PH 112: Quantum Physics and Applications

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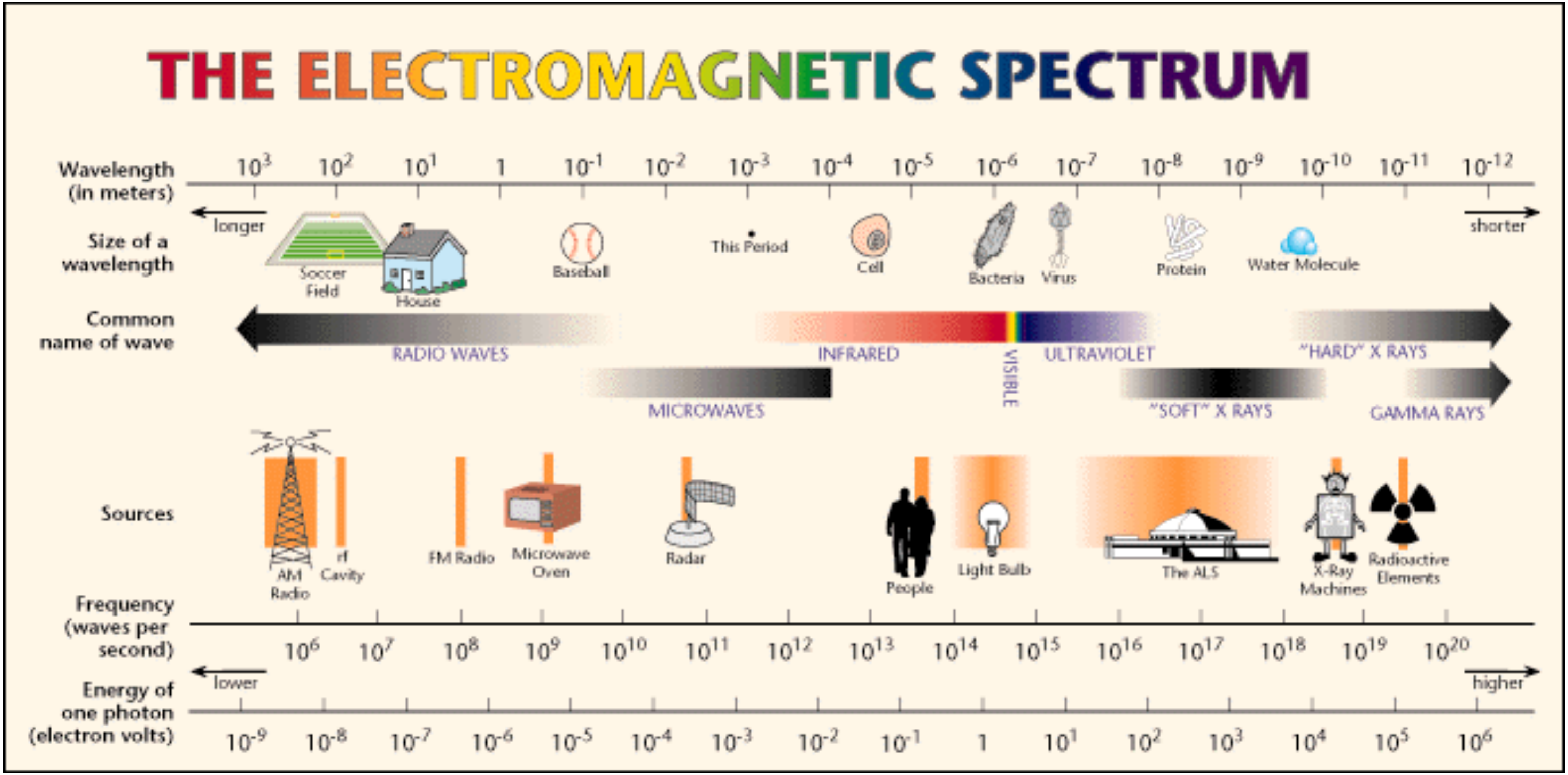
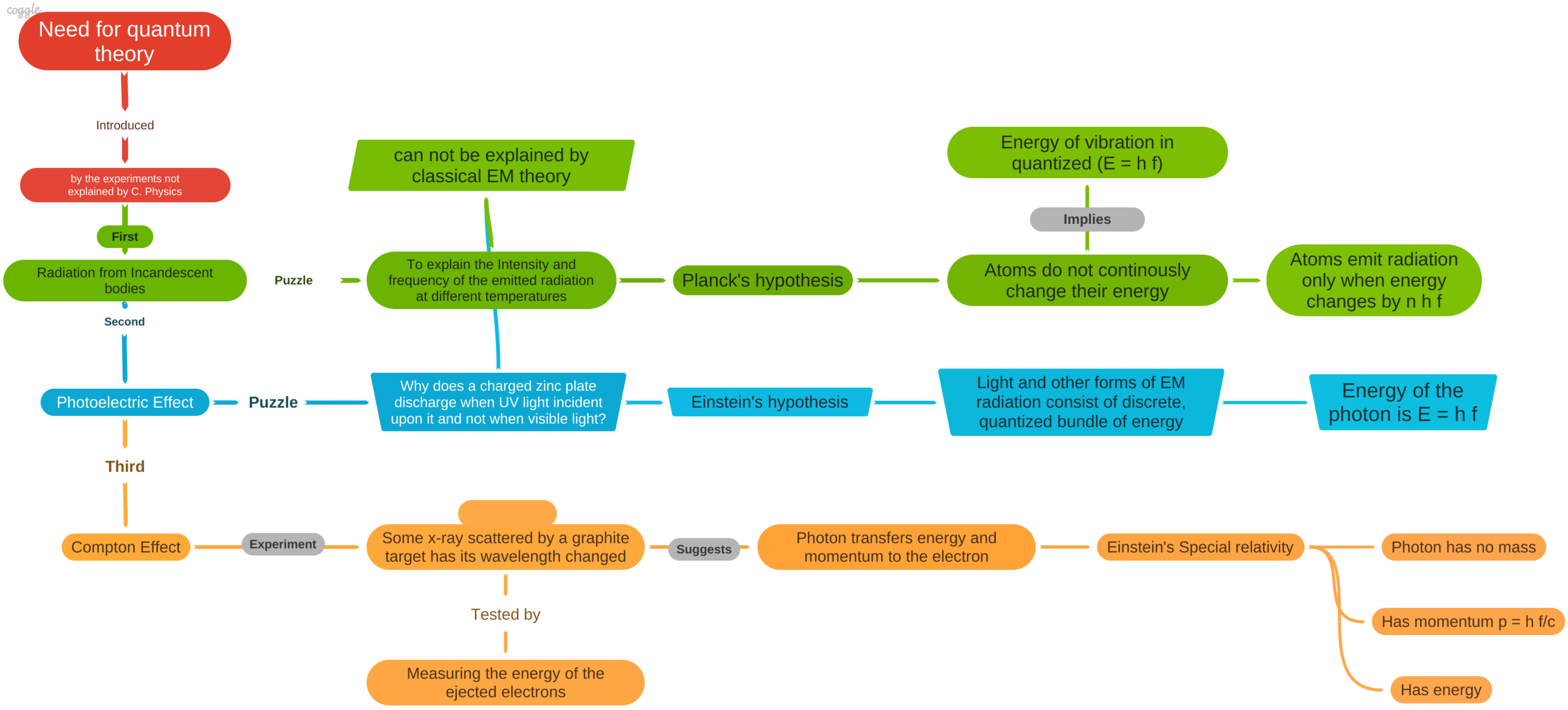
Week 01, Lecture 2 Compton Effect

