow \boxed{x_1 = x_2}$

$\therefore$ The given function $\boxed{f(x) = 1+x^2}$ is One-One.

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* Let y be an arbitrary element and $y \in R$ having a pre image $x \in R$ such that

$y = f(x) \Rightarrow f(x) = y$

$\Rightarrow y = 1+x^2$

$\Rightarrow x^2 = y-1$

$\boxed{x = \pm\sqrt{y-1}}$

now let us take

$\cancel{f(y)} = f(x) = 1+x^2$

$f(\pm\sqrt{y-1}) = 1+(\pm\sqrt{y-1})^2$

$= 1+y-1$

$= \boxed{y}$

$\therefore$ The given function is Onto

Hence $f: R \to R$ $f(x) = 1+x^2$ is bijective.

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53] given that $f: R \to R$
Ans given by $f(x) = 4x+3$

* Let $x_1, x_2 \in R$ be any elements such that

$f(x_1) = 4x_1 + 3$

$f(x_2) = 4x_2 + 3$

Let $f(x_1) = f(x_2)$

$\Rightarrow 4x_1 + 3 = 4x_2 + 3$

$4x_1 = 4x_2$

$\boxed{x_1 = x_2}$

∴ The given function is $\boxed{\text{One-One.}}$

* Let y be an arbitrary element and $y \in R$ having a pre image x under the function f. such that $f(x) = y$

$\Rightarrow y = 4x+3$

$\Rightarrow \boxed{x = \frac{y-3}{4}}$

now $f(x) = 4x+3$

$f[(y-3)/4] = \cancel{4}\left[\frac{y-\cancel{3}}{\cancel{4}}\right] \cancel{+3}$

$= \boxed{y}$

∴ The given function is Onto.

⇒ Since the function $f(x) = 4x+3$ is both One-One and onto, the function is invertible with $f^{-1}(y) = x$ and f^{-1} is $\boxed{\frac{y-3}{4}}$

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54] given that
Ans

$$A=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix} \quad B=\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{bmatrix}$$

$$C=\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}$$

computing.

* A+B

$$=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}+\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{bmatrix} \rightarrow (1)$$

* $B-C=$

$$\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{bmatrix} \rightarrow (2)$$

we need to verify $A+(B-C)=(A+B)-C$

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LHS = A+(B-C)

$$= \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} \quad [\text{from } ②]$$

$$= \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & +1 & 1 \end{bmatrix}$$

RHS = (A+B)-C

$$= \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -4 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} \quad [\text{from } ①]$$

$$= \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

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∴ LHS = RHS

i.e., $A+(B-C) = (A+B)-C$

$$= \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

Hence verified.

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57] Given that
Ans The lenght of a rectangle is x
The width of a rectangle is y.
and also change with respect to time

$\Rightarrow \frac{dx}{dt} = -3\,cm\,min^{-1}$

$\Rightarrow \frac{dy}{dt} = +2\,cm\,min^{-1}$

(i) W.K.T perimeter $(P) = 2(x+y)$

differentiating w.r.t time

$$\frac{dP}{dt} = 2\left[\frac{dx}{dt} + \frac{dy}{dt}\right]$$

$$\frac{dP}{dt} = 2\left[-3\,cm\,min^{-1} + 2\,cm\,min^{-1}\right]$$

$$\frac{dP}{dt} = 2(-1\,cm\,min^{-1})$$

$$\frac{dP}{dt} = \boxed{-2\,cm\,min^{-1}}$$

$\therefore$ The rate of change of perimeter is $\boxed{-2\,cm\,min^{-1}}$

(ii) W.K.T Area $(A) = xy$
differentiating w.r.t time

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{dA}{dt} = [(10)(2) + 6(-3)]\,cm\,min^{-1}$$

$$\frac{dA}{dt} = (20 - 18)\,cm\,min^{-1}$$

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$\Rightarrow \frac{dA}{dt} = \boxed{2\,cm\,min^{-1}}$

