Lecture 11

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Houston we have a PROBLEM

Lorentz force is a velocity dependent force and could depend on inertial frame of reference (ifor) used

Velocity Dependent Lorentz force:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

RECAP

The correct transformation of electric and magnetic fields as we go from one ifor to another cannot be obtained using Galilean relativity.

Another Problem

Speed of light is related to fundamental constants.

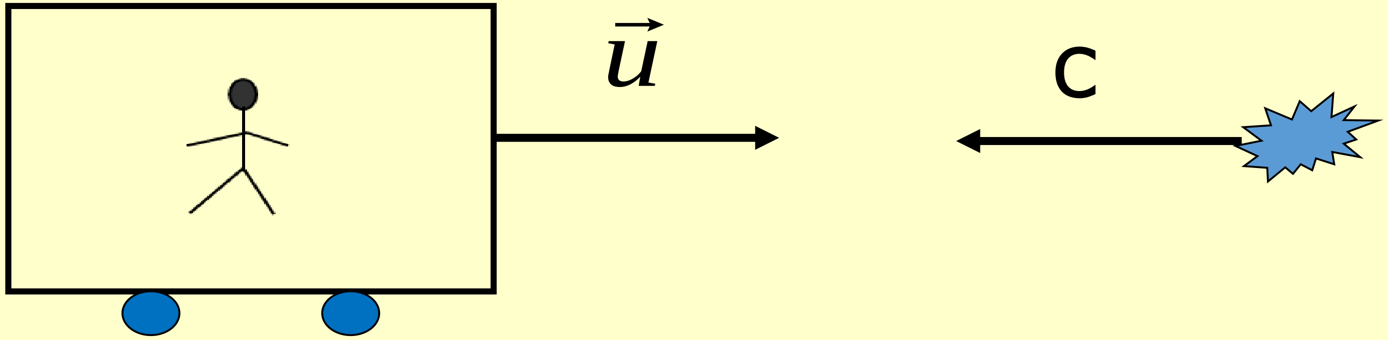
RECAP

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

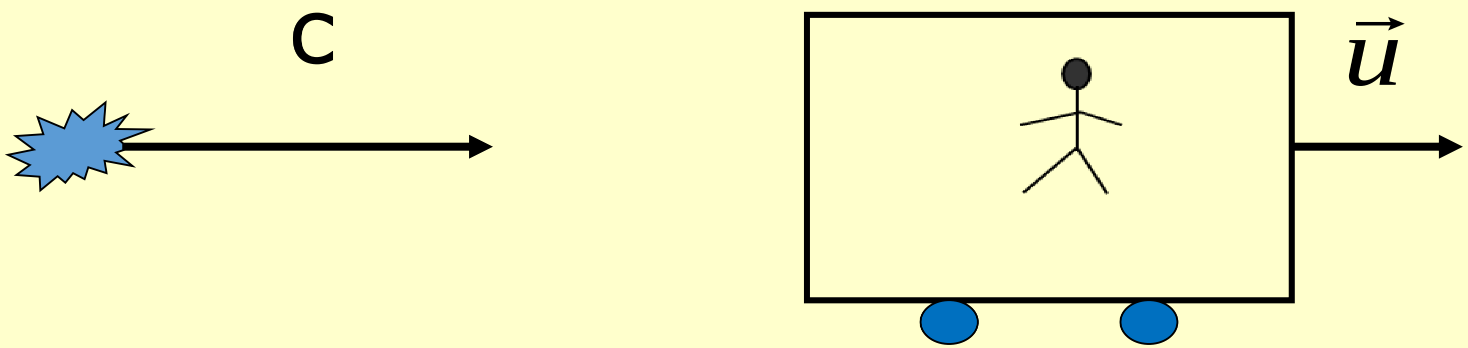
The speed of light in vaccum (c), depends on two fundamental constants, the permeability of free space μ_0 and permittivity of free space ε_0.

Why is this a problem ?

Speed of light frame dependent?



