Lecture 4

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Requirements/Consequence for/of a Conservative force: Summary

$$\oint \vec{F} \cdot d\vec{r} = 0$$

$$F(x) = -\frac{dV}{dx} = -\frac{dU}{dx}$$

$$\nabla \times \mathbf{F} = 0.$$

$$\mathbf{F}(\mathbf{r}) = -\nabla V$$

$$\frac{1}{2}mv_a^2 + V(\mathbf{r}_a) = \frac{1}{2}mv_b^2 + V(\mathbf{r}_b)$$

A consequence of work-energy theorem for conservative forces is that sum of kinetic and potential energies of a system is conserved

Non Inertial Frames and Pseudo/Fictitious Forces

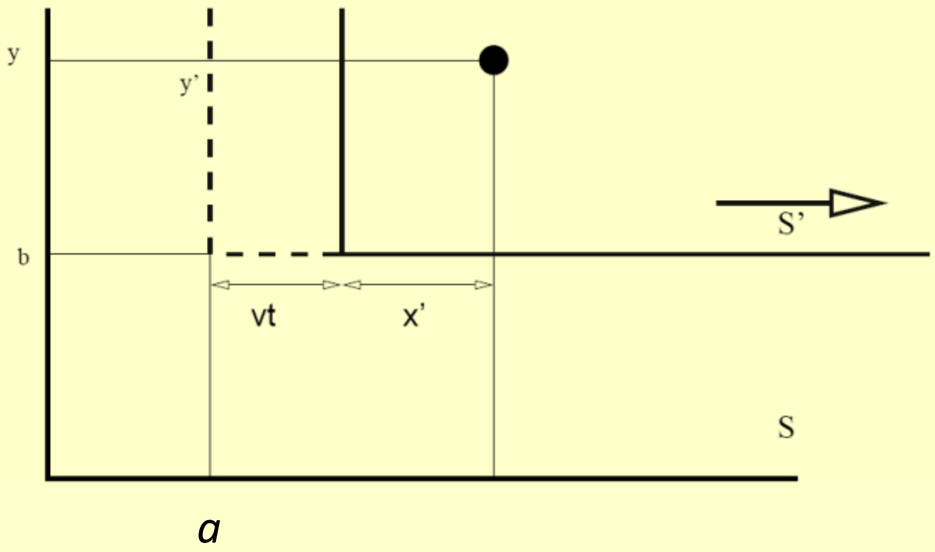
Translational Motion

Frame of Reference

Example

Coordinate Transformations

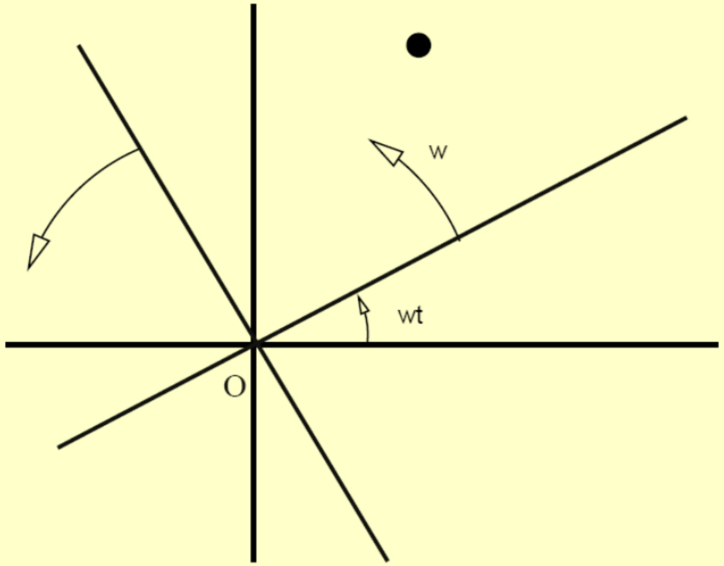
$$x' = x - a - vt$$
$$y' = y - b$$



Example

Coordinate Transformations

$$x' = x\cos(\omega t) + y\sin(\omega t)$$
$$y' = -x\sin(\omega t) + y\cos(\omega t)$$



