Q1 $\vec{a}_{fixed} = \vec{a}_{rot} + 2\vec{\Omega} \times \vec{v}_{rot} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$.

$\vec{a}_{fixed} = 0 \Rightarrow \vec{a}_{rot} = \underbrace{-2\vec{\Omega} \times v_{rot}}_{\text{Coriolis}} - \underbrace{\Omega \times (\Omega \times r)}_{\text{centrifugal}}$

a) Centrifugal part

$$-\vec{\Omega} \times \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \Omega \\ x & y & z \end{vmatrix} = -\vec{\Omega} \times (-\Omega y\,\hat{i} + \Omega x\,\hat{j})$$

$$= -\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \Omega \\ -\Omega y & -\Omega x & 0 \end{vmatrix} = \boxed{\Omega^2 x\,\hat{i} + \Omega^2 y\,\hat{j}}$$

b) Coriolis part $= -2\,\Omega \times v = -2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \Omega \\ \dot{x} & \dot{y} & 0 \end{vmatrix} = 2\Omega\dot{y}\,\hat{i} - 2\Omega\dot{x}\,\hat{j}$

$$\boxed{2\Omega\dot{y}\,\hat{i} - 2\Omega\dot{x}\,\hat{j}}$$

c) Equations of motion

$$\ddot{x} = \Omega^2 x + 2\Omega\dot{y}$$

$$\ddot{y} = \Omega^2 y - 2\Omega\dot{x}$$

Q2

a)

$$\frac{d^2u}{d\theta^2} + u = \frac{-\mu}{L^2u^2} f\left(\frac{1}{u}\right) .$$

here $\mu = m = 1$ kg.

$L = \frac{2}{\sqrt{3}}$ kg-m²/s

$f(r) = -\frac{\hat{r}}{r^3}$

$= -u^3$

$$\frac{d^2u}{d\theta^2} + u = -\frac{3}{4u^2}(-u^3) .$$

$$\boxed{\frac{d^2u}{d\theta^2} + \frac{u}{4} = 0} .$$

b) Solution of this is Simple Harmonic motion with $\omega^2 = \frac{1}{4}$ similar to $\ddot{x} + \omega^2 x = 0$

$$u = A\sin\frac{\theta}{2} + B\cos\frac{\theta}{2}$$

$$r = \frac{1}{A\sin\theta/2 + B\cos\theta/2}$$

A, B to be fixed using initial conditions.

3. The aerial velocity $= \frac{\frac{1}{2} r^2 \delta\theta}{\delta t} = \frac{1}{2} r^2 \dot{\theta} = K.$

$\Rightarrow \dot{\theta} = \frac{2K}{r^2}$ hence $L = m r^2 \dot{\theta} = 2mK.$ $\boxed{L = 2mK}$

(a) The orbit equation $\frac{d^2u}{d\theta^2} + u = \frac{-m}{L^2u^2} f\left(\frac{1}{u}\right)$

$r = a(1+\cos\theta) \Rightarrow \frac{1}{u} = a(1+\cos\theta).$

$-\frac{1}{u^2}\frac{du}{d\theta} = -a\sin\theta$

$\frac{du}{d\theta} = au^2 \sin\theta.$

$\frac{d^2u}{d\theta^2} = a \cdot 2u\frac{du}{d\theta} \cdot \sin\theta + au^2\cos\theta.$

$= 2au\,(au^2\sin\theta)\sin\theta + au^2\cos\theta$

$= 2a^2u^3\sin^2\theta + au^2\cos\theta$. But $\cos\theta = \frac{1}{ua} - 1$

$= 2a^2u^3\left[1 - \left(\frac{1}{ua}-1\right)^2\right] + au^2\left[\frac{1}{ua}-1\right].$

$= 2a^2u^3\left[\frac{2}{ua} - \frac{1}{u^2a^2}\right] + u - au^2.$

$\frac{d^2u}{d\theta^2} + u = 4au^2 - \cancel{2u} + \cancel{u} - au^2 + \cancel{u} = 3au^2$

$\Rightarrow f\left(\frac{1}{u}\right) = -\frac{L^2u^2}{m}\left[\frac{d^2u}{d\theta^2} + u\right] = -\frac{4m^2K^2}{m} \cdot u^2 \cdot 3au^2$

$f(r) = -\frac{12maK^2}{r^4}.$ $\boxed{f(r) = -\frac{12maK^2}{r^4}}$

(b) The potential satisfies $-\frac{\partial V}{\partial r} = f(r)$

$V(r) = 12maK^2\int_{\infty}^{r}\frac{1}{r^4}dr = 12 \cdot maK^2 \cdot \frac{1}{3} \cdot \frac{1}{r^3}\Big|_r^{\infty}$

$V(r) = -\frac{4maK^2}{r^3}$ $\boxed{V(r) = -\frac{4maK^2}{r^3}}$

Total energy $= V(r) + \frac{1}{2}mv^2$

ii) $T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)$.

