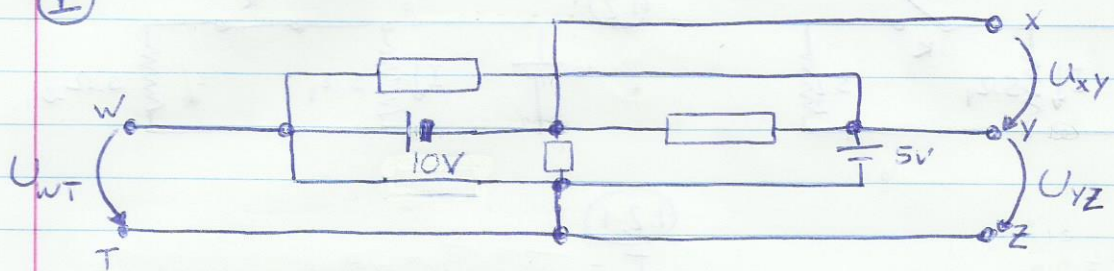


# Caderno de Exercícios - Aulas TP

\* 1ª Aula:

①



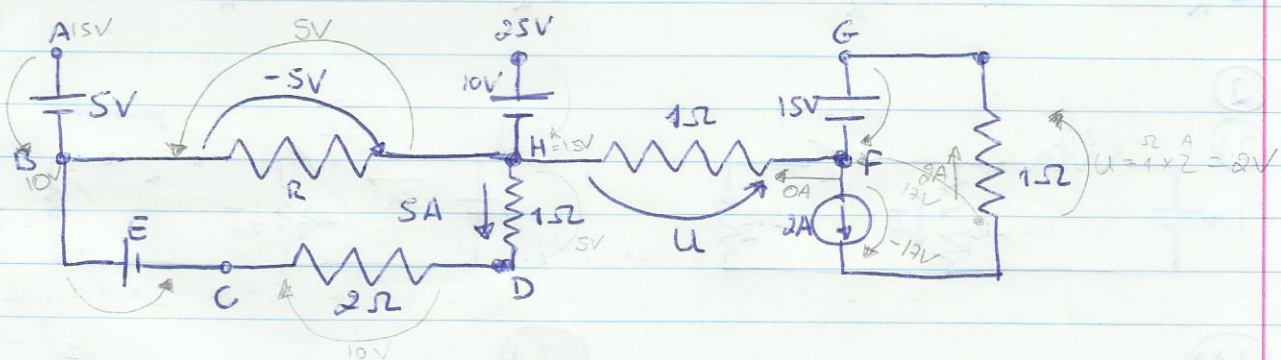
$$U_W = U_T \rightarrow U_{WT} = U_W - U_T = 0V$$

$$U_{xy} = -15$$

$$U_{yz} = +5$$

$U_y = \text{desconhecido}$

②



$$U_A = 15V$$

$$U_B = 10V$$

$$U_C = 0V$$

$$U_D = 10V$$

$$U_F = U_H = 15V$$

$$U_G = 30V$$

$$U_H = 15V$$

$$E = 10V$$

$$U = 0V \text{ (n\u00e3o passa corrente na resist\u00eancia)}$$

$$P_{2A} = U_{2A} \times I_{2A} = -17 \times 2 = -34W \rightarrow \text{fornece energia}$$

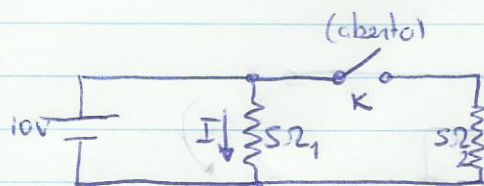
$$U_{HD} = 1 \times 5 = 5V$$

$$U_{DC} = 2 \times 5 = 10V$$

# \* 2ª Aula

①

1.1



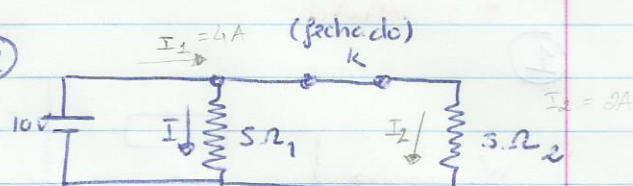
1.1.1

$$I = \frac{U}{R} = \frac{10}{5} = 2A$$

1.1.2

$U_{10V} = 10V$	$P_{10V} = 10 \times 2 = 20W$
$U_{5\Omega_1} = 10V$	$P_{5\Omega_1} = 10 \times 2 = 20W$
$U_{5\Omega_2} = 0V$	$P_{5\Omega_2} = 0 \times 0 = 0W$
$U_K = 10V$ (aberto)	$P_K = 10 \times 0 = 0W$

1.2



1.2.1

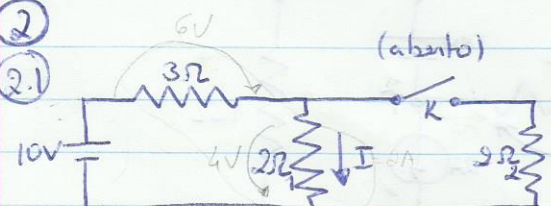
$$I = \frac{U}{R} = \frac{10}{5} = 2A$$

1.2.2

$U_{10V} = 10V$	$P_{10V} = 10 \times I_1 = 10 \times 4 = 40W$
$U_{5\Omega_1} = 10V$	$P_{5\Omega_1} = 10 \times I = 10 \times 2 = 20W$
$U_{5\Omega_2} = 10V$	$P_{5\Omega_2} = 10 \times I_2 = 10 \times 2 = 20W$
$U_K = 0V$	$P_K = 0 \times I_2 = 0 \times 2 = 0W$

②

2.1



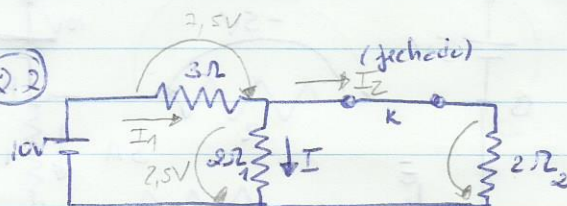
2.1.1

$$I = \frac{U}{R} = \frac{10}{R_{eq}} = \frac{10}{3+2} = \frac{10}{5} = 2A$$

2.1.2

$U_{10V} = 10V$	$P_{10V} = 10 \times 2 = 20W$
$U_{3\Omega} = 6V$	$P_{3\Omega} = 6 \times 2 = 12W$
$U_{2\Omega_1} = 4V$	$P_{2\Omega_1} = 4 \times 2 = 8W$
$U_{2\Omega_2} = 0V$	$P_{2\Omega_2} = 0 \times 0 = 0W$
$U_K = 4V$	$P_K = 4 \times 0 = 0W$