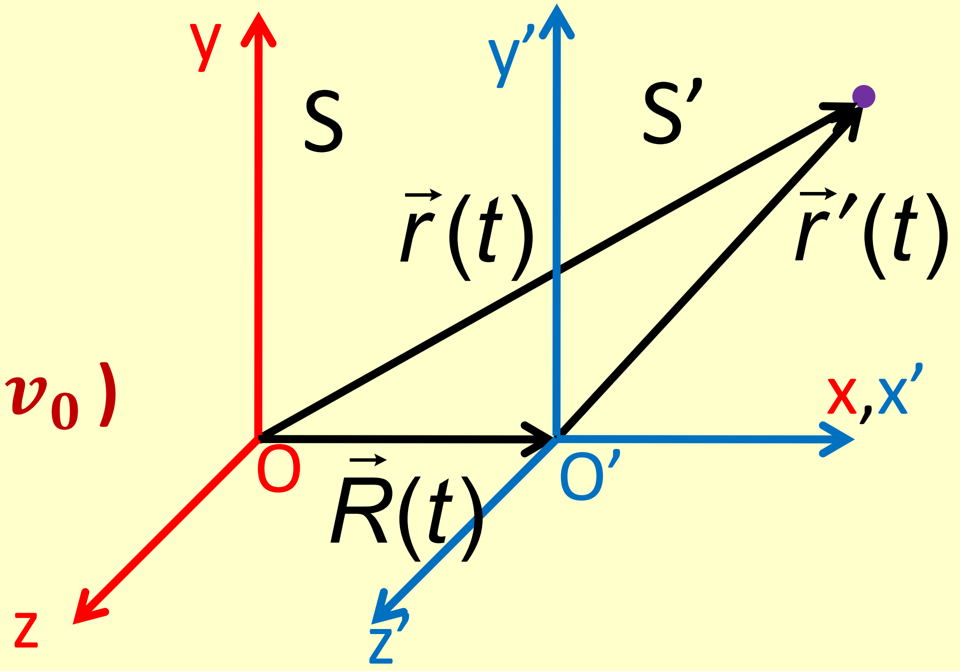
Coordinate transformations may be time dependent

Inertial Frames of Reference (I.F.O.R)

- **Isolated Particles (No net force) move with constant velocity.**

- Relative acceleration between any two inertial frames is zero, because both will find an isolated object to be moving with **constant velocity**.

Two Inertial Frames



constant velocity (v_0)

$$\boxed{\vec{r}(t) = \vec{R}(t) + \vec{r}'(t)}$$

Velocity and Acceleration

$$\vec{r}(t) = \vec{R}(t) + \vec{r}'(t)$$

$$\vec{v}(t) = \vec{v}_o + \vec{v}'(t)$$

$$\vec{a} = \vec{a}'$$

Thus acceleration of a particle in any inertial frame of reference is same, so we do not have to specify the frame, ***as long as it is inertial (constant velocity).***

S' must be inertial! (Galilean Relativity)

Non Inertial Frame (N.I.F) and Pseudo/Fictitious Forces

- If we have to work in a non-inertial frame of reference, Can we apply Newton’s laws of motion?

- Fortunately, we can apply these laws, but with some modifications.

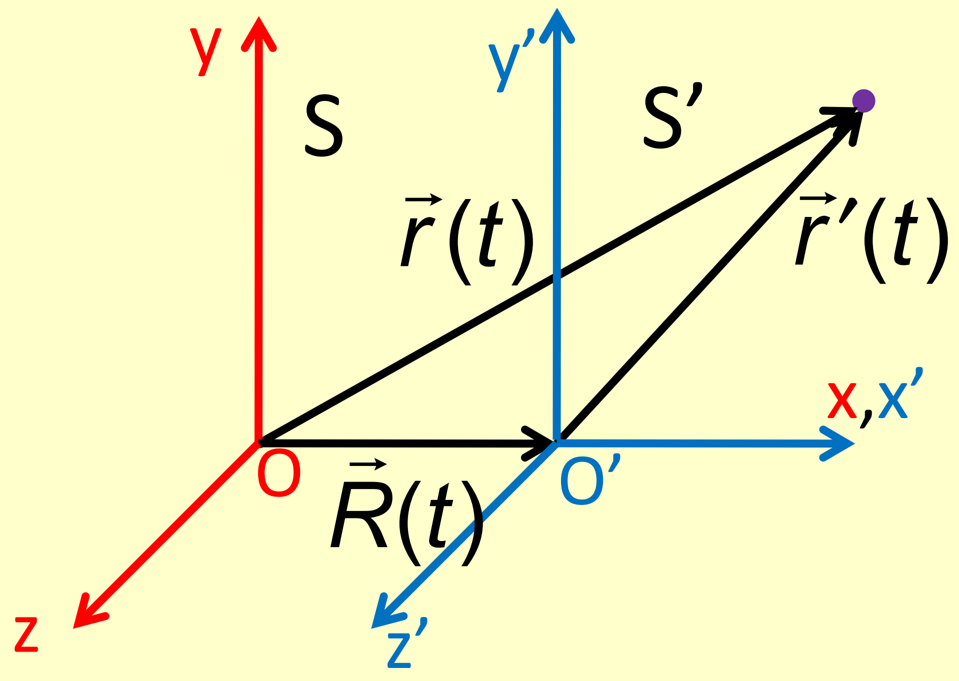
Non Inertial Frame of reference (N.I.F.O.R)