RECAP



B is stationary but is emitting light which is traveling with a speed of *c* in the same direction as the original direction of *B*. According to ***classical relative velocity formula*** the speed of light as seen by an observer on *A* would be $\boxed{(c+v).}$

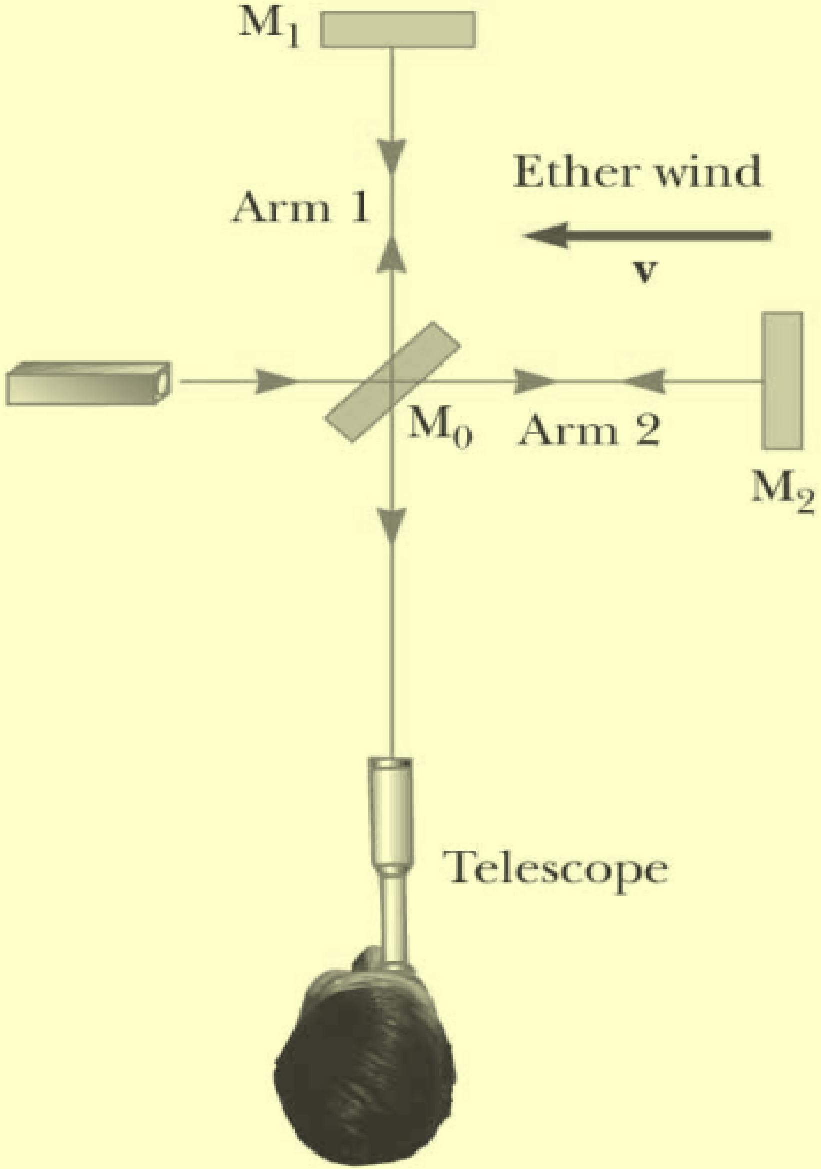
Ether RECAP

- Earlier ideas favored the concept of a special frame.
- It was imagined that the universe is filled with ocean of ether.
- All planets, stars, galaxies float in ether.

The Michelson-Morley Experiment

- Michelson and Morley did this experiment, but saw absolutely no shift in the fringes!
- Have to add a postulate to Galilean Relativity

RECAP



Summary

- Classical Physicists felt that electromagnetic theory required the presence of a special frame, which was termed as Ether.
- Experiments did not provide support for the concept of Ether.
- Einstein gave the postulates of Special Theory of Relativity (STR).

Einstein’s Postulates

- Laws of Physics are same in all inertial frames of references. No preferred inertial frame exists.

- The speed of light ‘c’ is same in all inertial frames.

Modified Formulation: Forthcoming

- ***Relative velocity formula requires modification*.**
- A new formulation is needed which can ensure that the speed of light is same in all the inertial frames.
- The new formulation must be consistent with the first postulate

