Folha Nº 1 /

FEUP FACULDADE DE ENGENHARIA
UNIVERSIDADE DO PORTO

Curso .... Data ..../..../....

Disciplina .... Ano .... Semestre ....

Nome ....

Espaço reservado para o avaliador

Produto Vectorial / Misto

25) $\vec{c} + \vec{d} = (3,0,3)$ $\quad$ $\vec{c} = (1,-1,2)$

$\vec{c} - \vec{d} = (-1,-2,1)$ $\quad$ $\vec{d} = (2,1,1)$

$$\vec{c} \times \vec{d} = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = (-3,3,3)$$

$\vec{a} \times \vec{c} \cdot \vec{b} = -1 \Leftrightarrow \vec{b} \times \vec{a} \cdot \vec{c} = -1 \Leftrightarrow \vec{a} \times \vec{b} \cdot \vec{c} = 1$

$\vec{d} \times \vec{d} \cdot \vec{b} = 7 \Leftrightarrow \vec{b} \times \vec{a} \cdot \vec{d} = 7 \Leftrightarrow \vec{a} \times \vec{b} \cdot \vec{d} = -7$

$\vec{a} \times \vec{b} = (x,y,z)$ $\quad$ $\begin{cases} x - y + 2z = 1 \\ 2x + y + z = -7 \end{cases}$ $\quad$ $\begin{cases} \text{---} \\ 3x + 3z = -6 \end{cases}$

$\begin{cases} y = x - 4 - 2x - 1 \\ z = -2 - x \end{cases}$ $\quad$ $\begin{cases} y = -x - 5 \\ z = -2 - x \end{cases}$

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \begin{vmatrix} x & -x-5 & -2-x \\ -3 & 3 & 3 \end{vmatrix} =$$

$$= (\cancel{-3x} - 15 + 6 + \cancel{3x}, \cancel{-3x} + 6 + \cancel{3x}, \cancel{3x} - \cancel{3x} - 15) =$$

$$= (-9, 6, -15)$$

33)

$$\cos(\theta) = \frac{\vec{b}\cdot\vec{c}}{\|\vec{b}\|\|\vec{c}\|} = \frac{\vec{b}\cdot\vec{c}}{\|\vec{b}\|^2}$$

$$\vec{b}\cdot\vec{c} = (\vec{d} - \vec{a}\times\vec{c})\cdot\vec{c} = \vec{c}\cdot\vec{d} - \underbrace{\vec{c}\cdot\vec{a}\times\vec{c}}_{=0} \checkmark$$

$$\vec{c}\cdot\vec{d} = \|\vec{c}\|\|\vec{d}\|\cos(\pi/3) = \frac{\|\vec{b}\|\|\vec{d}\|}{2} \checkmark$$

$$\|\vec{d}\|^2 = (\vec{a}\times\vec{c} + \vec{b})\cdot(\vec{a}\times\vec{c} + \vec{b}) = \|\vec{a}\times\vec{c}\|^2 + \|\vec{b}\|^2 + 2\underbrace{\vec{b}\cdot\vec{a}\times\vec{c}}_{=0}$$

$$\vec{b}\cdot\vec{a}\times\vec{c} = \vec{c}\cdot\vec{b}\times\vec{a} = -\vec{c}\cdot\vec{a}\times\vec{b} = 0$$

$$\|\vec{a}\times\vec{c}\|^2 = \|\vec{a}\|^2\|\vec{c}\|^2\,\text{sen}^2(\pi/6) = \frac{\|\vec{b}\|^2}{4}$$

$$\|\vec{d}\|^2 = \frac{\|\vec{b}\|^2}{4} + \|\vec{b}\|^2 = \frac{5\|\vec{b}\|^2}{4} \iff \|\vec{d}\| = \frac{\sqrt{5}\|\vec{b}\|}{2}$$

$$\vec{c}\cdot\vec{d} = \frac{\sqrt{5}\|\vec{b}\|^2}{4} \checkmark$$

$$\vec{b}\cdot\vec{c} = \frac{\sqrt{5}\|\vec{b}\|^2}{4} \checkmark \qquad \cos(\theta) = \frac{\sqrt{5}\cancel{\|\vec{b}\|^2}}{4\cancel{\|\vec{b}\|^2}} \iff \theta = \arccos\left(\frac{\sqrt{5}}{4}\right)$$

