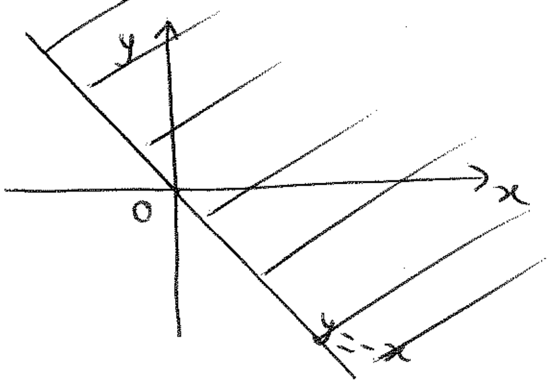
Ficha nº2

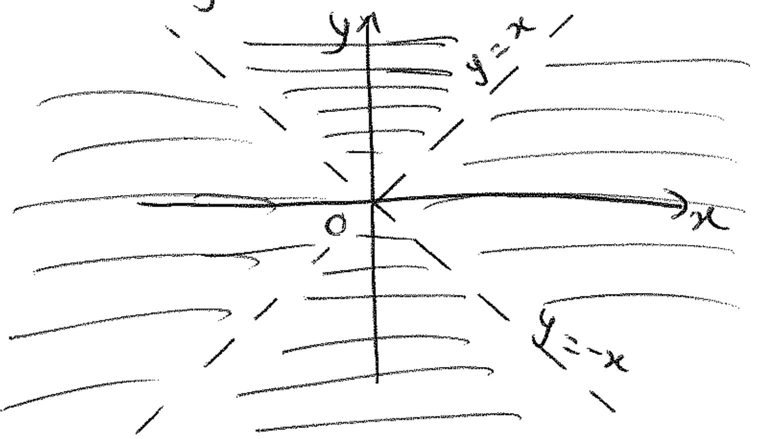
Funções Reais de várias variáveis

1 a) $f(x,y)=\sqrt{x+y}$, $D_f=\{(x,y)\in\mathbb{R}^2: x+y\geq 0\}=\{(x,y)\in\mathbb{R}^2: y\geq -x\}$



b) $f(x,y)=\dfrac{x}{x^2+y^2}$, $D_f=\{(x,y)\in\mathbb{R}^2: x^2+y^2\neq 0\}=\mathbb{R}^2\setminus\{(0,0)\}$

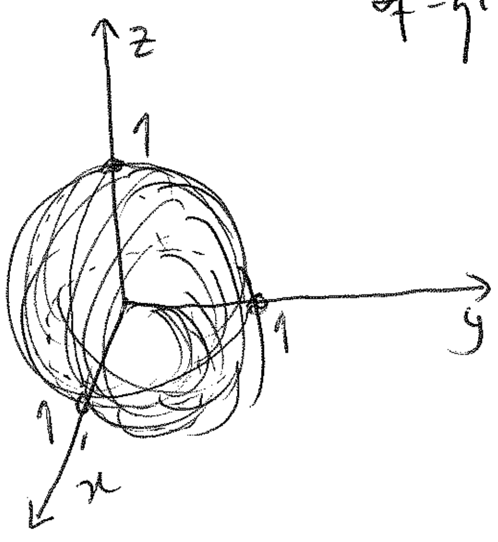
c) $f(x,y)=\dfrac{xy}{x^2-y^2}$, $D_f=\{(x,y)\in\mathbb{R}^2: x^2-y^2\neq 0\}=\{(x,y)\in\mathbb{R}^2: y\neq\pm x\}$



d) $g(x,y,z)=\dfrac{1}{x^2+y^2+z^2}$, $D_g=\{(x,y,z)\in\mathbb{R}^3: x^2+y^2+z^2\neq 0\}=\mathbb{R}^3\setminus\{(0,0,0)\}$

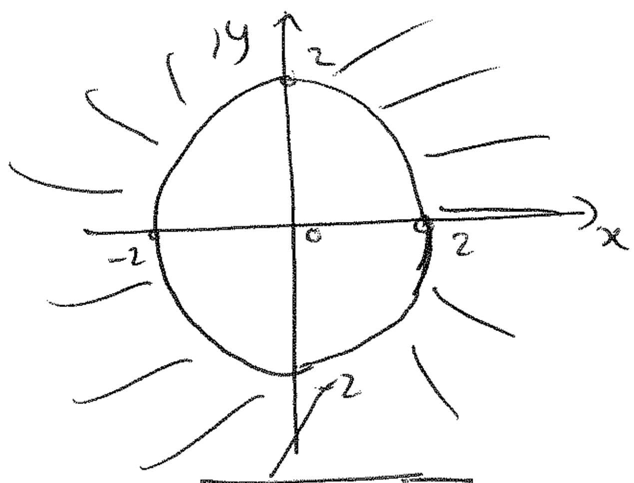
e) $f(x,y,z)=\sqrt{1-x^2-y^2-z^2}$, $D_f=\{(x,y,z)\in\mathbb{R}^3: 1-x^2-y^2-z^2\geq 0\}$