$\therefore$ The rate of change of area is $\boxed{2\,cm\,min^{-1}}$

58] To find the Integral of $\boxed{\frac{1}{x^2-a^2}}$

Ans

Let $I = \int \frac{1}{x^2-a^2}\,dx$

now $\frac{1}{x^2-a^2} = \frac{1}{2a}\left[\frac{(x+a)-(x-a)}{(x+a)(x-a)}\right]$

$= \frac{1}{2a}\left[\frac{-1}{(x+a)} + \frac{1}{(x-a)}\right]$

$= \frac{1}{2a}\left[\frac{1}{(x-a)} - \frac{1}{(x+a)}\right]$

now $I = \int \frac{1}{x^2-a^2}\,dx$

$= \int \frac{1}{2a}\left[\frac{1}{(x-a)} - \frac{1}{(x+a)}\right]dx$

$= \frac{1}{2a}\left[\log|(x-a)| - \log|(x+a)|\right] + C$

$[\because \int 1/x = \log x + c]$

$= \boxed{\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C}$

$\therefore \boxed{I = \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C}$ [Constant of integration]

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From the above result

let $I_1 = \int \frac{1}{x^2-16}\,dx$

$I_1 = \int \frac{1}{x^2-(4)^2}\,dx.$

as $a = 4$

$I_1 = \frac{1}{2\times 4} \log\left|\frac{x-4}{x+4}\right| + c$

$\boxed{I_1 = \frac{1}{8}\log\left|\frac{x-4}{x+4}\right| + c}$

where c is the constant of integration

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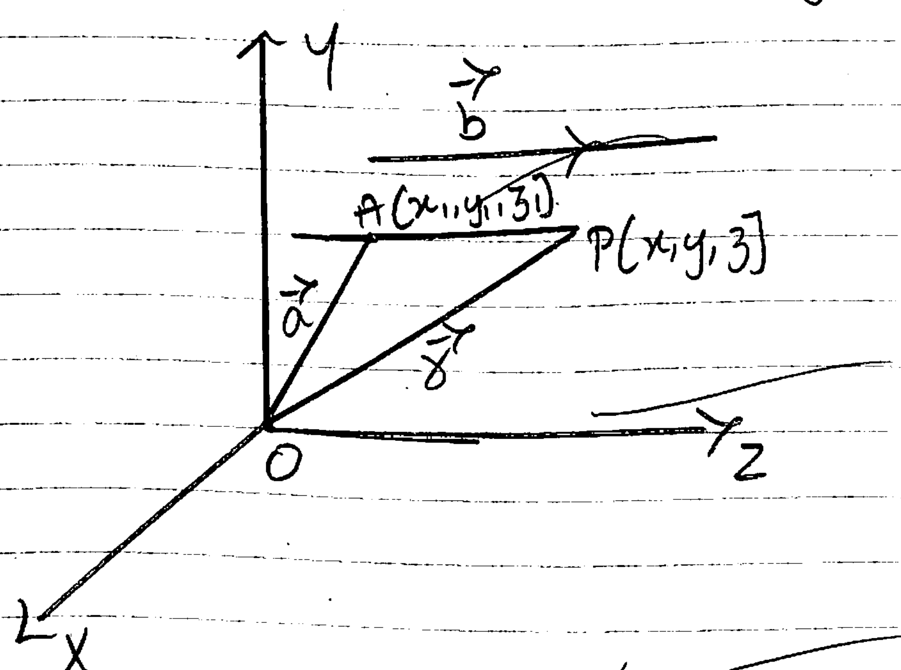
61] Ans



let A be a point with position vector $\vec{a} \Rightarrow \vec{a}$

and P be a point with position vector $\vec{r}$

5

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Vector form:

Let $\vec{b}$ be vector parallel to the line passing through the given point in space. (P)

i.e., $\boxed{\vec{AP} = \lambda\vec{b}} \rightarrow *$

now from the figure

$\vec{AP} = \vec{OP} - \vec{OA}$

$\vec{AP} = (\vec{r} - \vec{a}) \rightarrow (1)$

In (*) substitute equation (1)

i.e., $\vec{r} - \vec{a} = \lambda\vec{b}$

$\Rightarrow \boxed{\vec{r} = \vec{a} + \lambda\vec{b}} \rightarrow (2)$

is the required equation of the line.

