is to connect a zener diode with a positive temperature coefficient of about 2 mV/°C in series with a forward-conducting diode. Since the forward-conducting diode has a voltage drop of $\simeq 0.7$ V and a TC of about –2 mV/°C, the series combination will provide a voltage of $(V_Z + 0.7)$ with a TC of about zero.

EXERCISES

4.16 A zener diode whose nominal voltage is 10 V at 10 mA has an incremental resistance of 50 Ω. What voltage do you expect if the diode current is halved? Doubled? What is the value of V_{Z0} in the zener model?
Ans. 9.75 V; 10.5 V; 9.5 V

4.17 A zener diode exhibits a constant voltage of 5.6 V for currents greater than five times the knee current. I_{ZK} is specified to be 1 mA. The zener is to be used in the design of a shunt regulator fed from a 15-V supply. The load current varies over the range of 0 mA to 15 mA. Find a suitable value for the resistor R. What is the maximum power dissipation of the zener diode?
Ans. 470 Ω; 112 mW

4.18 A shunt regulator utilizes a zener diode whose voltage is 5.1 V at a current of 50 mA and whose incremental resistance is 7 Ω. The diode is fed from a supply of 15-V nominal voltage through a 200-Ω resistor. What is the output voltage at no load? Find the line regulation and the load regulation.
Ans. 5.1 V; 33.8 mV/V; –7 mV/mA

4.4.4 A Final Remark

Though simple and useful, zener diodes have lost a great deal of their popularity in recent years. They have been virtually replaced in voltage-regulator design by specially designed integrated circuits (ICs) that perform the voltage-regulation function much more effectively and with greater flexibility than zener diodes.

4.5 Rectifier Circuits

One of the most important applications of diodes is in the design of rectifier circuits. A diode rectifier forms an essential building block of the dc power supplies required to power electronic equipment. A block diagram of such a power supply is shown in Fig. 4.22. As indicated, the power supply is fed from the 120-V (rms) 60-Hz ac line, and it delivers a dc voltage V_O (usually in the range of 4 V to 20 V) to an electronic circuit represented by the *load* block. The dc voltage V_O is required to be as constant as possible in spite of variations in the ac line voltage and in the current drawn by the load.

The first block in a dc power supply is the **power transformer**. It consists of two separate coils wound around an iron core that magnetically couples the two windings. The **primary winding**, having N_1 turns, is connected to the 120-V ac supply, and the **secondary winding**, having N_2 turns, is connected to the circuit of the dc power supply. Thus an ac voltage v_S of $120(N_2/N_1)$ V (rms) develops between the two terminals of the secondary winding. By

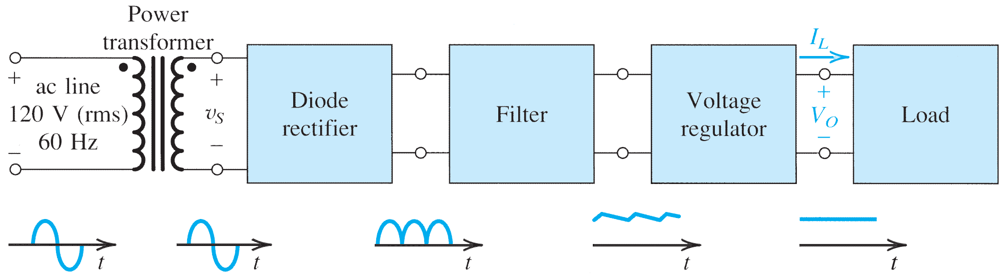

Figure 4.22 Block diagram of a dc power supply.

selecting an appropriate turns ratio (N_1/N_2) for the transformer, the designer can step the line voltage down to the value required to yield the particular dc voltage output of the supply. For instance, a secondary voltage of 8-V rms may be appropriate for a dc output of 5 V. This can be achieved with a 15:1 turns ratio.

In addition to providing the appropriate sinusoidal amplitude for the dc power supply, the power transformer provides electrical isolation between the electronic equipment and the power-line circuit. This isolation minimizes the risk of electric shock to the equipment user.

The diode rectifier converts the input sinusoid v_S to a unipolar output, which can have the pulsating waveform indicated in Fig. 4.22. Although this waveform has a nonzero average or a dc component, its pulsating nature makes it unsuitable as a dc source for electronic circuits, hence the need for a filter. The variations in the magnitude of the rectifier output are considerably reduced by the filter block in Fig. 4.22. In this section we shall study a number of rectifier circuits and a simple implementation of the output filter.

The output of the rectifier filter, though much more constant than without the filter, still contains a time-dependent component, known as **ripple**. To reduce the ripple and to stabilize the magnitude of the dc output voltage against variations caused by changes in load current, a voltage regulator is employed. Such a regulator can be implemented using the zener shunt regulator configuration studied in Section 4.4. Alternatively, and much more commonly at present, an integrated-circuit regulator can be used.