$D_f=\{(x,y,z)\in\mathbb{R}^3: x^2+y^2+z^2\leq 1\}$



1.f) $f(x,y)=\sqrt{x^2+y^2-4}$, $D_f=\{(x,y)\in\mathbb{R}^2 : x^2+y^2-4\geq 0\}$ (2)

$=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\geq 4\}$



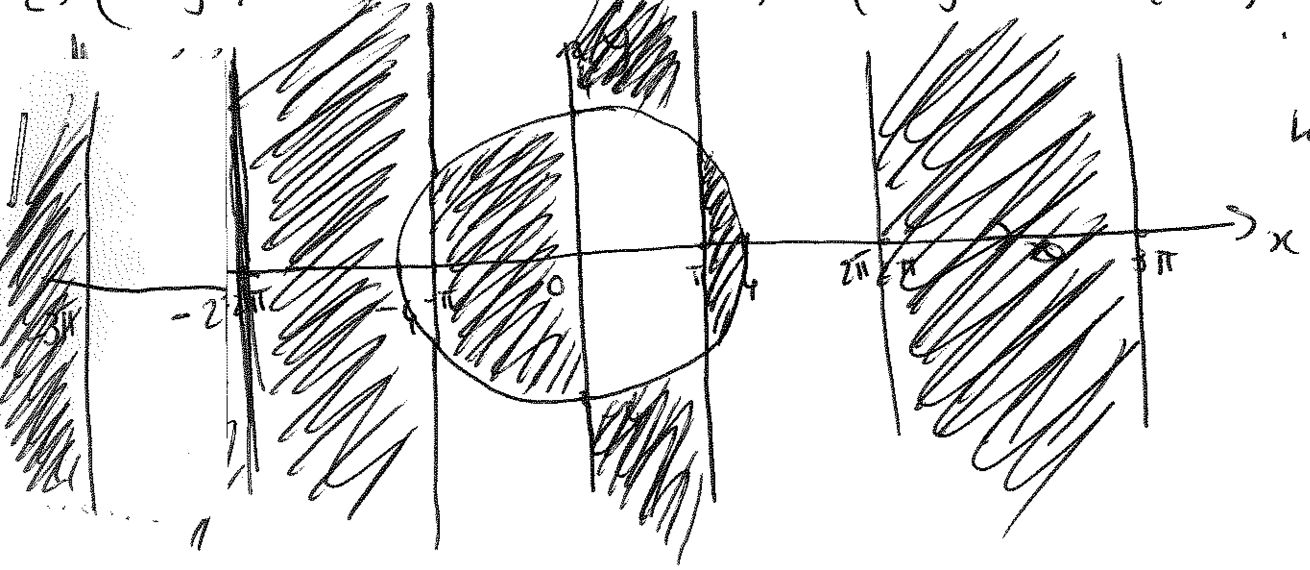
g) $f(x,y)=\sqrt{(x^2+y^2-16).\text{sen}\,x}$, $D_f=\{(x,y)\in\mathbb{R}^2 : (x^2+y^2-16).\text{sen}\,x\geq 0\}$

C.A.: $(x^2+y^2-16).\text{sen}\,x\geq 0 \iff$

$(x^2+y^2-16\geq 0 \wedge \text{sen}\,x\geq 0) \vee (x^2+y^2-16\leq 0 \wedge \text{sen}\,x\leq 0) \iff$

