TP5 Report

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First Part

In this part we had to solve 2 systems of congruences a) and b)

Notes:

- We'll consider the defenition of a function `gcd(a,b)` that gives the greater common divisor of two values *a* and *b*.
- *k*,*l* and *m* are positive integers
- The congruences will be conversible to equations in the form:

`X ≡ a (mod b)` -> `X = b*k + a` , *k* being an positive integer

a)

```
(1): X ≡ 48 (mod 13)
(2): X ≡ 57 (mod 23)
(3): X ≡ 39 (mod 27)
```

Firstly we begin with the congruency with largest modulus that is (3) `X ≡ 39 (mod 27)`

Then we substitute this congruences expression for *X* into the congruence with the next largest modulus (2):

```
27*m + 39 ≡ 57 (mod 23)
```

Solving this Linear Congruency ...

```
27*m ≡ 18 (mod 23)

note: gcd(27,23) = 1 so there is a solution

gcd(27,18) = 9

3*m ≡ 2 (mod 23)

3*m ≡ -21 (mod 23)

m ≡ -7 (mod 23)

m ≡ 16 (mod 23)

m = 23*k + 16
```

Replacing the expression form of this result in the expression for *X* we get:

```
X = 27*(23*l + 16) + 39
X = 621*l + 471
```

Then we replace this expression in the last congruency (1) and solve this Linear Congruency

```
621*l + 471 ≡ 48 (mod 13)

note: gcd(48,13) = 1 so there is a solution

621*l ≡ -423 (mod 13)

621*l ≡ 6 (mod 13)

207*l ≡ 2 (mod 13)

207*l ≡ -24 (mod 13)

gcd(24,207) = 3

69*l ≡ -8 (mod 13)

. . .

l ≡ 11 (mod 13)
```

Finally we replace this in the expression obtained previously and get the solution

```
X = 621*(13*m + 11) + 471

X = 8073*m + 7302

X = 7302
```

The smallest solution for this system is 7302

b)

For the second system the congruences we first simplified the each one to remove the coefficient

```
19*X ≡ 21 (mod 16)
37*X ≡ 100 (mod 15)
```

Solving this Linear Congruences ...

```
(1): X ≡ 7 (mod 16)
(2): X ≡ 10 (mod 15)
```

With the same intension as before, we begin with the congruency with largest modulus that is (1) `X ≡ 7 (mod 16)`

Then we substitute this congruences expression for X into the congruence with the next largest modulus (2):

```
16*k + 7 ≡ 10 (mod 15)
```

Solving this Linear Congruence ...

```
k ≡ 3 (mod 15)

k = 15*l + 3
```

Replacing the expression form of this result in the expression for X we get:

```

X = 16*(15*l + 3) + 7

X = 240*l + 55

X = 55
```

The smallest solution for this system is 55

Second Part

For this part