Assignment 4

K Vivek Kumar - CS21BTECH11026

I. Class-10-Probability-Exercise-15.1

Question 25: Which of the following arguments are correct and which are not correct? Give reasons for your answer.

(i) If coins are tossed simultaneously there are three possible outcomes-two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

(ii) If a die is thrown, there are two possible outcomes-an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Solution: We can individually find the probability using random variables.

(i) The possible events are (H,H); (H,T); (T,H) and (T,T). Lets take a random variable 'X', which maps to a real number and each possible outcomes can correspond to a real number.

$$\text{(H,H)} \to 0 \qquad \text{(H,T)} \to 1$$
$$\text{(T,H)} \to 2 \qquad \text{(T,T)} \to 3$$

$$\therefore X \in \{0,1,2,3\}$$

For each of the following case, finding the probability.

- **Case when both are heads:**
  As we know that,

  $$\text{(H,H)} \to 0$$

  Therefore, X here corresponds to the real number '0'.

  $$\text{P}\left(X=0\right) = \frac{\text{n}\left(X=0\right)}{\text{n}\left(X\right)} \tag{I.1}$$
  $$= \frac{1}{4} = 0.25 \tag{I.2}$$

  $\therefore$ The probability of obtaining both heads is 0.25.
- **Case when both are tails:**
  As we know that,

  $$\text{(T,T)} \to 3$$

  Therefore, X here corresponds to the real number '3'.

  $$\text{P}\left(X=3\right) = \frac{\text{n}\left(X=3\right)}{\text{n}\left(X\right)} \tag{I.3}$$
  $$= \frac{1}{4} = 0.25 \tag{I.4}$$

  $\therefore$ The probability of obtaining both tails is also 0.25.
- **Case when one is head and other is tail:**
  As we know that,

  $$\text{(H,T)} \to 1 \qquad \text{(T,H)} \to 2$$

  Therefore, X here corresponds to the real numbers, '1' and '2'.

  $$\text{P}\left(X=(1,2)\right) = \text{P}\left(X=1\right) + \text{P}\left(X=2\right) \tag{I.5}$$
  $$= \frac{\text{n}\left((X=1)\right) + \text{n}\left((X=2)\right)}{\text{n}\left(X\right)} \tag{I.6}$$
  $$= \frac{2}{4} = \frac{1}{2} = 0.5 \tag{I.7}$$

  $\therefore$ The probability of obtaining either one on each coin is 0.5.

Therefore, this statement is incorrect.

(ii) As the outcomes from a die are real number, therefore, we can assign a random variable 'Y', such that it maps to the possible outcomes from a die,i.e.,

$$\therefore Y \in \{1,2,3,4,5,6\}$$

Now for each of the following case, we can find the probability as follows.

- **Case when there is an odd number:** The possible outcomes are $\{1, 3, 5\}$,

$$\mathrm{P}\left(Y = \{1, 3, 5\}\right) = \mathrm{P}\left(Y = 1\right) + \mathrm{P}\left(Y = 3\right) + \left(\mathrm{P}\left(Y = 5\right) \tag{I.8}$$

$$\mathrm{P}\left(Y = \{1, 3, 5\}\right) = \frac{\mathrm{n}\left(\{1, 3, 5\}\right)}{\mathrm{n}\left(Y\right)} \tag{I.9}$$

$$= \frac{3}{6} = \frac{1}{2} = 0.5 \tag{I.10}$$

  $\therefore$ The probability of obtaining an odd number on the die is 0.5.

- **Case when there is an even number:** The possible outcomes are $\{2, 4, 6\}$,

$$\mathrm{P}\left(Y = \{2, 4, 6\}\right) = \mathrm{P}\left(Y = 2\right) + \mathrm{P}\left(Y = 4\right) + \left(\mathrm{P}\left(Y = 6\right) \tag{I.11}$$

$$\mathrm{P}\left(Y = \{2, 4, 6\}\right) = \frac{\mathrm{n}\left(\{2, 4, 6\}\right)}{\mathrm{n}\left(Y\right)} \tag{I.12}$$

$$= \frac{3}{6} = \frac{1}{2} = 0.5 \tag{I.13}$$

  $\therefore$ The probability of obtaining an even number on the die is also 0.5.

Therefore, this statement is correct.