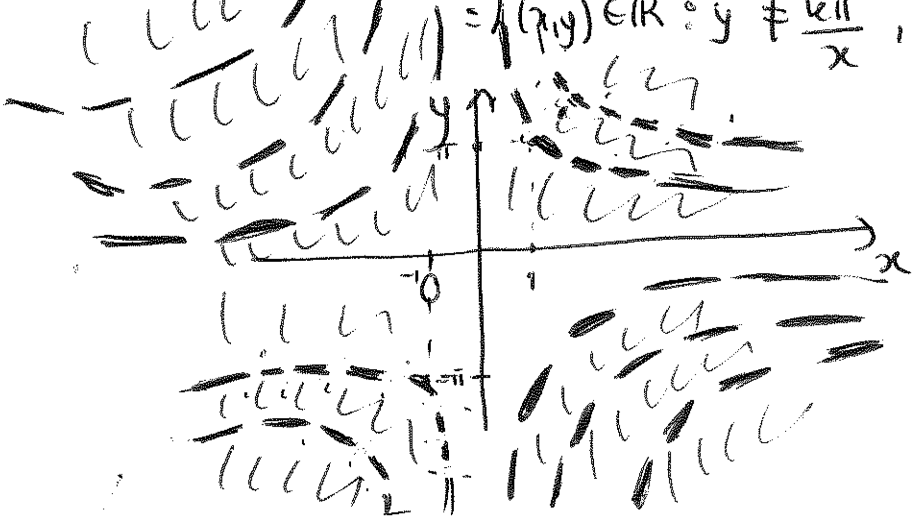
$\iff (x^2+y^2\geq 16 \wedge 2k\pi\leq x\leq(2k+1)\pi) \vee (x^2+y^2\leq 16 \wedge (2k-1)\pi\leq x\leq 2k\pi)$

$k\in\mathbb{Z}$



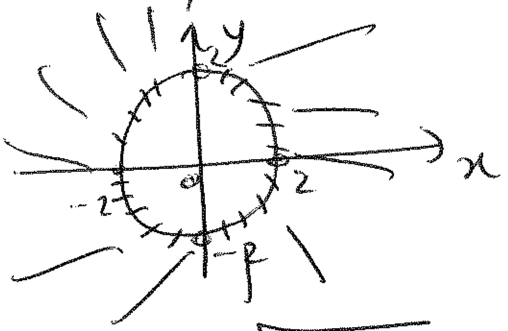
h) $f(x,y)=\dfrac{1}{\text{sen}(xy)}$, $D_f=\{(x,y)\in\mathbb{R}^2 : \text{sen}(xy)\neq 0\}=\{(x,y)\in\mathbb{R}^2 : xy\neq k\pi, k\in\mathbb{Z}\}$

$=\{(x,y)\in\mathbb{R}^2 : y\neq \dfrac{k\pi}{x}, k\in\mathbb{Z}\}$



i) $f(x,y) = \ln(x^2+y^2-4)$, $D_f = \{(x,y)\in\mathbb{R}^2 : x^2+y^2-4>0\}$

$= \{(x,y)\in\mathbb{R}^2 : x^2+y^2>4\}$

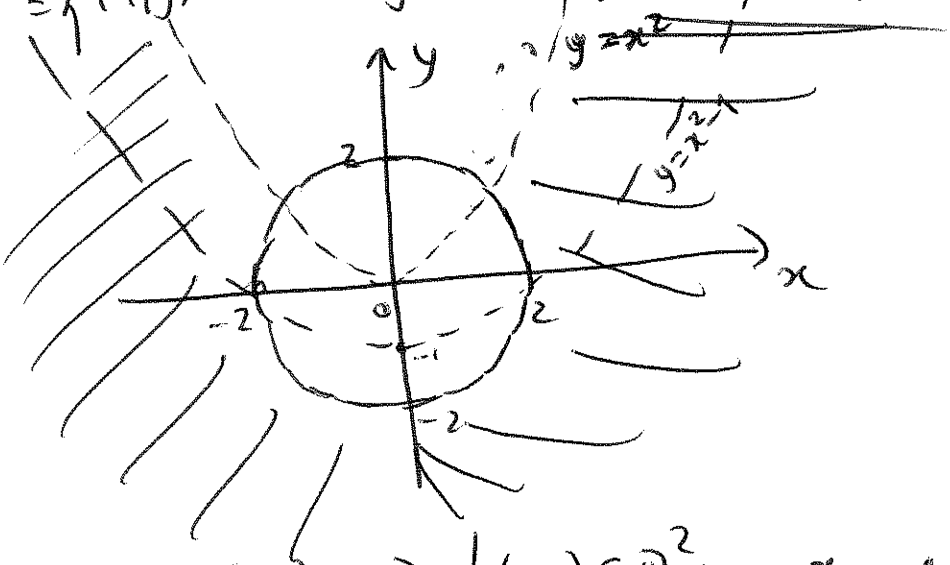


j) $f(x,y) = \dfrac{\sqrt{x^2+y^2-4}}{\ln(x^2-y)}$

$D_f = \{(x,y)\in\mathbb{R}^2 : x^2+y^2-4 \geq 0 \wedge x^2-y>0 \wedge \ln(x^2-y) \neq 0\}$