2.2



$$R_{eq} = (2 \parallel 2) + 3 = 1 + 3 = 4\Omega$$

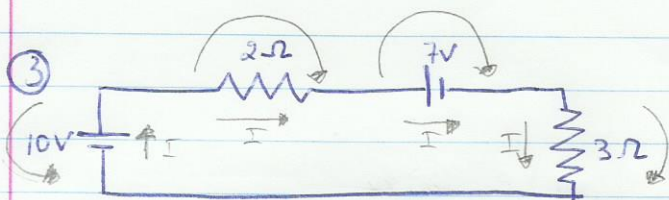
2.2.1

$$I = \frac{U}{R_{eq}} = \frac{10}{4} = 2.5A$$

2.2.2

$U_{10V} = 10V$	$P_{10V} = 10 \times I_1 = 10 \times 2.5 = 25W$
$U_{3\Omega} = 7.5V$	$P_{3\Omega} = 7.5 \times I_1 = 7.5 \times 2.5 = 18.75W$
$U_{2\Omega_1} = 2.5V$	$P_{2\Omega_1} = 2.5 \times 1.25 = 3.125W$
$U_{2\Omega_2} = 2.5V$	$P_{2\Omega_2} = 2.5 \times 1.25 = 3.125W$
$U_K = 0V$	$P_K = 0 \times 1.25 = 0W$





31 1 corrente

32 4 Tensões

33  $-10 + (2 \times I) + 7 + 3 \times I = 0$

( $\Rightarrow$ )  $-10 + 7 + 2I + 3I = 0$

( $\Rightarrow$ )  $-3 + 5I = 0$

( $\Rightarrow$ )  $I = \frac{3}{5} \approx 0,6 \text{ A}$  ← Corrente igual em todos os componentes

$U_{10V} = 10V$

$P_{10V} = -10 \times 0,6 = -6 \text{ W}$

$U_{2\Omega} = 2 \times 0,6 = 1,2V$

$P_{2\Omega} = 1,2 \times 0,6 = +0,72 \text{ W}$

$U_{7V} = 7V$

$P_{7V} = 7 \times 0,6 = +4,2 \text{ W}$

$U_{3\Omega} = 3 \times 0,6 = 1,8V$

$P_{3\Omega} = 1,8 \times 0,6 = +1,08 \text{ W}$

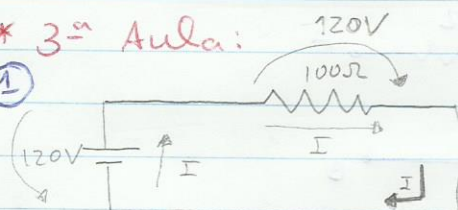
34

Absorve energia: Resistências ( $2\Omega$  e  $3\Omega$ ) e Fonte ( $7V$ )

Fornece energia: Fonte ( $10V$ )

\* 3ª Aula:

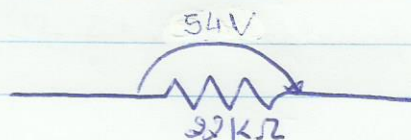
1



$P_{120V} = -(120 \times (-1,2)) = -144 \text{ W}$

$I = \frac{U}{R} = \frac{120V}{100\Omega} = -1,2 \text{ A}$

2



$P = \frac{U}{R}$

$E = P \times \Delta t$

( $\Rightarrow$ )  $E = U \times I \times \Delta t$

( $\Rightarrow$ )  $E = U \times \frac{U}{R} \times 2$

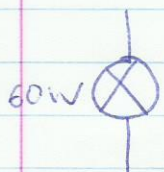
( $\Rightarrow$ )  $E = \frac{U^2}{R} \times 2$

( $\Rightarrow$ )  $E = \frac{(54)^2}{22k} \times 2$

( $\Rightarrow$ )  $E = 1,3255 \times 10^{-1} \times 2$

( $\Rightarrow$ )  $E = 2,6509 \times 10^{-1} \text{ Wh}$

③ 0,15 €/kWh



$$\Delta t = 8h \times 5d \times 4(\text{semanas}) = 160h$$

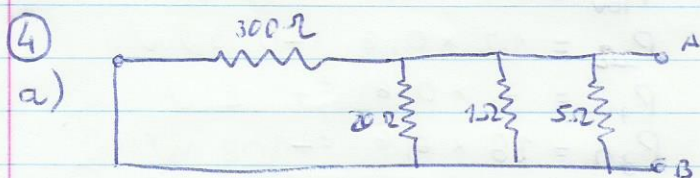
$$E = P \times \Delta t = 60 \times 160 = 9,6 \text{ kWh}$$

Custo mensal  $\rightarrow E \times 0,15 \text{ €/kWh}$

$$= 9,6 \text{ kWh} \times 0,15 \text{ €/kWh}$$

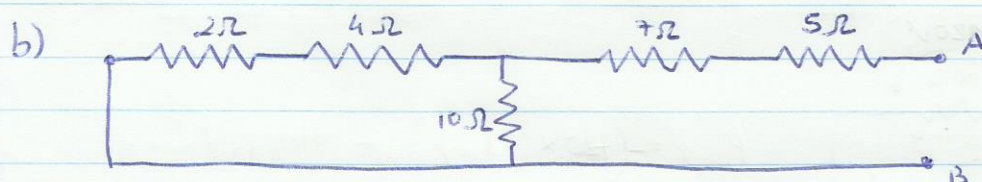
$$= 9,6 \times 0,15 \text{ €}$$

$$= 1,44 \text{ €}$$



$$\frac{1}{R_{eq}} = \frac{1}{300} + \frac{1}{20} + \frac{1}{12} + \frac{1}{5} \Leftrightarrow \frac{1}{R_{eq}} = 1,2533$$

