Three-Dimensional Rotations of the Eye

Eye motility

In principle the eyeball, like any rigid object, has six degrees of freedom: three for rotation, and three for translation. The adult human eyeball is about 24–25mm in diameter, and can rotate about ±50° horizontally, 42° up and 48° down, and about ±30° torsionally. In contrast, the amount of translation possible for the eye is very limited: over the whole horizontal range the eyeball translates no more than 2 mm along the antero-posterior axis and 0.7 mm in the frontal plane. Given the limited translational excursion, the globe can be treated as a spherical joint capable of rotating around its fixed center, and its motion has thus only three degrees of freedom.

Eye movements are easy to measure, in both the laboratory and the clinic. Technology for eye movement recording has been evolving for over 100 years, and although none are without their disadvantages, it is easy to find a commercial product to suit most needs (Box 8.1). Binocular recordings are especially important, because when both eyes fail to point in the same direction double vision (diplopia) results. Pathological conditions thus give rise to strabismus, the misalignment of the two gaze axes. This is commonly measured in the clinic with a Hess chart (Box 8.2), which gives a static measure of the misalignment. Binocular recordings can give both static and dynamic accounts of the misalignment as gaze angle changes.

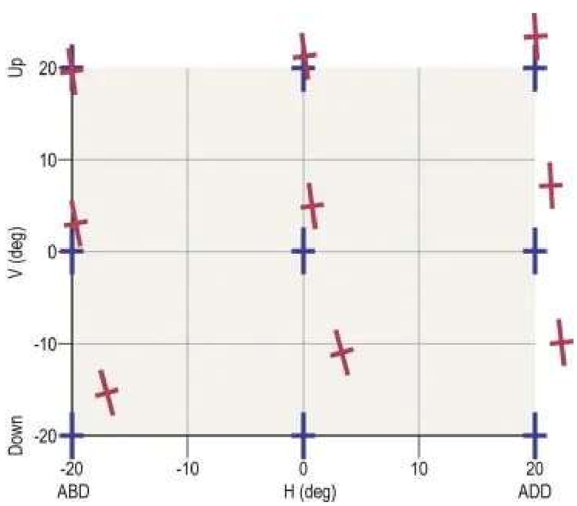
Box 8.1
Clinical eye movement recording methods

Axes	Range	Bandwidth	Resolution	Advantages	Disadvantages
Electro-oculography (EOG)					
H, V	• ~ ±25° H • ~ ±15° V • Not good for small movements (<5°)	~ 30 Hz	~0.5°	Easy to use, low cost	• 1, 2, 3, 4. • EMG from scalp or jaw muscles can interfere
Infrared Reflection Device (IRD)					
H, V	• ~ ±30° H • ~ ±20° V	~ 100 Hz	~ 0.02°	Easy to set up, moderate cost. Available for MRI, but limited range	• 1, 2, 5. • Risk of ocular drying from IR source
Video-oculography (VOG)					
H, V, T	• ~ ±30° H, +30° up, • −45 down, less for T	~ 25 Hz – 1 kHz H, V, but less for T	~ 0.05° H, V, ~ 0.1° T	Good for clinical use, good for children and infants, available for MEG & MRI	• 1, 2, 5, 6. • Limited range, high cost
Magnetic Field Scleral Coil					
H, V, T	• > ±45° in monkeys, but • ~ ±30° in people because	>1 KHz	~ 0.01°	Coil can be permanently implanted in animals	• 6, 7. • Contact lens causes discomfort, so

Axes	Range	Bandwidth	Resolution	Advantages	Disadvantages
	of eye coil limitations				inappropriate for children. High cost

- 1
  Eye closure or eye lid artifacts.
- 2
  Blink artifacts.
- 3
  Poor linear range.
- 4
  Signal drifts over time.
- 5
  Sensitive to translation of head relative to device.
- 6
  Torsion bandwidth and/or range reduced.
- 7
  Limited recording time (~30 min) for people.

