Exame de recurso

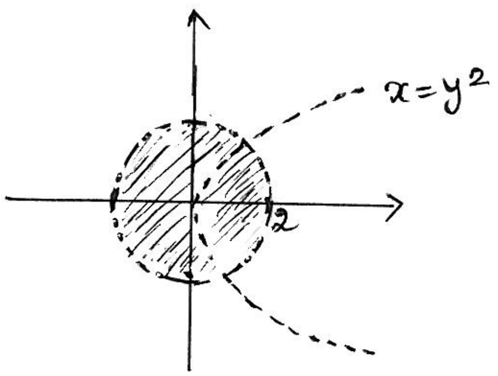
8 de julho de 2013

MEC, EICOM

Análise Matemática EE

1. $f(x,y) = \dfrac{y^2}{x-y^2} \ln(4-x^2-y^2)$

$$D_f = \{(x,y)\in\mathbb{R}^2 : x-y^2 \neq 0 \;\text{ e }\; 4-x^2-y^2>0\}$$
$$= \{(x,y)\in\mathbb{R}^2 : x \neq y^2 \;\text{ e }\; x^2+y^2<4\}$$



2.

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3yx^2}{x^2+y^4} = 0$, uma vez que

$\displaystyle\lim_{(x,y)\to(0,0)} 3y = 0$ e $\dfrac{x^2}{x^2+y^4} \leq \dfrac{x^2}{x^2} = 1$

(função limitada)

b) Domínio de continuidade : $\mathbb{R}^2 \setminus \{(0,0)\}$

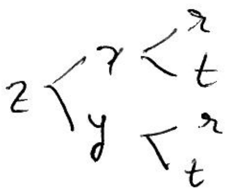
g é contínua em $(x,y)\neq(0,0)$ por ser o quociente de duas funções contínuas (funções polinomiais) e é descontínua em $(0,0)$ pois

$$f(0,0) = 1 \neq \lim_{(x,y)\to(0,0)} g(x,y)$$

3. $z = f(x,y)$

$x = r e^{-t}$

$y = r e^{t}$



$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} = \frac{\partial z}{\partial x}\cdot(-re^{-t}) + \frac{\partial z}{\partial y}(re^{t})$$

$$= r\left(-\frac{\partial z}{\partial x}e^{-t} + \frac{\partial z}{\partial y}e^{t}\right)$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$= \frac{\partial z}{\partial x}\cdot e^{-t} + \frac{\partial z}{\partial y}\cdot e^{t}$$

$$\frac{\partial z}{\partial r} + \frac{1}{r}\frac{\partial z}{\partial t} = \cancel{\frac{\partial z}{\partial x}e^{-t}} + \frac{\partial z}{\partial y}e^{t} - \cancel{\frac{\partial z}{\partial x}e^{-t}} + \frac{\partial z}{\partial y}e^{t}$$

$$= 2e^{t}\frac{\partial z}{\partial y}$$

4.

$u(x,y) = e^{-\alpha t}\operatorname{sen} x, \quad \alpha > 0$

$$\frac{\partial u}{\partial t} = -\alpha e^{-\alpha t}\operatorname{sen} x \qquad \frac{\partial u}{\partial x} = e^{-\alpha t}\cos x$$

$$\frac{\partial^2 u}{\partial x^2} = -e^{-\alpha t}\operatorname{sen} x$$

5. $f(x,y,z) = \operatorname{sen}(zx) + e^{-x^2+y^3-3z^2}$

a) $\dfrac{\partial f}{\partial y} = 3y^2 e^{-x^2+y^3-3z^2} \geq 0$, uma vez que $3y^2 \geq 0$ e

$e^{-x^2+y^3-3z^2} > 0$

b) $\vec{v} = Q - P = (2,1,2) - (1,1,0) = (1,0,2)$

$\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{1}{\sqrt{5}}(1,0,2)$

$D_{\vec{u}} f(P) = \nabla f(P) \cdot \vec{u}$

$\nabla f = \Big(z\cos(zx) - 2x\, e^{-x^2+y^3-3z^2} , 3y^2 e^{-x^2+y^3-3z^2} , x\cos(zx) - 6z\, e^{-x^2+y^3-3z^2}$

