13.11. 2014 (20+5) **MA010 midterm test B** Čas: / time: 80 min

Místnost: **Souřadnice:**

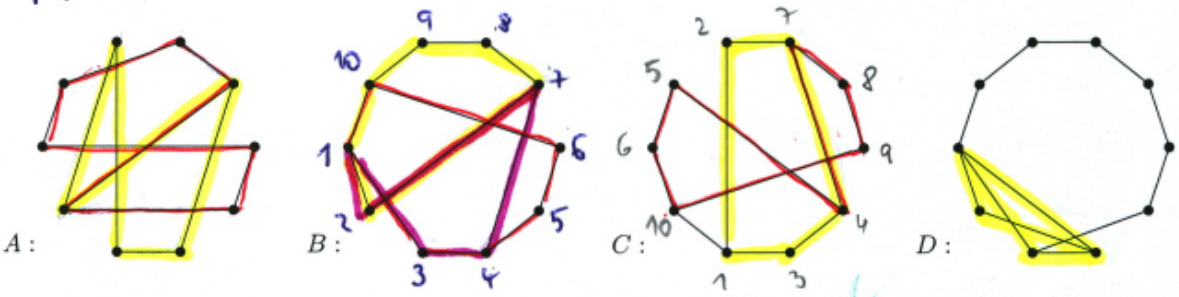
0007 *list* | *učo* *body* 10

Oblast strojově snímatelných informací. Své UČO vyplňte zleva dle přiloženého vzoru číslic. Jinak do této oblasti nezasahujte. 0123456789

Problem 1
10 points

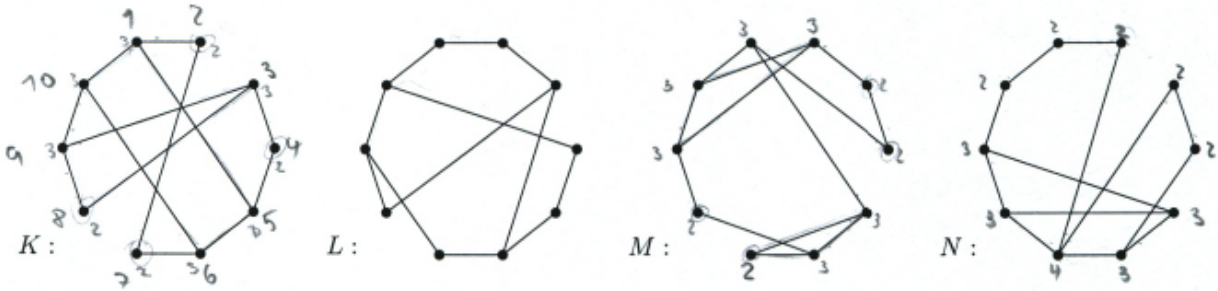
a) Among the following four graphs on 10 vertices, find and write down all the isomorphic pairs. For each isomorphic pair, show the respective isomorphism in the picture (preferably by corresponding numberings of the vertices).

2,2,3,2,2,2,2,3,2,2



C_5, C_7 C_6, C_6, C_5, C_7 $B \approx C$ C_3, C_3

b) Among the following four graphs on 10 vertices, find all isomorphic pairs. For each isomorphic pair, show the respective isomorphism in the picture, and for each non-isomorphic pair, give a short argument why the two graphs are not isomorphic.



C_4, C_3 C_6, C_6, C_5, C_7 C_3, C_3 C_3, C_4, C_4

a) $A \not\approx B$ - different score (degrees)
$A \not\approx C$ - — 11 —
$A \not\approx D$ - D contains $2 \times C_3$, different score
$B \approx C$ - proved by corresponding numbering
$B \not\approx D$ - D contains $2 \times C_3$
$C \not\approx D$ - — 11 —

b) no isomorphic pair
$K \not\approx L$ - L has no triangle
$K \not\approx M$ - M has $2 \times C_3$
$K \not\approx N$ - different score (degrees of nodes)
$L \not\approx M$ - L has no C_3
$L \not\approx N$ - L has no C_3

Write your final polished solution here!