4.5.1 The Half-Wave Rectifier

The half-wave rectifier utilizes alternate half-cycles of the input sinusoid. Figure 4.23(a) shows the circuit of a half-wave rectifier. This circuit was analyzed in Section 4.1 (see Fig. 4.3) assuming an ideal diode. Using the more realistic constant-voltage-drop diode model, we obtain

$$v_O = 0, \qquad v_S < V_D \tag{4.21a}$$

$$v_O = v_S - V_D, \qquad v_S \geq V_D \tag{4.21b}$$

The transfer characteristic represented by these equations is sketched in Fig. 4.23(b), where $V_D = 0.7$ V or 0.8 V. Figure 4.23(c) shows the output voltage obtained when the input v_S is a sinusoid.

In selecting diodes for rectifier design, two important parameters must be specified: the current-handling capability required of the diode, determined by the largest current the diode is expected to conduct, and the **peak inverse voltage** (PIV) that the diode must be able to

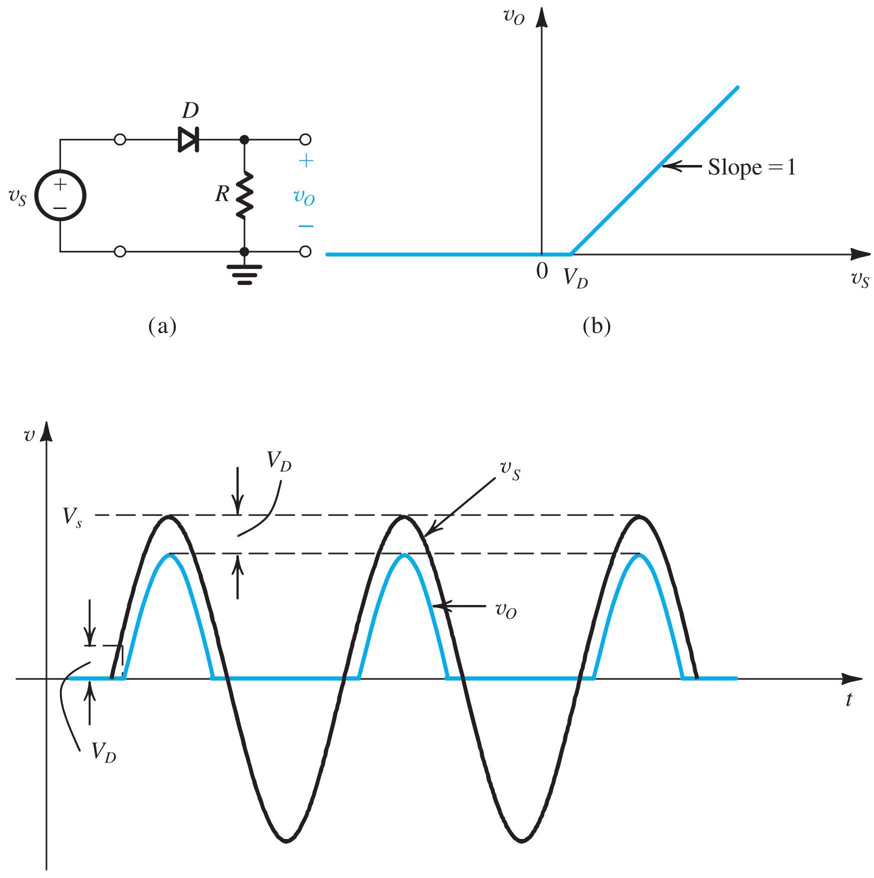

Figure 4.23 **(a)** Half-wave rectifier. **(b)** Transfer characteristic of the rectifier circuit. **(c)** Input and output waveforms.

withstand without breakdown, determined by the largest reverse voltage that is expected to appear across the diode. In the rectifier circuit of Fig. 4.23(a), we observe that when v_S is negative the diode will be cut off and v_O will be zero. It follows that the PIV is equal to the peak of v_S,

$$\text{PIV} = V_s \tag{4.22}$$

It is usually prudent, however, to select a diode that has a reverse breakdown voltage at least 50% greater than the expected PIV.

Before leaving the half-wave rectifier, the reader should note two points. First, it is possible to use the diode exponential characteristic to determine the exact transfer characteristic of the rectifier (see Problem 4.68). However, the amount of work involved is usually too great to be justified in practice. Of course, such an analysis can be easily done using a computer circuit-analysis program such as SPICE.

Second, whether we analyze the circuit accurately or not, it should be obvious that this circuit does not function properly when the input signal is small. For instance, this circuit cannot be used to rectify an input sinusoid of 100-mV amplitude. For such an application one resorts to a so-called precision rectifier, a circuit utilizing diodes in conjunction with op amps. One such circuit is presented in Section 4.5.5.