Lorentz Transformation

Lorentz Transformation: Expectation

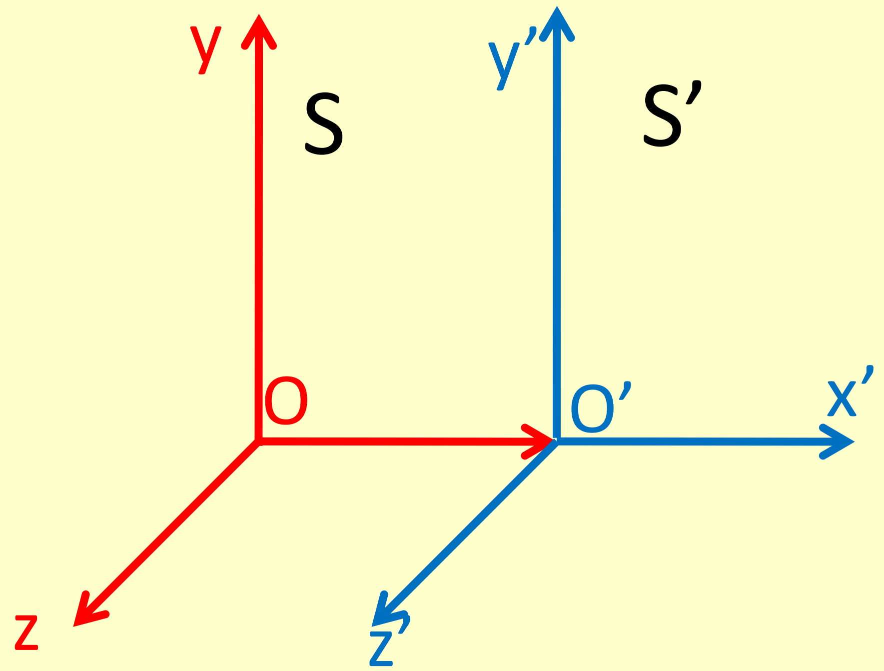
- A set of dynamical variables are known or measured for a particle in a given frame.
- We are interested in finding out the values of the same set of variables in a different frame.
- The equations that we shall use are called ***transformation*** equations.

Review: Types of Transformations

- Co-ordinates transformation
- Velocity transformations.

Recall: Galilean Transformation

- Take two inertial frames S and S'. Choose x and x' axes along the direction of relative velocity of the frames.
- Assume y and z axes to be parallel to y' and z' axes respectively.



- Frame S' moves with respect to S along x-axis with a velocity $\vec{v}$.
- Clock of each observer is set to zero when their origins coincide.

Galilean Transformation

Direct Transformation

$$x' = x - vt, \quad y' = y, \quad z' = z$$

$$t' = t$$

Inverse Transformation

$$x = x' + vt', \quad y = y', \quad z = z'$$

$$t = t'$$

Classical Velocity Transformation

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v,$$

$$\frac{dy'}{dt'} = \frac{dy}{dt}$$

$$\frac{dz'}{dt'} = \frac{dz}{dt}$$

$$u'_x = u_x - v, \quad u'_y = u_y \quad u'_z = u_z$$

1. This is same as the old relative velocity formula, which is inconsistent with second postulate.

2. The inverse velocity transformation can be similarly derived.

$$u_x = u'_x + v, \quad u_y = u'_y \quad u_z = u'_z$$

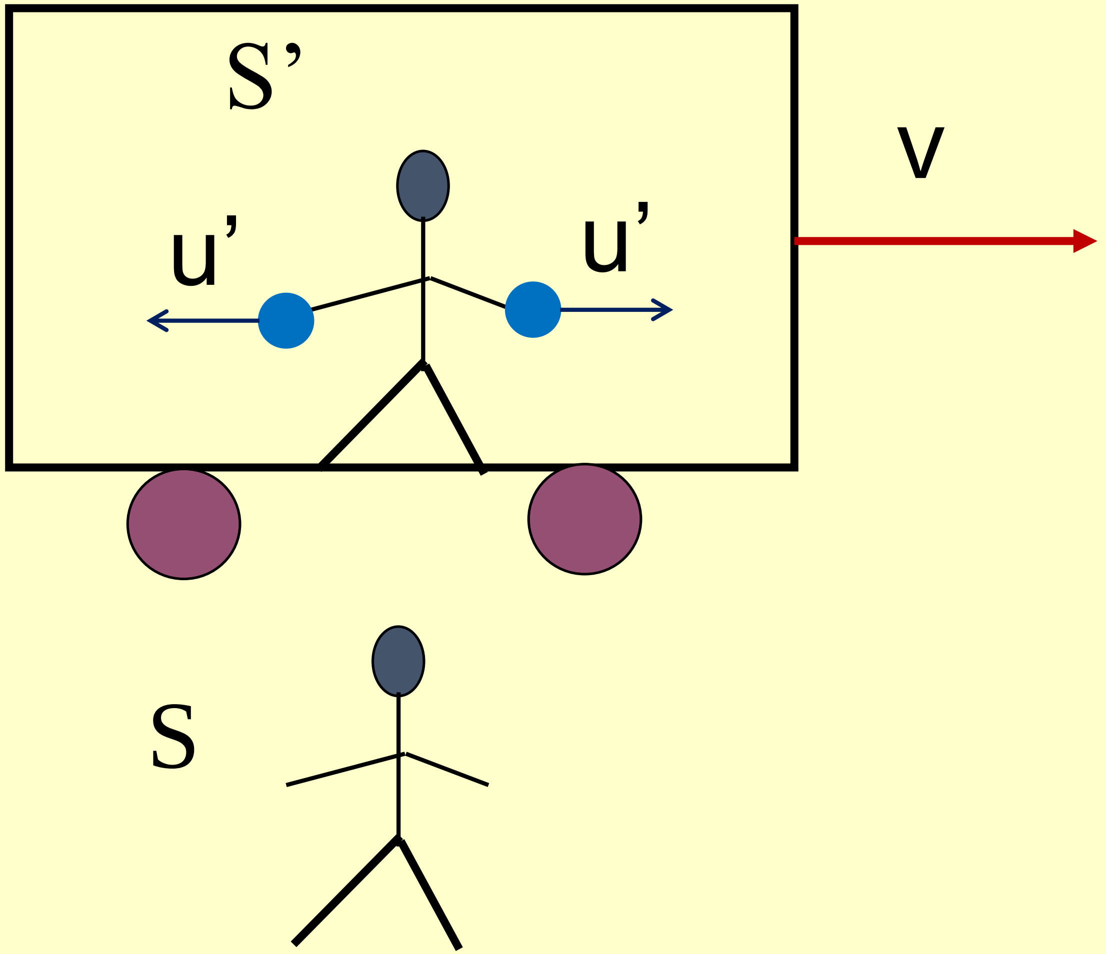
We have to now look for a transformation, which is consistent with the second postulate and can make the speed of light frame independent.

