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High School Assignment

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1 2018-ICSE-10th board-Problem

Problem 8(b): If the mean of the following distribution is 24, find the value of 'a'.

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	7	a	8	10	5

Solution: Given, the mean of the following distribution is, m = 24.

We know that,

$$m = \frac{\mathbf{f}^T \mathbf{x}}{\mathbf{1}^T \mathbf{f}} \tag{1}$$

where, f is the frequency vector and x is the midvalue's vector. As per the question,

Intervals	Frequencies	Mid-Values	
0-10	7	5	
10-20	a	15	
20-30	8	25	
30-40	10	35	
40-50	5	45	

Therefore, from the above table we can deduce the following vectors,

$$\mathbf{f} = \begin{pmatrix} 7 \\ a \\ 8 \\ 10 \\ 5 \end{pmatrix}; \mathbf{x} = \begin{pmatrix} 5 \\ 15 \\ 25 \\ 35 \\ 45 \end{pmatrix}$$

On substituting the following above values in the equation, we get

$$m = \frac{\begin{pmatrix} 7 \\ 8 \\ 10 \\ 5 \end{pmatrix}^{T} \begin{pmatrix} 5 \\ 25 \\ 35 \\ 45 \end{pmatrix} + (a)^{T} (15)}{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}^{T} \begin{pmatrix} 7 \\ 8 \\ 10 \\ 5 \end{pmatrix} + (1)^{T} (a)}$$
(4)

Taking the transpose, we get

$$m = \frac{\begin{pmatrix} 7 & 8 & 10 & 5 \end{pmatrix} \begin{pmatrix} 5 \\ 25 \\ 35 \\ 45 \end{pmatrix} + (a) (15)}{\begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 7 \\ 8 \\ 10 \\ 5 \end{pmatrix} + (1) (a)}$$
(5)

$$m = \frac{(35 + 200 + 350 + 225) + (15a)}{(7 + 8 + 10 + 5) + (a)} \tag{6}$$

$$m = \frac{810 + 15a}{30 + a} \tag{7}$$

$$24(30+a) = 810 + 15a \tag{8}$$

$$720 + 24a = 810 + 15a \tag{9}$$

$$9a = 90 \tag{10}$$

$$\therefore a = 10 \tag{11}$$

(2) Therefore, the required value(a) is 10.

To find the value of 'a', we can simplify the equation (1),

$$m = \frac{\mathbf{f}^T \mathbf{x}_{(i \neq a)} + \mathbf{f}^T \mathbf{x}_{(i=a)}}{\mathbf{1}^T \mathbf{f}_{(i \neq a)} + \mathbf{1}^T \mathbf{f}_{(i=a)}}$$
(3)