EXERCISE

4.19 For the half-wave rectifier circuit in Fig. 4.23(a), show the following: (a) For the half-cycles during which the diode conducts, conduction begins at an angle $\theta = \sin^{-1}(V_D/V_s)$ and terminates at $(\pi - \theta)$, for a total conduction angle of $(\pi - 2\theta)$. (b) The average value (dc component) of v_O is $V_O \simeq (1/\pi)V_s - V_D/2$. (c) The peak diode current is $(V_s - V_D)/R$.
Find numerical values for these quantities for the case of 12-V (rms) sinusoidal input, $V_D \simeq 0.7$ V, and $R = 100\ \Omega$. Also, give the value for PIV.
Ans. (a) $\theta = 2.4°$, conduction angle $= 175°$; (b) 5.05 V; (c) 163 mA; 17 V

4.5.2 The Full-Wave Rectifier

The full-wave rectifier utilizes both halves of the input sinusoid. To provide a unipolar output, it inverts the negative halves of the sine wave. One possible implementation is shown in Fig. 4.24(a). Here the transformer secondary winding is **center-tapped** to provide two equal voltages v_S across the two halves of the secondary winding with the polarities indicated. Note that when the input line voltage (feeding the primary) is positive, both of the signals labeled v_S will be positive. In this case D_1 will conduct and D_2 will be reverse biased. The current through D_1 will flow through R and back to the center tap of the secondary. The circuit then behaves like a half-wave rectifier, and the output during the positive half-cycles when D_1 conducts will be identical to that produced by the half-wave rectifier.

Now, during the negative half-cycle of the ac line voltage, both of the voltages labeled v_S will be negative. Thus D_1 will be cut off while D_2 will conduct. The current conducted by D_2 will flow through R and back to the center tap. It follows that during the negative half-cycles while D_2 conducts, the circuit behaves again as a half-wave rectifier. The important point, however, is that the current through R always flows in the same direction, and thus v_O will be unipolar, as indicated in Fig. 4.24(c). The output waveform shown is obtained by assuming that a conducting diode has a constant voltage drop V_D. Thus the transfer characteristic of the full-wave rectifier takes the shape shown in Fig. 4.24(b).

The full-wave rectifier obviously produces a more “energetic” waveform than that provided by the half-wave rectifier. In almost all rectifier applications, one opts for a full-wave type of some kind.

To find the PIV of the diodes in the full-wave rectifier circuit, consider the situation during the positive half-cycles. Diode D_1 is conducting, and D_2 is cut off. The voltage at the cathode of D_2 is v_O, and that at its anode is $-v_S$. Thus the reverse voltage across D_2 will be $(v_O + v_S)$, which will reach its maximum when v_O is at its peak value of $(V_s - V_D)$, and v_S is at its peak value of V_s; thus,

$$\text{PIV} = 2V_s - V_D$$

which is approximately twice that for the case of the half-wave rectifier.