Cartesian form:

Let the direction ratios of $\vec{b}$ be (a, b, c)

i.e., $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

and

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

From equation (2)

$x\hat{i} + y\hat{j} + z\hat{k} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$.

equating the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$

$\Rightarrow \boxed{\lambda = \frac{x - x_1}{a}}, \boxed{\lambda = \frac{y - y_1}{b}}, \boxed{\lambda = \frac{z - z_1}{c}}$

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Equating the three equations and eliminat-ing λ we get,

$$\Rightarrow \boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$$

is the required equation of the line.

56) $y=(\tan^{-1}x)^2$

differentiating w.r.t x.

$\frac{dy}{dx} = \frac{2\tan^{-1}x}{1+x^2}$

$(1+x^2)\frac{dy}{dx} = 2\tan^{-1}x$

again differentiating w.r.t x.

$(1+x^2)\frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right] = \frac{2}{1+x^2}$.

$\Rightarrow (1+x^2)^2\frac{d^2y}{dx^2} \quad 2x(x^2+1)\frac{dy}{dx} = 2$

writing $\frac{d^2y}{dx^2}$ as $\boxed{y_2}$

and $\frac{dy}{dx}$ as $\boxed{y_1}$

$\Rightarrow \boxed{(x^2+1)^2y_2 + 2x(x^2+1)y_1 = 2}$

Hence the proof.

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PART-E

64] (a) The objective function is given by $\boxed{Z = 4x+y}$

The given constraints are:

* $x+y \le 50$
* $3x+y \le 90$
* $x \ge 0, y \ge 0$

Equating the constraints we get.

* $x+y=50$

x	0	50
y	50	0

graph region is towards the base

* $3x+y=90$

x	0	30
y	90	0

graph region is towards the base.

* Since $x \ge 0, y \ge 0$, the region is in first quadrant.

∴ The feasible region $\boxed{OACBO}$ is bounded

Corner points	$Z = 4x+y$.
→ (0,0)	$Z=0$
→ (30,0]	$Z=120$
→ (0,50]	$Z=50$
→ (20,30]	$Z=110$.

∴ $\boxed{Z_{max} = 120}$ at the point (30,0]

$Z_{min} = 0$ at the point [0,0].

