数与二叉树

- 数是一种数据结构, 比如:目录结构
- 数是一种可以递归定义的数据结构
- 数是由n个节点组成的集合:
    - 如果n=0,那是一颗空树;
    - 如果n>0,那存在1个节点作为树的根节点,其他节点可以分为m个集合,每个集合本身又是一棵树.

数的实例:模拟文件系统

链式存储

```
class Node:
    def __init__(self, name, type='dir'):
        self.name = name
        self.type = type  #'dir' or 'file'
        self.children = []
        self.paent = None
        # 链式存储

    def __repr__(self):
        return self.name
```

```
class FileSystemTree:
    def __init__(self):
        self.root = Node('/')
        self.now = self.root

    def mkdir(self, name): # 当前目录下创建一个目录
        # name 以/结尾
        if name[-1] != '/':
            name += '/'
        node = Node(name)
        self.now.children.append(node)
        node.parent = self.now

    def ls(self): # 展示当前目录下的所有目录
        return self.now.children

    def cd(self, name): # 切换目录
        # '相对路径下面一层'
        # '../var/python/'
        if name[-1] != '/':
            name += '/'
        if name =='../':
            self.now = self.now.parent
            return
        for child in self.now.children:
            if child.name == name:
                self.now = child # 切换到下面一层
                return
        raise ValueError('invalid dir') # 否则才报错
```

```
tree = FileSystemTree()
tree.mkdir('var/')
tree.mkdir('bin/')
tree.mkdir('usr/')
tree.root.children
```

```
[var/, bin/, usr/]
```

```
tree.ls()
```

```
[var/, bin/, usr/]
```

```
tree.cd('bin/')
tree.mkdir('python/')
```

```
tree.cd('bin/')
```

二叉树

- 二叉树的链式存储:将二叉树的节点定义为一个对象,节点之间通过类似链表的链接方式来连接.
- 节点定义:

```
class BiTreeNode:
    def __init__(self, data):
        self.data = data
        self.lchild = None # 左孩子
        self.rchild = None # 右孩子
```

```
a = BiTreeNode('A')
b = BiTreeNode('B')
c = BiTreeNode('C')
d = BiTreeNode('D')
e = BiTreeNode('E')
f = BiTreeNode('F')
g = BiTreeNode('G')
```

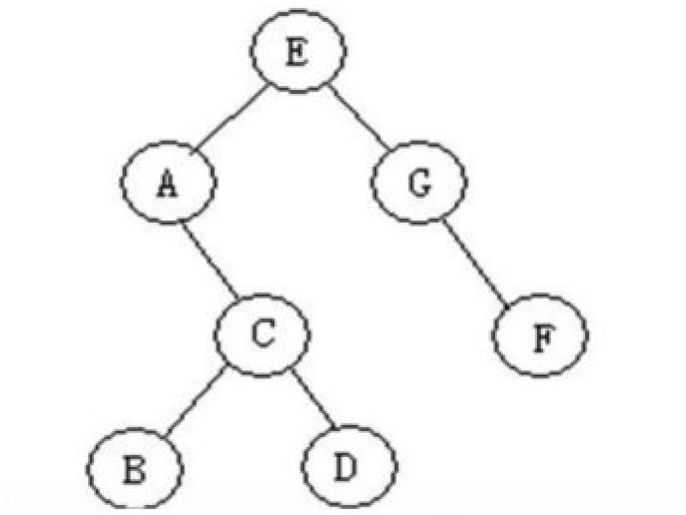
```
e.lchild = a
e.rchild = g
a.rchild = c
c.lchild = b
c.rchild = d
g.rchild = f
```

```
root = e
```

```
root.lchild.rchild.data
```

```
'C'
```

二叉树的遍历



- 二叉树的遍历方式:
    - 前序遍历:EACBDGF
    - 中序遍历:ABCDEGF
    - 后序遍历:BDCAFGE
    - 层次遍历:EAGCFBD
- 前序遍历

```
def pre_order(root): #先左后右
    if root:
        print(root.data, end=',')
        pre_order(root.lchild)
        pre_order(root.rchild)
```

```
pre_order(root)
```

```
E,A,C,B,D,G,F,
```

