$f : \mathbb{R} \longrightarrow \mathbb{R}$. $x_0 \in \mathbb{R}$.

$f'(x_0) \in \mathbb{R}$.

Consider way:

$$f'(x_0) \longleftrightarrow h \overset{F \,=\, Df}{\longmapsto} f'(x_0) \cdot h.$$

linear map.

$$\boxed{f(x_0+h) = f(x_0) + F(h) + O(h) \cdot h}$$

$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$. (x_0, y_0)

$Df(x_0) \longrightarrow$ derivative at (x_0, y_0).

$\longrightarrow$ linear map : $\mathbb{R}^2 \longrightarrow \mathbb{R}$.

$$f\big((x_0, y_0) + (h, k)\big) = f(x_0, y_0) + Df(x_0) + O((h,k))\,\|(h,k)\|$$

$\downarrow$

0 as $(h,k) \to (0,0)$.

Linear map

$f: \mathbb{R}^2 \longrightarrow \mathbb{R}$. $\longleftrightarrow$ 1×2 - matrix

$f\left((x_1, y_1) + (x_2, y_2)\right)$

$= f\left((x_1, y_1)\right) + f\left((x_2, y_2)\right)$

$f\left((\alpha x_1, \alpha y_1)\right) = \alpha \cdot f(x_1, y_1)$.

$[\alpha] \longleftrightarrow \alpha$.

$\left(h \longmapsto f'(x_0) \cdot h\right) \longleftrightarrow f'(x_0)$.