

# High School Assignment

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## 1 2018-ICSE-10th board-Problem : 8(b)

**Problem:** If the mean of the following distribution is 24, find the value of 'a'.

| Marks              | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|--------------------|------|-------|-------|-------|-------|
| Number of students | 7    | a     | 8     | 10    | 5     |

**Solution:** Given, the mean of the following distribution is,  $m = 24$ .

We know that,

$$mean(m) = \frac{\sum f_i x_i}{\sum f_i} \quad (1)$$

This can also be written as

$$mean(m) = \frac{\vec{f}^T \vec{x}}{\vec{1}^T \vec{f}} \quad (2)$$

As per the question,

| Intervals | Frequency ( $f_i$ ) | Mid-Value ( $x_i$ ) |
|-----------|---------------------|---------------------|
| 0-10      | 7                   | 5                   |
| 10-20     | a                   | 15                  |
| 20-30     | 8                   | 25                  |
| 30-40     | 10                  | 35                  |
| 40-50     | 5                   | 45                  |

Therefore, from the above table we can

deduce the following vectors,

$$\vec{f} = \begin{bmatrix} 7 \\ a \\ 8 \\ 10 \\ 5 \end{bmatrix}; \vec{x} = \begin{bmatrix} 5 \\ 15 \\ 25 \\ 35 \\ 45 \end{bmatrix}$$

To find the value of 'a', we can simplify the equation (2),

$$mean(m) = \frac{\vec{f}^T \vec{x}(\text{without 'a'}) + \vec{f}^T \vec{x}(\text{with 'a'})}{\vec{1}^T \vec{f}(\text{without 'a'}) + \vec{1}^T \vec{f}(\text{with 'a'})}$$

Taking the dot product,

$$\vec{f}^T \vec{x}(\text{without 'a'}) = \begin{bmatrix} 7 \\ 8 \\ 10 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 25 \\ 35 \\ 45 \end{bmatrix} = [35+200+350+225].$$

$$\vec{f}^T \vec{x}(\text{with 'a'}) = [a] \cdot [15] = [15a].$$

$$\vec{1}^T \vec{x}(\text{without 'a'}) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 8 \\ 10 \\ 5 \end{bmatrix} = [7+8+10+5].$$

$$\vec{1}^T \vec{x}(\text{with 'a'}) = [1] \cdot [a] = [a].$$

On substituting the following above values in the equation, we get

$$mean(m) = \frac{[35 + 200 + 350 + 225] + [15a]}{[7 + 8 + 10 + 5] + [a]}$$

$$mean(m) = \frac{810 + 15a}{30 + a}$$

$$24(30 + a) = 810 + 15a$$

$$720 + 24a = 810 + 15a$$

$$9a = 90$$

$$a = 10$$

Therefore, the required value(a) is 10.