Mathematical Foundations of Artificial Intelligence and Machine Learning (NCM-CEP Course)

Guest Lecture on Multi-agent AI

(a.k.a. **Game Theory and Mechanism Design**)

Swaprava Nath

CSE, IIT Bombay

February 21, 2023

Let us play a game: Neighboring King(Queen)dom's Dilemma

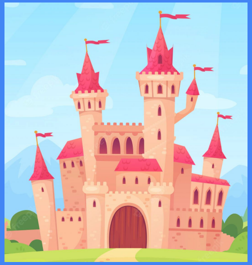
Each kingdom can invest either in Agriculture or War – but not both

- If both choose Agri – happiness is 5 for each
- If both choose War – happiness is 1 for each
- If one chooses Agri, but the other War – the Agri kingdom stand to lose everything and War kingdom gets happiness more than 5

What should the King / Queen do?

What is collectively best?

(Agriculture, Agriculture)



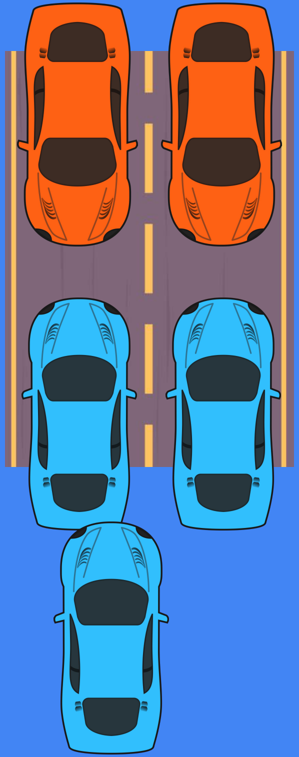
	Agriculture	War
Agriculture		
War		

Decisions are simultaneous

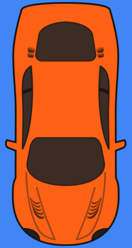
War is a Dominant Strategy for Queen as well as King

(War, War) is a Dominant Strategy Equilibrium

Another game: Traffic Movement



Does this game have a dominant strategy equilibrium?

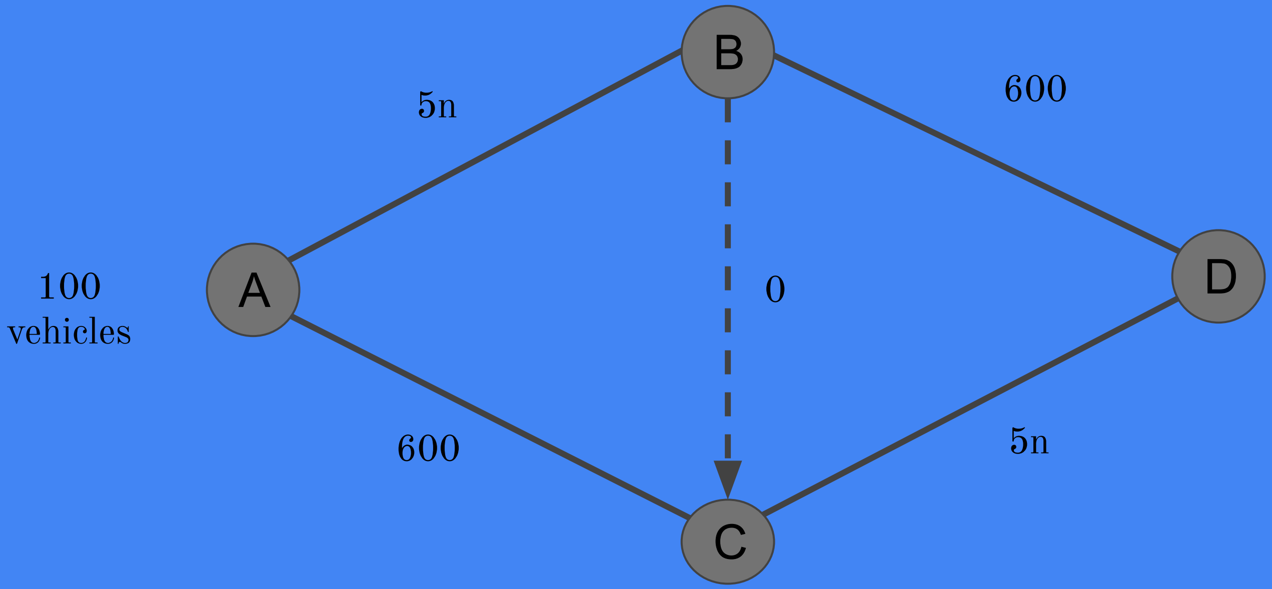


The Nash Equilibrium (John Nash, 1951)

