Assignment 2

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1 2018-ICSE-12th board-Problem

Problem 2: If the function $f(x) = \sqrt{2x-3}$ is invertible then find its inverse. Hence prove that $(fof^{-1})(x) = x$.

Solution:

- **Verifying the invertibility of the function:**
  Given function,

$$f(x) = \sqrt{2x-3} \tag{1.1}$$

  We know that, for a function to be invertible, it should be an injective as well as a subjective function too.

  – **Verification for injective function:**
  We can use ”Horizontal line test” for verifying this. Plotting the graph using the python functions, we get

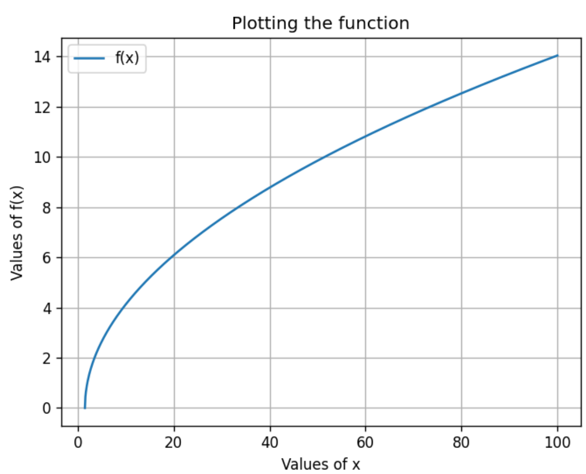

Fig. 0. Plotting the function, $f(x) = \sqrt{2x-3}$

  We can clearly see, by drawing any horizontal straight line, we can't make the line touch the graph twice.
  $\therefore$ $f(x)$ is an one to one, i.e., an injective function.

  – **Verification for subjective function:**
  We can verify, if the function is strictly increasing or decreasing by taking its ”Derivative test” with respect to 'x'.
  Given,

$$f(x) = \sqrt{2x-3} \tag{1.2}$$

Differentiating on both sides with respect to $'x'$,

$$\frac{df(x)}{dx} = \frac{d}{dx}\sqrt{2x-3} \tag{1.3}$$

$$\frac{df(x)}{dx} = \frac{1}{2\sqrt{2x-3}}(2) \tag{1.4}$$

$$\therefore \frac{df(x)}{dx} = \frac{1}{\sqrt{2x-3}} \tag{1.5}$$

Since, we know that $\sqrt{2x-3}$ is always positive. Therefore, this implies that $\frac{df(x)}{dx}$ is always positive.

$$\frac{1}{\sqrt{2x-3}} > 0 \tag{1.6}$$

$$\therefore \frac{df(x)}{dx} > 0 \tag{1.7}$$

Hence, we can say that the function, f(x) is always increasing. Therefore, f(x) covers all the values of its co-domain.
$\therefore$ $f(x)$ is an onto, i.e., a subjective function.

- **Finding the inverse of the function:**
  Since, we found out that the function, f(x) is a bijective function (both injective and subjective). Therefore, its inverse exists.
  We can find the inverse of the function, by swapping x with $f^{-1}(x)$ and $f(x)$ with x. Hence we get,

$$x = \sqrt{2(f^{-1}(x)) - 3} \tag{1.8}$$

$$x^2 = 2(f^{-1}(x)) - 3 \tag{1.9}$$

$$\therefore f^{-1}(x) = \frac{x^2+3}{2} \tag{1.10}$$

We can even verify, the above obtained inverse function by plotting and checking an inverse function property.

Inverse function property: It states that, the original function($f(x)$) and its inverse($f^{-1}(x)$) are always mirror images in the line, y = x.