Computing: Structures; Combinations; Algorithms

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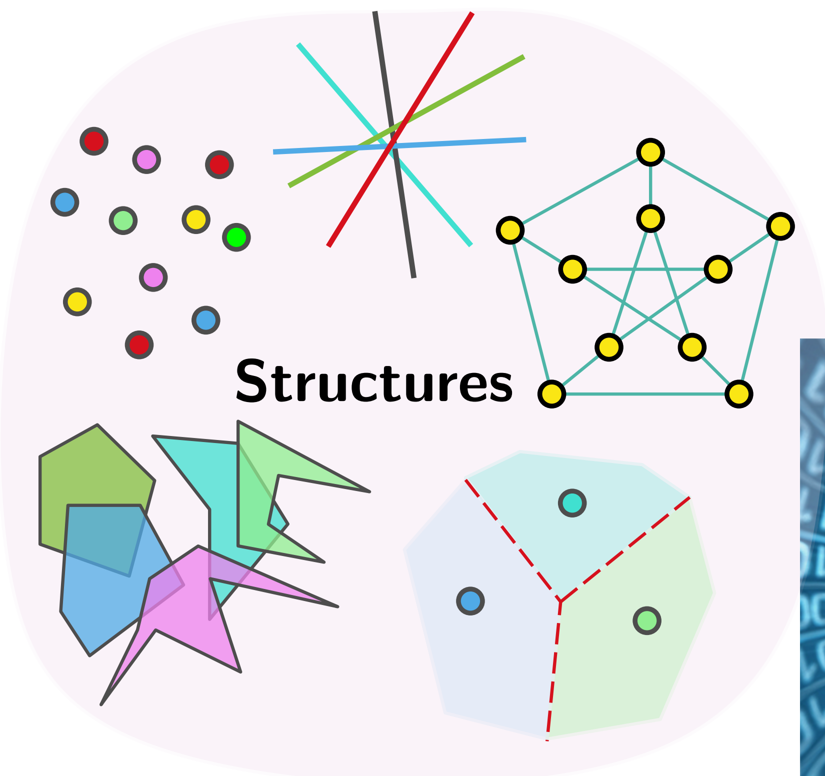
Indian Institute of Technology Bombay

Computing ...



$$f_{k-1} = \sum_{i=k}^{d} (-1)^{d-i} \binom{i}{k} f_{i-1}\,.$$

$$N(\mathcal{A}) = \textstyle\sum_{x \in L(\mathcal{A})} (-1)^{r(x)} \mu(0,x)$$

$$\#(tP \cap \mathbb{Z}^n) = \sum_{i=0}^{d} a_i t^i,$$

$$\chi_T(\phi+2) = (\phi+2)\,\phi^{3V(T)-10}\,(\chi_T(\phi+1))^2$$

$$\chi(KG_{n,k}) = n - 2k + 2$$

$$\frac{2(2n)!3^n}{n!(n+2)!}$$

$$\frac{\lambda_1 \lambda_2 \ldots \lambda_{n-1}}{n}$$

$$\frac{\alpha(G)}{n} \leq \frac{\lambda_{\min}}{d - \lambda_{\min}}$$

$$(1-\lambda_2)/2 \leq h(G) \leq \sqrt{2(1-\lambda_2)},$$

$$\Theta(C_5) = \sqrt{5}.$$

$$\mu_c := \lim_{n\to\infty} c_n^{1/n} = \sqrt{2+\sqrt{2}}.$$

$$\ell_n \sim 2\sqrt{n}.$$

$$\lim_{n\to\infty} \mathrm{Prob}\left(\frac{\mathrm{is}(\sigma) - 2\sqrt{n}}{n^{1/6}} \leq t\right) = F(t)$$

$$b \geq v.$$

$$\mathrm{per}(A) \geq \frac{n!}{n^n}$$

$$W(C^{\perp}; x, y) = \frac{1}{|C|} W(C; y-x, y+x).$$

$$|C| \leq \sum_{i=0}^{d} \binom{n}{i},$$

$$(2/e)(1+o(1))k2^{k/2} \leq r(k+1,k+1) \leq k^{-C\log k/\log\log k}\binom{2k}{k}.\ ^{*}$$

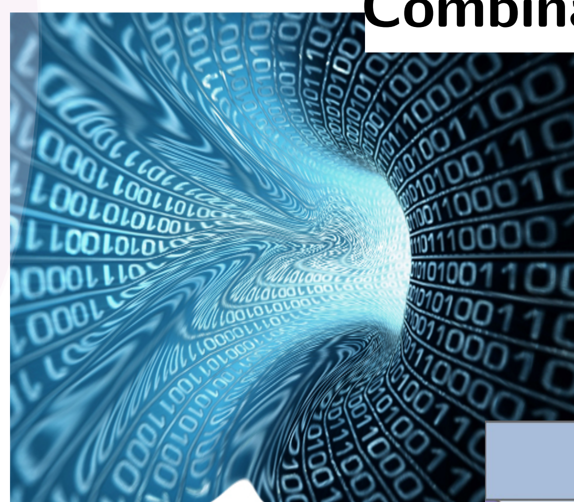
$$\aleph_\omega^{\aleph_0} < \max\{\aleph_{\omega_4}, (2^{\aleph_0})^+\}, \ ^{*}$$

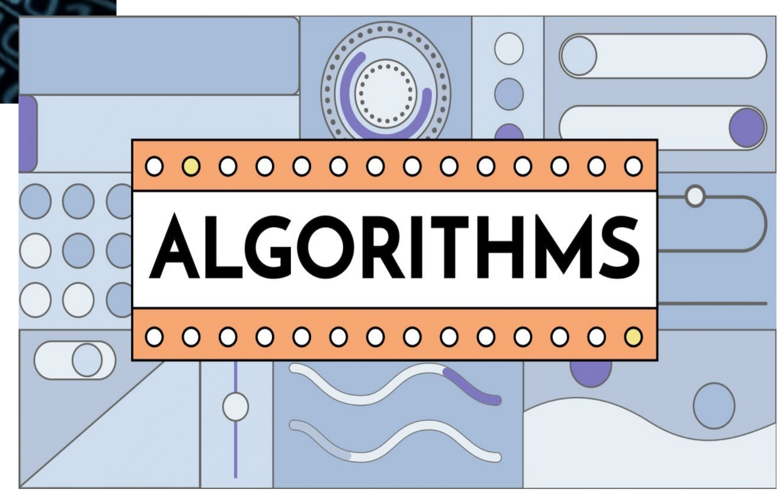
$$\max(|A+A|, |A \times A|) \geq \frac{1}{2}|A|^{4/3}(\log|A|)^{-1/3}\ ^{*}$$

$$\aleph_0 \leq^{*} \mathfrak{p} \iff \aleph_0 \leq 2^{\mathfrak{p}}$$

Combinations

$$Q_n = 2n(\ln n + \gamma - 2) + 2\ln n + 2\gamma + 1 + O\left(\frac{1}{n}\right)$$





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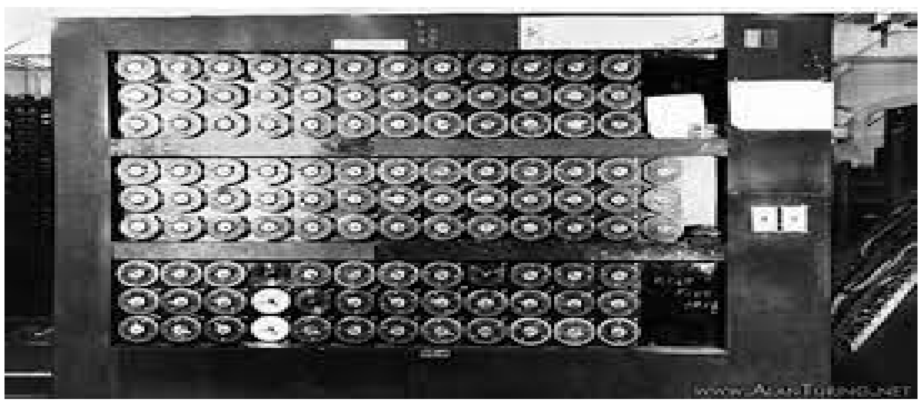

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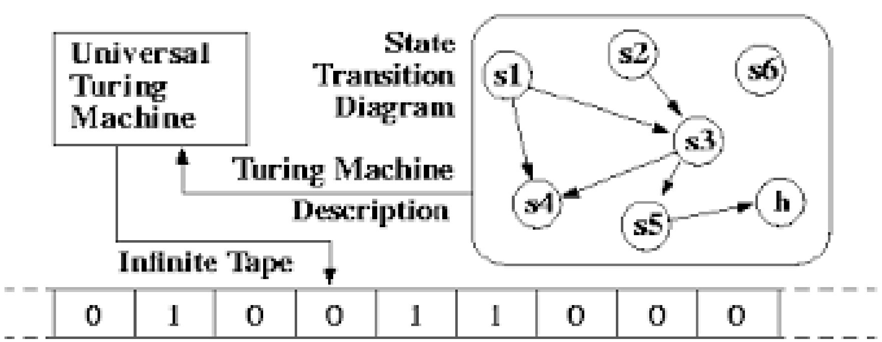

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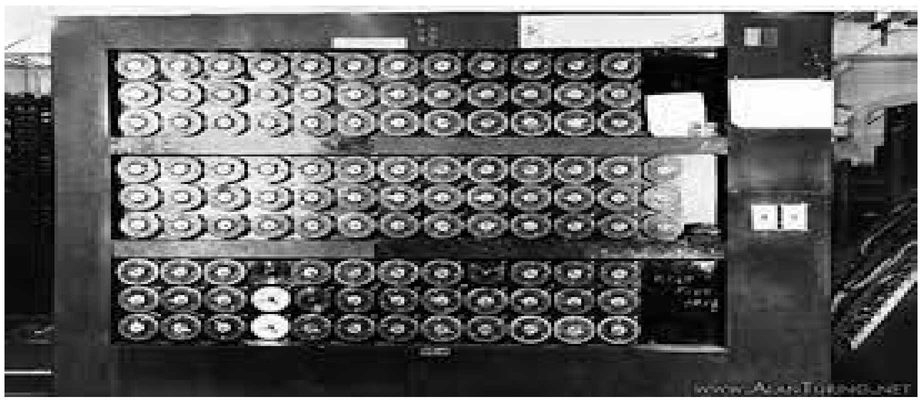