- 中序遍历

```
def in_order(root):
    if root:
        in_order(root.lchild)
        print(root.data, end=',')
        in_order(root.rchild)
```

```
in_order(root)
```

```
A,B,C,D,E,G,F,
```

- 后序遍历

```
def post_order(root):
    if root:
        post_order(root.lchild)
        post_order(root.rchild)
        print(root.data, end=',')
```

- 给出前序遍历和中序遍历，分析出数的结构，并给出后序遍历的结果。

  先根据前序遍历确定root，在根据中序遍历**确定左右子树**，再根据前序遍历确定左右子节点，不断反复分析，得出结果。
- 层次遍历

```
from collections import deque
```

```
def level_order(root):
    queue = deque() # 创建队列
    queue.append(root)
    while len(queue) > 0: # 只要队不空
        node = queue.popleft()
        print(node.data, end=',')
        if node.lchild:
            queue.append(node.lchild)
        if node.rchild:
            queue.append(node.rchild)
```

二叉搜索树

- 二叉搜索是一颗二叉树且满足性质：设x是二叉树的一个节点。如果y是x左子树的一个节点，那么y.key <= x.key；如果y是x右子树的一个节点，那么y.key >= x.key。
- 二叉搜索树的操作：查询、插入、删除

二叉搜索树:插入

```
class BiTreeNode:
    def __init__(self, data):
        self.data = data
        self.lchild = None # 左孩子
        self.rchild = None # 右孩子
        self.parent = None
```

```
class BST:
    def __init__(self, li=None):
        self.root = None
        if li:
            for val in li:
                self.insert_no_rec(val)

    def insert(self, node, val): # 相同的元素不插入
        if not node:
            node = BiTreeNode(val)
        elif val < node.data:
            node.lchild = self.insert(node.lchild, val)
            node.lchild.parent = node
        elif val > node.data:
            node.rchild = self.insert(node.rchild, val)
            node.rchild.parent = node
        return node

    def insert_no_rec(self, val):
        p = self.root
        if not p:   #空树
            self.root = BiTreeNode(val)
            return
        while True:
            if val < p.data:
                if p.lchild:
                    p = p.lchild # 往左子树走,继续循环
                else:  #左孩子不存在
                    p.lchild = BiTreeNode(val)
                    p.lchild.parent = p
                    return
            elif val > p.data:
                if p.rchild:
                    p = p.rchild
                else:
                    p.rchild = BiTreeNode(val)
                    p.rchild.parent = p
                    return
            else:
                return

    def query(self, node, val):
        if not node:
            return None
        if node.data < val:
            return self.query(node.rchild, val)
        elif node.data > val:
            return self.query(node.lchild, val)
        else:
            return node

    def query_no_rec(self, val):
        p = self.root
        while p:
            if p.data < val:
                p = p.rchild
            elif p.data > val:
                p = p.lchild
            else:
                return p
        return None

    def pre_order(self,root): #先左后右
        if root:
            print(self.root.data, end=',')
            pre_order(self.root.lchild)
            pre_order(self.root.rchild)

    def in_order(self,root): # 二叉树中中序序列一定是升序
        if root:
            in_order(self.root.lchild)
            print(self.root.data, end=',')
            in_order(self.root.rchild)

    def post_order(self,root):
        if root:
            post_order(self.root.lchild)
            post_order(self.root.rchild)
            print(self.root.data, end=',')

    def __remove_node_1(self, node): #情况1:node是叶子节点
        if not node.parent:
            self.root = None
        if node == node.parent.lchild: #node是他父亲的左孩子
            node.parent.lchild = None
            node.parent = None
        else: # 右孩子
            node.parent.rchild = None

    def __remove_node_21(self, node): #情况2.1:node只有一个左孩子
        if not node.parent: # 根节点
            self.root = node.lchild