Time is suspect

- Time is related to the simultaneity of two events.
  - Train leaves at 10:00 implies: ***train leaving*** and ***clock showing 10:00*** are simultaneous events.

- We shall show that simultaneity is relative under the second postulate.

An observer is exactly half way in a running compartment of length *L*. He throws two balls at the same time (t'=0) with same speed *u'* as measured by him, one towards the front wall and other toward back wall.



- Event1: The first ball reaches the front wall.
- Event 2: The second ball reaches back wall.

Question:

Are events 1 and 2 simultaneous, implying do they occur at the same time?

In *S'* frame

Time (t'_1) for event 1:

$$t'_1 = \frac{L}{2u'}$$

Time (t'_2) for event 2:

$$t'_2 = \frac{L}{2u'}$$

Hence the two events are simultaneous in this frame.

Same events in *S* frame

Recall: $u_x = u'_x + v, \quad u_y = u'_y \quad u_z = u'_z$

The speed of the first ball using inverse velocity transformation.

$$u_x = u'_x + v = u + v$$

The speed of the second ball

$$u_x = u'_x + v = -u + v$$

$$vt_1 + \frac{L}{2} = (u' + v)t_1$$

$$vt_2 - \frac{L}{2} = (v - u')t_2$$

$$t_1 = \frac{L}{2u'}$$

$$t_2 = \frac{L}{2u'}$$

$\boxed{\text{Thus } t_1 = t_2 = t'_1 = t'_2}$

ALL IS WELL

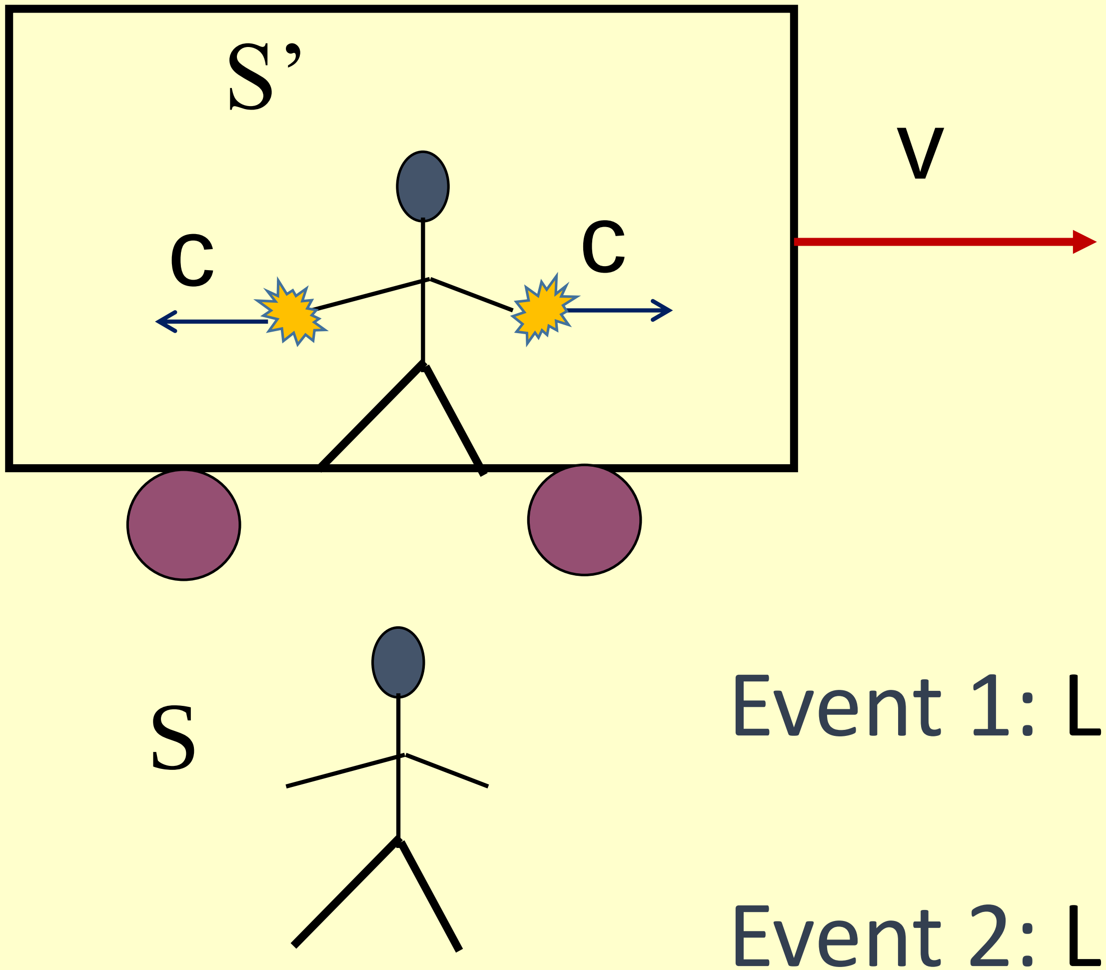
Use Light instead of Balls

Now imagine that the observer shines light instead of throwing balls which travels both in the front and the back direction.

Event 1: Light reaches the front wall

Event 2: Light reaches the back wall

Use Light instead of Balls



Event 1: Light reaches the front wall

Event 2: Light reaches the back wall

In *S'* frame

Time (t'_1) for event 1: $$t'_1 = \frac{L}{2c}$$

Time (t'_2) for event 2: $$t'_2 = \frac{L}{2c}$$

The two events are simultaneous because in S' the light covers the same distance in the front direction as in the back, and with the same speed.

Hence the two events are simultaneous in this frame as before. ($\Delta t'=0$)

In S Frame

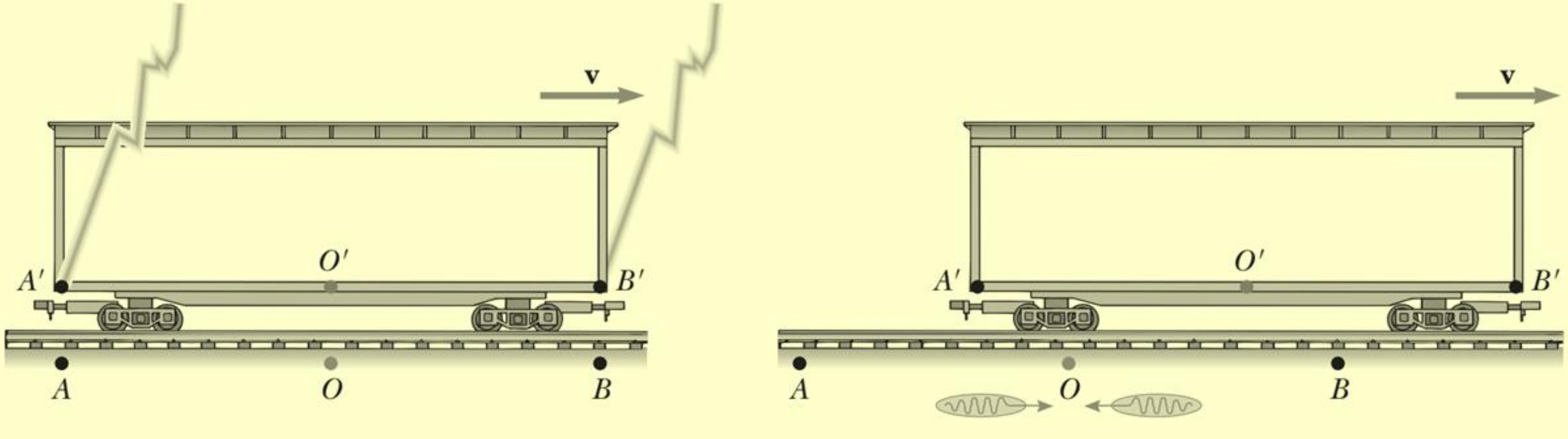
- The speed of light is still *c* in both the directions, according to the second postulate. But it has to travel a larger distance to reach the front wall than the back wall.
- $t_1 > t_2$ or $(t_2 - t_1)$ is negative.