$\nabla f(P) = (0-2, 3, \cos(0)) = (-2,3,1)$

$D_{\vec{u}} f(P) = (-2,3,1) \cdot \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right) = 0$

c) Direção de variação máxima: $\nabla f(P) = (-2,3,1)$

Taxa máxima de variação: $\|\nabla f(P)\| = \sqrt{14}$

6. $f(x,y) = x^2 + \frac{1}{2}y^2 + x^2 y + 4$

. Pontos críticos

$$\begin{cases} \dfrac{\partial f}{\partial x} = 0 \\ \dfrac{\partial f}{\partial y} = 0 \end{cases} \Leftrightarrow \begin{cases} 2x + 2xy = 0 \\ y + x^2 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x + 2x(-x^2) = 0 \\ y = -x^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x(1-x^2) = 0 \\ - \end{cases} \Leftrightarrow \begin{cases} x=0 \\ y=0 \end{cases} \vee \begin{cases} x=1 \\ y=-1 \end{cases} \vee \begin{cases} x=-1 \\ y=-1 \end{cases}$$

- Discriminante

$$D(x,y) = f_{xx} \cdot f_{yy} - (f_{xy})^2$$
$$= (2 + 2y) \cdot 1 - (2x)^2$$
$$= 2y - 4x^2 + 2$$

- Classificação dos pontos críticos

– $D(0,0) = 2 > 0$

$f_{xx}(0,0) = 2 > 0$

Logo, $(0,0)$ é minimizante local.

– $D(1,-1) = -2 - 4 + 2 = -4 < 0$

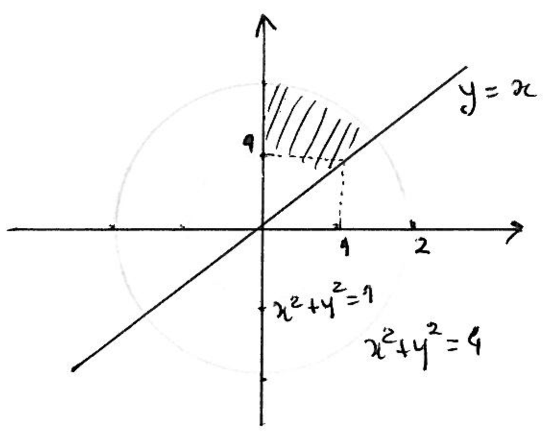
Logo, $(0,0)$ é ponto de sela.

– $D(-1,-1) = -2 - 4 + 2 = -4 < 0$

Logo, $(0,0)$ é ponto de sela

7. $$I = \iint_D \cos(x^2+y^2)\, dx\, dy$$

a)



$$D = \left\{(r,\theta) : 1 \le r \le 2 , \frac{\pi}{4} \le \theta \le \frac{\pi}{2}\right\}$$

b) $$\iint_D \cos(x^2+y^2)\,dx\,dy = \int_1^2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(r^2)\cdot r\,d\theta\,dr$$

$$= \int_1^2 \Big[\theta\cos(r^2)\cdot r\Big]_{\theta=\frac{\pi}{4}}^{\frac{\pi}{2}} dr = \int_1^2 \frac{\pi}{4} r\cos r^2\,dr$$

$$= \left[\frac{\pi}{4}\,\mathrm{sen}(r^2)\right]_{r=1}^{2} = \frac{\pi}{4}(\mathrm{sen}\,4 - \mathrm{sen}\,1)$$

8.

a) Região de integração:

$$-1 \leq x \leq 1$$

$$-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$$

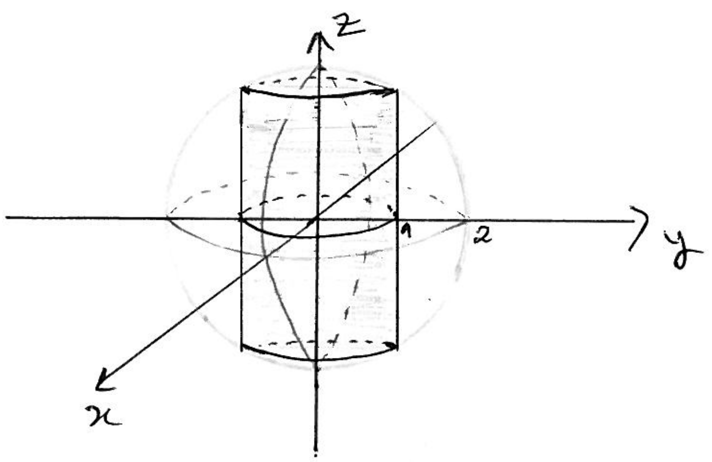
$$-\sqrt{4-x^2-y^2} \leq z \leq \sqrt{4-x^2-y^2}$$

$$y=\sqrt{1-x^2} \Rightarrow y^2 = 1-x^2 \Rightarrow x^2+y^2=1$$

$$z=\sqrt{4-x^2-y^2} \Rightarrow z^2 = 4-x^2-y^2 \Rightarrow x^2+y^2+z^2=4$$

$$x^2+y^2=1 \Leftrightarrow y=\pm\sqrt{1-x^2}$$

$$x^2+y^2+z^2=4 \Leftrightarrow z=\pm\sqrt{4-x^2-y^2}$$



b) e c)

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} \frac{z}{\sqrt{x^2+y^2}}\, dz\, dy\, dx =$$

$$= \int_0^1\int_0^{2\pi}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} \frac{z}{r} \cdot r\, dz\, d\theta\, dr$$

$$= \int_0^1\int_0^{2\pi}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} z\, dz\, d\theta\, dr = \int_0^1\int_0^{2\pi} \left[\frac{z^2}{2}\right]_{z=-\sqrt{4-r^2}}^{\sqrt{4-r^2}} d\theta\, dr$$

$$= \int_0^1\int_0^{2\pi} \frac{4-r^2-4-r^2}{2}\, d\theta\, dr = 0$$

9.

a) $r'(t) = (\cos t, -\operatorname{sen} t, 2)$ $\qquad r'(0) = (1, 0, 2)$

$r''(t) = (-\operatorname{sen} t, -\cos t, 0)$ $\qquad r''(0) = (0, -1, 0)$

b) $T(t) = \dfrac{r'(t)}{\|r'(t)\|} = \dfrac{1}{\sqrt{5}}(\cos t, -\operatorname{sen} t, 2)$

$T'(t) = \dfrac{1}{\sqrt{5}}(-\operatorname{sen} t, -\cos t, 0)$

$\|T'(t)\| = \dfrac{1}{\sqrt{3}}$

$k(t) = \dfrac{\|T'(t)\|}{\|r'(t)\|} = \dfrac{1/\sqrt{5}}{\sqrt{5}} = \dfrac{1}{5}$

c) $\displaystyle\int_C F \cdot ds = \int_0^2 F(r(t)) \cdot r'(t)\, dt$

$$= \int_0^2 F(\text{sen}t, \cos t, 2t+1) \cdot (\cos t, -\text{sen}t, 2)\, dt$$

$$= \int_0^2 (\text{sen}t - \cos t,\ \text{sen}t + \cos t,\ 2t+1) \cdot (\cos t, -\text{sen}t, 2)\, dt$$

$$= \int_0^2 [\cos t(\text{sen}t - \cos t) - \text{sen}t(\text{sen}t + \cos t) + 4t + 2]\, dt$$

$$= \int_0^2 (-1 + 4t + 2)\, dt$$

$$= \int_0^2 (4t+1)\, dt = \left[2t^2 + t\right]_0^2 = 10$$

d) $$\vec{\text{rot}}\, F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x-y & x+y & z \end{vmatrix} = (0,\ 0,\ 1-(-1)) = (0, 0, 2) \neq \vec{0}$$

10.

$$u'(t) = r'(t) \cdot [r'(t) \times r''(t)] + r(t) \cdot [r'(t) \times r''(t)]'$$

$$= r'(t) \cdot [r'(t) \times r''(t)] + r(t) \times [r'(t) \times r'''(t)]$$

(usando o resultado apresentado)

$$= 0 + r(t) \times [r'(t) \times r'''(t)]$$

$$= r(t) \times [r'(t) \times r'''(t)]$$