            node.lchild.parent = None
        elif node == node.parent.lchild:
            node.parent.lchild = node.lchild
            node.lchild.parent = node.parent
        else:
            node.parent.rchild = node.lchild
            node.lchild.parent = node.parent

    def __remove_node_22(self, node): #情况2.2:node只有一个左孩子
        if not node.parent:
            self.root = node.rchild
            node.rchild.parent = None
        elif node == node.parent.lchild:
            node.parent.lchild = node.rchild
            node.rchild.parent = node.parent
        else:
            node.parent.rchild = node.rchild
            node.rchild.parent = node.parent

    def delete(self, val):
        if self.root: #不是空树
            node = self.query_no_rec(val)
            if not node: # 不存在
                return False
            if not node.lchild and not node.rchild: # 1. 叶子节点
                self.__remove_node_1(node)
            elif not node.rchild:  # 2.1 只有一个左孩子
                self.__remove_node_21(node)
            elif not node.lchild:   # 2.2 只有一个右孩子
                self.__remove_node_22(node)
            else:   # 3. 两个孩子都有
                min_node = node.rchild # 找右子树最小的node
                while min_node.lchild:
                    min_node = min_node.lchild
                node.data = min_node.data #把原来的替换为min_node
                # 删除min_node
                if min_node.rchild:
                    self.__remove_node_22(min_node)
                else:
                    self.__remove_node_1(min_node)
```

```
tree = BST([4,6,7,9,2,1,3,5,8])
tree.pre_order(tree.root)
print('')
tree.in_order(tree.root)
print('')
tree.post_order(tree.root)
```

```
4,2,1,3,6,5,7,9,8,
1,2,3,4,5,6,7,8,9,
1,3,2,5,8,9,7,6,4,
```

```
import random
```

```
li = list(range(500))
random.shuffle(li)
tree = BST(li)
tree.in_order(tree.root) # 二叉树中中序序列一定是升序
```

```
0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,
30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56
,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,8
3,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,105,106,10
7,108,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,12
7,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,143,144,145,146,14
7,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,164,165,166,16
7,168,169,170,171,172,173,174,175,176,177,178,179,180,181,182,183,184,185,186,18
7,188,189,190,191,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,20
7,208,209,210,211,212,213,214,215,216,217,218,219,220,221,222,223,224,225,226,22
7,228,229,230,231,232,233,234,235,236,237,238,239,240,241,242,243,244,245,246,24
7,248,249,250,251,252,253,254,255,256,257,258,259,260,261,262,263,264,265,266,26
7,268,269,270,271,272,273,274,275,276,277,278,279,280,281,282,283,284,285,286,28
7,288,289,290,291,292,293,294,295,296,297,298,299,300,301,302,303,304,305,306,30
7,308,309,310,311,312,313,314,315,316,317,318,319,320,321,322,323,324,325,326,32
7,328,329,330,331,332,333,334,335,336,337,338,339,340,341,342,343,344,345,346,34
7,348,349,350,351,352,353,354,355,356,357,358,359,360,361,362,363,364,365,366,36
7,368,369,370,371,372,373,374,375,376,377,378,379,380,381,382,383,384,385,386,38
7,388,389,390,391,392,393,394,395,396,397,398,399,400,401,402,403,404,405,406,40
7,408,409,410,411,412,413,414,415,416,417,418,419,420,421,422,423,424,425,426,42
7,428,429,430,431,432,433,434,435,436,437,438,439,440,441,442,443,444,445,446,44
7,448,449,450,451,452,453,454,455,456,457,458,459,460,461,462,463,464,465,466,46
7,468,469,470,471,472,473,474,475,476,477,478,479,480,481,482,483,484,485,486,48
7,488,489,490,491,492,493,494,495,496,497,498,499,
```

二叉搜索树:查询

```python
tree = BST(li)
tree.query_no_rec(4).data
```

```
4
```

```python
tree.query_no_rec(3).data
```

```
3
```