35) a) $$\vec{a}\times\vec{d} = \underbrace{\vec{a}\times\vec{a}}_{=\vec{0}} + 3\,\vec{a}\times\vec{b} + 4\,\vec{a}\times\vec{c} = 3\vec{c} - 4\vec{b}$$

$$\|\vec{a}\times\vec{d}\|^2 = 9\|\vec{c}\|^2 + 16\|\vec{b}\|^2 - 24\underbrace{\vec{c}\cdot\vec{b}}_{=0} = 25$$

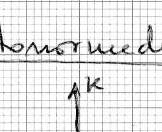
$$\|\vec{a}\times\vec{d}\| = 5$$

b) $$\vec{c}\times(\vec{a}\times\vec{d}) = \vec{c}\times(3\vec{c} - 4\vec{b}) = 3\underbrace{\vec{c}\times\vec{c}}_{=\vec{0}} - 4\,\vec{c}\times\vec{b} = 4\vec{a}$$

c) $$\cos(\beta) = \frac{\vec{c}\cdot\vec{a}\times\vec{d}}{\|\vec{c}\|\|\vec{a}\times\vec{d}\|} = \frac{3}{5} \iff \beta = \arccos(3/5)$$

$$\vec{c}\cdot(\vec{a}\times\vec{d}) = 3\|\vec{c}\|^2 - 4\underbrace{\vec{c}\cdot\vec{b}}_{=0} = 3$$

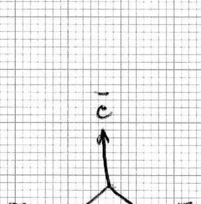
<u>Base ortonormada directa</u>:



$\vec{i}\times\vec{j} = \vec{k}$

$\vec{j}\times\vec{k} = \vec{i}$

$\vec{k}\times\vec{i} = \vec{j}$



$\|\vec{a}\| = \|\vec{b}\| = \|\vec{c}\| = 1$

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{a}\cdot\vec{c} = 0$

$\vec{a}\times\vec{b} = \vec{c}$

$\vec{b}\times\vec{c} = \vec{a}$

$\vec{c}\times\vec{a} = \vec{b}$

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28 c)

$E = (x,y,z) \qquad \overrightarrow{AE} = (x-1, y+1, z-4)$

$V = |\overrightarrow{AE} \cdot \overrightarrow{AB} \times \overrightarrow{AD}| = 24$

$\overrightarrow{AB} = (4,0,0)$
$\overrightarrow{AD} = (1,-2,0)$

$$\overrightarrow{AE} \cdot \overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} x-1 & y+1 & z-4 \\ 4 & 0 & 0 \\ 1 & -2 & 0 \end{vmatrix} = 32 - 8z$$

$|32-8z| = 24 \Leftrightarrow 32-8z = 24 \vee 32-8z = -24 \Leftrightarrow$

$\Leftrightarrow z = 1 \vee z = 7$

$E = (x,y,1),\ x,y \in \mathbb{R} \quad \vee \quad E = (x,y,7),\ x,y \in \mathbb{R}$

20 b) $S = \{\vec{a}, \vec{b}\}$ é linearmente independente $\Rightarrow$

$\Rightarrow \vec{a} \times \vec{b} \neq \vec{0} \wedge S_1 = \{\vec{a}, \vec{b}, \vec{a}\times\vec{b}\}$ é um conjunto linearmente independente, ou seja,

$\alpha_1 \vec{a} + \alpha_2 \vec{b} + \alpha_3 \vec{a}\times\vec{b} = \vec{0} \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0$

O conjunto U é linearmente independente se

$\beta_1(\vec{a} - 2\vec{b}) + \beta_2(\vec{a} + (\vec{a}\times\vec{b})) + \beta_3(2\vec{b} + (\vec{a}\times\vec{b})) = \vec{0} \Rightarrow$

