Folha Nº 1 /

FEUP FACULDADE DE ENGENHARIA
UNIVERSIDADE DO PORTO

Curso: MIEM / MIEGI — Data: / 12 / 20

Disciplina: Álgebra Linear e Geometria Analítica — Ano: 1º — Semestre: 1º

Nome: José Augusto Trigo Barbosa

Espaço reservado para o avaliador: AULAS: TEÓRICO - PRÁTICAS

Determinantes

14)

$$\Delta = \begin{vmatrix} x+3 & 1 & 2 & 1 \\ 0 & x & y & z \\ 4 & 4x+4 & 4y+2 & 4z \\ 9 & 3x+3 & 3y+6 & 3z+3 \end{vmatrix} \begin{matrix} \\ \\ \leftarrow L_3 - 4L_2 \\ \leftarrow L_4 - 3L_2 \end{matrix} =$$

$$= \begin{vmatrix} x+3 & 1 & 2 & 1 \\ 0 & x & y & z \\ 4 & 4 & 2 & 0 \\ 9 & 3 & 6 & 3 \end{vmatrix} \begin{matrix} \\ \\ \leftarrow L_3 / 2 \\ \leftarrow L_4 / 3 \end{matrix} =$$

$$= 6 \begin{vmatrix} x+3 & 1 & 2 & 1 \\ 0 & x & y & z \\ 2 & 2 & 1 & 0 \\ 3 & 1 & 2 & 1 \end{vmatrix} \begin{matrix} \leftarrow L_1 - L_4 \\ \\ \\ \\ \end{matrix} =$$

$$= 6 \begin{vmatrix} x & 0 & 0 & 0 \\ 0 & x & y & z \\ 2 & 2 & 1 & 0 \\ 3 & 1 & 2 & 1 \end{vmatrix} = 6\,(x)\,(-1)^2 \begin{vmatrix} x & y & z \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$

$$= 6\,(x)\left(\frac{1}{3}\right) = 2x$$

16)

$$|C| = \begin{vmatrix} 2 & 1 & 0 & -a & 3 \\ 1 & 2 & a-1 & 1 & a+2 \\ -1 & 0 & 2 & 0 & 2 \\ -2 & -1 & 3 & 2 & 3 \\ 1 & 0 & -2 & 1 & -2 \end{vmatrix} =$$

$$\uparrow \qquad\qquad \uparrow$$

$$C_3+2C_1 \qquad C_5+2C_1$$

$$= \begin{vmatrix} 2 & 1 & 4 & -a & 7 \\ 1 & 2 & a+1 & 1 & a+4 \\ -1 & 0 & 0 & 0 & 0 \\ -2 & -1 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & 0 \end{vmatrix} =$$

$$= (-1)(-1)^4 \begin{vmatrix} 1 & 4 & -a & 7 \\ 2 & a+1 & 1 & a+4 \\ -1 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 \end{vmatrix} =$$

$$= -(1)(-1)^7 \begin{vmatrix} 1 & 4 & 7 \\ 2 & a+1 & a+4 \\ -1 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 6 \\ 2 & a-1 & a+2 \\ -1 & 0 & 0 \end{vmatrix} =$$

$$\uparrow \qquad\qquad \uparrow$$

$$C_2-C_1 \qquad C_3-C_1$$

$$= (-1)(-1)^4 \begin{vmatrix} 3 & 6 \\ a-1 & a+2 \end{vmatrix} = -\left[3(a+2) - 6(a-1)\right] = 3a-12$$