Where $r = a(1+\cos\theta)$ $\qquad \dot{\theta} = \frac{2K}{r^2}$

$\Rightarrow \dot{r} = -a\sin\theta\,\dot{\theta} =$

$a\cos\theta = r-a$

$a^2\cos^2\theta = (r-a)^2$

$a^2\sin^2\theta = a^2-(r-a)^2$

$$T = \frac{m}{2}\left(a^2\sin^2\theta + r^2\right)\dot{\theta}^2$$

$$= \frac{m}{2}\left[a^2-(r-a)^2+r^2\right]\left(\frac{2K}{r^2}\right)^2$$

$$= \frac{m}{2}\left[\cancel{a^2}-\cancel{r^2}-\cancel{a^2}+2ar+\cancel{r^2}\right]\frac{4K^2}{r^4}$$

$$= \frac{4maK^2}{r^3}$$

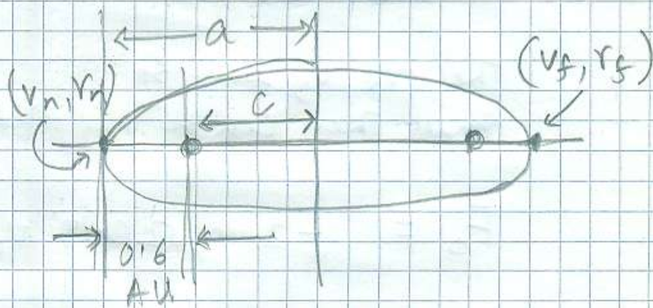
Total Energy $= T+V = \frac{4maK^2}{r^3} - \frac{4maK^2}{r^3} = 0$.

$\boxed{\text{Total Energy} = 0}$

(4) Earth's orbit is a circle 1 AU. Period 1 yr.

a) $\Rightarrow \left(\frac{T_{Halley}}{T_{earth}}\right)^2 = \left(\frac{a_{Halley}}{a_{earth}}\right)^3$ $\left.\begin{matrix} T_{earth} = 1 \\ a_{earth} = 1 \end{matrix}\right\}$ in these units.

$a = (76)^{2/3}$ in A.U.



Orbit must be an ellipse

nearest approach $= a - c$

Farthest approach $= a + c$

Since $a - c = 0.6$ A.U. $\Rightarrow c = a - 0.6$ A.U.

$\Rightarrow$ farthest approach $= a + a - 0.6$ AU $= \boxed{35.28 \text{ A.U.}}$

b) Ratio of K.E at nearest and farthest points:

At these points $\dot{r} = 0 \Rightarrow v_n r_n = v_f r_f$ $\left\{\begin{matrix} n = \text{near} \\ f = \text{far} \end{matrix}\right.$

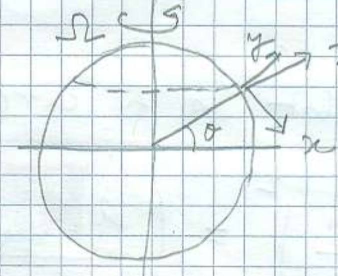
from angular momentum conservation

$\Rightarrow$ K.E. ratio $= \left(\frac{v_n}{v_f}\right)^2 = \left(\frac{r_f}{r_n}\right)^2 = \left(\frac{35.28}{0.6}\right)^2 \simeq \boxed{3457}$

c) The eccentricity $= \frac{c}{a} = \frac{a - 0.6}{a} = \boxed{0.967}$

5

The +ve direction of y is the "east" direction



P6

a) The Coriolis force

$$= -2m\vec{\Omega}\times\vec{v} = -2m\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -\Omega\cos\theta & 0 & \Omega\sin\theta\\ 0 & 0 & v_z\end{vmatrix}$$