$$\Leftrightarrow R_{eq} = 7,9787 \times 10^{-1} \Omega //$$



$$R_{eq} = [(2+4) // 10] + (7+5)$$

$$= (6 // 10) + 12$$

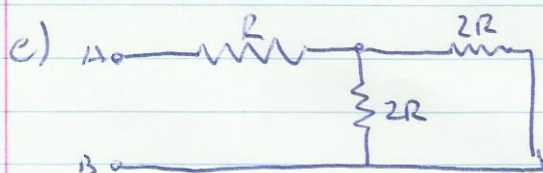
$$= 3,75 + 12$$

$$= 15,75 \Omega //$$

$$\frac{1}{R_{paralelo}} = \frac{1}{6} + \frac{1}{10}$$

$$\frac{1}{R_{paralelo}} = 2,6667 \times 10^{-1}$$

$$R_{paralelo} = 3,75 \Omega$$



$$R_{eq} = (2R // 2R) + R$$

$$= R + R$$

$$= 2R \Omega //$$

$$\frac{1}{R_{paralelo}} = \frac{1}{2R} + \frac{1}{2R}$$

$$\frac{1}{R_{paralelo}} = \frac{2}{2R}$$

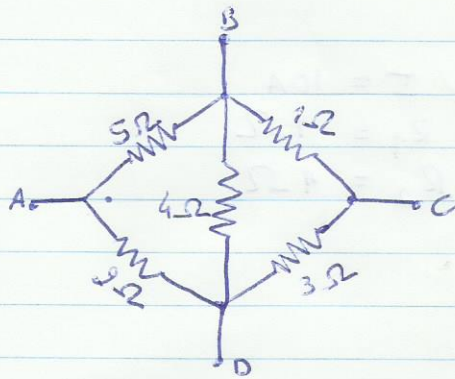
$$R_{paralelo} = \frac{2R}{2} = R$$

d) Repete-se 3 vezes o conjunto da alínea c), logo  $R_{eq} = 2R \Omega //$

e) Repete-se 5 vezes o conjunto da alínea c), logo  $R_{eq} = 2R \Omega //$



⑤



$$\begin{aligned}
 R_{AB} &= \left( \left[ (1+3) \parallel 4 \right] + 2 \right) \parallel 5 \\
 &= \left( (4 \parallel 4) + 2 \right) \parallel 5 \\
 &= (2 + 2) \parallel 5 \\
 &= 4 \parallel 5 \\
 &= 2,2222 \Omega
 \end{aligned}$$

$$\begin{aligned}
 4 \parallel 4: \frac{1}{R_{eq}} &= \frac{1}{4} + \frac{1}{4} \\
 \Leftrightarrow \frac{1}{R_{eq}} &= \frac{2}{4} = \frac{1}{2} \\
 \Leftrightarrow R_{eq} &= \frac{1 \times 2}{1} = 2 \Omega
 \end{aligned}$$

$$\begin{aligned}
 4 \parallel 5: \frac{1}{R_{eq}} &= \frac{1}{4} + \frac{1}{5} \\
 \Leftrightarrow \frac{1}{R_{eq}} &= 0,45 \\
 \Leftrightarrow R_{eq} &= 2,2222 \Omega
 \end{aligned}$$

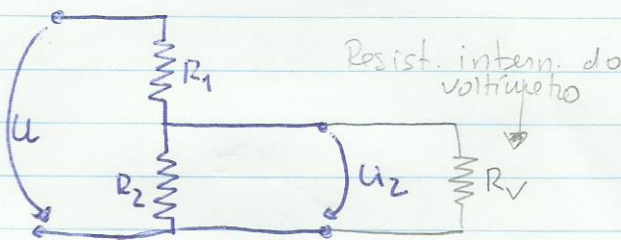
$$\begin{aligned}
 R_{BD} &= (5+2) \parallel 4 \parallel (1+3) \\
 &= 7 \parallel 4 \parallel 4 \\
 &= 1,5556 \Omega
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{R_{eq}} &= \frac{1}{7} + \frac{1}{4} + \frac{1}{4} \\
 \frac{1}{R_{eq}} &= 0,64786 \\
 R_{eq} &= 1,5556 \Omega
 \end{aligned}$$

$$R_{AC} = \text{Eq. } \Delta \text{ e } Y$$

\*4ª Aula

①



$$\begin{aligned}
 U &= 50V \\
 R_1 &= 1k\Omega \\
 R_2 &= 1k\Omega
 \end{aligned}$$

Divisor de Tensão

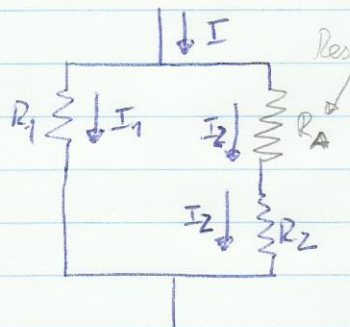
$$\begin{aligned}
 1.1 \quad R_V &= 1\Omega \\
 R_{eq} &= R_2 \parallel R_V \Leftrightarrow \frac{1}{R_{eq}} = \frac{1}{1000} + \frac{1}{1} \\
 \Leftrightarrow \frac{1}{R_{eq}} &= 1,001 \\
 \Leftrightarrow R_{eq} &= 0,999 \Omega
 \end{aligned}$$

$$\begin{aligned}
 U_2 &= \frac{R_{eq}}{R_1 + R_{eq}} \times U = \frac{0,999}{1000 + 0,999} \times 50 \\
 &= 0,0499V
 \end{aligned}$$

$$1.2 \quad R_V = 1k\Omega \quad R_{eq} = 500\Omega \quad U_2 = \frac{500}{1000 + 500} \times 50 = 16,667V$$

$$1.3 \quad R_V = 1M\Omega \quad R_{eq} = 999\Omega \quad U_2 = \frac{999}{1000 + 999} \times 50 = 24,988V$$

②



Resist. Intern. do Amperímetro  $I = 10A$

$$R_1 = 1\Omega$$

$$R_2 = 1\Omega$$

2.1  $R_A = 0,1\Omega$

$$R_{eq} = R_A + R_2 \Leftrightarrow R_{eq} = 0,1 + 1$$

$$\Leftrightarrow R_{eq} = 1,1\Omega$$

Divisão de corrente

$$I_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_{eq}}} \times I = \frac{\frac{1}{1}}{\frac{1}{1} + \frac{1}{1,1}} \times 10 = 4,7619A$$

