<u>Problema</u>: Considere as rectas

$$r : X(u) = P + uA \ , \quad P = (2,0,-1) \ \text{e} \ A = (1,1,1)$$

$$(x,y,z) = (2+u,\ u,\ -1+u)\ ,\ u \in \mathbb{R}$$

$$r_1 : X(t) = Q + tB \ , \quad Q = (1,1,-4) \ \text{e} \ B = (2,0,1)$$

$$(x,y,z) = (1+2t,\ 1,\ -4+t)\ ,\ t \in \mathbb{R}$$

a) Mostre que as rectas r e r_1 são não complanares (ou enviesadas).

As rectas são não complanares se o conjunto de vectores

$$\{ \overrightarrow{PQ}, A, B \}$$

for linearmente independente, isto é, se

$$\overrightarrow{PQ} \cdot A \times B \neq 0$$

Então

$$\overrightarrow{PQ} \cdot A \times B = \begin{vmatrix} -1 & 1 & -3 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = -1 + 0 + 2 - (-6 + 0 + 1) = 6 \neq 0$$

$$\begin{matrix} -1 & 1 & -3 \\ 1 & 1 & 1 \end{matrix}$$

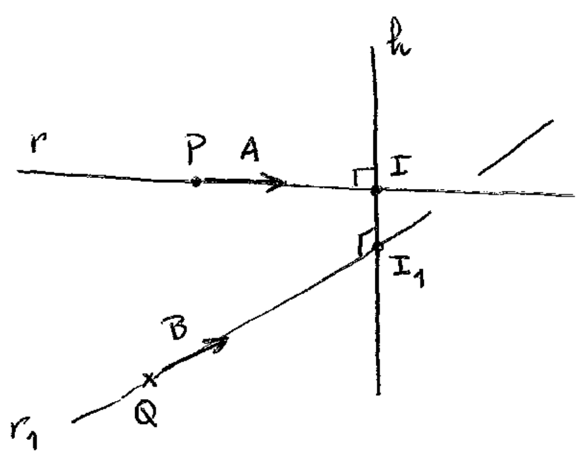
b) Determine a distância entre as duas rectas.

$$d_{r,r_1} = \frac{|\overrightarrow{PQ} \cdot A \times B|}{\| A \times B \|} = \frac{|6|}{\sqrt{6}} = \sqrt{6}$$

$$A \times B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = (1,1,-2) \Rightarrow \| A \times B \| = \sqrt{6}$$

c) Obtenha a equação vectorial da recta perpendicular comum às rectas r e r_1.

Seja h a recta perpendicular comum às rectas r e r_1.



Sejam os pontos $I = h \cap r$

$I_1 = h \cap r_1$

Atendendo a que

$$r \perp h \Rightarrow \overrightarrow{II_1} \perp A$$

$$r_1 \perp h \Rightarrow \overrightarrow{II_1} \perp B$$

então

$$\overrightarrow{II_1} \parallel A \times B \iff \overrightarrow{II_1} = \alpha\, A \times B \,,\ \alpha \neq 0$$

ou, em alternativa,

$$\overrightarrow{II_1} \parallel A \times B \iff \overrightarrow{II_1} \times (A \times B) = (0,0,0)$$

Determinemos os pontos I e I_1:

$$I \in r \Rightarrow I = (u+2, u, u-1)$$

$$I_1 \in r_1 \Rightarrow I_1 = (2t+1, 1, t-4)$$

$$\overrightarrow{II_1} = I_1 - I = (2t-u-1, 1-u, t-u-3)$$

$$\overrightarrow{II_1} \times (A \times B) = (0,0,0) \iff$$

$$\iff \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2t-u-1 & 1-u & t-u-3 \\ 1 & 1 & -2 \end{vmatrix} = (0,0,0) \iff$$

$$\iff \begin{bmatrix} -2+2u-t+u+3 \\ t-u-3+4t-2u-2 \\ 2t-\cancel{u}-1-1+\cancel{u} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \iff \begin{bmatrix} 3u-t+1 \\ 5t-3u-5 \\ 2t-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \iff$$

$$\iff \begin{cases} 0 = 0 \\ u = 0 \\ t = 1 \end{cases} \Rightarrow I = (2,0,-1) = P \text{ e } I_1 = (3,1,-3)$$

Sabendo que $\overrightarrow{II_1} = (1,1,-2)$, tem-se

$$h: X(\alpha) = I + \alpha\, \overrightarrow{II_1} \,,\ \alpha \in \mathbb{R}$$

$$(x,y,z) = (2+\alpha, \alpha, -1-2\alpha) \,,\ \alpha \in \mathbb{R}$$

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