	Left	Right
Left		
Right		

Equilibrium here is a strategy profile from where no player wants to unilaterally deviate

Adding resources (blindly) does not improve the society



n : number of vehicles on that road

Equilibrium (original)

50 vehicles each on paths ABD and ACD

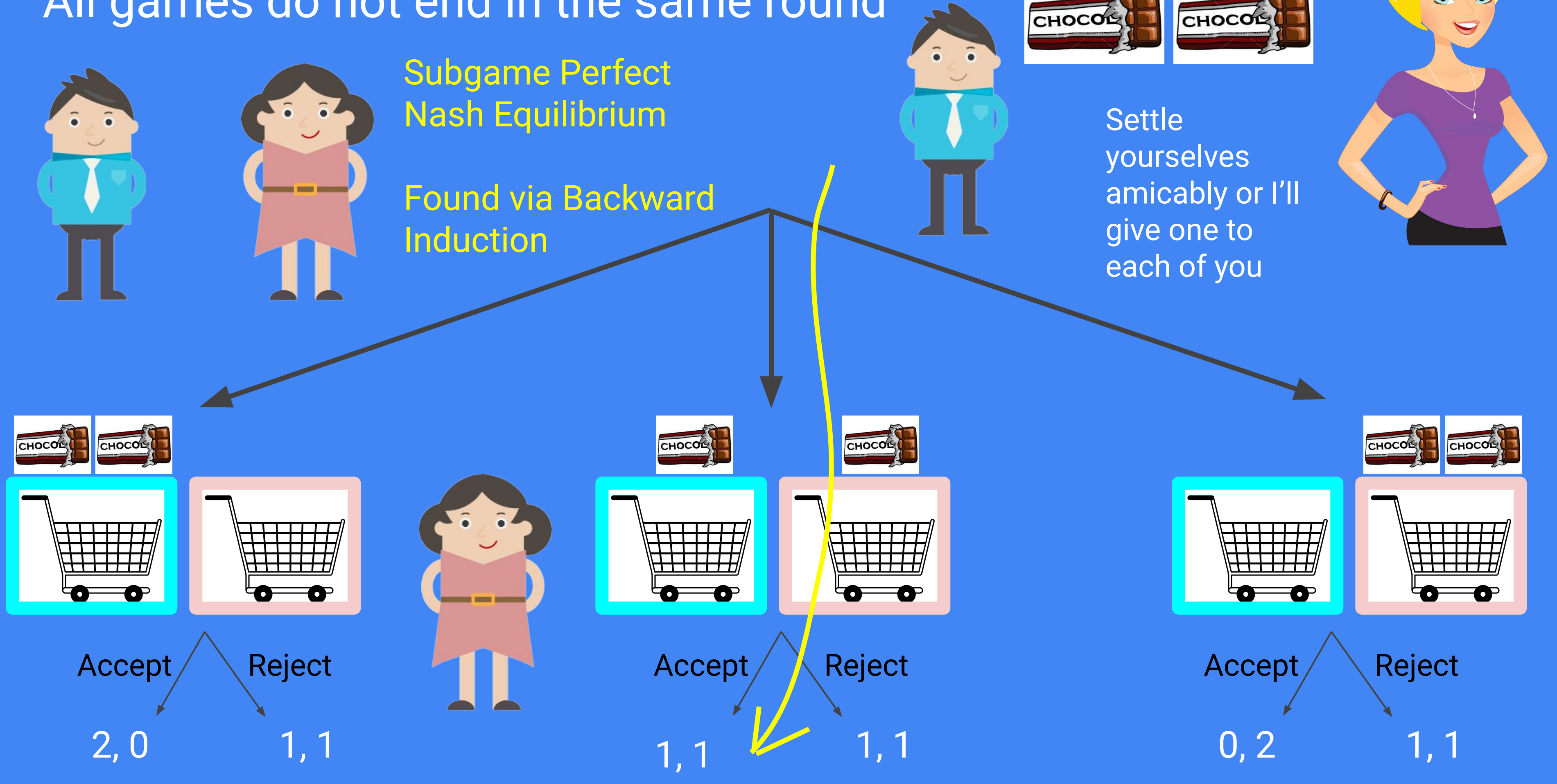
Time for each vehicle = 850

Equilibrium (after)

100 vehicles on path ABCD

Time for each vehicle = 1000

All games do not end in the same round



Can we design algorithms for a better society?