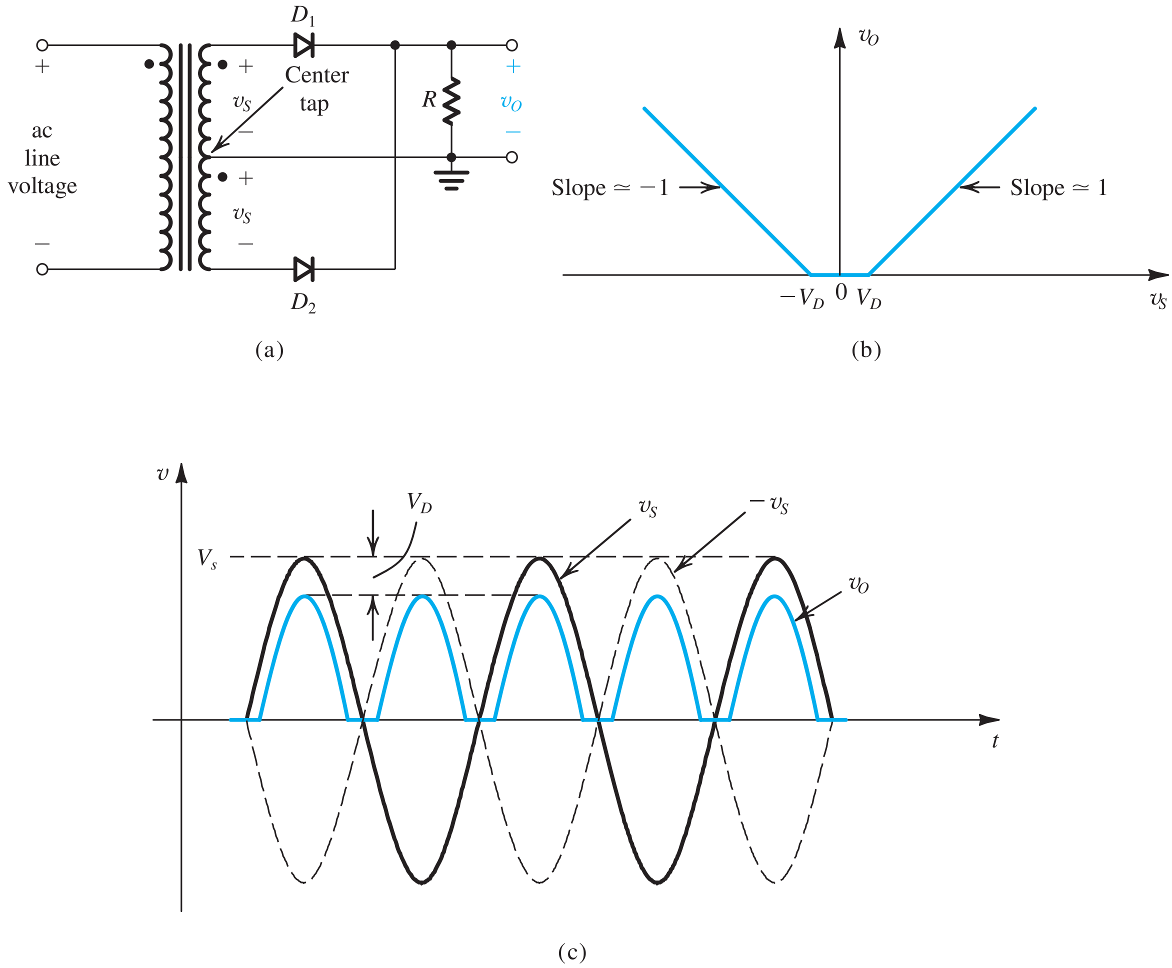

Figure 4.24 Full-wave rectifier utilizing a transformer with a center-tapped secondary winding: **(a)** circuit; **(b)** transfer characteristic assuming a constant-voltage-drop model for the diodes; **(c)** input and output waveforms.

EXERCISE

4.20 For the full-wave rectifier circuit in Fig. 4.24(a), show the following: (a) The output is zero for an angle of $2\sin^{-1}(V_D/V_s)$ centered around the zero-crossing points of the sine-wave input. (b) The average value (dc component) of v_O is $V_O \simeq (2/\pi)V_s - V_D$. (c) The peak current through each diode is $(V_s - V_D)/R$. Find the fraction (percentage) of each cycle during which $v_O > 0$, the value of V_O, the peak diode current, and the value of PIV, all for the case in which v_S is a 12-V (rms) sinusoid, $V_D \simeq 0.7$ V, and $R = 100\ \Omega$.

Ans. 97.4%; 10.1 V; 163 mA; 33.2 V

4.5.3 The Bridge Rectifier

An alternative implementation of the full-wave rectifier is shown in Fig. 4.25(a). This circuit, known as the bridge rectifier because of the similarity of its configuration to that of the Wheatstone bridge, does not require a center-tapped transformer, a distinct advantage over the full-wave rectifier circuit of Fig. 4.24. The bridge rectifier, however, requires four diodes as compared to two in the previous circuit. This is not much of a disadvantage, because diodes are inexpensive and one can buy a diode bridge in one package.

The bridge-rectifier circuit operates as follows: During the positive half-cycles of the input voltage, v_S is positive, and thus current is conducted through diode D_1, resistor R, and diode D_2. Meanwhile, diodes D_3 and D_4 will be reverse biased. Observe that there are two diodes in series in the conduction path, and thus v_O will be lower than v_S by two diode drops (compared to one drop in the circuit previously discussed). This is somewhat of a disadvantage of the bridge rectifier.

Next, consider the situation during the negative half-cycles of the input voltage. The secondary voltage v_S will be negative, and thus $-v_S$ will be positive, forcing current through D_3, R, and D_4. Meanwhile, diodes D_1 and D_2 will be reverse biased. The important point to note, though, is that during both half-cycles, current flows through R in the same direction (from right to left), and thus v_O will always be positive, as indicated in Fig. 4.25(b).

To determine the peak inverse voltage (PIV) of each diode, consider the circuit during the positive half-cycles. The reverse voltage across D_3 can be determined from the loop formed