It is many times easier to think and visualize the motion of bodies when observer is located in a non inertial frame of reference *(nifor)*. We shall discuss two types.

1) Uniformly accelerating frames of references *e.g. motion of a body in accelerating train.*

2) Rotating Frames of references *e.g. motion of a body on merry go round or for that matter on earth*.

Linearly accelerating N.I.F.O.R



Assume S is an inertial frame and S' is accelerating, with a constant acceleration $\vec{A}$

Acceleration

$$\vec{r}(t) = \vec{R}(t) + \vec{r}'(t)$$

$$\vec{v}(t) = \vec{v}_o + \vec{v}'(t)$$

$$\vec{a} = \vec{A} + \vec{a}'$$

According to S, acceleration of the particle is due to some real force

$$\vec{F}_{real} = m\vec{a}$$

Newton's Law in S'

Acceleration of the particle observed in S' is $\vec{a}'$

If Newton's law has to be applied in S'

$$\vec{F} = m\vec{a}'$$

$$= m\left(\vec{a} - \vec{A}\right)$$

$$= \vec{F}_{real} - \boxed{m\vec{A}}$$

Newton's Law in S'

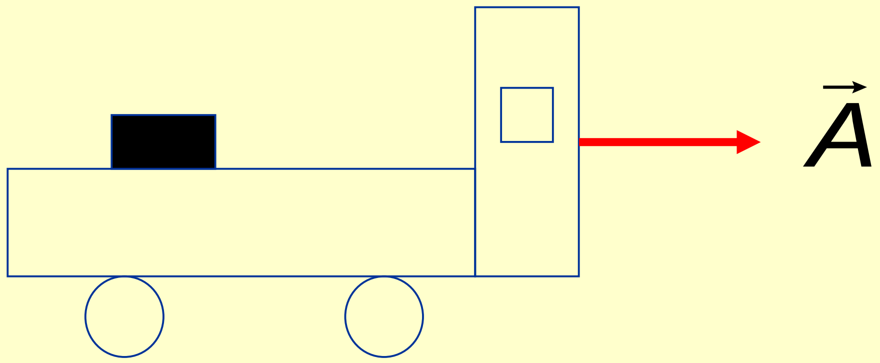
One can apply Newton's law if we assume that in addition to real forces an additional force of $-m\vec{A}$ is also applied to the particle.

Such a force is called ***Fictitious*** or ***Pseudo*** Force.

Fictitious/Pseudo Force: Properties

- Does not arise as a result of interaction of two bodies, unlike real forces.
- In a uniformly accelerating frame
  - Is uniform
  - Is proportional to mass, like gravitational force.
- We must know the acceleration of S’ in order to apply it.

A block kept on Frictionless Truck



Assume truck starts with an acceleration $\vec{A}$

Ground Frame assuming it to be inertial frame:

No force on the block. Hence it continues to remain stationary, even through truck is going ahead

Truck Frame:

The block accelerates backwards with an acceleration due to pseudo force $-m\vec{A}$

Assume there is enough friction between block and truck so that block moves with truck.