二叉搜索树:删除

- 1.如果要删除的节点是**叶子节点:直接删除**
- 2.如果要删除的节点**只有一个孩子:将此节点的父亲与孩子连接,然后删除该节点.**
- 3.如果要删除的节点有**两个孩子:将其右子树的最小节点(该节点最多有一个右孩子)删除,并替换当前节点.**

```
tree = BST([1,4,2,5,3,8,6,9,7])
tree.in_order(tree.root)
print('')

tree.delete(4)
tree.delete(1)
tree.in_order(tree.root)
```

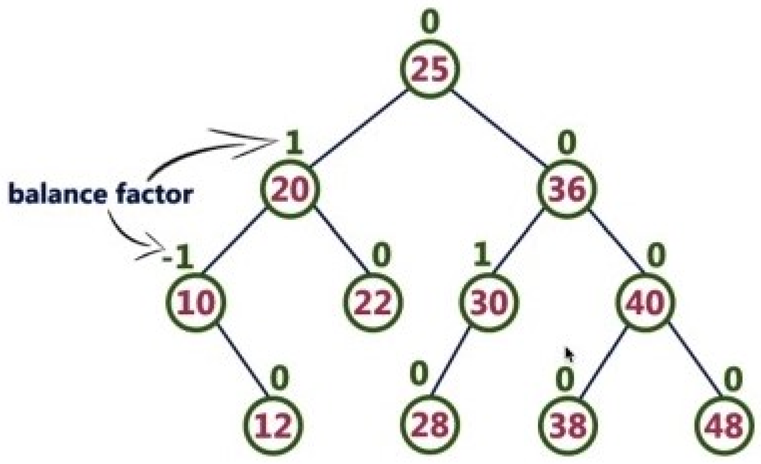
```
1,2,3,4,5,6,7,8,9,
2,3,5,6,7,8,9,
```

二叉搜索树的效率

- 平均情况下,二叉搜索树进行搜索的时间复杂度为$O(lgn)$
- 最坏情况下,二叉搜索树可能非常偏斜.
- 解决方案:
    - 随机化插入
    - AVL树

AVL树

- AVL树:是一棵自平衡的二叉搜索树.
- AVL树具有以下性质:
    - 根的左右子树的高度之差的绝对值不能超过1
    - 根的左右子树都是平衡二叉树



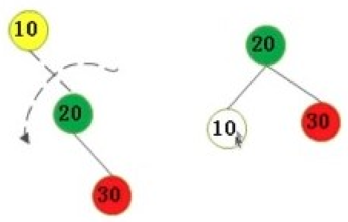
AVL:旋转

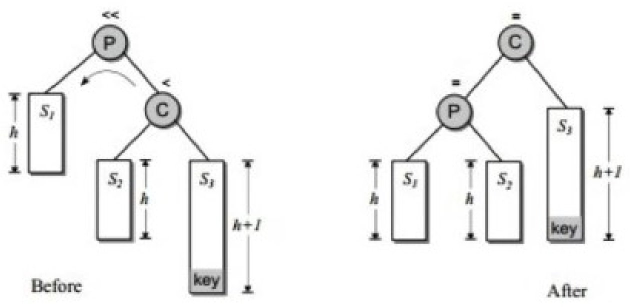
AVL树--插入

- 插入一个节点可能会破坏AVL树的平衡,可以通过**旋转**操作来进行修正.
- 插入一个节点后,只有从插入节点到根节点的路径上的节点的平衡可能被改变.我们需要找出**第一个破坏了平衡条件的节点**,称之为K. K的两颗子树的高度差2.
- 不平衡的出现可能有4种情况.