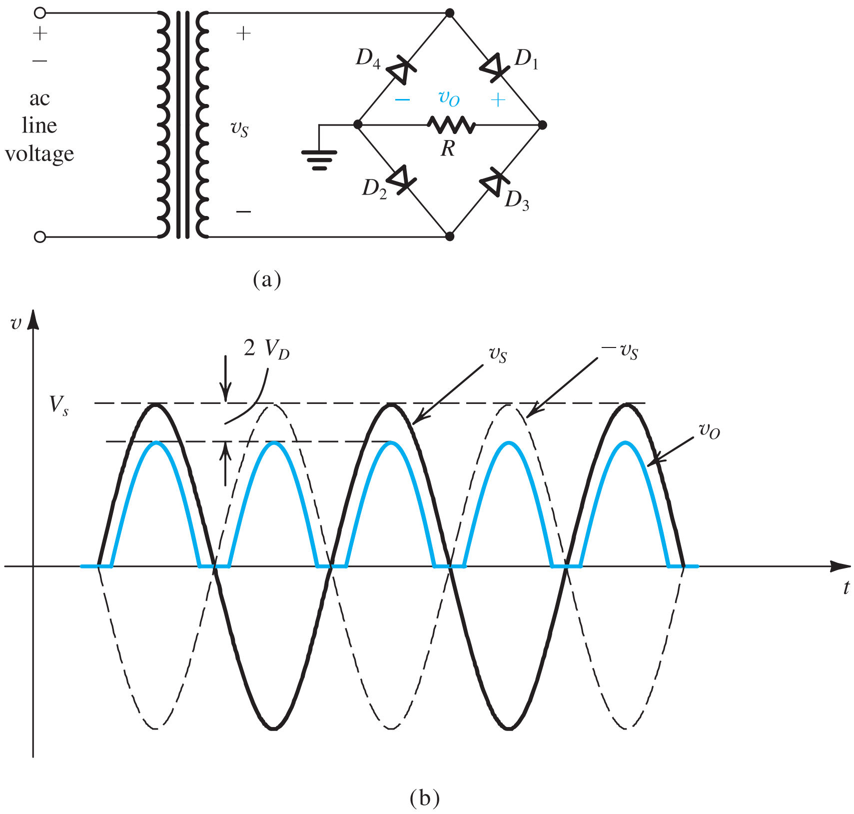

Figure 4.25 The bridge rectifier: **(a)** circuit; **(b)** input and output waveforms.

by D_3, R, and D_2 as

$$v_{D3}(\text{reverse}) = v_O + v_{D2}(\text{forward})$$

Thus the maximum value of v_{D3} occurs at the peak of v_O and is given by

$$\text{PIV} = V_s - 2V_D + V_D = V_s - V_D$$

Observe that here the PIV is about half the value for the full-wave rectifier with a center-tapped transformer. This is another advantage of the bridge rectifier.

Yet one more advantage of the bridge-rectifier circuit over that utilizing a center-tapped transformer is that only about half as many turns are required for the secondary winding of the transformer. Another way of looking at this point can be obtained by observing that each half of the secondary winding of the center-tapped transformer is utilized for only half the time. These advantages have made the bridge rectifier the most popular rectifier circuit configuration.

EXERCISE

4.21 For the bridge-rectifier circuit of Fig. 4.25(a), use the constant-voltage-drop diode model to show that (a) the average (or dc component) of the output voltage is $V_O \simeq (2/\pi)V_s - 2V_D$ and (b) the peak diode current is $(V_s - 2V_D)/R$. Find numerical values for the quantities in (a) and (b) and the PIV for the case in which v_S is a 12-V (rms) sinusoid, $V_D \simeq 0.7$ V, and $R = 100\ \Omega$.
Ans. 9.4 V; 156 mA; 16.3 V

4.5.4 The Rectifier with a Filter Capacitor—The Peak Rectifier

The pulsating nature of the output voltage produced by the rectifier circuits discussed above makes it unsuitable as a dc supply for electronic circuits. A simple way to reduce the variation of the output voltage is to place a capacitor across the load resistor. It will be shown that this **filter capacitor** serves to reduce substantially the variations in the rectifier output voltage.

To see how the rectifier circuit with a filter capacitor works, consider first the simple circuit shown in Fig. 4.26. Let the input v_I be a sinusoid with a peak value V_p, and assume the diode to be ideal. As v_I goes positive, the diode conducts and the capacitor is charged so that $v_O = v_I$. This situation continues until v_I reaches its peak value V_p. Beyond the peak, as v_I decreases, the diode becomes reverse biased and the output voltage remains constant at the value V_p. In fact, theoretically speaking, the capacitor will retain its charge and hence its voltage indefinitely, because there is no way for the capacitor to discharge. Thus the circuit provides a dc voltage output equal to the peak of the input sine wave. This is a very encouraging result in view of our desire to produce a dc output.

Next, we consider the more practical situation where a load resistance R is connected across the capacitor C, as depicted in Fig. 4.27(a). However, we will continue to assume the diode to be ideal. As before, for a sinusoidal input, the capacitor charges to the peak of the input V_p. Then the diode cuts off, and the capacitor discharges through the load resistance R. The capacitor discharge will continue for almost the entire cycle, until the time at which v_I