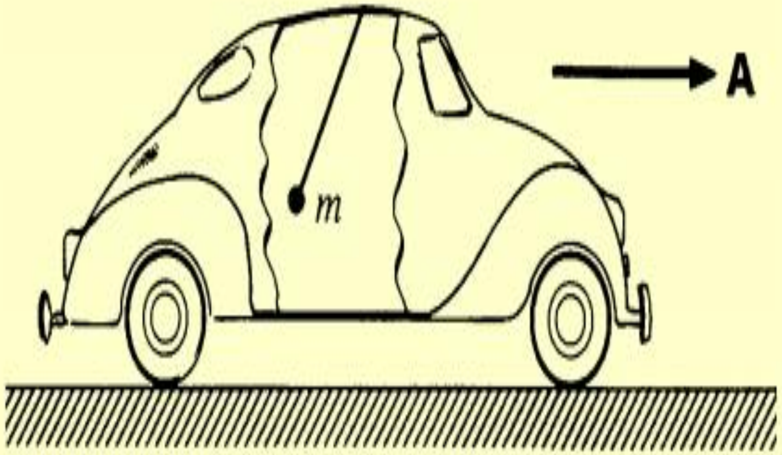
Ground Frame:

Real force of friction is causing the block to accelerate with truck.

Truck Frame:

The Pseudo force balances the real frictional force.

A small weight of mass m hangs from a string in an automobile which accelerates at rate $\boldsymbol{A}$. What is the static angle of the string from the vertical, and what is its tension?

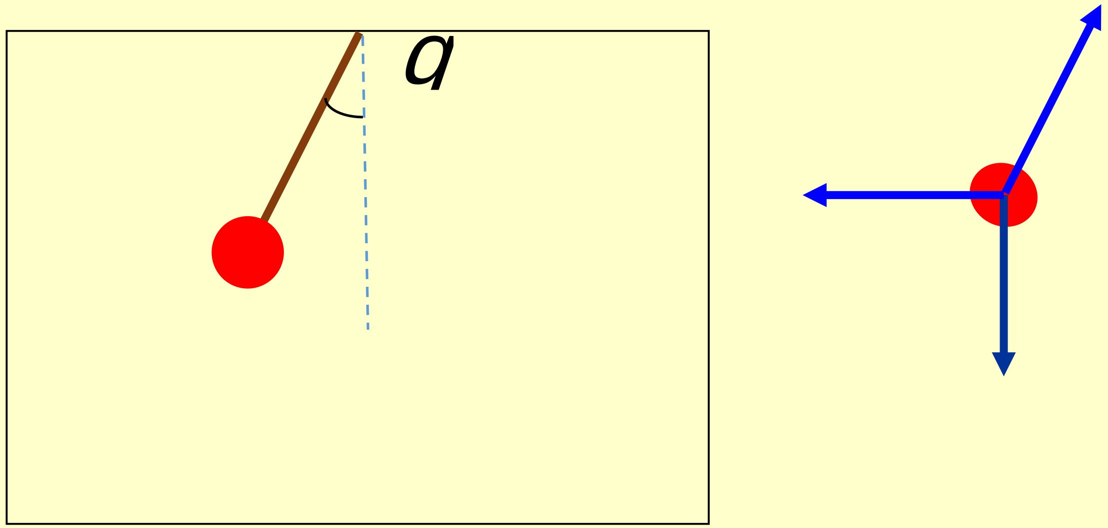


Let us analyze the problem both in an inertial frame and in a frame accelerating with the car.

Inertial system

System accelerating with auto

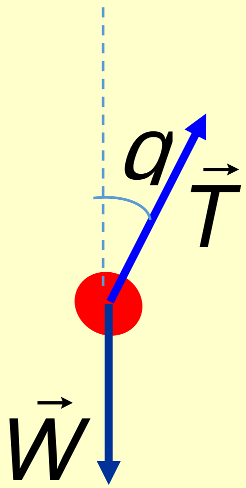
A pendulum in accelerating car:



We have to find equilibrium angle θ, if the car is

accelerating with an acceleration $\vec{A}$

Inertial Frame(Free body diagram)