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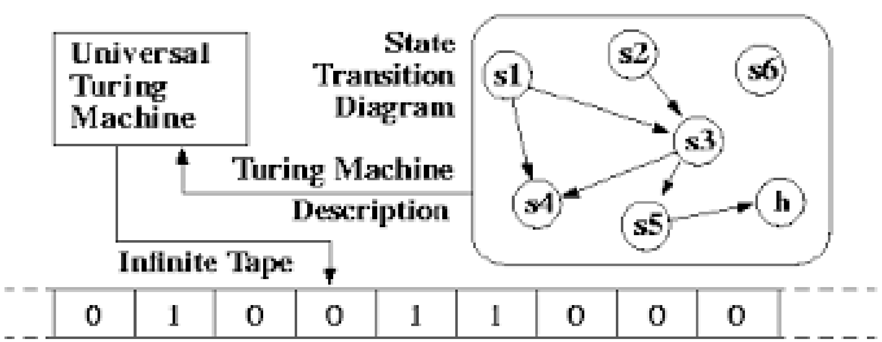

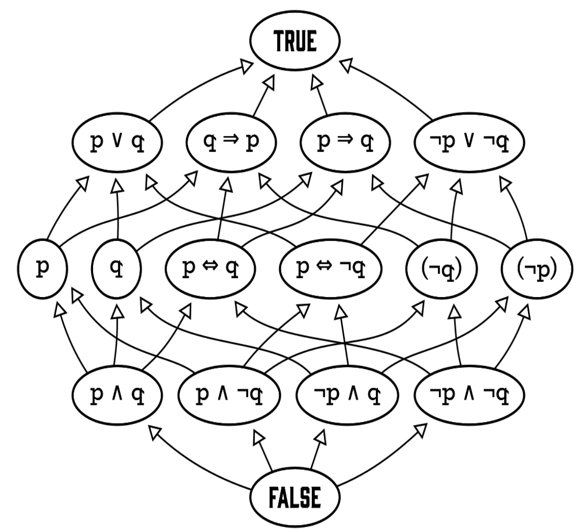

we are building upon the 2500 yrs. of knowledge ...

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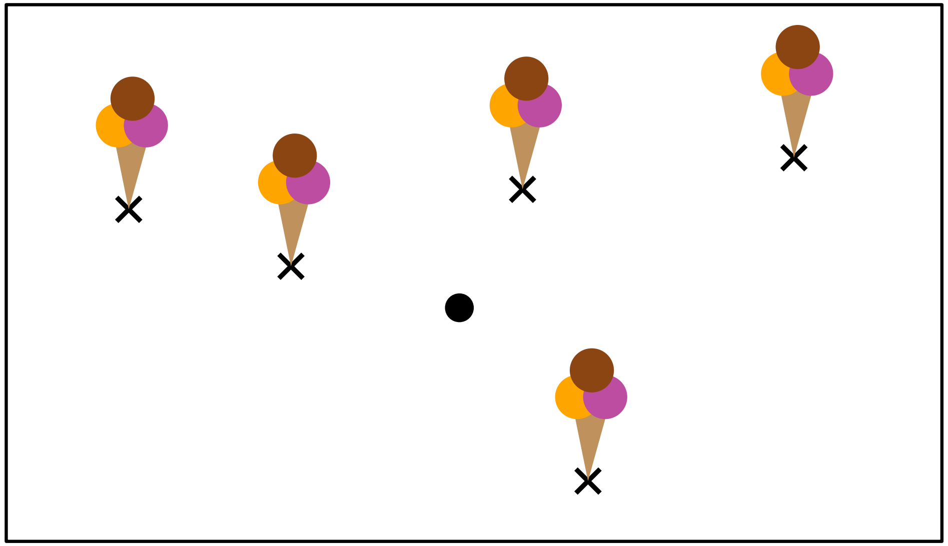
It's one of the last sunny days before winter.

Suppose you know the location of five ice cream shops in the city.

How can you determine the closest one for any location on a map?



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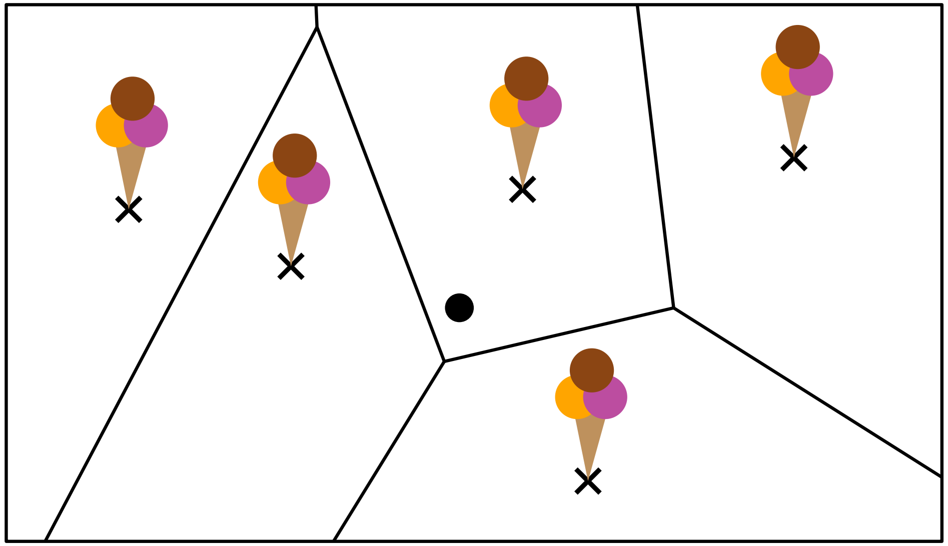

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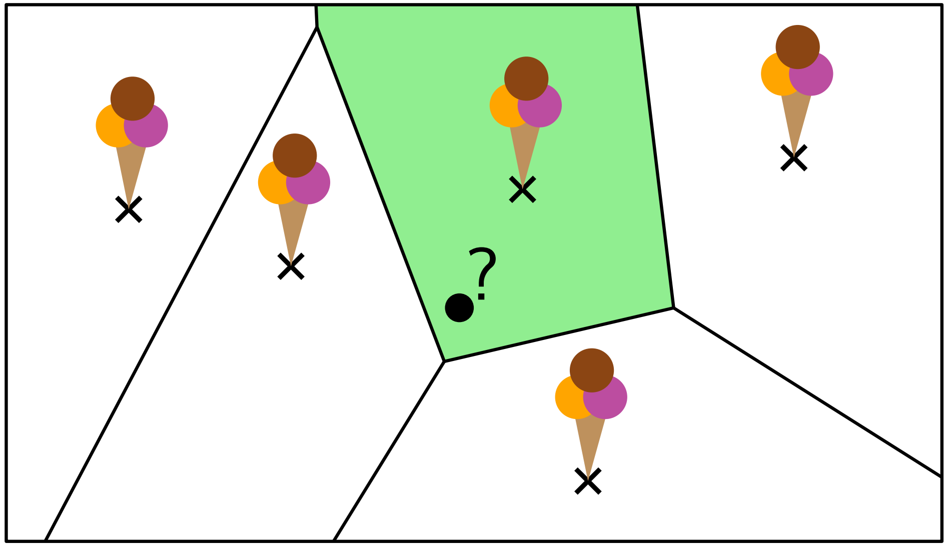

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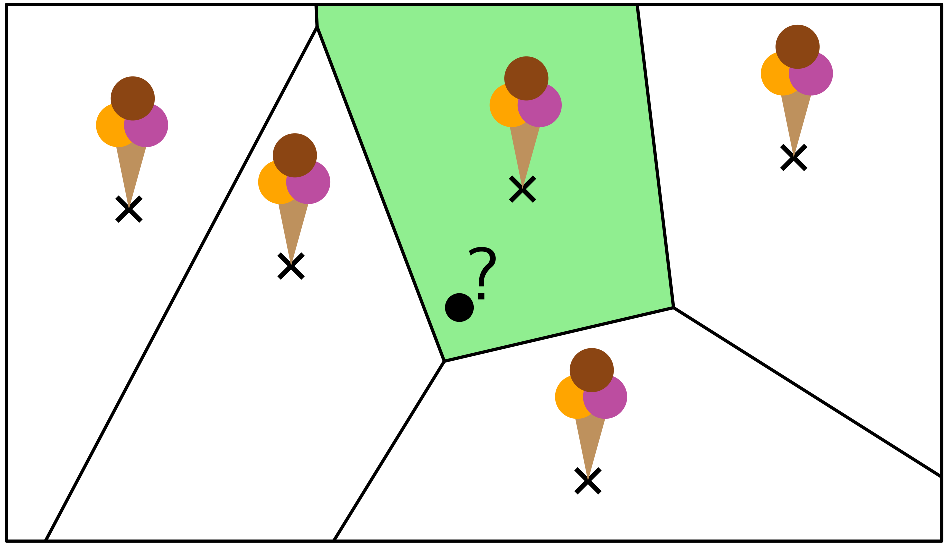

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Lots of applications, e.g.: site planning, facility placement, nearest-neighbor finding, wireless networks ...

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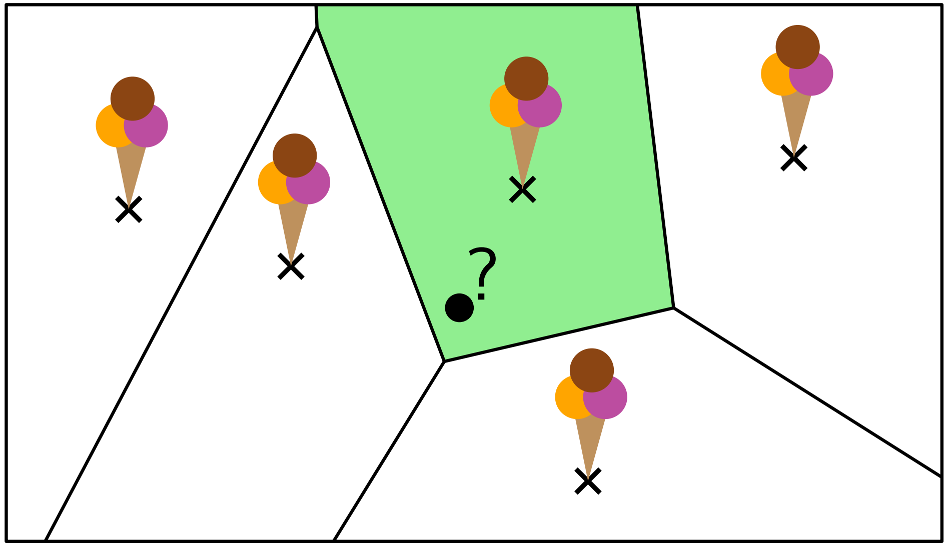

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Same approach applies to find some serious services - Hospitals, Bank, Metro Station, Institions, etc.

