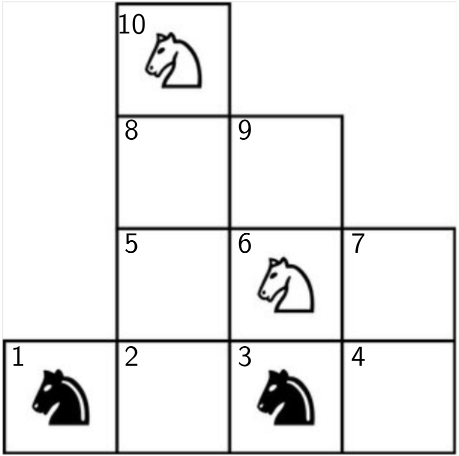
Puzzle: switch positions of dark and white knights



Solving computationally hard problems

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Computers solve problems

- Business processes
- Search
- Weather prediction

Problems are

Play chess

Easy, Hard, or Impossible

Best way to Bandra

Will my phone crash?

Hard problems

- Everywhere and important
- Class scheduling, optimal circuits, password cracking
- Needs solving

How do we solve?

- Clever algorithms
- Heuristics
- AI based approximate solving (e.g. AlphaGo)

Limited resources

- How much computation per second? 10^9
- How much time? 10^5
- How many computers? 10^5

$$\approx 10^{19} \text{ is a hard limit.}$$

Topic 1.1

Example hard problem: graph coloring

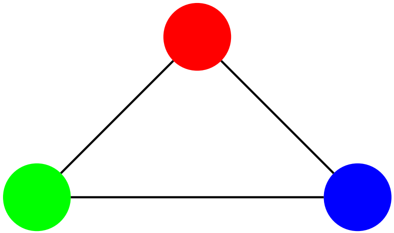
Graph coloringg problem

Given a graph and a set colors, assign a color to each node such that no edge has same color in both ends.

Example

Example 1.1

A graph coloring using three colors.

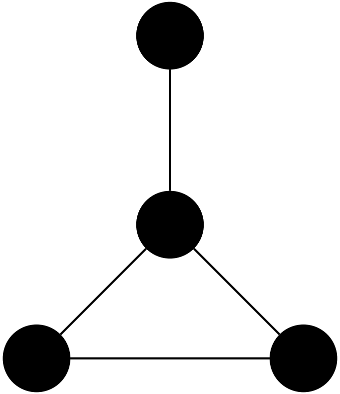


Can we color the above using two color?

Exercise

Example 1.2

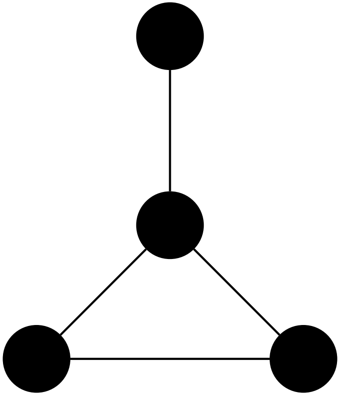
Can we color the following using three color?



Exercise

Example 1.3

How many ways can we color using three colors?



Let us look at a graph coloring solver.

How does this solver work?

- Make Boolean constraints
- Call a solver to solve the constraints
- The solvers are called SAT solvers.

Graph coloring: definition again

color a graph$(\{v_1, \dots, v_n\}, E)$ with d colors such that if $(v_i, v_j) \in E$ then the colors of v_i and v_j are different.

Graph coloring : Boolean variables

Variables: p_{ij} for $i \in 1..n$ and $j \in 1..d$.

p_{ij} is true if and only if v_i is assigned jth color.

Boolean constraints: Each vertex has at least one color

for each $i \in 1..n$

$$p_{i1} \text{ OR } \ldots \text{ OR } p_{id}$$

Boolean constraints: if $(v_i, v_j) \in E$ then color of v_i is different from v_j

for each $k \in 1..d, \quad (v_i, v_j) \in E$

$$(\text{NOT } p_{ik}) \text{ OR } (\text{NOT } p_{jk})$$

Exercise 1.1

Do we need "every vertex has at most one color" to solve the problem?

Lessons from experiments : hard problems are not always hard

- Not all instances are hard
- Real-world instances may be easy
- SAT solvers do solve real-world instances of hard problems

Final thought : Can you solve?

- 2-body problem
- 3-body problem
- 10^{23}-body problem

Medium sized problems are unsolved, e.g., biology.