12.11. 2015 **(20+5) MA010 midterm test D** Čas: / time: 80 min

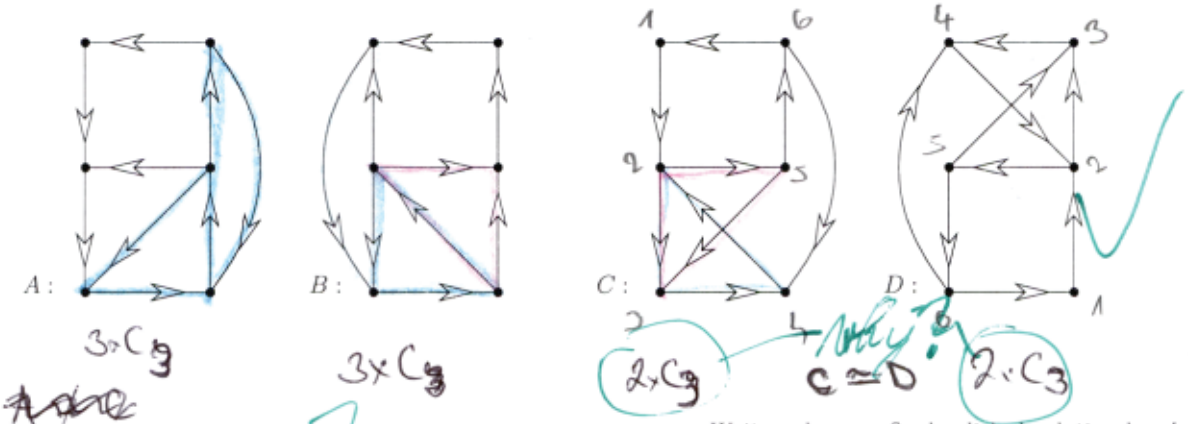
Jméno: **Místnost:** **Souřadnice:**

0007 *list* 1 *učo* *body* 9

Oblast strojově snímatelných informací. Své UČO vyplňte zleva dle přiloženého vzoru číslic. Jinak do této oblasti nezasahujte. 0 1 2 3 4 5 6 7 8 9

Problem 1
10 points

Recall that two directed graphs are isomorphic iff there is a bijection between their vertex sets which preserves each edge including the orientation.
Among the following four digraphs on 6 vertices, find and write down all the isomorphic pairs. For each isomorphic pair, show the respective isomorphism in the picture (preferably by corresponding numberings of the vertices). For each non-isomorphic pair, give a short but precise argument why the two graphs are not isomorphic. Do not be surprised if there is no isomorphic pair. You may write in Czech.



3×C3 3×C3 2×C3 C≅D 2×C3

Write only your final polished solution here!

Do not write here your notes, just the final solution. You may also write on the other side, but it will not be scanned into IS, or use an extra paper no. 5-8.

A ≇ C # of circles of length C3

A ≇ D — "

B ≇ C — "

B ≇ D — "

A ≇ B # of circles with edges like this: 
is different (marked with blue)

Why?

Do not write below. Only this page will be scanned, the other page will be seen only by graders.

12.11. 2015 (20+5) MA010 midterm test D Čas: / time: 80 min

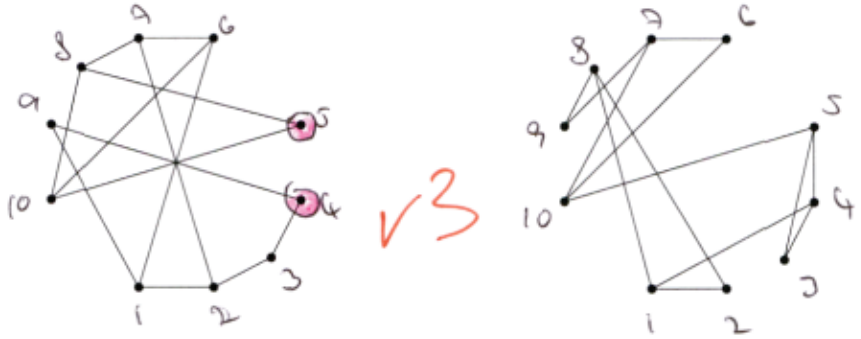
Jméno: Místnost: Souřadnice:

0007 list 2 učo body 6

Oblast strojově snímatelných informací. Své UČO vyplňte zleva dle přiloženého vzoru číslic. Jinak do této oblasti nezasahujte. 0 1 2 3 4 5 6 7 8 9

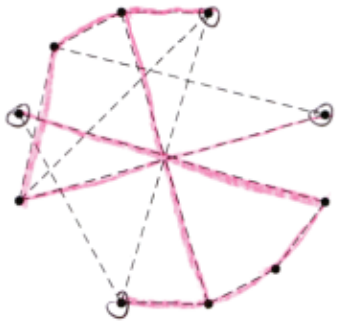
Problem 2
10 points

a) Find, in one of the following two graphs on 10 vertices, two vertices at distance 5 and mark them in the picture. *(Najděte dva vrcholy se vzdáleností 5 a vyznačte je.)*



v3

b) In the following graph on 10 vertices (with dashed edges), find and mark/draw an arbitrary spanning tree with exactly 4 leaves. *(Zakreslete kostru s přesně 4 listy.)*



v3

c) Find and draw here an arbitrary simple connected graph on 8 vertices which consists of exactly 3 blocks. *(Nakreslete souv. graf: 8 vrcholů a přesně 3 bloky.)*

Do not write below. Only this page will be scanned, the other page will be seen only by graders.

Jméno: Místnost: Souřadnice:

0007 list 3 učo body 0

Oblast strojově snímatelných informací. Své UČO vyplňte zleva dle přiloženého vzoru číslic. Jinak do této oblasti nezasahujte. 0 1 2 3 4 5 6 7 8 9

Problem 3
5 points

This is a bonus problem; it is harder than the previous questions, and your answer will be graded only if you provide here a clear and mathematically rigorous proof of your answer. Choose only one(!) of the following two problems I and II, as you will not receive points for both of them. You may write in Czech.

I. For each of the following two claims, either give a proof or show a counterexample:

- A graph containing two disjoint perfect matchings has a 2-factor.
- A graph on an even numer of vertices and containing a 2-factor has a perfect matching.

II. Prove that any 8-regular graph has a 2-factor.