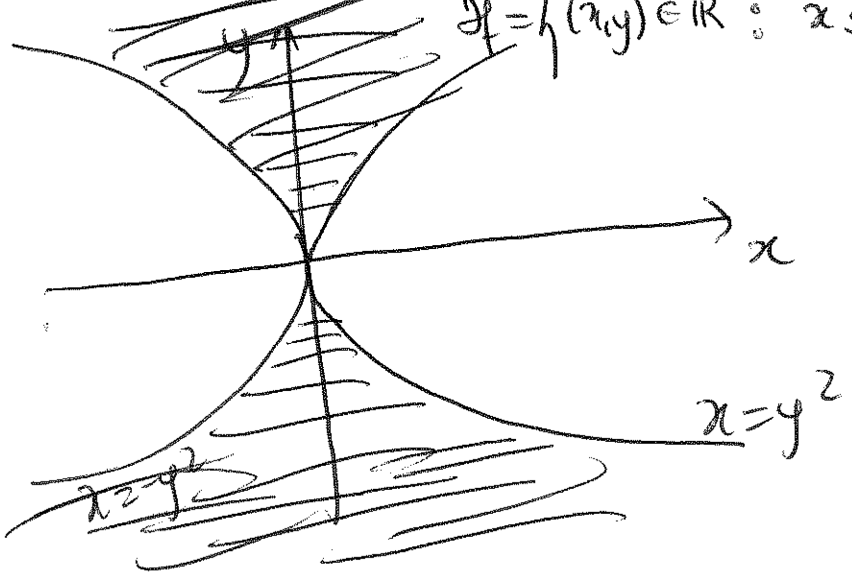
$= \{(x,y)\in\mathbb{R}^2 : x^2+y^2 \geq 4 \wedge y<x^2 \wedge x^2-y \neq 1\}$

$= \{(x,y)\in\mathbb{R}^2 : x^2+y^2 \geq 4 \wedge y<x^2 \wedge y \neq x^2-1\}$



k) $f(x,y) = \arccos\left(\dfrac{x}{y^2}\right)$, $D_f \{(x,y)\in\mathbb{R}^2 : -1 \leq \dfrac{x}{y^2} \leq 1 \wedge y \neq 0\}$

$D_f = \{(x,y)\in\mathbb{R}^2 : x \leq y^2 \wedge x \geq -y^2 \wedge y \neq 0\}$



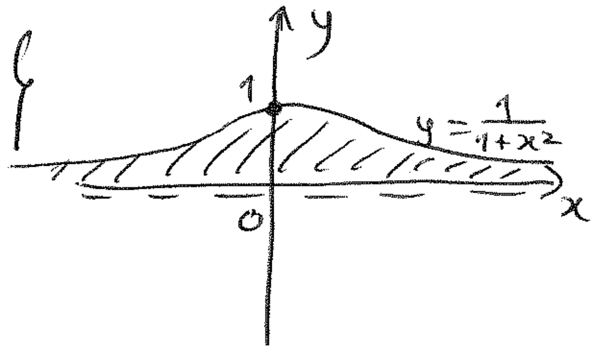
mi) $g(x,y) = \sqrt{\ln\left(\frac{1}{y} - x^2\right)}$

$D_g = \{(x,y) \in \mathbb{R}^2 : \ln\left(\frac{1}{y} - x^2\right) \geq 0 \wedge \frac{1}{y} - x^2 > 0 \wedge y \neq 0\}$

$= \{(x,y) \in \mathbb{R}^2 : \frac{1}{y} - x^2 \geq 1 \wedge \frac{1}{y} - x^2 > 0 \wedge y \neq 0\}$

$= \{(x,y) \in \mathbb{R}^2 : \frac{1}{y} - x^2 \geq 1 \wedge y \neq 0\}$

$= \{(x,y) \in \mathbb{R}^2 : y \leq \frac{1}{1+x^2} \wedge y > 0\}$



n) $f(x,y) = \begin{cases} \frac{x^2 - y^2}{x+y} & \text{se } x \neq y \\ 0 & \text{se } x = y \end{cases}$

