BITS, PILANI - K K BIRLA GOA CAMPUS
Second Semester-2023-24
MATH- II   Tutorial - 1

---

1. Determine whether the matrix is in row echelon form, reduced row echelon form, both or neither.

$$(a)\ \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\qquad (b)\ \begin{bmatrix}1&0&0\\0&0&1\\0&0&0\end{bmatrix}\qquad (c)\ \begin{bmatrix}0&1&2&0&1\\0&0&0&1&7\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\qquad (d)\ \begin{bmatrix}1&4&-2&3\\0&1&6&1\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

2. Solve the linear system by Gauss elimination method

$$(a)\quad \begin{aligned}2x_1+x_2+x_3&=5\\4x_1-6x_2&=-2\\-2x_1+7x_2+2x_3&=9\end{aligned}\qquad\qquad (b)\quad \begin{aligned}x_1+x_2+x_3&=2\\x_1+3x_2+3x_3&=0\\x_1+3x_2+5x_3&=2\end{aligned}$$

$$(c)\quad \begin{aligned}x_1+2x_2+x_3+2x_4+x_5&=1\\3x_1+6x_2+5x_3+8x_4+2x_5&=3\\3x_1+6x_2+x_3+4x_4+5x_5&=4\\2x_1+4x_2+4x_3+7x_4+2x_5&=5\\7x_1+14x_2+9x_3+17x_4+8x_5&=11.\end{aligned}$$

3. Use Gauss-Jordon method to find all possible solutions of problem 2.

4. Find the quadratic equation $y=ax^2+bx+c$ that goes through the points $(3,18)$, $(2,9)$ and $(1,3)$.

5. Find all values of λ for which the following systems has (a) unique solution (b) infinitely many solutions (c) no solution

$$(a)\quad \begin{aligned}x_1+2x_2-3x_3&=4\\3x_1-x_2+5x_3&=2\\4x_1+x_2+(\lambda^2-14)x_3&=\lambda+2\end{aligned}\qquad\qquad (b)\quad \begin{aligned}x_1+x_2&=3\\x_1+(\lambda^2-8)x_2&=\lambda\end{aligned}$$

6. Use Gauss-Jordan method to find the inverse of the coefficient matrix of the following system and then find its solution

$$\begin{aligned}8x_1+3x_2+6x_3&=4\\3x_1-x_2-7x_3&=-11\\x_1+x_2+2x_3&=2.\end{aligned}$$

7. Does the system of equations have any nontrivial solution? Justify your answer:

$$\begin{aligned}
&2x_1 - 3x_2 + x_3 = 0\\
&x_1 + 2x_2 - x_3 = 0\\
&3x_1 - x_2 = 0.
\end{aligned}$$

8. Show that if X_1 and X_2 are solutions to the linear system $AX = b$, then $X_1 - X_2$ is a solution to the associated homogeneous system $AX = 0$.

9. Use EROs to find the value(s) of λ for which the given matrix has rank two

$$\begin{bmatrix} -5 & -6 & 4 \\ -8 & -7 & 9 \\ -6 & 1 & \lambda \end{bmatrix}$$

10. Show that if there exists an inverse of a square matrix, it is always unique.

11. If A and B are square matrices of the same order and are invertible, then show that $(AB)^{-1} = B^{-1}A^{-1}$.