CH 107 Tutorial 3

Solve these problems BEFORE the tutorial session

1. The Schrödinger equation for a particle of mass m constrained to move on a circle of radius a is given by $-\frac{\hbar^2}{2I}\frac{d^2\psi(\theta)}{d^2\theta} = E_n\psi(\theta)$, where I = ma^2 is the moment of inertia and θ is the angle that describes the position of the particle on the circular ring.
   (a) Suggest an acceptable solution
   (b) Using appropriate boundary conditions, obtain the expression for energies E_n
   (c) What are the permissible values of quantum number (n)?

2. Evaluate the normalization constant A for 3D rigid rotor wavefunction $Y_1^0 = ACos\theta$

3. Check whether the normalized 3D rigid rotor wavefunction: Y_1^0 (in Q2) is an eigenfunction of L^2 (square of total angular momentum) operator. If so, what is the eigenvalue? Given, $L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$

4. a) Why is it necessary to use spherical polar coordinates to solve the time-independent Schrödinger equation for hydrogen atom?

   b) Both the rigid rotor and the H-atom are 3D systems (spatial coordinates). However, we require three quantum numbers to describe the eigenstates of H-atom, while only two are necessary to specify the states of rigid rotor. *Justify!*

5. a) Evaluate the normalization constant N for the ground state wave function of hydrogen atom. $\Psi_{1s}(r,\theta,\phi) = N\exp\left(\frac{-r}{a_0}\right)$. If you need, use $\int x^n \cdot e^{-ax}dx = n!/a^{n+1}$

   b) For the lowest energy state of H-atom, evaluate the average distance of the electron from the nucleus.

Additional Question for students to practice (not to be done during tutorial 3):

5. Express the Laplacian operator of H-atom in spherical polar coordinates ($\nabla^2_{r\theta\varphi}$ in relative coordinates) in terms of the square of the total angular momentum operator (L^2) of 3D rigid rotor, and other terms which do not involve angular coordinates.

6. Verify whether the normalized 3D rigid rotor wavefunctions: $Y_1^{\pm1} = \sqrt{3/8\pi}\cdot Sin\theta\cdot\exp(\pm i\varphi)$ and $Y_2^0 = \sqrt{5/16\pi}\left(3\cos^2\theta - 1\right)$ are eigenfunctions of $\boldsymbol{L^2}$ and $\boldsymbol{L_z}$. What are the eigenvalues?