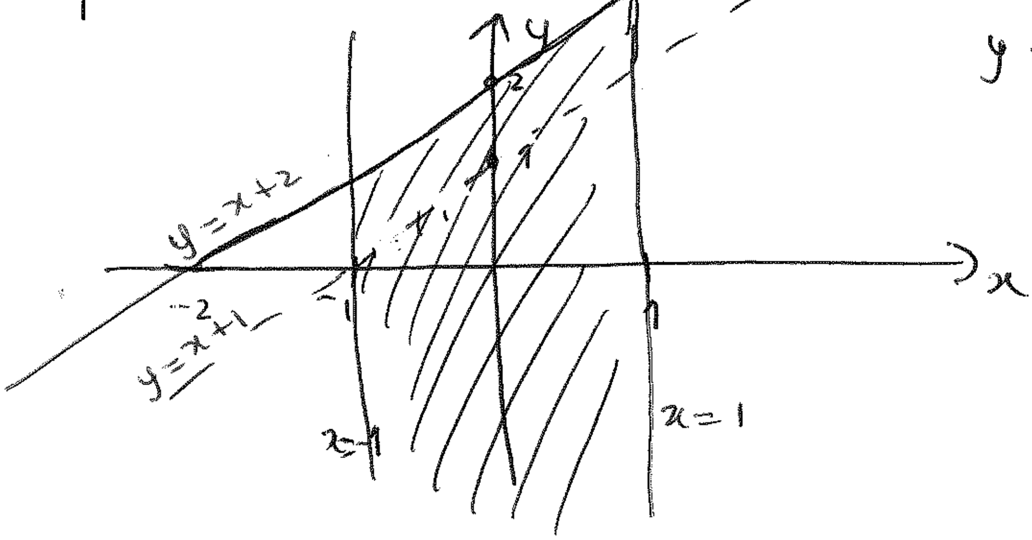
$D_f = \{(x,y) \in \mathbb{R}^2 : x + y \neq 0\} = \{(x,y) \in \mathbb{R}^2 : y \neq -x\}$

o) $f(x,y) = \begin{cases} \frac{\sqrt{1-x^2}}{\ln(x-y+2)} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$

$D_f = \{(x,y) \in \mathbb{R}^2 : 1 - x^2 \geq 0 \wedge x - y + 2 > 0 \wedge \ln(x-y+2) \neq 0\} \cup \{(0,0)\}$

$= \{(x,y) \in \mathbb{R}^2 : (1-x)(1+x) \geq 0 \wedge y < x + 2 \wedge x - y + 2 \neq 1\}$

$y \neq x + 1$



2. $f(x,y) = 2{,}5\,x + 1{,}4\,y$

$f(100,10) = 264$ → o dinheiro da venda de 100 produtos A e de 10 produtos B

3. $P(A,t) = A \exp(-0{,}05t)$

$P(100,10) = 100 \exp(-0{,}5)$ → o valor de 100 euros actuais daqui a 10 anos será de $100 \cdot e^{-0{,}5}$.

4. $f(R,M,S) = \frac{RM}{S}$

$f(400,400,80) = 2000$ → estimativa do nº de animais existentes nessa área quando se recolhe inicialmente 400 animais, da 2ª vez recolhem-se outros 400 animais e contaram-se 80 animais marcados.

5. $f(x,y) = 60\,x^{3/4}\,y^{1/4}$

a) $f(81,16) = 3240$ unidades de produto

b) $f(2x,2y) = 60\,\sqrt[4]{2^3 \cdot x^3}\,\sqrt[4]{2y} = 60 \times \sqrt[4]{2^4} \cdot x^{3/4} \cdot y^{1/4} =$

$= 2 \times 60 \cdot x^{3/4} \cdot y^{1/4} = 2f(x,y)$

6. $P(3,4) = \dfrac{4!\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)}{3!\,1!} = 4 \times \frac{3}{4} \times \left(\frac{1}{4}\right)^3 = \frac{3}{4^3}$

7. $f(x,y,z) = 10xy + 8zy + 6zy + 10xz + 5xz + xy$

a) $f(x,y,z) = 11xy + 14zy + 15xz$

b) $z = 50$

$x = 100$

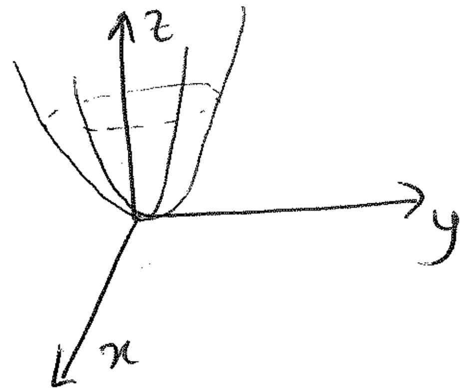
$y = 70$

$f(100,70,50) = 11 \times 7000 + 14 \times 3500 + 15 \times 5000$

$= 77000 + 49000 + 75000$

$= 201.000$ unidades de calor.

