<u>Problema</u> : $S = \{A, B\} \subset \mathbb{R}^3$ é um conjunto ortogonal $\Rightarrow$ $A \cdot B = 0$

$C \in L(S) \Rightarrow C = \alpha_1 A + \alpha_2 B, \quad \alpha_1, \alpha_2 \in \mathbb{R}$

$\|B\| = 1$

$D = C + A \times B$ e $\|D\| = \sqrt{6}$

$\theta = \measuredangle(C, D) = \frac{\pi}{3}$

Pretende-se determinar $\|A\|$.

$$\|D\|^2 = D \cdot D = (C + A\times B)\cdot(C + A \times B) = \|C\|^2 + 2\,C\cdot A\times B + \|A\times B\|^2 \Leftrightarrow$$

$$\Leftrightarrow \|C\|^2 + 2\,C\cdot A \times B + \|A\times B\|^2 = 6 \qquad (1)$$

Verifica-se que

$$C \cdot A\times B = (\alpha_1 A + \alpha_2 B)\cdot A\times B = \alpha_1 \underbrace{A\cdot A\times B}_{=0} + \alpha_2 \underbrace{B\cdot A\times B}_{=0} = 0$$

$$\|A\times B\|^2 = \|A\|^2\,\|B\|^2 - (\underbrace{A\cdot B}_{=0})^2 = \|A\|^2$$

$$C\cdot D = \|C\|\,\|D\| \cos\frac{\pi}{3} = \frac{\sqrt{6}}{2}\|C\| \Leftrightarrow$$

$$\Leftrightarrow C\cdot(C + A\times B) = \frac{\sqrt{6}}{2}\|C\| \Leftrightarrow \|C\|^2 + \underbrace{C\cdot A\times B}_{=0} = \frac{\sqrt{6}}{2}\|C\| \Leftrightarrow$$

$$\Leftrightarrow \|C\| = \frac{\sqrt{6}}{2}$$

Obtém-se, finalmente, a partir da equação (1)

$$\frac{6}{4} + \|A\|^2 = 6 \Leftrightarrow \|A\|^2 = \frac{18}{4} \Leftrightarrow \|A\| = \frac{3\sqrt{2}}{2}$$

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