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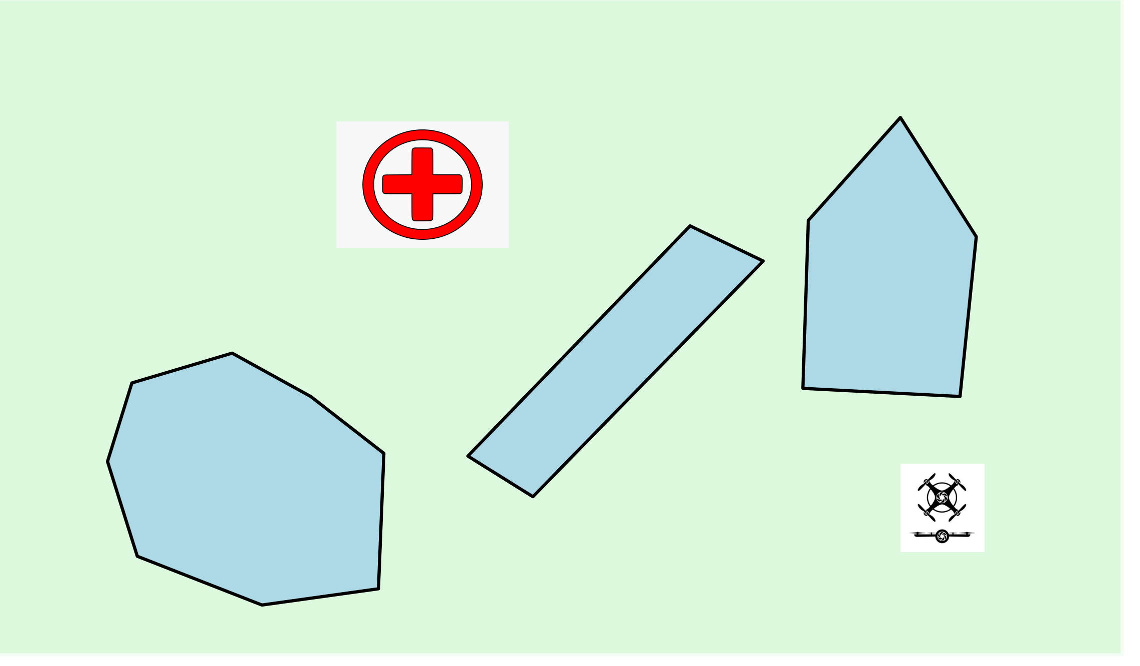
We want to send a robot to go to a hospital/ medical store. How can the robot reach the destination without passing through houses, park benches, and trees?



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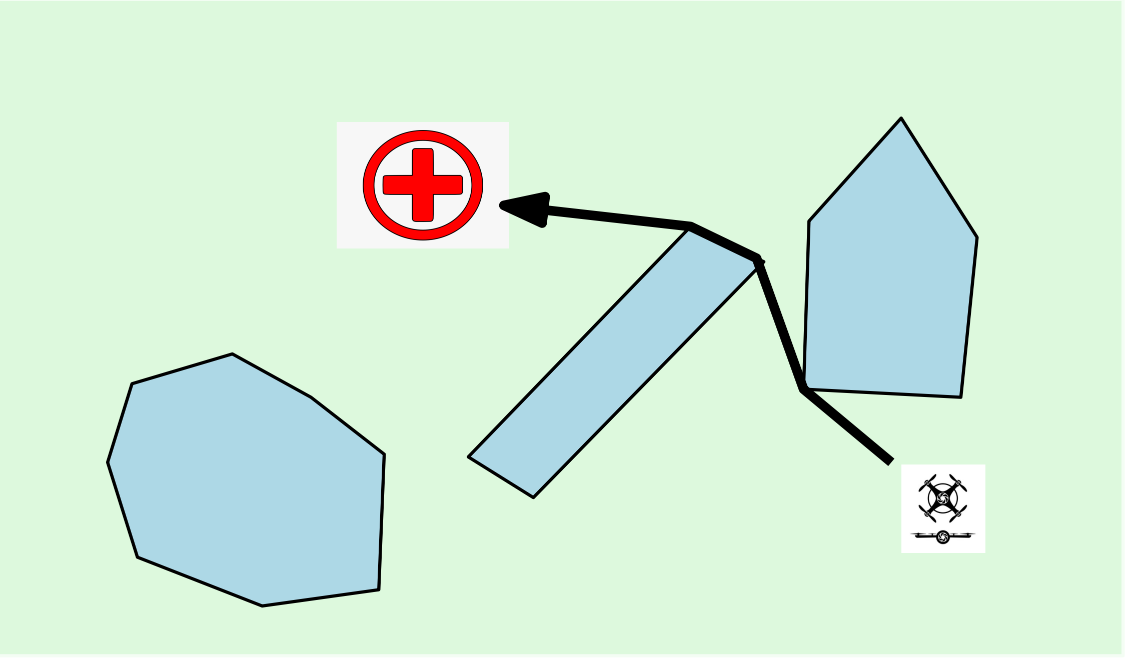

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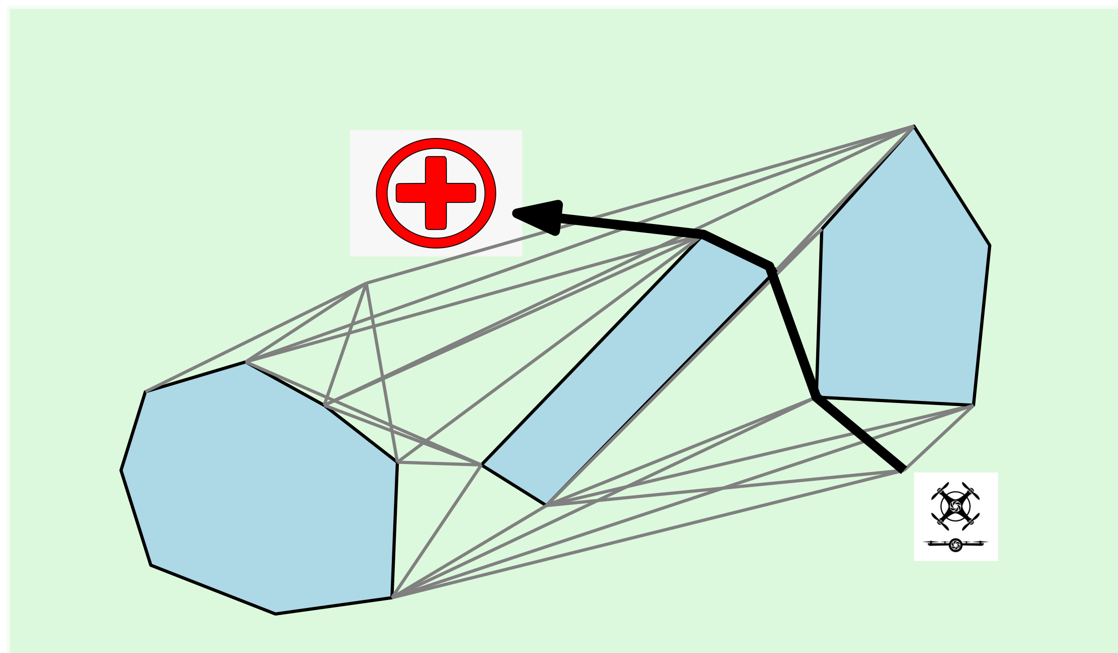
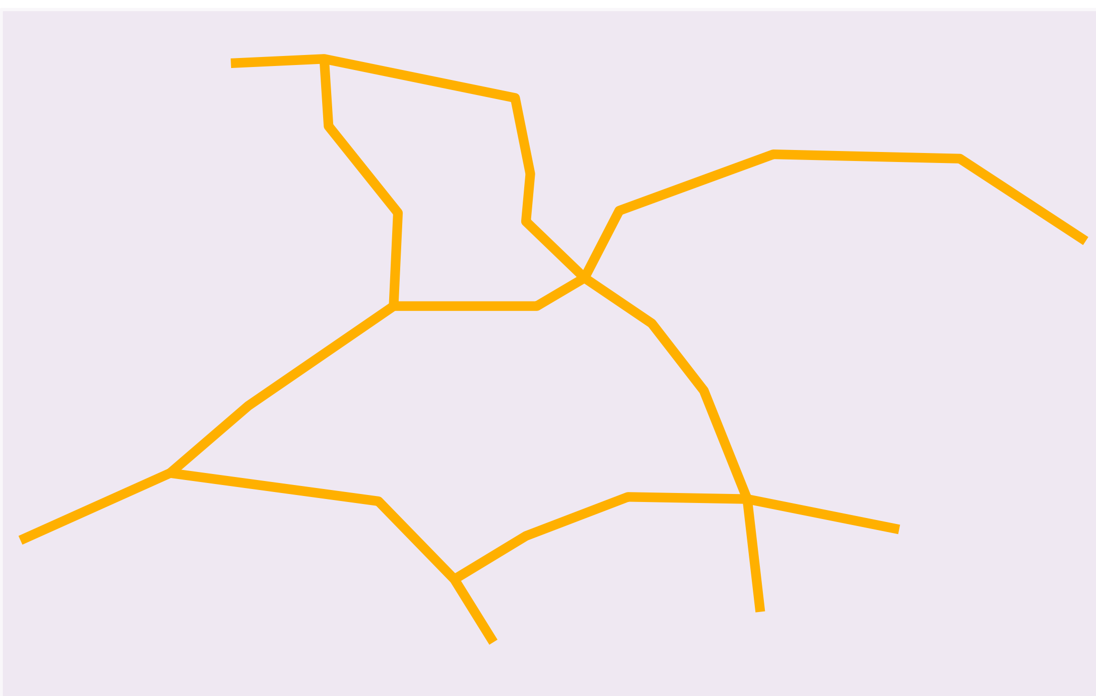


Motion planning problem in robotics:
Given a set of obstacles with a start and destination point,
find collision-free shortest route, e.g., using the **visibility graph.**

Problem 3

Maps in geographic information systems consist of several layers (e.g., roads, rivers, borders, etc.). When superimposing several layers, where are the intersection points?

One example is to to view all roads and rivers as a set of line segments and ask for the river crossings. For these, you have to find all intersections between the two layers.