左旋

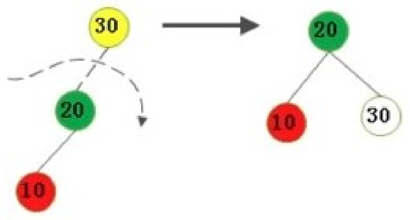
- 不平衡是由于对K的右孩子的右子树插入导致的：左旋

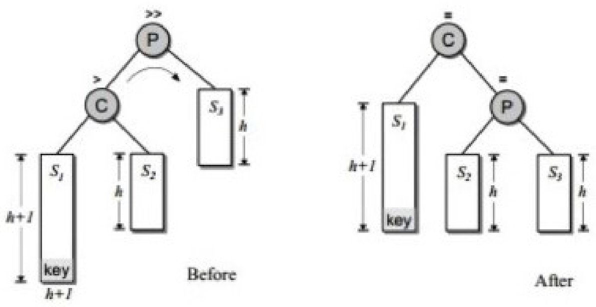




右旋

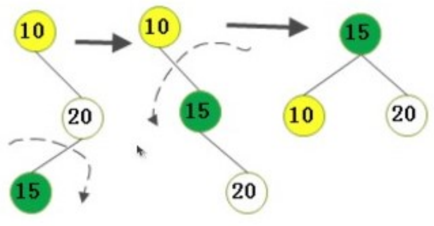
- 不平衡是由于对K的左孩子的左子树插入导致的：右旋

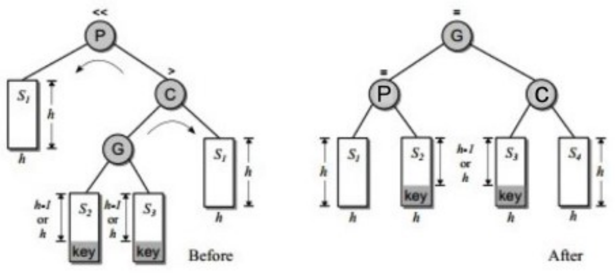




右旋-左旋

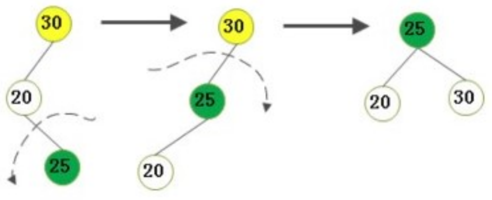
- 不平衡是由于对K的右孩子的左子树插入导致的：右旋-左旋

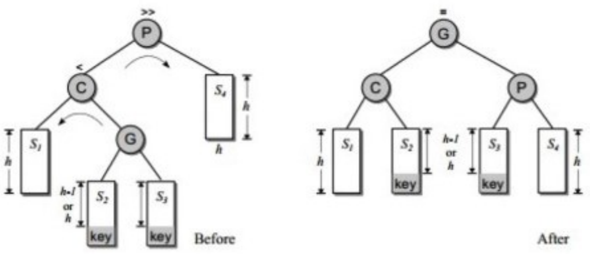




左旋-右旋

- 不平衡是由于对K的左孩子的右子树插入导致的：左旋-右旋





```python
class AVLNode(BiTreeNode):
    def __init__(self):
        BiTreeNode.__init__(self, data)
        self.bf = 0  # balance_factor
```

```python
class AVLTree(BST):
    def __init__(self, li=None):
        BST.__init__(self, li)


    def rotate_left(self, p, c): # 左旋，方法不适用于删除
        s2 = c.lchild
        p.rchild = s2
        if s2:
            s2.parent = p

        c.lchild = p
        p.parent = c
        # 更新balance factor
        p.bf = 0
        c.bf = 0
        return c

    def rotate_right(self, p, c): # 右旋
        s2 = r.rchild
        p.lchild = s2
        if s2:
            s2.parent = p

        c.rchild = p
        p.parent = c

        p.bf = 0
        c.bf = 0
        return c

    def rotate_right_left(self, p, c):
        g= c.lchild

        s3 = g.rchild
        c.lchild = s3

        if s3:
            s3.parent = c
        g.rchild = c
        c.paent = g

        s2 = g.lchild
        p.rchild = s2
        if s2:
            s2.parent = p
        g.lchild = p
        p.parent = g

        # 更新bf
        if g.bf > 0:
            p.bf = -1
            c.bf = 0
        elif g.bf < 0:
            p.bf = 0
            c.bf = 1
        else: # 插入的是g
            p.bf = 0
            c.bf = 0
        g.bf = 0
        return g

    def rotate_left_right(self, p, c):
        g = c.rchild

        s2 = g.lchild
        c.rchild = s2

        if s2:
            s2.parent = c
        g.lchild = c
        c.parent = g

        s3 = g.rchild
        p.lchild = s3
        if s3:
            s3.parent = p
        g.rchild = p
        p.parent = g

        # 更新bf
        if g.bf < 0:
            p.bf = 1
            c.bf = 0
        elif g.bf > 0:
            p.bf = 0
            c.bf = -1
        else:
            p.bf = 0
            c.bf = 0
        g.bf = 0
        return g

    def insert_no_rec(self, val):
        # 1. 和BST一样，插入
        p = self.root
        if not p:
            self.root = BiTreeNode(val)
            return
        while True:
            if val < p.data:
                if p.lchild:
                    p = p.lchild
                else: # 左孩子不存在
                    p.lchild = BiTreeNode(val)
