Assignment-9

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Papoulis-Chapter-15

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Problem 15-7

Show that the sums $s_n = x_1 + x_2 + ... + x_n$ of independent zero mean random variables form a martingale.

Solution : Property involved

The following property would be involved in the problem.

Property:

A random sequence x_n is called a martingale if $E\{x_n = 0\}$ and

$$E\{x_n | x_{n-1}, x_{n-2}, ..., x_1\} = x_{n-1} \tag{1}$$

Solution : I

Given,

$$s_n = x_1 + x_2 + ... + x_n \tag{2}$$

where, x_n are i.i.d. random variables. We have

$$s_{n+1} = s_n + x_{n+1} \tag{3}$$

Solution : II

So from the property we can say that,

$$E\{s_{n+1}|s_n\} = E\{s_n + x_{n+1}|s_n\} \tag{4}$$
$$= s_n + E\{x_{n+1}\} \tag{5}$$
$$= s_n \tag{6}$$

Hence, $\{s_n\}$ represents a Martingale.