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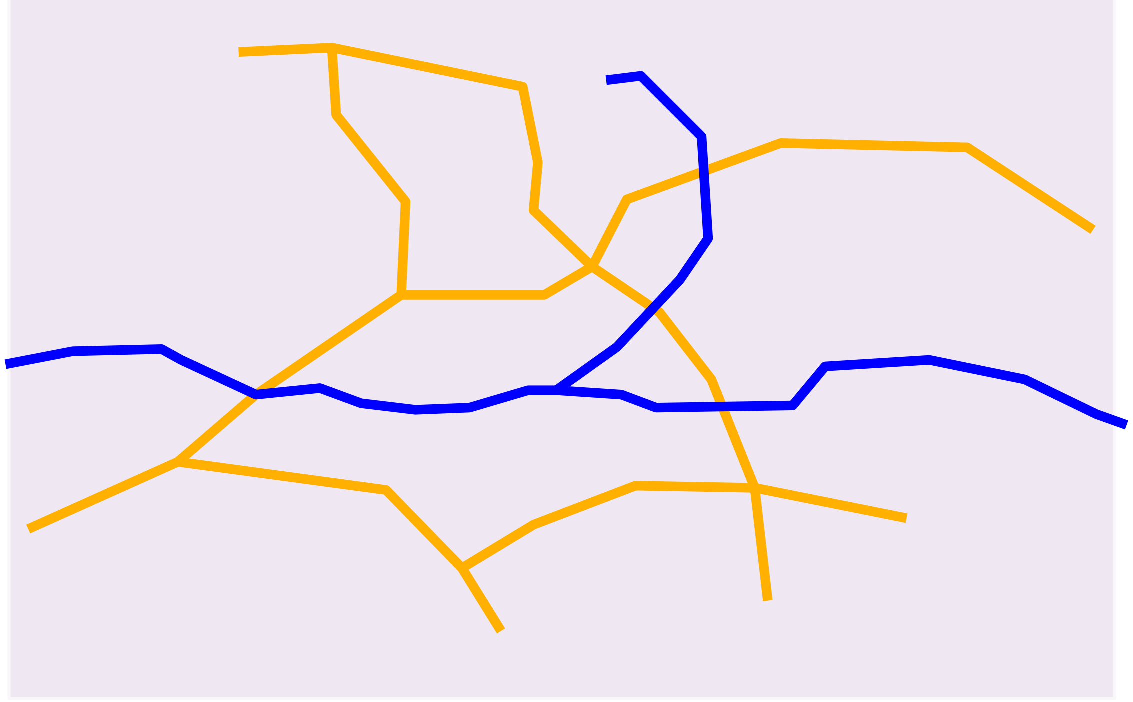

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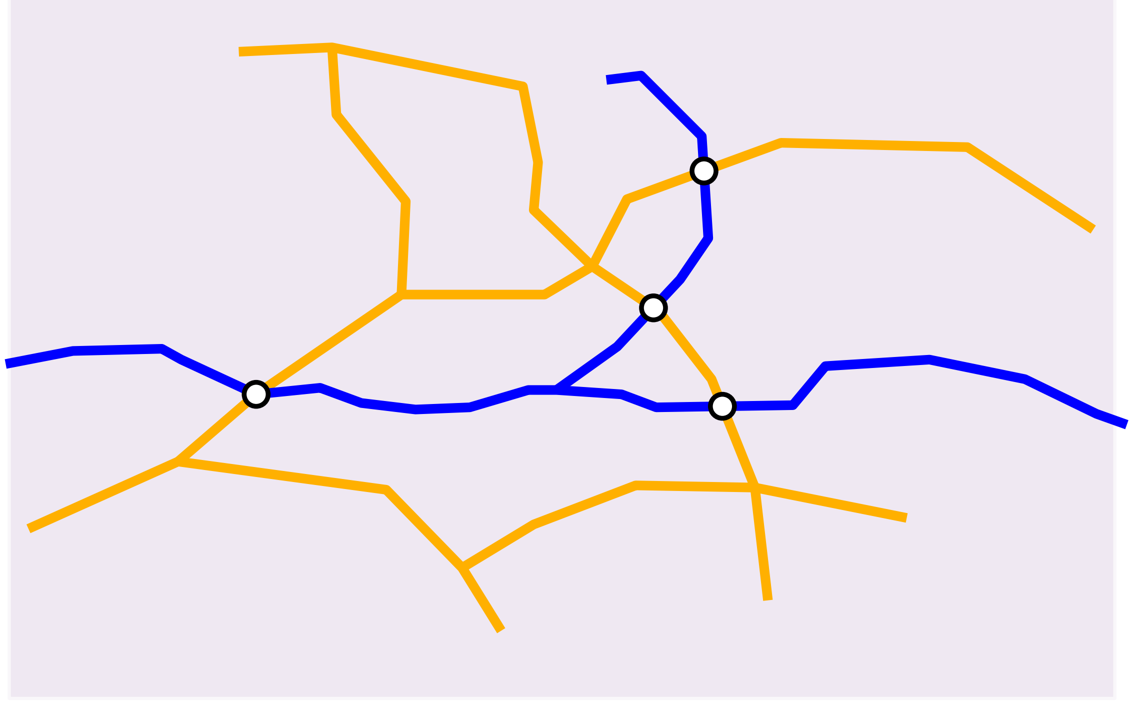

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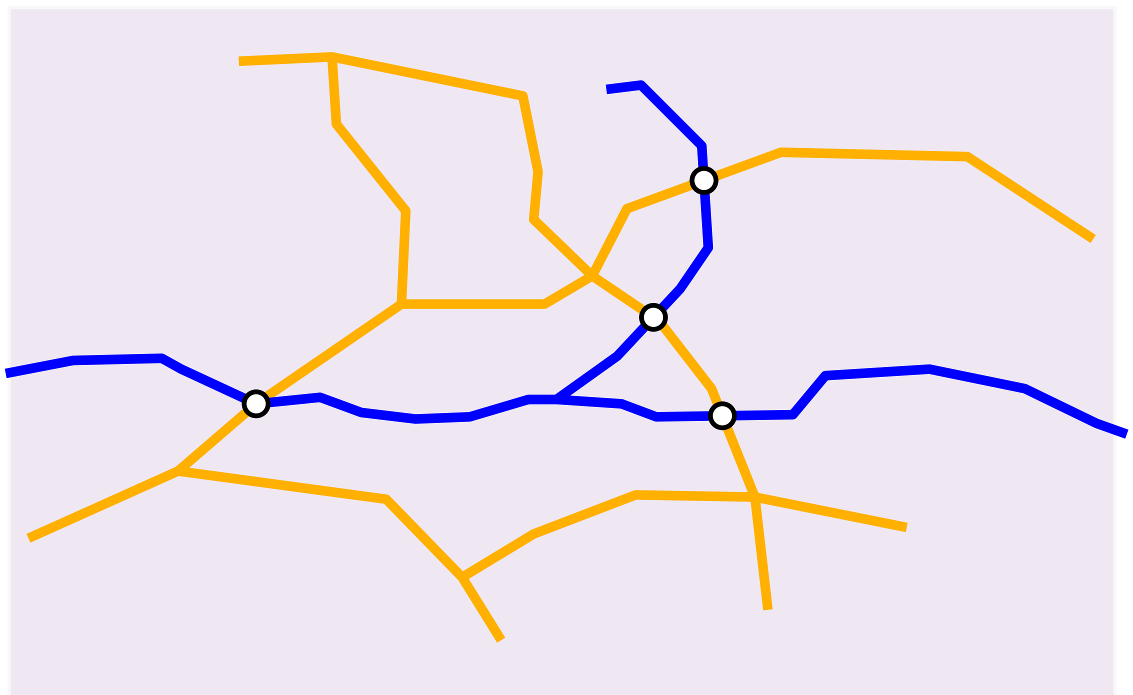

Testing all edge pairs is slow.

Q: How can you quickly find all intersections?

Problem 4

Given a map and a query point q (e.g., a mouse click), determine the country containing q.

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We want to obtain efficient & robust data structures for answering point queries.

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Evaluating each map feature is unrealistic.

→ We want efficient & robust data structures for answering such range queries

Mixing Ratios

We are given -

Mixture	fraction A	fraction B
S_1	10%	35%
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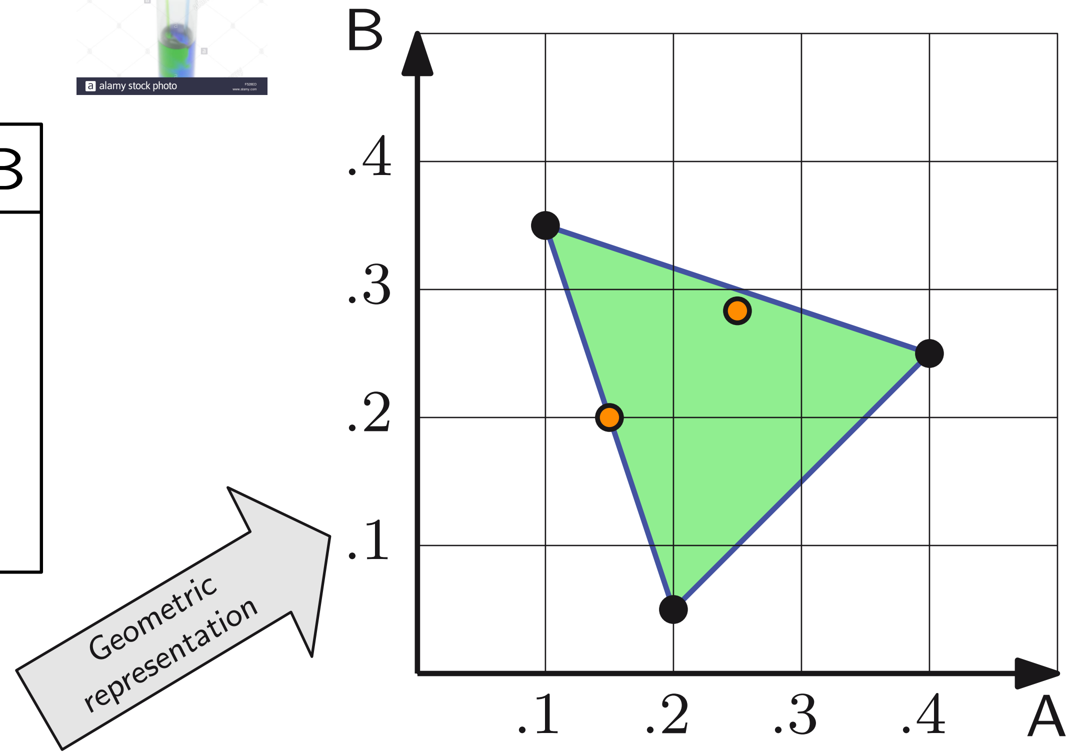