- With respect to lab frame, mass m has an acceleration A
- As shown, it experiences two forces, tension T of the string, and its own weight $W = mg$
- We want to find angle of inclination θ, and T
- Application of Newton's law in vertical and horizontal directions, yields

$$T\cos q - W = 0; W = mg$$

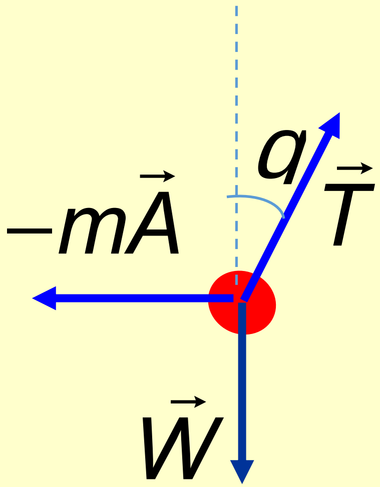
$$T\sin q = mA$$

$$q = \tan^{-1}\left(\frac{A}{g}\right)$$

$$T = m\sqrt{g^2 + A^2}$$

Non-Inertial Frame(Free body diagram)

Acceleration = 0



- In the NI frame, particle is stationary, and in equilibrium
- But it is acted upon by three forces, instead of two
- Additional force is the fictitious (or pseudo) force $F_{fict} = -mA$

$$T\cos\theta - mg = 0$$

$$T\sin\theta - mA = 0$$

$$\theta = \tan^{-1}\left(\frac{A}{g}\right)$$

$$T = m\sqrt{g^2 + A^2}$$

An elevator is accelerating upwards with an acceleration of $A = 2\ m/s^2$. A ball is thrown vertically upwards such that it just touches ceiling at the height of $4\ m$. **Find initial velocity u relative to elevator.**

Non-Inertial Frame(Elevator)

Net acceleration

$= 10+2=12\ m/s^2$ (Taking g=10 m/s^2)

$$\therefore \quad u^2 = 2 \times 12 \times s = 96$$

$$\Rightarrow \quad u = \sqrt{96} m/s$$

Inertial Frame

Let the velocity of the elevator be *v* at the instant ball is thrown. The ball velocity is *(v+u).* Let the ball hit the ceiling after time *t*.

Velocity of elevator after time *t*

$$= \quad v + At = v + 2t$$

Velocity of ball after time *t*

$$= \quad (v + u) - gt$$

Inertial Frame

Just touching implies:

$$v + At = \left(v + u\right) - gt$$

$$\Rightarrow \quad u = \left(A + g\right)t$$

Also the distances travelled by lift and ball differ by 4*m* implies:

$$\left((u+v)t - \frac{1}{2}gt^2\right) - \left(vt + \frac{1}{2}At^2\right) = 4$$

Substituting and solving we get

$$\frac{1}{2}\left(A+g\right)t^2 = 4$$

Þ $$t = \sqrt{\frac{2}{3}}$$

Further substitution gives

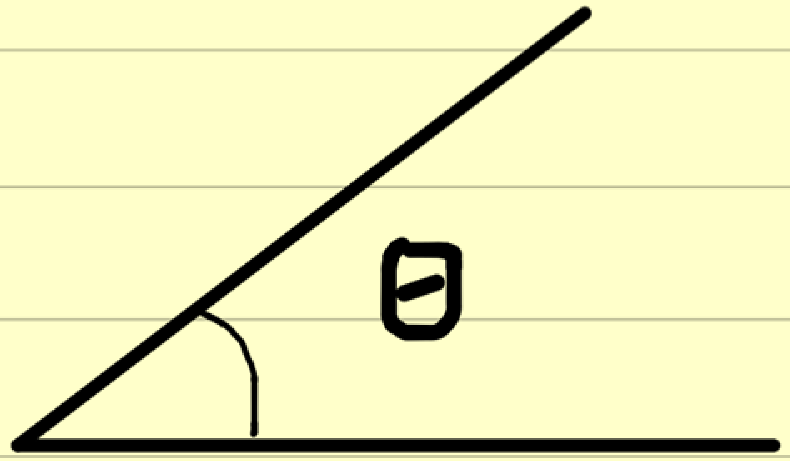
$$u = \sqrt{96} m / s$$

Points to Note

1. Initial velocity of elevator.
2. Touching the ceiling does not mean the velocity of ball is zero.
3. The distance traveled by ball in vertical direction is not 4*m.*

Q.1: A particle moving with constant velocity. An observer in frame S locates this particle at origin at t=0, and finds that it is moving with a constant velocity of 5 m/s making an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ with x-axis in x-y plane. Find the co-ordinates of the particle in S at t=2 s. Also find that the distance that the particle travels according to an observer in S. Assume another observer in S' frame, which is moving relative to S with a speed of 1 m/s along x-axis of S frame. Find the speed of the particle and its co-ordinates at t=2 s in S' frame. Also find the angle that the velocity direction makes with x'-axis in frame S' and the distance covered by the particle in S' frame.