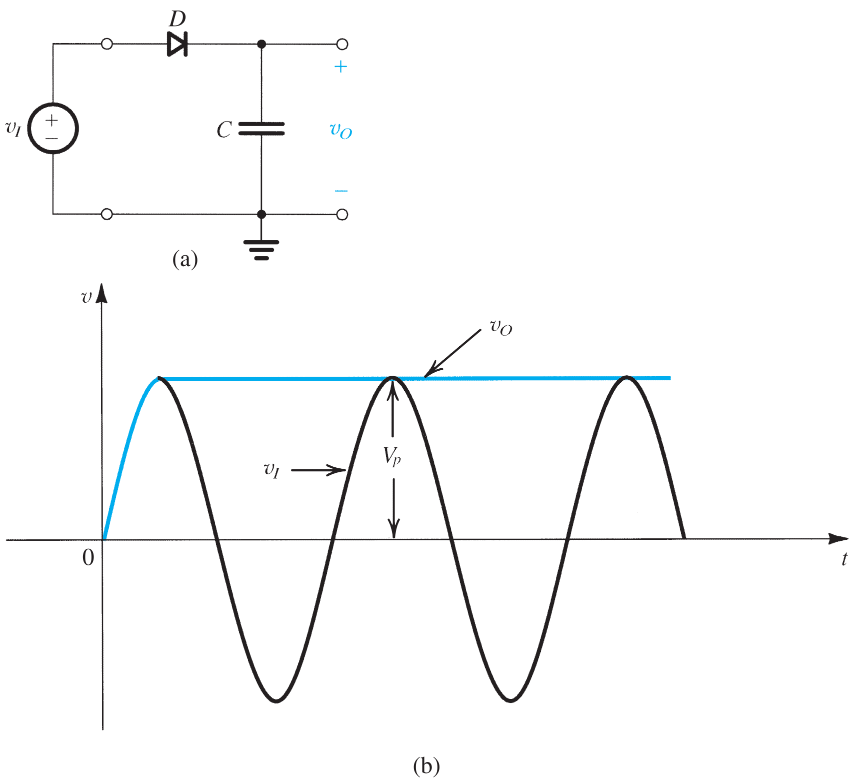

Figure 4.26 **(a)** A simple circuit used to illustrate the effect of a filter capacitor. **(b)** Input and output waveforms assuming an ideal diode. Note that the circuit provides a dc voltage equal to the peak of the input sine wave. The circuit is therefore known as a *peak rectifier* or a *peak detector*.

exceeds the capacitor voltage. Then the diode turns on again and charges the capacitor up to the peak of v_I, and the process repeats itself. Observe that to keep the output voltage from decreasing too much during capacitor discharge, one selects a value for C so that the time constant CR is much greater than the discharge interval.

We are now ready to analyze the circuit in detail. Figure 4.27(b) shows the steady-state input and output voltage waveforms under the assumption that $CR \gg T$, where T is the period of the input sinusoid. The waveforms of the load current

$$i_L = v_O/R \tag{4.23}$$

and of the diode current (when it is conducting)

$$i_D = i_C + i_L \tag{4.24}$$

$$= C\frac{dv_I}{dt} + i_L \tag{4.25}$$

are shown in Fig. 4.27(c). The following observations are in order:

1. The diode conducts for a brief interval, Δt, near the peak of the input sinusoid and supplies the capacitor with charge equal to that lost during the much longer discharge interval. The latter is approximately equal to the period T.