D3, Spring 2023

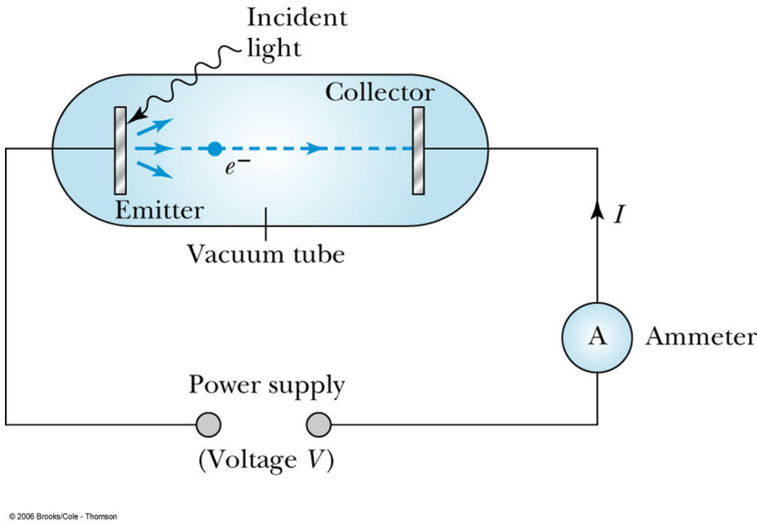
Electromagnetic spectrum



Three experiments that C.Physics can not explain



Photoelectric Effect



Experimental observations:

- Incident EM wave shining on the material transfers energy to electrons, allows electrons to escape.
- Only sufficiently energetic light makes an effect.
  Light with enormous intensity and insufficient energy has no effect.
- Different materials had different threshold frequency.

Inferences:

- The kinetic energy of the photoelectrons are independent of the light intensity.
- Classical theory predicts that the total amount of energy in a light wave increases as the light intensity increases. Threshold frequency is inexplicable in classical.
- In Photoelectric effect, light behaves like a particle rather than like wave.

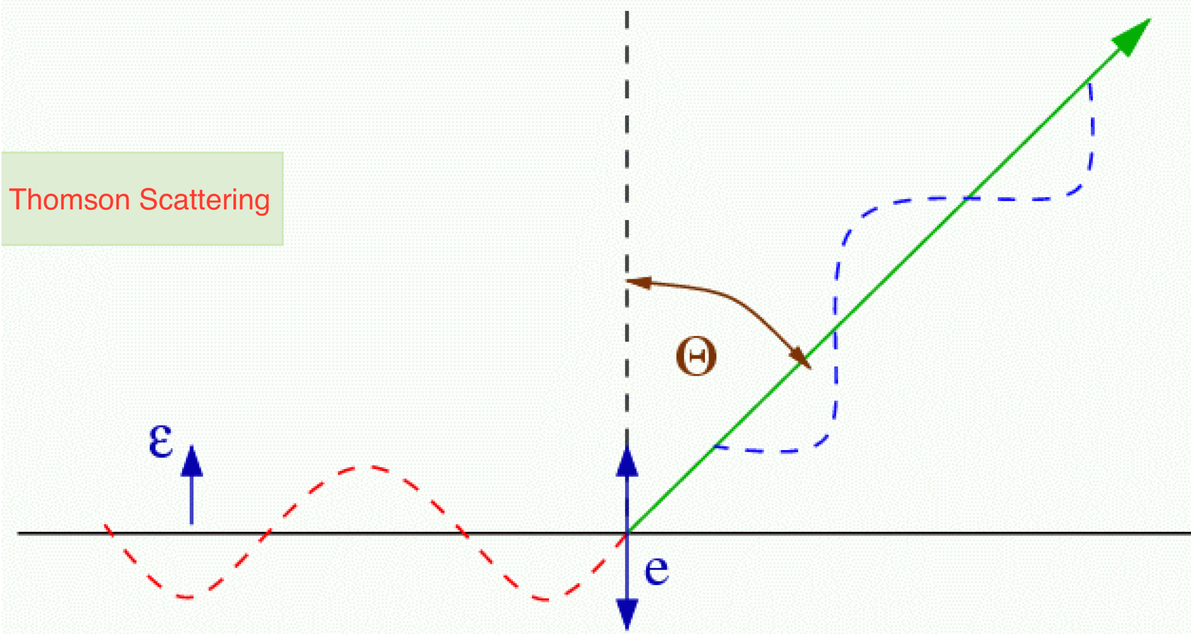
This lecture will discuss

Another experimental evidence for particle like properties of EM wave.

How does light scatter from an electron?

Thomson Scattering

Classically, the process is referred to as Thomson scattering



In Thomson scattering an electromagnetic (EM) wave of frequency *f* is incident on an electron.

What happens to the electron?

The electron absorbs energy from the EM wave and scatters it in a different direction.

Thomson Scattering

Experimental observations

- Wavelength of the scattered wave is the same as the incident wave.
- Scattered waves are in-phase with the incident wave.