Box 8.2
3-D Effects of Muscle Weakness



3-D Hess chart of ocular alignment following unilateral trochlear (IVth) nerve damage. The denervated superior oblique muscle can only develop a passive force. The graph shows the effects of a superior oblique palsy (SOP) in a monkey. These measurements were made with 3-D eye coils in each eye. The normal eye views the target, but the palsied eye is covered. The monkey follows a light as it jumps to each of the nine target positions. The blue cross shows the attitude of the normal eye. The red cross shows the attitude of the palsied eye. Note that as the animal looks down in adduction (the main effect of the SO muscle), the deficit worsens. When the deviation between the two eyes is not constant with gaze angle, it is called an *incomitant strabismus* .

Note that the error is not only a displacement of the cross (up and out), but also includes a CCW twist. Similar recordings can be made in human patients with eye coils on a contact lens. In a lower technology method subjects wear (red or blue) colored filters over each eye. A blue cross is projected onto the screen, and is seen only by the normal eye. A red cross is also projected, which can only be seen by the affected eye. The subject moves and twists the red cross until it is aligned, perceptually, with the blue cross.

Quantifying eye rotations

Describing translations is simple and intuitive. Once three orthogonal axes (e.g. Cartesian coordinates) are defined, a translation of a rigid object can be specified by simply providing the amount of translation of any one point of the object along each of the three axes. Importantly, the final position reached is independent of the order in which the three axes are considered (e.g., *x* -axis followed by *y* -axis movement yields the same position as *y* followed by *x,* i.e. translations are *commutative*). This happens because the familiar Euclidean space of translations is flat.

In contrast, rotations and their resulting orientations can not be described by any simple (i.e. intuitive) set of three coordinates. One of the fundamental reasons for this complexity is that the space of all rotations is curved. This can be easily noted by considering that if one keeps rotating an object around the same axis it will eventually (after 360°) get back to its initial orientation. One of the implications of this feature is that the final orientation, or attitude, reached after a sequence of rotations around different axes depends on their order. In the two panels of Figure 8.1 a camera, starting from the same initial orientation (left column), is rotated around the same pair of axes (arrows in the figure), but in different order. Clearly, the final orientation (right column) is different for the two sequences of rotations; unlike translations, rotations are *non-commutative.*

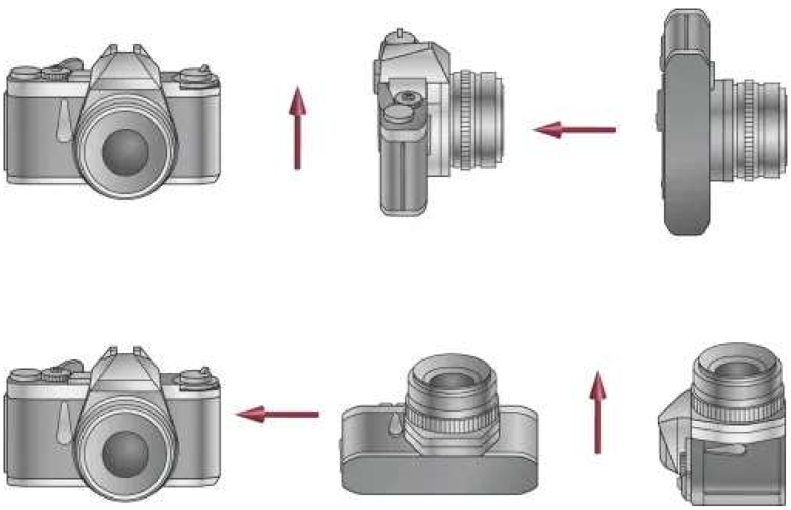


Figure 8.1
Non-commutativity of rotations. The image on the right of each arrow is obtained by rotating the image on its left around an axis collinear with the arrow. (**A**) The camera first rotates 90° around a vertical axis, and then 90° around a horizontal axis. (**B**) The order of rotations is reversed. The final orientation of the camera is clearly different in the two cases.
(Redrawn from Quaia C, Optican LM. Commutative saccadic generator is sufficient to control a 3-D ocular plant with pulleys. J Neurophysiol 1998 Jun; 79(6):3197–3215. Used with permission.)