$\Rightarrow \beta_1 = \beta_2 = \beta_3 = 0$

$$\beta_1(\bar{a}-2\bar{b})+\beta_2(\bar{a}+(\bar{a}\times\bar{b}))+\beta_3(2\bar{b}+(\bar{a}\times\bar{b}))=\bar{0} \Rightarrow$$

$$\Rightarrow \underbrace{(\beta_1+\beta_2)}_{=0}\vec{a}+\underbrace{(-2\beta_1+2\beta_3)}_{=0}\bar{b}+\underbrace{(\beta_2+\beta_3)}_{=0}\bar{a}\times\bar{b}=\bar{0} \Rightarrow$$

$$\Rightarrow \begin{cases}\beta_1+\beta_2=0\\ -2\beta_1+2\beta_3=0\\ \beta_2+\beta_3=0\end{cases} \Rightarrow \begin{cases}0=0\\ \beta_1=-\beta_2\\ \beta_3=-\beta_2\end{cases} \quad \forall_{\beta_2\in\mathbb{R}}$$

Conclui-se que U é linearmente dependente

17 a) $\bar{b}+\bar{c}=\bar{a}\times\bar{b}$

$\bar{a}\cdot(\bar{b}+\bar{c})=\bar{a}\cdot\bar{a}\times\bar{b}=0$ ✓

b) $$\cos(\beta)=\frac{\bar{b}\cdot\bar{c}}{\|\bar{b}\|\|\bar{c}\|}=\frac{-\|\bar{b}\|^2}{\|\bar{b}\|\|\bar{c}\|}=\frac{-\|\bar{b}\|}{\|\vec{c}\|}$$

$$\bar{b}\cdot\bar{c}=\bar{b}\cdot[(\bar{a}\times\bar{b})-\bar{b}]=\underbrace{\bar{b}\cdot(\bar{a}\times\bar{b})}_{=0}-\|\bar{b}\|^2 \qquad \|\bar{b}\|>0$$

$$\|\bar{b}\|>0 \wedge \|\bar{c}\|>0 \Rightarrow \cos(\beta)\neq 0 \Rightarrow \beta\neq\frac{\pi}{2}$$

$$\|\vec{c}\|^2=[(\bar{a}\times\bar{b})-\bar{b}]\cdot[(\bar{a}\times\bar{b})-\bar{b}]=$$

$$=\|\bar{a}\times\bar{b}\|^2-\underbrace{\bar{b}\cdot\bar{a}\times\bar{b}}_{=0}-\underbrace{(\bar{a}\times\bar{b})\cdot\bar{b}}_{=0}+\|\bar{b}\|^2=$$

$\{\bar{a},\bar{b}\}$ é linearmente independente $\Rightarrow \bar{a}\times\bar{b}\neq 0 \Rightarrow \|\bar{a}\times\bar{b}\|>0$

$$\|\bar{c}\|^2>\|\bar{b}\|^2 \Rightarrow \|\bar{c}\|>\|\bar{b}\| \Rightarrow$$

$$\Rightarrow \cos(\beta)>-1 \Rightarrow \beta\neq\pi$$

$$-1<\cos(\beta)<0 \Rightarrow \beta\in\,]\pi/2,\pi[$$

c) $\|\bar{c}\|^2=\|\bar{a}\times\bar{b}\|^2+\|\bar{b}\|^2=9+1 \Leftrightarrow \|\bar{c}\|=\sqrt{10}$

[illegible]

27.

a) $V = |\vec{c} \cdot \vec{a} \times \vec{b}| = \|\vec{a} \times \vec{b}\|^2 = 12$

$\vec{c} \cdot \vec{a} \times \vec{b} = [(\vec{a} \times \vec{b}) - \vec{a}] \cdot \vec{a} \times \vec{b} = \|\vec{a} \times \vec{b}\|^2 - \underbrace{\vec{a} \cdot \vec{a} \times \vec{b}}_{=0}$

$\|\vec{c}\|^2 = 16 \Leftrightarrow [(\vec{a} \times \vec{b}) - \vec{a}] \cdot [(\vec{a} \times \vec{b}) - \vec{a}] = 16 \Leftrightarrow$

$\Leftrightarrow \|\vec{a} \times \vec{b}\|^2 - \underbrace{\vec{a} \cdot \vec{a} \times \vec{b}}_{=0} - \underbrace{(\vec{a} \times \vec{b}) \cdot \vec{a}}_{=0} + \|\vec{a}\|^2 = 16 \Leftrightarrow$

