# Collections, Part Four

# From the Last 3 Lectures...

- `Stack`: **Ordered** sequence of data, access from **top**
- `Vector`: **Ordered** sequence of data
- `Grid`: **2D Ordered** sequence of data
- `Lexicon`: **Large**, **unordered** container of **`strings`** that doesn't change over time
- `Set`: **Unordered** container of **distinct** elements
- `Map`: **Unordered** container of **key-value** pairs. Keys are **distinct**

# Queue

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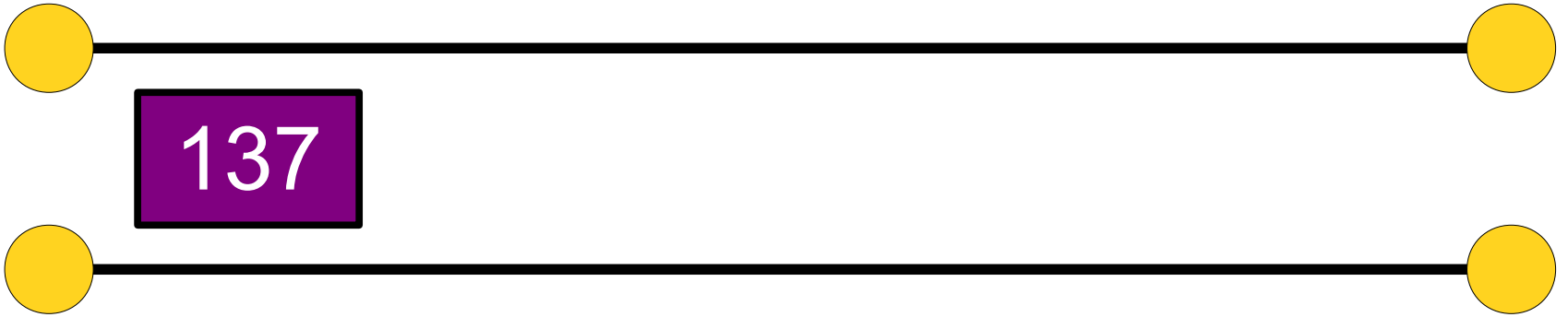
- A **`Queue`** is a data structure representing a waiting line.
- Objects can be **enqueued** to the back of the line or **dequeued** from the front of the line.
- No other objects in the queue are visible.
- Example: A checkout counter.



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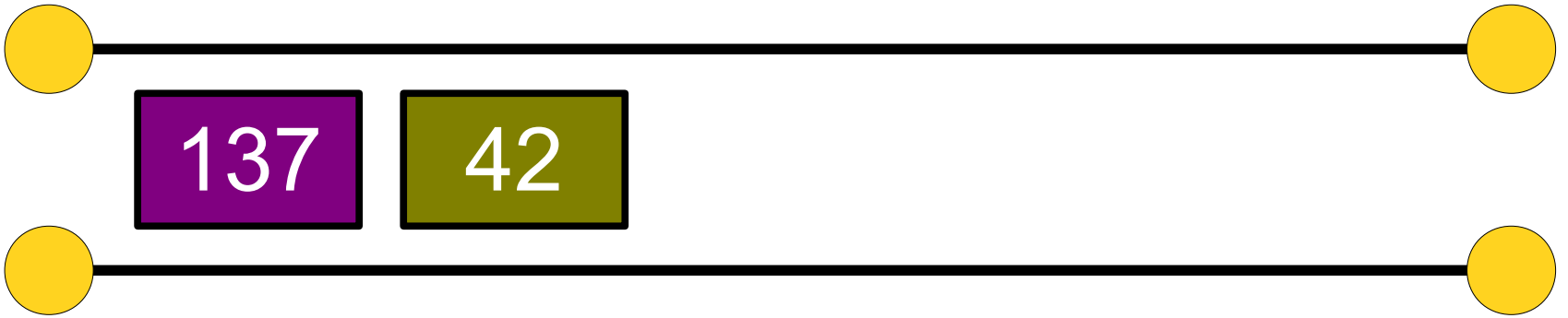

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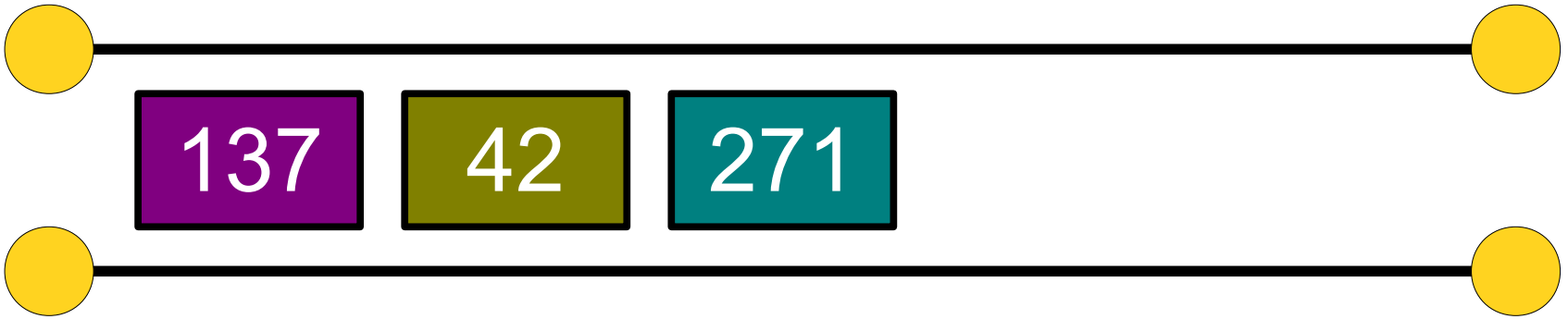

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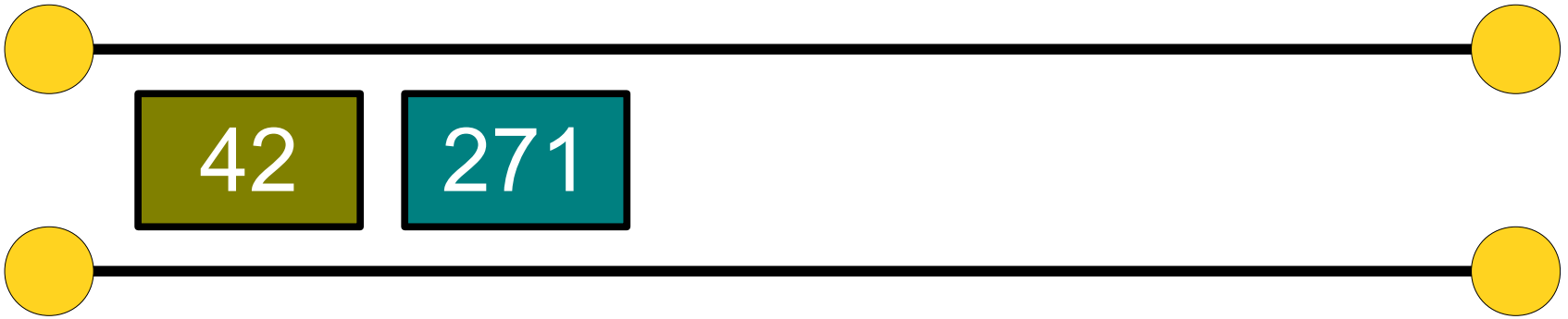

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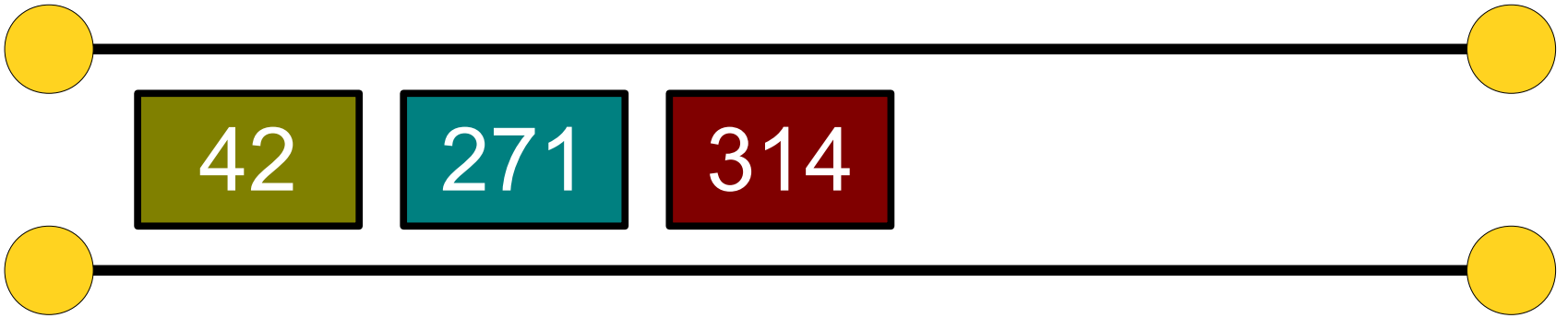

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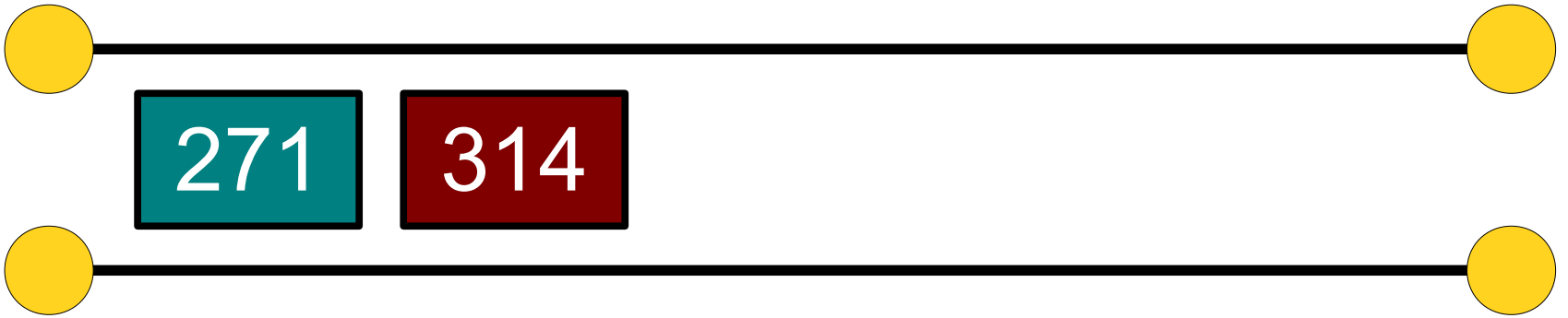

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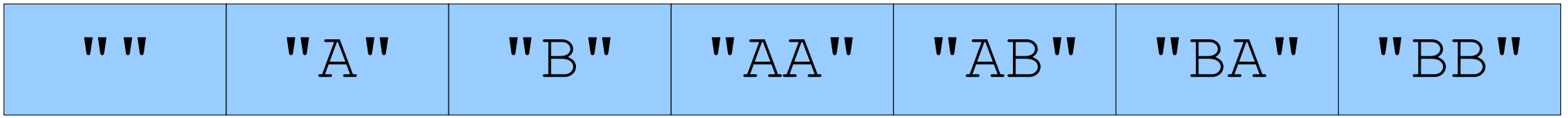
# Listing All Strings