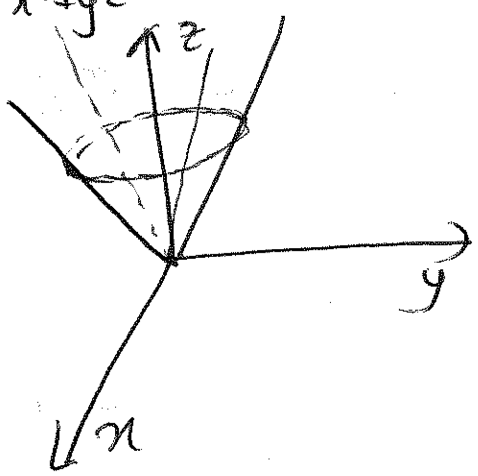
8. a) $z = x^2 + y^2$



paraboloide ao longo do eixo Oz

$z \geq 0$

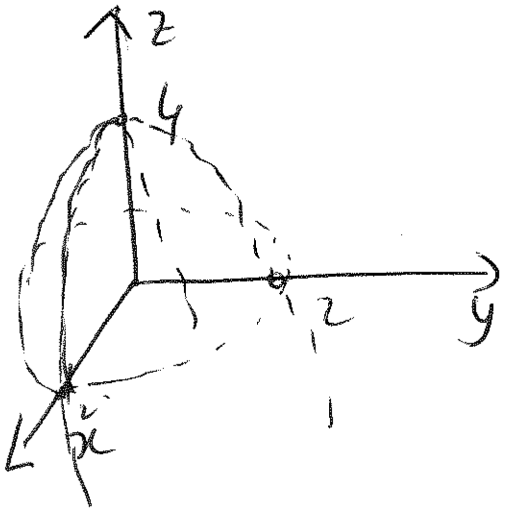
b) $f(x,y) = \sqrt{x^2+y^2}$



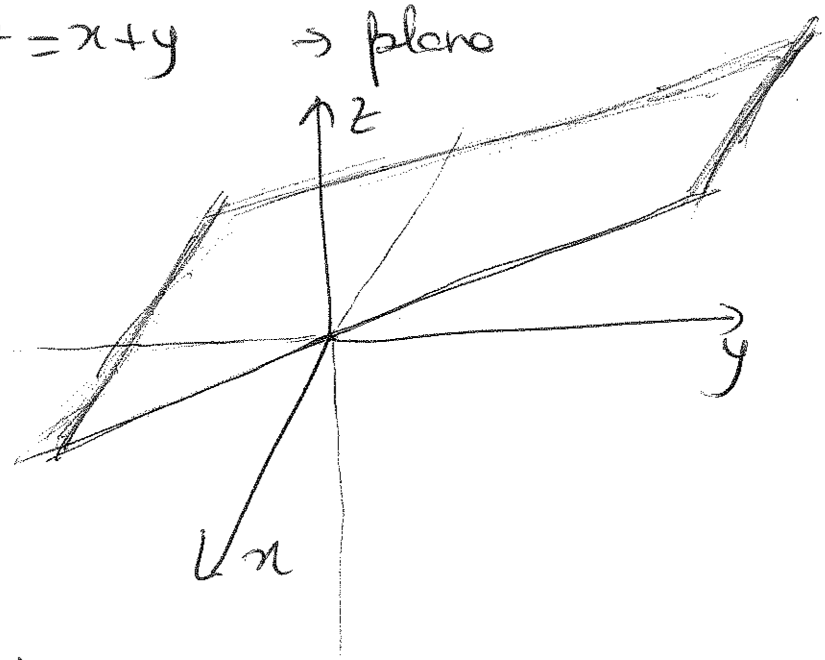
cone ao longo do eixo Oz, $z \geq 0$

c) $f(x,y) = 4 - x^2 - y^2$

paraboloide virado para baixo ao longo do eixo OX, $z \leq 4$.



d) $z = x + y \rightarrow$ plano



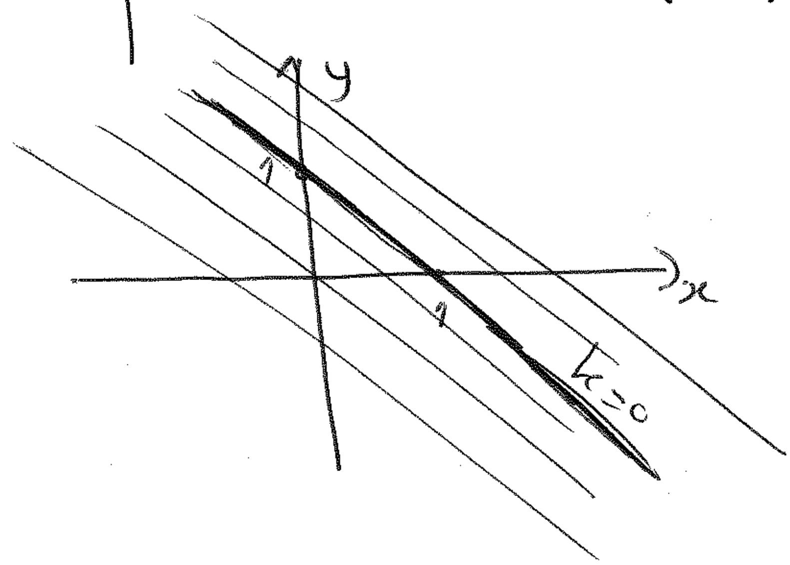
9. a) $z = 9 - x^2 - y^2$ — 3)
b) — 1)
c) — 4)
d) — 2)