To address the inherent complexity of rotations, many mathematical tools have been developed over the last 150 years, such as quaternions, sequences of rotations, rotation matrices, rotation vectors, spinors, rotors, motors, etc. Although all these methods must be equivalent (they all describe the same rotations), each method has both advantages and disadvantages in different applications. Accordingly, we can choose the mathematical formalism that best facilitates our thinking about eye rotations.

The first issue that must be addressed when describing rotations or orientations of the eye is the selection of three orthogonal axes. Obviously, they must pass through the center of the eye, but other than that there are few constraints. Of course, keeping the axes fixed in space, so that they point in the same spatial direction regardless of the movements of the eye or the head, would not be very useful, as the six extraocular muscles rotate the eye relative to the head. One reasonable arrangement is to fix them to the eye, so that their spatial orientation changes whenever either the eye or the head move. Alternatively, they could be fixed to the head, so that their orientation in space would change only when the head moves. These two possibilities might sound radically different, but they are tightly connected if the head is fixed: starting from an initial orientation, a sequence of two eye rotations within one system leads to the same eye orientation as the reversed sequence of rotations in the other system. A final possibility is represented by the use of nested axes, in which one axis is fixed to the head, the second one rotates with it, and the third one is fixed to the eye and thus rotates with the other two. Unfortunately, these nested systems are often inappropriately called eye-fixed.

In oculomotor research only head-fixed and nested-axes systems have been extensively used. The decision of which one to use is affected by personal preference, and by the specific oculomotor task under study.

Nested-axes coordinates

Nested-axes coordinate systems are inspired by the classical mechanical method for mounting a rotating object, such as a camera. The simplest way to mount a camera is to have one axis for panning the camera left or right (yaw or Z-axis), one for tilting it up or down (pitch or Y-axis), and one for twisting it clockwise or counter-clockwise about the lens's optical axis (roll or X-axis). These axes are nested, one within the other, in a system of gimbals (since there are three rotation axes, there are six possible nesting sequences). As noted above, final orientation depends crucially on the order of rotations. However, in a gimbal system the mathematical order of the rotations is determined by the nesting order of the gimbals, and not by the order in which the mounted object is moved within the gimbals. Note that this mechanical coupling of the axes does not render rotations commutative; the space of rotations is still curved.

Two such nested-axes systems have been extensively used in the field of oculomotor research. The Fick system starts with a *horizontal* rotation around the vertical axis, followed by a *vertical* rotation around the new horizontal axis, and finally a *torsional* rotation about the new line of sight. The Helmholtz system starts with a *vertical* rotation around the horizontal axis, followed by a *horizontal* rotation around the new vertical axis, and finally a *torsional* rotation about the new line of sight. The leftmost column of Figure 8.2 shows a Fick gimbal system, and the rightmost column shows a Helmholtz gimbal system (torsional axes not shown). Initially (top row) the eye looks straight ahead, in *primary* position. When the eye rotates from primary position around the head-fixed horizontal or vertical axis, it is said to move into a *secondary* position (note: all secondary positions lie along the horizontal or vertical meridian of the globe; cf. Fig. 8.5 below). This is shown in he middle row for Fick and Helmholtz gimbals: the first Fick rotation turns the eye to the left, whereas the first Helmholtz rotation turns the eye upward. The bottom row shows the eye rotated away from the horizontal or vertical meridian, into what is called a *tertiary* position. Note that, in spite of the magnitude of the two rotations being the same, the final orientation of the eye is different in the two cases.