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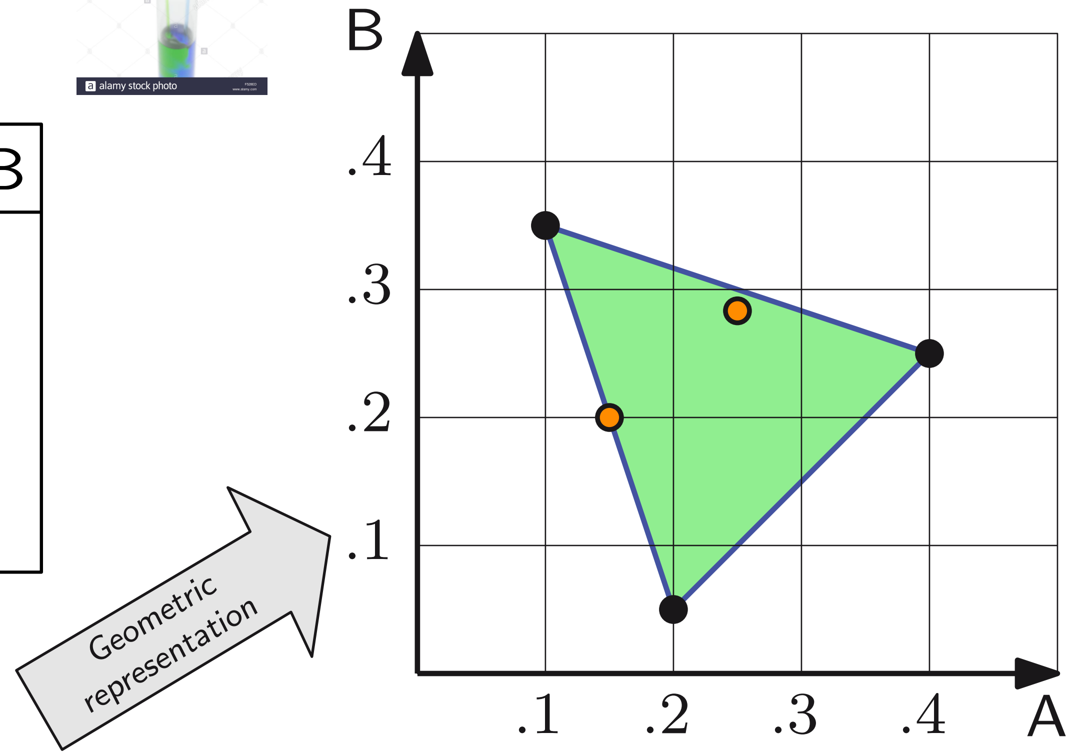

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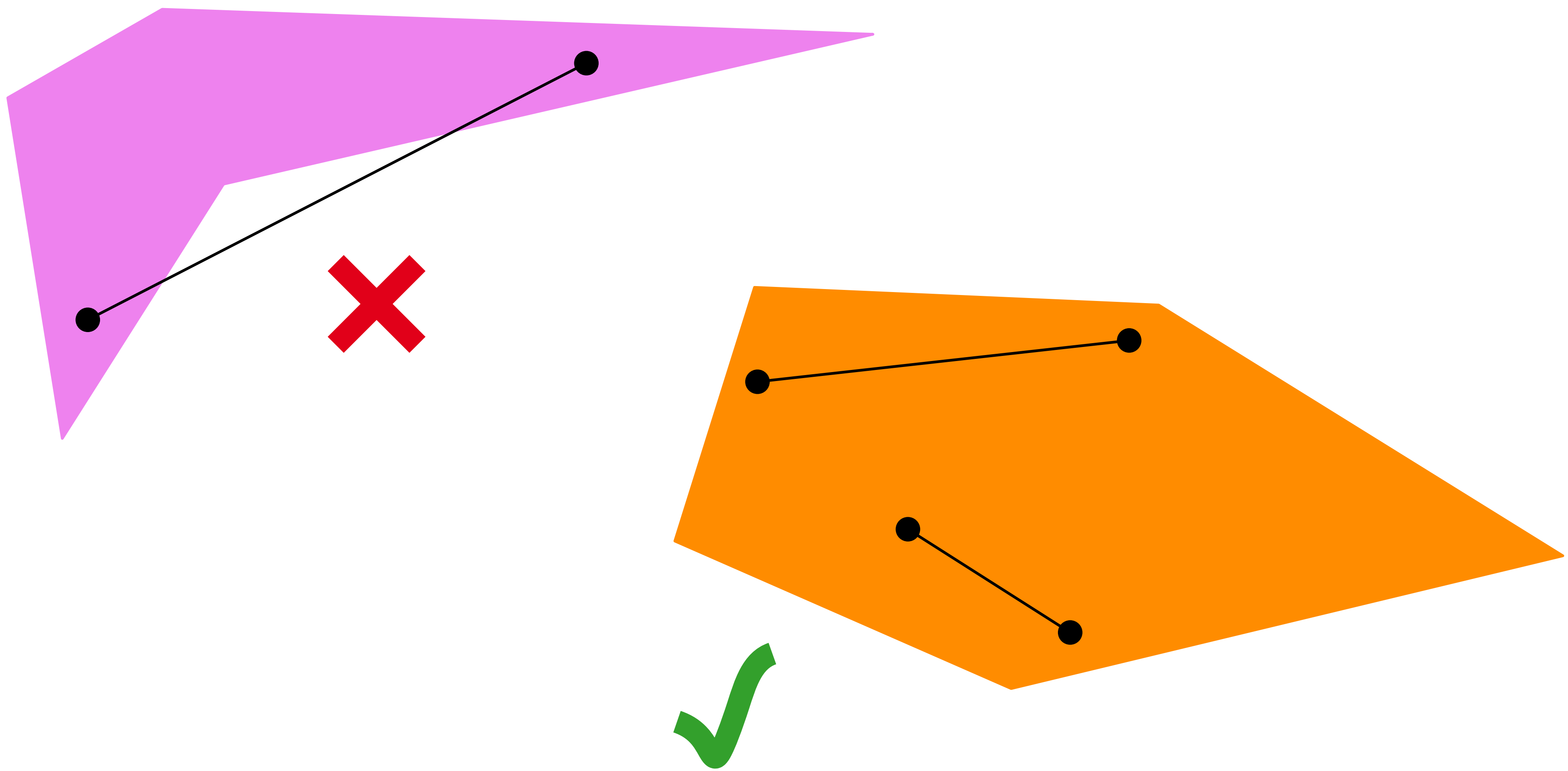
Observation. Given a set $S \subseteq \mathbb{R}^2$ mixtures, it is possible to make another mixture $q \in \mathbb{R}^2$ using $S \iff q \in$ ConvexHull $CH(S)$.
$q = \sum_i \lambda_i s_i$ with $\sum_i \lambda_i = 1$.

Definition of Convex Hull

Definition. A region $S \subseteq \mathbb{R}^2$ is called **convex**, if for any two points $p, q \in S$ the line segment $\overline{pq} \in S$. The **convex hull** $CH(S)$ of S is the smallest convex region containing S.



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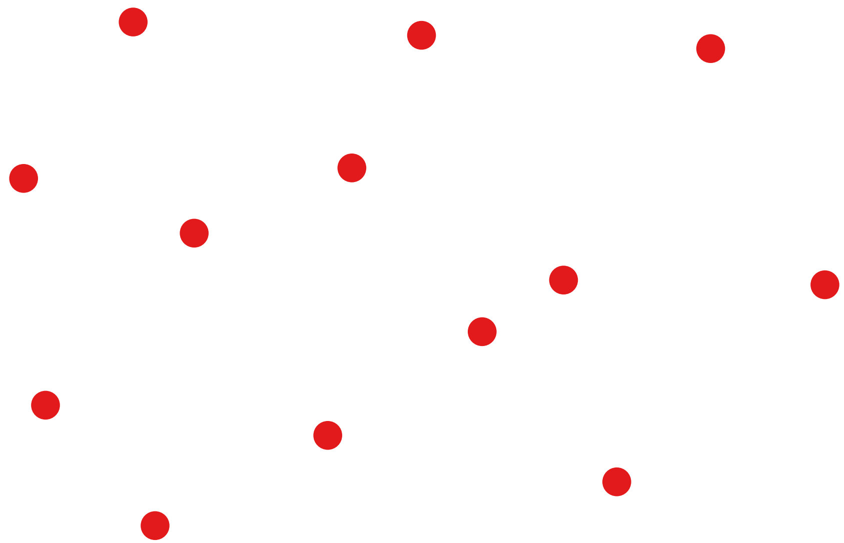

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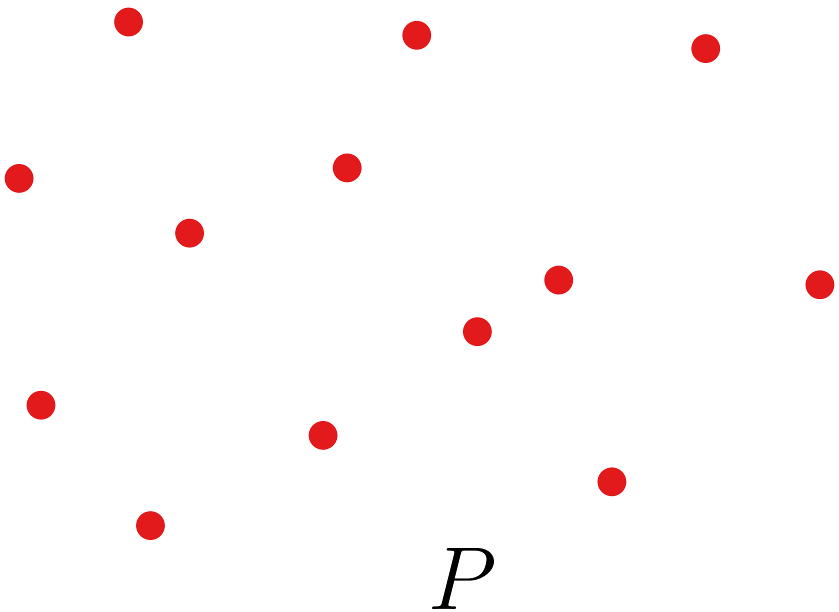