2.2  $R_A = 1\Omega$

$$R_{eq} = 2\Omega$$

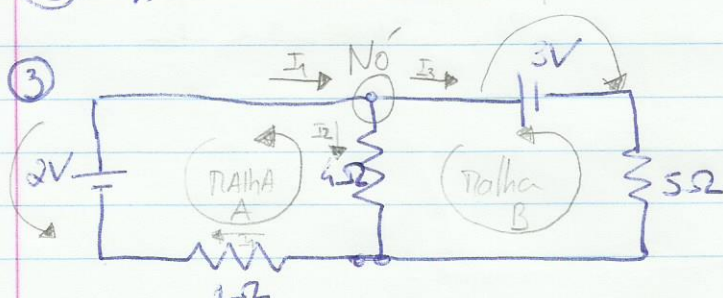
$$I_2 = 3,3333A$$

2.3  $R_A = 1K\Omega$

$$R_{eq} = 1001\Omega$$

$$I_2 = 0,00998A$$

③



$$A: \begin{cases} 2 - I_1 - 4I_2 = 0 \end{cases}$$

$$B: \begin{cases} 4I_2 - 5I_3 - 3 = 0 \end{cases}$$

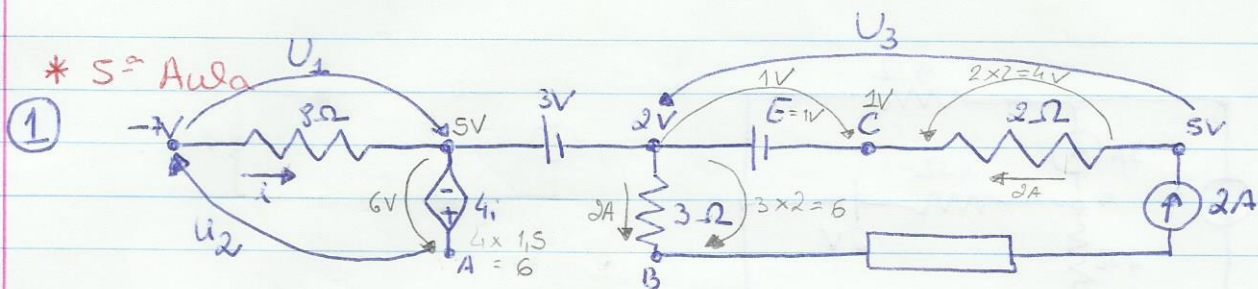
$$No: \begin{cases} I_1 = I_2 + I_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2 - I_1 - 4I_2 = 0 \\ 4I_2 - 5I_3 - 3 = 0 \\ I_1 = I_2 + I_3 \end{cases} \Leftrightarrow \begin{cases} -I_1 = -2 + 4I_2 \\ -5I_3 = 3 - 4I_2 \end{cases} \Leftrightarrow \begin{cases} I_1 = 2 - 4I_2 \\ I_3 = -\frac{3}{5} + \frac{4}{5}I_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} -4I_2 - I_2 - \frac{4}{5}I_2 = -2 - \frac{3}{5} \end{cases} \Leftrightarrow \begin{cases} -\frac{29}{5}I_2 - \frac{4}{5}I_2 = -\frac{10}{5} - \frac{3}{5} \end{cases}$$

$$\begin{cases} -\frac{29}{5}I_2 = -\frac{13}{5} \\ -29I_2 = -13 \end{cases} \begin{cases} I_1 = 2 - \frac{4 \times 13}{29} = 0,20690A \\ I_3 = -\frac{3}{5} + \frac{4 \times 13}{5 \times 29} = -0,24138A \\ I_2 = \frac{13}{29} = 0,44828A \end{cases}$$





$$U_A = (5) - (6) = -1 \text{ V}$$

$$U_B = (2) - (6) = -4 \text{ V}$$

$$U_C = (5) - (4) = 1 \text{ V}$$

$$i = \frac{U_A}{R} = \frac{-12}{8} = -1,5 \text{ A}$$

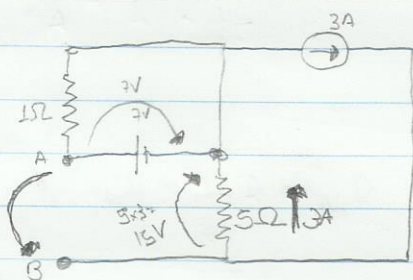
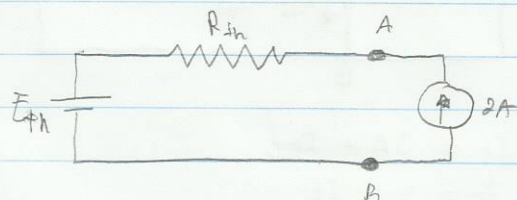
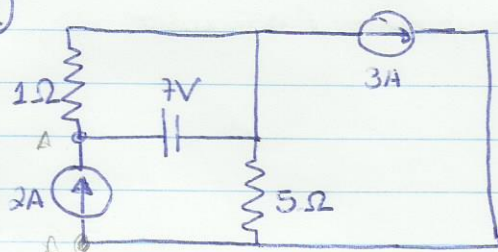
$$E = 1 \text{ V}$$

$$U_1 = (-7) - (5) = -12 \text{ V}$$

$$U_2 = (-1) - (-7) = 6 \text{ V}$$

$$U_3 = (5) - (2) = 3 \text{ V}$$

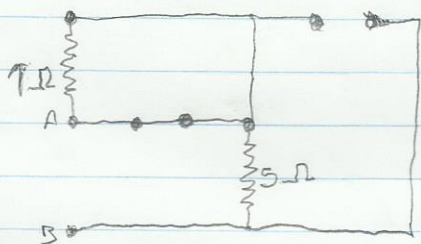
2



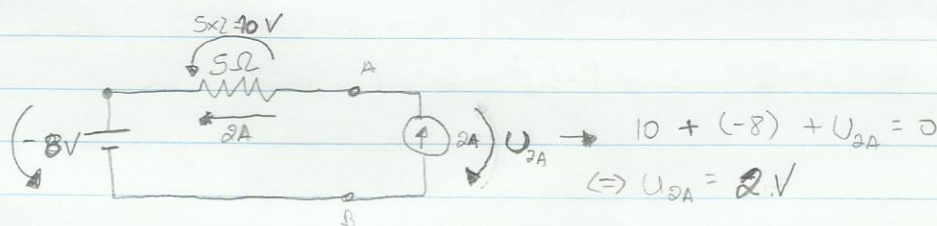
$$U_{AB} - 7 \text{ V} + 15 \text{ V} = 0 \Leftrightarrow$$