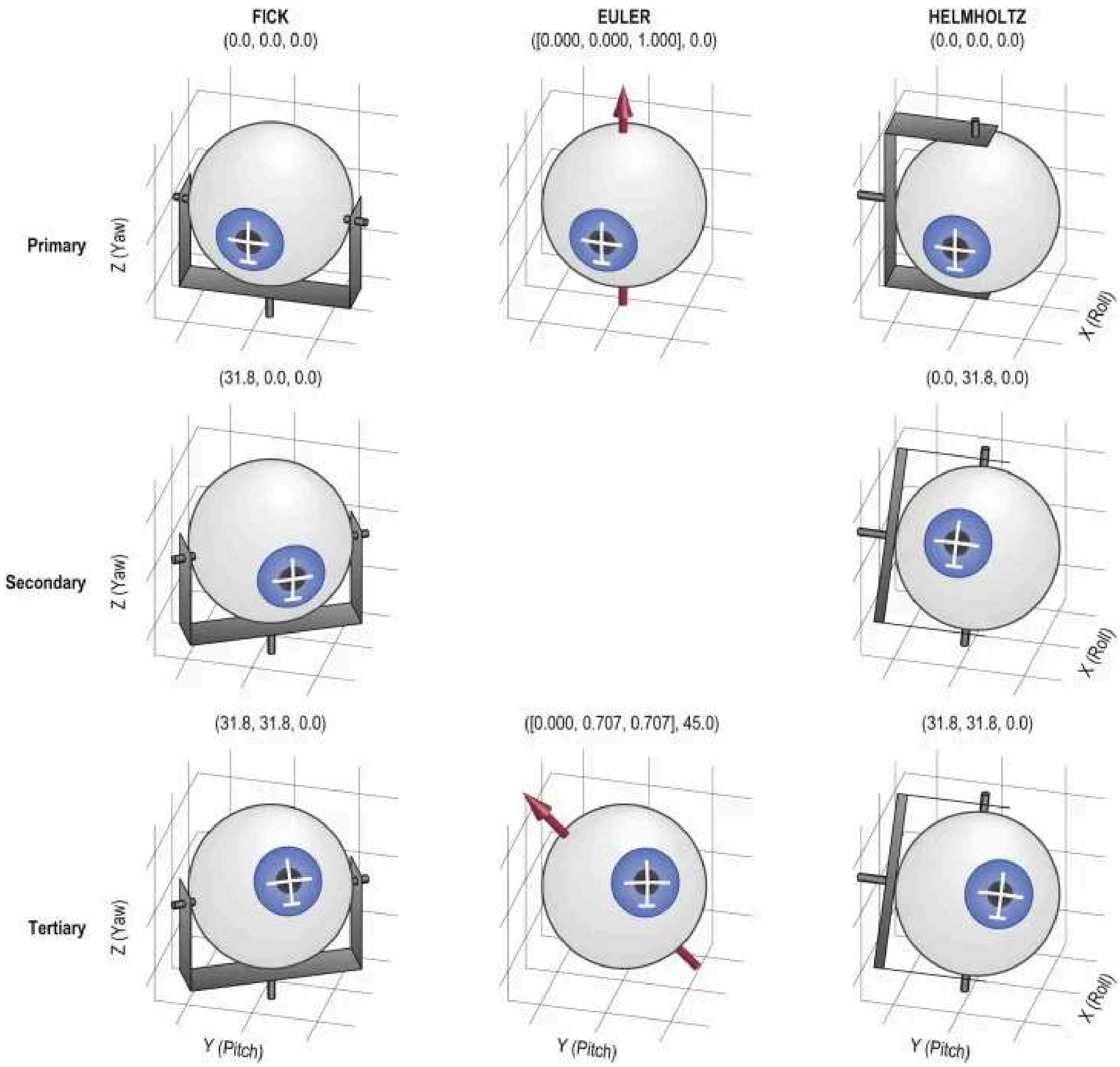


Figure 8.2

Head-fixed and nested-axes coordinate systems. In the left column Fick's nested-axes coordinate system is illustrated (torsion axis not shown). The axes can be represented by a gimbal system. The nesting order is: vertical axis, horizontal axis, torsional axis. In the middle column rotations are described in a head-fixed coordinate system, and Euler's axis is shown. Any orientation is represented by a single axis, tilted appropriately. In the right column another nested-axes system is shown (Helmholtz). In the Helmholtz system, the nesting order is: horizontal axis, vertical axis, torsional axis. Orientations are referred to as primary (looking straight ahead, top row), secondary (on the horizontal or vertical meridian, middle row), or tertiary (off both the horizontal and vertical meridians, bottom row). In all three cases we applied a rotation of 31.8° around the vertical and horizontal axes. In Fick coordinates, that corresponds to (31.8, 31.8, 0), in Helmholtz coordinates to (31.8, 31.8, 0), and in head-fixed coordinates to {(0, 0.707, 0.707), 45}. NB: even when the coordinates in two systems are the same numerically, the final orientations are different because rotations do not commute. In Euler coordinates, these Fick and Helmholtz rotations would be {(−0.198, 0.693, 0.693), 44.7} and {(0.198, 0.693, 0.693), 44.7}, respectively (note the opposite sign for the torsional component).