- Suppose we want to generate all strings of letters `A` and `B` of length at most three.
- How might we do this?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"AA"
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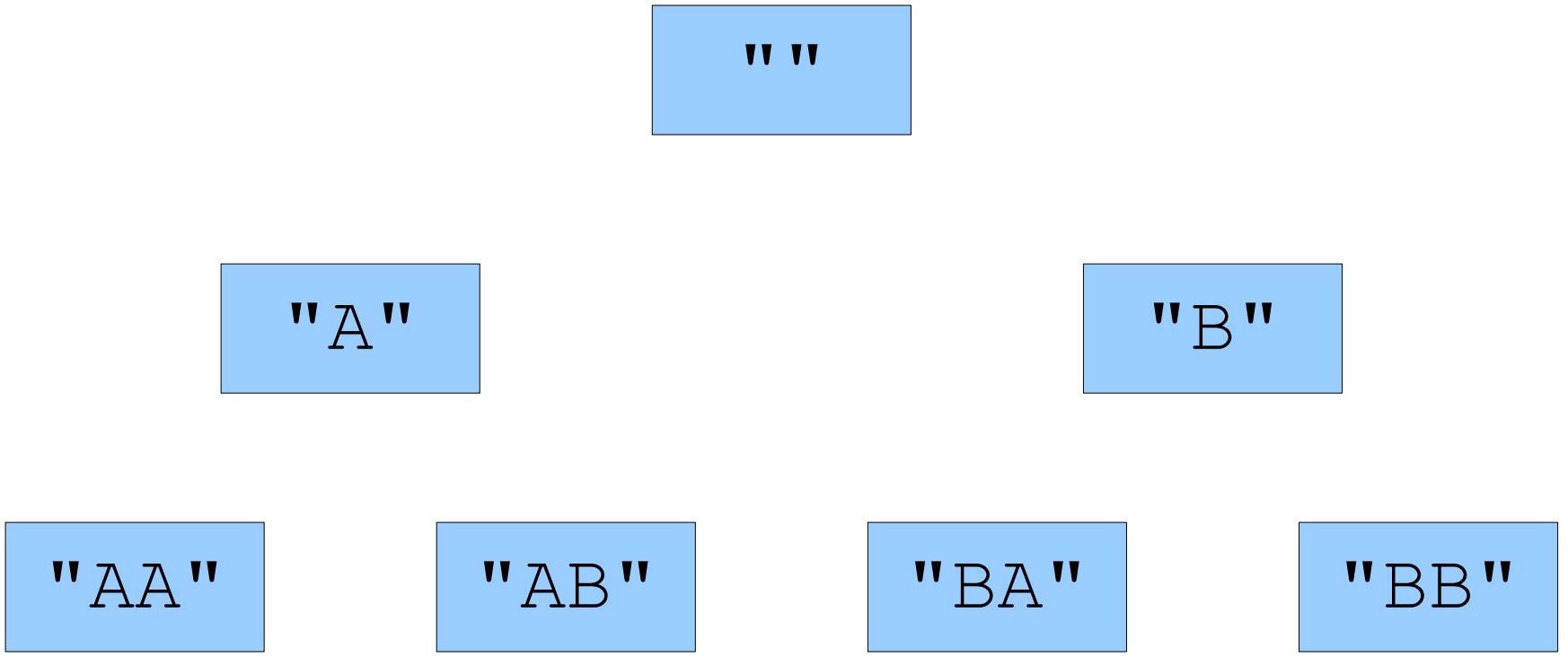
"BA"
"BB"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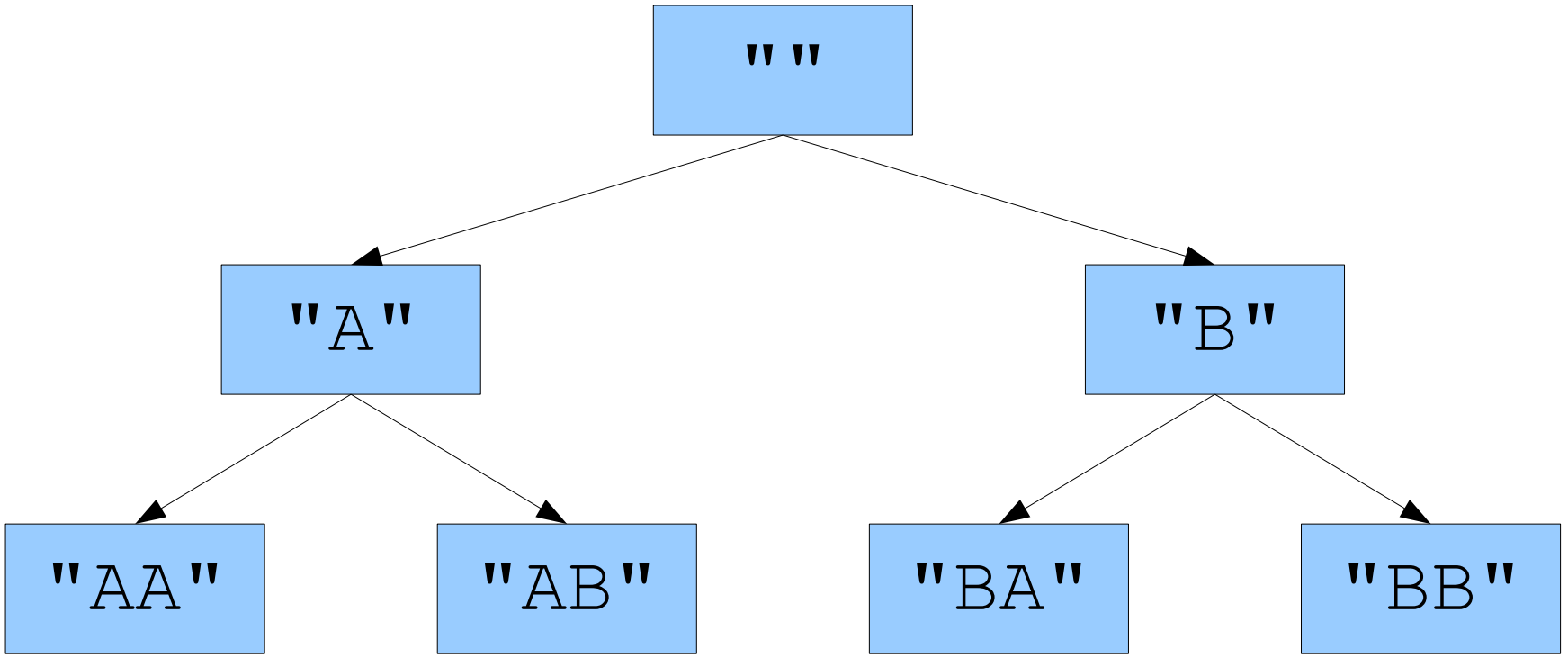
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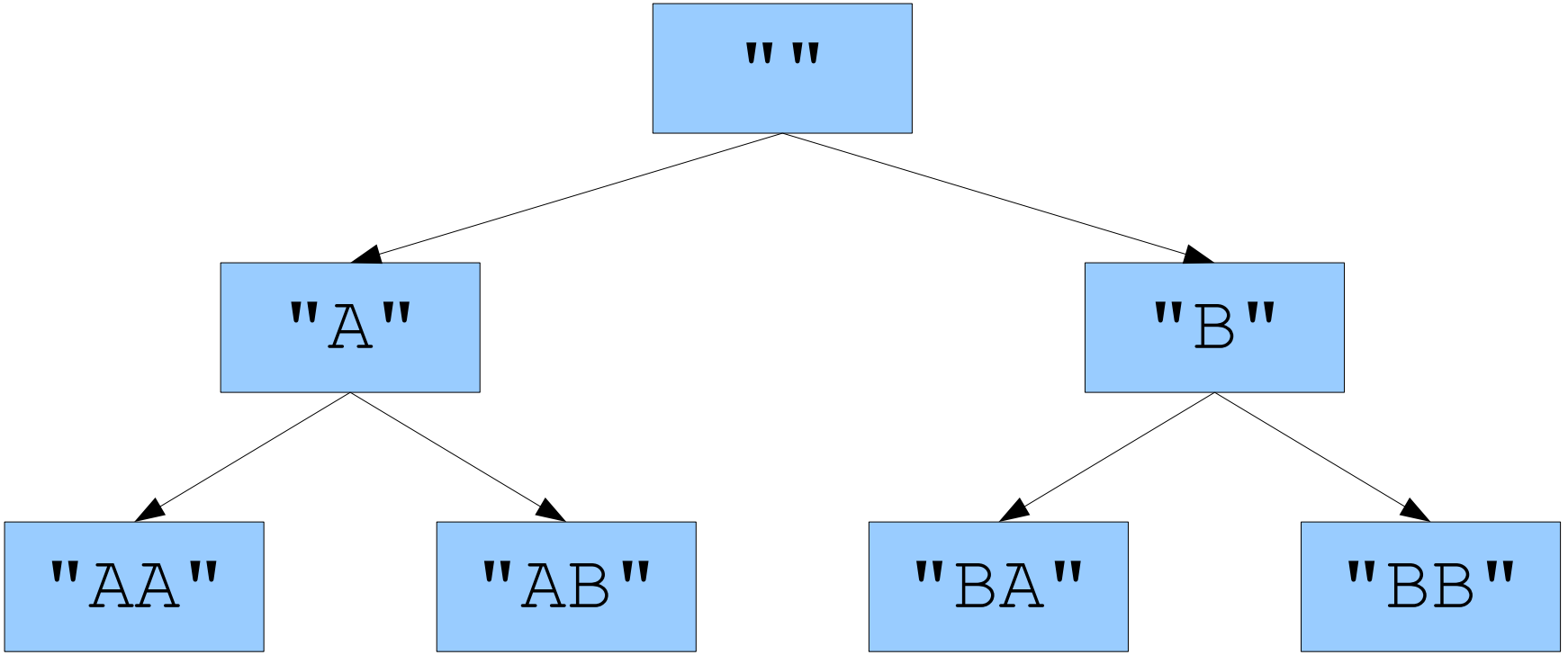
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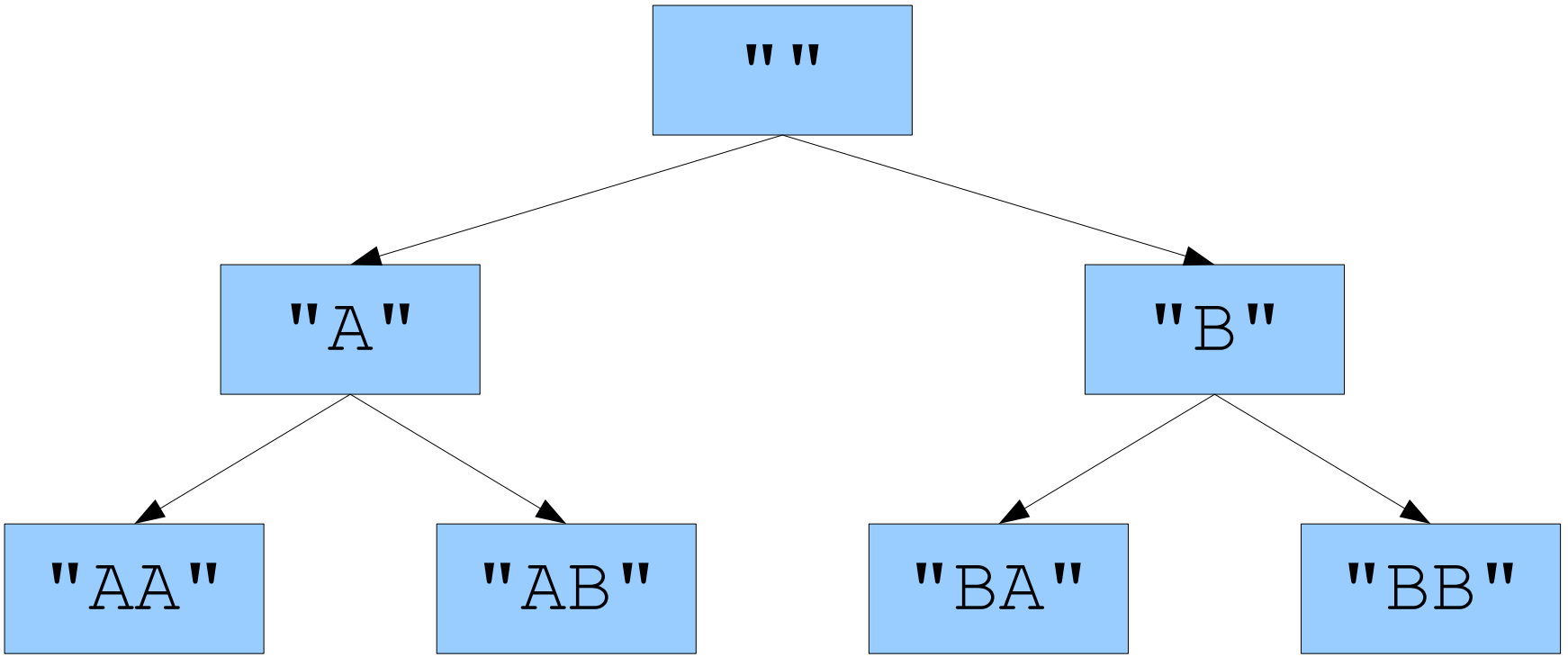
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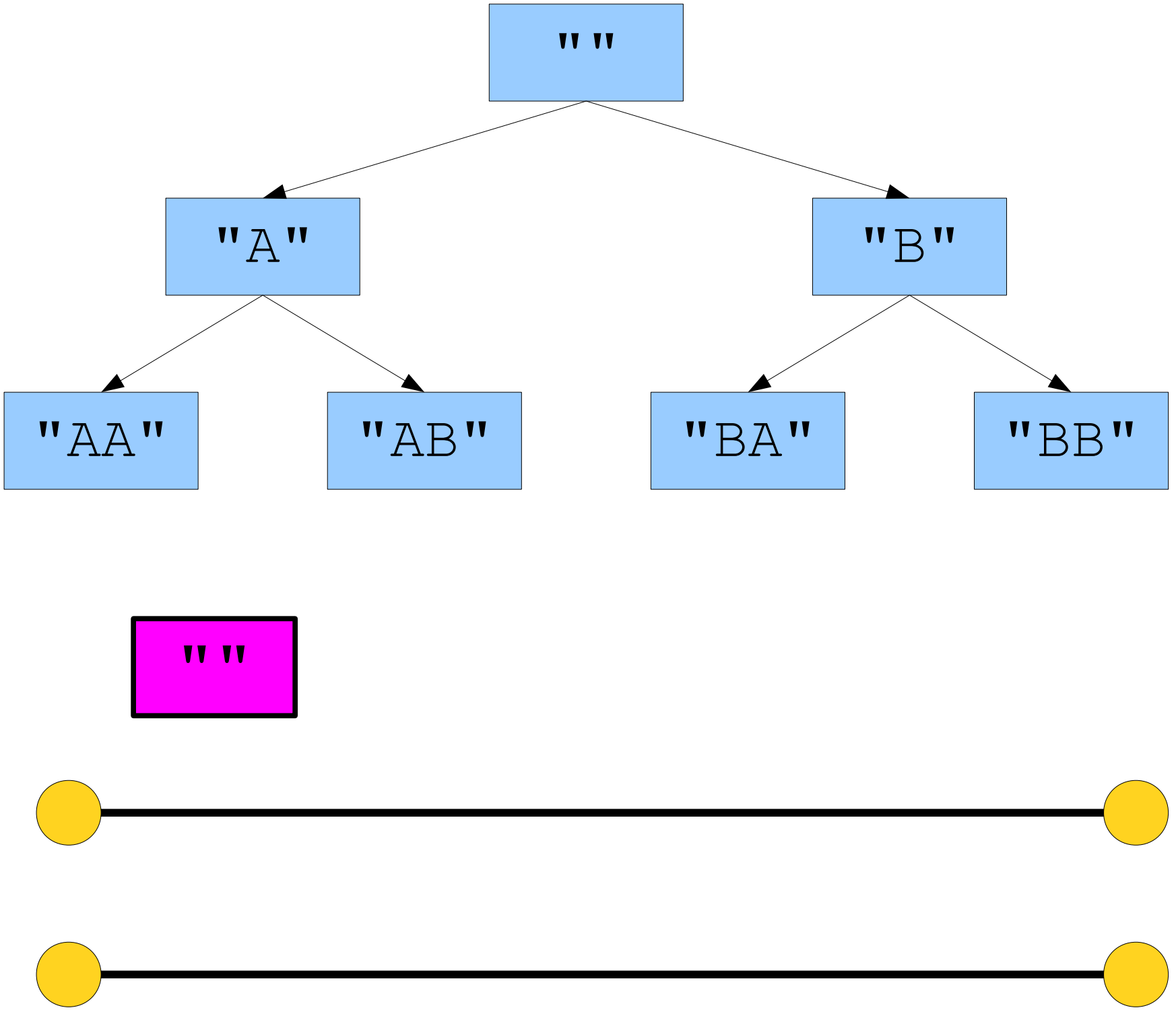
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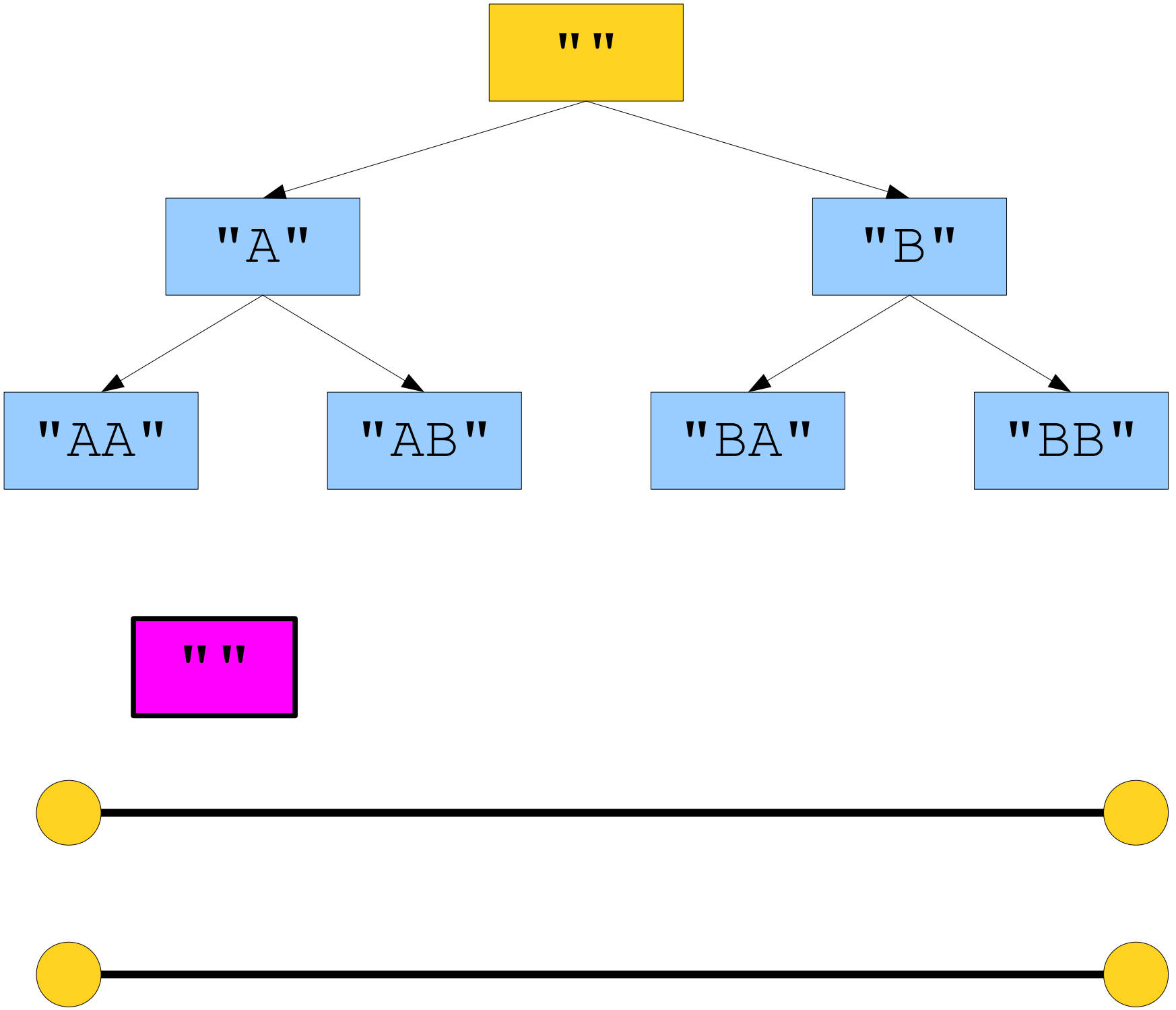
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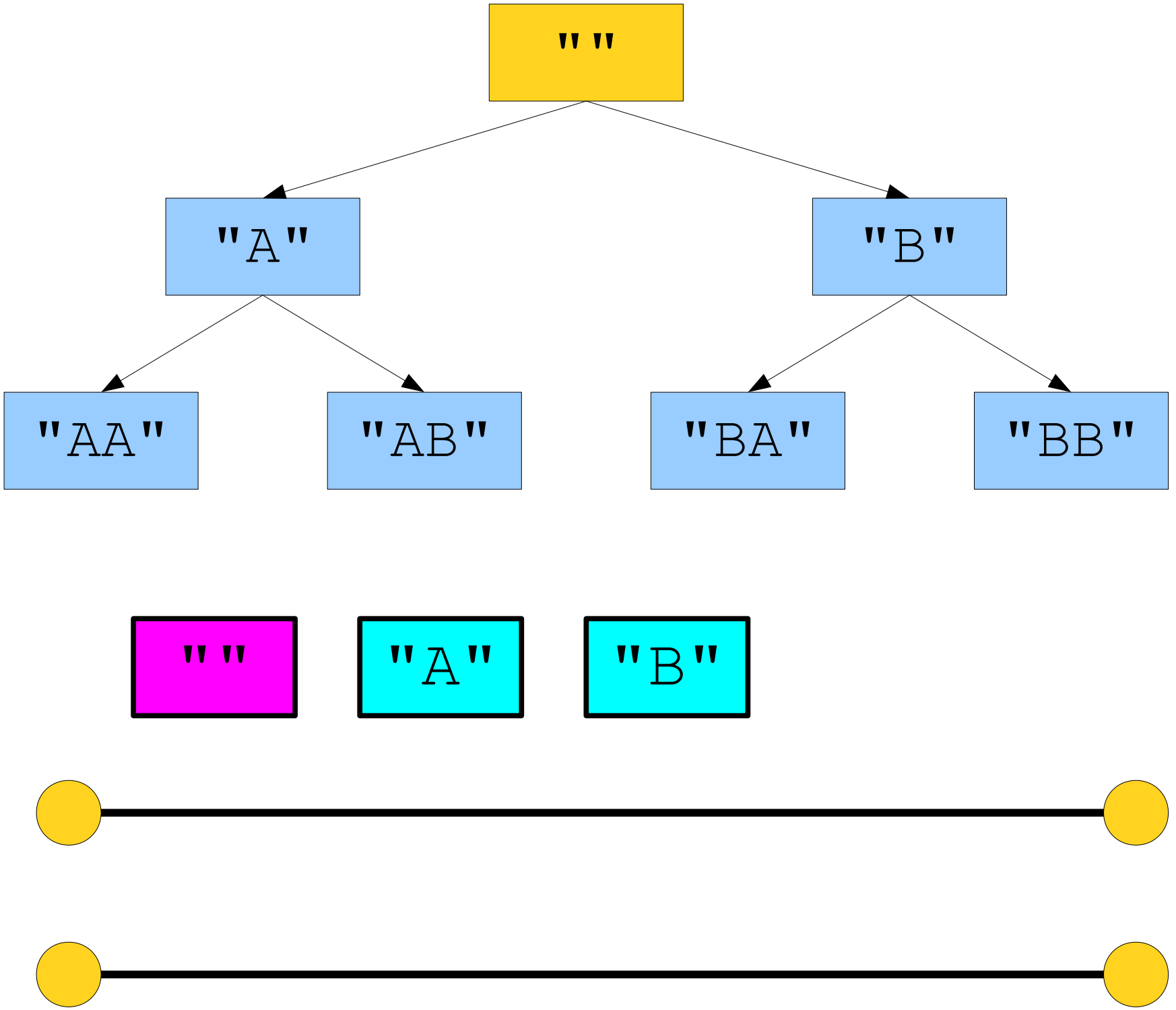
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"A"
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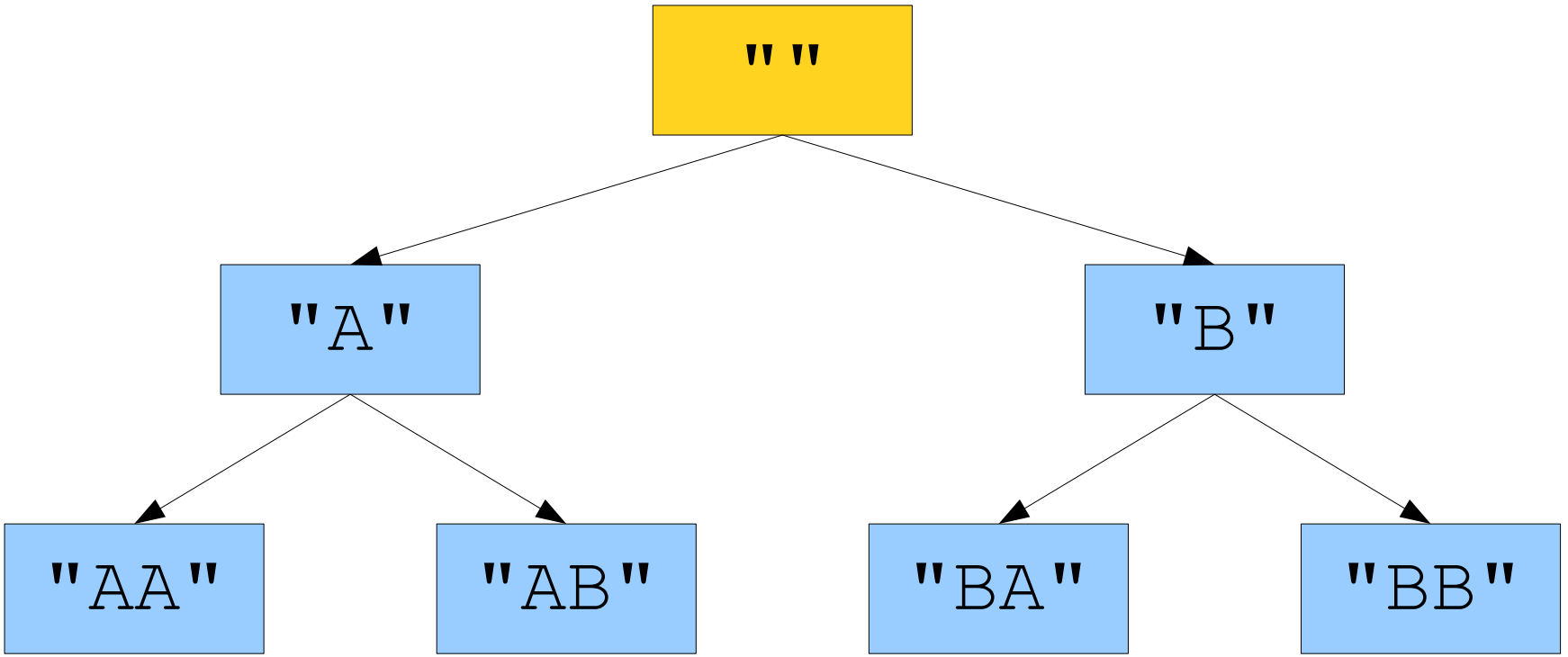
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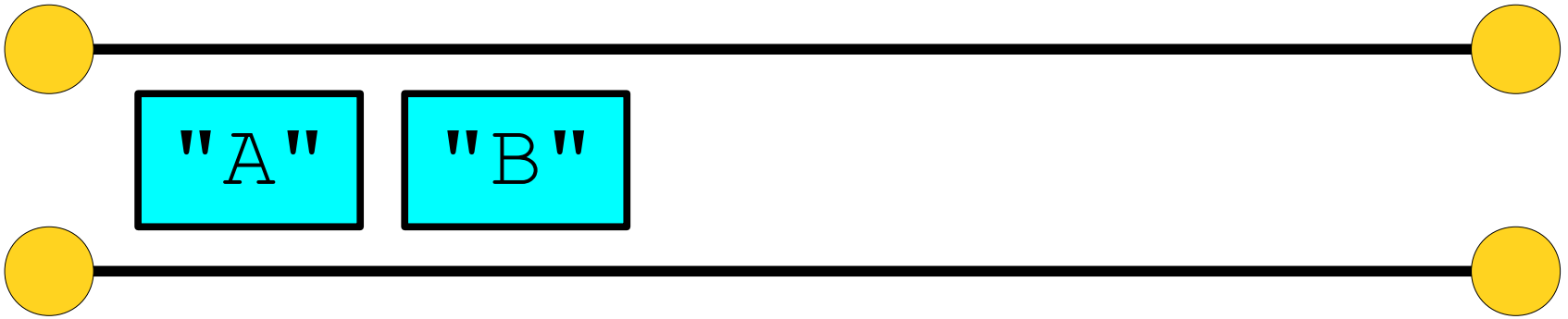
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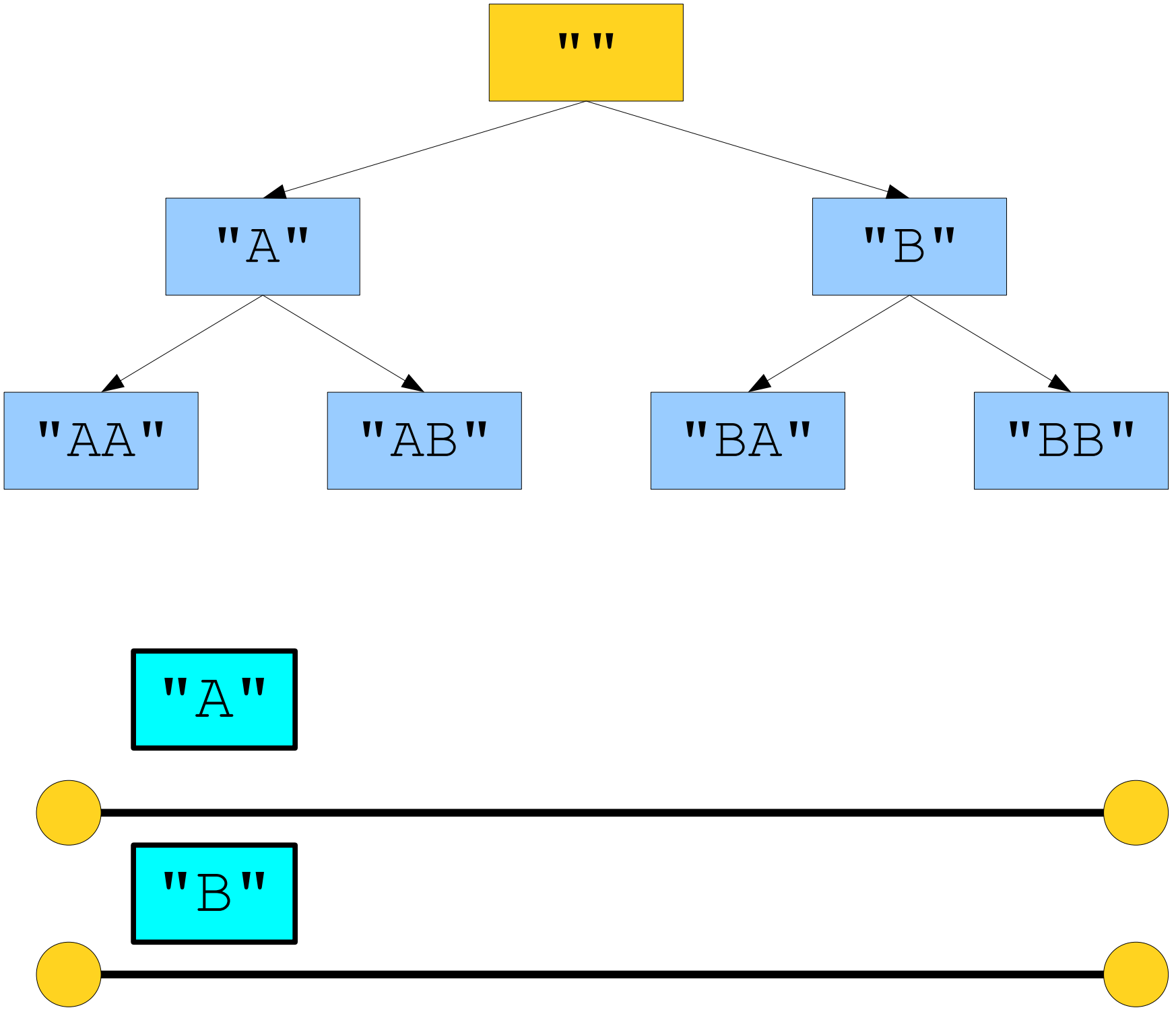
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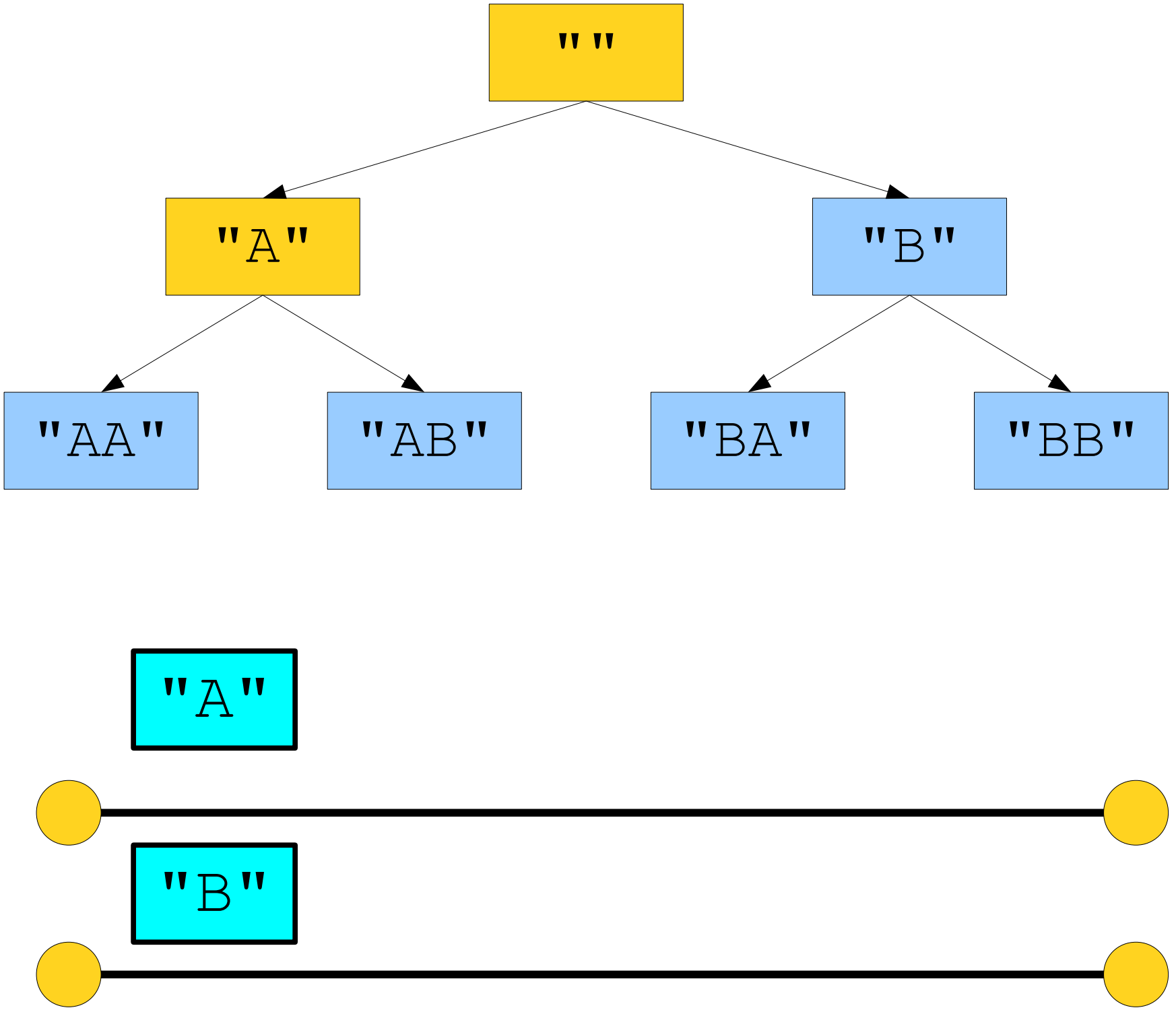
"B"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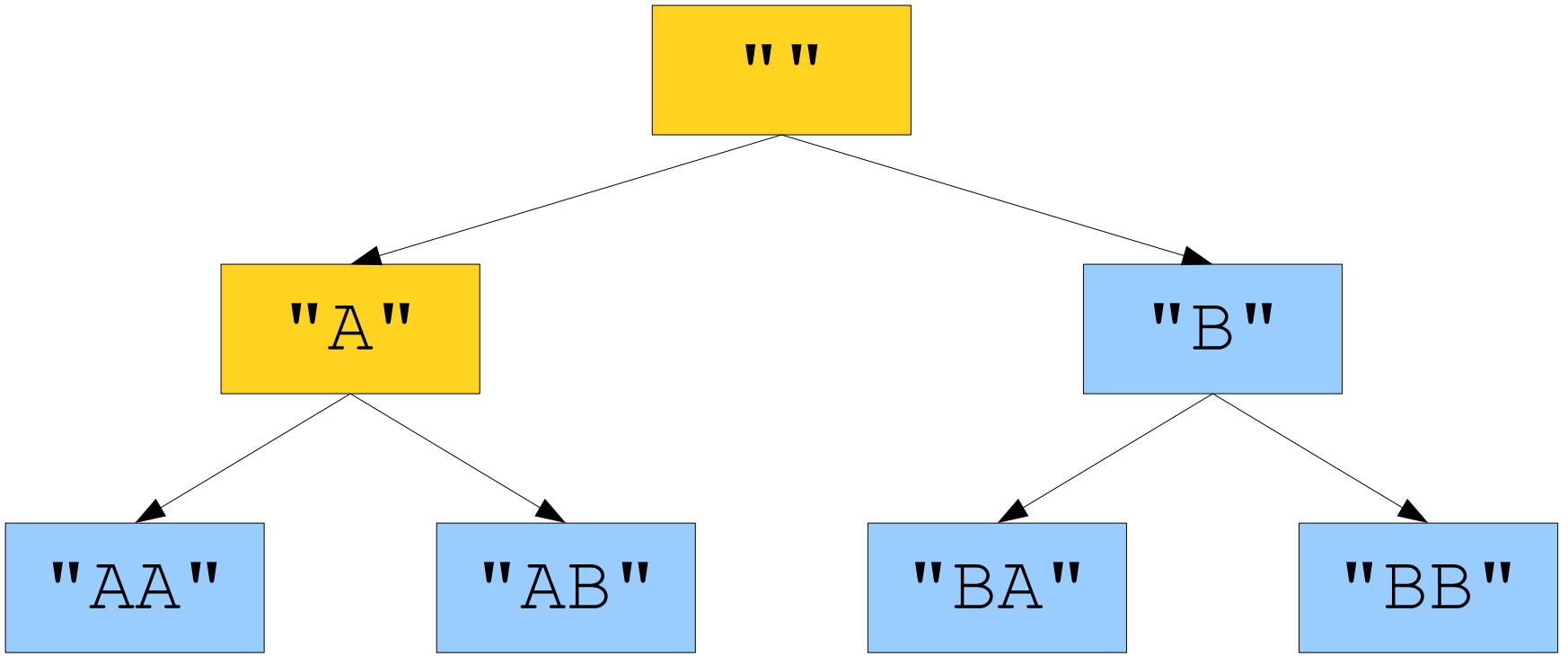
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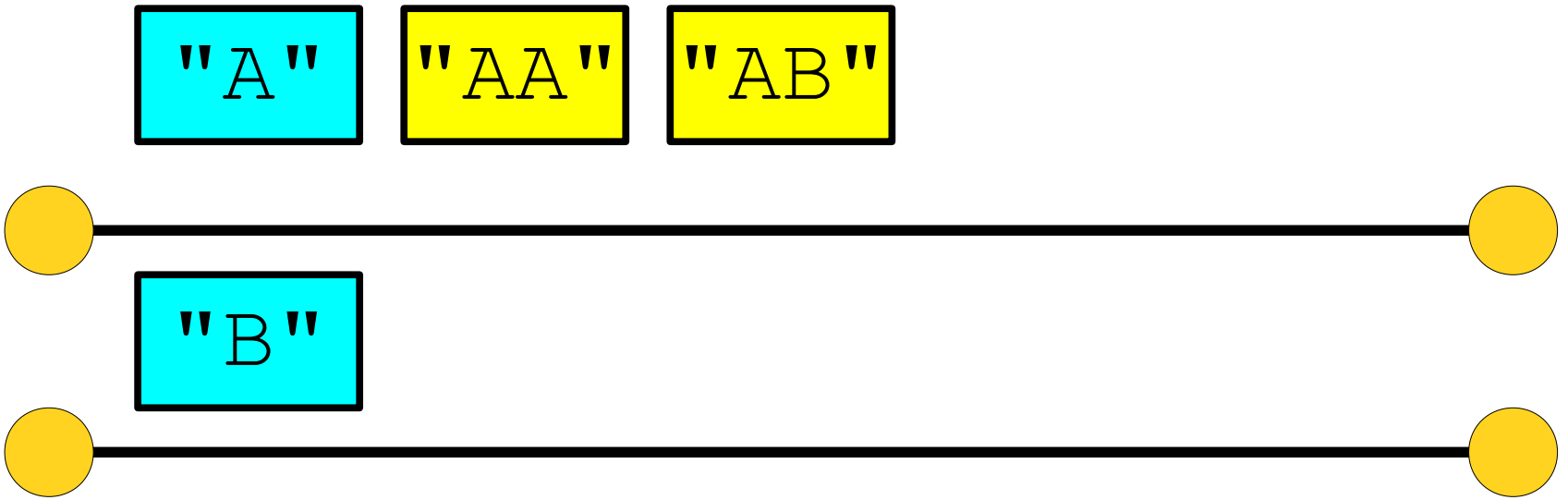
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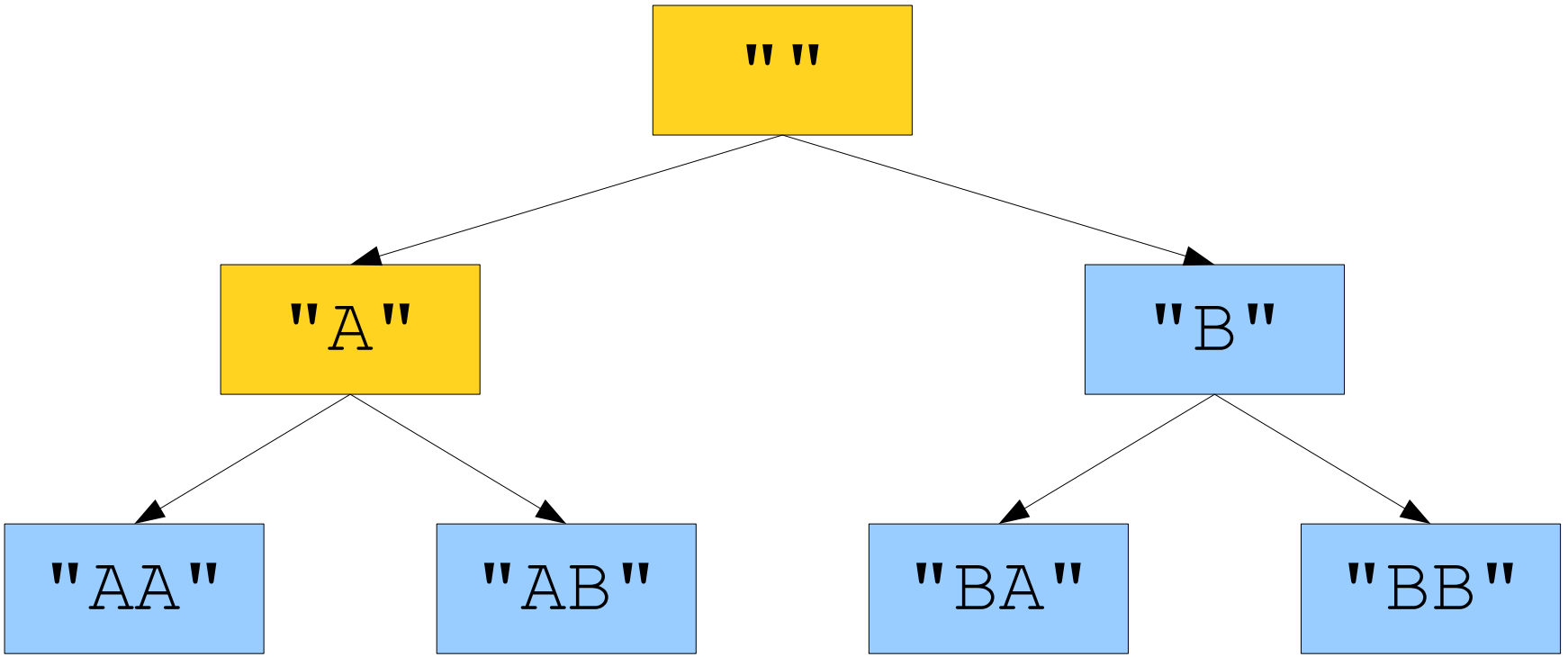
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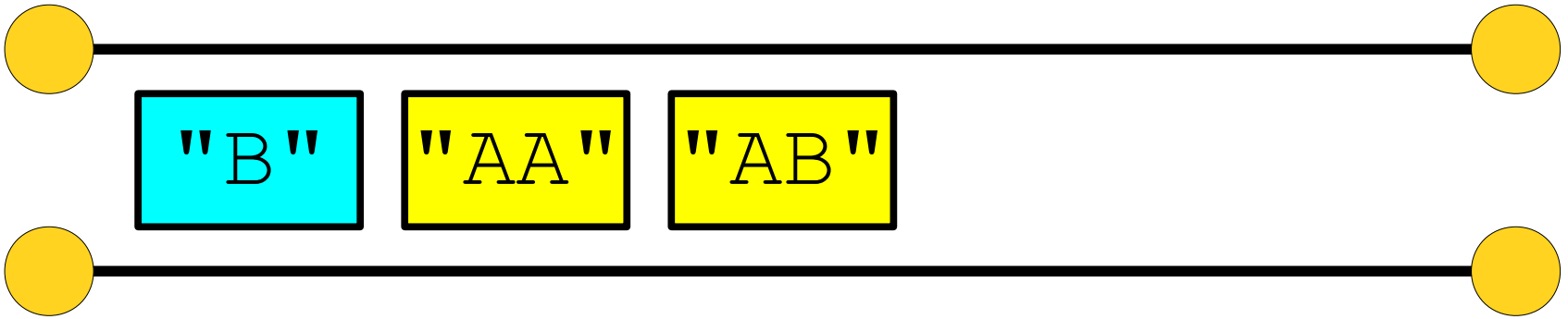
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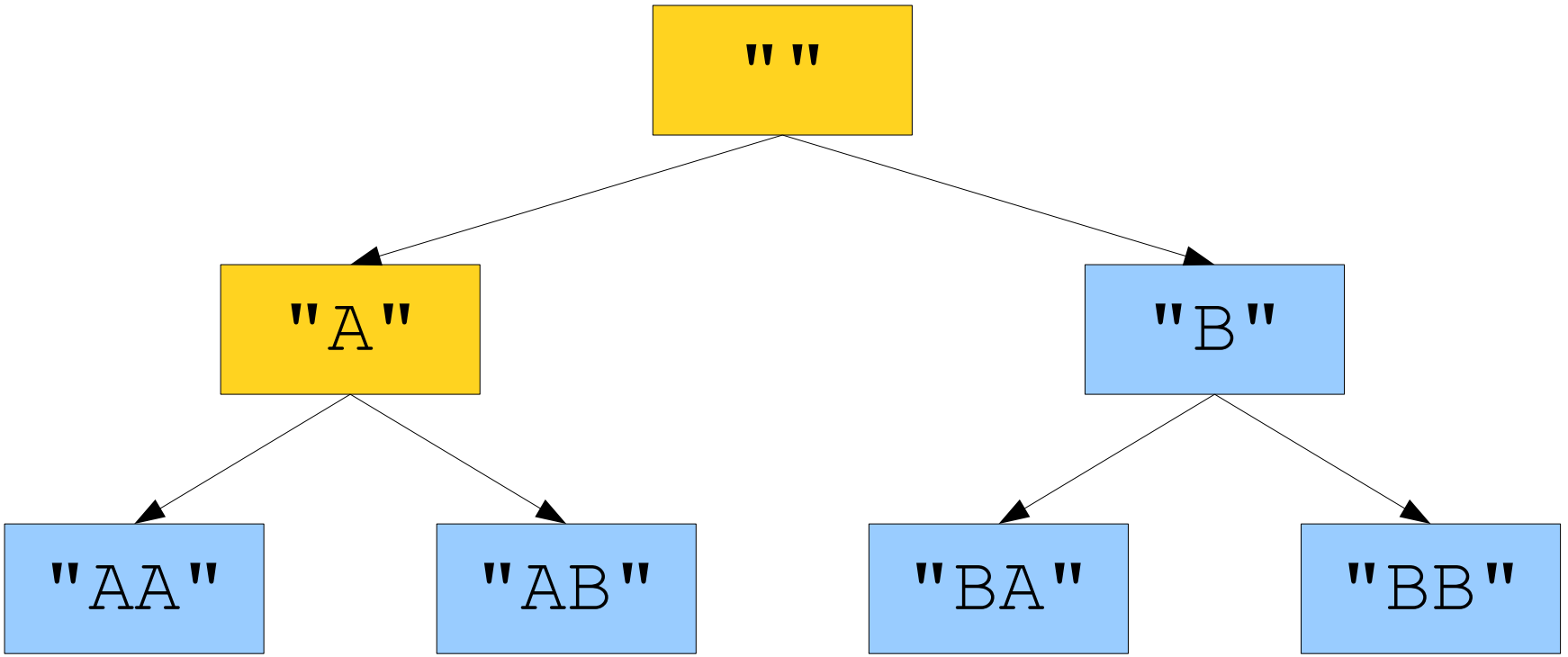