$$\Leftrightarrow U_{AB} = +7 - 15$$

$$\Leftrightarrow U_{AB} = -8 \text{ V} = E_{th}$$



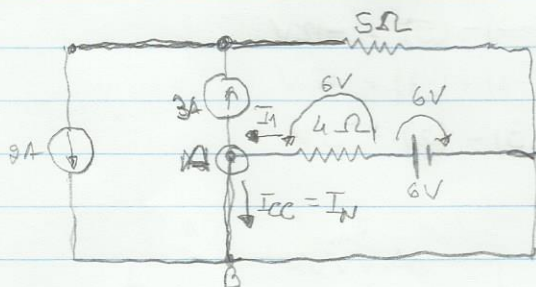
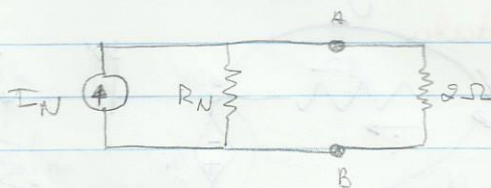
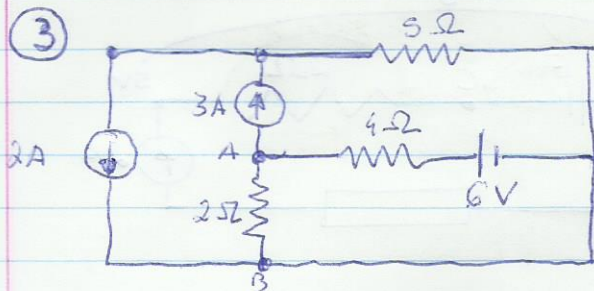
$$R_{AB} = (1/0) + 5 = 0 + 5 = 5 \Omega = R_{th}$$



$$P_{2A} = U_{2A} \times I_{2A} = -(2 \times 2) = -4 \text{ W}$$

A fonte de 2A tem uma potência negativa, logo fornece energia.

3



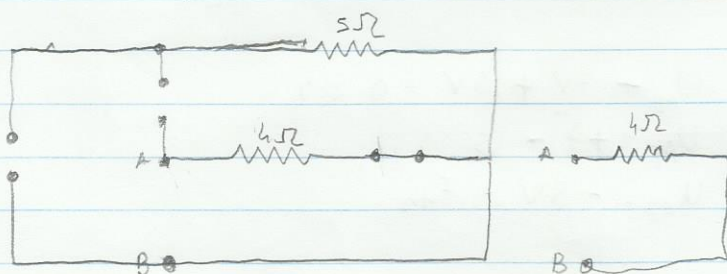
Os terminais da resistência são os mesmos da fonte de 6V logo a tensão existente entre os terminais dos dois componentes é o mesmo.

$$A: I_1 = 3A + I_{CC}$$

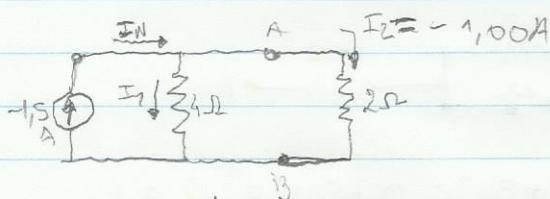
$$\Leftrightarrow \frac{6}{4} = 3 + I_{CC}$$

$$\Leftrightarrow 1,5 = 3 + I_{CC}$$

$$\Rightarrow I_{CC} = -1,5A = I_N$$



$$R_{AB} = 4\Omega = R_N$$



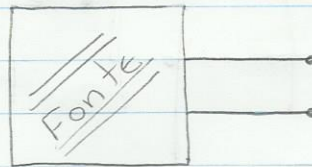
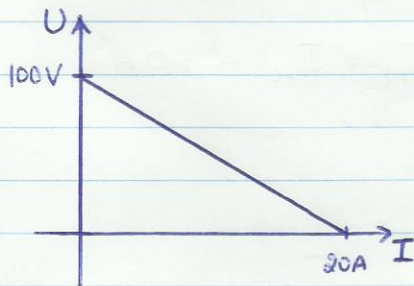
$$I_2 = \frac{\frac{1}{2}}{\frac{1}{4} + \frac{1}{2}} \times (-1,5) = \frac{0,5}{0,75} \times (-1,5) = -1,00A$$

$$P_{2\Omega} = U \times I_2 = R \times I \times I = 2 \times (-1)^2 = 2W$$

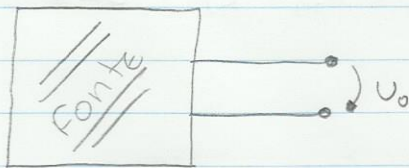


\* 6ª Aula

1



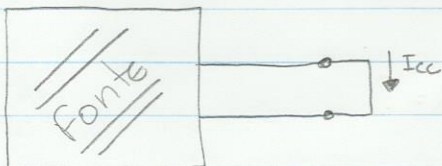
1.1



( $I = 0A \rightarrow$  circuito aberto)

$$\therefore U_0 = 100V$$

1.2



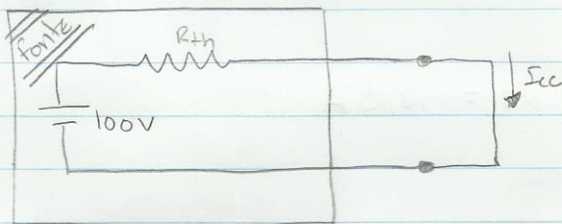
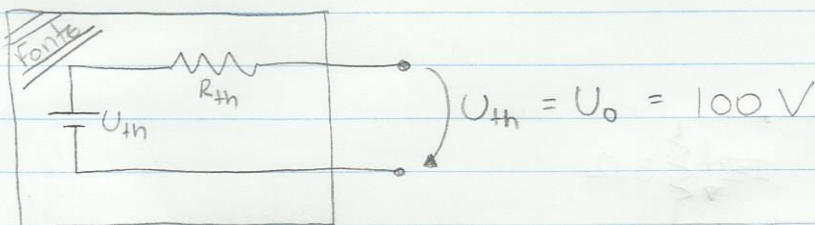
( $U = 0V \rightarrow$  curto-circuito)

$$\therefore I_{cc} = 20A$$

1.3

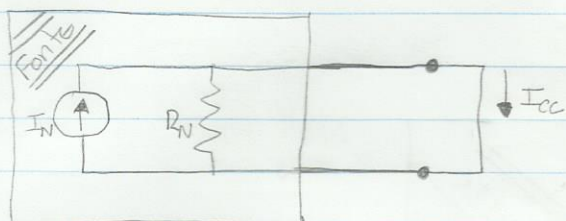
$$R_i = \frac{U_0}{I_{cc}} = \frac{100}{20} = 5\Omega \quad (R_i = r_m = \frac{\Delta U}{\Delta I})$$