10. a) $z = 3(1-x-y)$

Curvas de nível, resultam da intersecção da superfície $z = 3(1-x-y)$ com os planos $z = k$, $k \in \mathbb{R}$ horizontais.

$$\begin{cases} z = 3(1-x-y) \\ z = k \end{cases} \qquad \begin{cases} k = 3(1-x-y) \Leftrightarrow y = \left(1 - \frac{k}{3}\right) - x \end{cases}$$

$\rightarrow$ rectas

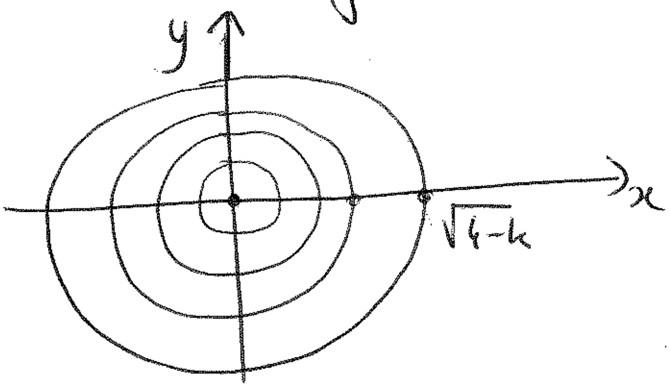


10 b) $f(x,y) = 4 - x^2 - y^2$

$$\begin{cases} z = 4 - x^2 - y^2 \\ z = k \quad , k \in \mathbb{R} \end{cases} \quad \begin{cases} k = 4 - x^2 - y^2 \Leftrightarrow x^2 + y^2 = 4 - k \end{cases}$$

$x^2 + y^2 = 4 - k \quad , k \in \mathbb{R}$

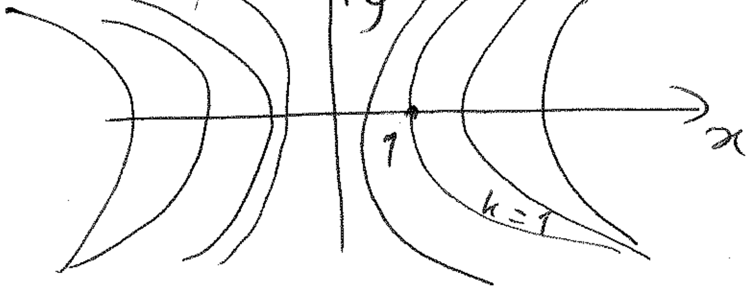
- se $4 - k > 0$, $x^2 + y^2 = 4 - k$ são circunferências de C $(0,0)$
  $k < 4$ e raio $\sqrt{4-k}$
- se $4 - k = 0 \Leftrightarrow k = 4$ $\quad x^2 + y^2 = 0 \rightarrow$ ponto $(0,0)$
- se $4 - k < 0 \Leftrightarrow k > 4$ conj. vazio



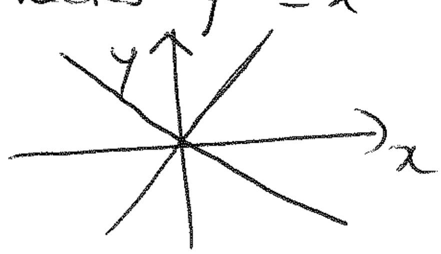
10.c) $z = x^2 - y^2$

$$\begin{cases} z = x^2 - y^2 \\ z = k \end{cases} \quad \begin{cases} x^2 - y^2 = k \end{cases}$$

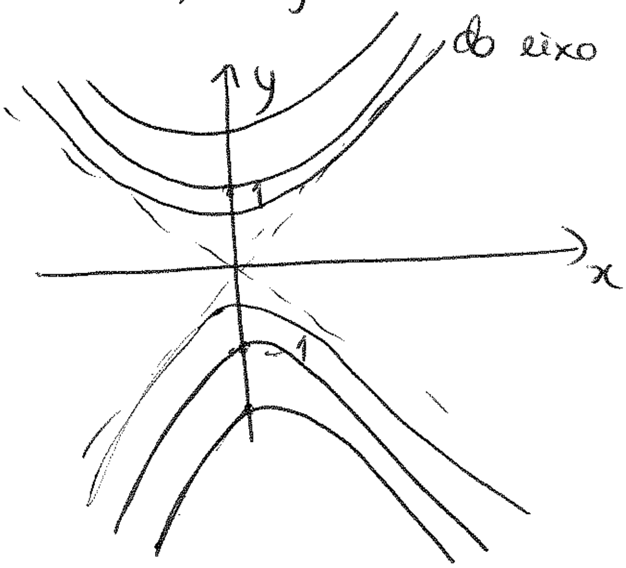
se $k > 0$, $x^2 - y^2 = k$ são hipérboles ao longo do eixo Ox