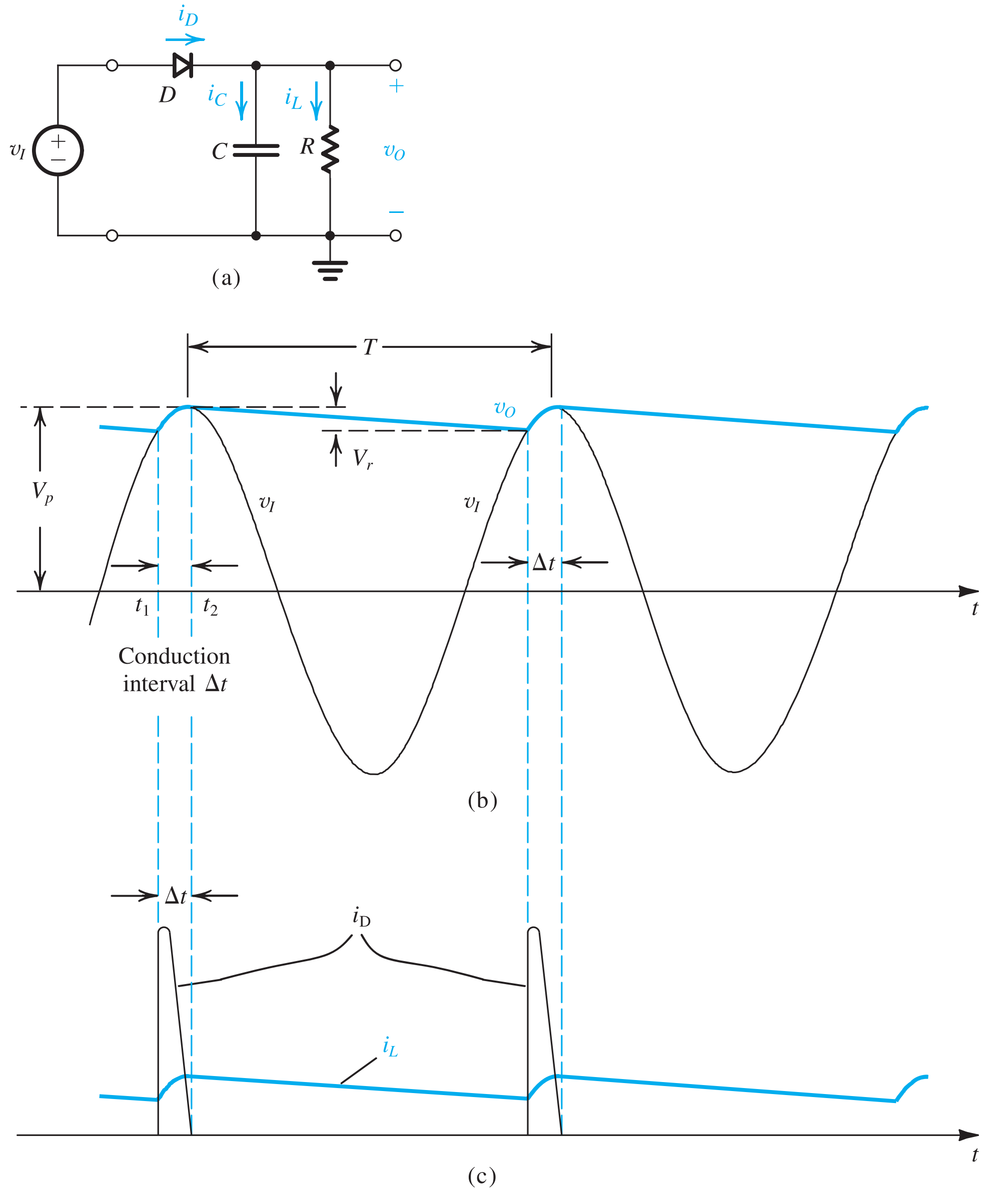

Figure 4.27 Voltage and current waveforms in the peak-rectifier circuit with $CR \gg T$. The diode is assumed ideal.

2. Assuming an ideal diode, the diode conduction begins at time t_1, at which the input v_I equals the exponentially decaying output v_O. Conduction stops at t_2 shortly after the peak of v_I; the exact value of t_2 can be determined by setting $i_D = 0$ in Eq. (4.25).
3. During the diode-off interval, the capacitor C discharges through R, and thus v_O decays exponentially with a time constant CR. The discharge interval begins just past the peak of v_I. At the end of the discharge interval, which lasts for almost the entire period T, $v_O = V_p - V_r$, where V_r is the peak-to-peak ripple voltage. When $CR \gg T$, the value of V_r is small.

4. When V_r is small, v_O is almost constant and equal to the peak value of v_I. Thus the dc output voltage is approximately equal to V_p. Similarly, the current i_L is almost constant, and its dc component I_L is given by

$$I_L = \frac{V_p}{R} \tag{4.26}$$

If desired, a more accurate expression for the output dc voltage can be obtained by taking the average of the extreme values of v_O,

$$V_O = V_p - \tfrac{1}{2}V_r \tag{4.27}$$

With these observations in hand, we now derive expressions for V_r and for the average and peak values of the diode current. During the diode-off interval, v_O can be expressed as

$$v_O = V_p e^{-t/CR}$$

At the end of the discharge interval we have

$$V_p - V_r \simeq V_p e^{-T/CR}$$

Now, since $CR \gg T$, we can use the approximation $e^{-T/CR} \simeq 1 - T/CR$ to obtain

$$V_r \simeq V_p \frac{T}{CR} \tag{4.28}$$

We observe that to keep V_r small we must select a capacitance C so that $CR \gg T$. The **ripple voltage** V_r in Eq. (4.28) can be expressed in terms of the frequency $f = 1/T$ as

$$V_r = \frac{V_p}{fCR} \tag{4.29a}$$

Using Eq. (4.26) we can express V_r by the alternate expression

$$V_r = \frac{I_L}{fC} \tag{4.29b}$$

Note that an alternative interpretation of the approximation made above is that the capacitor discharges by means of a constant current $I_L = V_p/R$. This approximation is valid as long as $V_r \ll V_p$.

Assuming that diode conduction ceases almost at the peak of v_I, we can determine the **conduction interval** Δt from

$$V_p \cos(\omega \Delta t) = V_p - V_r$$

where $\omega = 2\pi f = 2\pi/T$ is the angular frequency of v_I. Since $(\omega\Delta t)$ is a small angle, we can employ the approximation $\cos(\omega\Delta t) \simeq 1 - \frac{1}{2}(\omega\Delta t)^2$ to obtain

$$\omega \Delta t \simeq \sqrt{2V_r/V_p} \tag{4.30}$$

We note that when $V_r \ll V_p$, the conduction angle $\omega\Delta t$ will be small, as assumed.

To determine the average diode current during conduction, i_{Dav}, we equate the charge that the diode supplies to the capacitor,

$$Q_{\text{supplied}} = i_{Cav}\Delta t$$

where from Eq. (4.24),

$$i_{Cav} = i_{Dav} - I_L$$

to the charge that the capacitor loses during the discharge interval,

$$Q_{lost} = CV_r$$

to obtain, using Eqs. (4.30) and (4.29a),

$$i_{Dav} = I_L\left(1 + \pi\sqrt{2V_p/V_r}\right) \tag{4.31}$$

Observe that when $V_r \ll V_p$, the average diode current during conduction is much greater than the dc load current. This is not surprising, since the diode conducts for a very short interval and must replenish the charge lost by the capacitor during the much longer interval in which it is discharged by I_L.