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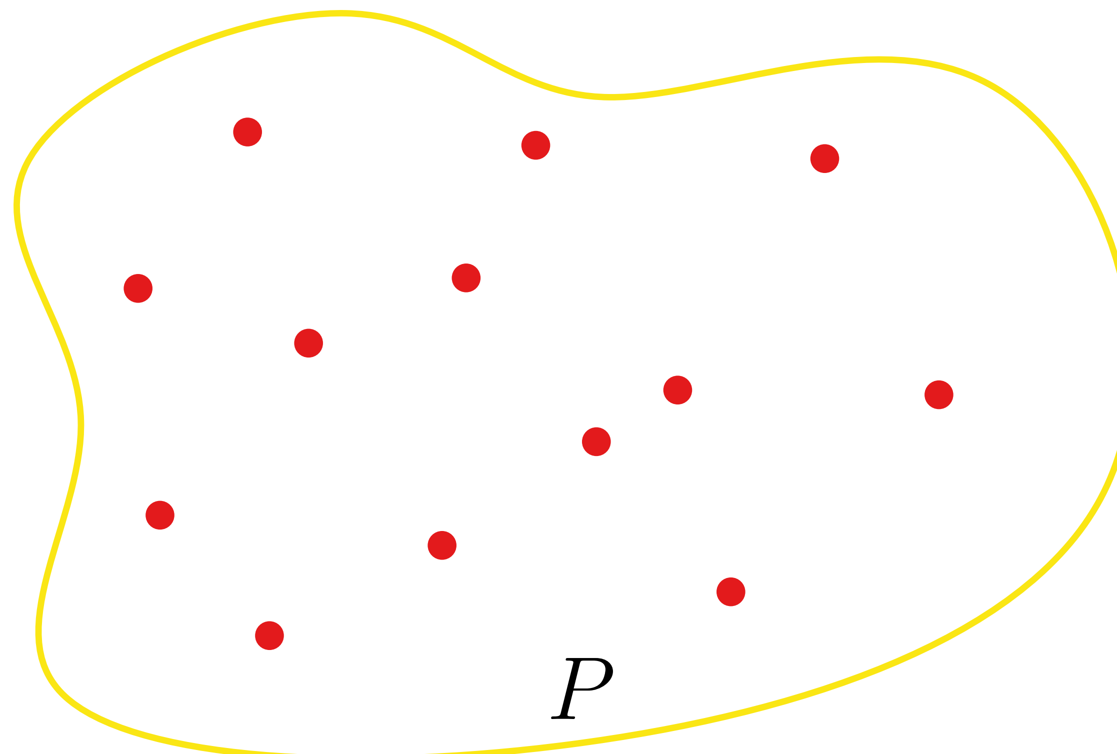

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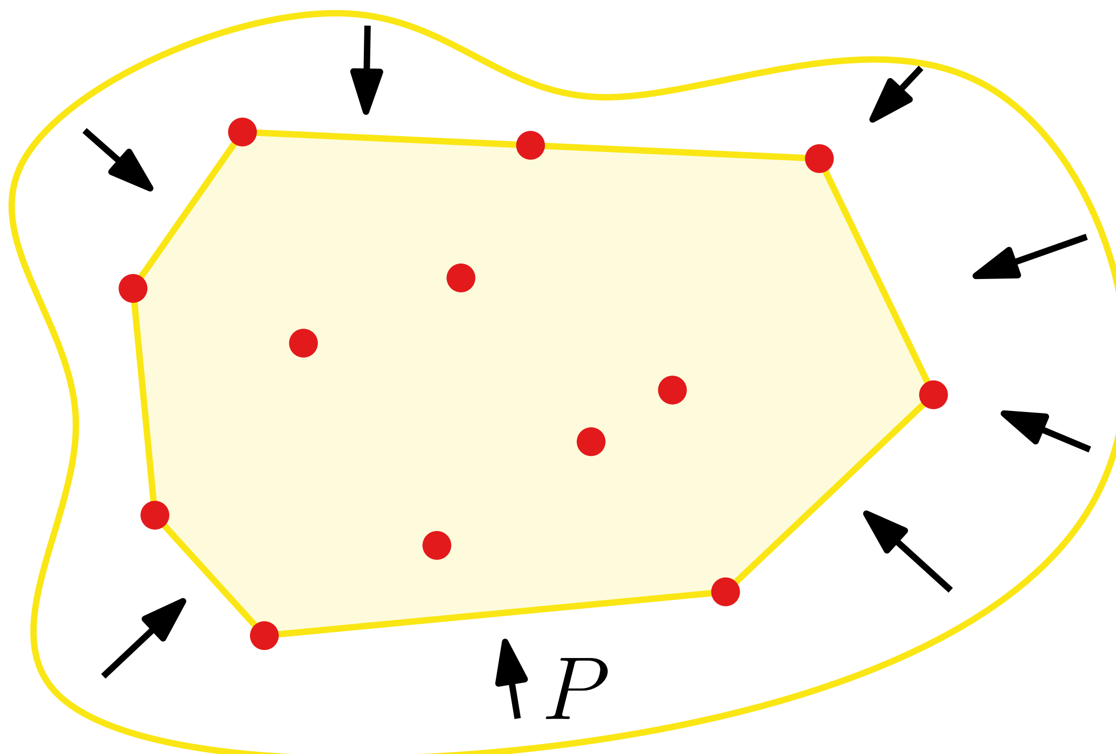

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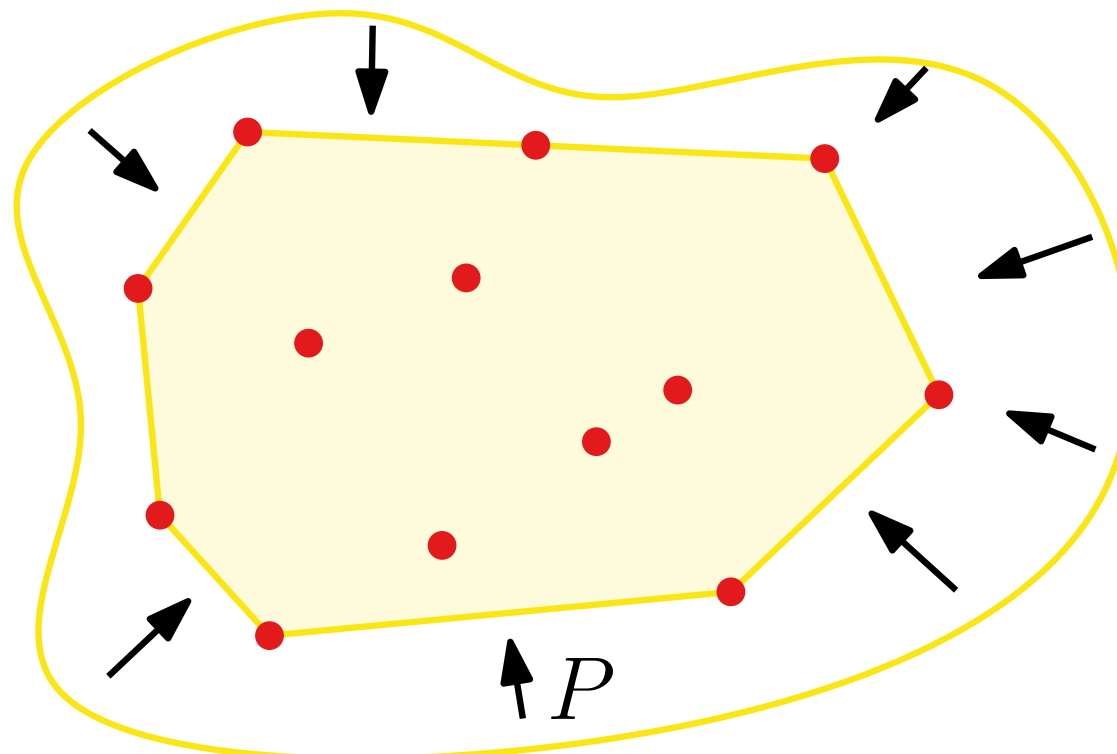

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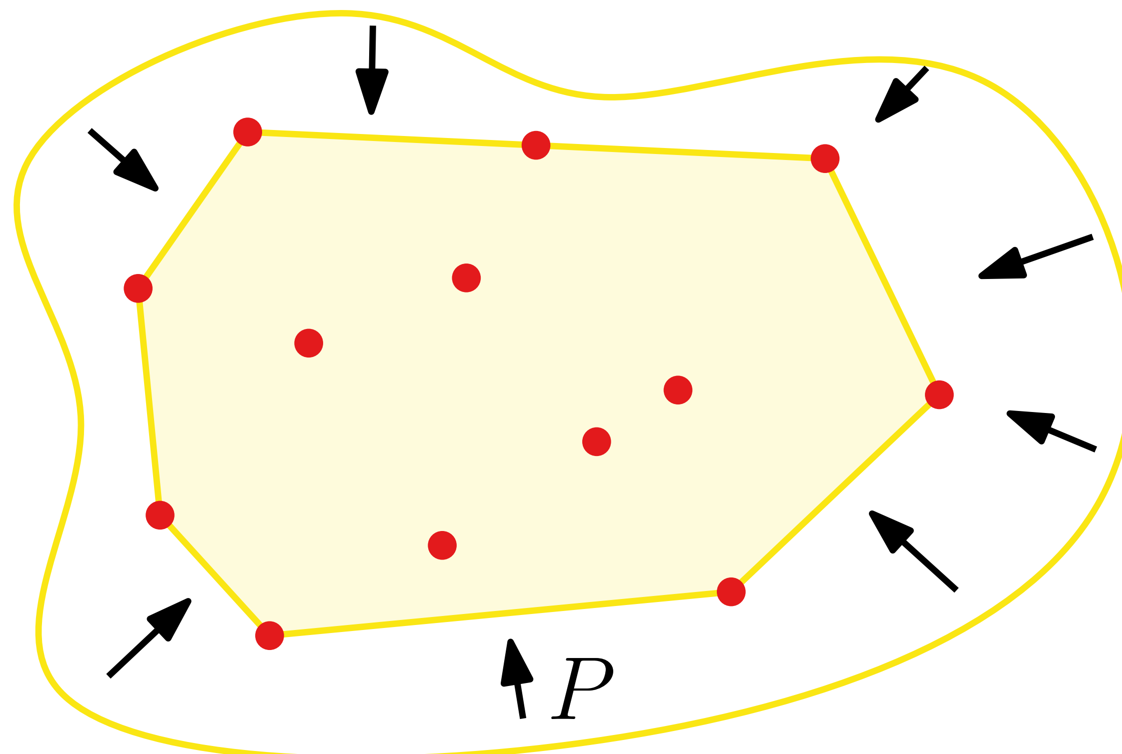