The v_x, v_y are so small they can be neglected

$$F_c = -2m\Omega\cos\theta\, v_z\hat{j} \Rightarrow \boxed{a_y = -2\Omega\cos\theta\, v_z}$$

b) while going up & coming down exactly same values of V_z will occur with opposite sign, so $\int a_y dt = 0$

explicitly $V_z = V_0 - gt \quad t=0 \xrightarrow{\text{rise}} \frac{V_0}{g} \xrightarrow{\text{fall}} \frac{2V_0}{g}$

$$a_y = -2\Omega\cos\theta(V_0 - gt)$$

$$\Delta V_y = -2\Omega\cos\theta\int_0^{2V_0/g}(V_0 - gt)\,dt = -2\Omega\cos\theta\left(V_0 t - \frac{gt^2}{2}\right)\Bigg|_0^{\frac{2V_0}{g}}$$

$$= -2\Omega\cos\theta\left(V_0\cdot\frac{2V_0}{g} - \frac{g}{2}\cdot\frac{4V_0^2}{g^2}\right) = \boxed{0}$$

c)

$$\Delta y = \int_0^{2V_0/g} V_y\,dt = -2\Omega\cos\theta\, V_0\frac{t^2}{2}\Bigg|_0^{2V_0/g} + \Omega\cos\theta\, g\frac{t^3}{3}\Bigg|^{2V_0/g}$$

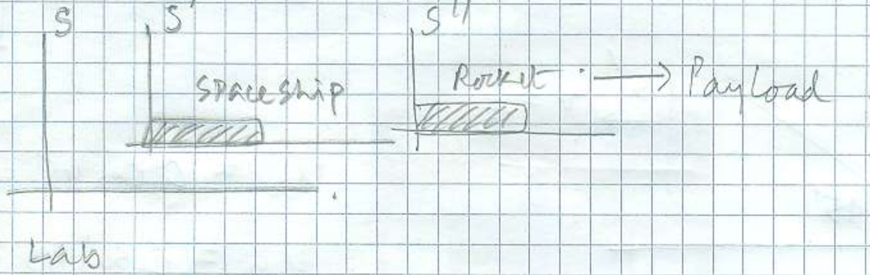
$$= -2\Omega\cos\theta\frac{V_0}{2}\left(\frac{2V_0}{g}\right)^2 + \frac{\Omega}{3}\cos\theta\, g\left(\frac{2V_0}{g}\right)^3$$

$$= -4\Omega\cos\frac{V_0^3}{g^2} + \frac{\Omega}{3}\cos\theta\frac{8V_0^3}{g^2}$$

$$= \boxed{-\frac{4}{3}\Omega\cos\theta\frac{V_0^3}{g^2}}$$

$-$ve sign $\Rightarrow$ WESTWARD DEFLECTION

Q6



Rocket's velocity is v w.r.t S'

Spaceship's velocity is v w.r.t. S

a) $$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$$ here $u_x' = v$ as well.

$$u_x = \frac{2v}{1 + v^2/c^2}$$

$$\boxed{\text{Rocket w.r.t. } S \quad \frac{2v}{1 + v^2/c^2}}$$

b) The velocity of S'' w.r.t S is then $v_2 = \frac{2v}{1+v^2/c^2}$

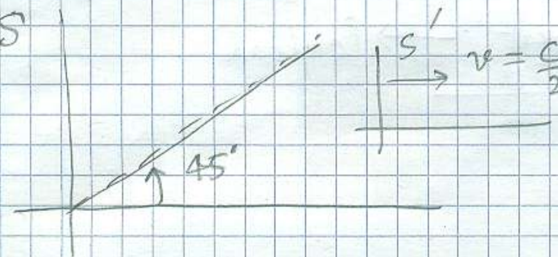
Payload's speed is v w.r.t to S''

$$u_x = \frac{u_x'' + v_2}{1 + \frac{u_x'' v_2}{c^2}} = \frac{v + \frac{2v}{1+v^2/c^2}}{1 + \frac{2v^2/c^2}{1+v^2/c^2}} = \frac{v\left(1+\frac{v^2}{c^2}\right) + 2v}{1 + \frac{3v^2}{c^2}}$$

$$= \frac{3v + \frac{v^3}{c^2}}{1 + \frac{3v^2}{c^2}} = \frac{1 + \frac{v^2}{3c^2}}{1 + \frac{3v^2}{c^2}} \cdot 3v$$

$$\boxed{\text{Payload w.r.t. } S : \quad \frac{1 + \frac{v}{3c^2}}{1 + 3v^2/c^2} \cdot 3v}$$

Q7.