Assumptions

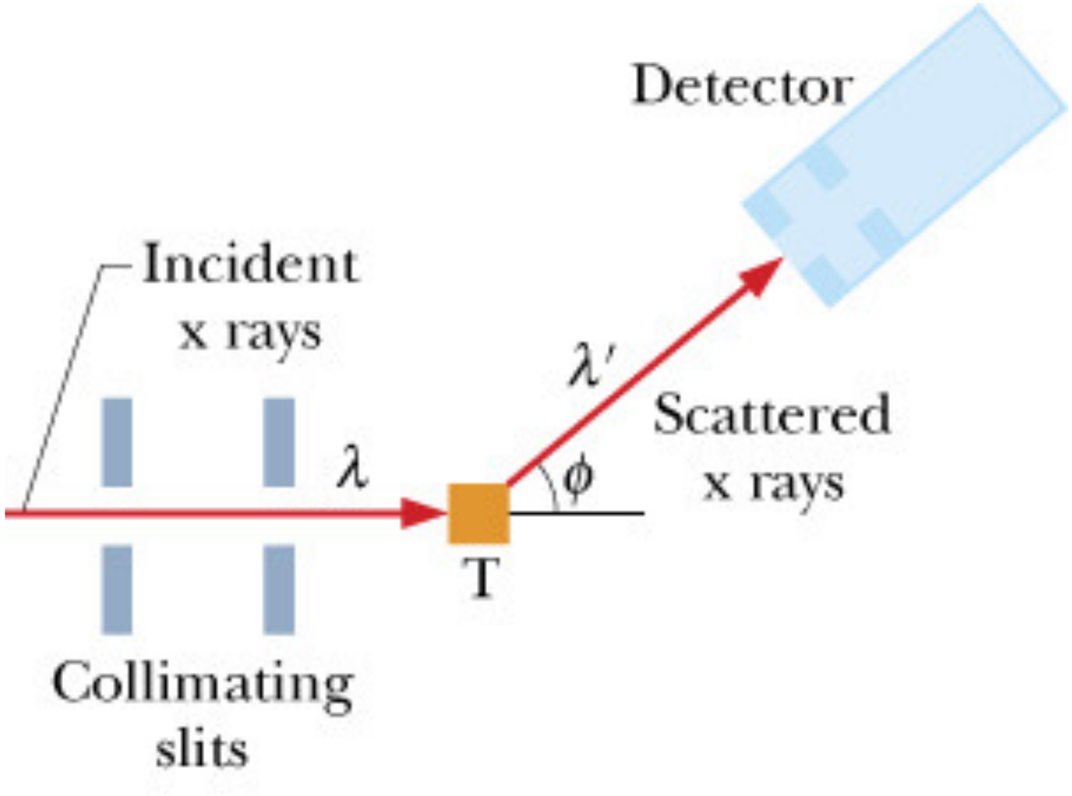
1. Wavelength of EM wave is large compared the size of an atom
2. Energy of EM wave $(h\ f)$ is much smaller than energy of electron (0.5 MeV m_ec^2)

What happens if the energy of EM wave is comparable to electron energy?

Compton’s experimental setup

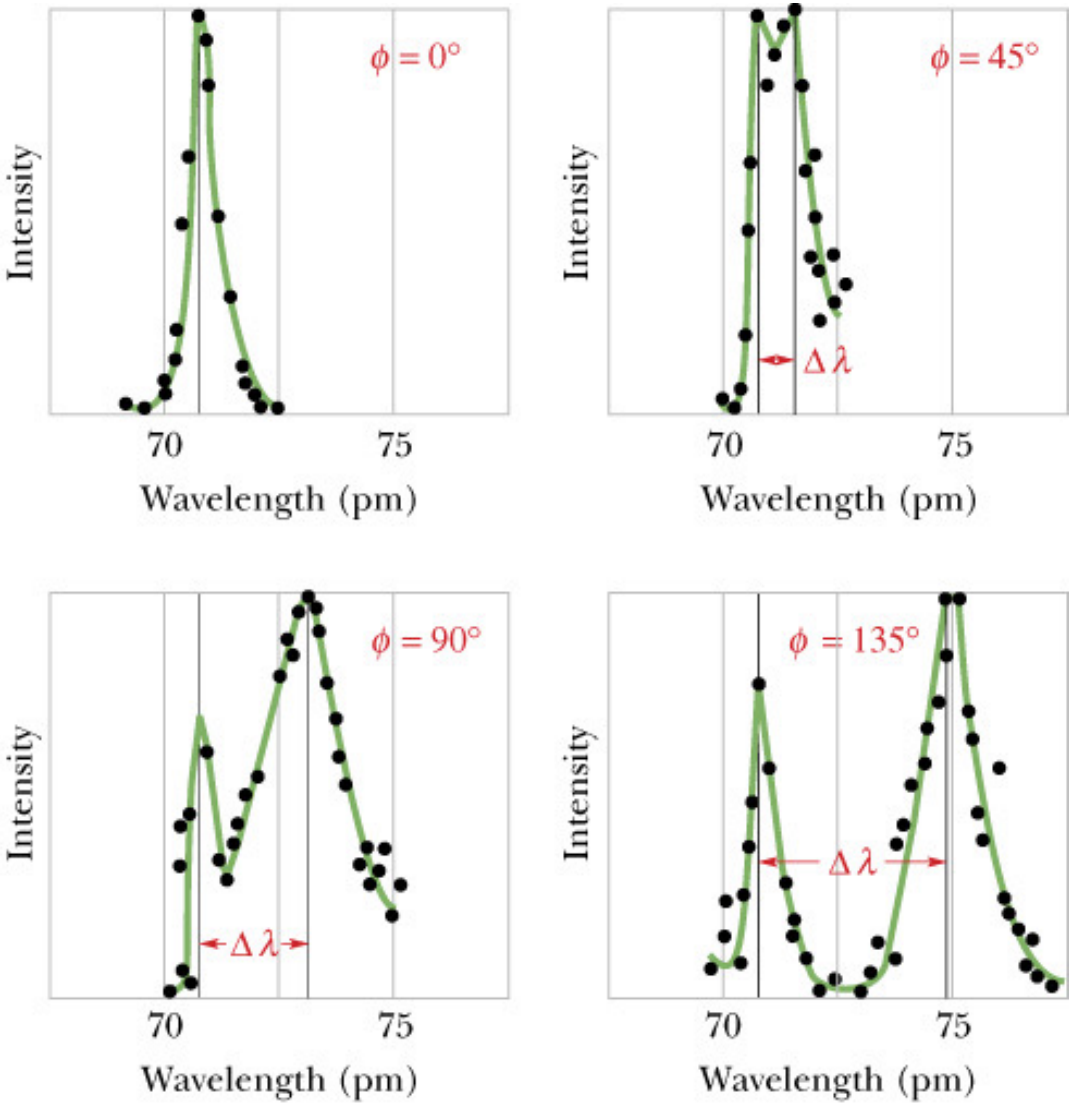
Compton's experimental setup

- A beam of x rays of wavelength 71.1 pm (10^{18} Hz) is directed onto a carbon target T.
- Photon's energy is very high (100 KeV) compared to binding energy of the electron (~ 10 eV).
- The x rays scattered from the target are observed at various angle ϕ to the direction of the incident beam.
- The detector measures both the intensity of the scattered x rays and their wavelength