Q.1:



$$\theta = \tan^{-1}(3/4)$$

$\therefore \sin\theta = 3/5$, $\cos\theta = 4/5$

$u_x = 5 \times \frac{4}{5} = 4$ m/s ; $u_y = 5 \times \frac{3}{5} = 3$ m/s

$x = 4 \times 2 = 8$ m ; $y = 3 \times 2 = 6$ m $\therefore$ Distance travelled $= \sqrt{8^2 + 6^2} = 10$ m

$u_x' = u_x - V = 4 - 1 = 3$ m/s

$u_y' = u_y = 3$ m/s

The speed is $3\sqrt{2}$ m/s

Using Galilean transformation:

$x' = 8 - 1 \times 2 = 6\,m$

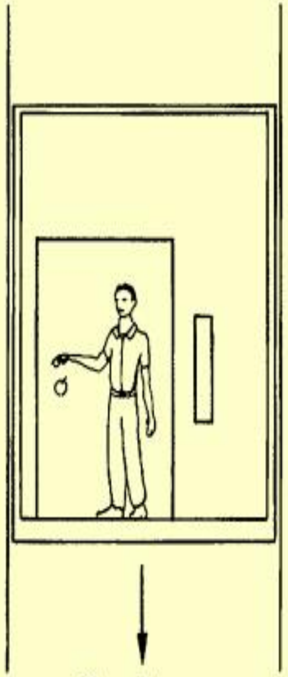
$y' = 6\,m$ **= y**

Velocity
The angle in $S' = \tan^{-1}(3/3) = 45^{0}$

$u_y' = u_y = 3\,m/s$

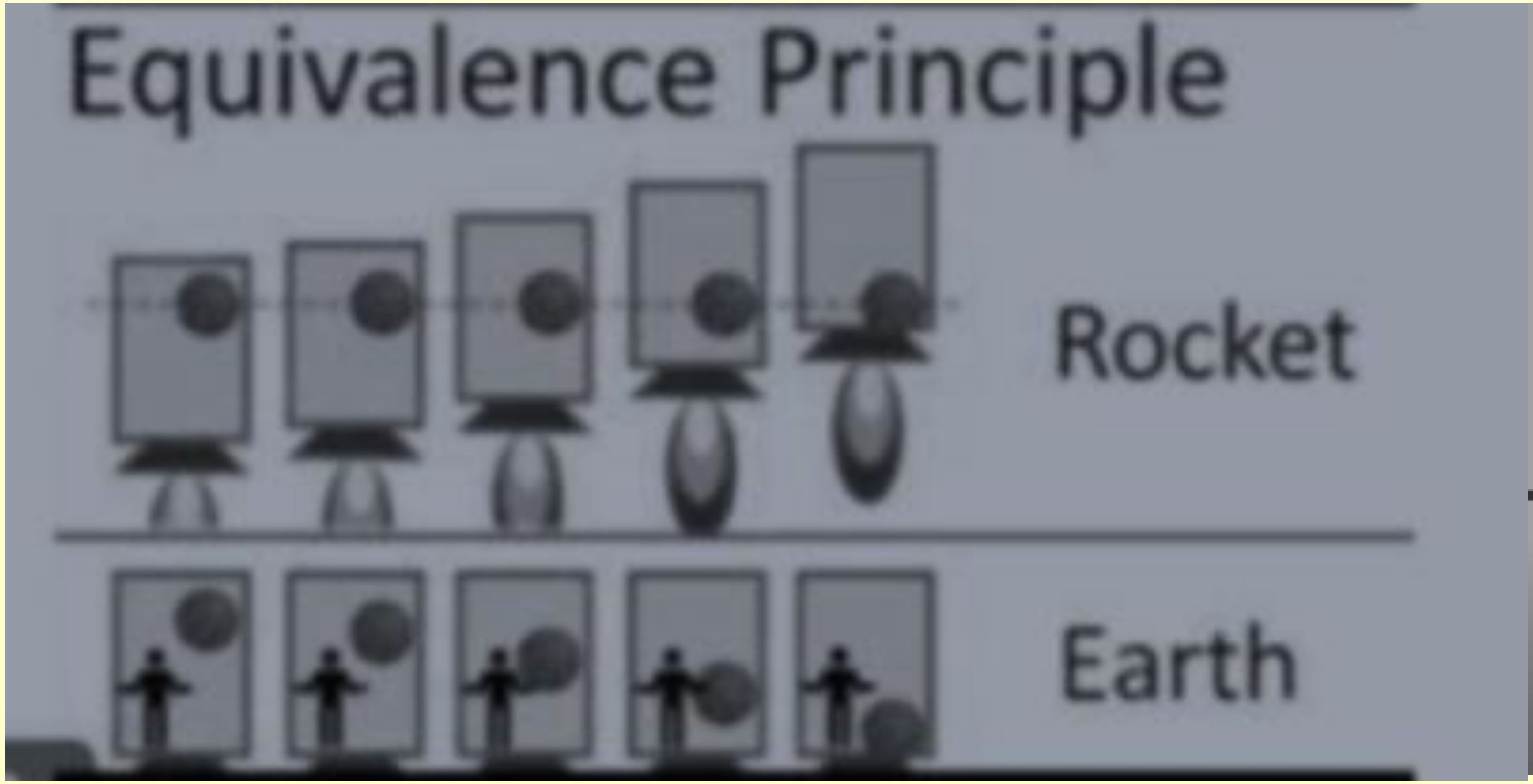
Distance covered in $S' = 6\sqrt{2}\,m$

The Principle of Equivalence



A man is holding an apple in an elevator at rest in a gravitational field g. He lets go of the apple, and it falls with a downward acceleration $a = g$. Now consider the same man in the same elevator, but let the elevator be in free space accelerating upward at rate $a = g$. The man again lets go of the apple, and it again appears to him to accelerate down at rate g. From his point of view the two situations are identical. He cannot distinguish between acceleration of the elevator and a gravitational field.

