Livro : CAP. 3

33) a)

$$|B| = \begin{vmatrix} -2 & 4 \\ -3 & 5 \end{vmatrix} = -10 + 12 = 2 \neq 0 \Rightarrow$$ B é não singular (regular)

$$|C| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 0 \end{vmatrix} = 0 + 0 + 0 - (0 - 1 + 0) = 1 \neq 0 \Rightarrow$$

$$\begin{array}{ccc} \uparrow & \uparrow \\ C_2 - C_1 & C_3 - C_1 \end{array} \qquad \begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & -1 \end{array}$$

$\Rightarrow$ C é não singular (regular)

$$|D| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 4 & -1 & 1 & 0 \\ 3 & 2 & 0 & 1 \\ 4 & 4 & 0 & 2 \end{vmatrix} \leftarrow L_4/2 \quad = 2 \begin{vmatrix} 1 & 1 & 1 & 1 \\ 4 & -1 & 1 & 0 \\ 3 & 2 & 0 & 1 \\ 2 & 2 & 0 & 1 \end{vmatrix} \begin{matrix} \\ \\ \leftarrow L_3 - L_1 \\ \leftarrow L_4 - L_1 \end{matrix} =$$

$$= 2 \begin{vmatrix} 1 & 1 & 1 & 1 \\ 4 & -1 & 1 & 0 \\ 2 & 1 & -1 & 0 \\ 1 & 1 & -1 & 0 \end{vmatrix} = 2 \times (1) \times (-1)^5 \begin{vmatrix} 4 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -1 \end{vmatrix} =$$

$$\begin{array}{c} \uparrow \\ C_3 + C_2 \end{array}$$

$$= -2 \begin{vmatrix} 4 & -1 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0 \Rightarrow$$ D é singular (não regular)

b)

$$\text{Adj } B = \begin{bmatrix} 5 & 3 \\ -4 & -2 \end{bmatrix} \Rightarrow B^{-1} = \frac{1}{|B|} [\text{Adj } B]^T = \frac{1}{2} \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$$

$$B^{-1} B = \frac{1}{2} \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ -3 & 5 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$\text{Adj } C = \begin{bmatrix} 5 & -3 & -1 \\ 1 & 0 & -1 \\ -2 & 1 & 1 \end{bmatrix} \Rightarrow C^{-1} = \frac{1}{|C|} [\text{Adj } C]^T = \begin{bmatrix} 5 & 1 & -2 \\ -3 & 0 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$C^{-1} C = \begin{bmatrix} 5 & 1 & -2 \\ -3 & 0 & 1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Exercício 191 c) : Sebenta

Desdobremos a 1ª coluna (Propriedade 9 dos determinantes)

$$\begin{bmatrix} a+3 \\ 0 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 2 \\ 3 \end{bmatrix}$$

Então

$$\begin{vmatrix} a+3 & 1 & 2 & 1 \\ 0 & a & b & c \\ 2 & 2a+2 & 2b+1 & 2c \\ 3 & a+1 & b+2 & c+1 \end{vmatrix} = \underbrace{\begin{vmatrix} a & 1 & 2 & 1 \\ 0 & a & b & c \\ 0 & 2a+2 & 2b+1 & 2c \\ 0 & a+1 & b+2 & c+1 \end{vmatrix}}_{(*)} + \underbrace{\begin{vmatrix} 3 & 1 & 2 & 1 \\ 0 & a & b & c \\ 2 & 2a+2 & 2b+1 & 2c \\ 3 & a+1 & b+2 & c+1 \end{vmatrix}}_{(**)}$$

O determinante (**) é nulo

$$\begin{array}{r} \\ \\ \\ L_4-L_1 \rightarrow \end{array}\begin{vmatrix} 3 & 1 & 2 & 1 \\ 0 & a & b & c \\ 2 & 2a+2 & 2b+1 & 2c \\ 3 & a+1 & b+2 & c+1 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 2 & 1 \\ 0 & a & b & c \\ 2 & 2a+2 & 2b+1 & 2c \\ 0 & a & b & c \end{vmatrix} = 0 \quad \text{(duas linhas iguais)}$$

Aplicando o Teorema de Laplace à 1ª coluna do determinante (*)

$$\begin{vmatrix} a & 1 & 2 & 1 \\ 0 & a & b & c \\ 0 & 2a+2 & 2b+1 & 2c \\ 0 & a+1 & b+2 & c+1 \end{vmatrix} = a \times (-1)^{1+1} \begin{vmatrix} a & b & c \\ 2a+2 & 2b+1 & 2c \\ a+1 & b+2 & c+1 \end{vmatrix} =$$

$$= a \begin{vmatrix} a & b & c \\ 2a+2 & 2b+1 & 2c \\ a+1 & b+2 & c+1 \end{vmatrix}\begin{array}{l} \\ \leftarrow L_2-2L_1 \\ \leftarrow L_3-L_1 \end{array} = a \underbrace{\begin{vmatrix} a & b & c \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix}}_{\Delta = 1} = a$$

Sebenta : 195 e)

$$|E| = \begin{vmatrix} -1 & 0 & 0 & 1 & 1 \\ 2 & 1 & -a & 2 & 1 \\ 1 & 2 & 1 & a & 2 \\ -2 & -1 & 2 & 1 & 2 \\ 1 & 0 & 1 & -1 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 & 0 & 0 \\ 2 & 1 & -a & 4 & 3 \\ 1 & 2 & 1 & a+1 & 3 \\ -2 & -1 & 2 & -1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{vmatrix} =$$

$$\begin{array}{ccccc} & & & \uparrow & \uparrow \\ & & & C_4+C_1 & C_5+C_1 \end{array}$$

$$= (-1) \times (-1)^2 \begin{vmatrix} 1 & -a & 4 & 3 \\ 2 & 1 & a+1 & 3 \\ -1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} = -3 \begin{vmatrix} 1 & -a & 4 & 1 \\ 2 & 1 & a+1 & 1 \\ -1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} =$$

$$\begin{array}{cccc} & & & \uparrow \\ & & & C_4/3 \end{array}$$

$$= -3 \times 1 \times (-1)^6 \begin{vmatrix} 1 & 4 & 1 \\ 2 & a+1 & 1 \\ -1 & -1 & 0 \end{vmatrix} \leftarrow L_2 - L_1 = -3 \begin{vmatrix} 1 & 4 & 1 \\ 1 & a-3 & 0 \\ -1 & -1 & 0 \end{vmatrix} =$$

$$= -3 \times 1 \times (-1)^4 \begin{vmatrix} 1 & a-3 \\ -1 & -1 \end{vmatrix} = -3\left[-1 + (a-3)\right] = -3(a-4) =$$

$$= 12 - 3a$$