                    p.lchild.paent = p
                    node = p.lchild # node 存储的是插入的节点
                    break
            elif val > p.data:
                if p.rchild:
                    p = p.rchild
                else:
                    p.rchild = BiTreeNode(val)
                    p.rchild.parent = p
                    node = p.rchild
                    break
            else: # val == p.data
                return

        # 2 更新balance factor
        while node.parent: # node.paent不空
            if node.parent.lchild == node: # 传递是从左子树来的。左子树更沉了
                # 更新node.parent的bf -= 1
                if node.parent.bf < 0: # 原来node.parent.bf == -1, 更新后变成-2
                    # 做旋转
                    # 看node哪边沉
                    g = node.parent.parent # 为了连接旋转之后的子树
                    x = node.parent # 旋转前子树的根
                    if node.bf > 0:
                        # 旋转之后子树新的根
                        n = self.rotate_left_right(node.parent, node)
                    else:
                        n = self.rotate_right(node.parent, node)
                    # 记得：把n和g连起来
                elif node.parent.bf > 0: # 原来node.parent.bf = 1,更新之后变成0
                    node.parent.bf = 0
                    break
                else: # 原来node.parent.bf = 0, 更新之后变成-1
                    node.parent.bf = -1
                    node = node.parent
                    continue
            else: # 传递是从右子树来的，右子树更沉了
                # 更新node.parent.bf += 1
                if node.parent.bf > 0: # 原来node.parent.bf == 1,更新后变成2
                    # 做旋转
                    # 看node哪边沉
                    g = node.parent.parent
                    x = node.parent # 旋转前子树的根
                    if node.bf < 0: # node.bf = -1
                        n = self.rotate_right_left(node.parent, node)
                    else: # node.bf = 1
                        n = self.rotate_left(node.parent, node)
                    # 记得连起来
                elif node.parent.bf < 0: # 原来node.parent.bf == -1, 更新后变成0
                    node.parent.bf = 0
                    break
                else:
                    node.parent.bf = 1
                    node = node.parent
                    continue

            # 连接旋转后的子树
            n.parent = g
            if g: # g不是空
                if x == g.lchild:
                    g.lchild = n # node.parent
                else:
                    g.rchild = n
                break
            else:
                self.root = n
                break
```