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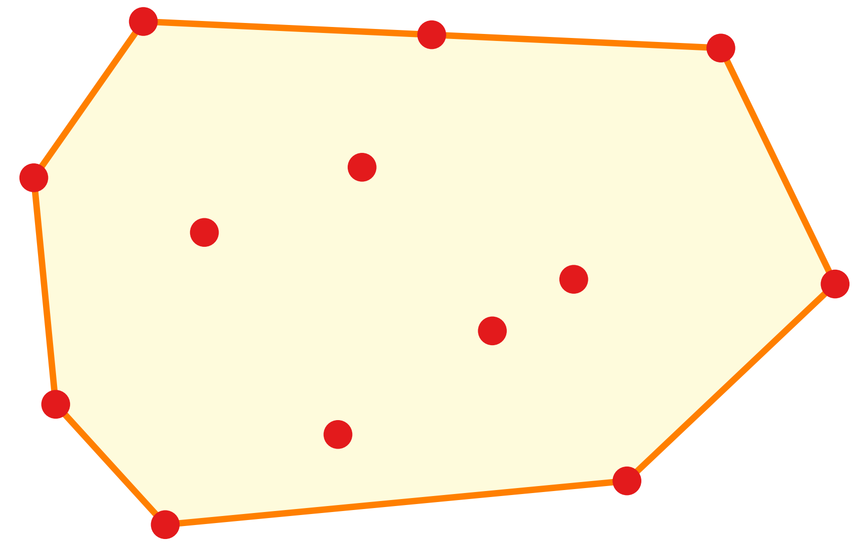
Now, some Mathematics -

- define $CH(S) = \bigcap_{C \supseteq S :\, C \text{ convex}} C$
- does not help either.

Algorithmic Approach

Lemma. For a set of points $P \subseteq \mathbb{R}^2$, $CH(P)$ is a convex polygon that contains P and whose vertices are a subset of P.



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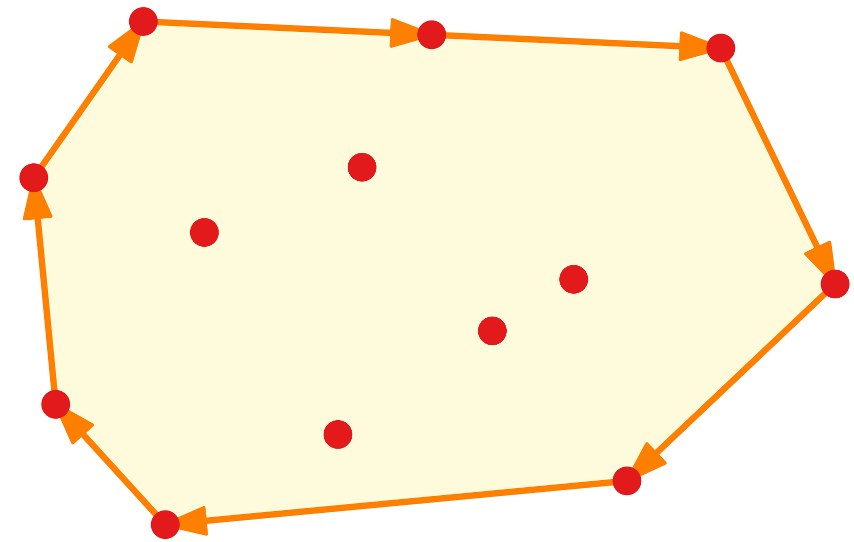

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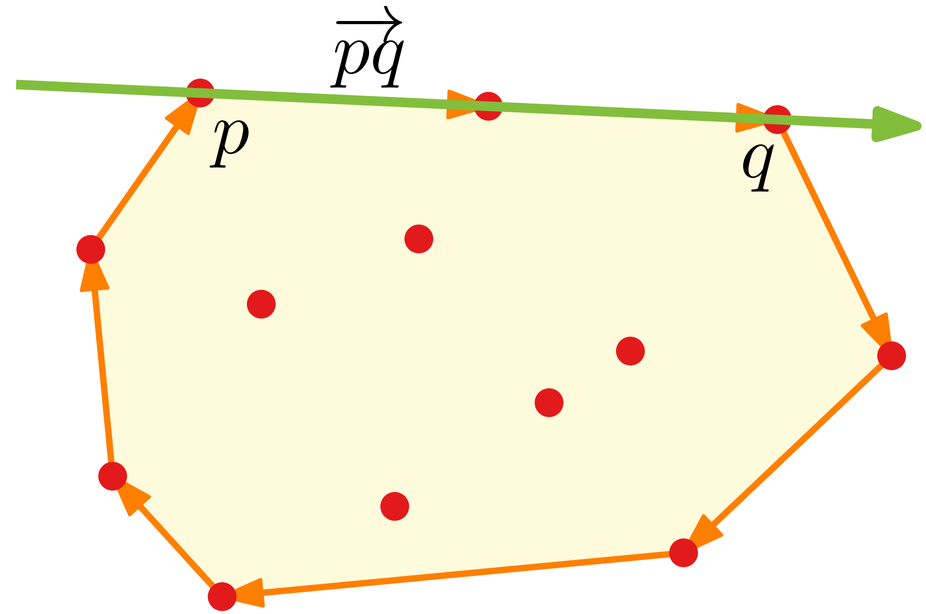

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Observation. (p, q) is an edge of $CH(P) \Leftrightarrow$ each point $r \in P \setminus \{p, q\}$

- strictly right of the oriented line $\overrightarrow{pq}$ or
- on the line segment $\overline{pq}$

Computing Convex Hull

ConvexHull(P)

$E \leftarrow \emptyset$
foreach $(p, q) \in P \times P$ with $p \neq q$ **do**
  valid $\leftarrow$ *true*
  foreach $r \in P$ **do**
    if not (r strictly right of $\overrightarrow{pq}$ **or** $r \in \overline{pq}$) **then**
      valid $\leftarrow$ *false*
  if *valid* **then**
    $E \leftarrow E \cup \{(p, q)\}$
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Test in $O(1)$ time with

$$\begin{vmatrix} x_r & y_r & 1 \\ x_p & y_p & 1 \\ x_q & y_q & 1 \end{vmatrix} < 0$$

$\rightarrow$ think (exercise-1)

Limits of Computation ...

What computers can not do ... ?

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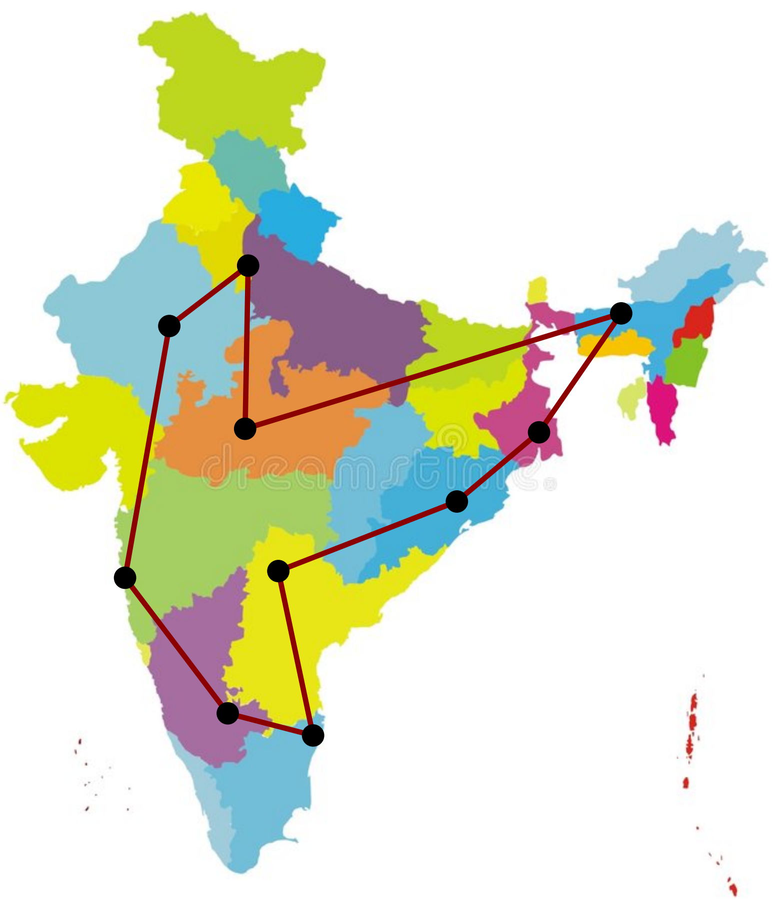

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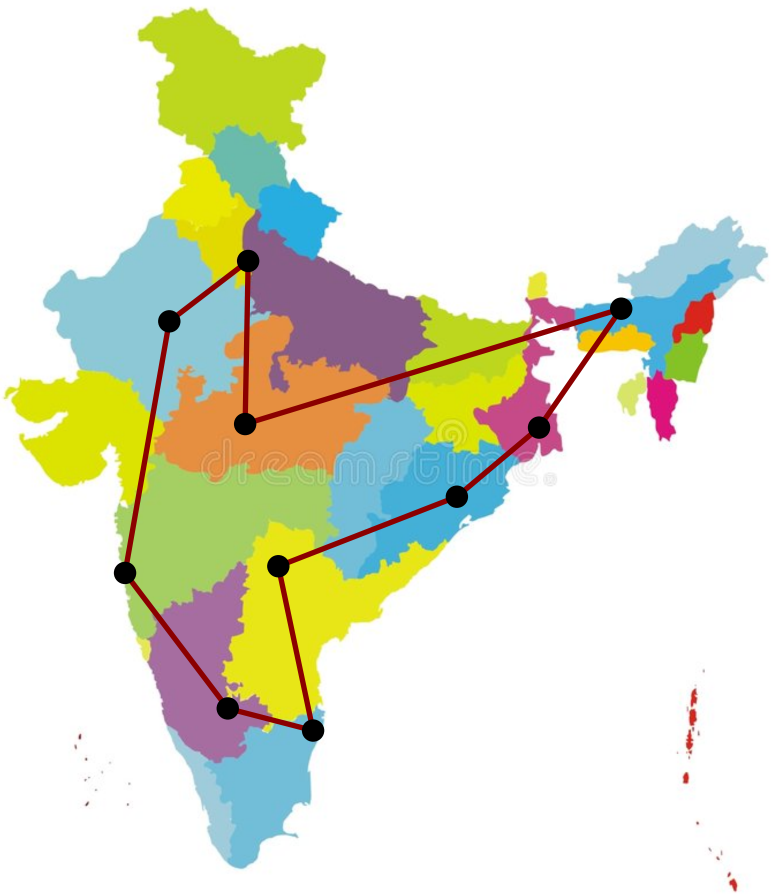

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- main issue is Ordering!

