Assignment 9

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I. PAPOULIS-CHAPTER-15

Question 15-7: Show that the sums $s_n = x_1 + x_2 + ... + x_n$ of independent zero mean random variables form a martingale.

Solution: Given,

$$s_n = x_1 + x_2 + ... + x_n \qquad (\text{I.1})$$

where, x_n are i.i.d. random variables. We have

$$s_{n+1} = s_n + x_{n+1} \qquad (\text{I.2})$$

Property: A random sequence x_n is called a martingale if $E\{x_n = 0\}$ and

$$E\{x_n | x_{n-1}, x_{n-2}, ..., x_1\} = x_{n-1} \qquad (\text{I.3})$$

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so that,

$$E\{s_{n+1}|s_n\} = E\{s_n + x_{n+1}|s_n\} \qquad (\text{I.4})$$
$$= s_n + E\{x_{n+1}\} \qquad (\text{I.5})$$
$$= s_n \qquad (\text{I.6})$$

Hence, $\{s_n\}$ represents a Martingale.