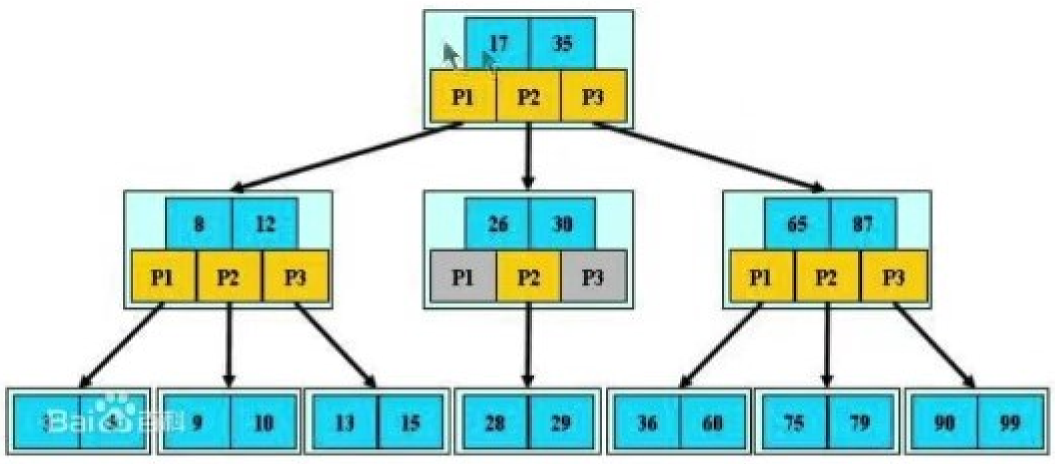
```
tree = AVLTree([10,8,7,6,5,4,3,2,1])
```

```
tree.pre_order(tree.root)
print("")
tree.in_order(tree.root)
```

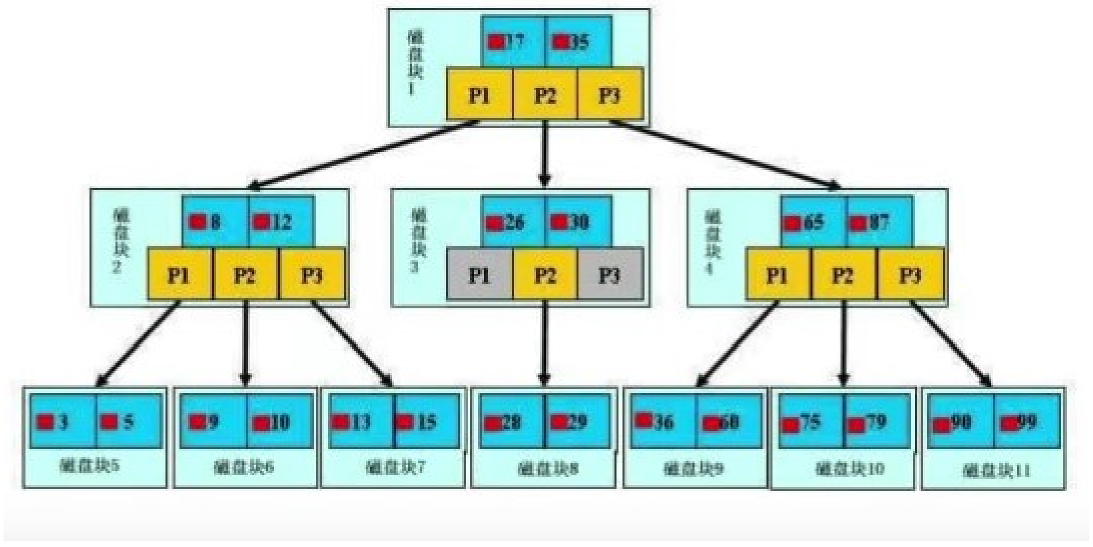
```
10,8,7,6,5,4,3,2,1,
1,2,3,4,5,6,7,8,10,
```

二叉搜索树扩展应用--B树

- B树(B-Tree)：B树是一棵自平衡的多路搜索树。常用于数据库的索引。



17 35
P1 P2 P3
8 12
26 30
65 87
3 5
9 10
13 15
28 29
36 60
75 79
90 99
Baidu百科



磁盘块1
17 35
P1 P2 P3
磁盘块2
8 12
磁盘块3
26 30
磁盘块4
65 87
3 5
磁盘块5
9 10
磁盘块6
13 15
磁盘块7
28 29
磁盘块8
36 60
磁盘块9
75 79
磁盘块10
90 99
磁盘块11