Conclusion

- There is simultaneity of two events in S' but not in S.
- Simultaneity of two events thus depends on the frame chosen.
- In other words, two events that are simultaneous in a given frame will not be seen to be simultaneous in another frame. That means, ***time is a frame dependent quantity!*** ***<u>Or time is no longer absolute!</u>***
- ***<u>This is a big deviation from the Galilean transformation.</u>***

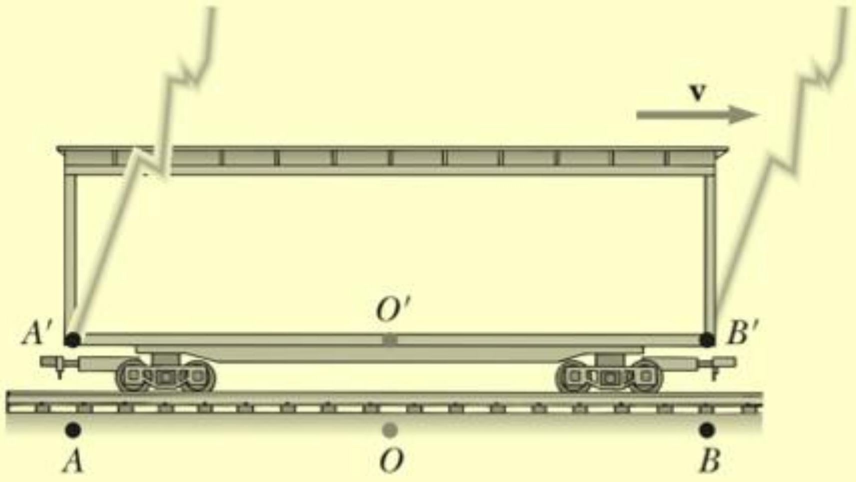
SIMILAR EXAMPLE

Einstein the Experimentalist !!!



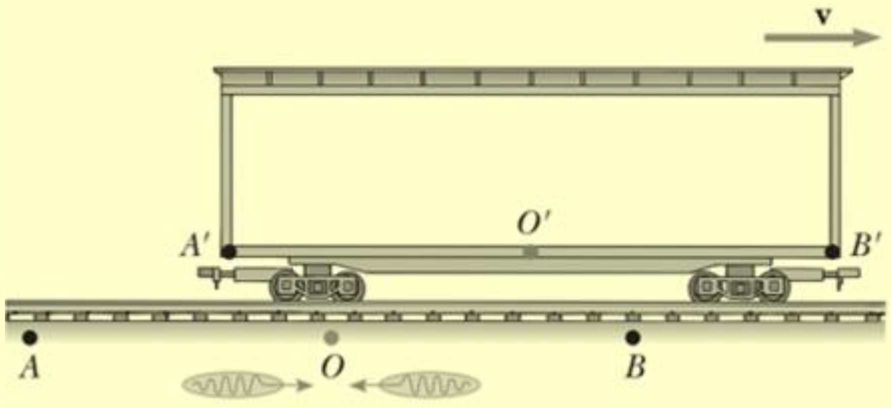
(a) (b)

Einstein devised the following thought experiment to illustrate this point. A boxcar moves with uniform velocity, and two lightning bolts strike the ends of the boxcar, as in Figure above leaving marks on the boxcar and ground. The marks left on the boxcar are labeled A' and B'; those on the ground are labeled A and B. An observer at O' moving with the boxcar is midway between A' and B', and a ground observer at O is midway between A and B. The events recorded by the observers are the light signals from the lightning bolts.