Head-fixed coordinates

Over the years several different mathematical formalisms have been used to quantify eye rotations using head-fixed coordinate systems. The formalism that we prefer (because we consider it the most intuitive), and the only one that we describe here, is the so-called axis-angle form (Fig. 8.2 , middle column), which follows from Euler's theorem. This theorem states that *any orientation of a rigid body with one point fixed can be achieved, starting from a reference orientation, by a single rotation around an axis (through the fixed point) along a unit-length vector* $\hat{n}$ *by an angle* Φ . Euler's theorem highlights an aspect common to all the methods that can be used to represent rotary motion: the need to define a *reference* , or *primary* , orientation. Although its choice is totally arbitrary, the one most commonly adopted in eye movement research is the orientation with the head upright and the eye looking straight ahead. (Note: when eye orientations are discussed in the context of Listing's law, a somewhat different reference orientation is chosen for convenience; this will be discussed below.) The three main axes of rotation then point straight ahead (X-axis, roll or torsional rotations), straight to the left (Y-axis, pitch or vertical rotations) and straight up (Z-axis, yaw or horizontal rotations). The X-, Y-, and Z-axes define a right-handed system of head-fixed coordinates (x, y, z), that describe, for each eye orientation, Euler's axis of rotation. In a right-handed coordinate system, positive rotations are in the direction that the fingers of the right hand curl when the thumb points along the axis $\hat{n}$.

With this convention, for example, after a 45° rotation to the left, the orientation is described by {(0, 0, 1), 45}, as that orientation is achieved, starting from the reference orientation, by rotating 45° around the vertical axis (0,0,1) (Fig. 8.3A ; note that we are looking at the camera from in front, so the X, Y, and Z axes point out of the page, to the right, and up, respectively). Similarly, after a rotation 45° up and to the left, the orientation is {(0, 0.707, 0.707), 45} (Fig. 8.3B , and Fig. 8.2 middle column, bottom row).

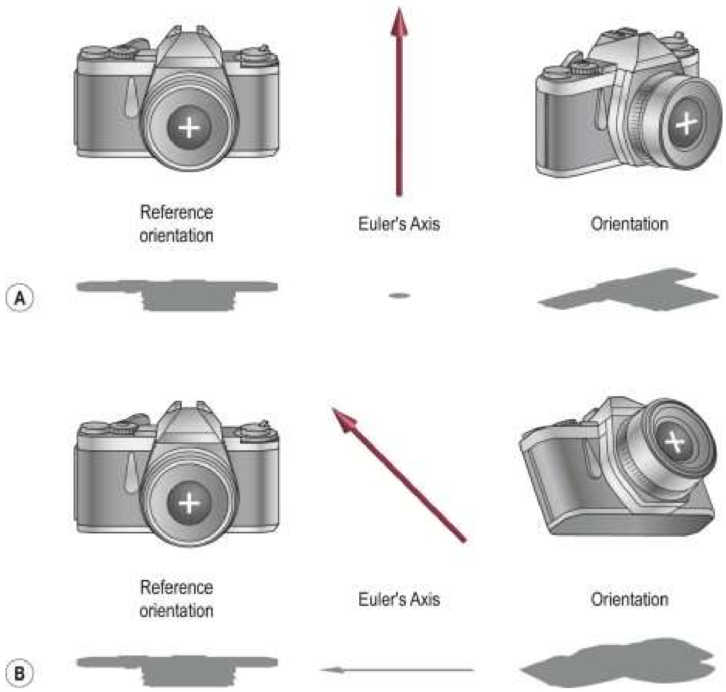
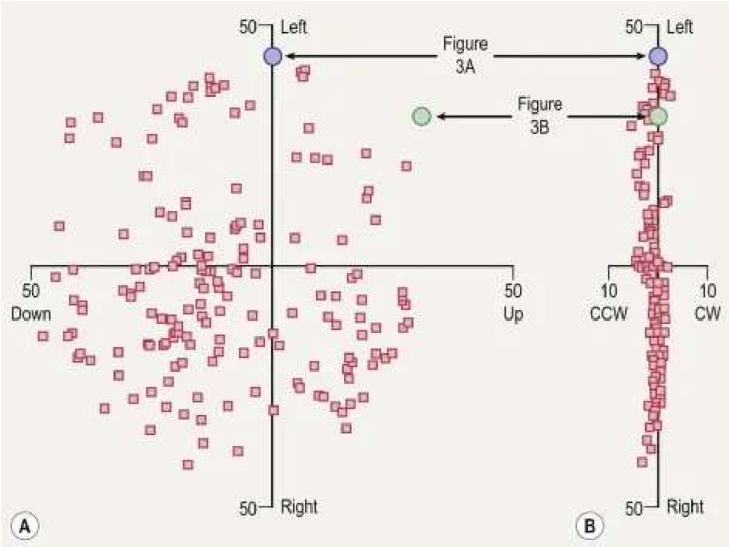


