Assignment 7

K Vivek Kumar - CS21BTECH11026

I. Papoulis-Exercise-5

Question 5-7: We place at random 200 points in the interval (0, 100). The distance from 0 to the first random point is a random variable z. Find $F_z(z)$.

1) exactly, and
2) using the Poisson approximation.

Solution: Clearly, $z' \leq z$ iff the number n$(0,z)$ of the points in the interval $(0,z)$ is atleast one. Hence,

$$F_z(z) = \mathrm{P}\left\{z' \leq z\right\} \tag{I.1}$$
$$= \mathrm{P}\left\{\text{n'}(0,z) > 0\right\} \tag{I.2}$$
$$F_z(z) = 1 - \mathrm{P}\left\{\text{n'}(0,z) = 0\right\} \tag{I.3}$$

The probability p that a particular point is in the interval $(0,z)$ equals $\frac{z}{100}$. With $n = 200, k = 0$ and $p = \frac{z}{100}$, from the fundamental theorem

FUNDAMENTAL THEOREM:

$$p_n(k) = \text{P}(A \text{ occurs } k \text{ times in any order}) \tag{I.4}$$
$$= \binom{n}{k} p^k q^{n-k} \tag{I.5}$$

it yields that $\mathrm{P}\left\{\text{n'}(0,z) = 0\right\} = (1-p)^{200}$. Hence,

1) $$F_z(z) = 1 - \left(1 - \frac{z}{100}\right)^{200} \tag{I.6}$$
2) From the following theorem statement,

POISSON THEOREM: If

$$n \to \infty; \qquad p \to 0; \qquad \text{such that } np \to \lambda$$

then for $k = 0, 1, 2..$,

$$\frac{n!}{k!(n-k)!} p^k q^{n-k} \xrightarrow[n \to \infty]{} e^{-\lambda} \frac{\lambda^k}{k!} \tag{I.7}$$

It follows that for $z \ll 100$,

$$F_z(z) \simeq 1 - e^{-2z} \tag{I.8}$$