There is no way to distinguish locally between a uniform gravitational acceleration $\boldsymbol{g}$ and an acceleration of the coordinate system $\boldsymbol{A} = -\boldsymbol{g}$. This is known as the principle of equivalence. However, such indistinguishable nature of two forces is valid only for point objects .



Observer floating outside rocket

Observer inside rocket

A ball is thrown vertically upwards by an observer P’ sitting in a cart, which is moving with a constant speed of 4m/s in a straight line on the ground. The ball is caught back by the observer P’ sitting in the same place in the cart. An observer P on the ground finds that the speed of the ball at the time of touching the ground of bullock cart is 5 m/s. Find the speed of the ball at the time of throwing according to P and P’ observers. What is the horizontal distance travelled by the ball according to the two observers and what is the height to which the ball reaches? (Take $g=10\ m/s^2$)

Rotating Frame of Reference

- Intrinsically non-inertial. Pseudo forces is must for describing dynamics.
- Pseudo Forces do not have such simple form as in uniformly accelerating frames.

Rotating Coordinate System

We denoted the position of a particle as a vector

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

Can we similarly specify the angular position of a particle

$$\boldsymbol{\theta} = \theta_x\hat{\mathbf{i}} + \theta_y\hat{\mathbf{j}} + \theta_z\hat{\mathbf{k}}\,?$$