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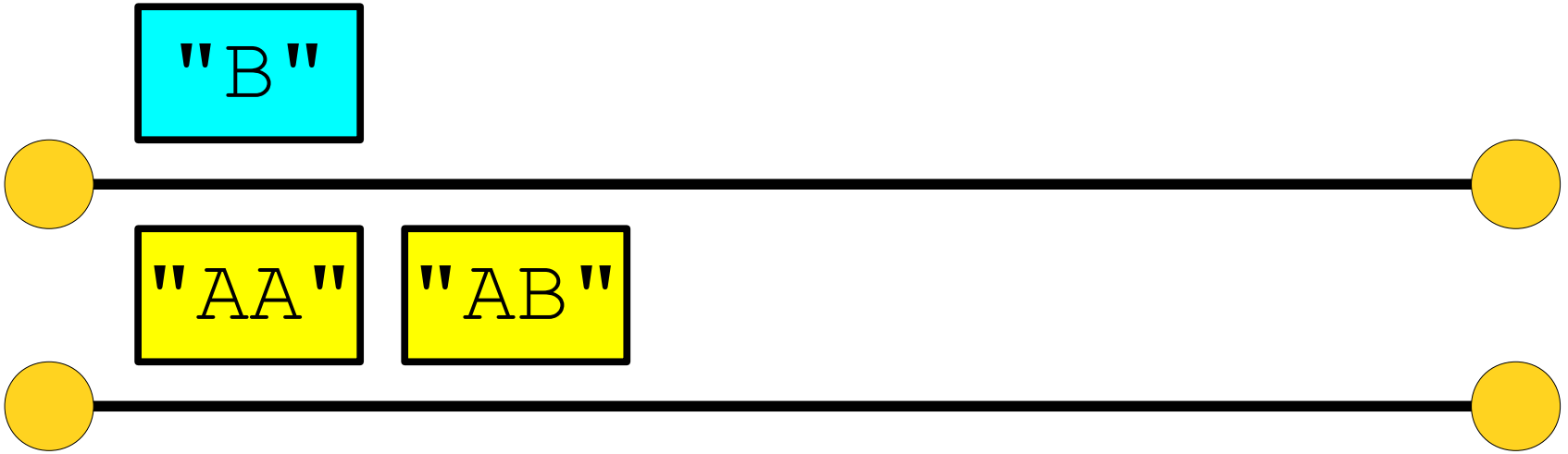
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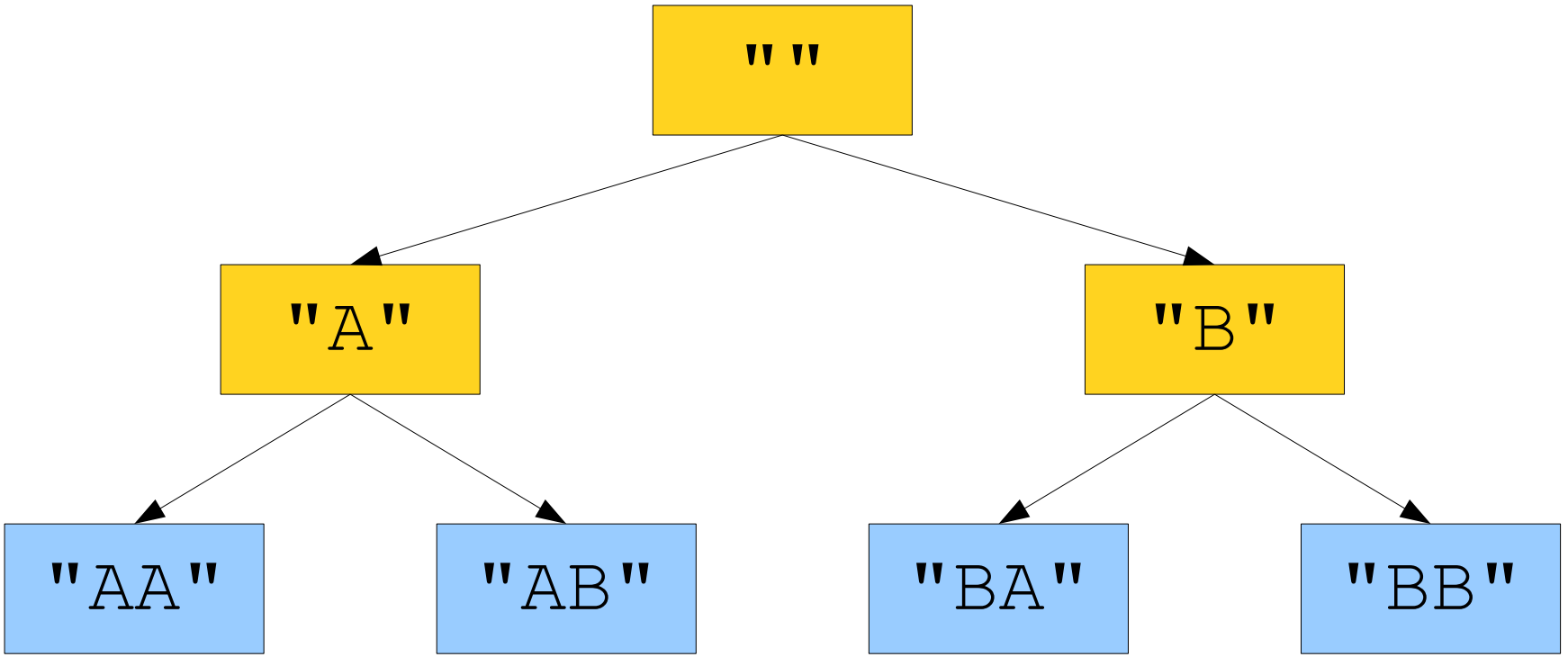
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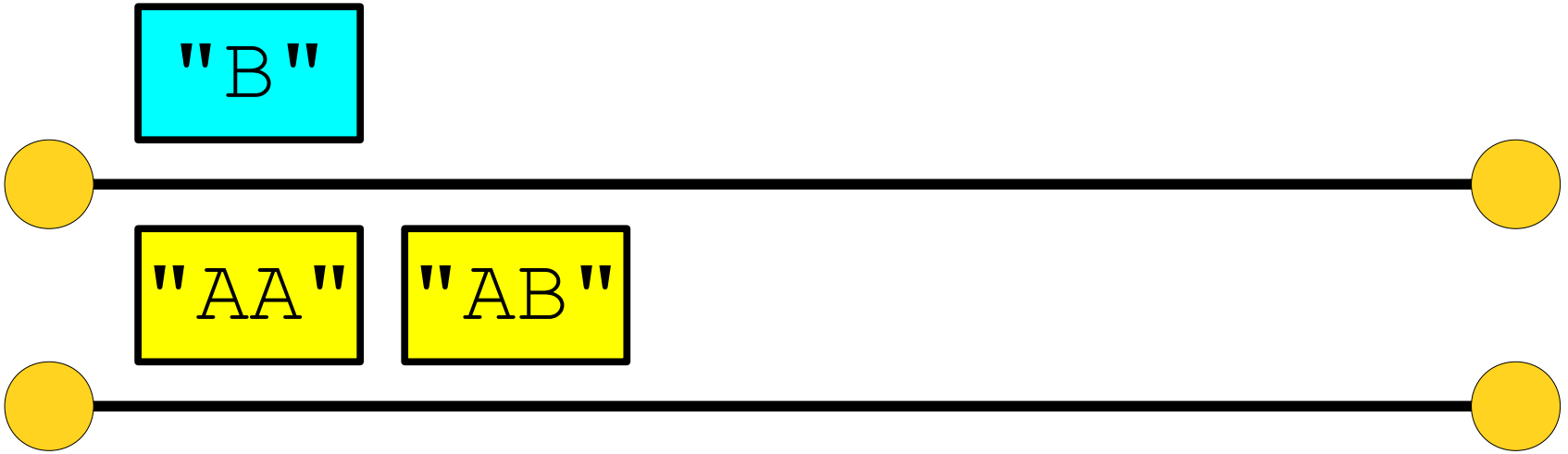
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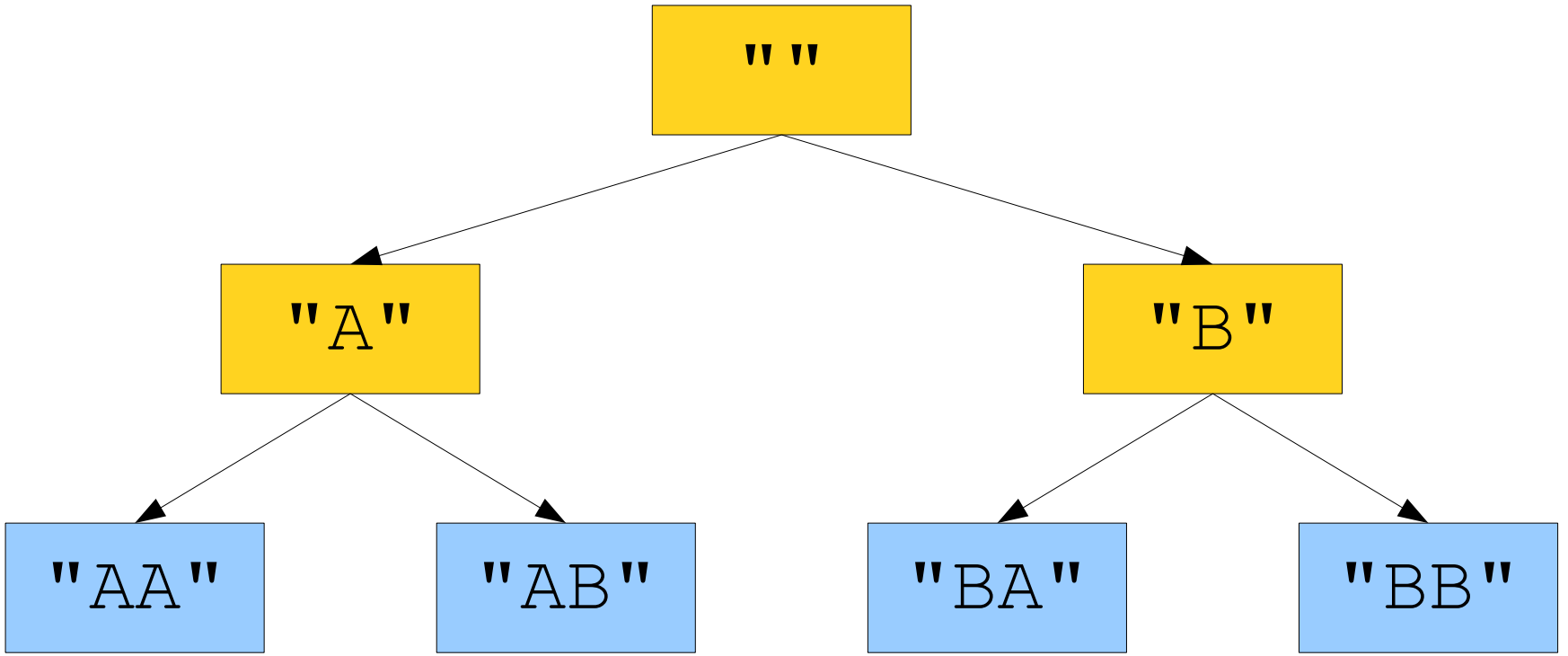
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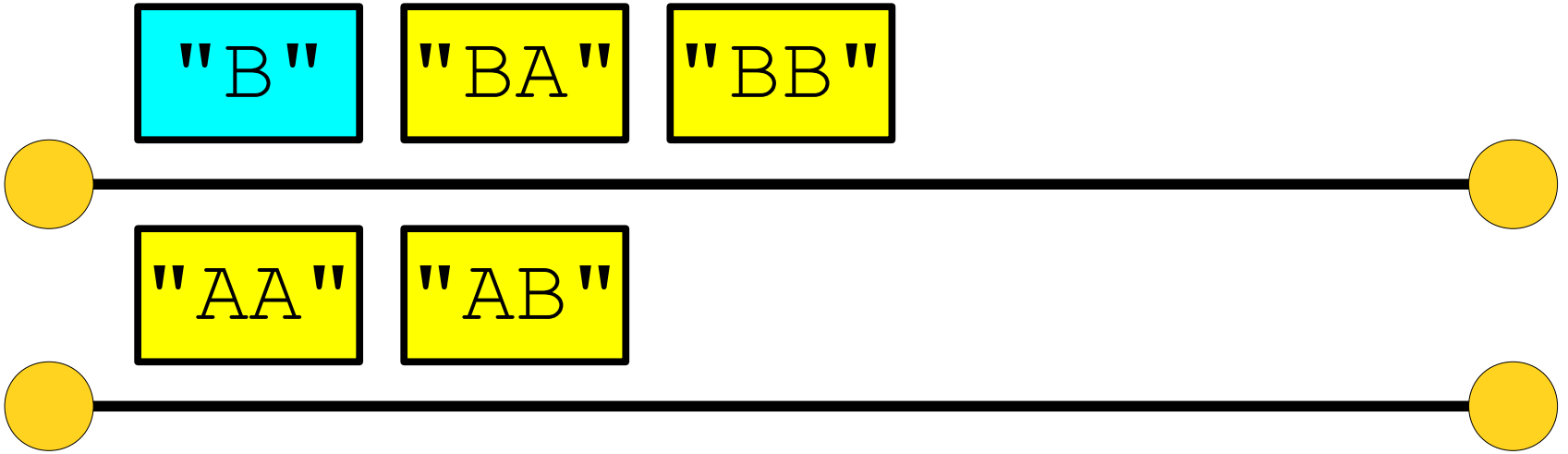
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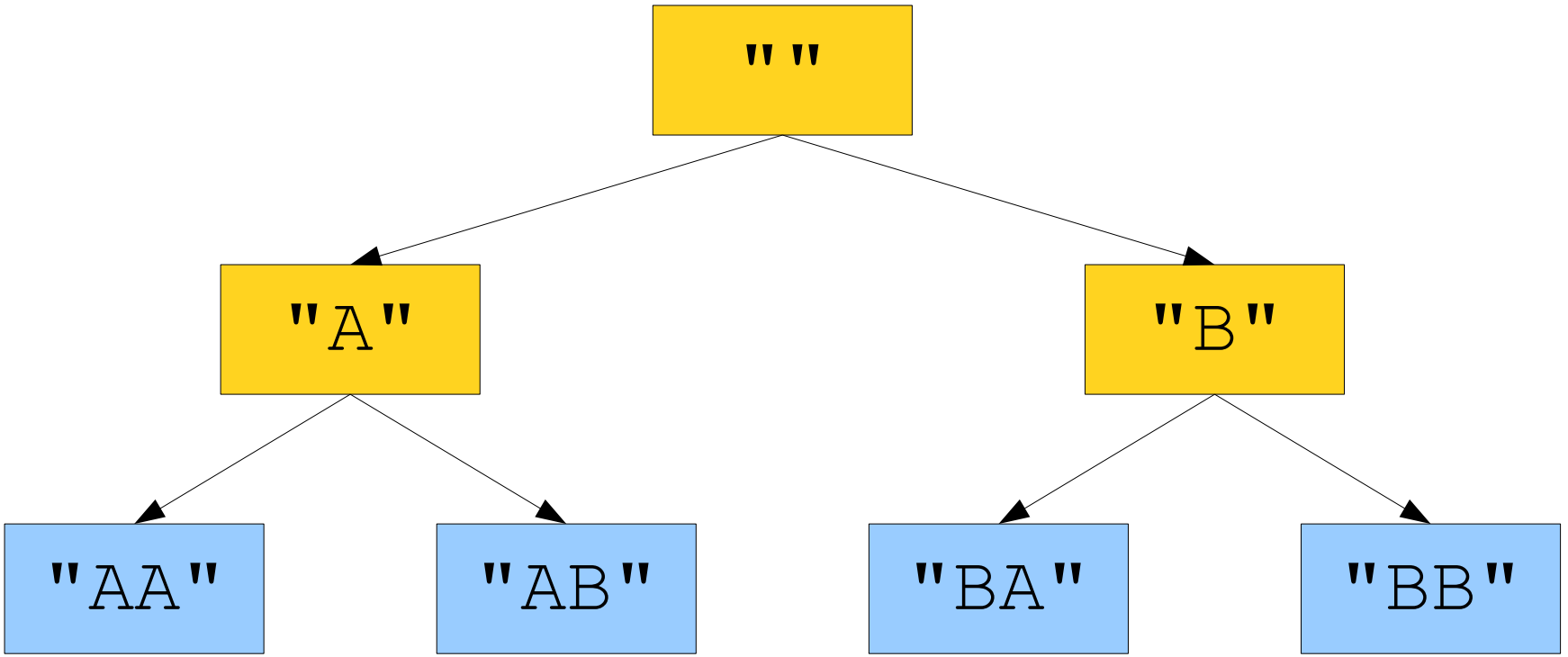
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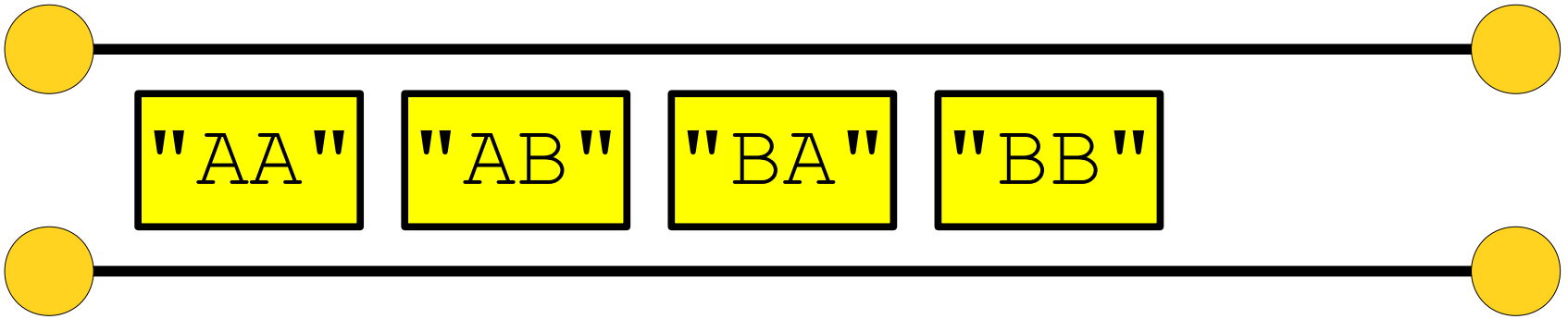
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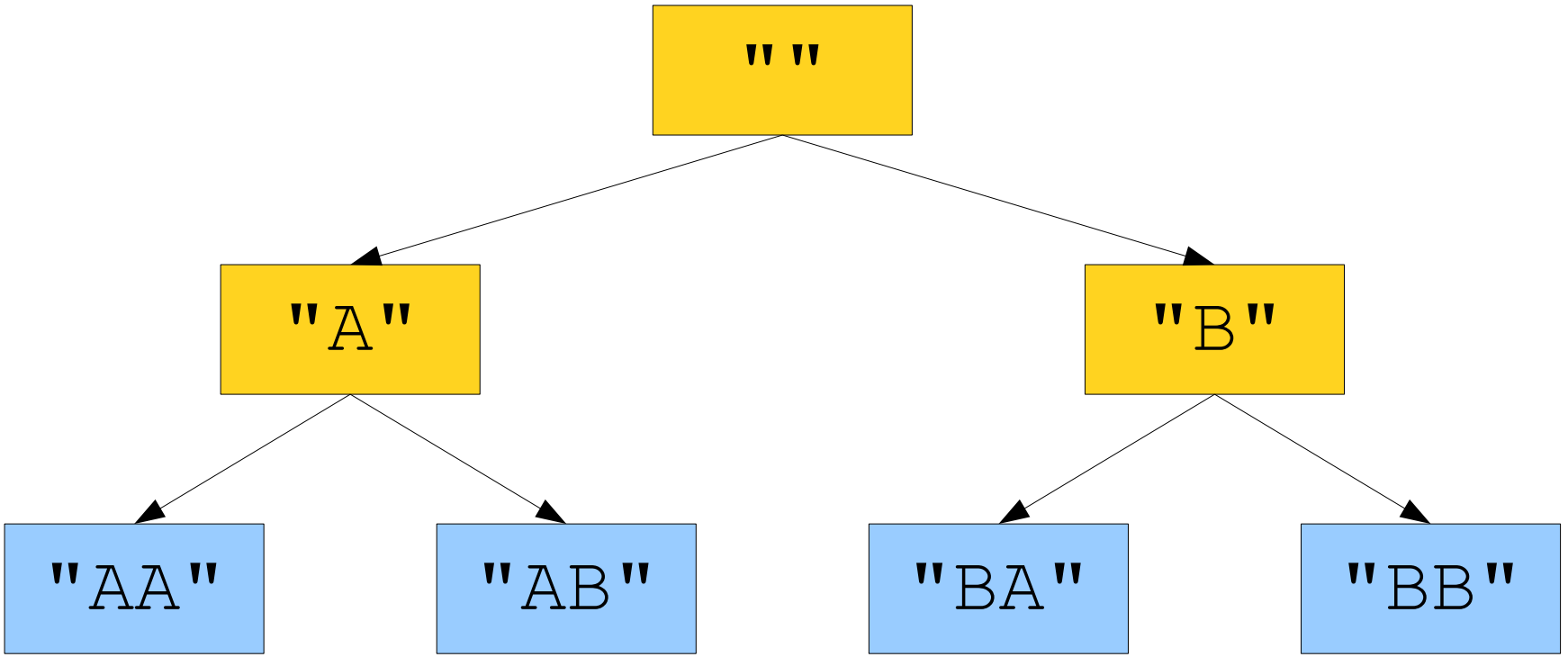
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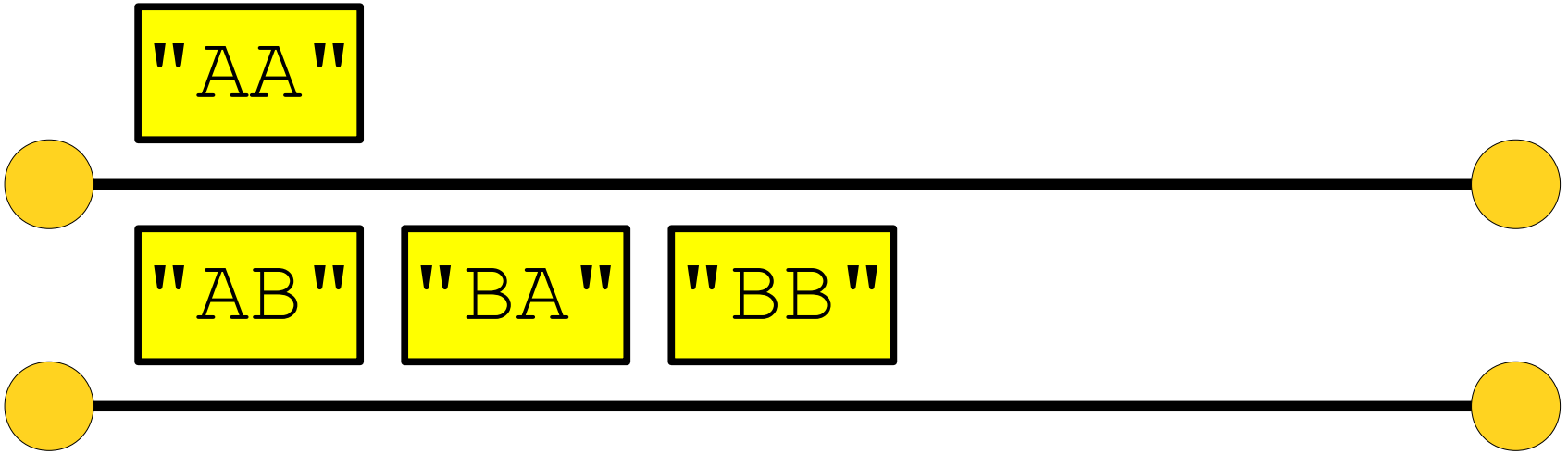
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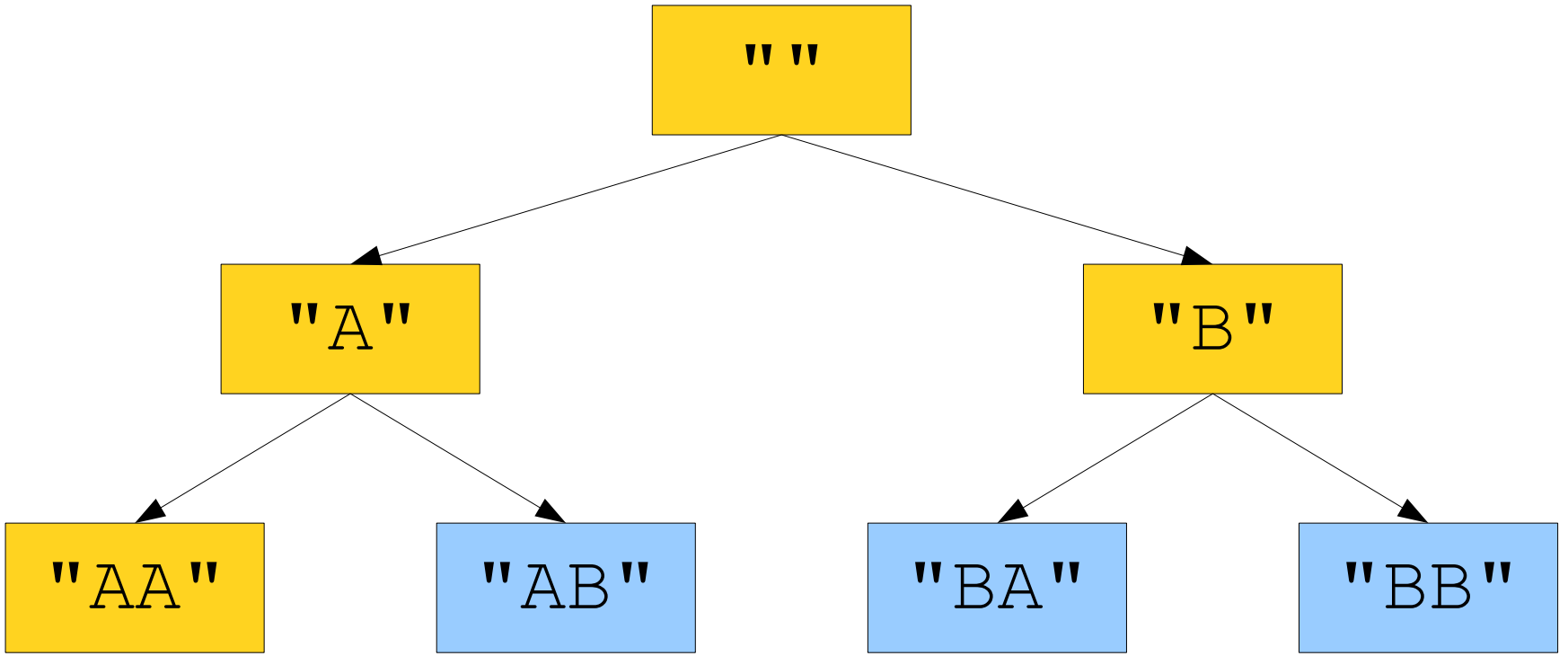
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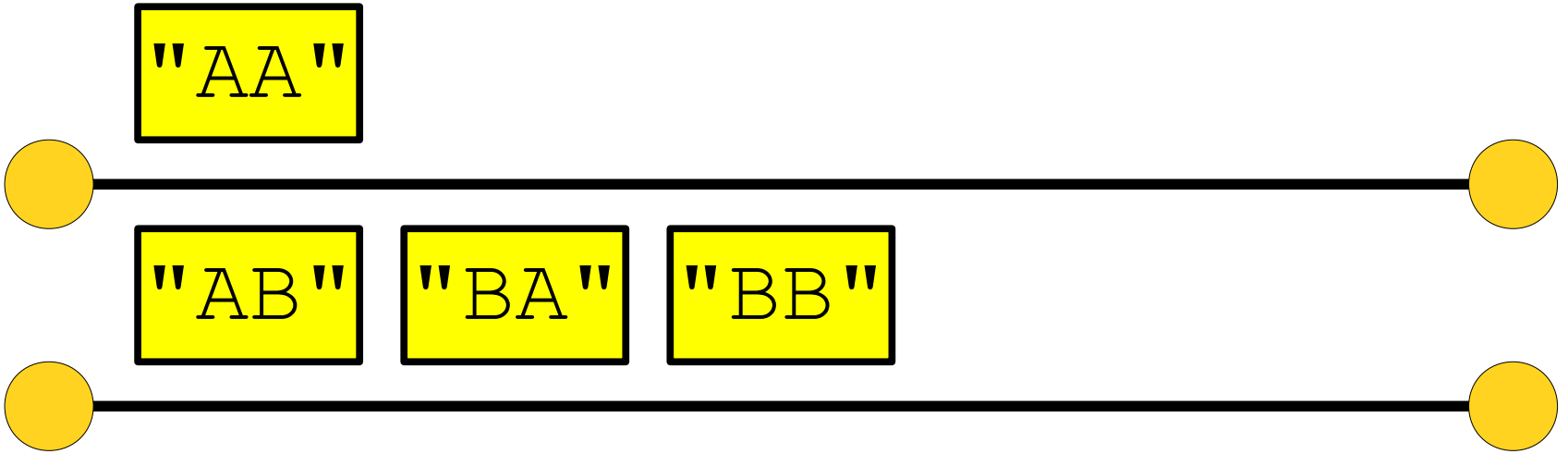
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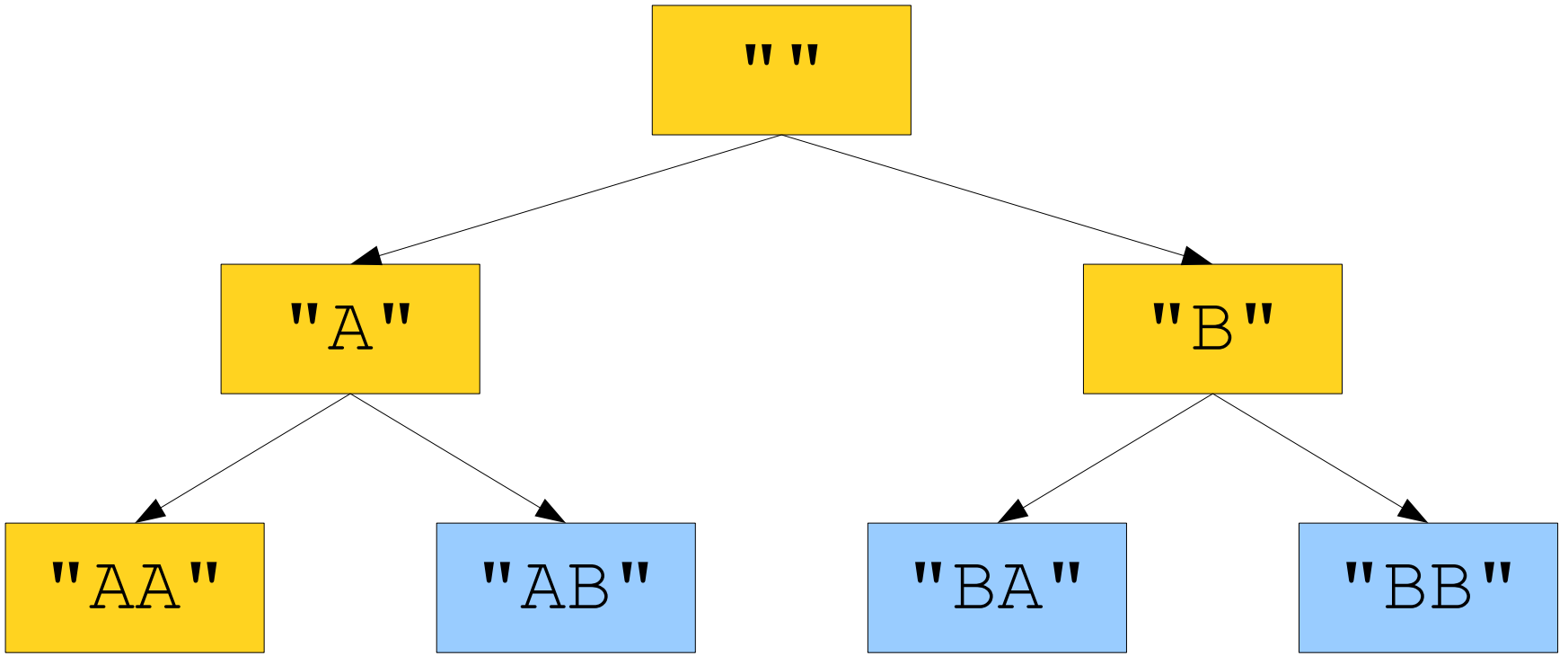
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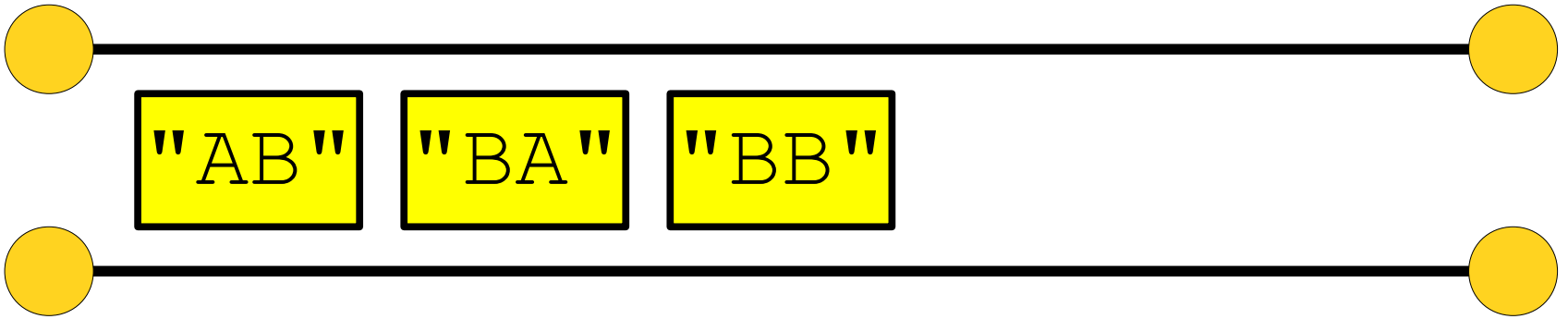
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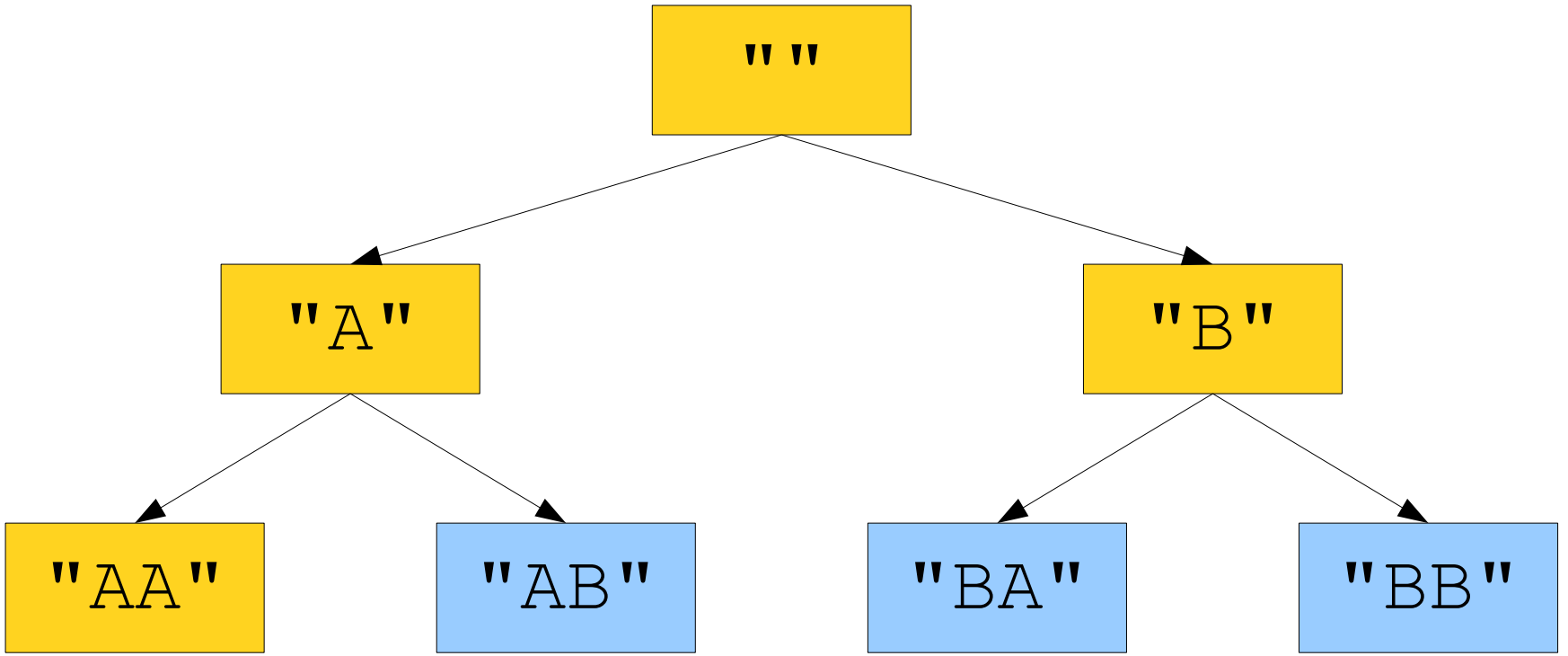
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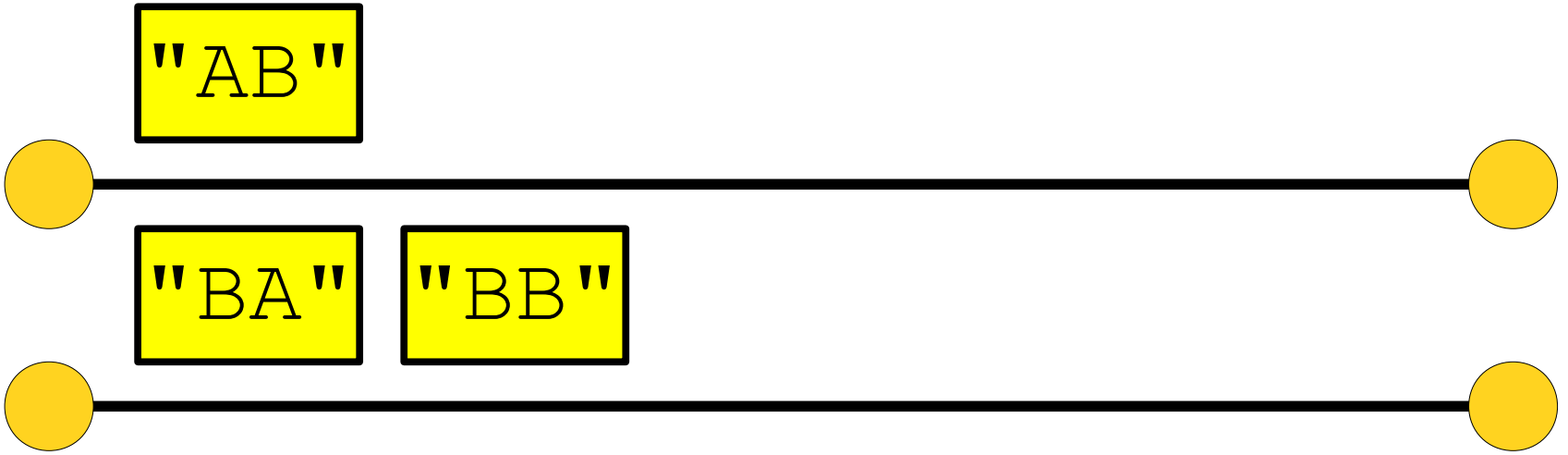
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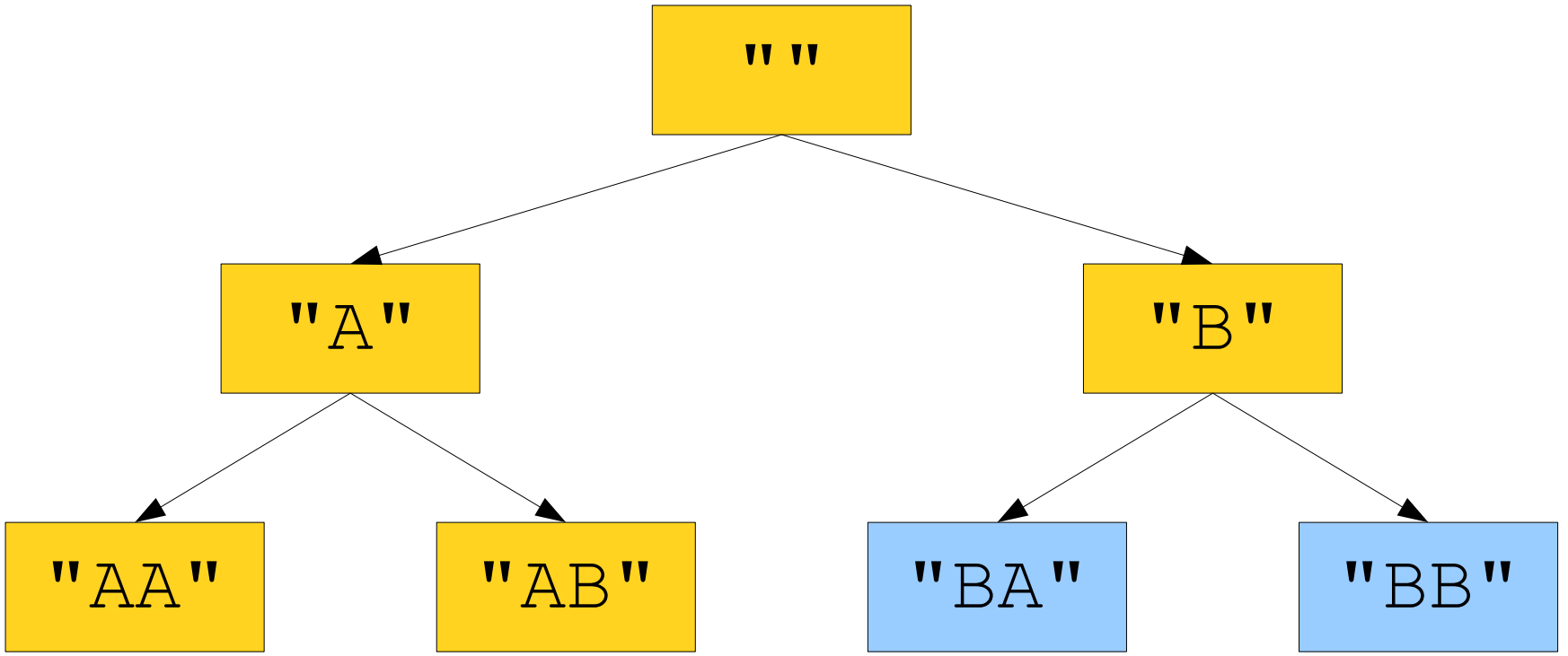
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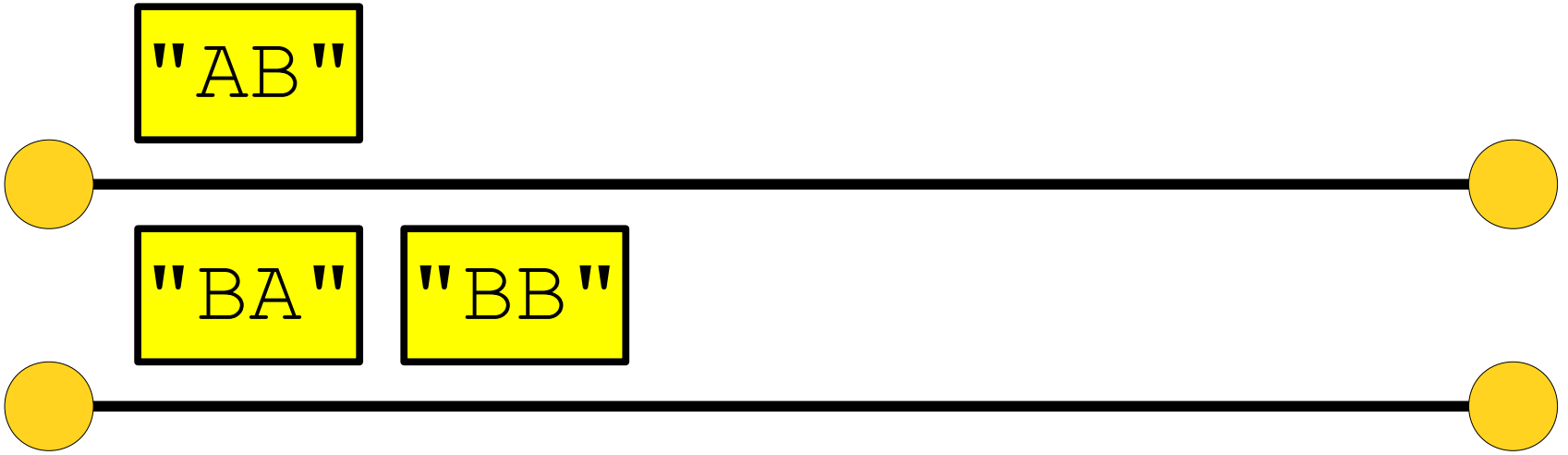
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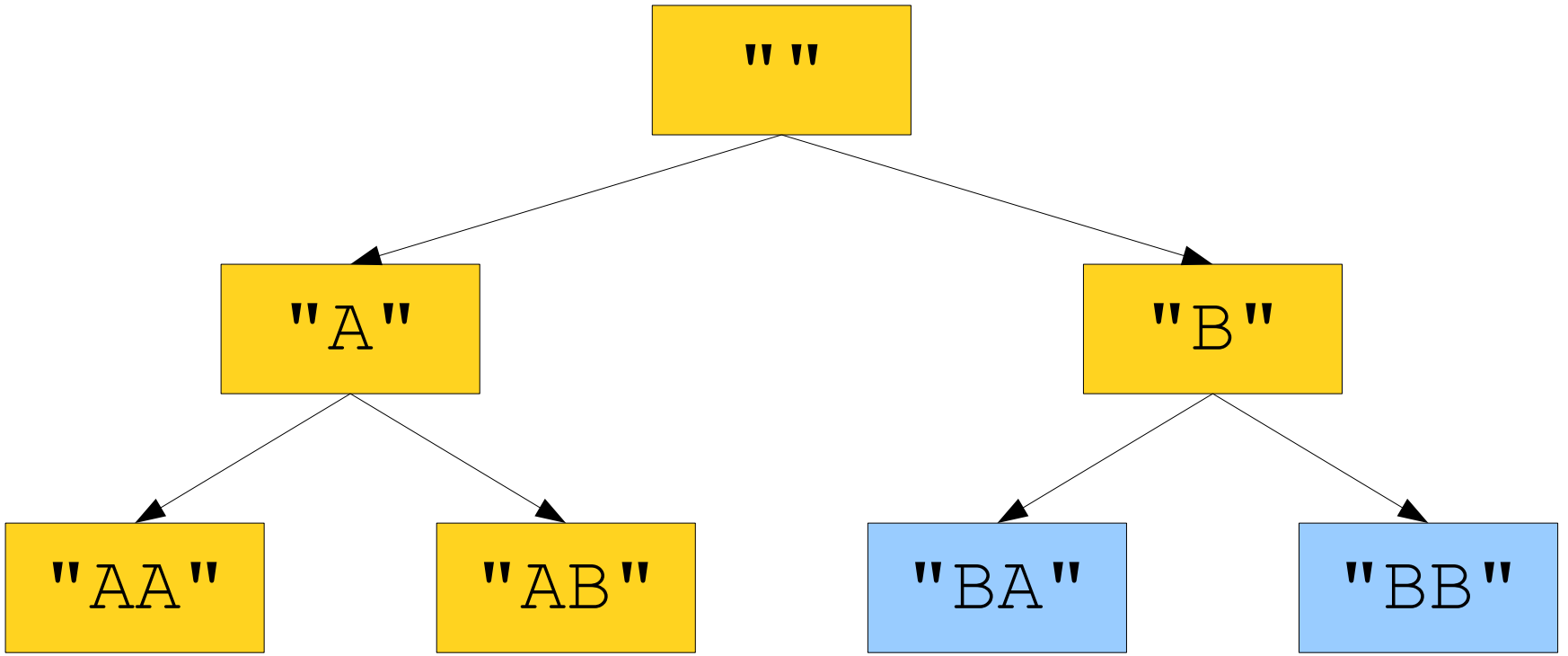
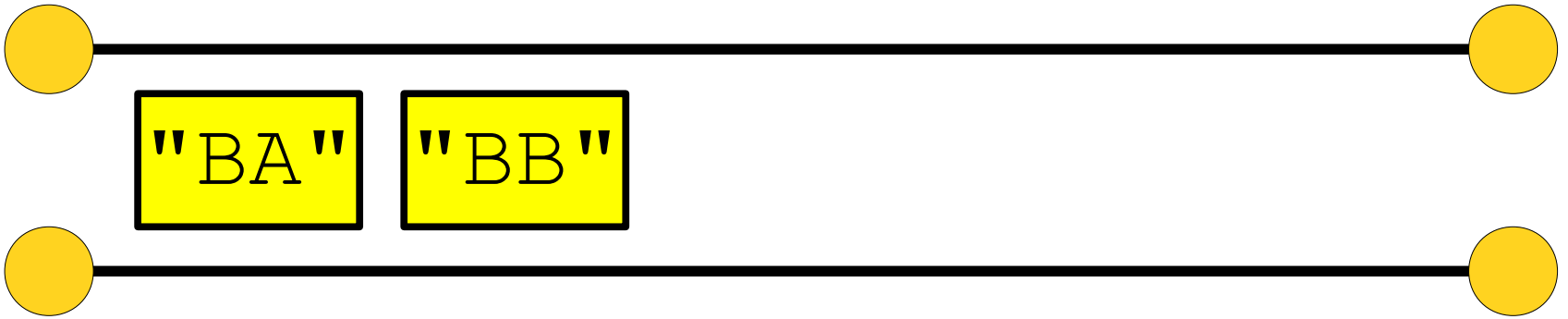
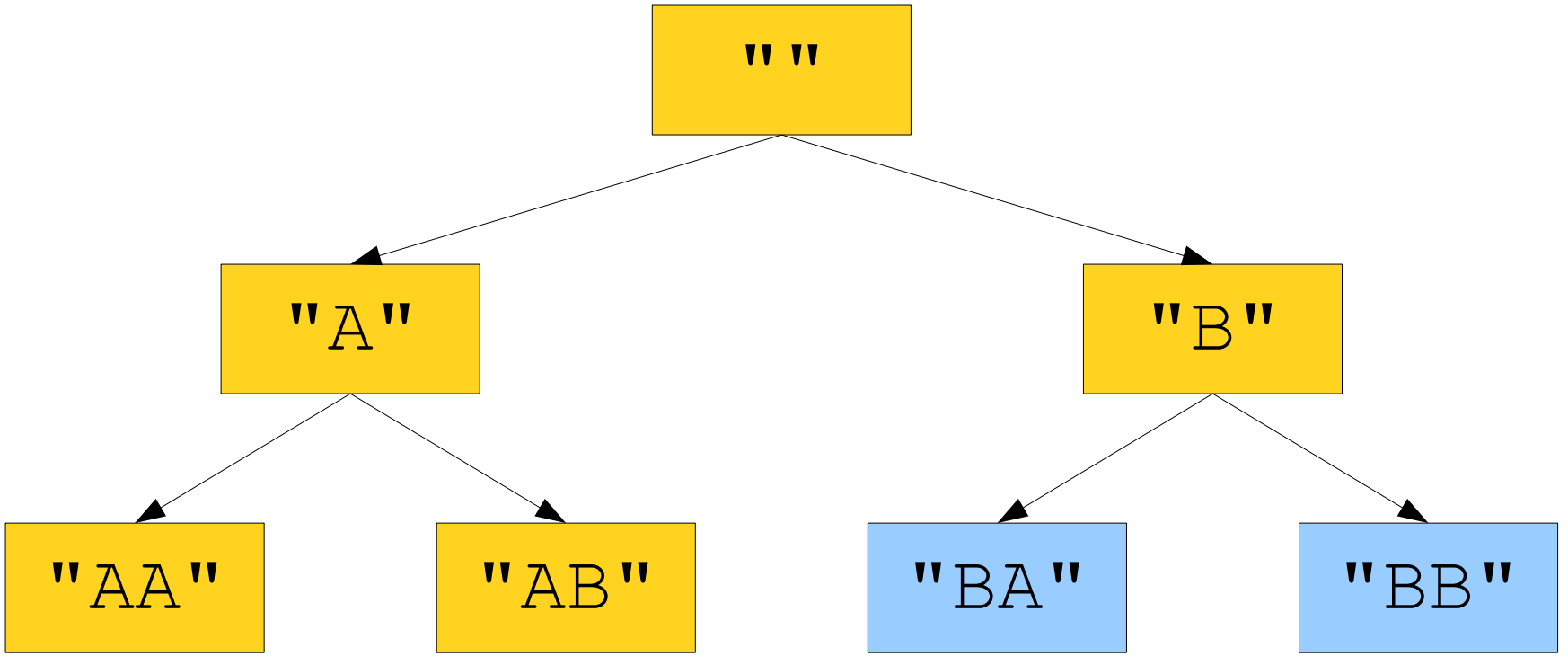
"BA"
"BB"