$u_x = c\cos 45 = \frac{c}{\sqrt{2}}$

$u_y = c\sin 45 = \frac{c}{\sqrt{2}}$

a) $$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{\frac{c}{\sqrt{2}} - \frac{c}{2}}{1 - \frac{1}{2\sqrt{2}}} = \boxed{\frac{2-\sqrt{2}}{2\sqrt{2}-1}\, c} = 0{\cdot}32\,c$$

$$u_y' = \frac{u_y\sqrt{1 - v^2/c^2}}{1 - \frac{u_x v}{c^2}} = \frac{\frac{c}{\sqrt{2}}\sqrt{1 - \frac{1}{4}}}{1 - \frac{1}{2\sqrt{2}}} = \boxed{\frac{\sqrt{3}}{2\sqrt{2}-1}\, c} = 0{\cdot}95\,c$$

b) Angle observed by S'

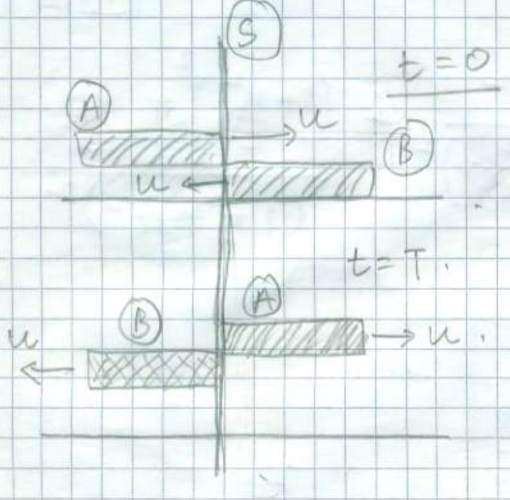
$$\tan\phi = \frac{u_y'}{u_x'} = \frac{\sqrt{3}}{2-\sqrt{2}} = 2{\cdot}957 \qquad \boxed{\phi = 71{\cdot}32^{0}}$$

c) $$u_x'^2 + u_y'^2 = \frac{(2-\sqrt{2})^2 + (\sqrt{3})^2}{(2\sqrt{2}-1)^2}\, c^2$$

$$= \frac{4 + 2 - 4\sqrt{2} + 3}{8 + 1 - 4\sqrt{2}}\, c^2 = \boxed{c^2}$$

Since the particle had a speed c in S
it must have the same speed in S'

Q8



$\beta = \frac{u}{c}$

rest length of A, B $\rightarrow L_0$

a) at $t=0$ S will observe the rear end of train B to be at $x = L_0\sqrt{1-\beta^2}$

when they cross $\boxed{T = \frac{L_0\sqrt{1-\beta^2}}{u}}$

b) In the frame of A (say S'), B approaches with a velocity

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \quad \text{with} \quad \begin{cases} u_x = -u \\ v = u \end{cases}$$

$$= \frac{-2u}{1 + u^2/c^2} \qquad \text{define } \beta' = \frac{u_x'}{c} = \frac{2u/c}{1 + u^2/c^2}$$

In A's frame when the engines just cross, B's rear end will be at $x' = L_0\sqrt{1-\beta'^2}$

This point will have to reach $x' = -L_0$ for crossing to be complete

$$T' = \frac{L_0 + L_0\sqrt{1-\beta'^2}}{u_x'} = \frac{L_0}{c}\,\frac{1+\sqrt{1-\beta'^2}}{\beta'}$$

$$1-\beta'^2 = 1-\frac{u'^2}{c^2} = 1-\frac{4u^2/c^2}{(1+u^2/c^2)^2} = \frac{\left(1+\frac{u^2}{c^2}\right)^2-\frac{4u^2}{c^2}}{(1+u^2/c^2)^2}$$

$$= \frac{(1-u^2/c^2)^2}{(1+u^2/c^2)^2}$$

$$\frac{1+\sqrt{1-\beta'^2}}{\beta'} = \frac{1+\frac{1-u^2/c^2}{1+u^2/c^2}}{\frac{2u/c}{1+u^2/c^2}} = \frac{\cancel{2}}{\cancel{2}\frac{u}{c}}$$

$$T' = \frac{L_0}{c}\frac{1+\sqrt{1-\beta'^2}}{\beta'} = \frac{L_0}{c}\cdot\frac{c}{u} = \frac{L_0}{u}$$

$$\boxed{T' = \frac{L_0}{u}}$$