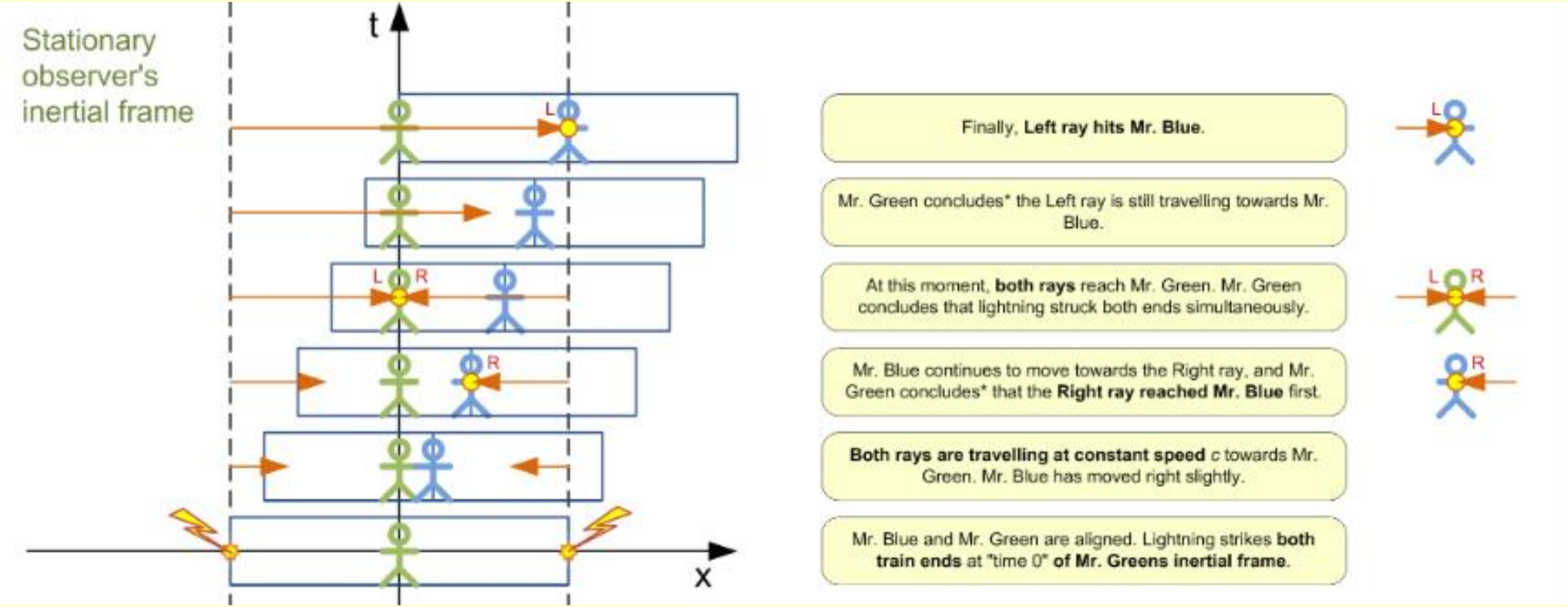
(a)

In the inertial frame of the standing observer, there are three events which are spatially dislocated, but simultaneous: standing observer facing the moving observer (i.e., the center of the train), lightning striking the front of the train car, and lightning striking the back of the car.