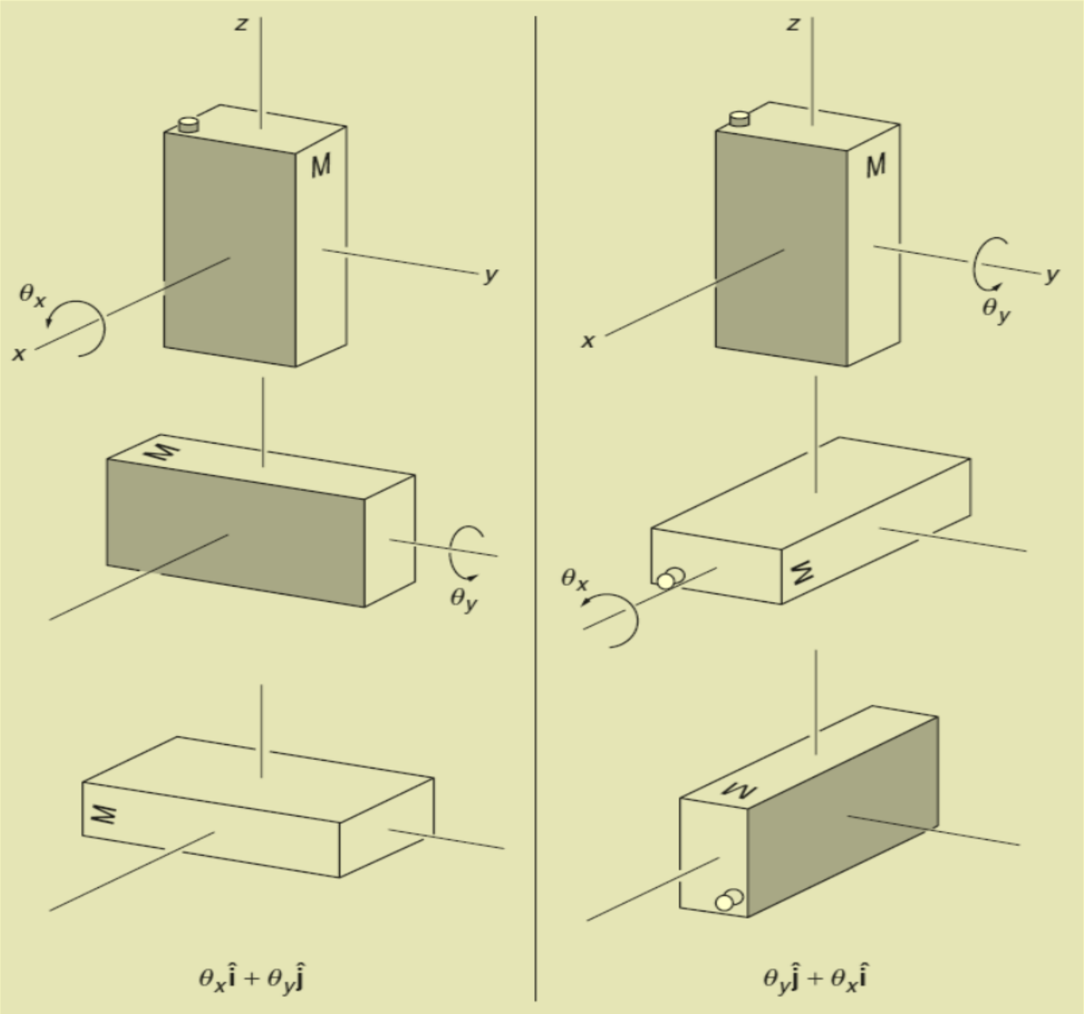
The answer is no because such an expression does not satisfy commutative law of vector addition

$$\boldsymbol{\theta}_1 + \boldsymbol{\theta}_2 \neq \boldsymbol{\theta}_2 + \boldsymbol{\theta}_1$$

Let us rotate a block first around the x axis, and then around the y axis. Compare that to the same operations performed in the reverse order

Rotating Coordinate System

Consider those two rotations, with each one of them being $\pi/2$



Clearly $\theta_x\hat{\mathbf{i}} + \theta_y\hat{\mathbf{j}} \neq \theta_y\hat{\mathbf{j}} + \theta_x\hat{\mathbf{i}}$

On the other hand, one can verify that infinitesimal rotations commute to first order terms

$$\Delta\theta_x\hat{\mathbf{i}} + \Delta\theta_y\hat{\mathbf{j}} \approx \Delta\theta_y\hat{\mathbf{j}} + \Delta\theta_x\hat{\mathbf{i}}$$

Thus, infinitesimal rotations can be represented as vectors

Because angular velocity is defined in terms of infinitesimal rotations

$$\boldsymbol{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\boldsymbol{\theta}}{\Delta t},$$

Relation between v and ω

Angular velocity can be denoted as a vector

$$\boldsymbol{\omega} = \omega_x\hat{\mathbf{i}} + \omega_y\hat{\mathbf{j}} + \omega_z\hat{\mathbf{k}}$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \omega \times \mathbf{r}$$

And, in general,

$$\boldsymbol{\omega} = \omega\hat{\mathbf{n}},$$

where $\hat{\mathbf{n}}$ is the direction of the axis of rotation, and ω is the magnitude of the angular velocity.