Assignment-6

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TABLE of CONTENTS

1. Question
2. Solution : Theorems involved - 1
3. Solution : Theorems involved - 2
4. Solution : deductions
5. Solution (i)
6. Solution (ii)

Problem 5-7

We place at random 200 points in the interval (0, 100). The distance from 0 to the first random point is a random variable z. Find $F_z(z)$.

1. exactly, and
2. using the Poisson approximation.

Solution

The following theorem would be involved in the problem.

FUNDAMENTAL THEOREM:

$$p_n(k) = \mathrm{P}(A \text{ occurs } k \text{ times in any order}) \tag{1}$$
$$= \binom{n}{k} p^k q^{n-k} \tag{2}$$

Solution

The following theorem would be involved in the problem.

POISSON THEOREM:

If

$$n \to \infty; \qquad p \to 0; \qquad \text{such that } np \to \lambda$$

then for $k = 0, 1, 2..,$

$$\frac{n!}{k!(n-k)!} p^k q^{n-k} \xrightarrow[n \to \infty]{} e^{-\lambda} \frac{\lambda^k}{k!} \tag{3}$$

Solution : deductions

Clearly, $z' \leq z$ iff the number $\mathrm{n}(0, z)$ of the points in the interval $(0, z)$ is atleast one.
Hence,

$$F_z(z) = \mathrm{P}(z' \leq z) \tag{4}$$
$$= \mathrm{P}(\mathrm{n'}(0, z) > 0) \tag{5}$$
$$F_z(z) = 1 - \mathrm{P}(\mathrm{n'}(0, z) = 0) \tag{6}$$

The probability p that a particular point is in the interval $(0, z)$ equals $\frac{z}{100}$.
With $n = 200, k = 0$ and $p = \frac{z}{100}$, from the fundamental theorem it gives $\mathrm{P}(\mathrm{n'}(0, z) = 0) = (1 - p)^{200}$.

Solution (i)

Hence from the fundamental theorem, we can conclude that

$$F_z(z) = 1 - (1 - \frac{z}{100})^{200} \tag{7}$$

Solution (ii)

From the poisson's theorem, it follows that for $z \ll 100$,

$$F_z(z) \simeq 1 - e^{-2z} \tag{8}$$