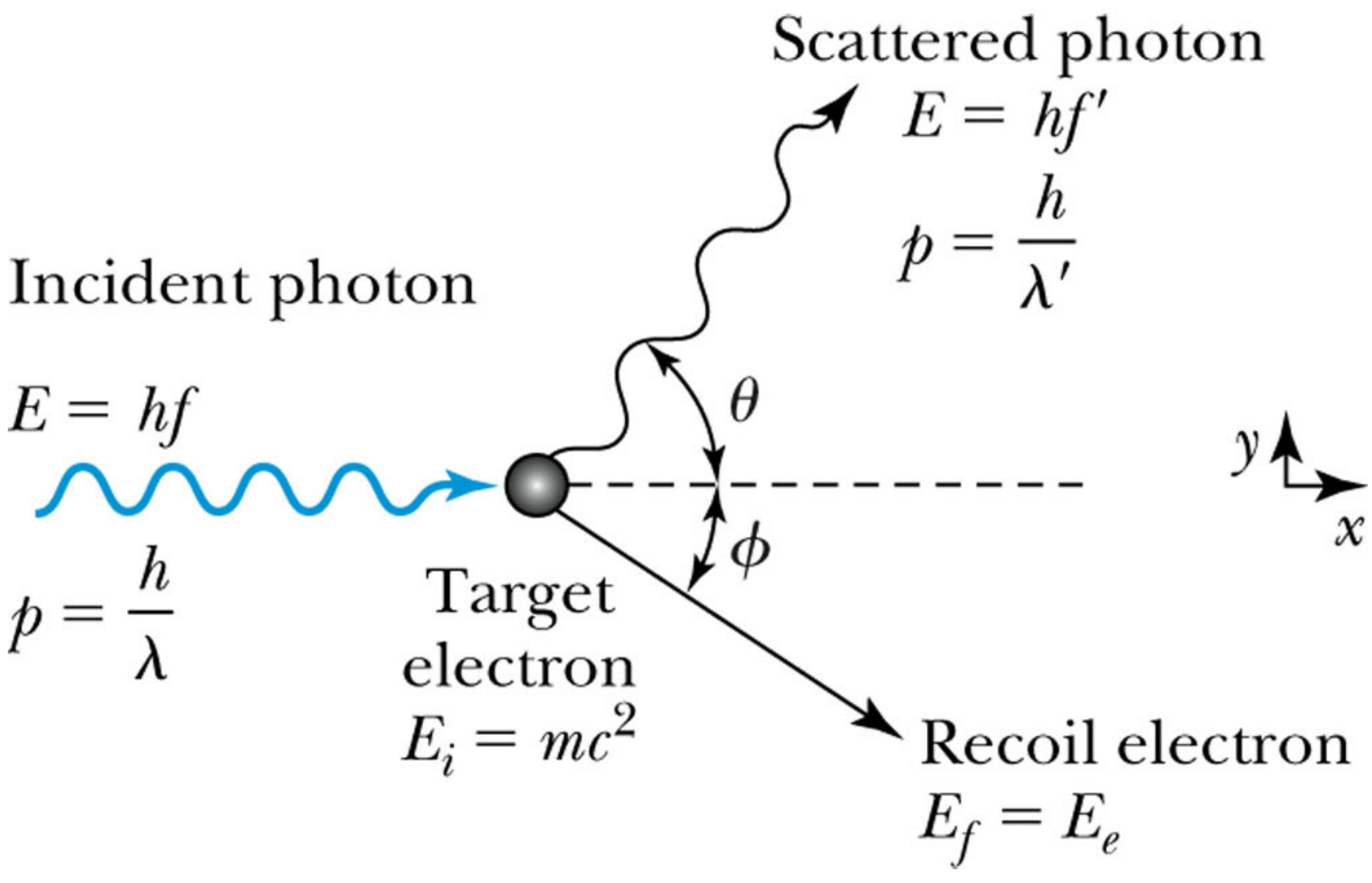
Experimental data and inference

- The incident beam consists of a single well-defined wavelength (λ).
- The scattered x-rays at an angle (ϕ) have intensity peaks at two wavelengths (λ, λ'). $\Delta\lambda = \lambda' - \lambda > 0 \Longrightarrow \lambda' > \lambda$.
- $\Delta\lambda$ varies with ϕ.
- *According to classical wave theory, there should not be any shift in the frequency.*
- *Compton effect can not be explained by classical wave theory of light.*



Modeling Compton shift

Modeling as particle-particle collision



- Compton explained this in-terms of collision between collections of (particle-like) Photon, each with energy ($E = h\nu = pc$) *with free electrons in the target.*
- Photons energy is very high (100 KeV) compared to binding energy of the electron (~ 10 eV).
- Even if you shine these photons on bound electrons, it is almost like shining these photons on free electrons.

Energy-momentum Relation in Special relativity

Energy is $E = \gamma_u mc^2$ Momentum is $p = \gamma_u mu$

Squaring and subtracting the second equation from the first:

$$E^2 - p^2c^2 = m^2c^4\gamma^2{}_u\left(1 - \frac{u^2}{c^2}\right) = m^2c^4$$

Energy-momentum relation: $E^2 = p^2c^2 + (mc^2)^2$

For particles at rest *(u = 0)* we have $p = 0, E = mc^2$

For particles with zero mass (Photons), total energy is $E = pc$

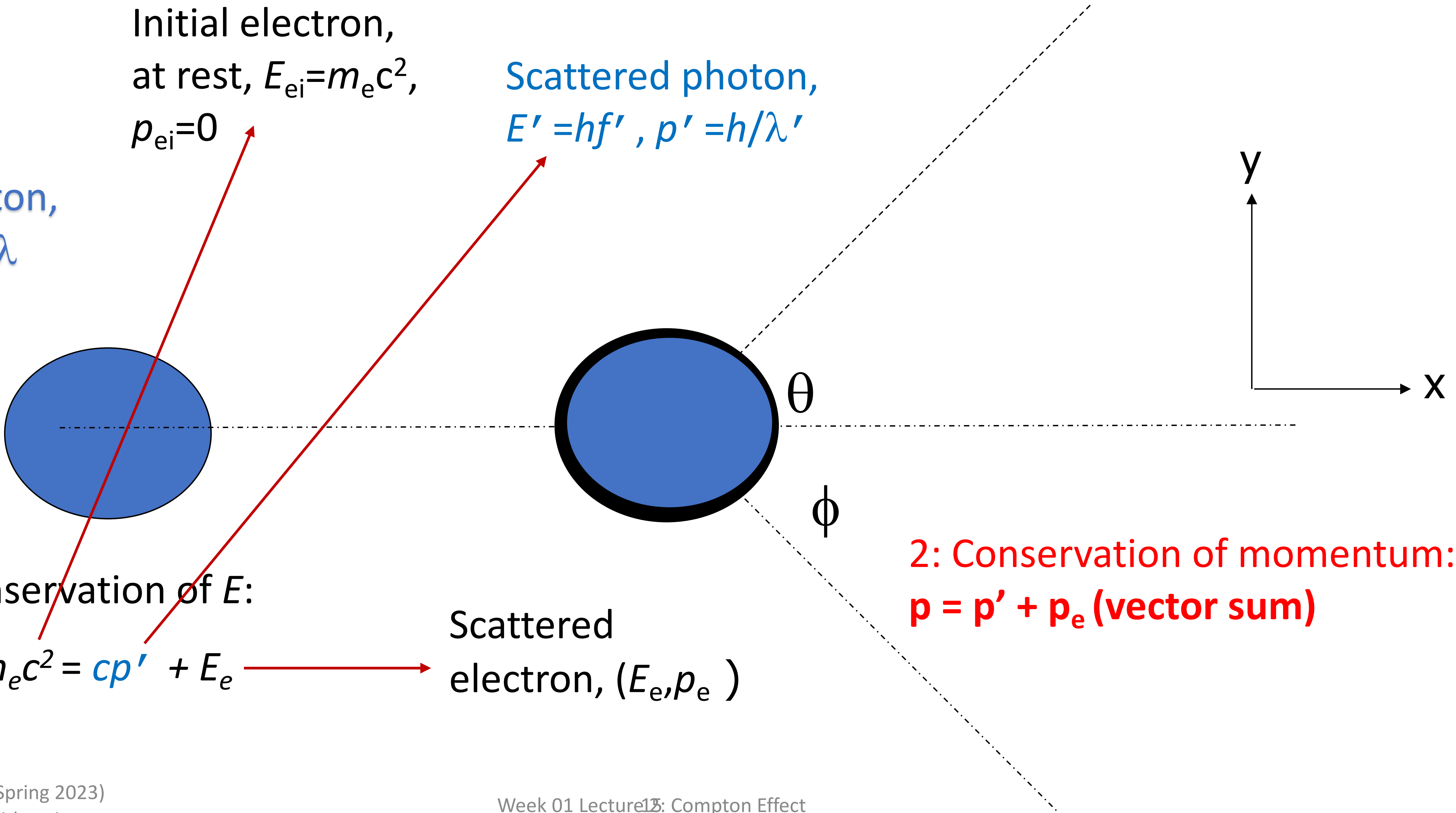
$$E = h\nu = \left(\frac{h}{\lambda}\right)c \Longrightarrow p = \frac{h}{\lambda}$$

Two particle collision in 2-D

Initial electron,
at rest, $E_{ei}=m_e c^2$,
$p_{ei}=0$

Scattered photon,
$E' = hf'$, $p' = h/\lambda'$

Initial photon,
$E=hf$, $p=h/\lambda$



1: Conservation of E:

$cp + m_e c^2 = cp' + E_e$

Scattered
electron, (E_e, p_e)

2: Conservation of momentum:
$\mathbf{p} = \mathbf{p'} + \mathbf{p}_e$ (vector sum)

Compton Effect: Mathematical setup

- Momentum conservation: Comprises of two equations.
  For conversation along x- and y-direction:

$$\mathrm{x-axis} \quad p = p' cos\theta + p_e \cos\phi$$

$$\mathrm{y-axis} \quad p' \sin\theta = p_e \sin\phi$$

- Energy conservation

$$hf + m_e c^2 = hf' + E_e = hf' + \left(p_e^2 c^2 + m_e^2 c^4\right)^{1/2}$$

Compton Effect: Some algebra

- From momentum conservation

$$p_e^2 = p^2 + p'^2 - 2\vec{p}\cdot\vec{p}' = p^2 - p'^2 - 2p\cdot p'\cos\theta$$

$$p_e^2 = \left(\frac{hf}{c}\right)^2 + \left(\frac{hf'}{c}\right)^2 - 2\frac{hf}{c}\frac{hf'}{c}\cos\theta$$

- From energy conservation

$$m_e^2c^4 + (hf - hf')^2 + 2m_ec^2\left(hf - hf'\right) = m_e^2c^4 + p_e^2c^2$$

$$p_e^2 = \left(\frac{hf}{c}\right)^2 + \left(\frac{hf'}{c}\right)^2 - \frac{2hfhf'}{c^2} + 2m_e\left(hf - hf'\right)$$

- Eliminating p_e^2

$$m_ec^2\left(hf - hf'\right) = hfhf'\left(1 - \cos\theta\right)$$

Compton Effect

- Rewriting we have

$$\frac{f-f'}{ff'} = \frac{h}{m_e c^2}(1-\cos\theta)$$

- Using *f=c/λ* we arrive at the Compton effect

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$$

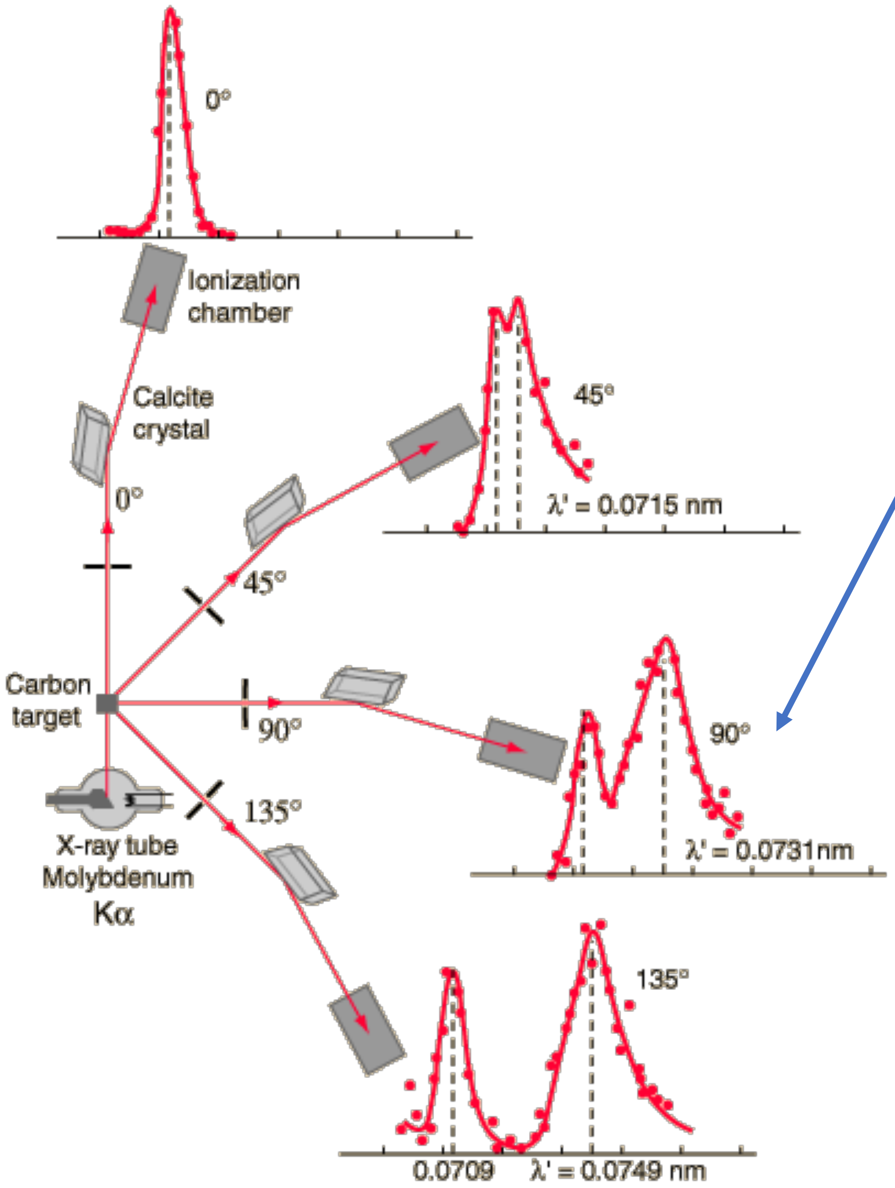
- Consider two limits:
  - $\theta = 0$ $\quad \lambda' = \lambda$ (No change)
  - $\theta = \pi$ $\quad \lambda' = \lambda + 2\,\lambda_C$ (maximum change)