$\Leftrightarrow \|\vec{a} \times \vec{b}\|^2 + 4 = 16 \Leftrightarrow \|\vec{a} \times \vec{b}\|^2 = 12 \Leftrightarrow \|\vec{a} \times \vec{b}\| = 2\sqrt{3}$

b) $\cos(\varphi) = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \dfrac{\vec{a} \cdot \vec{b}}{4}$

$\|\vec{a} \times \vec{b}\|^2 = \|\vec{a}\|^2 \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2 \Leftrightarrow 12 = 16 - (\vec{a} \cdot \vec{b})^2 \Leftrightarrow$

$\Leftrightarrow (\vec{a} \cdot \vec{b})^2 = 4 \Leftrightarrow \vec{a} \cdot \vec{b} = 2 \vee \vec{a} \cdot \vec{b} = -2$

Se $\vec{a} \cdot \vec{b} = 2 \Rightarrow \cos(\varphi) = \frac{1}{2} \Rightarrow \varphi = \pi/3$

Se $\vec{a} \cdot \vec{b} = -2 \Rightarrow \cos(\varphi) = -1/2 \Rightarrow \varphi = \pi - \pi/3 = 2\pi/3$

19

a) $\bar{a}\cdot(\bar{b}+\bar{c}) = \bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c} = 2 + \bar{a}\cdot\bar{c}$

$\bar{a}\cdot\bar{c} = \bar{a}\cdot\left[\sqrt{3}\,\bar{a}\times\bar{b} - \sqrt{2}\,\bar{b}\right] = \sqrt{3}(\underbrace{\bar{a}\cdot\bar{a}\times\bar{b}}_{=0}) - \sqrt{2}(\bar{a}\cdot\bar{b}) = -2\sqrt{2}$

$\bar{a}\cdot(\bar{b}+\bar{c}) = 2 - 2\sqrt{2}$

b) $\|\bar{c}\|^2 = \bar{c}\cdot\bar{c} = \left[\sqrt{3}\,(\bar{a}\times\bar{b}) - \sqrt{2}\,\bar{b}\right]\cdot\left[\sqrt{3}(\bar{a}\times\bar{b}) - \sqrt{2}\,\bar{b}\right] =$

$= 3\,\|\bar{a}\times\bar{b}\|^2 - \sqrt{6}\,\underbrace{\bar{a}\times\bar{b}\cdot\bar{b}}_{=0} - \sqrt{6}\,\underbrace{\bar{b}\cdot\bar{a}\times\bar{b}}_{=0} + 2\,\|\bar{b}\|^2 =$

$= 3\,\|\bar{a}\times\bar{b}\|^2 + 4$

$\|\bar{a}\times\bar{b}\|^2 = \|\bar{a}\|^2\,\|\bar{b}\|^2 - (\bar{a}\cdot\bar{b})^2 = (3)(2) - 4 = 2$

$\|\bar{c}\|^2 = 3\,(2) + 4 = 10 \Leftrightarrow \|\bar{c}\| = \sqrt{10}$

c) $\bar{b}\cdot\bar{c} = \bar{b}\cdot\left[\sqrt{3}\,(\bar{a}\times\bar{b}) - \sqrt{2}\,\bar{b}\right] = \sqrt{3}\,\underbrace{\bar{b}\cdot\bar{a}\times\bar{b}}_{=0} - \sqrt{2}\,\|\bar{b}\|^2 = -2\sqrt{2}$

$\cos(\alpha) = \dfrac{\bar{b}\cdot\bar{c}}{\|\bar{b}\|\,\|\bar{c}\|} = \dfrac{-2\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\,\sqrt{10}} = \dfrac{-2\sqrt{10}}{10} = \dfrac{-\sqrt{10}}{5} \Rightarrow$

$\Rightarrow \alpha = \arccos\left(\dfrac{-\sqrt{10}}{5}\right)$

18)

$$\|\bar{d}\|^2 = \bar{d}\cdot\bar{d} = 8 \Leftrightarrow \left[\bar{c} - 2(\bar{a}\times\bar{b})\right]\cdot\left[\bar{c} - 2(\bar{a}\times\bar{b})\right] = 8 \Leftrightarrow$$

$$\Leftrightarrow \|\bar{c}\|^2 - 2(\bar{a}\times\bar{b})\cdot\bar{c} - 2\,\bar{c}\cdot(\bar{a}\times\bar{b}) + 4\,\|\bar{a}\times\bar{b}\|^2 = 8$$

