High School Assignment

K Vivek Kumar

1 2018-ICSE-10TH BOARD-PROBLEM

Problem 8(b): If the mean of the following distribution is 24, find the value of 'a'.

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	7	a	8	10	5

Solution: Given, the mean of the following distribution is, m = 24.
We know that,

$$m = \frac{\mathbf{f}^T \mathbf{x}}{\mathbf{1}^T \mathbf{f}} \tag{1.1}$$

where, $\mathbf{f}$ is the frequency vector and $\mathbf{x}$ is the mid-value's vector. As per the question,

Intervals	Frequencies	Mid-Values
0-10	7	5
10-20	a	15
20-30	8	25
30-40	10	35
40-50	5	45

Therefore, from the above table we can deduce the following vectors,

$$\mathbf{f} = \begin{pmatrix} 7 \\ a \\ 8 \\ 10 \\ 5 \end{pmatrix} ; \mathbf{x} = \begin{pmatrix} 5 \\ 15 \\ 25 \\ 35 \\ 45 \end{pmatrix} \tag{1.2}$$

To find the value of 'a', we can simplify the equation (1.1),

$$m = \frac{\mathbf{f}^T \mathbf{x}_{(i \neq a)} + \mathbf{f}^T \mathbf{x}_{(i = a)}}{\mathbf{1}^T \mathbf{f}_{(i \neq a)} + \mathbf{1}^T \mathbf{f}_{(i = a)}} \tag{1.3}$$

On substituting the following above values in the equation, we get

$$m = \frac{\begin{pmatrix} 7 \\ 8 \\ 10 \\ 5 \end{pmatrix}^T \begin{pmatrix} 5 \\ 25 \\ 35 \\ 45 \end{pmatrix} + (a)^T (15)}{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} 7 \\ 8 \\ 10 \\ 5 \end{pmatrix} + (1)^T (a)} \tag{1.4}$$

Taking the transpose, we get

$$m = \frac{\begin{pmatrix} 7 & 8 & 10 & 5 \end{pmatrix} \begin{pmatrix} 5 \\ 25 \\ 35 \\ 45 \end{pmatrix} + (a)(15)}{\begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 7 \\ 8 \\ 10 \\ 5 \end{pmatrix} + (1)(a)} \tag{1.5}$$

$$m = \frac{(35 + 200 + 350 + 225) + (15a)}{(7 + 8 + 10 + 5) + (a)} \tag{1.6}$$

$$m = \frac{810 + 15a}{30 + a} \tag{1.7}$$

$$24(30 + a) = 810 + 15a \tag{1.8}$$

$$720 + 24a = 810 + 15a \tag{1.9}$$

$$9a = 90 \tag{1.10}$$

$$\therefore a = 10 \tag{1.11}$$

Therefore, the required value(a) is 10.