""
"A"
"B"
"AA"
"AB"
"BA"
"BB"


"BA"
"BB"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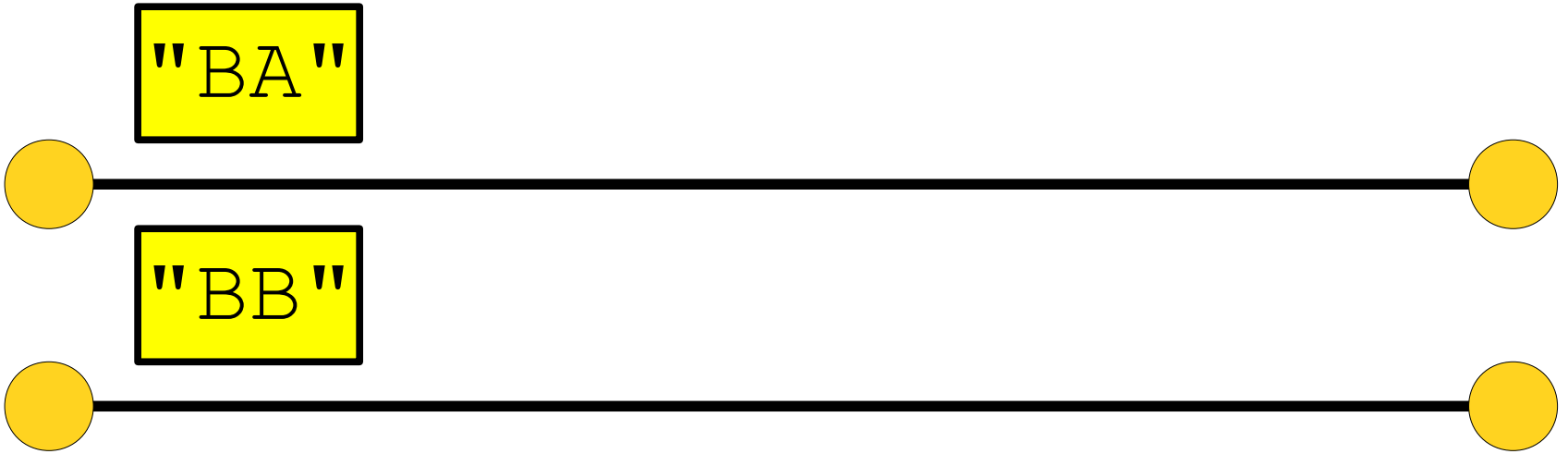
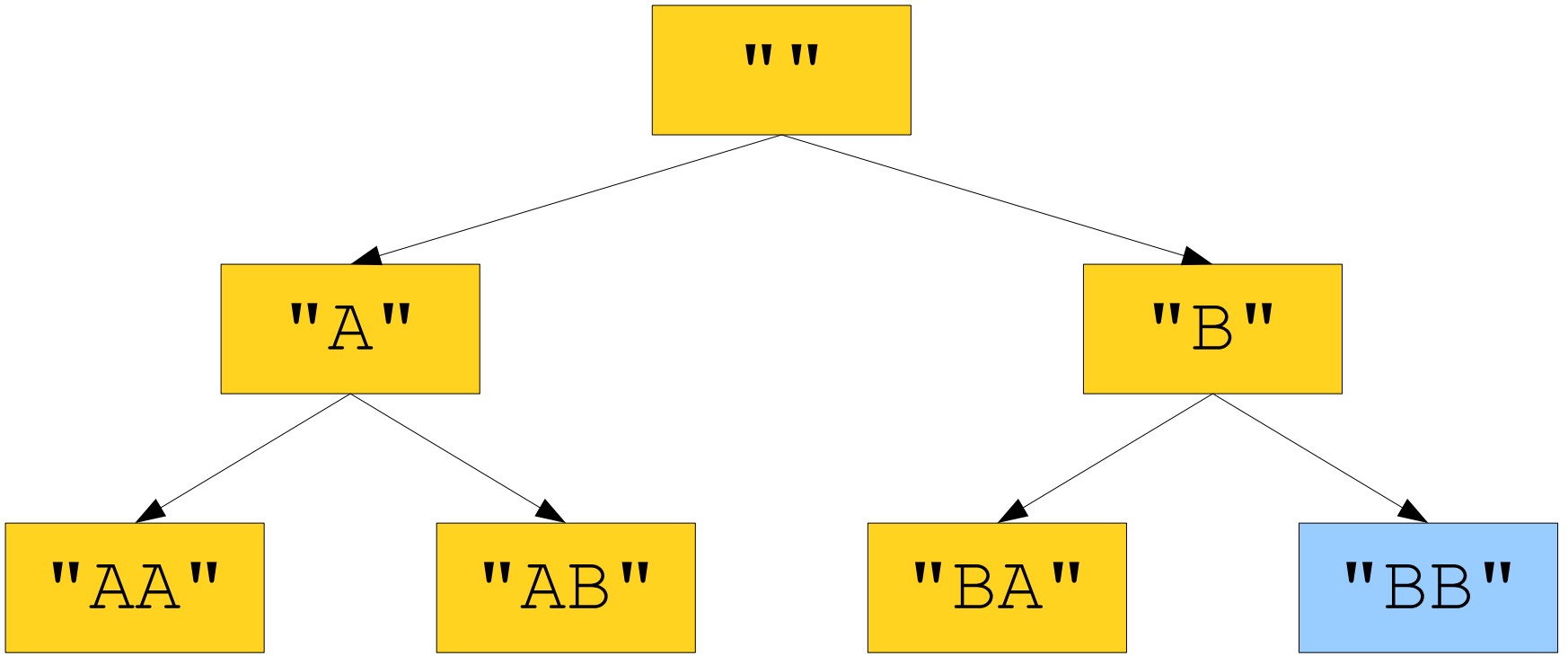
"A"
"B"
"AA"
"AB"
"BA"
"BB"


"BA"
"BB"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"A"
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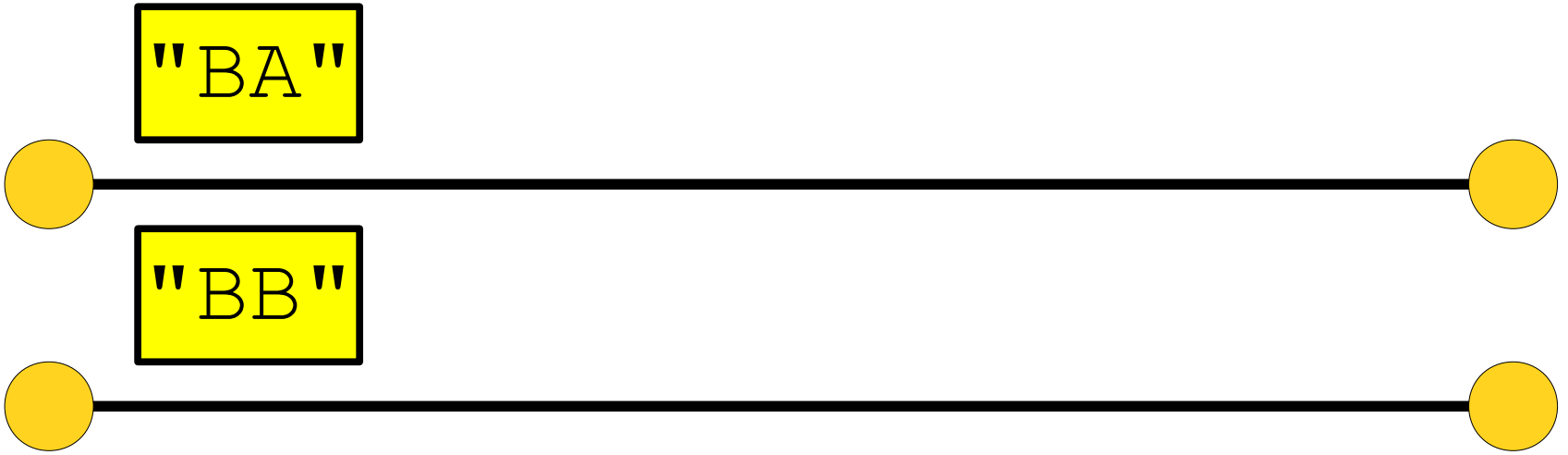
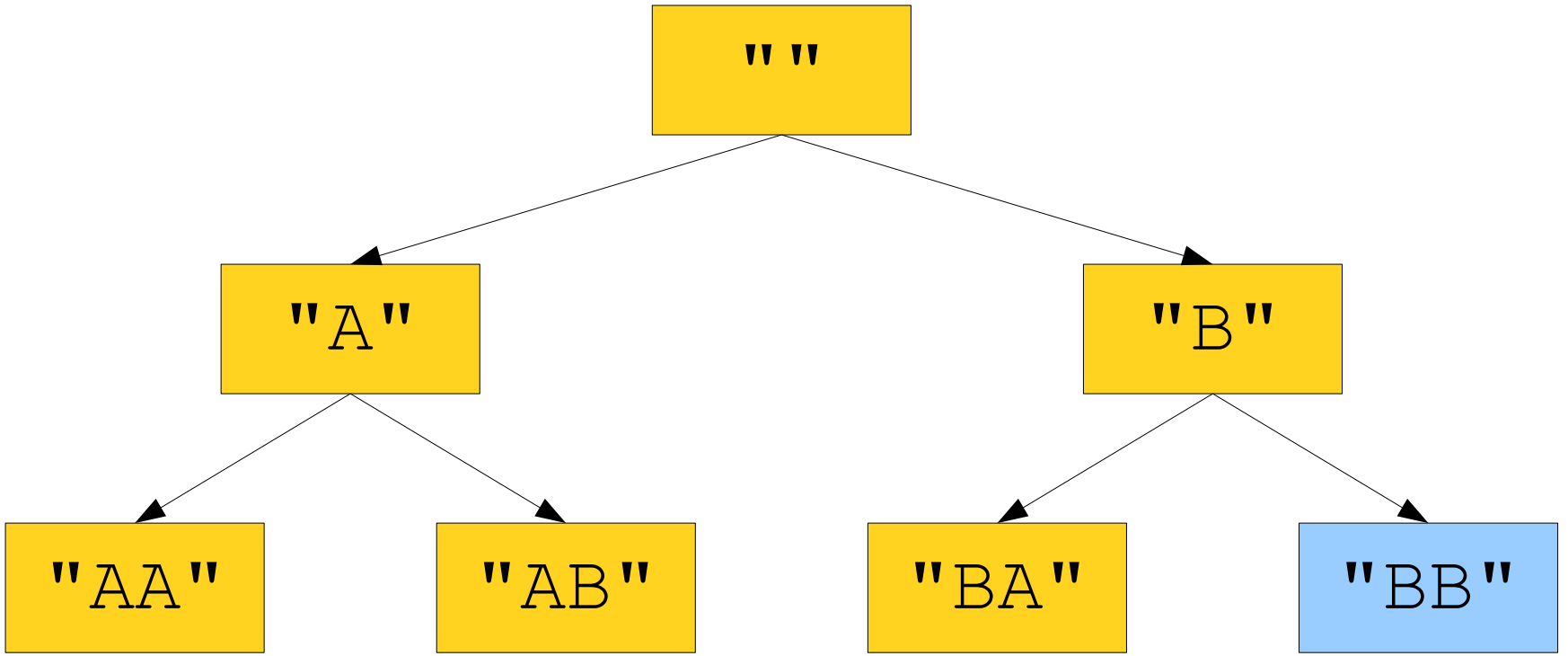
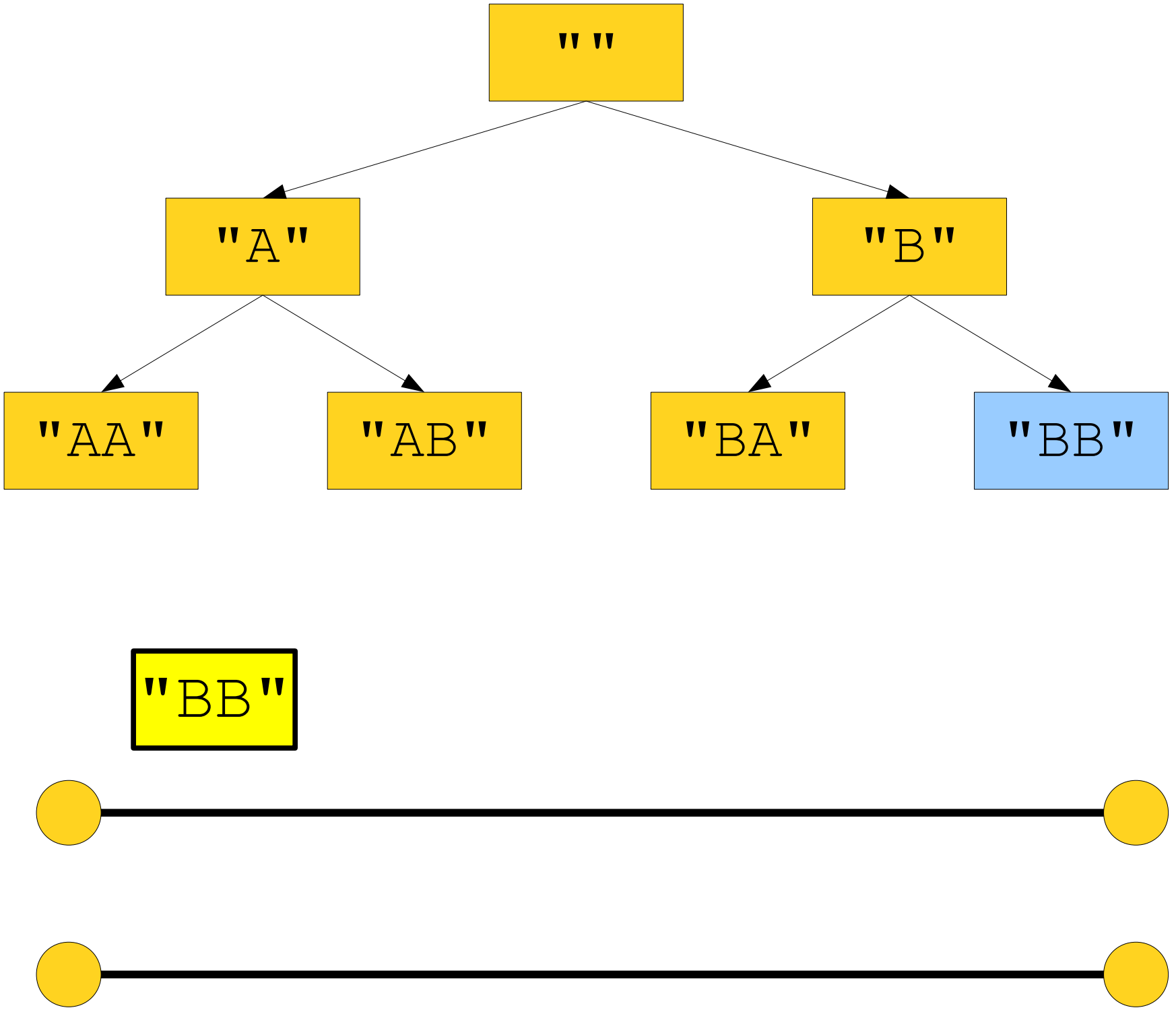
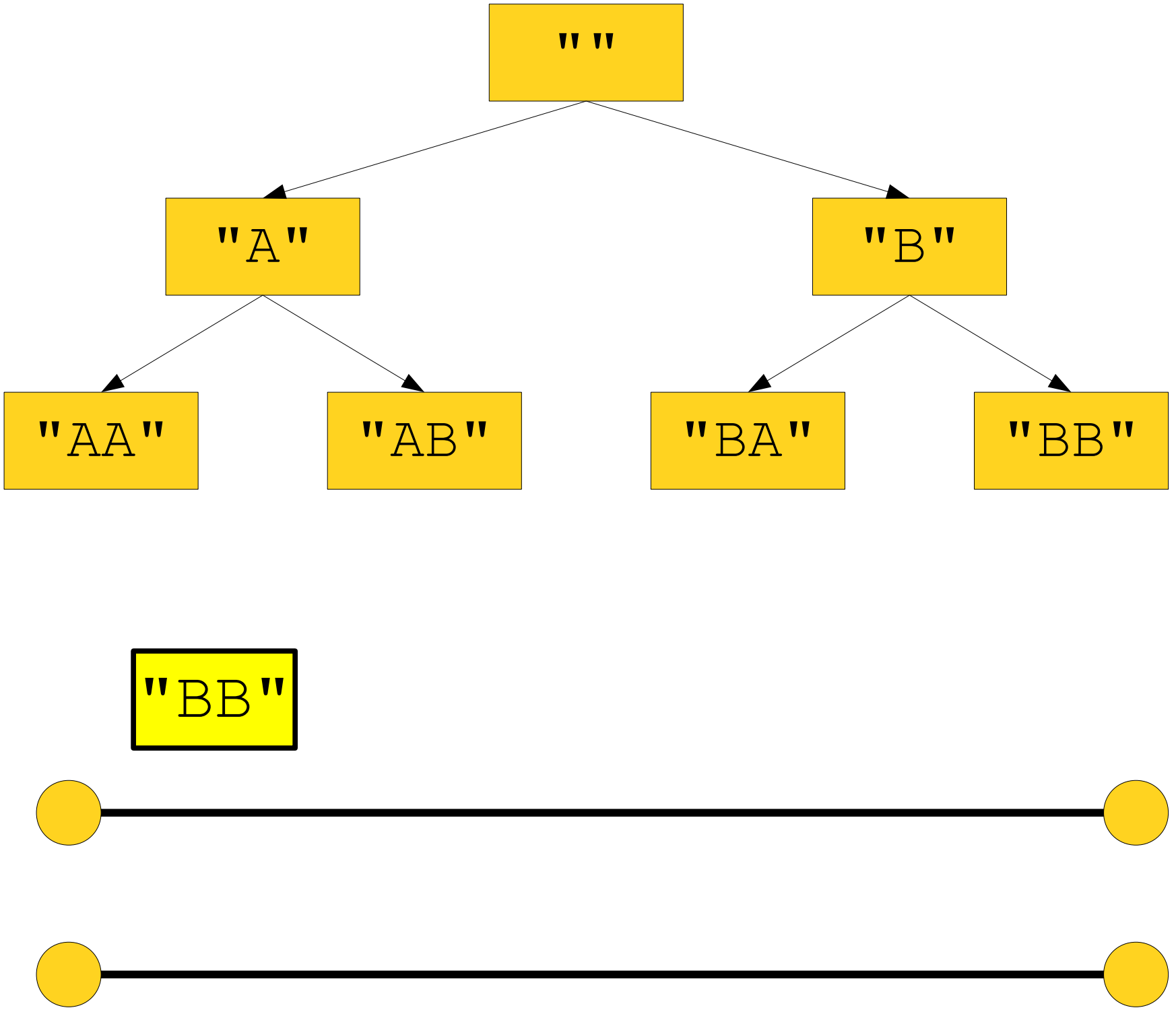
"AA"
"AB"
"BA"
"BB"

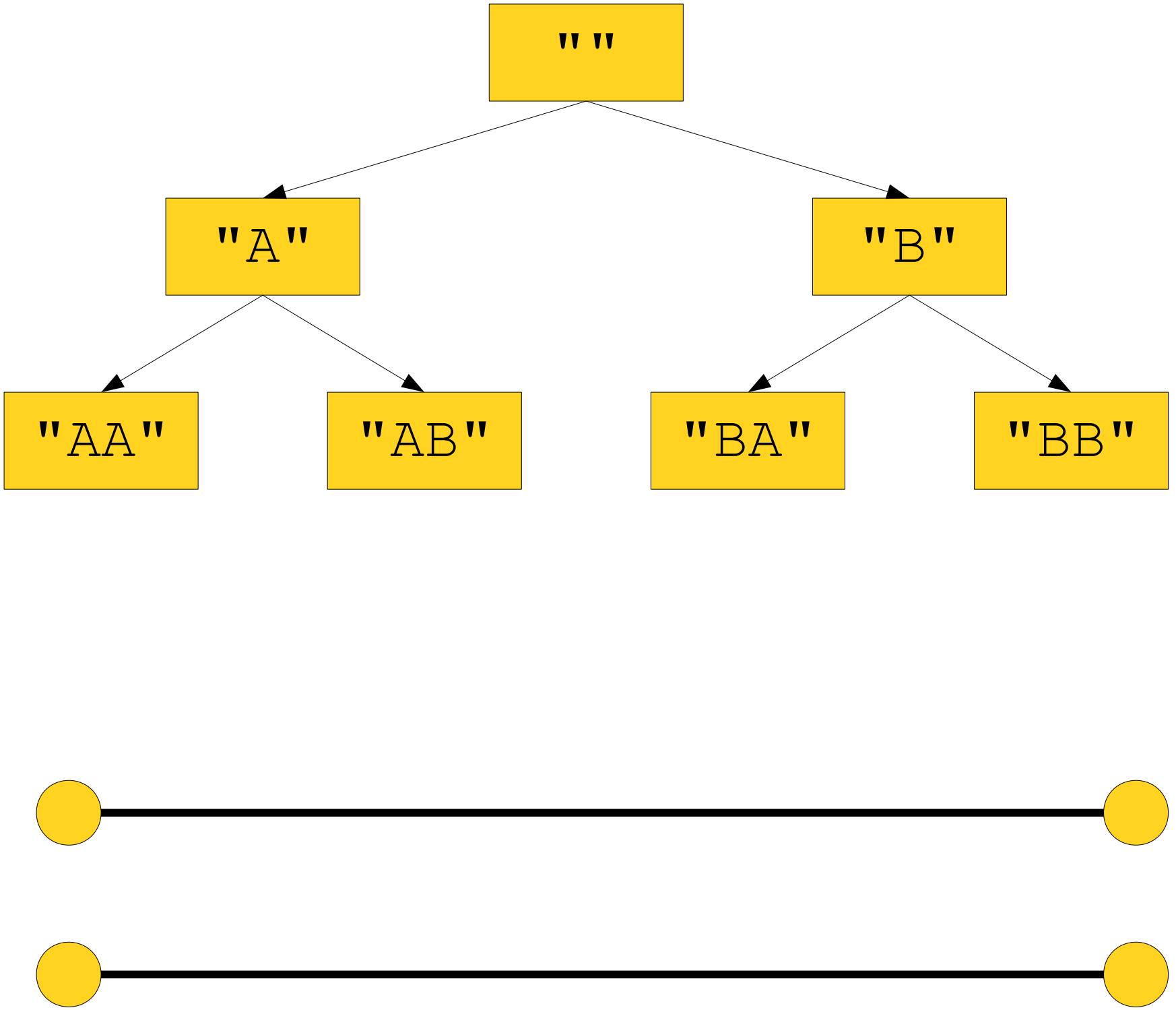

"BA"
"BB"


""
"A"
"B"
"AA"
"AB"
"BA"
"BB"


"BB"


""
"A"
"B"
"AA"
"AB"
"BA"
"BB"
"BB"


""
"A"
"B"
"AA"
"AB"
"BA"
"BB"
"BB"


""
"A"
"B"
"AA"
"AB"
"BA"
"BB"

# Listing All Strings

- Using a Queue we can list all strings of length less than 3 **in order of string length**!
  - Keep this in mind moving forward.