$$\lambda_C = \frac{h}{m_e c} = 2.43 \times 10^{-12}\, m$$

Compton wavelength

Expt. Verification of Compton Effect

Compton Effect: Interpretation of the result



1. Let us first look at the second peak.

2. According to the calculations, the second peak must be due to Compton effect! At 90 degrees, the Compton formula leads to

   $$(0.0731 - 0.07029)\mathrm{nm} = 0.0024\ \mathrm{nm} = \lambda_C$$

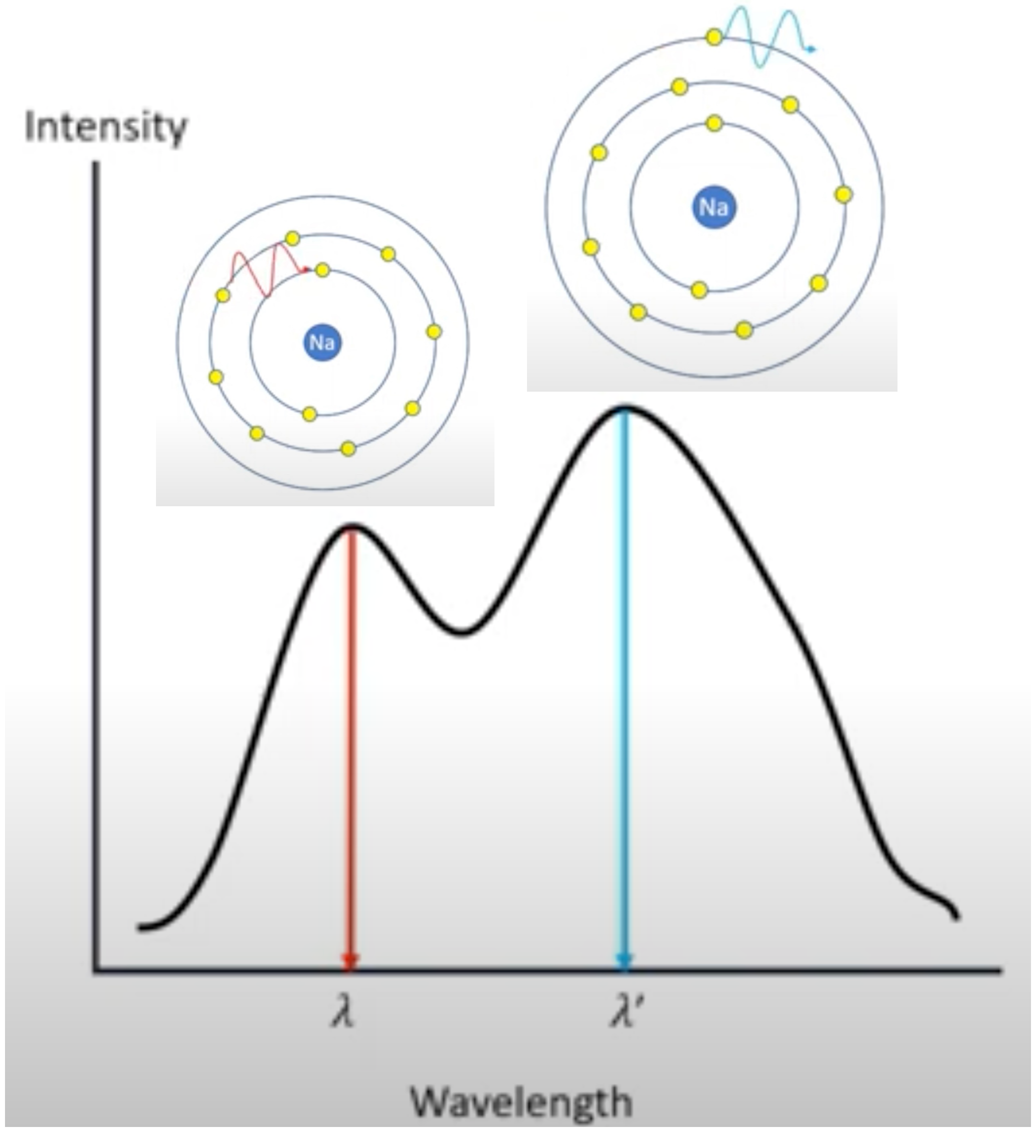
3. The difference between the initial and final wavelength is close to the Compton wavelength!

   Thus, second peak is due to Compton scattering of electrons.