- even the fastest computers can not afford exponential computation.

- time and space are invaluable.

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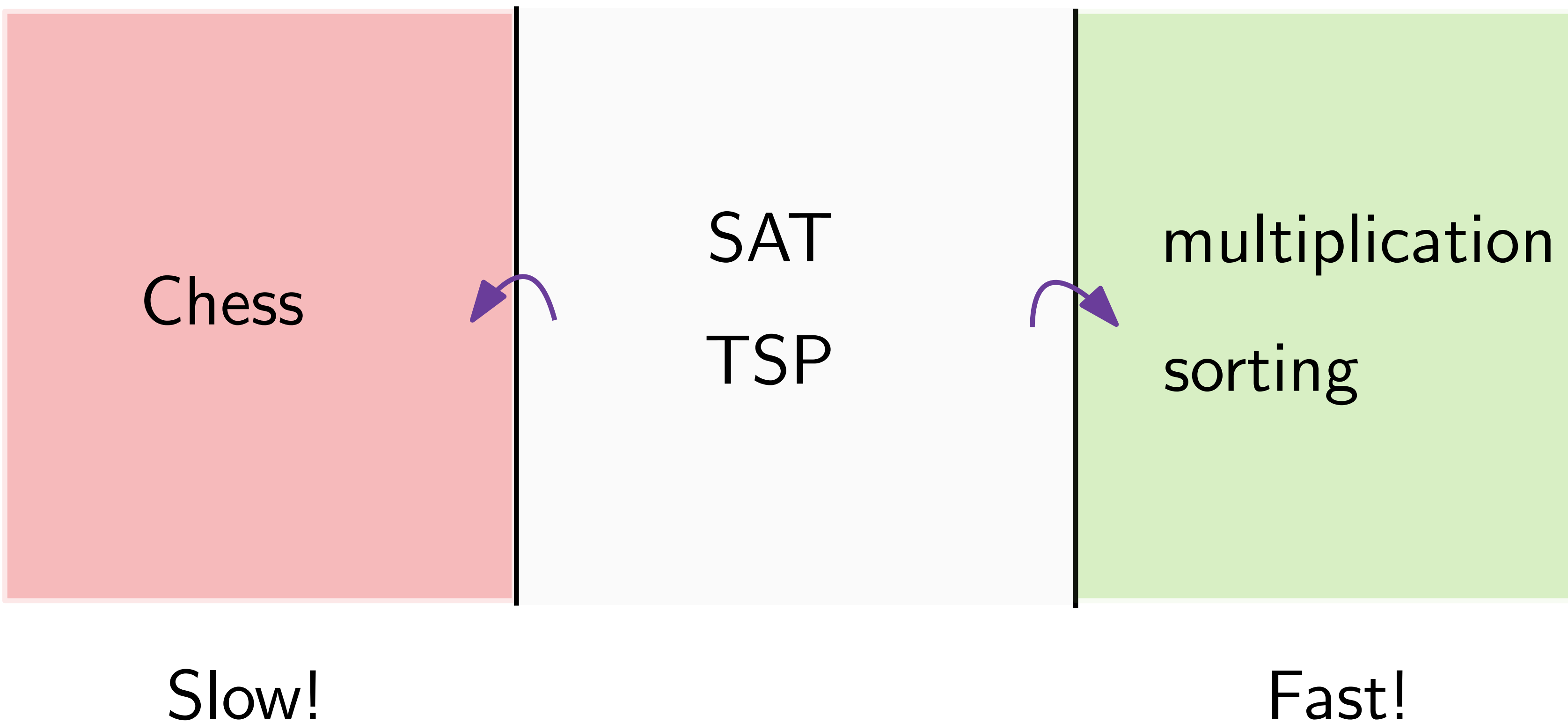
Categories of problems



Complexity Classes ...

P $= \{$Problems that are solvable in polynomial time.$\}$

If the problem has size n, then it can be solved in $n^{O(1)}$ time.

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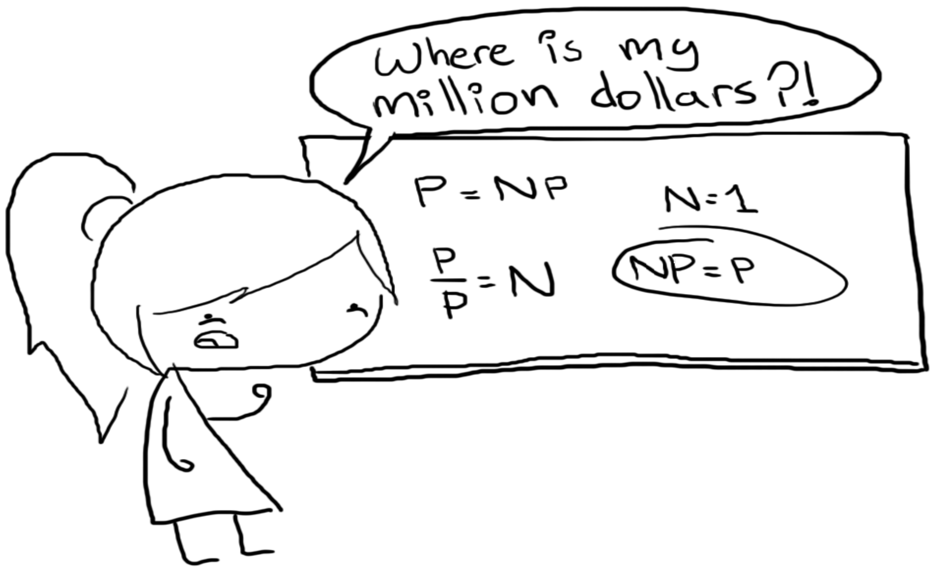
Output is **YES** or **NO**.

In $O(1)$ time can "guess" among polynomial number of choices & if any guess leads to YES, then the nondeterministic algorithm will make that guess.

There is an asymmetry between YES and NO inputs.

The BIG problem !!!

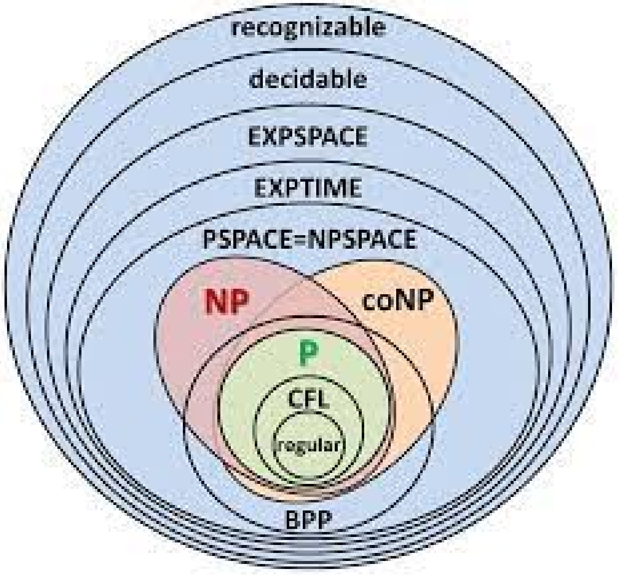


on a hilarious note!

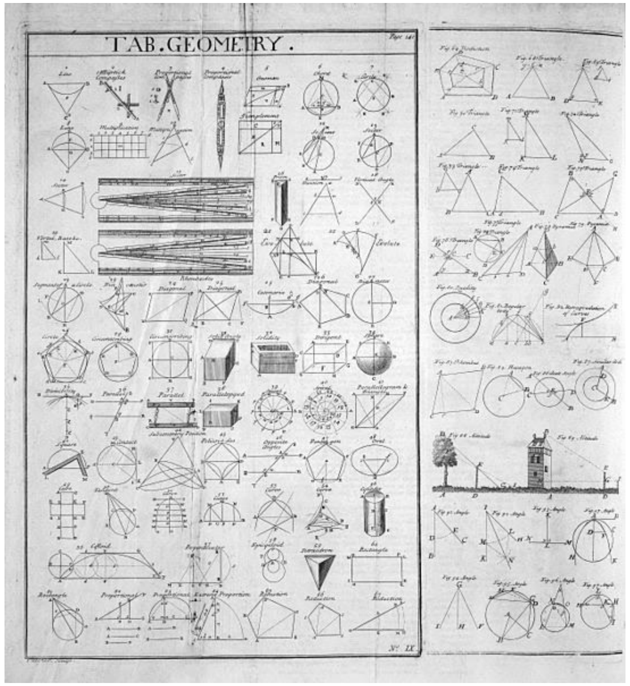
The BIG problem !!!

on a hilarious note!

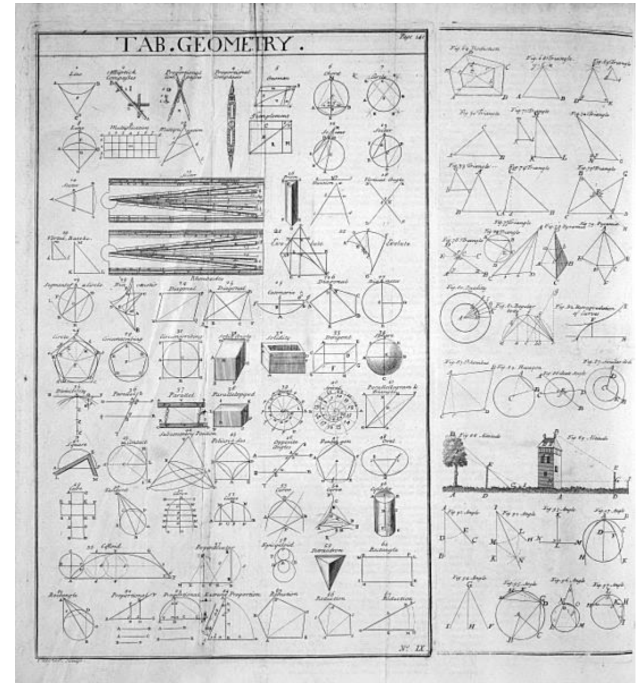
on a serious note!



Let's begin the learning together ...



Let's begin the learning together ...



Thank you!