Assignment-8

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Papoulis-Chapter-10

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Problem 10-17

Find the power spectrum $S(\omega)$ of a process $x(t)$ if $S(\omega) = 0$ for $|\omega| > \pi$ and

$$E\{x(n+m)x(n)\} = N\delta[m]$$

Solution : Properties involved - 1

The following property is involved in the problem.

Property 1:

As we know from $T = \frac{\pi}{\sigma}$,

$$R(mT) = E\{x(nT + mT)x(nT)\} = \begin{cases} I & m = 0 \\ \eta^2 & m \neq 0 \end{cases} \quad (1)$$

Solution : Properties involved - 2

The following property is involved in the problem.

Property 2:

$$R(\tau) = \sum_{n=-\infty}^{\infty} R(nT) \frac{\sin(\sigma(\tau - nT))}{\sigma(\tau - nT)} \tag{2}$$

Solution - I

Given,

$$E\{x(n+m)x(n)\} = N\delta[m] \tag{3}$$

Therefore, from the property 2, we can deduce the following equation,

$$R(\tau) = \sum_{m=-\infty}^{\infty} R(mT)\frac{\sin(\sigma(\tau - mT))}{\sigma(\tau - mT)} \tag{4}$$

$$= \eta^2 + (I - \eta^2)\frac{\sin \sigma\tau}{\pi\tau} \tag{5}$$

Solution - II

Therefore,

$$S(\omega) = 2\pi\eta^2\delta(\omega) + 2\pi(I - \eta^2)p_\sigma(\omega) \tag{6}$$

As it is said that $S(\omega) = 0$ for $|\omega| > \pi$, therefore we can say from the equation 6, $\eta = 0$ and $I = N$. On substituting it in the equation,

$$S(\omega) = 2\pi(0)^2\delta(\omega) + 2\pi(N - (0)^2)p_\sigma(\omega) \tag{7}$$

$$= 2\pi N p_\sigma(\omega) \tag{8}$$

$\therefore$ The power spectrum $S(\omega)$ is $2\pi N p_\sigma(\omega)$.