Assignment 8

K Vivek Kumar - CS21BTECH11026

I. Papoulis-Chapter-10

Question 10-17: Find the power spectrum $S(\omega)$ of a process $x(t)$ if $S(\omega) = 0$ for $|\omega| > \pi$ and

$$E\{x(n+m)x(n)\} = N\delta[m]$$

Solution: Given,

$$E\{x(n+m)x(n)\} = N\delta[m] \tag{I.1}$$

As we know, with $T = \frac{\pi}{\sigma}$

$$R(mT) = E\{x(nT+mT)x(nT)\} = \begin{cases} I & m = 0 \\ \eta^2 & m \neq 0 \end{cases} \tag{I.2}$$

Hence, from the following equation,

$$R(\tau) = \sum_{n=-\infty}^{\infty} R(nT)\frac{\sin(\sigma(\tau - nT))}{\sigma(\tau - nT)} \tag{I.3}$$

We can write,

$$R(\tau) = \sum_{m=-\infty}^{\infty} R(mT)\frac{\sin(\sigma(\tau - mT))}{\sigma(\tau - mT)} \tag{I.4}$$

$$= \eta^2 + (I - \eta^2)\frac{\sin\sigma\tau}{\pi\tau} \tag{I.5}$$

Therefore,

$$S(\omega) = 2\pi\eta^2\delta(\omega) + 2\pi(I - \eta^2)p_\sigma(\omega) \tag{I.6}$$

As it is said that $S(\omega) = 0$ for $|\omega| > \pi$, therefore we can say from the equation I.6, $\eta = 0$ and $I = N$. On substituting it in the equation,

$$S(\omega) = 2\pi(0)^2\delta(\omega) + 2\pi(N - (0)^2)p_\sigma(\omega) \tag{I.7}$$

$$= 2\pi N p_\sigma(\omega) \tag{I.8}$$

$\therefore$ The power spectrum $S(\omega)$ is $2\pi N p_\sigma(\omega)$.