4. What is the cause of the first peak?

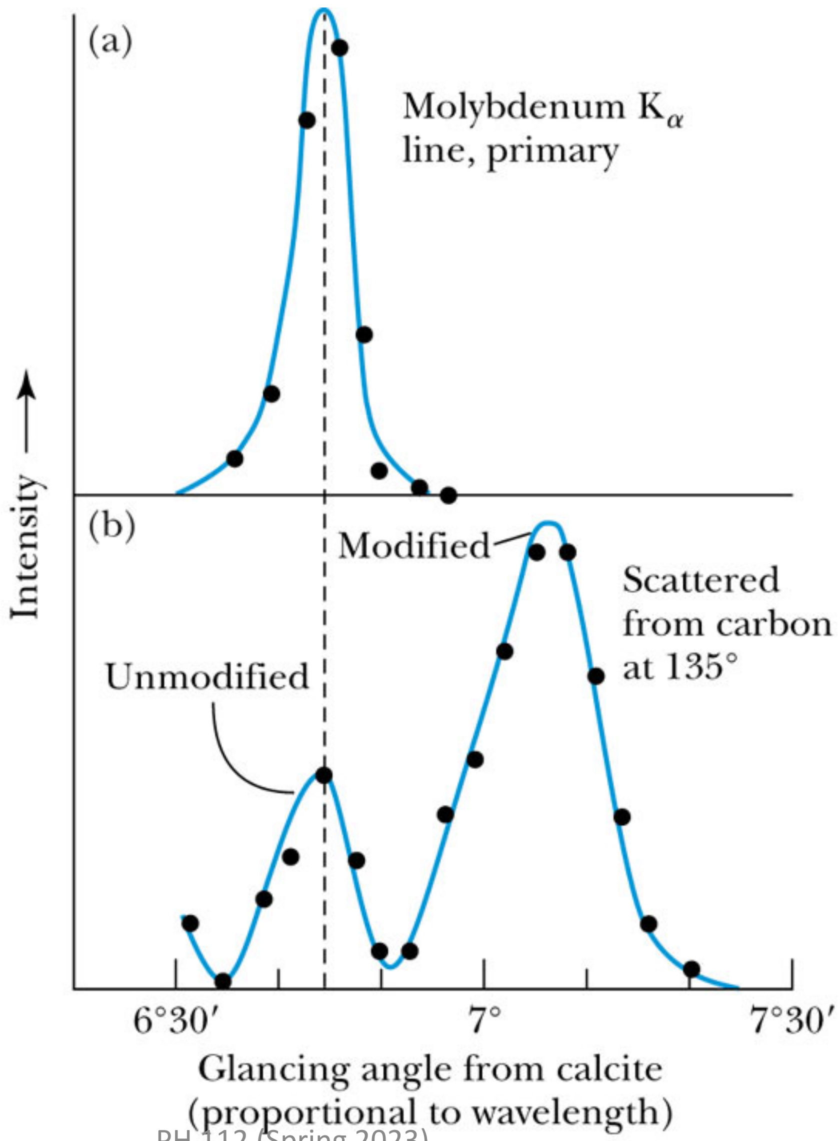
   Hint: The first peak occurs at the same wavelength as λ_i

Compton Effect: Understanding first peak



- Compton scattering occurs because the photon strikes the outer electron.
- First peak occurs because the photon strikes the inner electron.
- Binding energy of the Inner electron is much larger than the photon energy.
- Hence, the electron does not escape the atom.
  $\Longrightarrow$ Photon scatters off the entire atom (not electron). Since the atom is much more massive than the electron, momentum transfer from photon to atom is small and atom hardly moves!
- Thus, the change in the wavelength of the photon is negligible (like in the case of Thomson scattering).
- Hence, we see a first peak with identical initial and final momentum.

Compton Effect: Experimental verification



1. Intensities drop significantly as variable angle ϕ increases, but this is just an experimental difficulty!

2. In (a) only one peak, transmitted X-rays plus inelastically scattered X-rays with have a slightly longer wavelength from both atoms and electrons overlap

3. In (d) , the inelastically scattered X-rays from the atoms and electrons clearly separate the spacing's between these two peaks is in ***good*** agreement with Compton result:

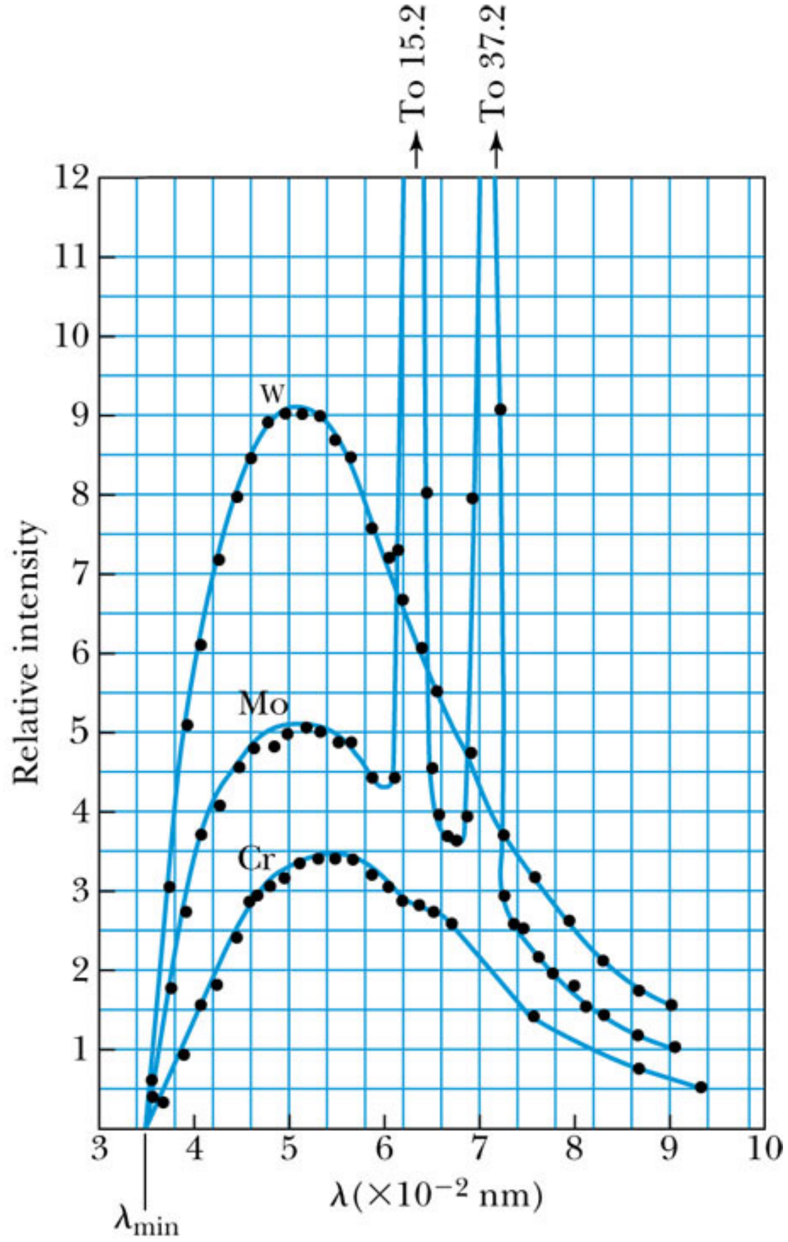
$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Compton Scattering

X-ray spectrum produced by bombarding a metal with electrons

1. Line spectra correspond to atomic electron transitions in an excited atoms
2. Continuum corresponds to the emission of radiation from accelerated electrons (scattered by the Coulomb force of atomic nuclei)

$$S = \frac{2}{3}\frac{q^2 a^2}{c^3}$$



Why were X-rays required for Compton's scattering experiments?