- se $k = 0$, $x^2 - y^2 = 0$ são as retas $y = \pm x$



- Se $k<0$, $x^2-y^2=k$ são hipérboles ao longo do eixo Oy



e)

$$f(x,y)=\sqrt{x^2+(y-2)^2}$$

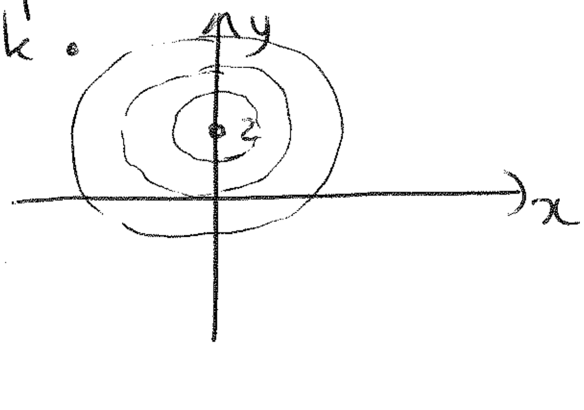
Assim,

$$\begin{cases} z=\sqrt{x^2+(y-2)^2} \\ z=k \end{cases} \quad \left\{ k=\sqrt{x^2+(y-2)^2} \right.$$

$k=0 \Rightarrow x^2+(y-2)^2=0$ ponto $(0,2)$

$k<0 \Rightarrow$ conj. vazio

$k>0 \Rightarrow$ $k=\sqrt{x^2+(y-2)^2}$ é a circunferência $k^2=x^2+(y-2)^2$ de centro $(0,2)$ e raio k.



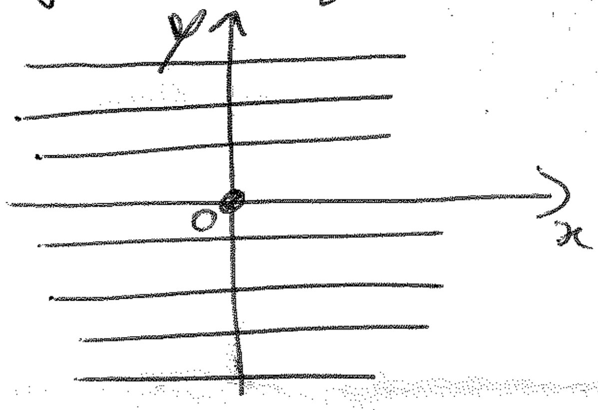
10.f) $f(x,y) = \operatorname{sen} y$

$\begin{cases} z = \operatorname{sen} y \\ z = k \end{cases}$ $\Big\{$ $\operatorname{sen} y = k$

- $-1 \leq k \leq 1 \Rightarrow \operatorname{sen} y = k$ são as rectas horizontais.

  $y = \operatorname{arcsen} k_0$

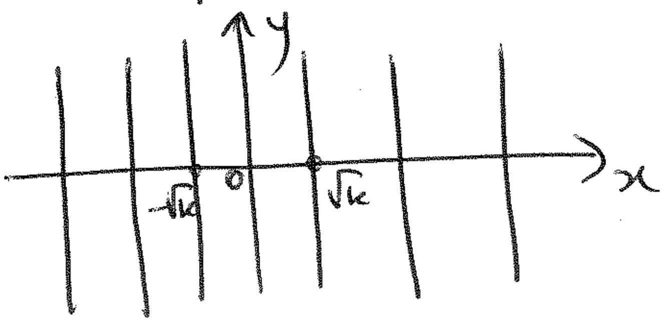
- $k \leq -1 \vee k \geq 1 \Rightarrow$ conj. vazio



10.d) $z = x^2$

$\begin{cases} z = x^2 \\ z = k \end{cases}$ $\Big\{$ $x^2 = k$

se $k > 0 \Rightarrow x = \pm\sqrt{k}$ $\rightarrow$ rectas verticais



- se $k = 0$ ... $x = 0$ $\rightarrow$ recta - eixo Oy
- se $k < 0$ $\Rightarrow$ conj. vazio.