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320715

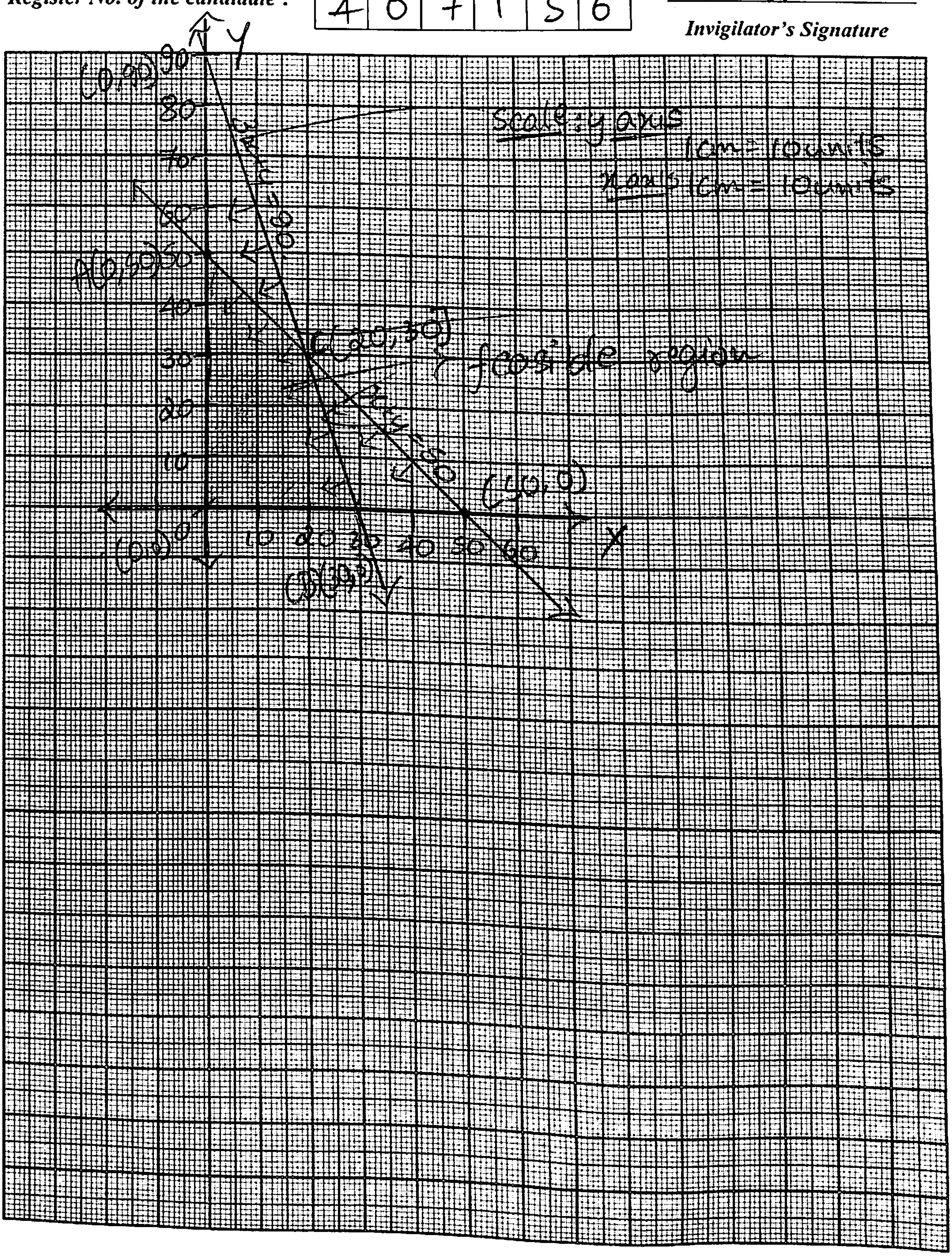
March, 2022

Subject Name : MATHEMATICS

Subject Code : 35 (NS)

Register No. of the candidate : 4 0 7 1 5 6

Invigilator's Signature



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Ans (b) given $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

satisfies the equation $A^2-4A+I=0$

multiplying A^{-1} on both sides

$\Rightarrow (AA^{-1})A-4(A)(A^{-1})+IA^{-1}=0$

$IA-4I+A^{-1}=0 \qquad [\because AA^{-1}=I]$

$\Rightarrow \boxed{A^{-1}=4I-A}$

$\Rightarrow A^{-1}=4\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

$A^{-1}=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}-\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

$\therefore A^{-1}=\begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$

4

PART-C

34] The relation R in the Z is given by

$R=\{(a,b): 2 \text{ divides } (a-b)\}$

$\Rightarrow$* for the relation R,

$(a,a)\in R \Rightarrow (a-a)=0$

and 0 is divisible by 2

$\Rightarrow (a,a)\in R$

$\therefore$ The relation R is $\boxed{\text{reflexive.}}$

$\Rightarrow$* for the relation R,

$(a,b)\in R \Rightarrow (a-b)$ is divisible by 2

and ~~(a-b)~~ $(b-a)$ is also divisible by 2

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i.e, $(b-a)$ is divisible by 2
For example: if $[4,2]$ divisble by 2 $\in R$
the $[2-4]$ is also divisible by 2
and $\therefore (b,a) \in R$.
$\therefore$ The relation is $\boxed{\text{symmetric}}$

* for the relation R
let $(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow$ $(a-b)$ is divisible by 2
$(b-c)$ is divisible by 2.
Since the addition of two numbers divisible by 2 is also divisible by 2
$(a-b)+(b-c) = a-b+b-c$
$= \boxed{(a-c)}$ is also divisible by 2
i.e, $(a,c) \in R$
$\therefore$ The Relation is $\boxed{\text{transitive}}$

Hence the given relation R is an equivalence relation.