The peak value of the diode current, i_{Dmax}, can be determined by evaluating the expression in Eq. (4.25) at the onset of diode conduction—that is, at $t = t_1 = -\Delta t$ (where $t = 0$ is at the peak). Assuming that i_L is almost constant at the value given by Eq. (4.26), we obtain

$$i_{Dmax} = I_L\left(1 + 2\pi\sqrt{2V_p/V_r}\right) \tag{4.32}$$

From Eqs. (4.31) and (4.32), we see that for $V_r \ll V_p$, $i_{Dmax} \simeq 2i_{Dav}$, which correlates with the fact that the waveform of i_D is almost a right-angle triangle (see Fig. 4.27c).

Example 4.8

Consider a peak rectifier fed by a 60-Hz sinusoid having a peak value $V_p = 100$ V. Let the load resistance $R = 10\ \text{k}\Omega$. Find the value of the capacitance C that will result in a peak-to-peak ripple of 2 V. Also, calculate the fraction of the cycle during which the diode is conducting and the average and peak values of the diode current.

Solution

From Eq. (4.29a) we obtain the value of C as

$$C = \frac{V_p}{V_r fR} = \frac{100}{2 \times 60 \times 10 \times 10^3} = 83.3\ \mu\text{F}$$

The conduction angle $\omega\Delta t$ is found from Eq. (4.30) as

$$\omega\Delta t = \sqrt{2 \times 2/100} = 0.2 \text{ rad}$$

Thus the diode conducts for $(0.2/2\pi) \times 100 = 3.18\%$ of the cycle. The average diode current is obtained from Eq. (4.31), where $I_L = 100/10 = 10$ mA, as

$$i_{Dav} = 10\left(1 + \pi\sqrt{2 \times 100/2}\right) = 324 \text{ mA}$$

The peak diode current is found using Eq. (4.32),

$$i_{Dmax} = 10\left(1 + 2\pi\sqrt{2 \times 100/2}\right) = 638 \text{ mA}$$

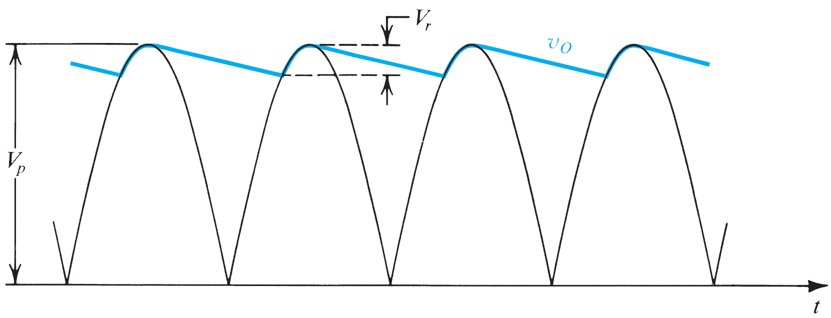

Figure 4.28 Waveforms in the full-wave peak rectifier.

The circuit of Fig. 4.27(a) is known as a half-wave **peak rectifier**. The full-wave rectifier circuits of Figs. 4.24(a) and 4.25(a) can be converted to peak rectifiers by including a capacitor across the load resistor. As in the half-wave case, the output dc voltage will be almost equal to the peak value of the input sine wave (Fig. 4.28). The ripple frequency, however, will be twice that of the input. The peak-to-peak ripple voltage, for this case, can be derived using a procedure identical to that above but with the discharge period T replaced by $T/2$, resulting in

$$V_r = \frac{V_p}{2fCR} \tag{4.33}$$

While the diode conduction interval, Δt, will still be given by Eq. (4.30), the average and peak currents in each of the diodes will be given by

$$i_{D\mathrm{av}} = I_L\left(1 + \pi\sqrt{V_p/2V_r}\right) \tag{4.34}$$

$$i_{D\mathrm{max}} = I_L\left(1 + 2\pi\sqrt{V_p/2V_r}\right) \tag{4.35}$$

Comparing these expressions with the corresponding ones for the half-wave case, we note that for the same values of V_p, f, R, and V_r (and thus the same I_L), we need a capacitor half the size of that required in the half-wave rectifier. Also, the current in each diode in the full-wave rectifier is approximately half that which flows in the diode of the half-wave circuit.

The analysis above assumed ideal diodes. The accuracy of the results can be improved by taking the diode voltage drop into account. This can be easily done by replacing the peak voltage V_p to which the capacitor charges with $(V_p - V_D)$ for the half-wave circuit and the full-wave circuit using a center-tapped transformer and with $(V_p - 2V_D)$ for the bridge-rectifier case.

We conclude this section by noting that peak-rectifier circuits find application in signal-processing systems where it is required to detect the peak of an input signal. In such a case, the circuit is referred to as a **peak detector**. A particularly popular application of the peak detector is in the design of a demodulator for amplitude-modulated (AM) signals. We shall not discuss this application further here.