



# Language Modeling

# Advanced: Kneser-Ney Smoothing



## Resulting Good-Turing numbers

- Numbers from Church and Gale (1991)
- 22 million words of AP Newswire

$$c^* = \frac{(c+1)N_{c+1}}{N_c}$$

- It sure looks like  $c^* = (c - .75)$

Count c	Good Turing c*
0	.0000270
1	0.446
2	1.26
3	2.24
4	3.24
5	4.22
6	5.19
7	6.21
8	7.24
9	8.25



# Absolute Discounting Interpolation

- Save ourselves some time and just subtract 0.75 (or some  $d$ )!

$$P_{\text{AbsoluteDiscounting}}(w_i | w_{i-1}) = \frac{\overset{\text{discounted bigram}}{c(w_{i-1}, w_i) - d}}{c(w_{i-1})} + \overset{\text{Interpolation weight}}{\lambda(\overset{\swarrow}{w_{i-1}})} \underset{\nwarrow \text{unigram}}{P(w)}$$

- (Maybe keeping a couple extra values of  $d$  for counts 1 and 2)
- But should we really just use the regular unigram  $P(w)$ ?



# Kneser-Ney Smoothing I

- Better estimate for probabilities of lower-order unigrams!
  - Shannon game: *I can't see without my reading* Francisco ?
  - "Francisco" is more common than "glasses"
  - ... but "Francisco" always follows "San"
- The unigram is useful exactly when we haven't seen this bigram!
- Instead of  $P(w)$ : "How likely is  $w$ "
- $P_{\text{continuation}}(w)$ : "How likely is  $w$  to appear as a novel continuation?"
  - For each word, count the number of bigram types it completes
  - Every bigram type was a novel continuation the first time it was seen

$$P_{\text{CONTINUATION}}(w) \propto |\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$



# Kneser-Ney Smoothing II

- How many times does  $w$  appear as a novel continuation:

$$P_{CONTINUATION}(w) \propto |\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

- Normalized by the total number of word bigram types

$$|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}|$$

$$P_{CONTINUATION}(w) = \frac{|\{w_{i-1} : c(w_{i-1}, w) > 0\}|}{|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}|}$$



## Kneser-Ney Smoothing III

- Alternative metaphor: The number of # of word types seen to precede  $w$

$$|\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

- normalized by the # of words preceding all words:

$$P_{CONTINUATION}(w) = \frac{|\{w_{i-1} : c(w_{i-1}, w) > 0\}|}{\sum_{w'} |\{w'_{i-1} : c(w'_{i-1}, w') > 0\}|}$$

- A frequent word (Francisco) occurring in only one context (San) will have a low continuation probability



# Kneser-Ney Smoothing IV

$$P_{KN}(w_i | w_{i-1}) = \frac{\max(c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1})P_{CONTINUATION}(w_i)$$

$\lambda$  is a normalizing constant; the probability mass we've discounted

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})} |\{w : c(w_{i-1}, w) > 0\}|$$

the normalized discount

The number of word types that can follow  $w_{i-1}$   
 = # of word types we discounted  
 = # of times we applied normalized discount



# Kneser-Ney Smoothing: Recursive formulation

$$P_{KN}(w_i | w_{i-n+1}^{i-1}) = \frac{\max(c_{KN}(w_{i-n+1}^i) - d, 0)}{c_{KN}(w_{i-n+1}^{i-1})} + \lambda(w_{i-n+1}^{i-1}) P_{KN}(w_i | w_{i-n+2}^{i-1})$$

$$c_{KN}(\bullet) = \begin{cases} \textit{count}(\bullet) & \text{for the highest order} \\ \textit{continuationcount}(\bullet) & \text{for lower order} \end{cases}$$

Continuation count = Number of unique single word contexts for •





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