35] $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$
Ans

W.K.T $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$

$\Rightarrow \boxed{\tan^{-1}\left[\frac{2x+3x}{1-(2x)(3x)}\right] = \frac{\pi}{4}}$

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$$\Rightarrow \tan^{-1}\left[\frac{5x}{1-6x^2}\right] = \frac{\pi}{4}$$

$$\frac{5x}{1-6x^2} = \tan\frac{\pi}{4}$$

$$\frac{5x}{1-6x^2} = 1 \qquad [\because \tan\pi/4 = 1]$$

$$\Rightarrow 5x = 1-6x^2$$

$$6x^2+5x-1=0$$

$$6x^2+6x-x-1=0$$

$$\cancel{6x(x+1)-6x}$$

2

$$\Rightarrow 6x(x+1)-(x+1)=0$$

$$(6x-1)(x+1)=0$$

$$\therefore \boxed{x=\frac{1}{6}} \text{ or } \boxed{x=-1}$$

36] Ans given that $A=\begin{bmatrix}1&5\\-1&2\end{bmatrix}$

let $\boxed{P=\frac{1}{2}(A+A^T)}$

$$A^T=\begin{bmatrix}1&-1\\5&2\end{bmatrix}$$

$$P=\frac{1}{2}\left[\begin{bmatrix}1&5\\-1&2\end{bmatrix}+\begin{bmatrix}1&-1\\5&2\end{bmatrix}\right]$$

$$P=\frac{1}{2}\begin{bmatrix}2&4\\4&4\end{bmatrix}=\begin{bmatrix}1&2\\2&2\end{bmatrix}$$

$$\Rightarrow P'=\begin{bmatrix}1&2\\2&2\end{bmatrix}=\boxed{P}$$

3

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$\therefore$ P is a symmetric matrix.

* let $Q = \frac{1}{2}(A - A^T)$

$Q = \frac{1}{2}\left[\begin{bmatrix}1 & 5\\-1 & 2\end{bmatrix} - \begin{bmatrix}1 & -1\\5 & 2\end{bmatrix}\right]$

$Q = \frac{1}{2}\left[\begin{bmatrix}0 & 6\\-6 & 0\end{bmatrix}\right]$

$Q = \begin{bmatrix}0 & 3\\-3 & 0\end{bmatrix}$

$\Rightarrow Q' = \begin{bmatrix}0 & -3\\3 & 0\end{bmatrix} \Rightarrow \boxed{-Q}$

$\therefore$ Q is a skew symmetric matrix.

now $A\cancel{B} = P + Q$ [To verify]

$P + Q = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$= \begin{bmatrix}1 & 2\\2 & 2\end{bmatrix} + \begin{bmatrix}0 & 3\\-3 & 0\end{bmatrix}$

$= \begin{bmatrix}1 & 5\\-1 & 2\end{bmatrix} = \cancel{B}$

3

$\therefore$ $P + Q = \cancel{B}\boxed{A}$

$\Rightarrow \cancel{B = \frac{1}{2}(B + B^T) + \frac{1}{2}(B - B^T)}$

$\therefore$ $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

is expressed as sum of symmetric and skew-symmetric matrix.

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37] wehave the property of determinant
$C_2 \to 9C_2 + C_1$

LHS = $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

applying $C_2 \to 9C_2 + C_1$

$= \begin{vmatrix} 2 & 7\times9+2 & 65 \\ 3 & 8\times9+3 & 75 \\ 5 & 9\times9+5 & 86 \end{vmatrix}$

$= \begin{vmatrix} 2 & 65 & 65 \\ 3 & 75 & 75 \\ 5 & 86 & 86 \end{vmatrix}$

since two coloums are equal the value of the determinant is zero

$= \boxed{0}$

= RHS

Hence the proof

2

38] Ans $y = \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$

put $x = \cos\theta$
such that $\theta = \cos^{-1}x. \to (1)$

$y = \cos^{-1}\left[\frac{1-\cos^2\theta}{1+\cos^2\theta}\right]$

$y = \cos^{-1}[\cos 2\theta]$ $\left[\because \cos 2\theta = \frac{1-\cos^2\theta}{1+\cos^2\theta}\right]$

$\boxed{y = 2\theta}$ $[\because \cos(\cos^{-1}\theta) = \theta]$.

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From (1) ~~we~~ get.