1.4

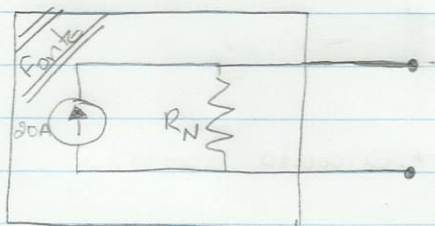


$$R_{th} = \frac{U_{th}}{I_{cc}} = \frac{100}{20} = 5\Omega = R_i$$

15

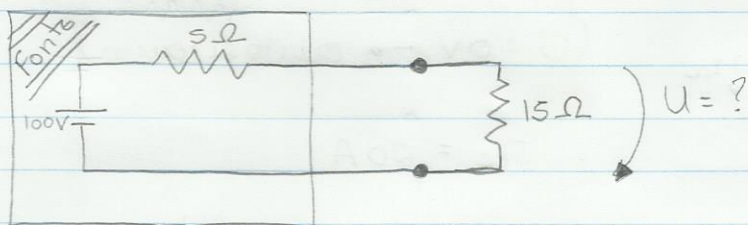


$$I_N = I_{CC} = 20 \text{ A}$$



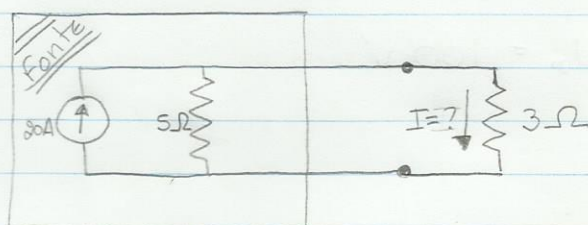
$$R_N = R_{th} = R_i = 5 \Omega$$

16



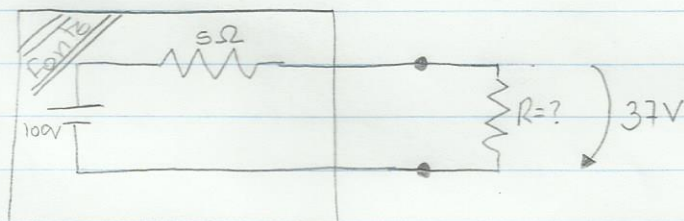
Divisor de tensão:  $U = \frac{15}{15 + 5} \times 100 = 75 \text{ V}$

17



Divisor de corrente:  $I = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{5}} \times 20 = 12,5 \text{ A}$

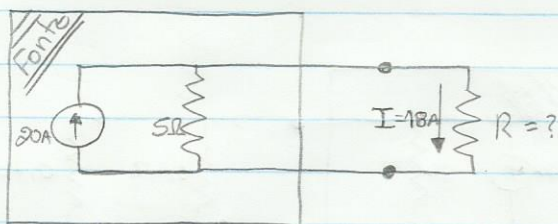
18



Divisor de tensão:  $37 = \frac{R}{R + 5} \times 100 \Leftrightarrow \frac{37}{100} = \frac{R}{R + 5}$   
 $\Leftrightarrow R = \frac{37(R + 5)}{100} \Leftrightarrow R = \frac{37R}{100} + \frac{185}{100}$   
 $\Leftrightarrow \frac{100R}{100} - \frac{37R}{100} = 1,85 \Leftrightarrow \frac{63R}{100} = 1,85$   
 $\Leftrightarrow 63R = 1,85 \times 100 \Leftrightarrow R = \frac{185}{63} = 2,9365 \Omega$



19



Divisor de corrente:  $18 = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{5}} \times 20 \Leftrightarrow \frac{18}{20} = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{5}}$

$$\Leftrightarrow \frac{1}{R} = \frac{18(\frac{1}{R} + \frac{1}{5})}{20}$$

$$\Leftrightarrow \frac{1}{R} = \frac{0,9}{R} + \frac{0,9}{5}$$

$$\Leftrightarrow \frac{1}{R} - \frac{0,9}{R} = \frac{0,9}{5}$$

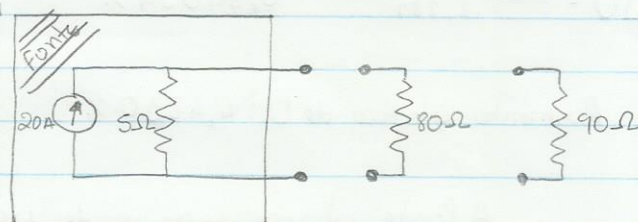
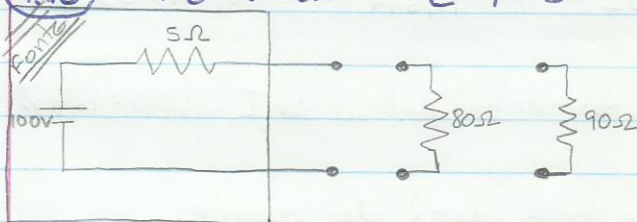
$$\Leftrightarrow \frac{0,1}{R} = \frac{0,9}{5}$$

$$\Leftrightarrow R = \frac{0,1 \times 5}{0,9}$$

$$\Leftrightarrow R = \frac{0,5}{0,9}$$

$$\Leftrightarrow R = 0,55556 \Omega$$

140 •  $R_e$  varia em  $[80, 90] \Omega$



Divisor de tensão:  $U = \frac{R_e}{R_e + R_i} \times U_0$

$$U_{80\Omega} = \frac{80}{80 + 5} \times 100 = 94,12 V$$

$$U_{90\Omega} = \frac{90}{90 + 5} \times 100 = 94,74 V$$

$$\Delta U = \frac{|94,12 - 94,74|}{94,12} = 0,7 \%$$

$$\Delta U = \frac{|94,12 - 94,74|}{94,74} = 0,7 \%$$

Aumento Relativo de  $U = 0\%$

Divisor de corrente:  $I = \frac{\frac{1}{R_e}}{\frac{1}{R_e} + \frac{1}{R_i}} \times I_{cc}$

$$I_{80\Omega} = \frac{\frac{1}{80}}{\frac{1}{80} + \frac{1}{5}} \times 20 = 1,18 A$$

$$I_{90\Omega} = \frac{\frac{1}{90}}{\frac{1}{90} + \frac{1}{5}} \times 20 = 1,05 A$$