$$\bar{c} = \alpha_1\,\bar{a} + \alpha_2\,\bar{b}\ ,\quad \alpha_1, \alpha_2 \in \mathbb{R}$$

$$\bar{c}\cdot\bar{a}\times\bar{b} = \alpha_1\,\underbrace{\bar{a}\cdot\bar{a}\times\bar{b}}_{=0} + \alpha_2\,\underbrace{\bar{b}\cdot\bar{a}\times\bar{b}}_{=0} = 0$$

$$\|\bar{c}\|^2 + 4\,\|\bar{a}\times\bar{b}\|^2 = 8$$

$$\bar{c}\cdot\bar{d} = \|\bar{c}\|\,\|\bar{d}\|\cos\left(\pi/3\right) \Leftrightarrow$$

$$\Leftrightarrow \bar{c}\cdot\left[\bar{c} - 2(\bar{a}\times\bar{b})\right] = \frac{\|\bar{c}\|\,\|\bar{d}\|}{2} \Leftrightarrow$$

$$\Leftrightarrow \|\bar{c}\|^2 - 2(\underbrace{\bar{c}\cdot\bar{a}\times\bar{b}}_{=0}) = \frac{\|\bar{c}\|\,\|\bar{d}\|}{2} \Leftrightarrow \|\bar{c}\| = \frac{\|\bar{d}\|}{2} = \sqrt{2}$$

$$\|\bar{c}\| > 0$$

$$4\,\|\bar{a}\times\bar{b}\|^2 = 8 - 2 \Leftrightarrow \|\bar{a}\times\bar{b}\|^2 = \frac{3}{2} \Leftrightarrow$$

$$\Leftrightarrow \|\bar{a}\|^2\,\underbrace{\|\bar{b}\|^2}_{=1} - (\underbrace{\bar{a}\cdot\bar{b}}_{=0})^2 = \frac{3}{2} \Leftrightarrow \|\bar{a}\|^2 = \frac{3}{2} \Leftrightarrow \|\bar{a}\| = \frac{\sqrt{6}}{2}$$

32 a) $\|\bar{a}+\bar{b}\|^2 = (\bar{a}+\bar{b}) \cdot (\bar{a}+\bar{b}) =$

$= \|\bar{a}\|^2 + \underbrace{\bar{a} \cdot \bar{b}}_{=0} + \underbrace{\bar{b} \cdot \bar{a}}_{=0} + \|\bar{b}\|^2 = 2 \Leftrightarrow \|\bar{a}+\bar{b}\| = \sqrt{2}$

b) $\bar{d} = \bar{c} + \bar{a} - \bar{a} \times \bar{b}$

$\|\bar{d}\|^2 = (\bar{c}+\bar{a}-\bar{a}\times\bar{b}) \cdot (\bar{c}+\bar{a}-\bar{a}\times\bar{b}) =$

$= \|\bar{c}\|^2 + \|\bar{a}\|^2 + \|\bar{a}\times\bar{b}\|^2 + 2\underbrace{\bar{c}\cdot\bar{a}}_{=0} - 2\bar{c}\cdot\bar{a}\times\bar{b} - 2\underbrace{\bar{a}\cdot\bar{a}\times\bar{b}}_{=0}$

$\vec{c} \parallel \bar{a}\times\bar{b} \Rightarrow \bar{c}\cdot\bar{a} = 0 \wedge \bar{c}\cdot\bar{b} = 0$

$\bar{c}\cdot\bar{a}\times\bar{b} = \|\bar{c}\| \, \|\bar{a}\times\bar{b}\| \cos(\alpha)$

$\bar{c} \parallel \bar{a}\times\bar{b} \Rightarrow \bar{c} = k\, \bar{a}\times\bar{b}, \quad k \neq 0$

$\bar{a}\cdot\bar{c}\times\bar{b} = \bar{b}\cdot\bar{a}\times\bar{c} = \bar{c}\cdot\bar{b}\times\bar{a} > 0 \Rightarrow \bar{c}\cdot\bar{a}\times\bar{b} < 0$

$\bar{c} = k\,\bar{a}\times\bar{b}, \quad k<0 \quad \wedge \quad \alpha = \measuredangle(\bar{c}, \bar{a}\times\bar{b}) = \pi$