# **Application**: Cracking Passwords

# The Setup

- Suppose that you have a mystery function

```
bool login(string& username,
           string& password);
```

- This function takes in a username and password, then tells you whether it is correct.
- Suppose that you know someone's username. How might you break in to their account?

# A Brute-Force Attack

- **Idea:** Try logging in with all possible passwords!
  - Try logging in with all strings of length 0, then all strings of length 1, then all strings of length 2, ...
- Eventually, this will indeed crack the password.
- What would this look like in code?
- How fast will it be?

# `all-strings.cpp`
# (Computer)

# Why Brute-Forcing is Hard

# Analyzing Efficiency

- How long will it take for our program to crack a password?
- Let's make the following assumptions:
  - The password consists purely of lowercase letters.
  - The password has length at most $n$.
- There are $26^k$ possible lowercase strings of length $k$.
- Might have to try all strings of length 0, 1, …, $n$.
- To break the password, we need to try at most

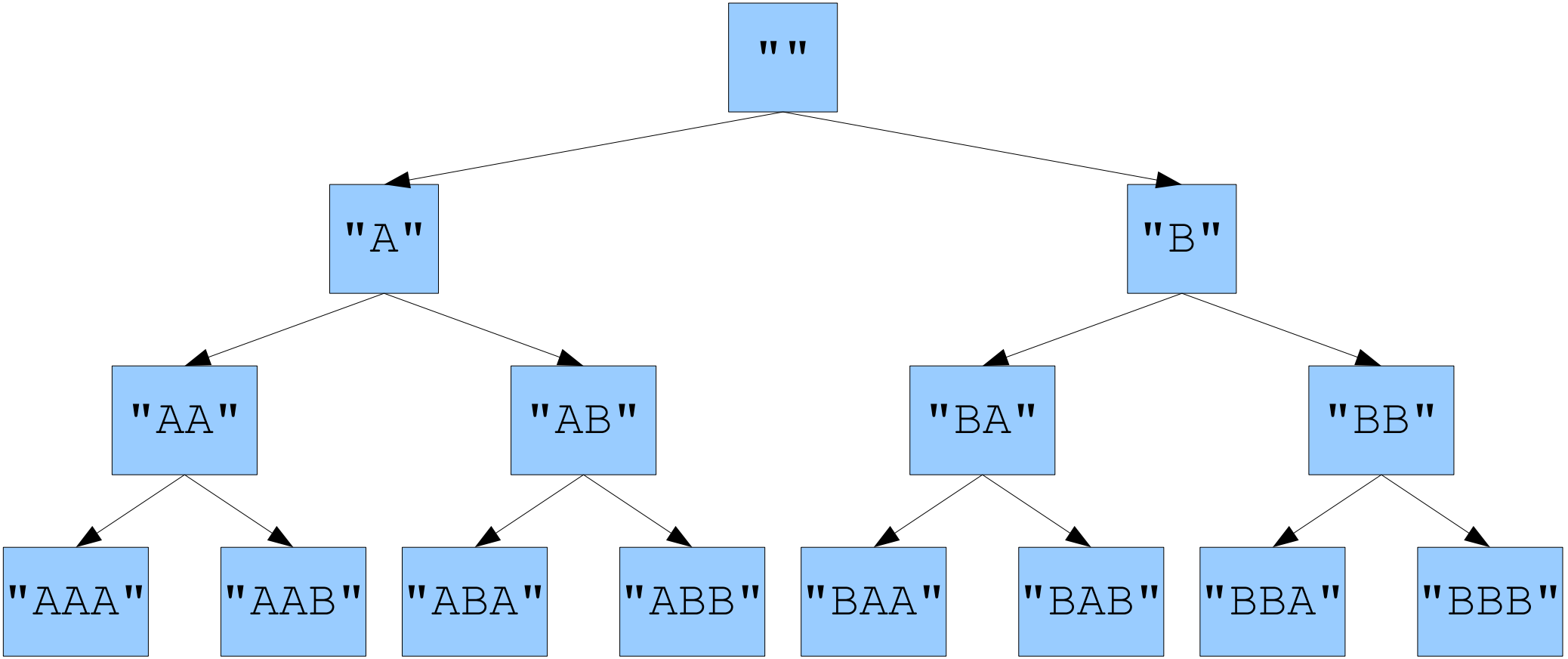
$$1 + 26 + 26^2 + 26^3 + 26^4 + \ldots + 26^n = \frac{26^{n+1} - 1}{25}$$

different strings.

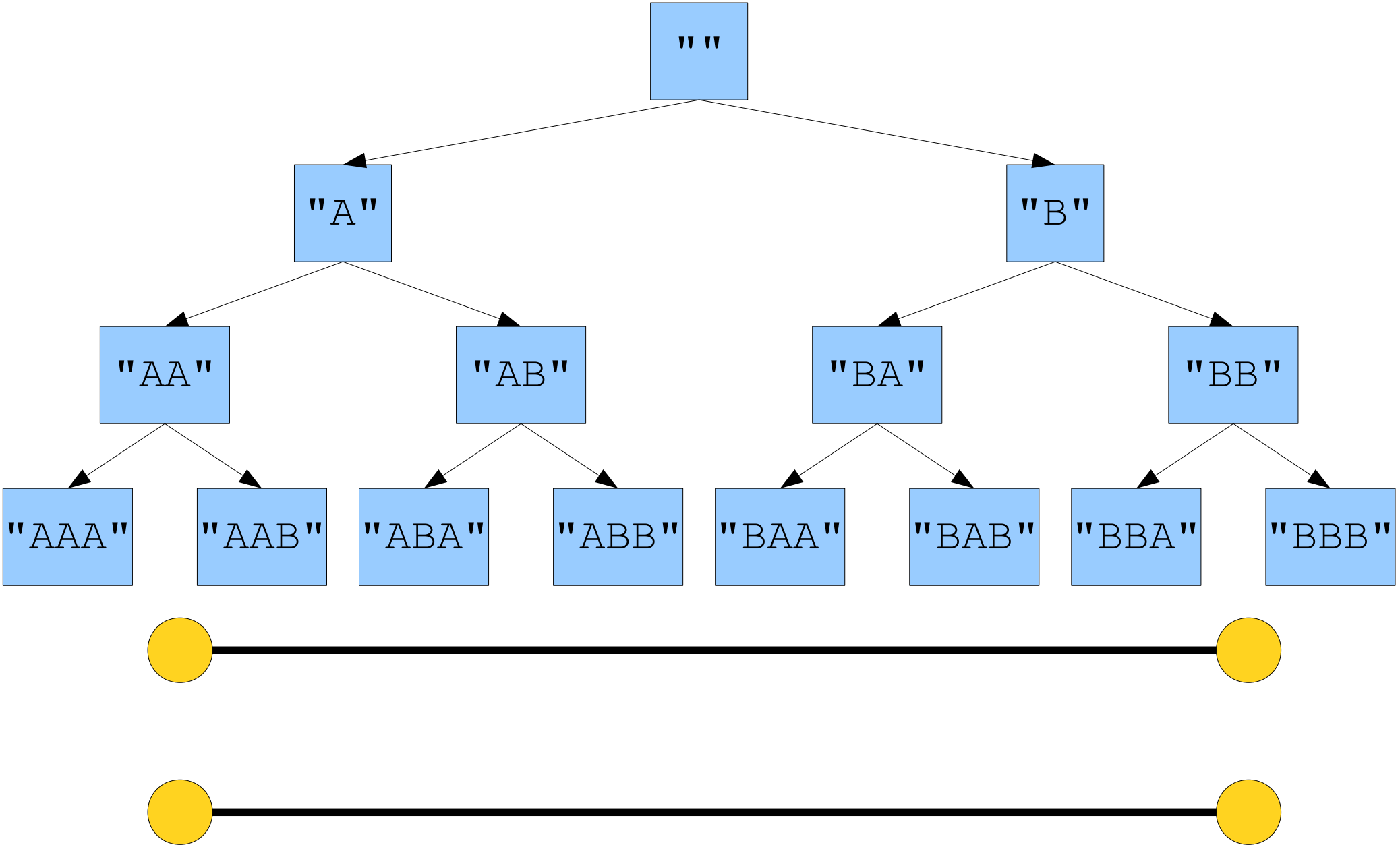
# How Many Strings?

- If $n = 0$, might have to try 1 string.
- If $n = 1$, might have to try 27 strings.
- If $n = 2$, might have to try 703 strings.
- If $n = 3$, might have to try 18,279 strings.
- …
- If $n = 8$, might have to try 217,180,147,159 strings.
- …
- If $n = 15$, might have to try 1,744,349,715,977,154,962,391 strings.

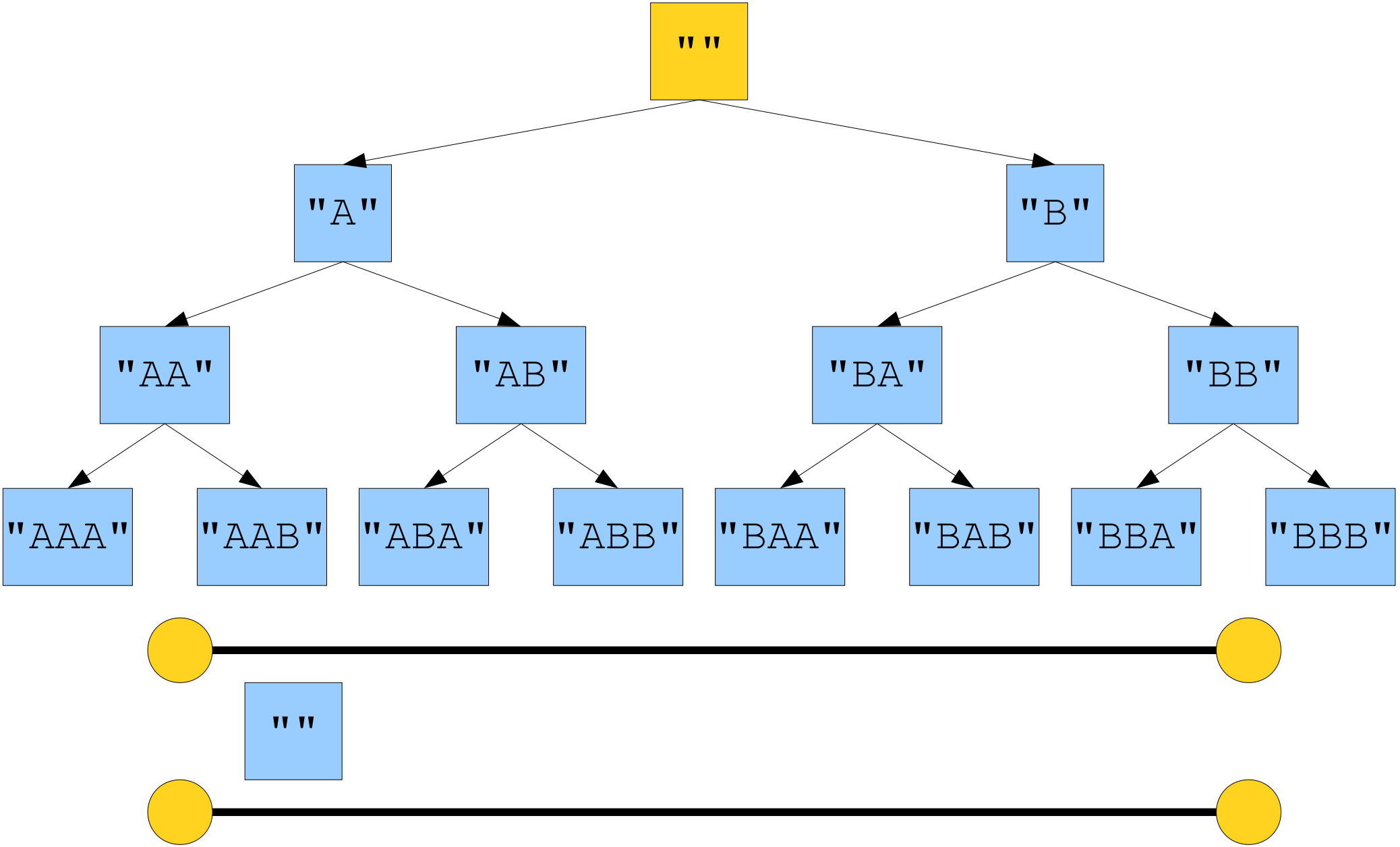
# Memory Usage



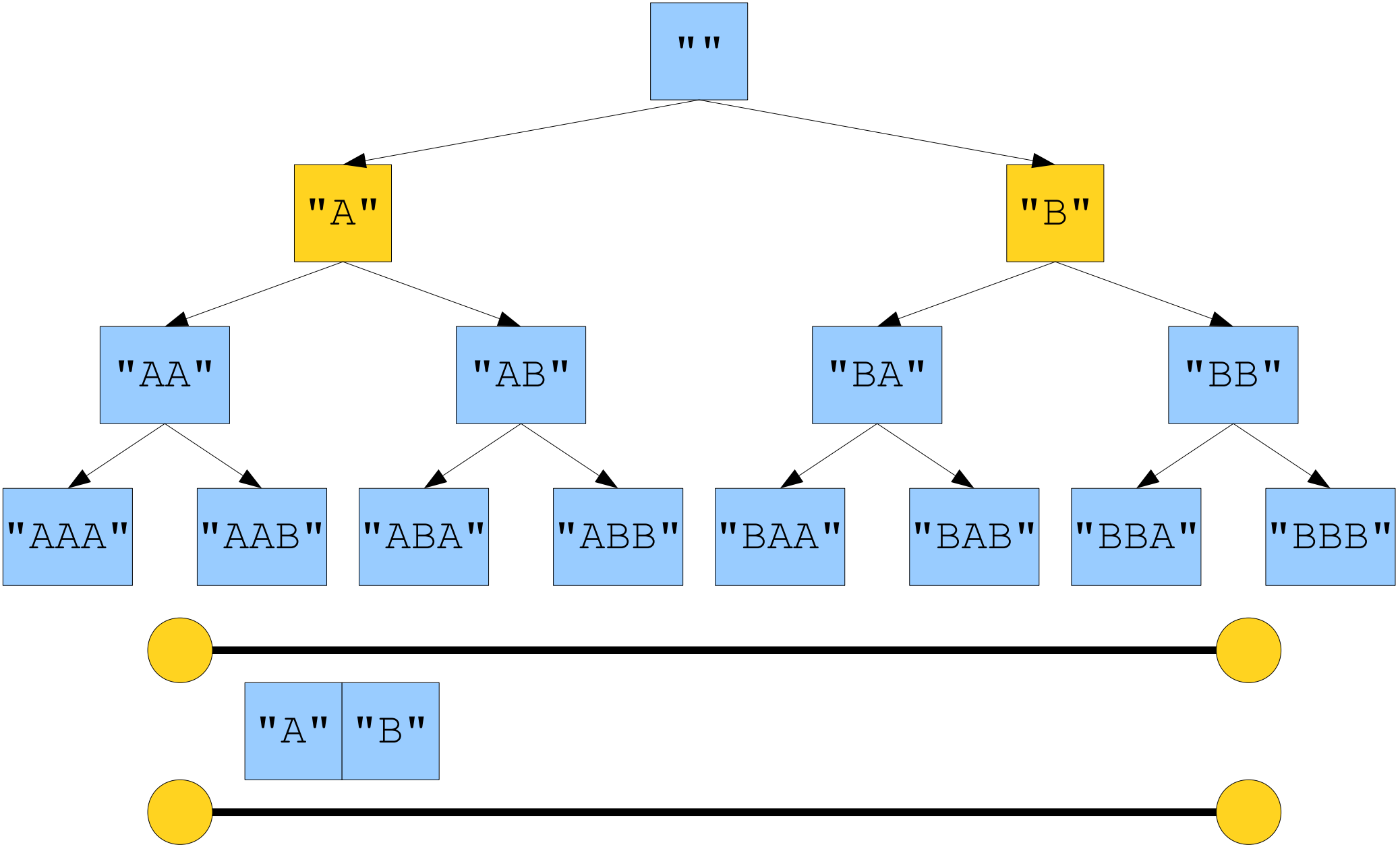
# Memory Usage



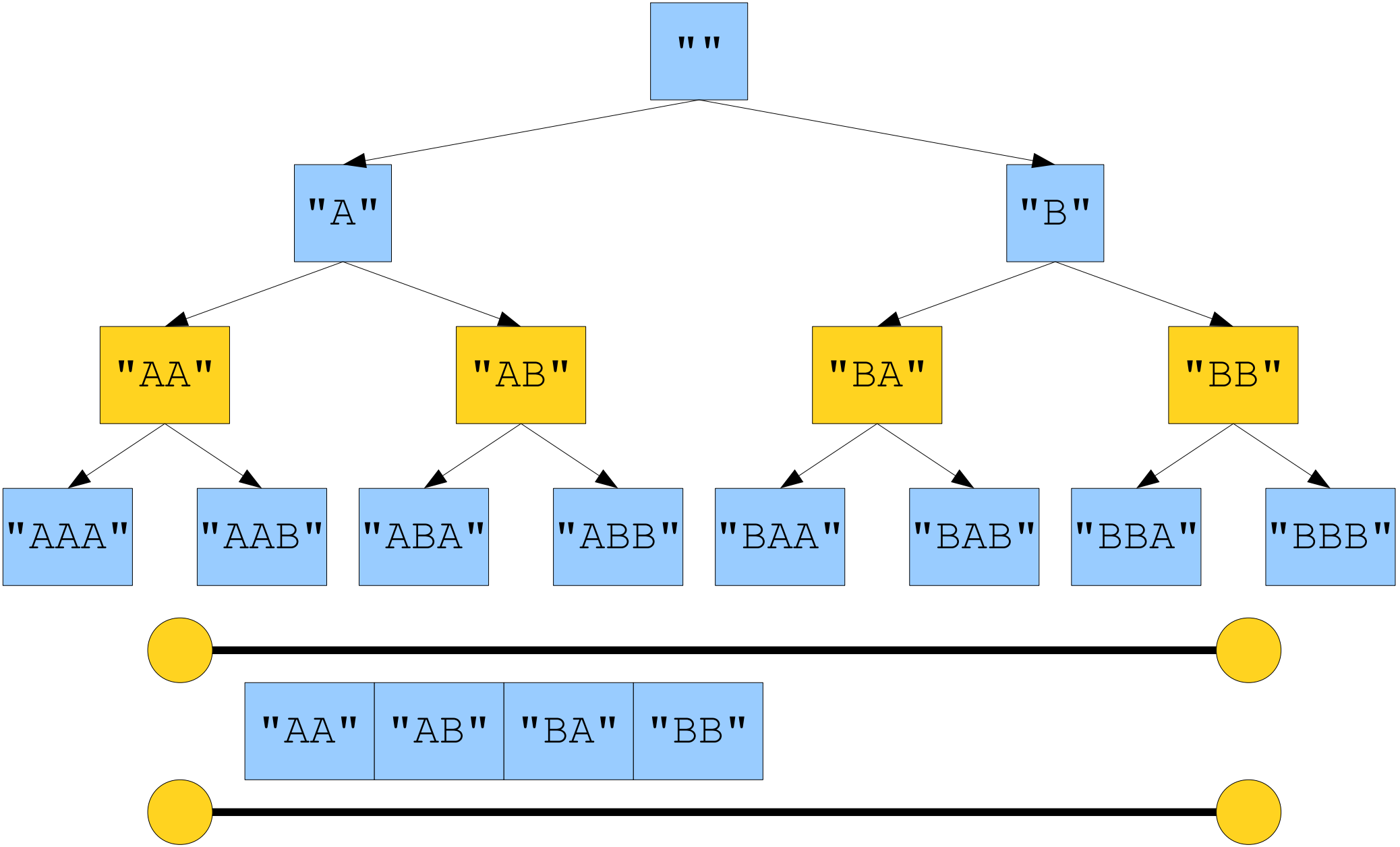
# Memory Usage



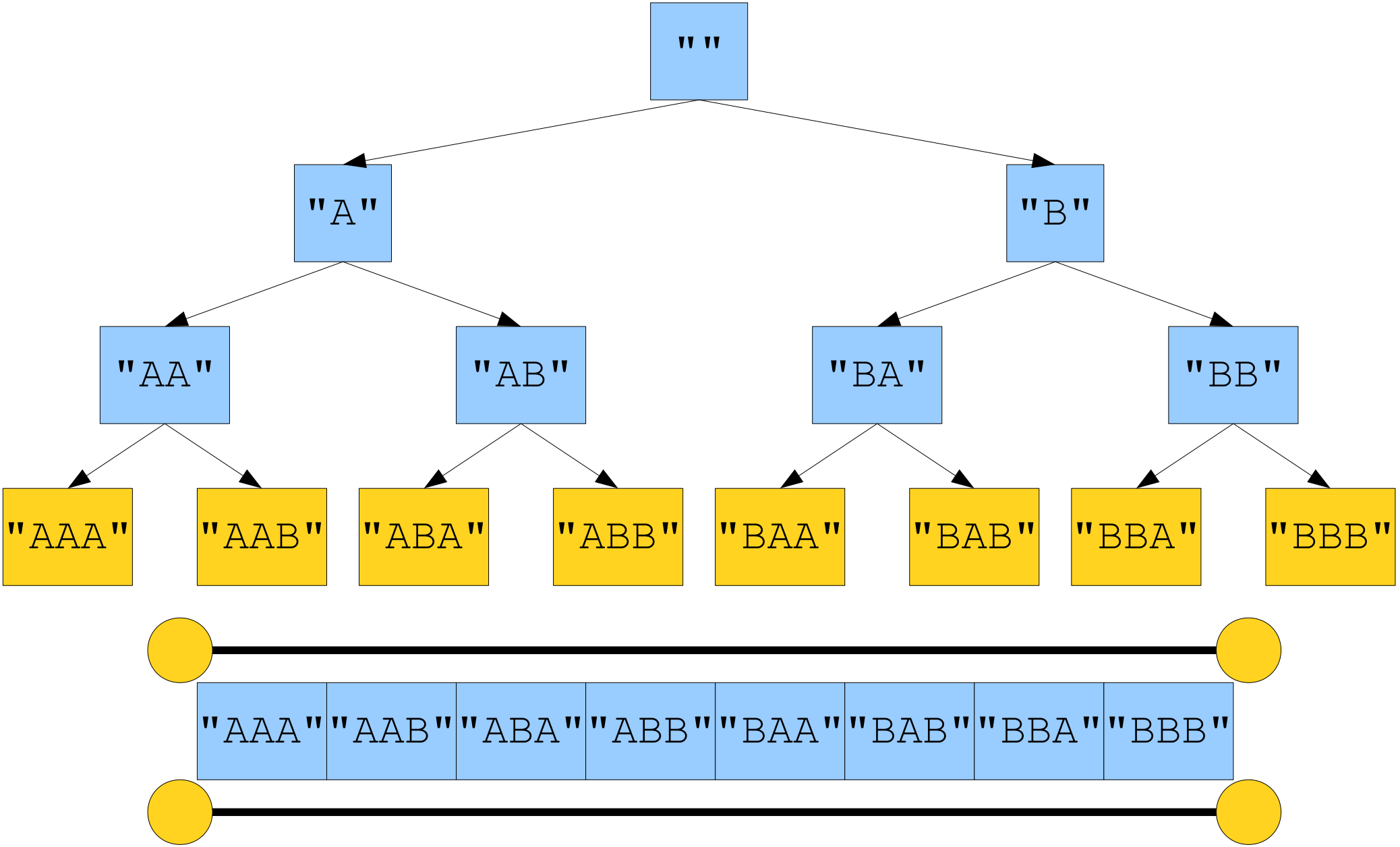
# Memory Usage



# Memory Usage



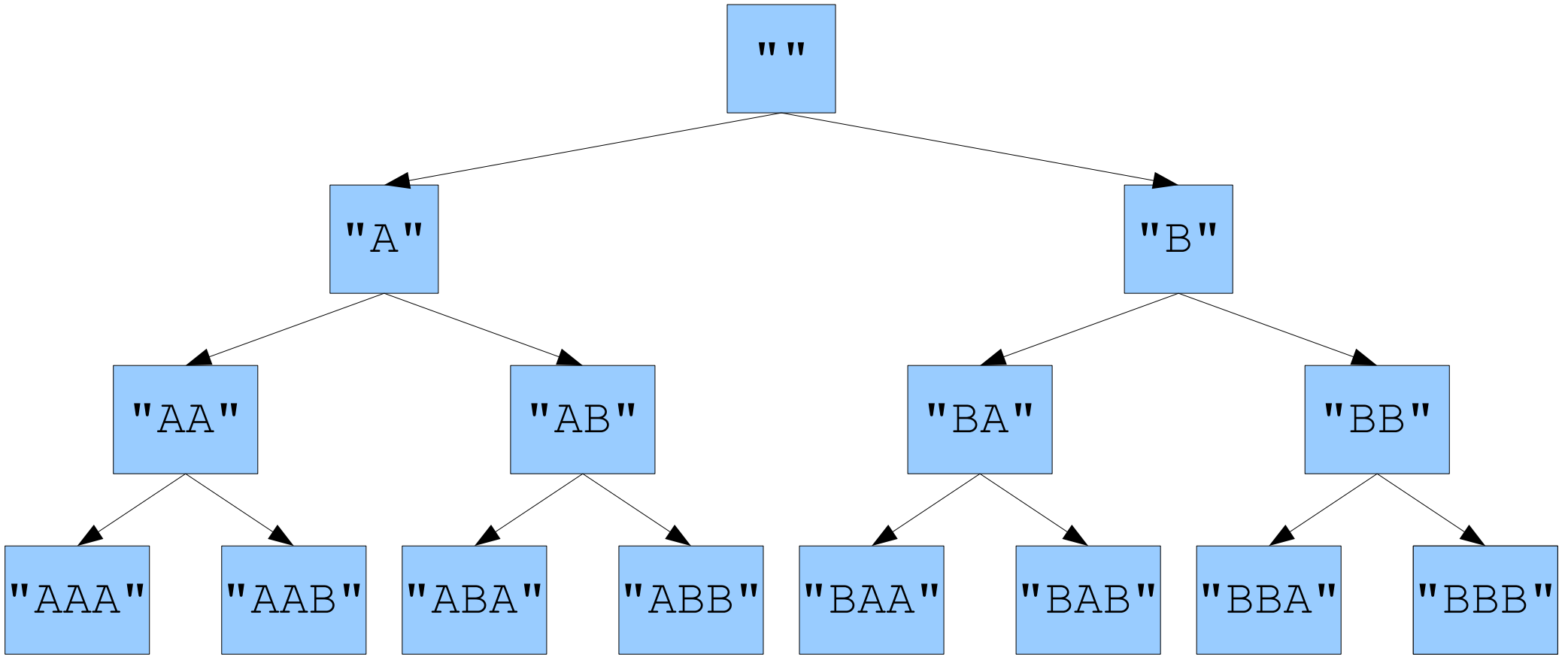
# Memory Usage



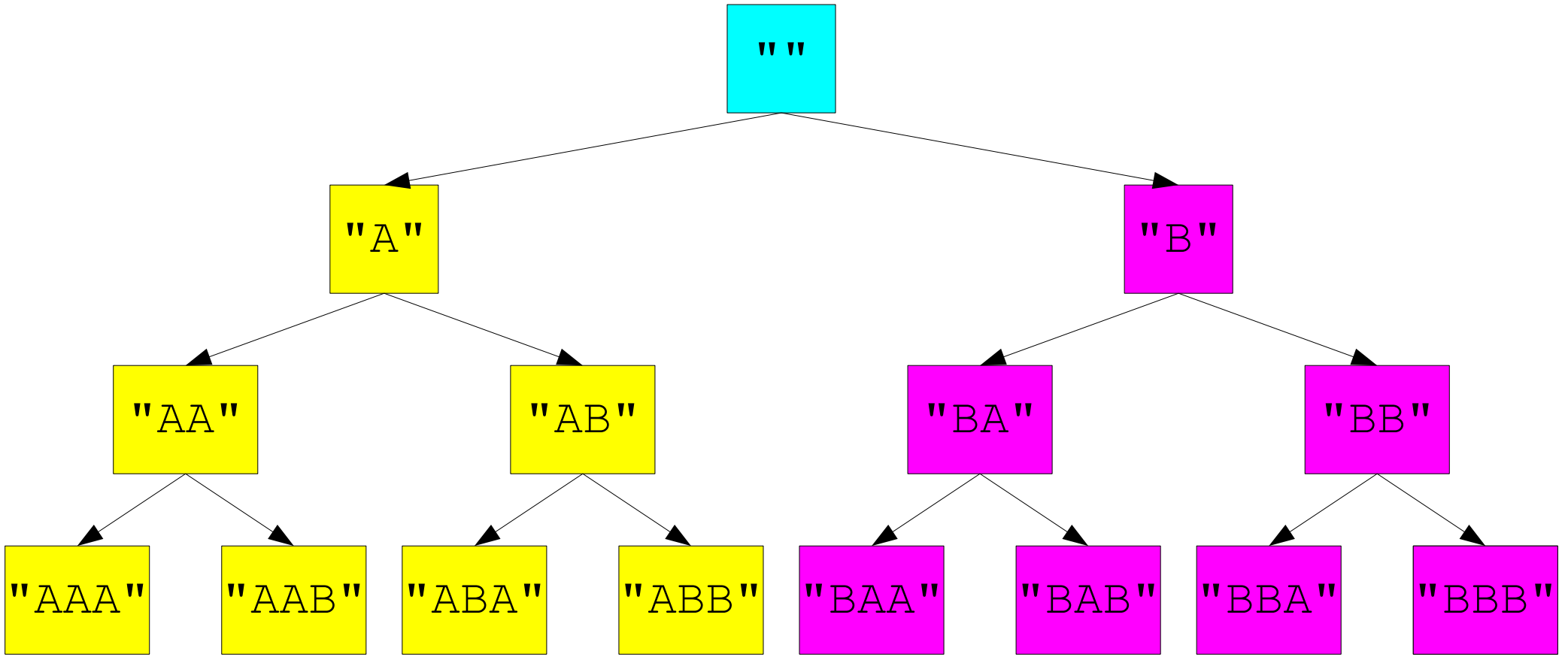
# Memory Usage

- Our current brute-force attack requires us to have about $26^n$ strings in memory when attacking a password of length $n$.
- When $n$ gets large, this is an *enormous* amount of memory.
- On a computer with only 4GB or 8GB of memory, we might run out of memory trying to do a brute-force attack!