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$

EXAMPLE 3.8 X-ray Photons versus Visible Photons

(a) Why are x-ray photons used in the Compton experiment, rather than visible-light photons? To answer this question, we shall first calculate the Compton shift for scattering at 90° from graphite for the following cases: (1) very high energy γ-rays from cobalt, $\lambda = 0.0106$ Å; (2) x-rays from molybdenum, $\lambda = 0.712$ Å; and (3) green light from a mercury lamp, $\lambda = 5461$ Å.

Solution In all cases, the Compton shift formula gives $\Delta\lambda = \lambda' - \lambda_0 = (0.0243\text{ Å})(1 - \cos 90°) = 0.0243\text{ Å} = 0.00243$ nm. That is, regardless of the incident wavelength, the same small shift is observed. However, the fractional change in wavelength, $\Delta\lambda/\lambda_0$, is quite different in each case:

γ-rays from cobalt:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{0.0243\text{ Å}}{0.0106\text{ Å}} = 2.29$$

X-rays from molybdenum:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{0.0243\text{ Å}}{0.712\text{ Å}} = 0.0341$$

Visible light from mercury:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{0.0243\text{ Å}}{5461\text{ Å}} = 4.45 \times 10^{-6}$$

Because both incident and scattered wavelengths are simultaneously present in the beam, they can be easily resolved only if $\Delta\lambda/\lambda_0$ is a few percent or if $\lambda_0 \leq 1$ Å.

(b) The so-called free electrons in carbon are actually electrons with a binding energy of about 4 eV. Why may this binding energy be ignored for x-rays with $\lambda_0 = 0.712$ Å?

Solution The energy of a photon with this wavelength is

$$E = hf = \frac{hc}{\lambda} = \frac{12\,400\text{ eV}\cdot\text{Å}}{0.712\text{ Å}} = 17\,400\text{ eV}$$

Therefore, the electron binding energy of 4 eV is negligible in comparison with the incident x-ray energy.

Nobel prize 1927 for Arthur Holly Compton, so there is no doubt that the photons indeed behave like particles in collisions

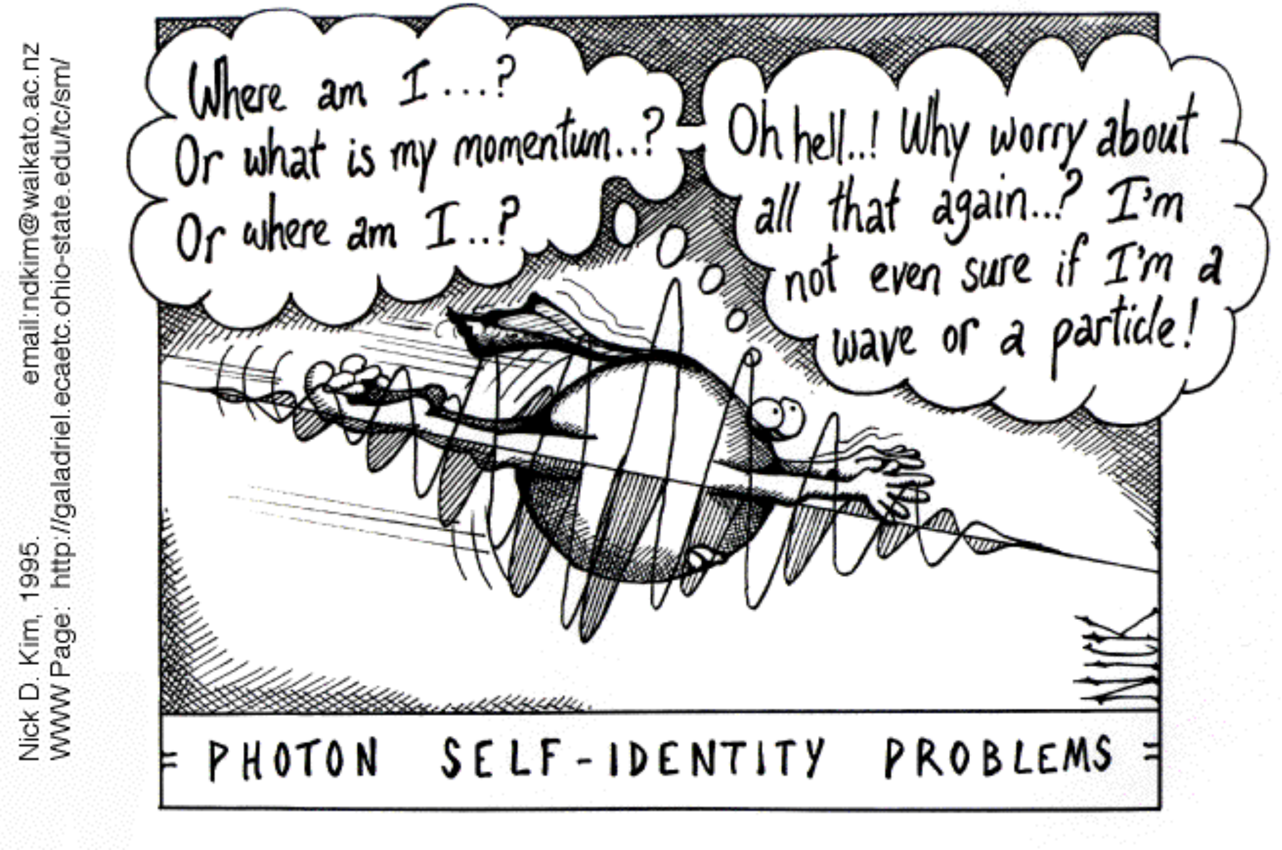
As Einstein had predicted already in 1906: *"If a bundle of radiation causes a molecule to emit or absorb an energy packet hf, then momentum of quantity* $^{hf}/_{c}$ *is transferred to the molecule, directed along the line of motion of the bundle for absorption and opposite to the bundle for emission"*

(for photons, $p = {}^{E}/_{c}$ from special relativity 1905)

As people had enormous difficulties with this, Einstein wrote in 1911: *"I insist on the provisional nature of this concept which does not seem reconcilable with the experimentally verified consequences of the wave theory."*

What is a photon?

- Like an EM wave, photons travel with speed of light *c.*
- They have zero mass and rest energy.
- The carry energy and momentum, which are related to the frequency and wavelength of the EM wave by $E=hf$ and $p = h/\lambda$
- They can have particle-like collisions with other particles such as electrons.



Summary: Phenomena can not be explained from classical physics

- Blackbody radiation
  - Rayleigh-Jeans classical formula clearly incorrect at explaining spectrum
  - Planck: oscillators with fixed energies
- Compton scattering
  - Scattered photons have different wavelength in contrast to classical description

Recommended Reading

The Compton effect and X-rays, section 3.5 in page 86.