Do not write here your notes, just the final solution. You may also write on the other side, but it will not be scanned into IS, or use an extra paper no. 5-8.

$M \not\approx N$ - different score

Do not write below. Only this page will be scanned, the other page will be seen only by graders.

13.11. 2014 (20+5) **MA010 midterm test B** Čas: / time: 80 min

Místnost: **Souřadnice:**

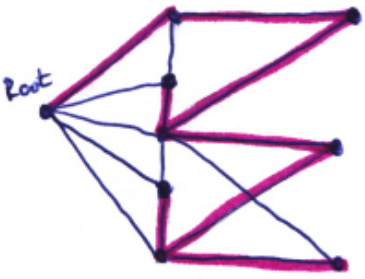
0007 *list* 2 *učo* *body* 9

Oblast strojově snímatelných informací. Své UČO vyplňte zleva dle přiloženého vzoru číslic. Jinak do této oblasti nezasahujte. 0 1 2 3 4 5 6 7 8 9

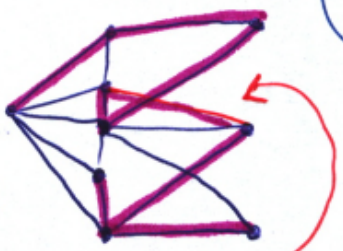
Problem 2
10 points

a) Find and draw here a simple vertex 2-connected graph G on 9 vertices such that G has a DFS search tree with exactly 3 leaves. Mark this search tree in the picture as well.
(Jednoduchý 2-souvislý graf na 9 vrch. s DFS prohledávacím stromem s přesně 3 listy.)

3/3



- I suppose, that the root is not a leaf

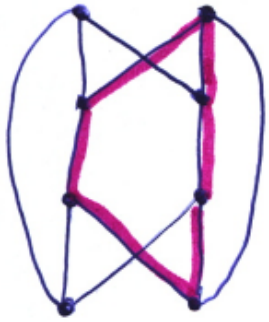


If root is considered as leaf, the I would add red line

a) Find and draw here an arbitrary simple connected 3-regular graph on 8 vertices
- containing at least one cycle of length 6,
- but having no triangle.

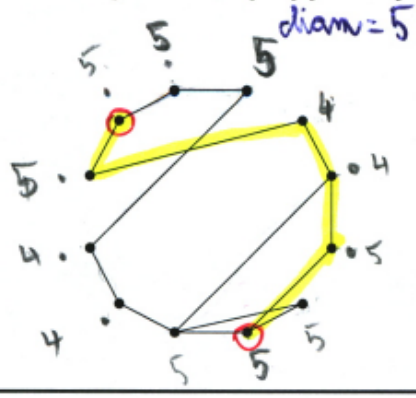
(Jednod. souv. 3-reg. graf na 8 vrch. mající kružnici(e) délky 6, ale bez trojúhelníku.)

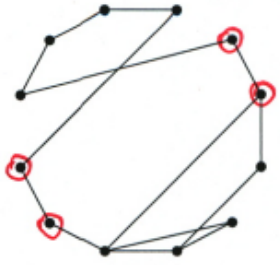
3/3



c) In the following graph on 12 vertices, determine the diameter and mark in the left-hand picture a vertex pair achieving the diameter. Then determine the radius and mark in the right-hand picture all the vertices in the center of the graph.
(Nalevo – určit průměr a dvojici jej dosahující, napravo – poloměr a vyznačit centrum.)

3/4





radius = 4
center = all vertices with the excentricity value 4 (radius)

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Místnost: Souřadnice:

0007 list 3 učo body 0

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Problem 3
5 points

This is a bonus problem; it is harder than the previous questions, and your answer will be graded only if you provide here a clear and mathematically rigorous proof of your answer. Choose only one(!) of the following two problems I and II, as you will not receive points for both of them. You may write in Czech here (and only here).

I. For each of the following two claims, either give a proof or show a counterexample:

a) – a 3-regular bipartite graph is vertex 2-connected.

b) – a 3-regular bipartite graph is vertex 3-connected.

II. Prove that a graph on n vertices has less than n bridges.