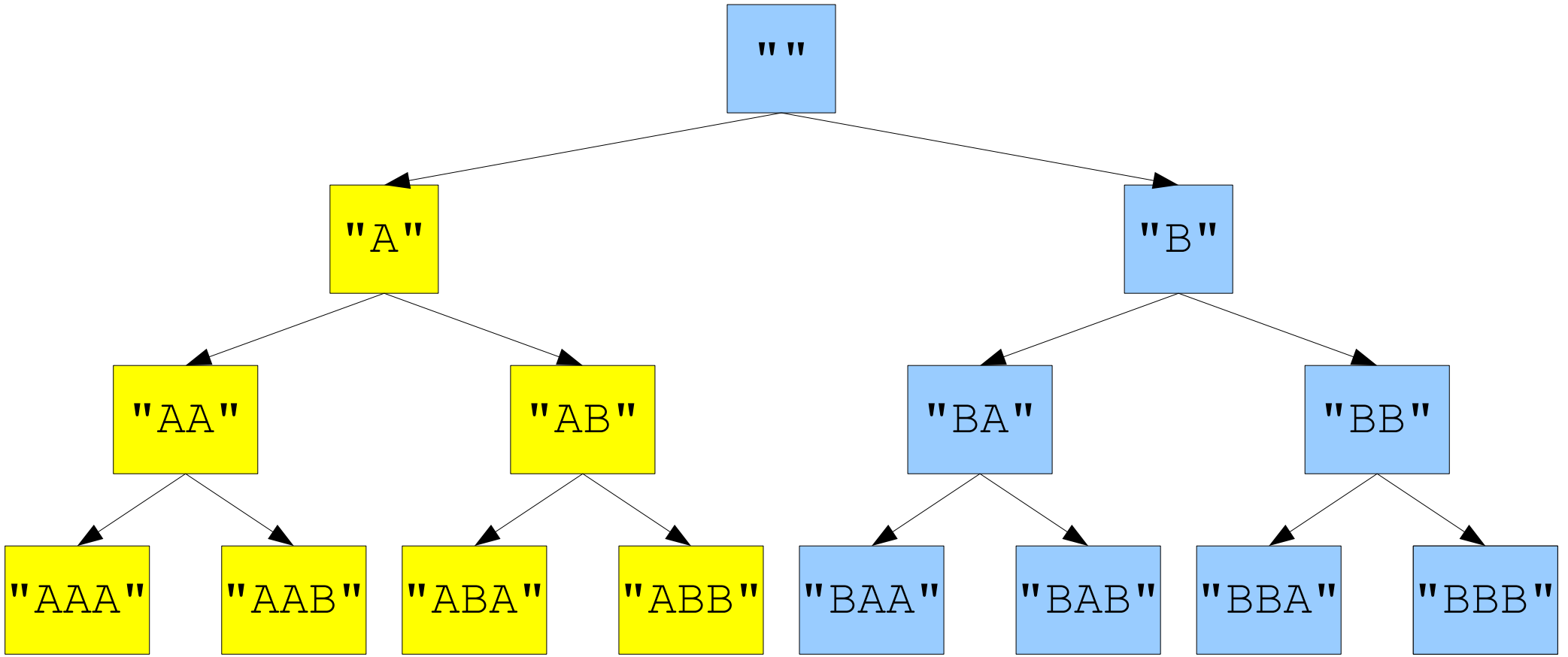
# A Recursive Approach



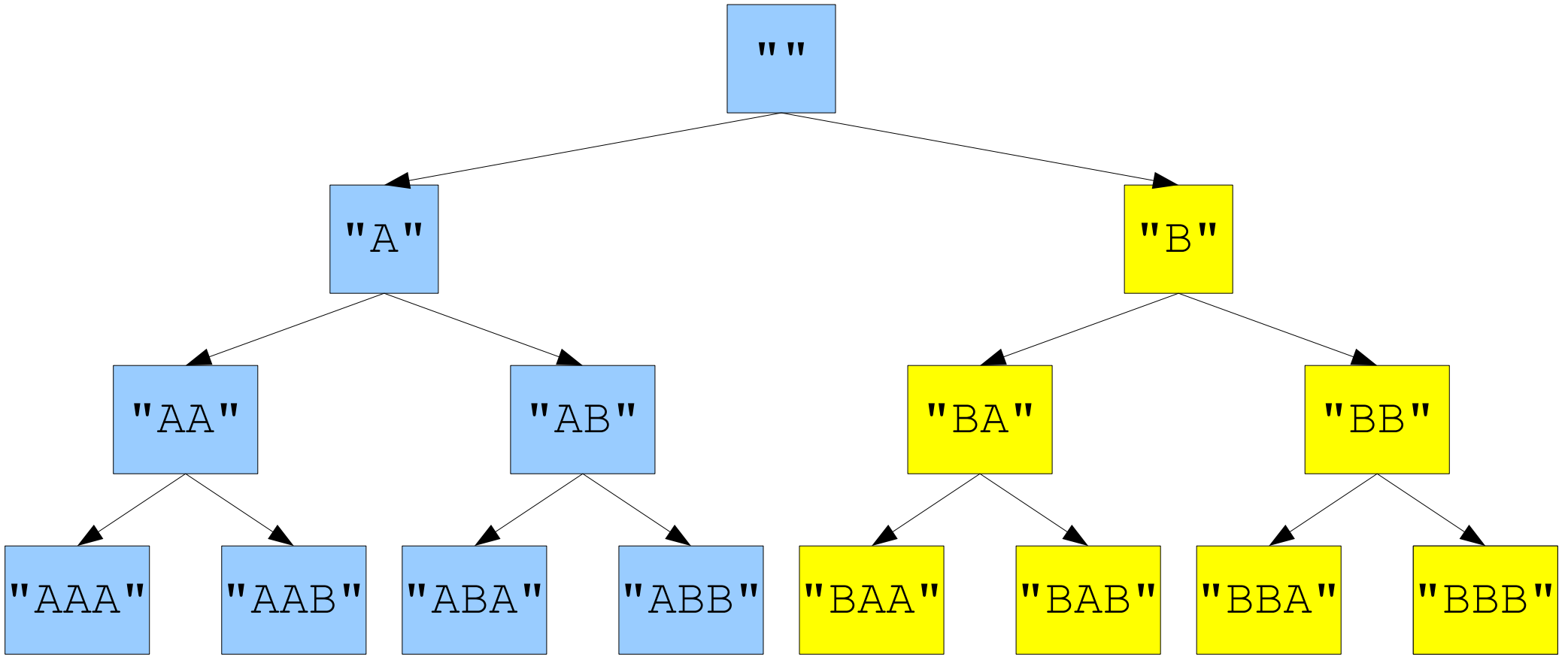
# A Recursive Approach



# A Recursive Approach



# A Recursive Approach



# `recursive-search.cpp`
# (Computer)

# A Recursive Approach

# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  doSomethingWith(soFar);
  if (soFar.length() < maxLength) {
    for (char ch = 'A'; ch <= 'B'; ch++) {
      generate(soFar + ch, maxLength);
    }
  }
}
```

soFar `""`  maxLength `3`

# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  doSomethingWith(soFar);
  if (soFar.length() < maxLength) {
    for (char ch = 'A'; ch <= 'B'; ch++) {
      generate(soFar + ch, maxLength);
    }
  }
}
```

soFar `""`  maxLength `3`

# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  doSomethingWith(soFar);
  if (soFar.length() < maxLength) {
    for (char ch = 'A'; ch <= 'B'; ch++) {
      generate(soFar + ch, maxLength);
    }
  }
}
```

soFar: ""  maxLength: 3

""

# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  doSomethingWith(soFar);
  if (soFar.length() < maxLength) {
    for (char ch = 'A'; ch <= 'B'; ch++) {
      generate(soFar + ch, maxLength);
    }
  }
}
```

soFar: ""  maxLength: 3

""

# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  doSomethingWith(soFar);
  if (soFar.length() < maxLength) {
    for (char ch = 'A'; ch <= 'B'; ch++) {
      generate(soFar + ch, maxLength);
    }
  }
}
```

soFar: `""`  maxLength: 3

`""`

# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  doSomethingWith(soFar);
  if (soFar.length() < maxLength) {
    for (char ch = 'A'; ch <= 'B'; ch++) {
      generate(soFar + ch, maxLength);
    }
  }
}
```

soFar: ""  maxLength: 3

""

# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  void generate(string soFar, int maxLength) {
    doSomethingWith(soFar);
    if (soFar.length() < maxLength) {
      for (char ch = 'A'; ch <= 'B'; ch++) {
        generate(soFar + ch, maxLength);
      }
    }
  }
}
```

soFar "A"  maxLength 3

""

# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  void generate(string soFar, int maxLength) {
    doSomethingWith(soFar);
    if (soFar.length() < maxLength) {
      for (char ch = 'A'; ch <= 'B'; ch++) {
        generate(soFar + ch, maxLength);
      }
    }
  }
}
```

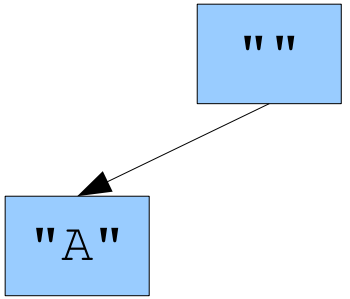
soFar "A"  maxLength 3

""

# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  void generate(string soFar, int maxLength) {
    doSomethingWith(soFar);
    if (soFar.length() < maxLength) {
      for (char ch = 'A'; ch <= 'B'; ch++) {
        generate(soFar + ch, maxLength);
      }
    }
  }
}
```

soFar **"A"**  maxLength **3**



# A Recursive Approach

```
void generate(string soFar, int maxLength) {
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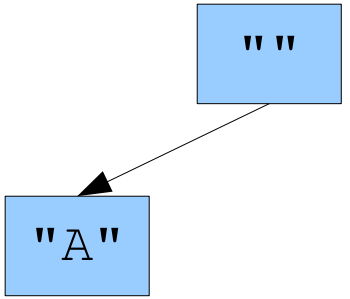
    if (soFar.length() < maxLength) {
      for (char ch = 'A'; ch <= 'B'; ch++) {
        generate(soFar + ch, maxLength);
      }
    }
  }
}
```

soFar **"A"**  maxLength 3



# A Recursive Approach

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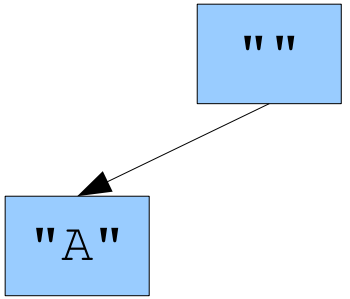
      for (char ch = 'A'; ch <= 'B'; ch++) {
        generate(soFar + ch, maxLength);
      }
    }
  }
}
```

soFar **"A"**  maxLength **3**



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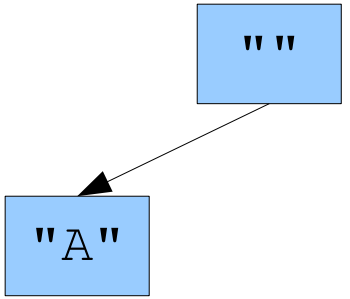
      for (char ch = 'A'; ch <= 'B'; ch++) {
        generate(soFar + ch, maxLength);
      }
    }
  }
}
```

soFar **"A"**  maxLength **3**



# A Recursive Approach

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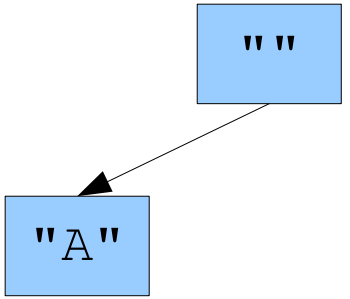
      if (soFar.length() < maxLength) {
        for (char ch = 'A'; ch <= 'B'; ch++) {
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

soFar **"AA"**  maxLength 3



# A Recursive Approach

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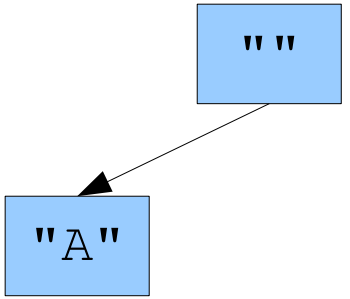
      if (soFar.length() < maxLength) {
        for (char ch = 'A'; ch <= 'B'; ch++) {
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

soFar **"AA"**  maxLength 3



# A Recursive Approach

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void generate(string soFar, int maxLength) {
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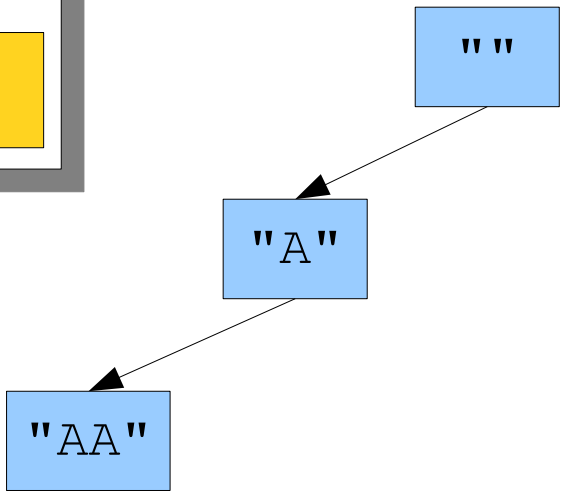
      doSomethingWith(soFar);
      if (soFar.length() < maxLength) {
        for (char ch = 'A'; ch <= 'B'; ch++) {
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

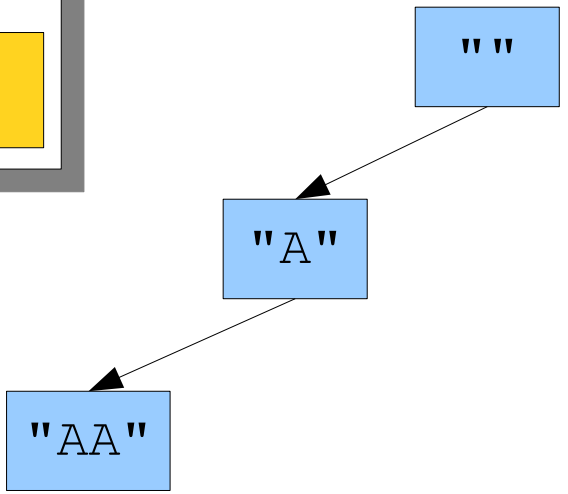
soFar **"AA"**  maxLength 3



# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  void generate(string soFar, int maxLength) {
    void generate(string soFar, int maxLength) {
      doSomethingWith(soFar);
      if (soFar.length() < maxLength) {
        for (char ch = 'A'; ch <= 'B'; ch++) {
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

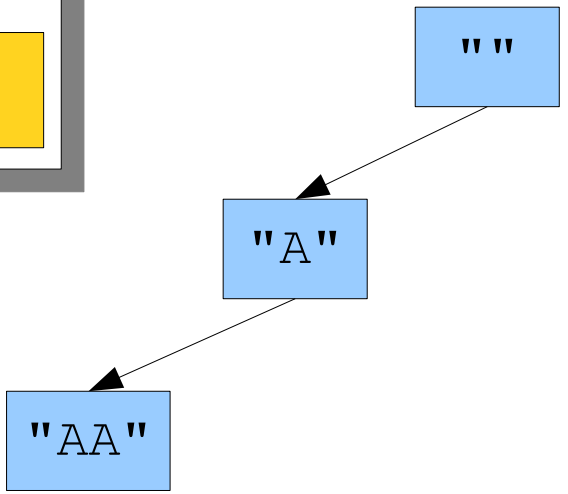
soFar **"AA"**  maxLength 3



# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  void generate(string soFar, int maxLength) {
    void generate(string soFar, int maxLength) {
      doSomethingWith(soFar);
      if (soFar.length() < maxLength) {
        for (char ch = 'A'; ch <= 'B'; ch++) {
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

soFar **"AA"**  maxLength 3



# A Recursive Approach

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void generate(string soFar, int maxLength) {
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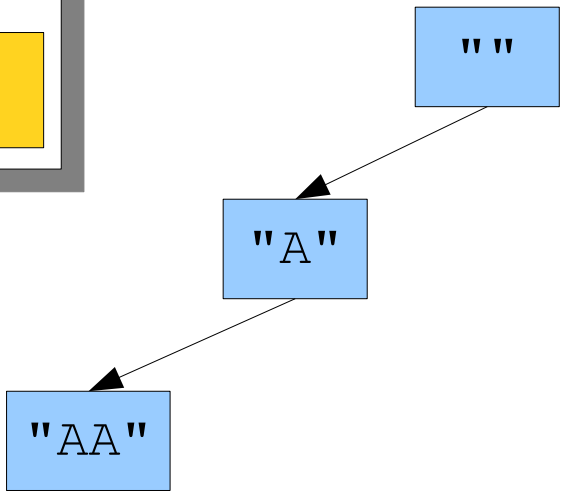
      if (soFar.length() < maxLength) {
        for (char ch = 'A'; ch <= 'B'; ch++) {
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

soFar **"AA"**  maxLength 3



# A Recursive Approach

```
void generate(string soFar, int maxLength) {
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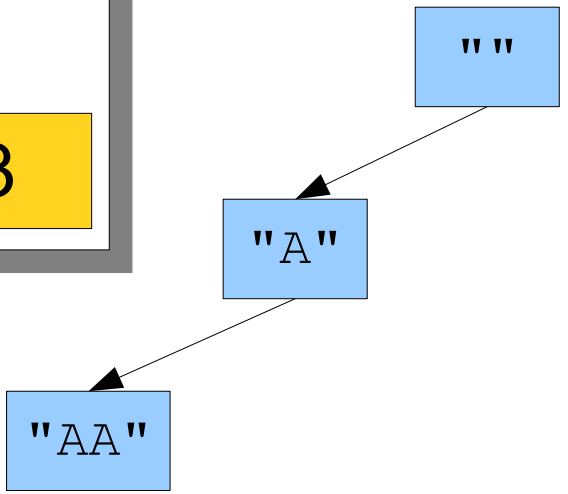
    void generate(string soFar, int maxLength) {
      void generate(string soFar, int maxLength) {
        doSomethingWith(soFar);
        if (soFar.length() < maxLength) {
          for (char ch = 'A'; ch <= 'B'; ch++) {
            generate(soFar + ch, maxLength);
          }
        }
      }
    }
  }
}
```

soFar **"AAA"**  maxLength 3



# A Recursive Approach

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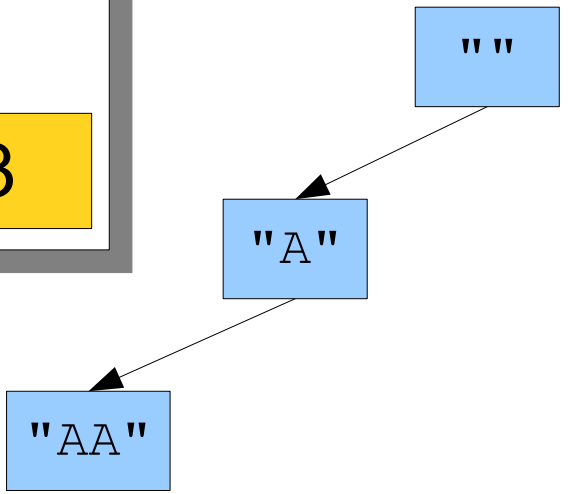
      void generate(string soFar, int maxLength) {
        doSomethingWith(soFar);
        if (soFar.length() < maxLength) {
          for (char ch = 'A'; ch <= 'B'; ch++) {
            generate(soFar + ch, maxLength);
          }
        }
      }
    }
  }
}
```

soFar **"AAA"**  maxLength 3



# A Recursive Approach

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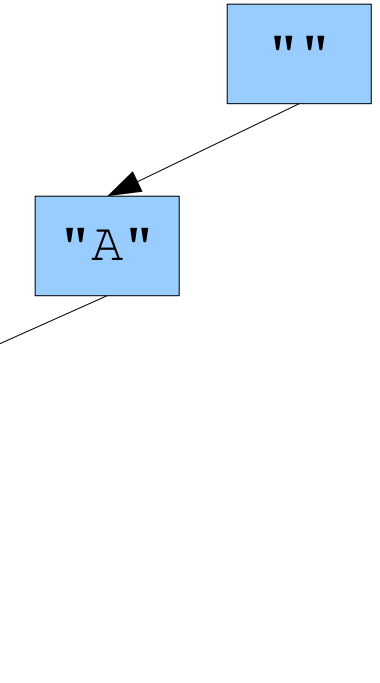
            generate(soFar + ch, maxLength);
          }
        }
      }
    }
  }
}
```

soFar **"AAA"**  maxLength 3



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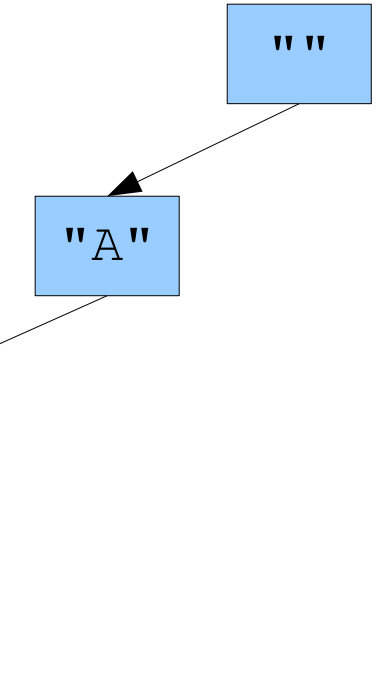
          }
        }
      }
    }
  }
}
```

soFar **"AAA"**  maxLength 3



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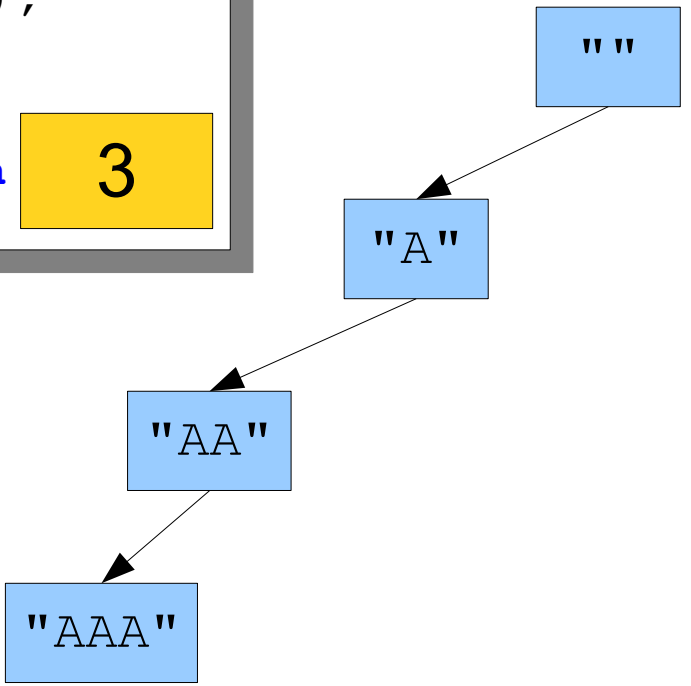
          }
        }
      }
    }
  }
}
```

soFar **"AAA"**  maxLength 3



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void generate(string soFar, int maxLength) {
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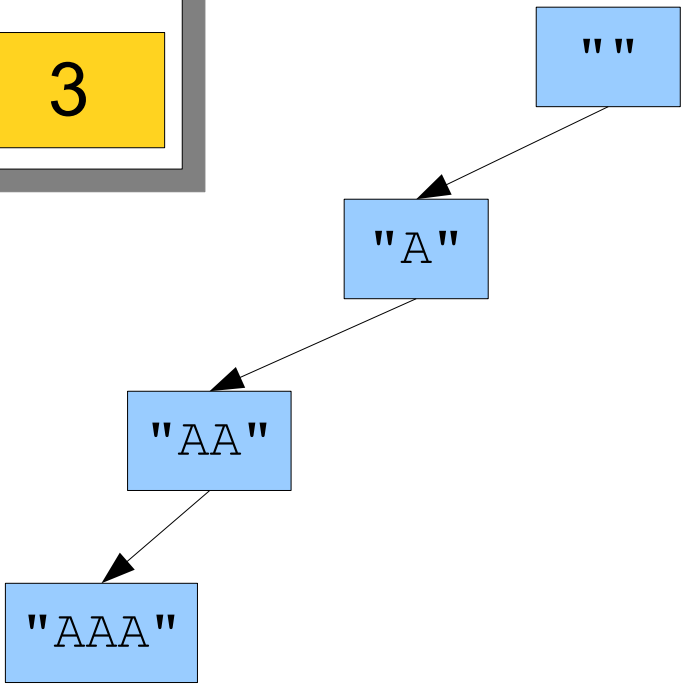
    void generate(string soFar, int maxLength) {
      doSomethingWith(soFar);
      if (soFar.length() < maxLength) {
        for (char ch = 'A'; ch <= 'B'; ch++) {
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

soFar **"AA"**  maxLength 3



# A Recursive Approach

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void generate(string soFar, int maxLength) {
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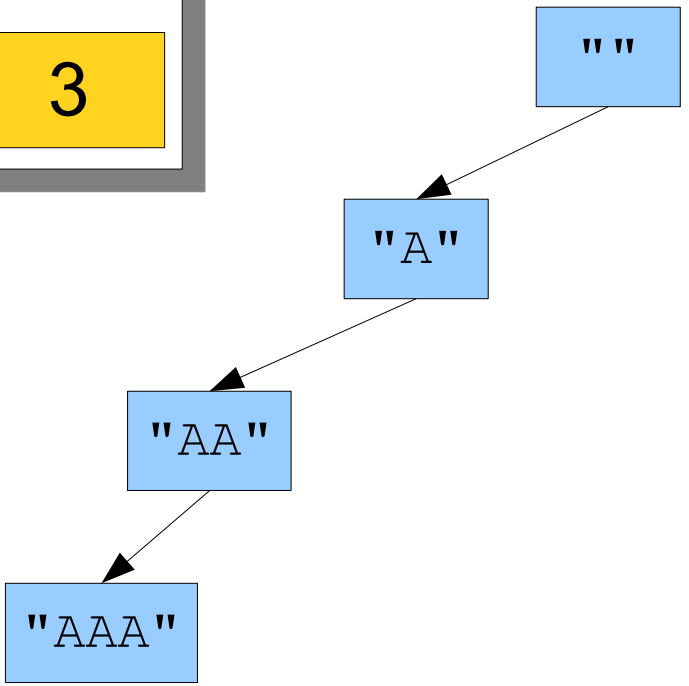
      doSomethingWith(soFar);
      if (soFar.length() < maxLength) {
        for (char ch = 'A'; ch <= 'B'; ch++) {
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

soFar **"AA"**  maxLength 3



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      doSomethingWith(soFar);
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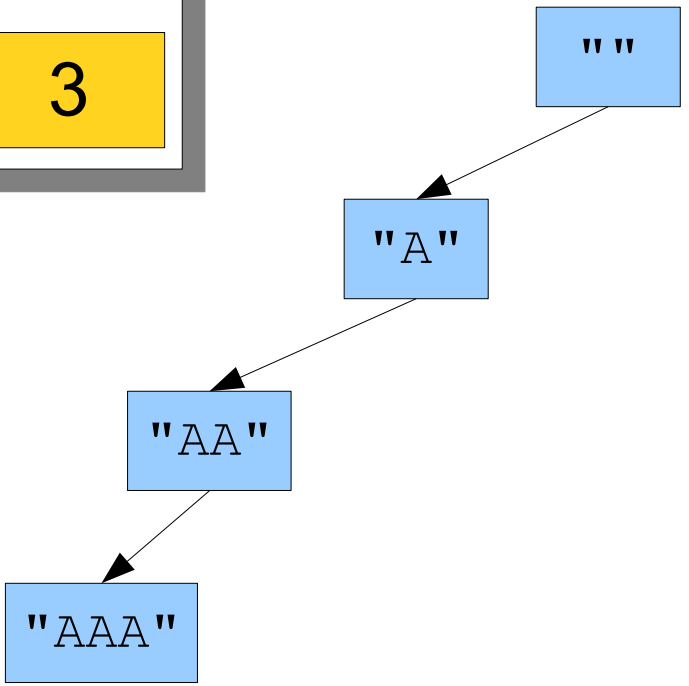
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

soFar **"AA"**  maxLength 3



# A Recursive Approach

```
void generate(string soFar, int maxLength) {
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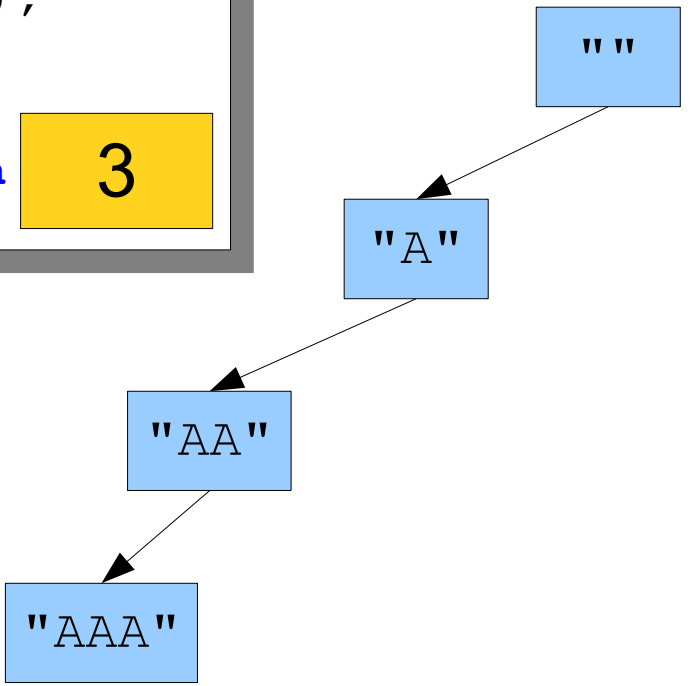
      void generate(string soFar, int maxLength) {
        doSomethingWith(soFar);
        if (soFar.length() < maxLength) {
          for (char ch = 'A'; ch <= 'B'; ch++) {
            generate(soFar + ch, maxLength);
          }
        }
      }
    }
  }
}
```

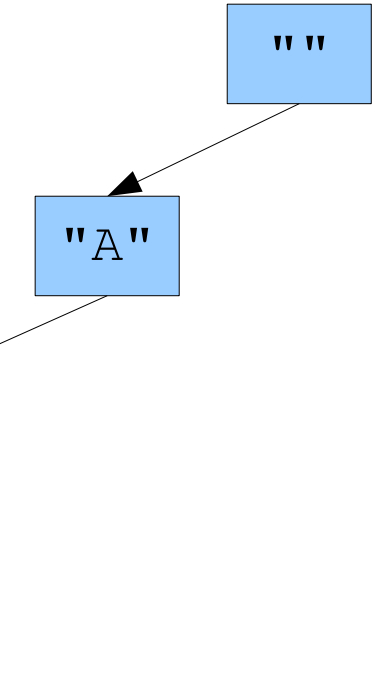
soFar **"AAB"**  maxLength 3



# A Recursive Approach