$\|\bar{c}\|^2 = \bar{c}\cdot\bar{c} = k^2 \|\bar{a}\times\bar{b}\|^2 = 9$

$\|\bar{a}\times\bar{b}\|^2 = \|\bar{a}\|^2\|\bar{b}\|^2 - (\underbrace{\bar{a}\cdot\bar{b}}_{=0})^2 = 1$

$\|\bar{c}\|^2 = k^2\|\bar{a}\times\bar{b}\|^2 = 9 \Leftrightarrow k^2 = 9 \Leftrightarrow \cancel{k=3} \vee k=-3$

$\bar{c} = -3\,\bar{a}\times\bar{b}$

$\bar{c}\cdot\bar{a}\times\bar{b} = 3\,(1)\cos(\pi) = -3$

$\|\bar{d}\|^2 = 9 + 1 + 1 + 6 \Leftrightarrow \|\bar{d}\| = \sqrt{17}$

c) $\bar{d}\cdot(\bar{a}+\bar{b}) = (\bar{c}+\bar{a}-\bar{a}\times\bar{b})\cdot(\bar{a}+\bar{b}) =$

$= \underbrace{\bar{c}\cdot\bar{a}}_{=0} + \underbrace{\bar{c}\cdot\bar{b}}_{=0} + \|\bar{a}\|^2 + \underbrace{\bar{a}\cdot\bar{b}}_{=0} - \underbrace{\bar{a}\times\bar{b}\cdot\bar{a}}_{=0} - \underbrace{\bar{a}\times\bar{b}\cdot\bar{b}}_{=0} = 1$

$\cos(\theta) = \dfrac{\bar{d}\cdot(\bar{a}+\bar{b})}{\|\bar{d}\|\,\|\bar{a}+\bar{b}\|} = \dfrac{1}{\sqrt{17}\sqrt{2}} = \dfrac{\sqrt{34}}{34} \Leftrightarrow \theta = \arccos\left(\dfrac{\sqrt{34}}{34}\right)$

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30 a) $\bar{b} = (\bar{c} \times \bar{d}) - \bar{a}$

$\|\bar{b}\|^2 = \bar{b} \cdot \bar{b} = [(\bar{c} \times \bar{d}) - \bar{a}] \cdot [(\bar{c} \times \bar{d}) - \bar{a}] =$

$= \|\bar{c} \times \bar{d}\|^2 - \bar{a} \cdot (\bar{c} \times \bar{d}) - (\bar{c} \times \bar{d}) \cdot \bar{a} + \|\bar{a}\|^2 =$

$= \|\bar{c} \times \bar{d}\|^2 - 2\,\bar{a} \cdot (\bar{c} \times \bar{d}) + \underbrace{\|\bar{a}\|^2}_{\|\bar{b}\|^2} \Leftrightarrow$

$\Leftrightarrow \|\bar{c} \times \bar{d}\|^2 - 2\bar{a} \cdot (\bar{c} \times \bar{d}) = 0$

$\|\bar{c} \times \bar{d}\|^2 = \|\bar{c}\|^2 \|\bar{d}\|^2 \operatorname{sen}^2(\pi/3) = \frac{3}{4} \Leftrightarrow \|\bar{c} \times \bar{d}\| = \frac{\sqrt{3}}{2}$

$\bar{a} \cdot (\bar{c} \times \bar{d}) = \|\bar{a}\| \|\bar{c} \times \bar{d}\| \cos(\pi/6) = \|\bar{b}\| \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) =$

$= \frac{3\|\bar{b}\|}{4}$

$\frac{3}{4} - \frac{3}{2}\|\bar{b}\| = 0 \Leftrightarrow \|\bar{b}\| = 1/2$

b) $\bar{a} \cdot \bar{b} = \bar{a} \cdot ((\bar{c} \times \bar{d}) - \bar{a}) = \bar{a} \cdot (\bar{c} \times \bar{d}) - \|\bar{a}\|^2 = \frac{3}{8} - \frac{1}{4} = \frac{1}{8}$

$\cos(\theta) = \frac{\bar{a} \cdot \bar{b}}{\|\bar{a}\|\|\bar{b}\|} = \frac{1/8}{1/4} = \frac{1}{2} \Rightarrow \theta = \pi/3$