Fair division

Equal amount may have different values for an agent

Heterogeneous

Any fractional allocation is feasible

Divisible

Differing preferences

Different agents may have different preferences for the same piece

Proportional division

For each agent i

$v_i(A_i) \geq 1/n$

Each agent gets at least the average share

Normalization: for each i, $v_i([0,1]) = 1$

"I cut, you choose" algorithm

Envy free division

For each pair of agents i, j

$v_i(A_i) \geq v_i(A_j)$

Each agent likes her own share than others

Proportional?

Yes, agent 1 cuts ½, and agent 2 picks the larger

Envy-free?

Yes, agent 1 gets ½, which is the same as the other piece in his view
agent 2 picks first, can't envy the other piece

Fair division of indivisible objects

All are indivisible objects

Faculty retires and wants to give away his/her belongings to the department staff / existing faculty

Items:

1. Books
2. Shelves
3. Furnitures
4. Wall decor
5. Table decor
6. Electronic gadgets
7. Many more …

Notice that Envy-free allocation is no longer possible

Consider a single item and two agents

Envy free upto one good allocation

For each pair of agents i, j

$v_i(A_i) \geq v_i(A_j \setminus x_j)$, for some $x_j \in A_j$

Each agent likes her own share upto all but one item of every other agent

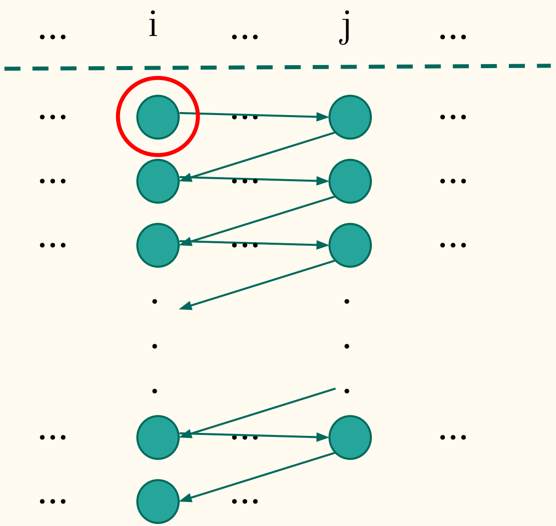
Envy free upto one good (EF-1)

All are indivisible objects

Always exists and computable in polynomial time!

Example for additive valuations: Round-Robin Algorithm

Place the agents in any arbitrary order, and ask them to pick their favorite remaining item



i does not envy j, since it picks before j

j may envy i, but not if the first item i picked is dropped

Round-Robin achieves EF-1 for additive valuations

Envy free upto one good allocation

For each pair of agents i, j

$v_i(A_i) \geq v_i(A_j \setminus x_j)$, for some $x_j \in A_j$

Each agent likes her own share upto all but one item of every other agent

- A game is an interaction between agents who want to maximize their utilities
- **Game theory** predicts the outcome of a game
- This is a predictive approach
- **Mechanism design** tries to design the game with desirable outcomes
- This is a prescriptive approach

References:

1. Lecture notes and videos from the modules 1, 2, 5, 6, 7 of course CS 6001 (webpage), and
2. For the fair division, lectures 7 and 8 of course CS 6002 (webpage)

General fun reading:
Nash equilibrium, Braess's paradox

If you are interested in probing further, two courses in the CSE department are relevant – lecture materials (and videos for CS 6001) are available

1. CS 6001: Game Theory and Algorithmic Mechanism Design
2. CS 6002: Selected Areas of Mechanism Design

Thanks!

swaprava@cse.iitb.ac.in

http://www.cse.iitb.ac.in/~swaprava