```
void generate(string soFar, int maxLength) {
  void generate(string soFar, int maxLength) {
    void generate(string soFar, int maxLength) {
      void generate(string soFar, int maxLength) {
        doSomethingWith(soFar);
        if (soFar.length() < maxLength) {
          for (char ch = 'A'; ch <= 'B'; ch++) {
            generate(soFar + ch, maxLength);
          }
        }
      }
    }
  }
}
```

soFar **"AAB"**  maxLength 3



# A Recursive Approach

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void generate(string soFar, int maxLength) {
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      void generate(string soFar, int maxLength) {
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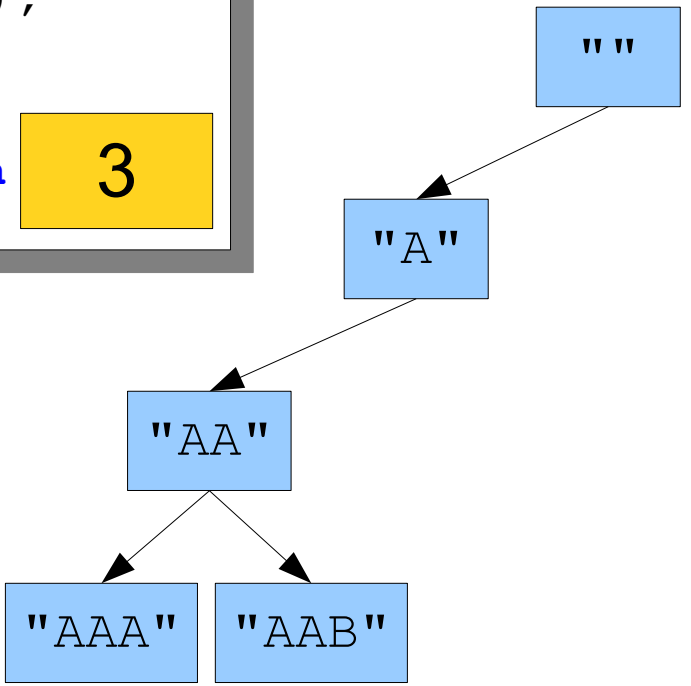
            generate(soFar + ch, maxLength);
          }
        }
      }
    }
  }
}
```

soFar **"AAB"**  maxLength 3



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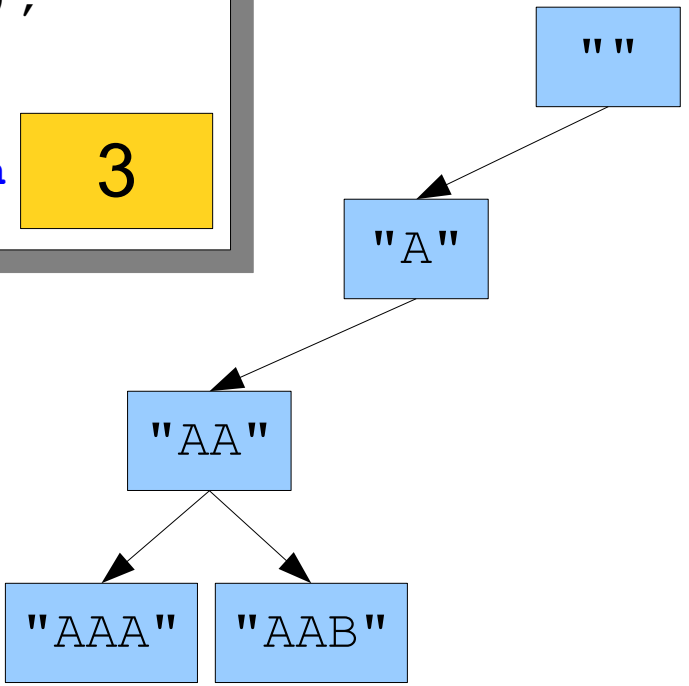
    }
  }
}
```

soFar **"AAB"**  maxLength 3



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        }
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  }
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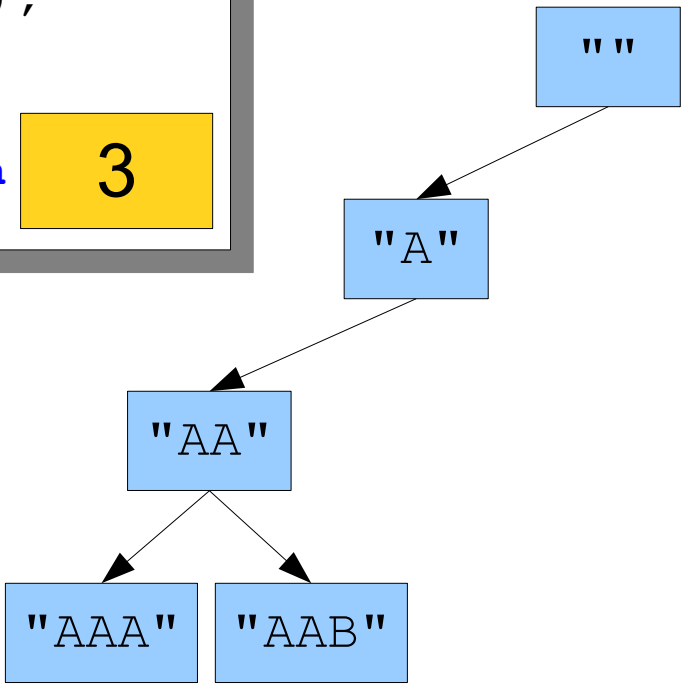
soFar **"AAB"**  maxLength 3



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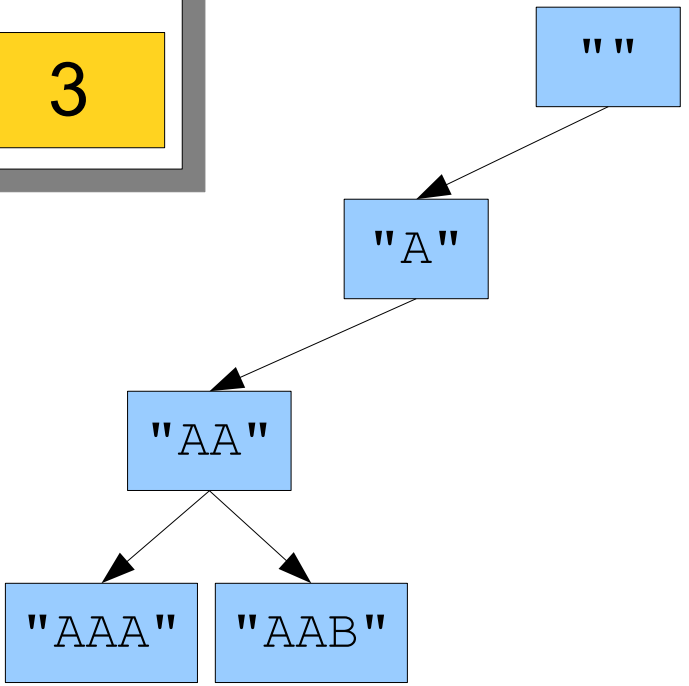
soFar **"AA"**  maxLength 3



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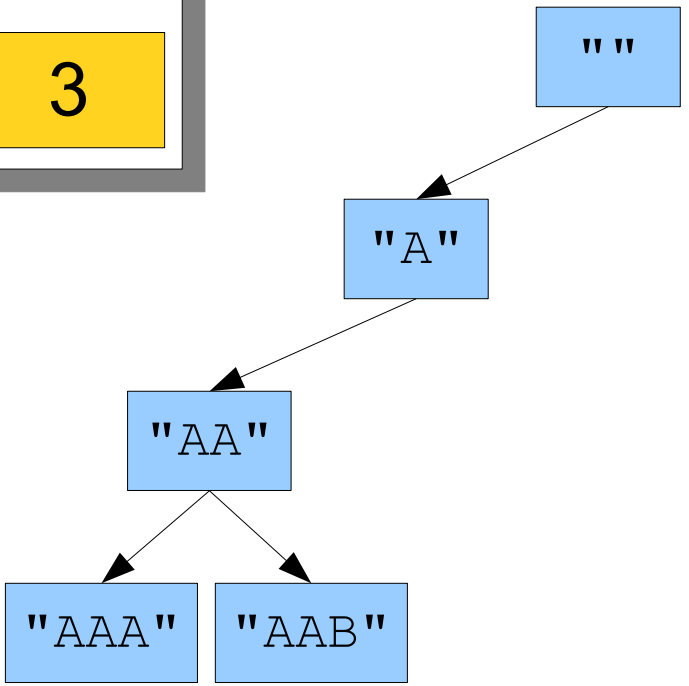
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

soFar **"AA"** maxLength 3



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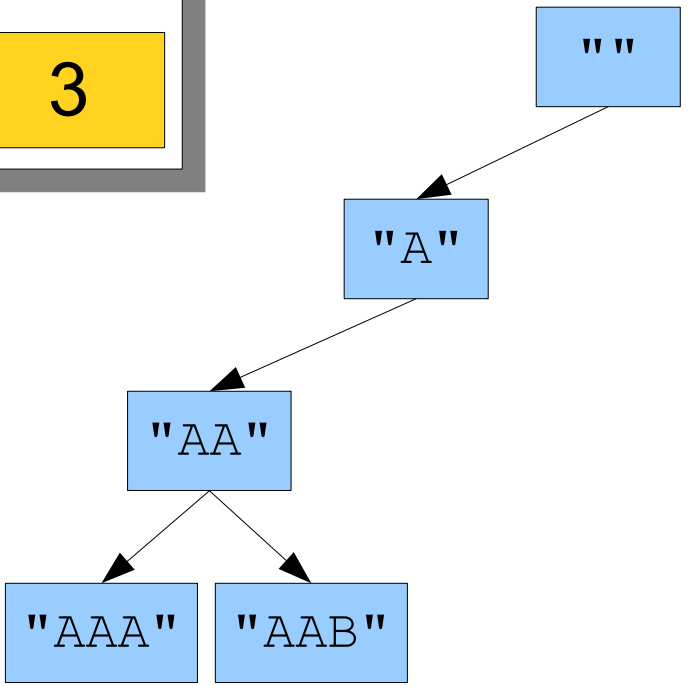
          generate(soFar + ch, maxLength);
        }
      }
    }
  }
}
```

soFar **"AA"** maxLength 3



# A Recursive Approach

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    if (soFar.length() < maxLength) {
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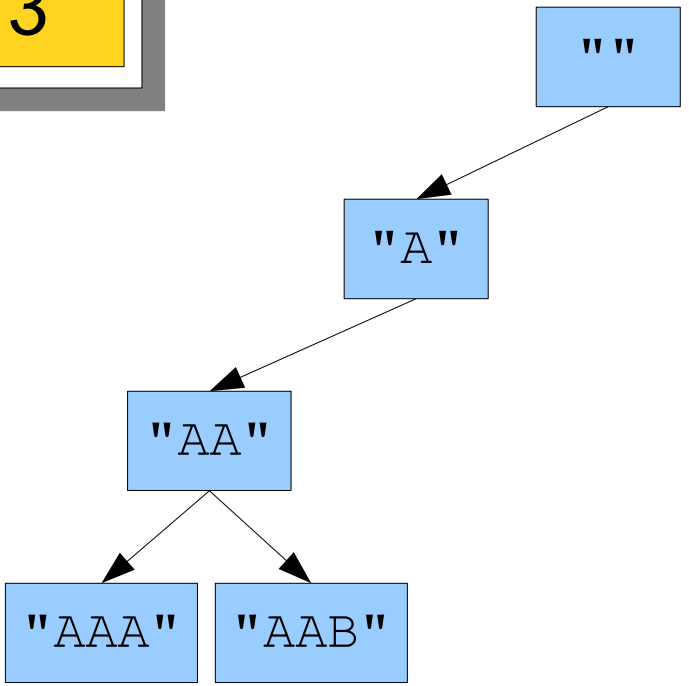
        generate(soFar + ch, maxLength);
      }
    }
  }
}
```

soFar **"A"**  maxLength 3



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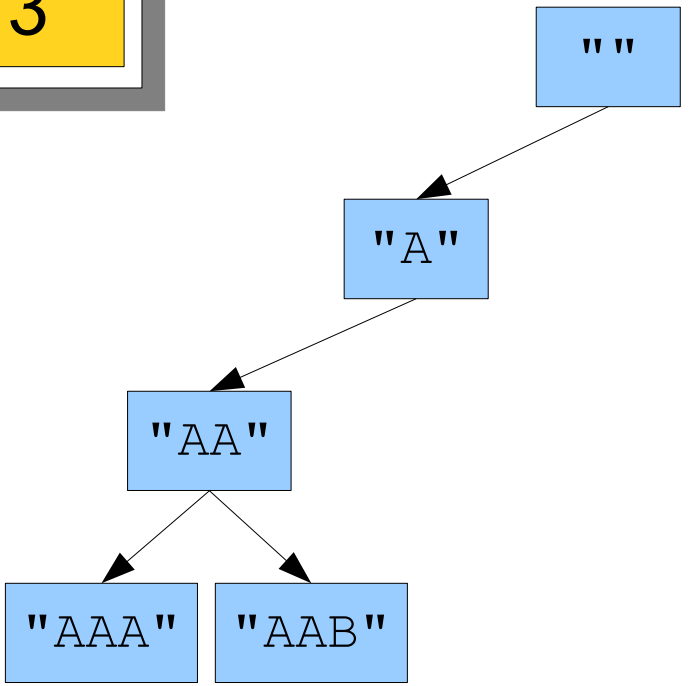
        generate(soFar + ch, maxLength);
      }
    }
  }
}
```

soFar **"A"**  maxLength **3**



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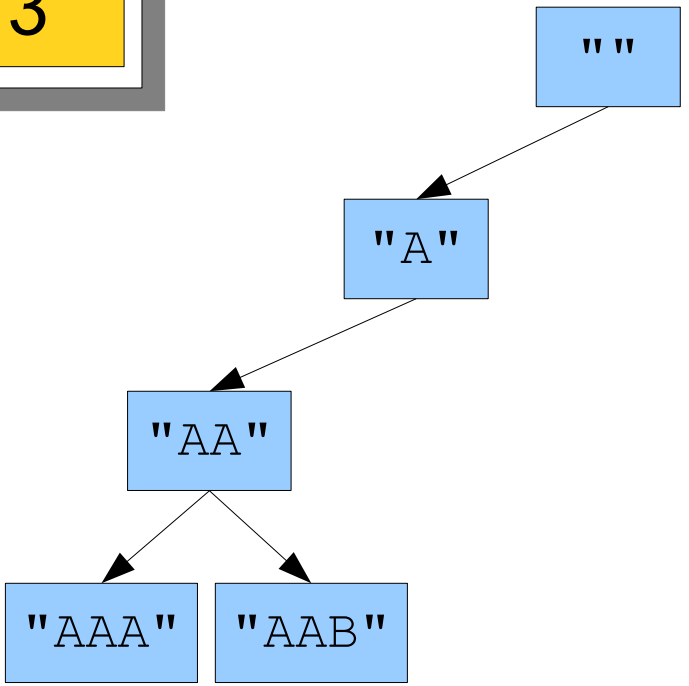
      }
    }
  }
}
```

soFar **"A"**  maxLength 3



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      }
    }
  }
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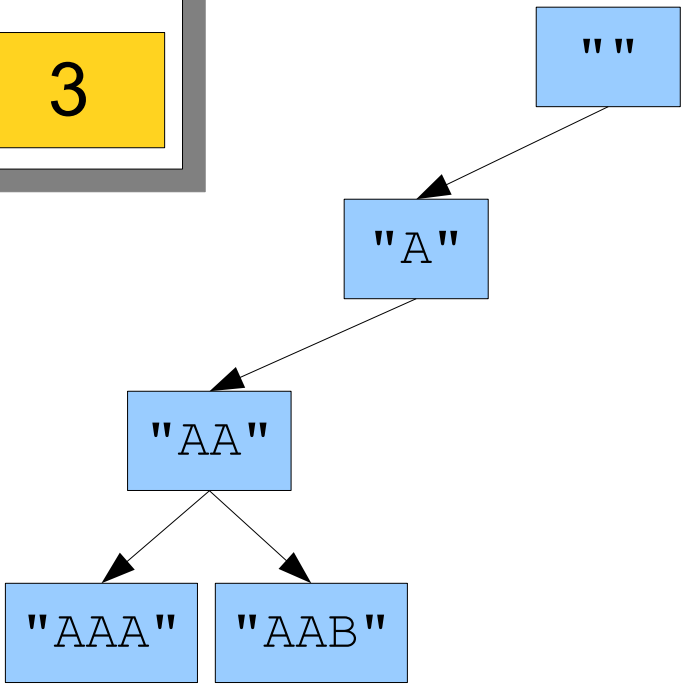
```

soFar **"AB"**  maxLength 3



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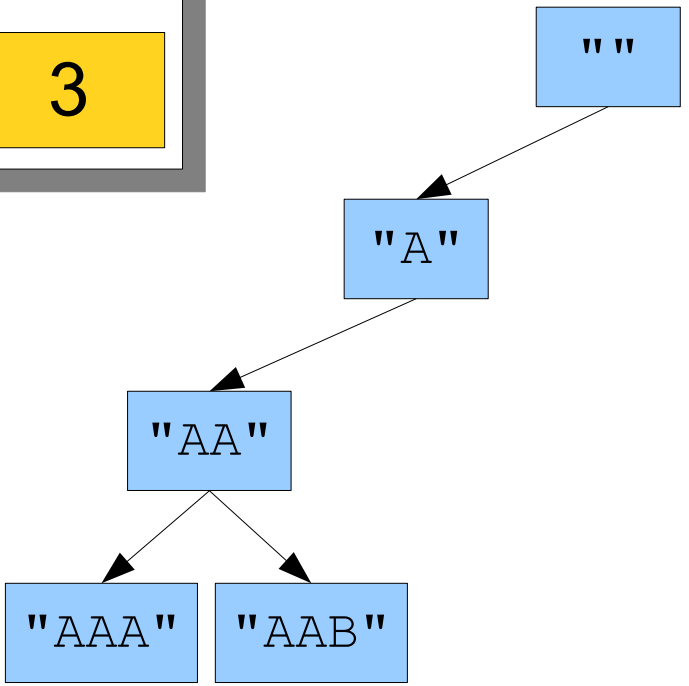
soFar **"AB"**  maxLength 3



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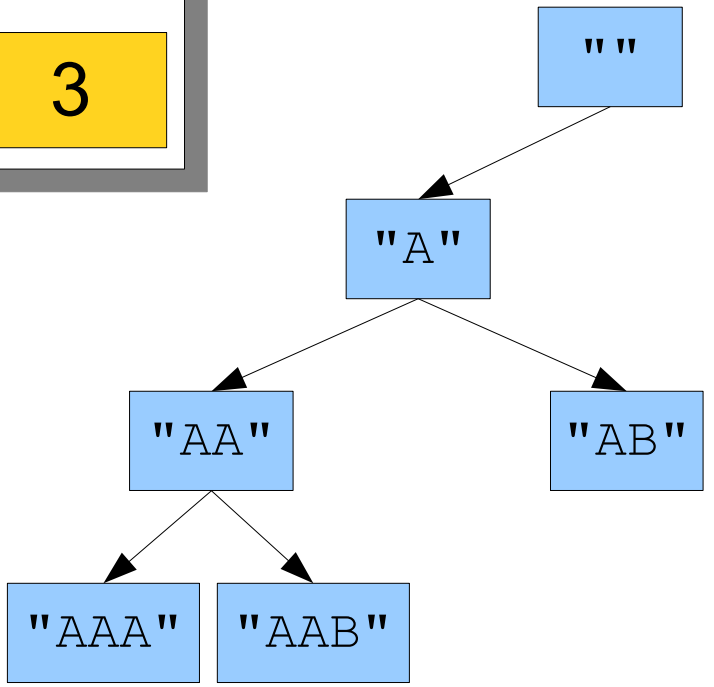
soFar **"AB"**  maxLength 3



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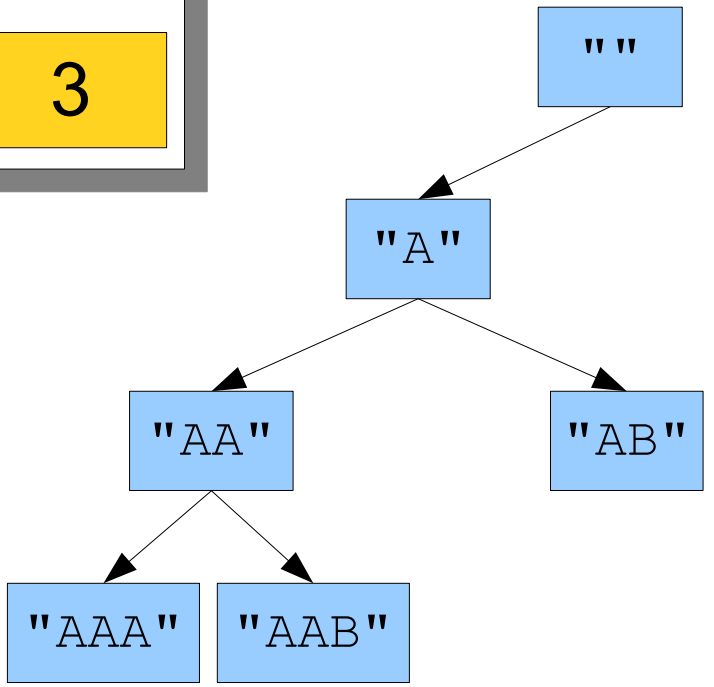
soFar **"AB"**  maxLength 3



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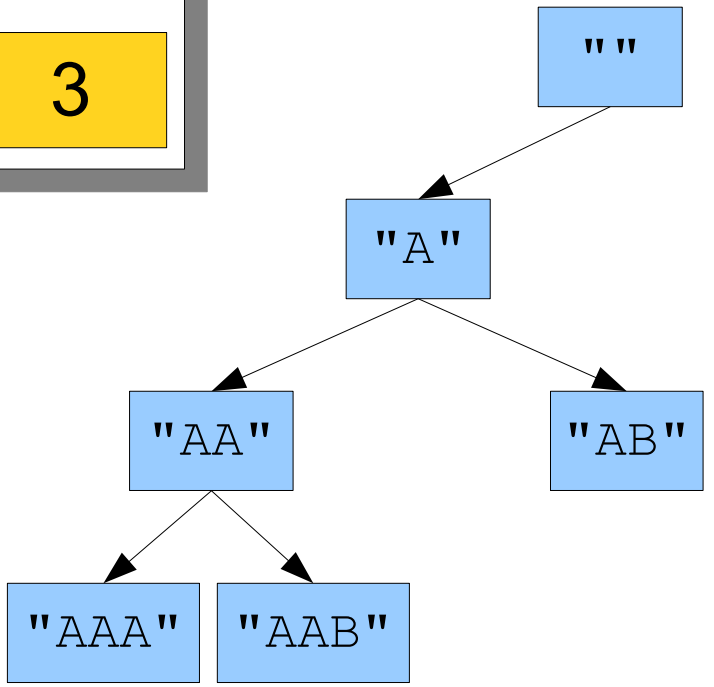
      }
    }
  }
}
```

soFar **"AB"**  maxLength **3**



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      }
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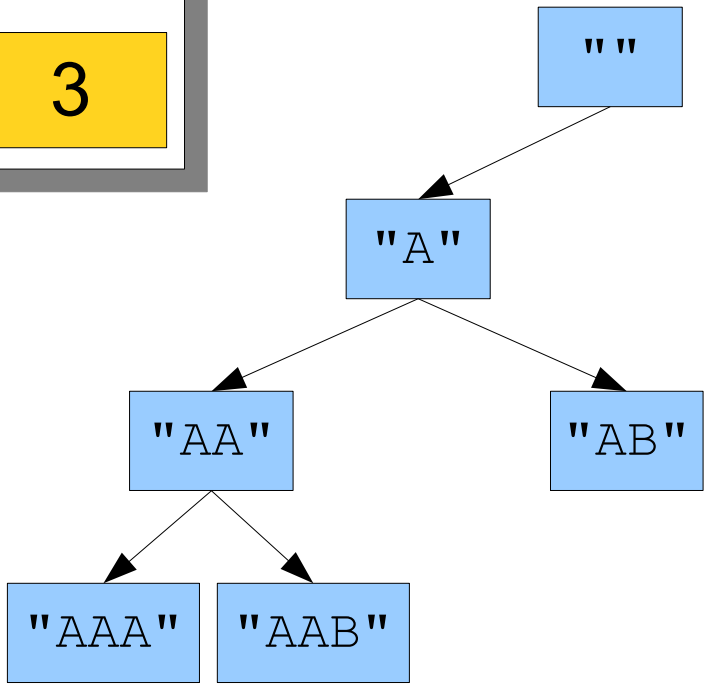
```

soFar **"AB"**  maxLength 3



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void generate(string soFar, int maxLength) {
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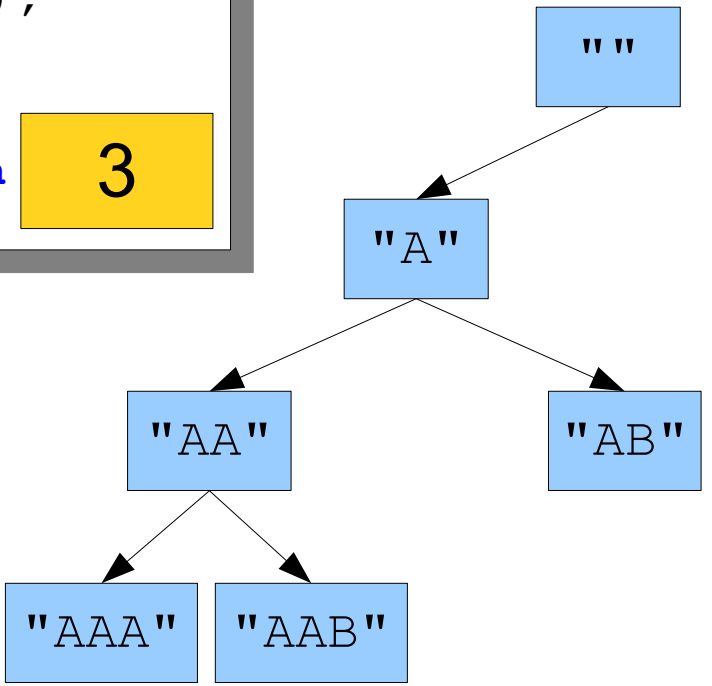
        }
      }
    }
  }
}
```

soFar **"ABA"**  maxLength 3



# A Recursive Approach

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    }
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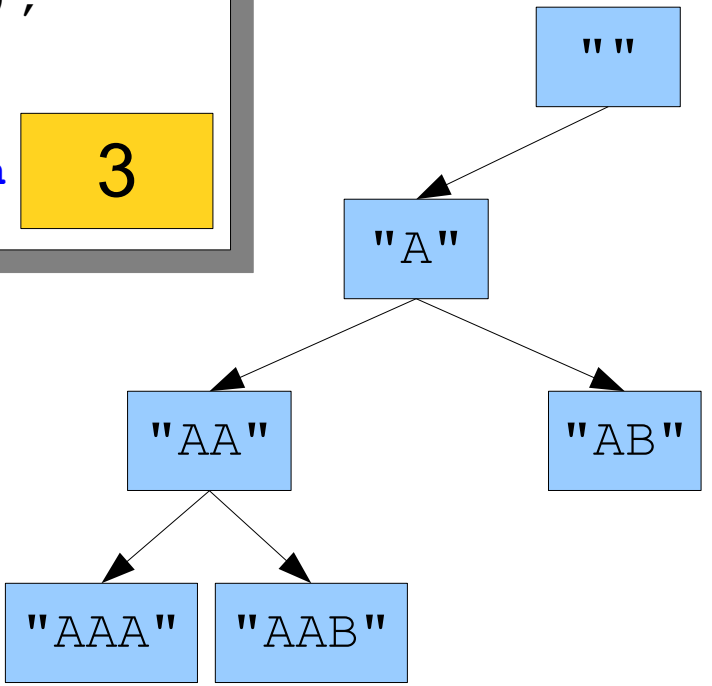
```

soFar **"ABA"**  maxLength 3



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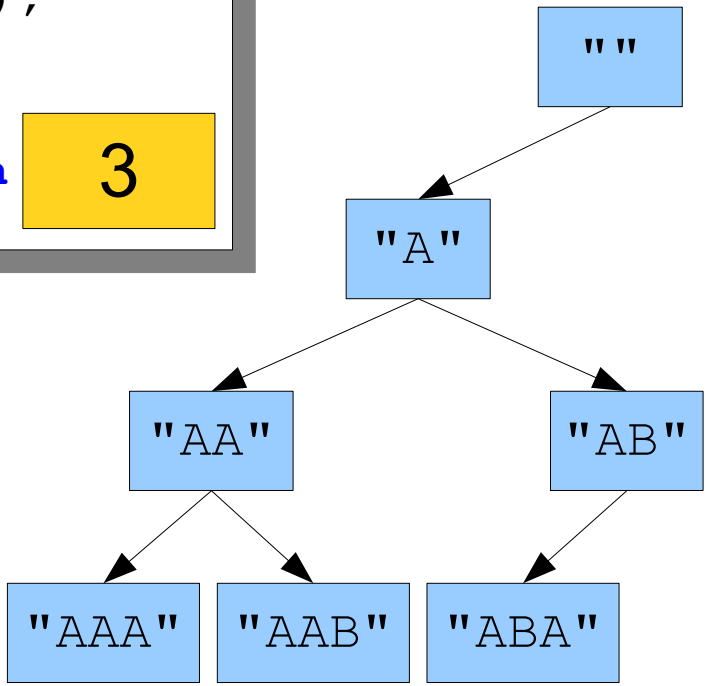

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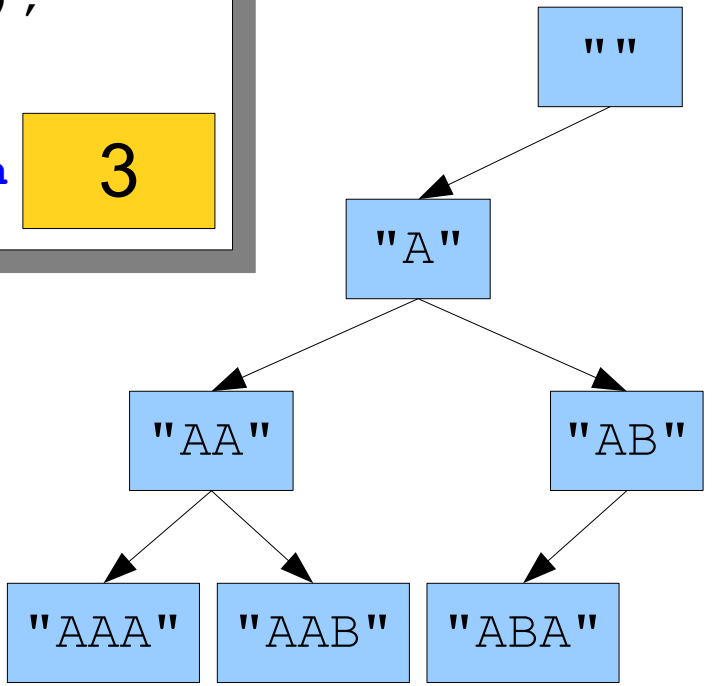

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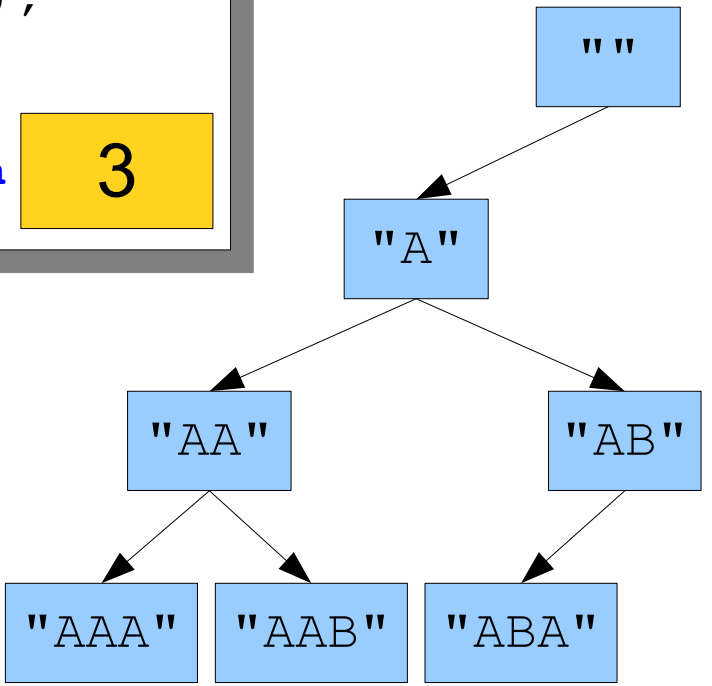

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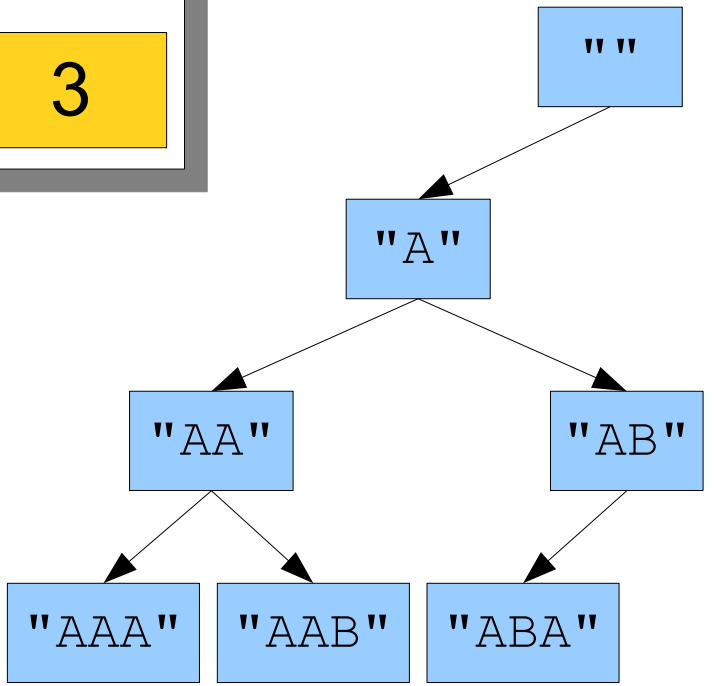