$\boxed{y = 2\cos^{-1}x.}$

differentiating w.r.t x we get,

$$\boxed{\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}}$$

~~38]~~

39] Ans given $x = a(\theta - \sin\theta)$ $y = a(1+\cos\theta)$

differentiating ~~both~~ x and y with respect to θ

$$\boxed{\frac{dx}{d\theta} = a(1-\cos\theta)}$$

$$\boxed{\frac{dy}{d\theta} = a(-\sin\theta)}$$

W.K.T $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$= \frac{a(-\sin\theta)}{a(1-\cos\theta)} \Rightarrow \frac{\sin\theta}{\cos\theta - 1}$

$= \frac{2\sin\theta/2 . \cos\theta/2}{(1-2\sin^2\theta/2)-1}$ $[\because \cos 2\theta = \cos^2\theta - 1$
$= 1 - \sin^2\theta$

$= \frac{2\sin\theta/2 . \cos\theta/2}{-2\sin^2\theta/2} = \frac{-\cos\theta/2}{\sin\theta/2}$

$= -\cot\frac{\theta}{2}$

$\therefore \boxed{\frac{dy}{dx} = -\cot\frac{\theta}{2}}$

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40] given that $f(x)=x^2$
To verify for mean value theorem

* $f(x)=x^2$ is a continuous function as every polynomial function is continuous in closed interval $[a,b]$
$\therefore$ $f(x)=x^2$ is a continuous in the closed interval $\boxed{[2,4]}$

* $f(x)=x^2$ is differentiable in the interval $(2,4)$
and also $\boxed{f'(x)=2x.}$

* For mean value theorem, there exists $c \in (2,4)$
such that $\boxed{f'(c)=\frac{f(b)-f(a)}{b-a}.}$

here $b=4; a=2$
$f(b)=16$ and $f(a)=4.$

$\therefore f'(c)=\frac{16-4}{4-2} \Rightarrow \frac{12}{2} \Rightarrow \boxed{6}$

$f'(c)= 2c=6$
$\boxed{c=3}$
and $c \in (2,4)$

$\therefore$ Mean value theorem for the function $f(x)=x^2$ in the interval $[2,4]$ is verified.

8

3

41) given $f(x) = x^2 - 4x + 6$

differentiating w.r.t x

$f'(x) = 2x - 4$

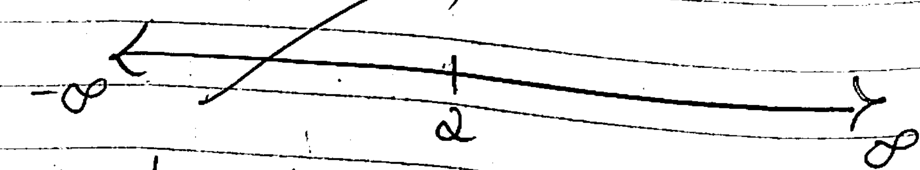
Let us equate $f'(x) = 0$

$2x - 4 = 0$

$2x = 4$

$\boxed{x = 2}$

$\therefore$ This divides the line segment into two parts $(-\infty, 2)$ and $(2, \infty)$



now at point < 2 say $x = 1$

$f'(1) = 2(1) - 4$

$= -2 < 0$

$\therefore$ The function is decreasing in the interval $(-\infty, 2)$

now at point > 2 say $x = 3$

$f'(3) = 2(3) - 4$

$= 2 > 0$

$\therefore$ The function is increasing in the interval $(2, \infty)$

(i) $(2, \infty)$

(ii) $(-\infty, 2)$

2

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42) Let $I = \int \frac{x}{(x+1)(x+2)} dx$

we can express $\frac{x}{(x+1)(x+2)}$ as

$\Rightarrow \frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$\Rightarrow x = A(x+2) + B(x+1)$

$x = Ax + Bx + 2A + B$

$x = x(A+B) + 2A+B$

equating the co-efficients we get

$A+B=1$ and $2A+B=0$.

solving the equations

$\boxed{A=-1}$ and $\boxed{B=2}$

3

$\therefore I = \int \left[\frac{2}{(x+2)} - \frac{1}{(x+1)} \right] dx$

$\boxed{I = 2\log|x+2| - \log|x+1| + C}$

where C is the constant of integration

43) Let $I = \int_0^1 \frac{\tan^{-1}x}{1+x^2} dx$.

