High School Assignment

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1 2018-ICSE-10TH BOARD-PROBLEM

Problem 8(b): If the mean of the following distribution is 24, find the value of 'a'.

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	7	a	8	10	5

Solution: Given, the mean of the following distribution is, m = 24.
We know that,

$$mean(m) = \frac{\sum f_i x_i}{\sum f_i} \tag{1.1}$$

This can also be written as

$$mean(m) = \frac{\mathbf{f}^T\mathbf{x}}{\mathbf{1}^T\mathbf{f}} \tag{1.2}$$

As per the question,

Intervals	Frequencies	Mid-Values
0-10	7	5
10-20	a	15
20-30	8	25
30-40	10	35
40-50	5	45

Therefore, from the above table we can deduce the following vectors,

$$\mathbf{f} = \begin{pmatrix} 7 \\ a \\ 8 \\ 10 \\ 5 \end{pmatrix}; \mathbf{x} = \begin{pmatrix} 5 \\ 15 \\ 25 \\ 35 \\ 45 \end{pmatrix}$$

To find the value of 'a', we can simplify the equation (2),

$$mean(m) = \frac{\mathbf{f}^T\mathbf{x}\text{(without 'a')} + \mathbf{f}^T\mathbf{x}\text{(with 'a')}}{\mathbf{1}^T\mathbf{f}\text{(without 'a')} + \mathbf{1}^T\mathbf{f}\text{(with 'a')}}$$

Taking the dot product,

$$\mathbf{f}^T\mathbf{x}\text{(without 'a')} = \begin{pmatrix} 7 \\ 8 \\ 10 \\ 5 \end{pmatrix} . \begin{pmatrix} 5 \\ 25 \\ 35 \\ 45 \end{pmatrix} = \begin{pmatrix} 810 \end{pmatrix}.$$

$$\mathbf{f}^T\mathbf{x}\text{(with 'a')} = \begin{pmatrix} a \end{pmatrix} . \begin{pmatrix} 15 \end{pmatrix} = \begin{pmatrix} 15a \end{pmatrix}.$$

$$\mathbf{1}^T\mathbf{x}\text{(without 'a')} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} . \begin{pmatrix} 7 \\ 8 \\ 10 \\ 5 \end{pmatrix} = \begin{pmatrix} 7+8+10+5 \end{pmatrix}.$$

$$\mathbf{1}^T\mathbf{x}\text{(with 'a')} = \begin{pmatrix} 1 \end{pmatrix} . \begin{pmatrix} a \end{pmatrix} = \begin{pmatrix} a \end{pmatrix}.$$

On substituting the following above values in the equation, we get

$$mean(m) = \frac{(35+200+350+225)+(15a)}{(7+8+10+5)+(a)}$$

$$mean(m) = \frac{810+15a}{30+a}$$

$$24(30+a) = 810+15a$$

$$720+24a = 810+15a$$

$$9a = 90$$

$$a = 10$$

Therefore, the required value(a) is 10.