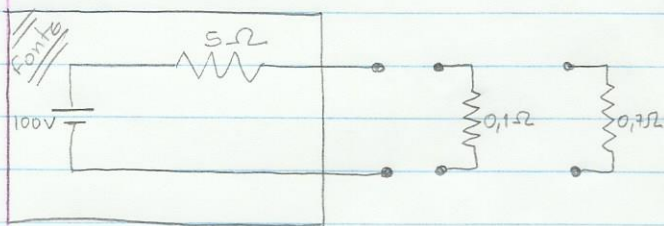
$$\Delta I = \frac{|1,18 - 1,05|}{1,18} = 11 \%$$

$$\Delta I = \frac{|1,18 - 1,05|}{1,05} = 12 \%$$

Aumento Relativo de  $I = 1\%$

A fonte aproxima-se de uma fonte ideal de tensão, quando as cargas variam  $[80, 90] \Omega$

1.10 •  $R_e$  varia em  $[0,1; 0,7] \Omega$



Divisor de tensão:  $U = \frac{R_e}{R_e + R_i} \times U_0$

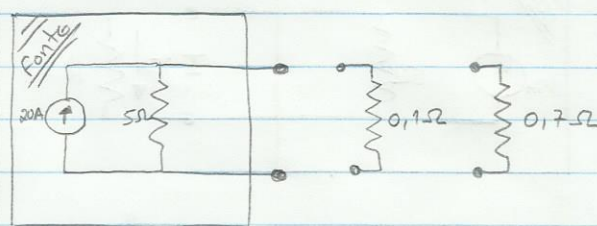
$$U_{0,1\Omega} = \frac{0,1}{0,1 + 5} \times 100 = 1,9608 \text{ V}$$

$$U_{0,7} = \frac{0,7}{0,7 + 5} \times 100 = 12,281 \text{ V}$$

$$\Delta U = \frac{12,281 - 1,9608}{1,9608} = 5,2633 \%$$

$$\Delta U = \frac{12,281 - 1,9608}{12,281} = 0,84034 \%$$

Aumento Relativo de U: 4,4230 %



Divisor de corrente:  $I = \frac{\frac{1}{R_e}}{\frac{1}{R_e} + \frac{1}{R_i}} \times I_{cc}$

$$I_{0,1\Omega} = \frac{\frac{1}{0,1}}{\frac{1}{0,1} + \frac{1}{5}} \times 20 = 19,608 \text{ A}$$

$$I_{0,7\Omega} = \frac{\frac{1}{0,7}}{\frac{1}{0,7} + \frac{1}{5}} \times 20 = 17,544 \text{ A}$$

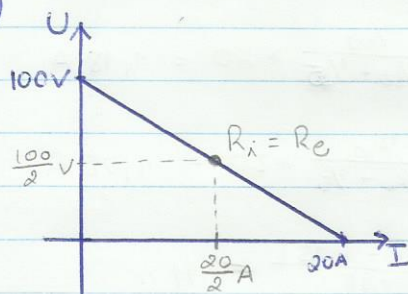
$$\Delta I = \frac{19,608 - 17,544}{19,608} = 0,10526 \%$$

$$\Delta I = \frac{19,608 - 17,544}{17,544} = 0,11765 \%$$

Aumento Relativo de I: 0,007124 %

A fonte aproxima-se de uma fonte ideal de corrente quando as cargas variam em  $[0,1; 0,7] \Omega$

1.11

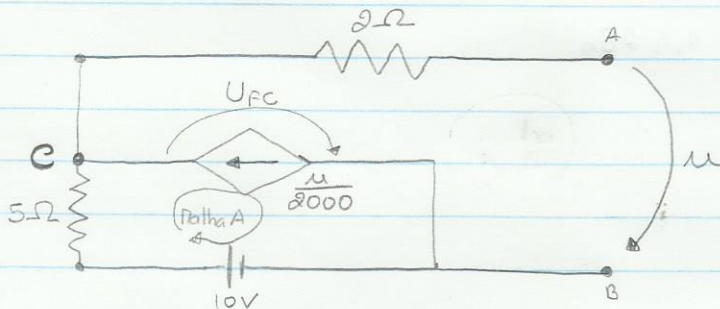
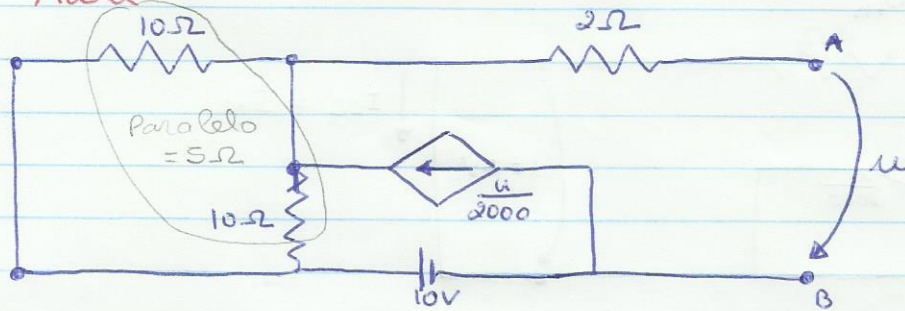


$$\begin{aligned} P_{\text{MAX}} = U \times I &= \frac{U_0}{2} \times \frac{I_{cc}}{2} \\ &= \frac{100}{2} \times \frac{20}{2} \\ &= \frac{100 \times 20}{2 \times 2} \\ &= \frac{2000}{4} \\ &= 500 \text{ W} \end{aligned}$$