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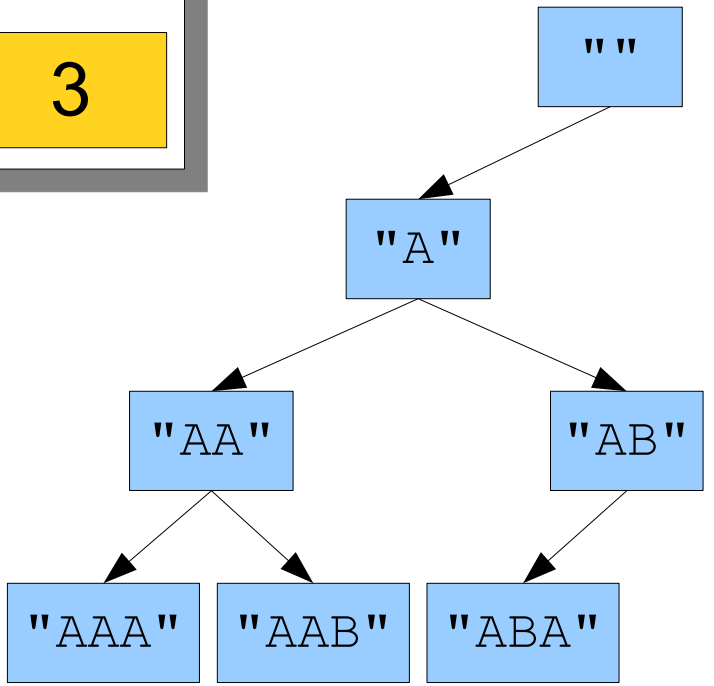

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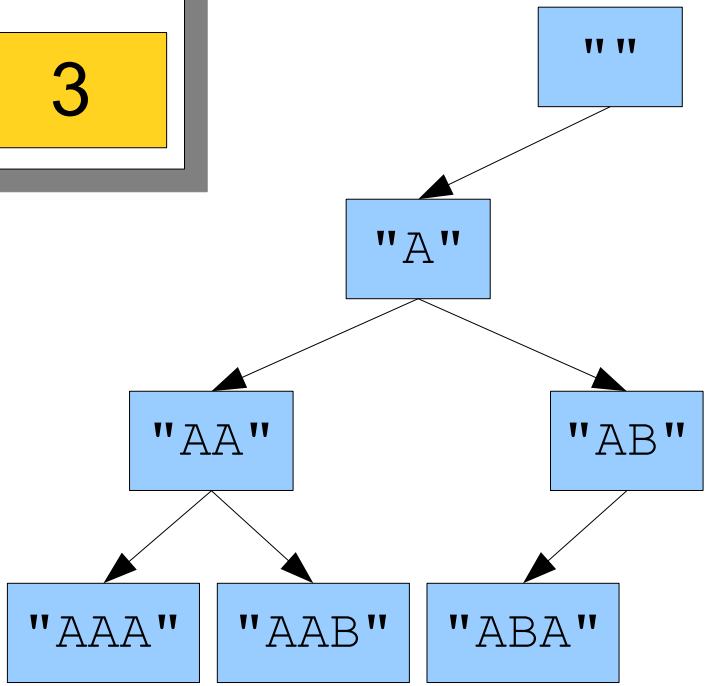

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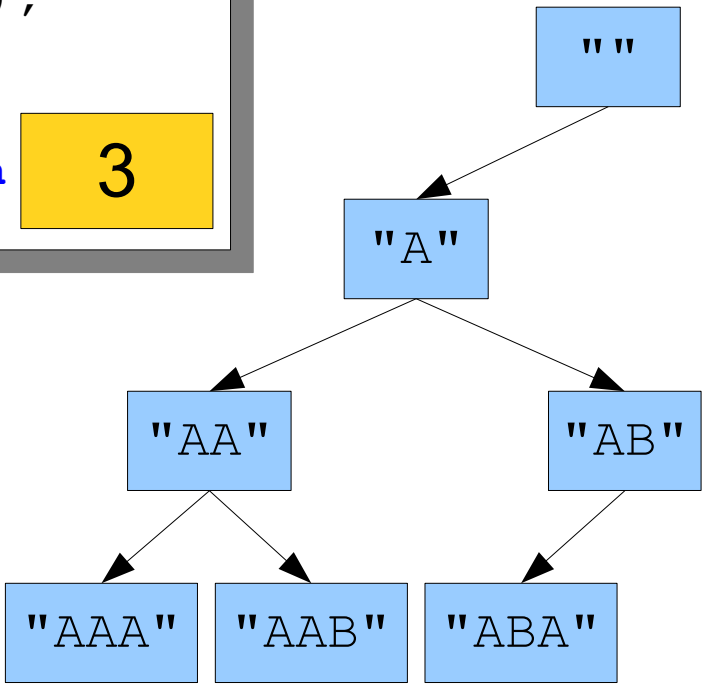
```

soFar **"ABB"**  maxLength **3**



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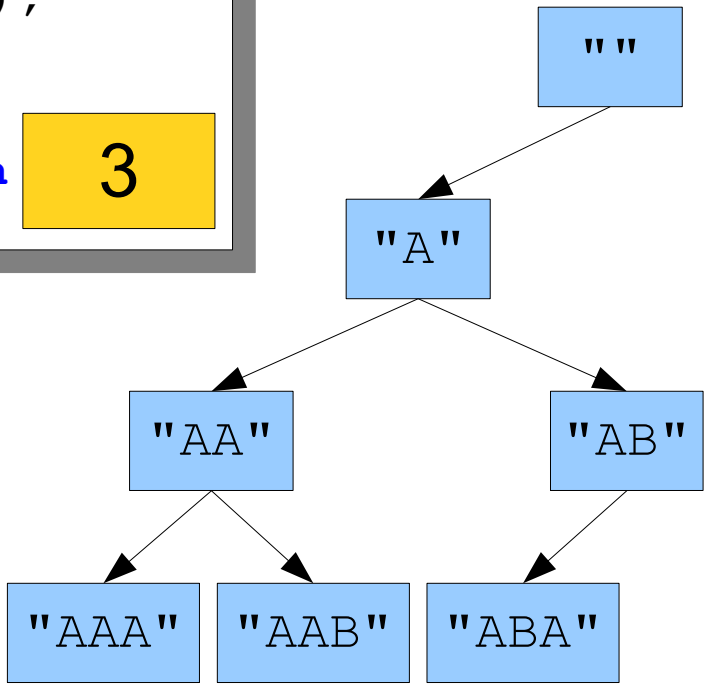
soFar **"ABB"**  maxLength **3**



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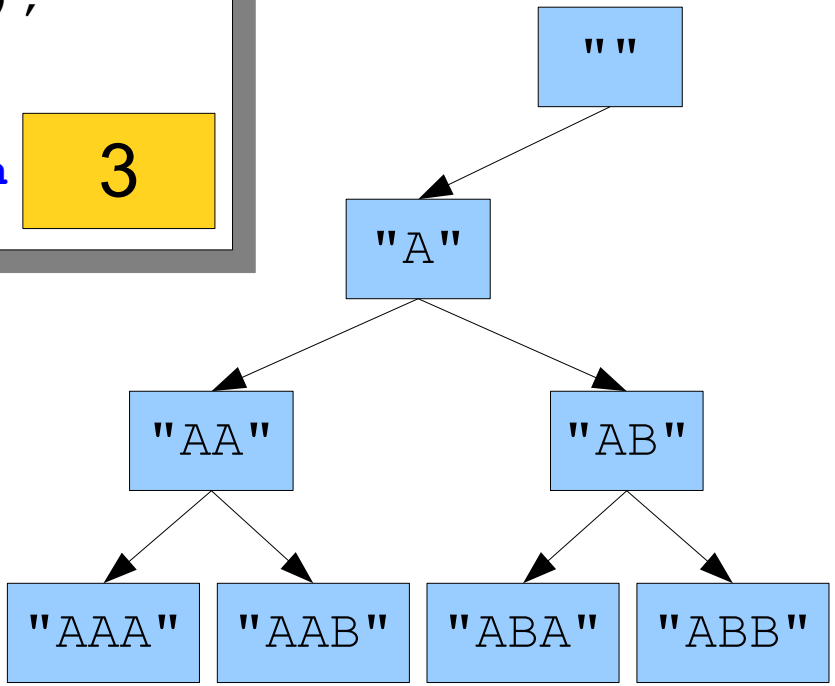

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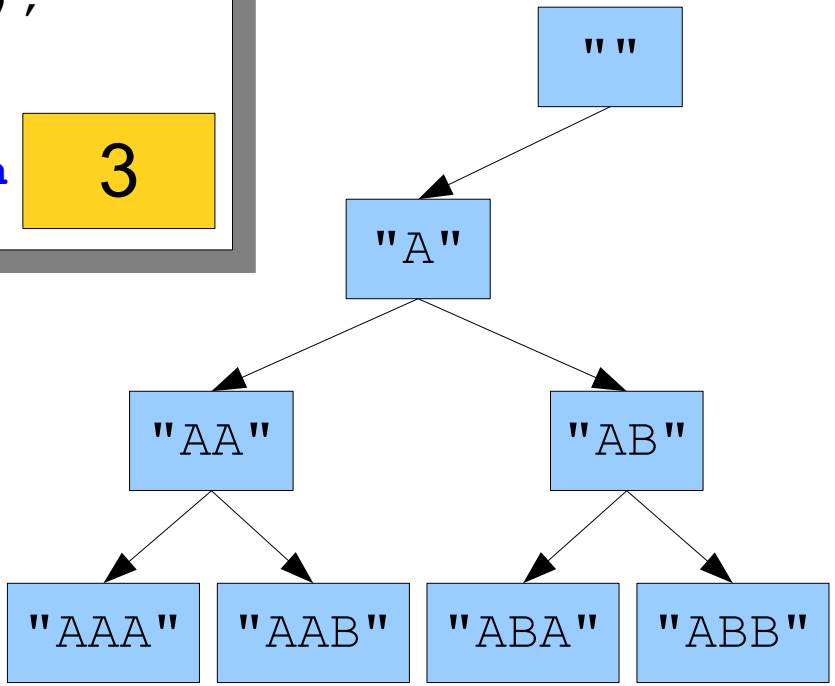
soFar **"ABB"**  maxLength **3**



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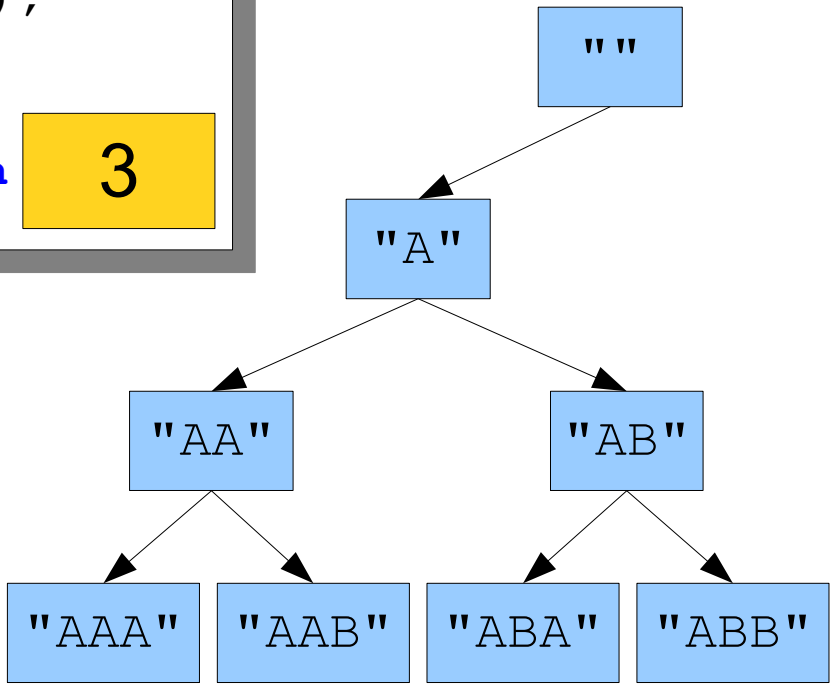

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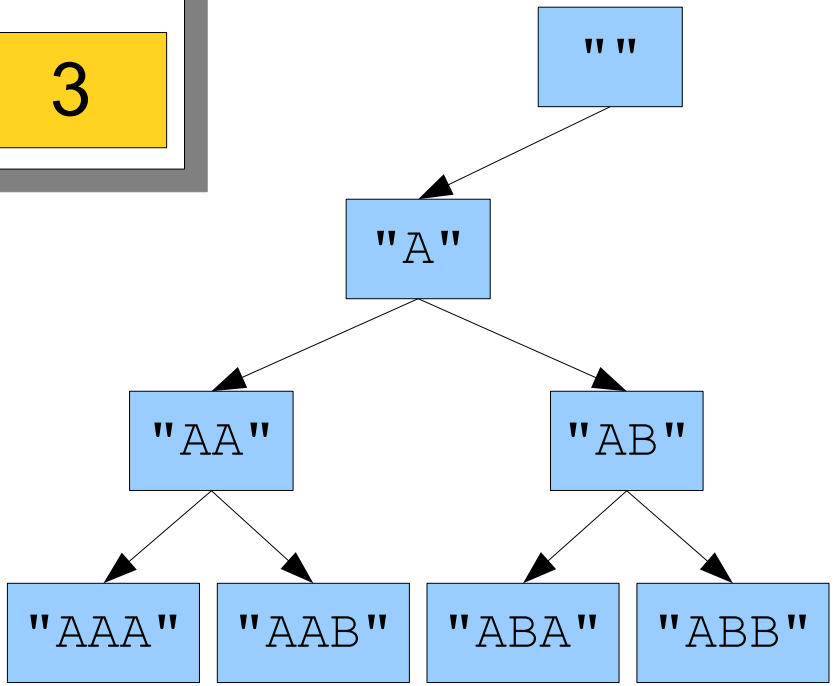
soFar **"AB"**  maxLength 3



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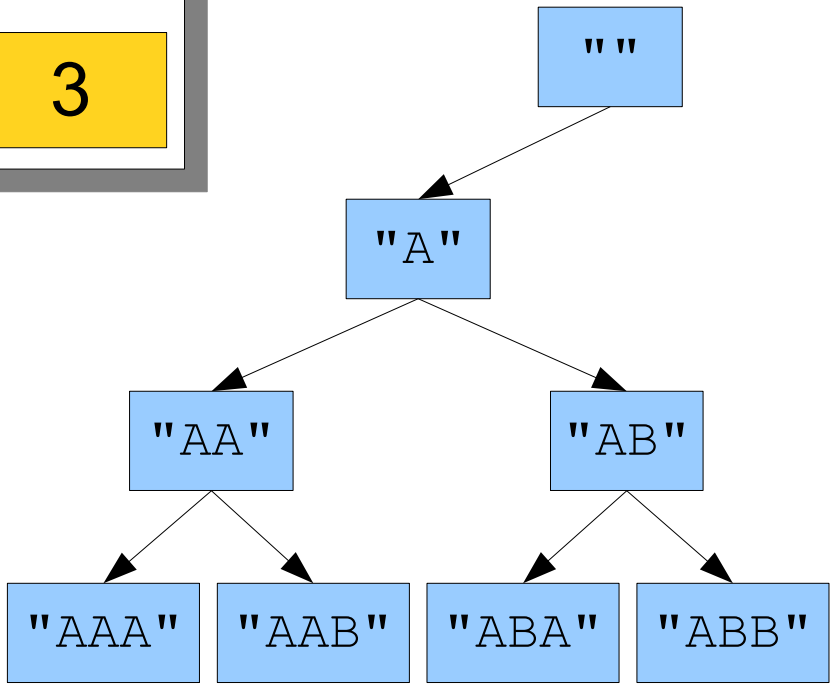
soFar **"AB"**  maxLength **3**



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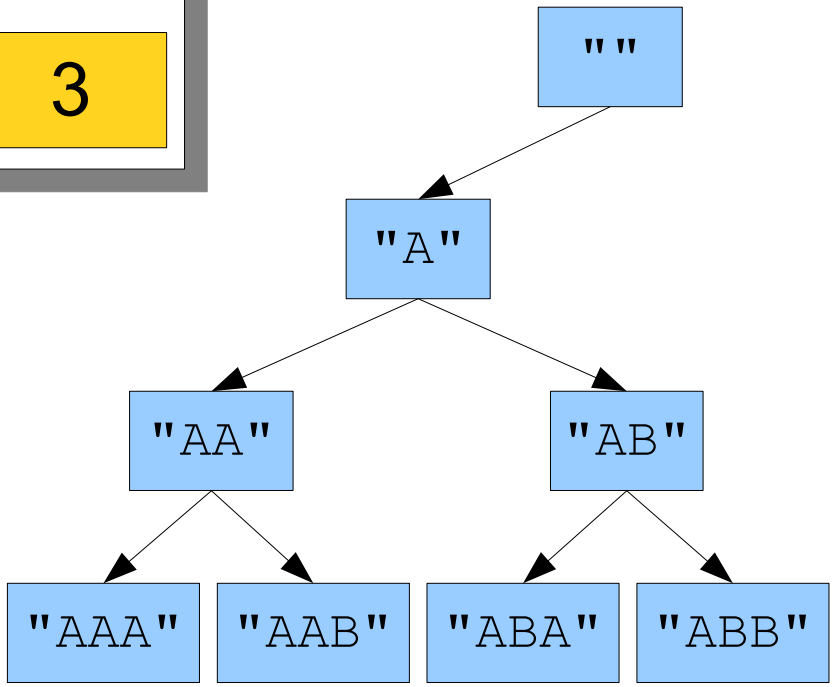
soFar **"AB"**  maxLength **3**



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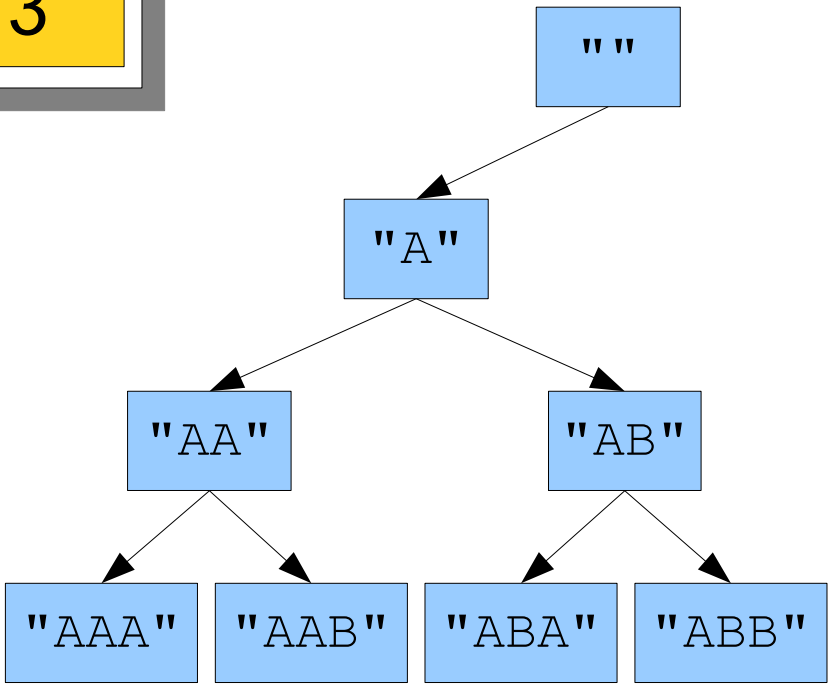
soFar **"A"**  maxLength 3



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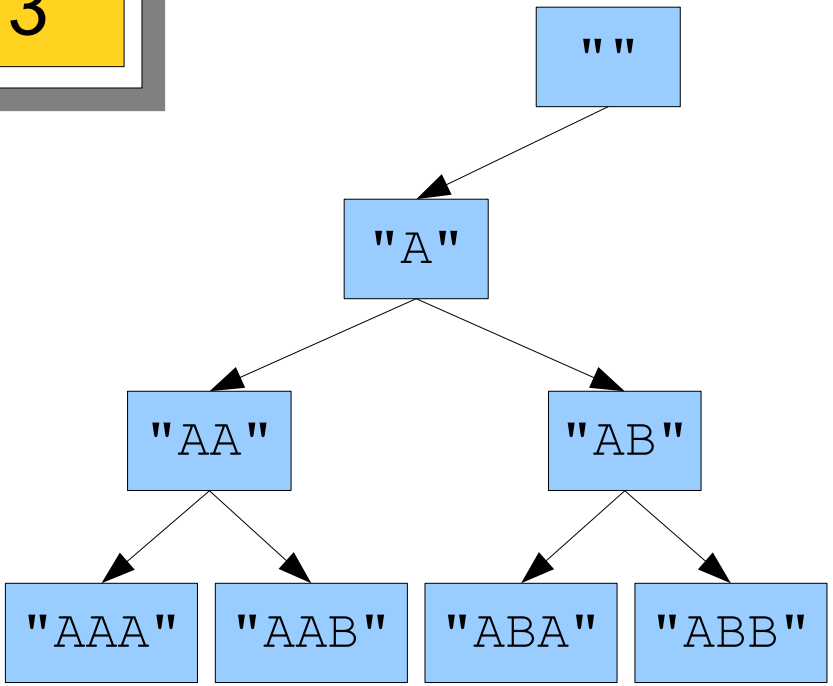
```

soFar **"A"**

maxLength 3



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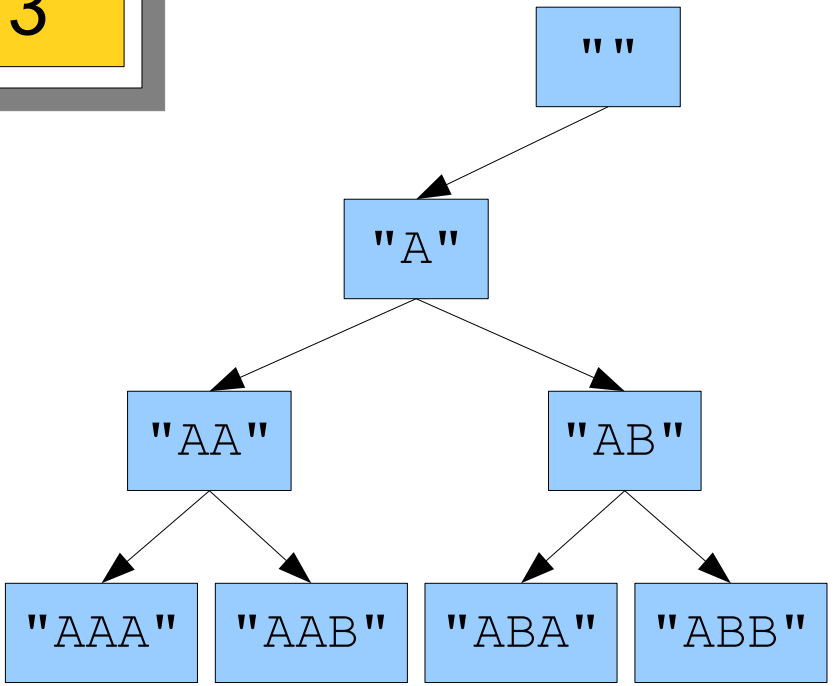
soFar **"A"**  maxLength 3



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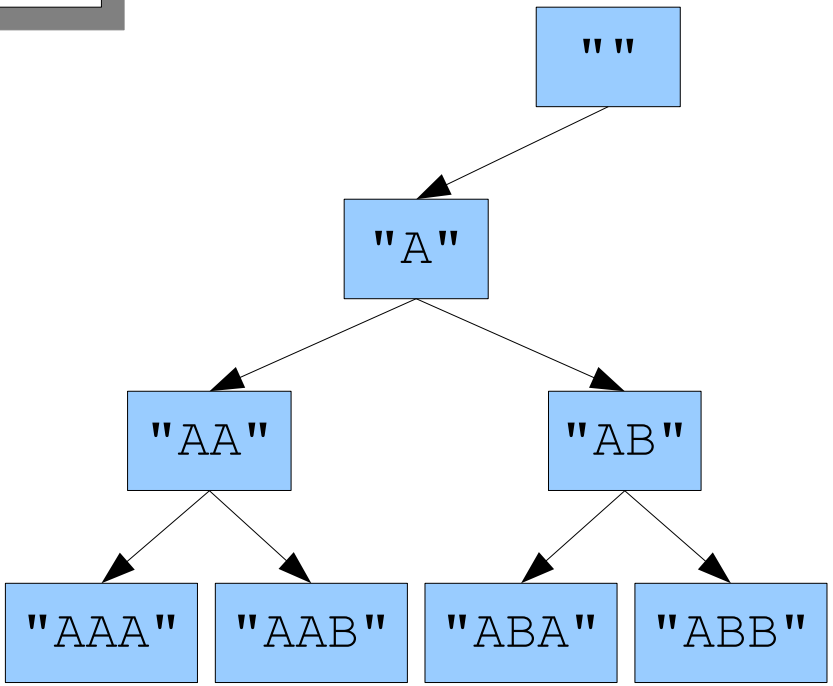
soFar `""`  maxLength `3`



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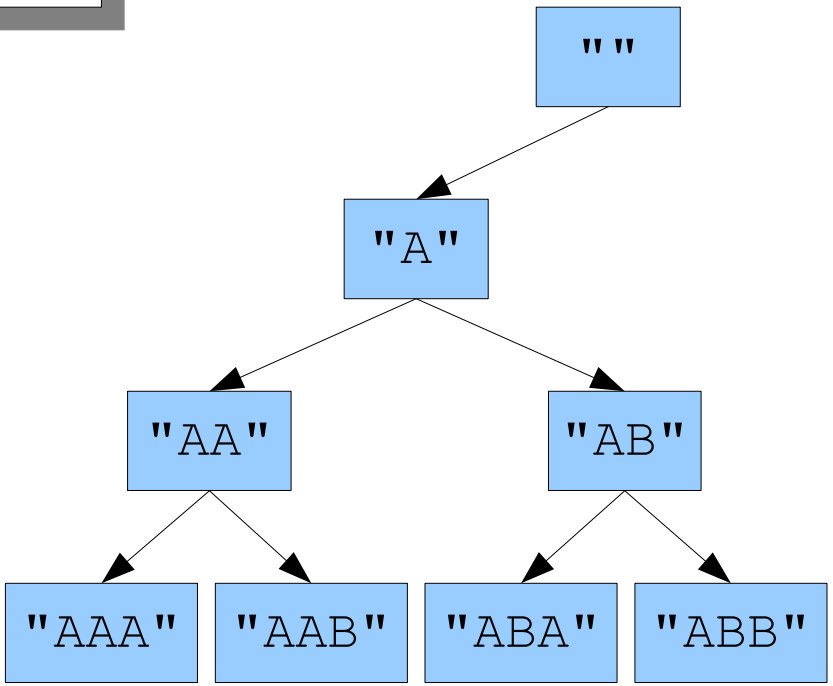

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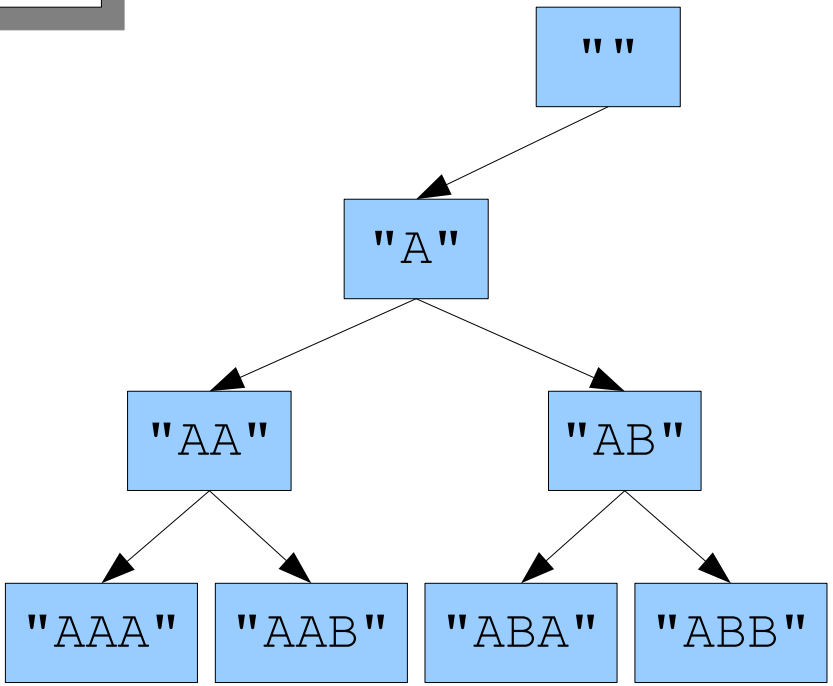

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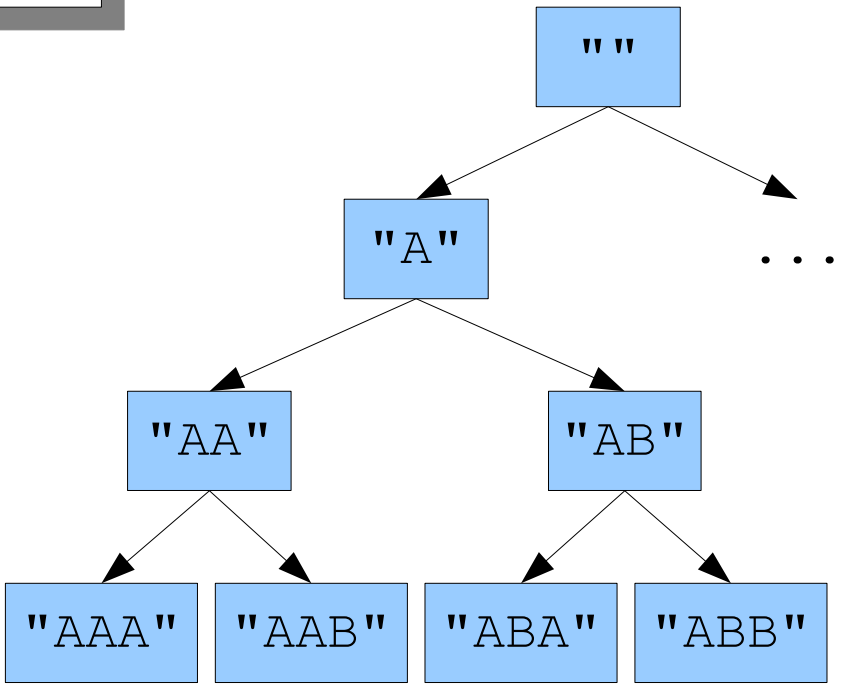

# Why This Matters

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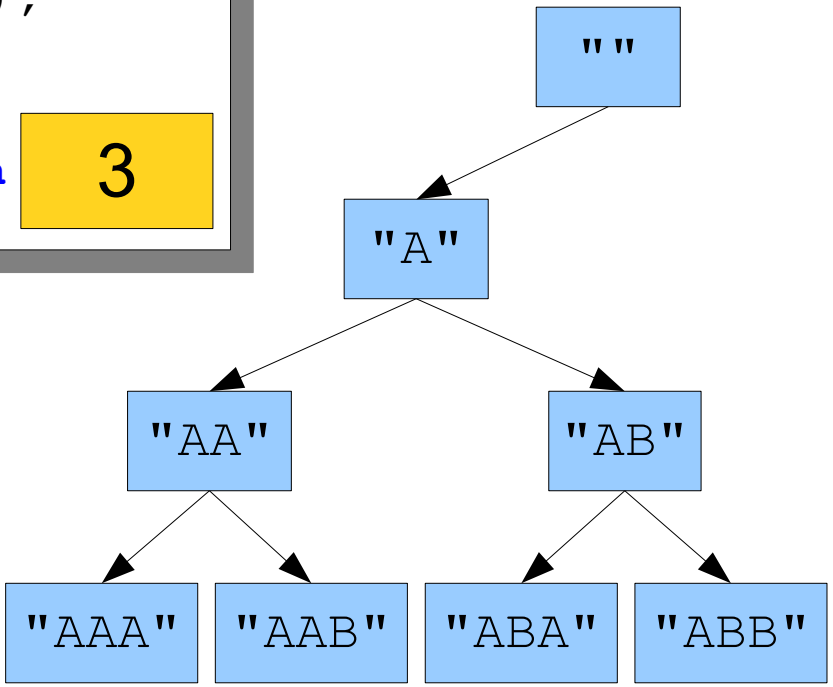
soFar **"ABB"**  maxLength 3



# A Comparison

- **`Queue`**-based approach:
  - Time: $\approx 26^{n+1}$
  - Space: $\approx 26^{n+1}$
- This algorithm is called ***breadth-first search*** (or just **BFS**).
- Although less useful here, BFS has many applications; you'll see one in Assignment 2.

- Recursive approach:
  - Time: $\approx 26^{n+1}$
  - Space: $\approx n$
- This algorithm is called ***depth-first search*** (or just **DFS**).
- Also quite useful; we'll see more applications later on.

# A Comparison

- Why would we *ever* use the `Queue`-based approach
- Remember the `Queue` approach visits strings *in order of length*
  - What if there are multiple answers to a question, and we want to find the shortest?
    - This is what you'll be doing for Assignment 2
  - What if know ahead of time that shorter answers are more likely?

# Dictionary Attacks

- Passwords are not random strings; people often use
  - words from the dictionary,
  - simple combinations of numbers,
  - their birthdays,
  - etc.
- A **dictionary attack** is an attack on a password system in which the attacker runs through a list of likely password candidates to try to break in.
- Watch how easy this is...

# `dictionary-attack.cpp`
# (Computer)

# `Vector` or `Queue`?

- Similar to the `Stack`, anytime we use a `Queue` we can use a `Vector` instead.
- What are the advantages of using a `Queue` instead of a `Vector`?
- We have the same two advantages as the Stack:
  - Easier to read
  - Protects against programmer mistakes
- Surprisingly, `Queue`s also have a speed advantage

queue-speed-test.cpp

# `Vector` or `Queue`?

- Why is the `Queue` so much faster?
  - Intuition: When removing to the beginning of the Vector all the existing elements need to "slide over"
  - *Somehow* the `Queue` doesn't do this. We'll find out how in a couple weeks.

# Next Time

- **Thinking Recursively**
  - How can you best solve problems using recursion?
  - What techniques are necessary to do so?
  - And what problems yield easily to a recursive solution?