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Determinantes

21) a)

$$A^{-1} = \frac{1}{|A|}\left[\mathrm{Cof}(A)\right]^T \Rightarrow \left(A^{-1}\right)^T = \frac{1}{|A|}\left[\mathrm{Cof}(A)\right]$$

$$\left(A^T\right)^{-1} = \frac{1}{|A^T|}\left[\mathrm{Cof}(A^T)\right]^T \Leftrightarrow$$

$$\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$$

$$|A^T| = |A|$$

$$\Leftrightarrow \left(A^{-1}\right)^T = \frac{1}{|A|}\left[\mathrm{Cof}(A^T)\right]^T$$

$$\left[\mathrm{Cof}(A)\right] = \left[\mathrm{Cof}(A^T)\right]^T \Leftrightarrow$$

$$\Leftrightarrow \left[\mathrm{Cof}(A)\right]^T = \mathrm{Cof}(A^T)$$

b)

$$|A^{-1}| = \left|\frac{1}{|A|}\left[\mathrm{Cof}(A)\right]^T\right| \Leftrightarrow$$

$$|A^{-1}| = \frac{1}{|A|}$$

$$|kA| = k^n|A|$$

$A \hookrightarrow n \times n$

$$\Leftrightarrow \frac{1}{|A|} = \frac{1}{|A|^n}\left|\left[\mathrm{Cof}(A)\right]^T\right| \Leftrightarrow$$

$$\Leftrightarrow |A|^{n-1} = |\mathrm{Cof}(A)|$$

28)

$$|G| = \begin{vmatrix} k+x & g_{12} & g_{13} & \cdots & g_{1n} \\ x & k & 0 & \cdots & 0 \\ x & k & k & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x & k & k & \cdots & k \end{vmatrix} =$$

$$\text{Coluna } 1 = \begin{bmatrix} k \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} x \\ x \\ x \\ \vdots \\ x \end{bmatrix} \quad (\text{Propriedade } 9)$$

$$= \begin{vmatrix} k & g_{12} & g_{13} & \cdots & g_{1n} \\ 0 & k & 0 & \cdots & 0 \\ 0 & k & k & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & k & k & \cdots & k \end{vmatrix} + \begin{array}{c} \begin{vmatrix} x & g_{12} & g_{13} & \cdots & g_{1n} \\ x & k & 0 & \cdots & 0 \\ x & k & k & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x & k & k & \cdots & k \end{vmatrix} \end{array} \begin{array}{l} \\ \leftarrow L_2 - L_1 \\ \leftarrow L_3 - L_1 \\ \vdots \\ \leftarrow L_n - L_1 \end{array} =$$

$$= k(-1)^2 \underbrace{\begin{vmatrix} k & 0 & \cdots & 0 \\ k & k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ k & k & \cdots & k \end{vmatrix}}_{(n-1)\times(n-1)} + \begin{vmatrix} x & g_{12} & g_{13} & \cdots & g_{1n} \\ 0 & k-g_{12} & -g_{13} & \cdots & -g_{1n} \\ 0 & k-g_{12} & k-g_{13} & \cdots & -g_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & k-g_{12} & k-g_{13} & \cdots & k-g_{1n} \end{vmatrix} =$$

$$= k\, k^{n-1} + x(-1)^2 \underbrace{\begin{vmatrix} k-g_{12} & -g_{13} & \cdots & -g_{1n} \\ k-g_{12} & k-g_{13} & \cdots & -g_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ k-g_{12} & k-g_{13} & \cdots & k-g_{1n} \end{vmatrix}}_{(n-1)\times(n-1)} \begin{array}{l} \\ \leftarrow L_2 - L_1 \\ \vdots \\ \leftarrow L_{n-1} - L_1 \end{array} =$$

$$= k^n + x \underbrace{\begin{vmatrix} k-g_{12} & -g_{13} & \cdots & -g_{1n} \\ 0 & k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & k & \cdots & k \end{vmatrix}}_{(n-1)\times(n-1)} =$$

$$= k^n + x(k-g_{12})(-1)^2 \underbrace{\begin{vmatrix} k & \cdots & 0 \\ \vdots & \ddots & \vdots \\ k & \cdots & k \end{vmatrix}}_{(n-2)\times(n-2)} = k^n + x(k-g_{12})\, k^{n-2}$$