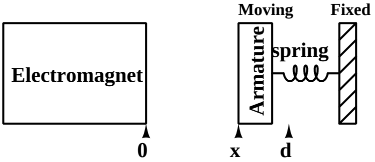
Relay Operation



Balance of Magnetic and Spring force:

- Let the origin be at the edge of (fixed) electro-magnet.
- Let the resting position of armature (when Spring force = 0) be d.

We want to analyse the balance of Magnetic and Spring forces when due to current I flowing in the electromagnet, the armature is at x.

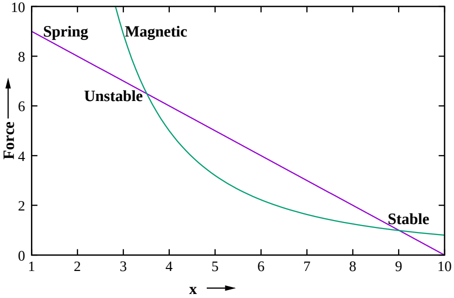
$$\text{Spring Force } = K_1(d-x), \quad \text{Magnetic Force } = \frac{K_2 I^2}{x^2}$$

Where K_1 is the spring constant, I is the current through the coil to bring the armature to x and K_2 is a constant dependent on the construction of the electro-magnet. In equilibrium:

$$K_1(d-x) = \frac{K_2 I^2}{x^2}$$

Solution in Equilibrium

In equilibrium: $K_1(d - x) = K_2 I^2/x^2$. Let us plot the two sides of this equation for $d = 10, K_1 = 1$ and $K_2 I^2 = 80$.



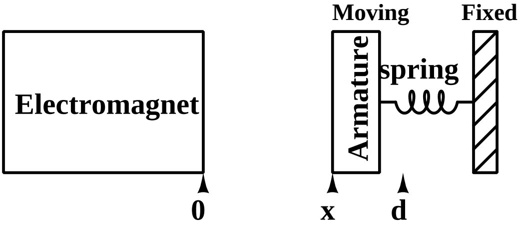
Magnetic and spring forces are equal at two values of x in the range 0 to d (=10).
(The third solution gives -ve x).

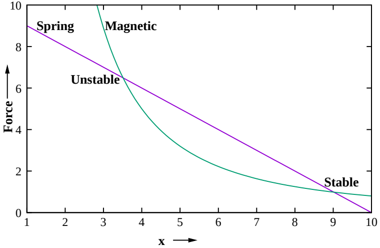
However, the equilibrium is stable only at the crossing point on the right.

- To check for stability at these two points, we consider a small perturbation in the value of x.
- Equilibrium at the crossing points will be stable if the residual force at the perturbed point brings it back to its original position.

Stable Equilibrium point

Consider the crossing point on the right (around $x \approx 9$).





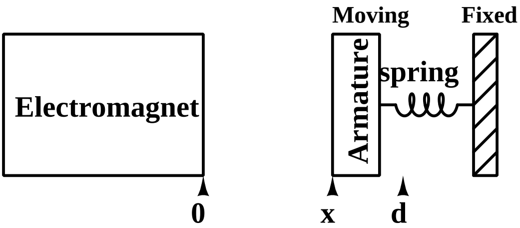
If we perturb the equilibrium point to the left,
spring force is $>$ magnetic force,
which will restore the equilibrium point back towards right.

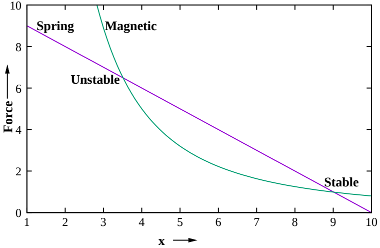
If we perturb the equilibrium point to the right,
spring force is $<$ magnetic force,
which will restore the equilibrium point back towards left.

Thus the equilibrium at the right crossing point is stable.

Unstable Equilibrium point

Now consider the crossing point on the left (around $x \approx 3.5$).