put $t = \tan^{-1}x$

such that $dt = \frac{dx}{1+x^2}$

now, when $x=0 \Rightarrow t = \tan^{-1} 0$

$\boxed{t=0}$

when $x=1 \Rightarrow t = \tan^{-1} 1$

$\boxed{t = \pi/4}$

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$\therefore I = \int_0^{\pi/4} t\, dt.$

$I = \left[\frac{t^2}{2}\right]_0^{\pi/4}$

$I = \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^2 - 0^2\right]$

$\boxed{I = \frac{\pi^2}{32}} \Rightarrow \boxed{I = \frac{\pi^2}{32}}$

48] Ans given that

$|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2$

$\vec{a} + \vec{b} + \vec{c} = 0$

squaring on both sides

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}.\vec{a}) = 0.$

$1^2 + 4^2 + 2^2 + 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}.\vec{a}) = 0$

$\therefore \boxed{\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}.\vec{a} = -\frac{21}{2}}$

49] Ans LHS $= [\vec{a}, \vec{b}, \vec{c} + \vec{d}]$

$= \vec{a}.[\vec{b} \times (\vec{c} + \vec{d})]$ [By scalar triple product]

$= \vec{a}[\vec{b} \times \vec{c} + \vec{b} \times \vec{d}]$

$\Rightarrow \vec{a}.(\vec{b} \times \vec{c}) + \vec{a}(\vec{b} \times \vec{d})$

$\Rightarrow [\vec{a}, \vec{b}, \vec{c}] + [\vec{a}, \vec{b}, \vec{d}] = \boxed{RHS}$

2 3 2 3 2 3 6

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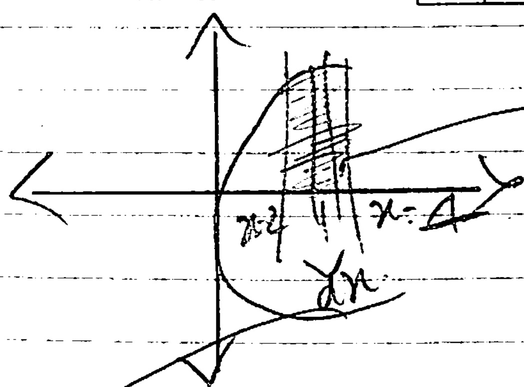
45]
Ans



$\text{area} = \int_2^4 \sqrt{x}\,dx.$

2

$\text{area} = \frac{2}{3}\left(x^{3/2}\right)_2^4$

$= \boxed{\frac{2}{3}\left[(4^2)^{3/2} - 2^{3/2}\right]}$

PART-B

17]
Ans

$\sin^{-1}(2x\sqrt{1-x^2})$

~~since~~, $\frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

put $\boxed{x = \sin\theta}$

~~such that~~ $\theta = \sin^{-1}x. \rightarrow (1)$

$\sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta}]$

$= \sin^{-1}(2\sin\theta\cos\theta]$

$\Rightarrow \sin^{-1}(2\sin 2\theta] \Rightarrow \boxed{2\theta}$ $(\because 2\sin\theta\cos\theta = \sin 2\theta]$

2

$\Rightarrow \boxed{2\sin^{-1}x}$ (From (1)]

Hence the proof

2

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18] $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$

Ans w.k.T

$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$\cot^{-1}(-\sqrt{3}) = -\frac{\pi}{6}$

0

$\Rightarrow \frac{\pi}{3} - \left[-\frac{\pi}{6}\right]$

$\Rightarrow \boxed{\frac{\pi}{2}}$

19] $X+Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \rightarrow ①$ $X-Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \rightarrow ②$

adding ① and ②

1

$X+Y+X-Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

~~So~~ substituting it in ~~2~~ ①

$\cancel{X = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}}$ $Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

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20] vertices $(2,7]$ $(1,1]$ $(10,8]$