\* 7ª Aula

①



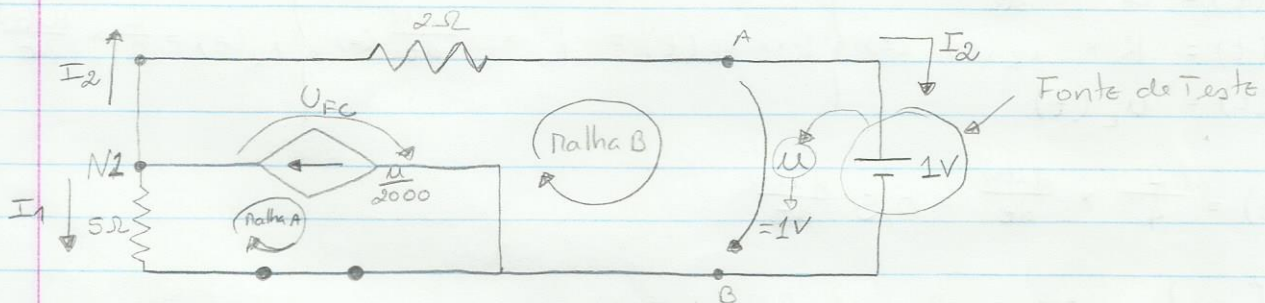
$$U_{AC} = 0V$$

$$U_{th} = U_{AB} = U$$

$$\text{Malha A: } \begin{cases} -5 \cdot \frac{\mu}{2000} + U_{FC} - 10 = 0 \\ \mu = U_{FC} \end{cases} \Leftrightarrow \begin{cases} -5 \cdot \frac{\mu}{2000} + \mu - 10 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} -\frac{5\mu}{2000} + \mu = 10 \end{cases} \Leftrightarrow \begin{cases} \frac{-5\mu + 2000\mu}{2000} = 10 \end{cases}$$

$$\Leftrightarrow \begin{cases} 1995\mu = 2000 \times 10 \end{cases} \Leftrightarrow \begin{cases} \mu = \frac{20000}{1995} = 10,025V \end{cases}$$



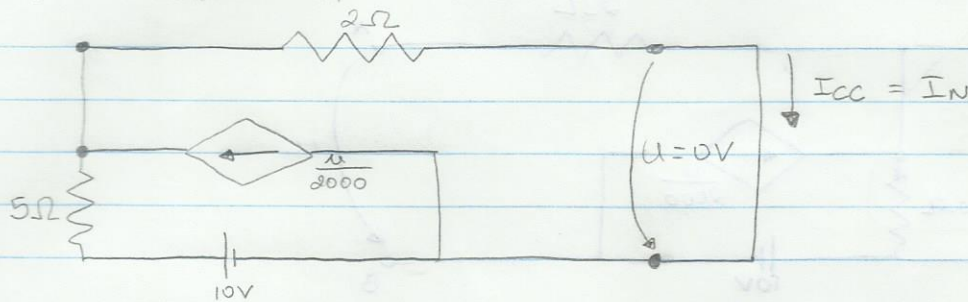
$$R_{th} = R_{AB} \rightarrow$$

$$\begin{cases} \text{Malha A: } -5 \cdot I_1 + U_{FC} = 0 \\ \text{Malha B: } -5 \cdot I_1 + 2I_2 + 1 = 0 \\ \text{Nó 1: } \frac{1}{2000} = I_1 + I_2 \end{cases} \dots \begin{cases} U_{FC} = 0,715 \\ I_1 = 0,143 \\ I_2 = -0,1425 \end{cases}$$

$$R_{th} = R_{AB} = \frac{1}{-I_2}$$

ou

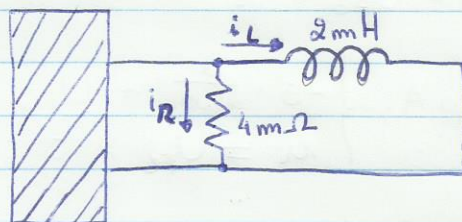
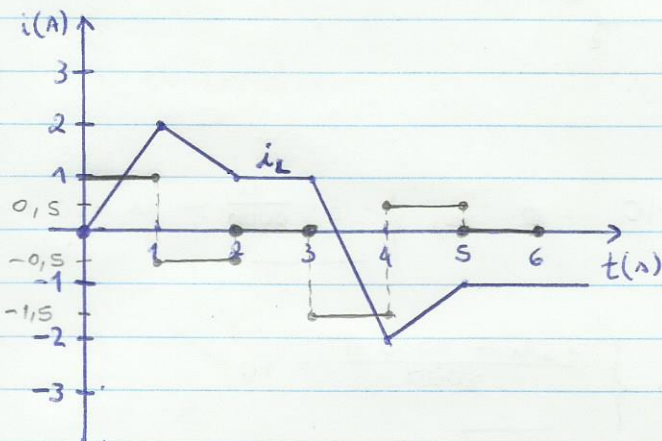
$$R_{th} = \frac{U_{th}}{I_N} = \frac{I_{cc}}{U_{th}}$$



$$I_{cc} = \frac{U}{R_{eq}} = \frac{10}{5+2} = \frac{10}{7} = 1,4286$$

$$R_{th} = \frac{10,025}{1,4286} = 7,0175 \Omega$$

2



$$u_L = \int (i_L)$$

$$\begin{cases} u_L(t) = L \times \frac{di(t)}{dt} \\ u_R(t) = R \times i_R(t) \Leftrightarrow R \times i_R(t) = L \times \frac{di(t)}{dt} \Leftrightarrow i_R(t) = \frac{L}{R} \times \frac{di(t)}{dt} \\ u_R(t) = u_L(t) \end{cases}$$

$$i_R(t) = \frac{2}{4} \times \frac{di(t)}{dt} = 0,5 \frac{di(t)}{dt}$$

- $t \in [0, 1] \rightarrow 0,5 \times 2 = 1 A$
- $t \in [1, 2] \rightarrow 0,5 \times (-1) = -0,5 A$
- $t \in [2, 3] \rightarrow 0,5 \times 0 = 0 A$
- $t \in [3, 4] \rightarrow 0,5 \times (-3) = -1,5 A$
- $t \in [4, 5] \rightarrow 0,5 \times 1 = 0,5 A$
- $t \in [5, 6] \rightarrow 0,5 \times 0 = 0 A$