Figure 8.3
Representing orientation using the axis-angle form. For each panel, the reference orientation is shown on the left. (**A**) The camera is rotated 45° to the left, and the corresponding Euler's axis points straight up, (0, 0, 1) in the XYZ coordinate system (see text). (**B**) The camera is rotated 45° around the tilted axis (0, 0.707, 0.707). Note that the central cross on the camera's lens appears twisted with respect to the vertical axis, even though the Euler axis has no torsional component.
(Redrawn from Quaia C, Optican LM. Commutative saccadic generator is sufficient to control a 3-D ocular plant with pulleys. J Neurophysiol 1998 Jun;79(6):3197–3215. Used with permission.)

Listing's law

The purpose of voluntary eye movements is to point the region of highest visual acuity (the fovea) at the object of interest. Because the eye can rotate about the line of sight without changing the direction of gaze, the latter has only two degrees of freedom, whereas eye orientation has three. This situation, called *kinematic redundancy* , implies that an infinite number of different eye orientations correspond to each direction of gaze. Despite this redundancy, observation of voluntary eye movements reveals that the brain constrains the torsion to be a function of the horizontal and vertical gaze direction. This reduces the number of degrees of freedom of eye orientation from three to two, so that each gaze direction (achieved with saccadic or smooth pursuit movements) corresponds to a unique eye orientation, regardless of previous movements and orientations. This observation is known as *Donder's law* .

Donder's law states that the torsional component of the eye's attitude is a function of the horizontal and vertical components, but does not specify the relationship. Listing extended Donder's law, by specifying the torsional angle for any line of sight. *Listing's law* states that if the Euler vectors describing the eye orientations attained by a subject looking around with his head fixed in space are plotted, they lie in or near a plane (the so-called *Listing's plane*). In Figure 8.4 orientations measured in a human subject are shown; each point indicates the orientation of the eye during a period of fixation. Figure 8.4A shows the vertical and horizontal components of the Euler axis (from the subject's point of view), whereas Figure 8.4B shows the vertical and torsional components (the components of the unit-length axis are multiplied by the eccentricity). As an example, on top of the human data we have added two symbols indicating where the Euler axes for the camera orientations shown in Figure 8.3A (blue disks) and Figure 8.3B (green disks) would be in this graph (note that the horizontal component is reversed, as in Figure 8.3 we were looking at the vectors from the front, and not from the camera's point of view). Obviously the points in Figure 8.4 form a thin cloud; Listing's plane is defined as the plane that best fits this cloud.



Related Posts:

1. The Extraocular Muscles
2. The Lens
3. Electroretinogram of Human, Monkey and Mouse
4. The Effects of Visual Deprivation After Infancy

Jan 23, 2019 | Posted by drzezo in OPHTHALMOLOGY | Comments Off