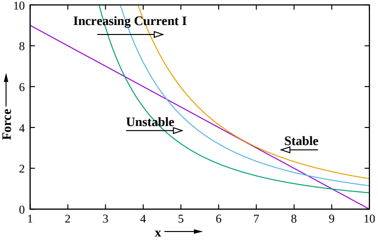
If we perturb the equilibrium point to the left,
spring force is $<$ magnetic force, which will pull the armature further towards left.

If we perturb the equilibrium point to the right,
spring force is $>$ magnetic force,
which will take the equilibrium point further towards right.

Thus the equilibrium at the left crossing point is unstable.

Armature position with Increasing current

As we increase the current, the armature will be pulled more and more towards the electromagnet.



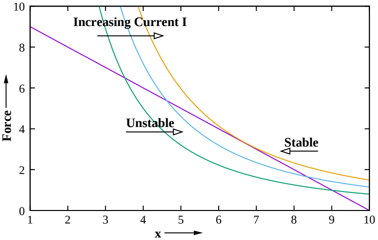
Increasing current will make the value of $K_2 I^2$ higher and higher, reducing the value of x in stable solution, and increasing it for the unstable solution.

In the critical case, the value of x for stable and unstable solutions will coincide.

If the current is made any higher than this critical value, there will be no solution in the range 0 to d.
Then the magnetic force will always be higher than the spring force and the armature will be pulled all the way to the electromagnet.

Critical current to pull the armature all the way

What is the value of the critical current which will ensure that the armature is pulled in all the way to the electromagnet?



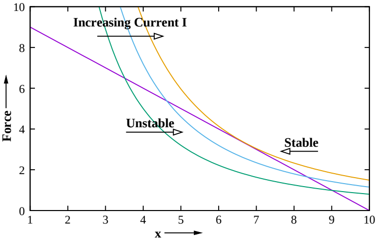
In the critical case, the spring force represented by the straight line $y = K_1 x$ will be a tangent to the inverse square law magnetic force, touching it at a single point.
At this point, the value as well as the derivative of the two forces must match.

Equating values and derivatives of the two forces:

$$K_1(d - x) = \frac{K_2 I^2}{x^2} \quad \text{and} \quad -K_1 = -2\frac{K_2 I^2}{x^3} \quad \text{So } K_2 I^2 = K_1 x^3/2$$

Critical current to pull the armature all the way

In the critical case, value as well as the slopes of the two forces must be equal.



$$K_1(d-x) = \frac{K_2 I^2}{x^2} \text{ and } -K_1 = -2\frac{K_2 I^2}{x^3}$$

$$\text{So } K_2 I^2 = K_1 x^3/2$$

$$\text{Therefore } K_1(d-x) = \frac{K_2 I^2}{x^2} = \frac{K_1 x}{2}$$

$$\text{Hence } d - x = \frac{x}{2} \quad \text{Which gives} \quad \frac{3}{2}x = d \quad \text{or} \quad x = 2d/3$$

This result is independent of values of K_1 or K_2!

To pull in the armature all the way, we need a current just greater than the value which will reduce the gap to two thirds of the zero current gap.

Pull in current of a Relay

At the pull in condition,

$$K_2 I^2 = K_1 x^3/2 \quad \text{so} \quad I^2 = \frac{K_1}{2K_2}\, x^3 = \frac{K_1}{2K_2}\left(\frac{2d}{3}\right)^3$$

$$\text{Thus } I^2 = \frac{4K_1}{27K_2}\, d^3 \qquad \text{Hence} \quad I = \sqrt{\frac{4K_1}{27K_2}}\, d^{3/2}$$

This gives the pull in current of a relay in terms of the spring constant K_1, electro-magnetic properties of the solenoid represented by K_2 and the gap between the electromagnet and the armature d.

The minimum voltage required to drive the relay will be given by

$$V = RI = R\sqrt{\frac{4K_1}{27K_2}}\, d^{3/2}$$

where R is the coil resistance.