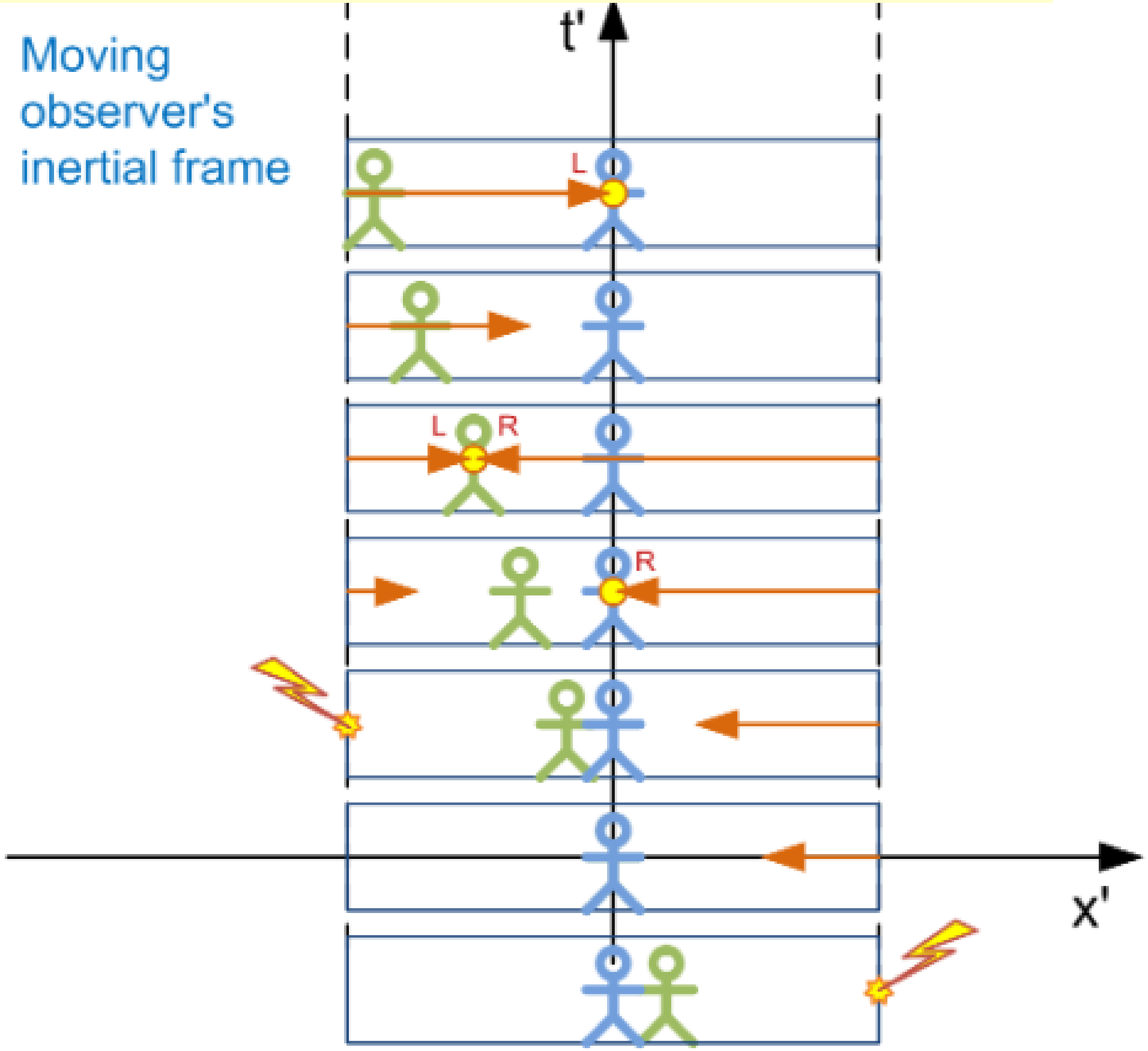
(b)

Since the events are placed along the axis of train movement, their time coordinates become projected to different time coordinates in the moving train's inertial frame. Events which occurred at space coordinates in the direction of train movement happen *earlier* than events at coordinates opposite to the direction of train movement. In the moving train's inertial frame, this means that lightning will strike the front of the train car *before* the two observers align (face each other) and then finally the lightning strikes the back of the car.

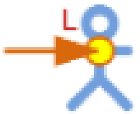


Stationary observer's inertial frame
t
x
L
R
Finally, **Left ray hits Mr. Blue**.
Mr. Green concludes* the Left ray is still travelling towards Mr. Blue.
At this moment, **both rays** reach Mr. Green. Mr. Green concludes that lightning struck both ends simultaneously.
Mr. Blue continues to move towards the Right ray, and Mr. Green concludes* that the **Right ray reached Mr. Blue** first.
Both rays are travelling at constant speed *c* towards Mr. Green. Mr. Blue has moved right slightly.
Mr. Blue and Mr. Green are aligned. Lightning strikes **both train ends** at "time 0" **of Mr. Greens inertial frame**.

Moving observer's inertial frame

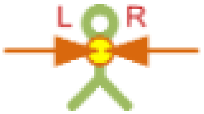


Finally, **Left ray hits Mr. Blue**.

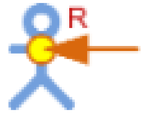


Left ray is still travelling towards Mr. Blue.

At this moment, **both rays** reach Mr. Green. Mr. Blue is still waiting for the other ray, since it struck later.



Right ray hits Mr. Blue first. Mr. Blue concludes that the front side has been struck first.



Sometime after that, lightning strikes the back (left) side of the train.

Mr. Blue and Mr. Green are aligned at this moment. One side of the train has already been struck, but the **other one hasn't**.

In this inertial frame, lightning strikes the front (right) side of the train **before** Mr. Green and Mr. Blue are even aligned!

Yet again; Conclusion

Two events that are simultaneous in one frame are in general not simultaneous in a second frame moving with respect to the first. That is, simultaneity is not an absolute concept, but one that depends on the state of motion of the observer.