Ans $x_1\ y_1$ $x_2\ y_2$ $x_3\ y_3$

$$\text{area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}$$

$$= \frac{1}{2}\left[(2)(1-8] - 7(1-10]+1(8-10]\right]$$

$$= \frac{1}{2}[-14-63-2]$$

$$= \boxed{\frac{-79}{2}}$$

Since area cannot be negative

$$\boxed{\text{area} = \frac{79}{2}\ \text{sq units}}$$

21] $2x+3y=\sin x$

differentiate w.r.t x

$$2+3\frac{dy}{dx} = \cos x$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x - 2}{3}}$$

3

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22] Let $y = x^{\sin x}$

& apply log on both sides

$\log y = \sin x . \log x$

2

$$\frac{dy}{dx} = y\left[\cos x . \log x + \frac{\sin x}{x}\right]$$

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{x\cos x . \log x + \sin x}{x}\right]$$

28] Order = 2
degree = Not defined.

2

29] W.K.T

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

$$\vec{r} = \frac{2(-\hat{i} + \hat{j} + \hat{k}) + 1(\hat{i} + 2\hat{j} - \hat{k})}{2+1}$$

2

$$\vec{r} = \frac{-2\hat{i} + 2\hat{j} + 2\hat{k} + \hat{i} + 2\hat{j} - \hat{k}}{3}$$

$$\boxed{\vec{r} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}}$$

27] $\int_0^{\pi/4} \sin 2x\,dx$

Ans

$$\Rightarrow \left[-\frac{\cos 2x}{2}\right]_0^{\pi/4}$$

2

$$\Rightarrow -\frac{1}{2}\left[+\cos 2\left(\frac{\pi}{4}\right) - \cos 0\right] = \boxed{\frac{1}{2}}$$

8

25] $I = \int x^2 \log x \, dx$

$u = \log x$ and $v = x^2$

$\log x \int x^2 dx - \int \left[\frac{d}{dx}(\log x) \int x^2 dx\right] dx.$

$\frac{\log x \cdot x^3}{3} - \int \left(\frac{1}{x} \times \frac{x^3}{3}\right) dx$

$\frac{x^3 \log x}{3} - \frac{1}{3} \times \frac{x^3}{3} + C$

$\Rightarrow \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$

$\Rightarrow \boxed{\frac{3x^3 \log x - x^3}{9} + C}$

2

PART-A

1] In the relation R

~~(2,2) ∈ R and (2,3) ∈ R~~

~~but (~~

$(1,2) \in R$ and $(2,3) \in R$

but $(1,3) \notin R$

$\therefore$ Not transitive.

2] $5 * 7$ is the LCM of 5 and 7

i.e., $\boxed{35}$

4] W.K.T $\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$.

$\cos(\sec^{-1}x + \csc^{-1}x) = \cos\frac{\pi}{2}$

$= \boxed{0}$

5

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5] A matrix in which its non-diagonal elements are equal to zero is called a diagonal matrix.

0

6] $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$10-12 = 5x-6x.$

$\boxed{x=2}$

7] $y=\sin(ax+b)$

$\boxed{\frac{dy}{dx} = a\cos(ax+b)}$

8] e^{x^3}

differentiating it w.r.t x.

$\boxed{3x^2e^{x^3}}$

9] $\int \sec x(\sec x+\tan x]dx.$

$\int(\sec^2x+\sec x\tan x)dx$

$\Rightarrow \tan x+\sec x+C$

where C is constant of integration

10] $\int_2^3 x^2dx$

$\Rightarrow \frac{1}{3}[x^3]_2^3$

$\Rightarrow \frac{1}{3}(27-8) \Rightarrow \boxed{\frac{19}{3}}$

5

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11] $|\vec{a}| = \sqrt{4+9+1}$

$|\vec{a}| = \sqrt{14}$

$$\boxed{\hat{a} = \frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k}}$$

1

12] Two vectors, if parallel to the same line, irrespective of their magnitude and direction is called collinear vectors

1

13] $[0, 1, 0]$

1

15] $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{0.2}{0.3} = \dfrac{2}{3} \approx \boxed{0.67}$

1

ಮುಕ್ತಾಯವಾಗಿದೆ / THE END.

Signature of the Invigilator ....

2