

$$f: \mathbb{R} \rightarrow \mathbb{R}. \quad x_0 \in \mathbb{R}.$$

$$\underline{f'(x_0) \in \mathbb{R}.}$$

Consider map:

$$\begin{array}{ccc} \text{---} f'(x_0) \text{---} & \xleftrightarrow{F = DF} & \text{---} f'(x_0) \cdot h \text{---} \\ \text{---} \text{---} \text{---} & & \text{---} \text{---} \text{---} \\ & & \text{Linear map.} \end{array}$$

$$f(x_0 + h) = f(x_0) + \underline{f'(h)} + o(h) \cdot h.$$

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}. \quad (x_0, y_0)$$

$$\underline{DF(x_0)} \rightarrow \text{derivative at } (x_0, y_0).$$

$$\rightarrow \text{Linear map: } \mathbb{R}^2 \rightarrow \mathbb{R}.$$

$$f(x_0, y_0) + (h, k) = f(x_0, y_0) + DF(x_0) + O(h, k) \|(h, k)\|$$

$$\begin{array}{c} \downarrow \\ 0 \end{array} \quad \begin{array}{c} \alpha (h, k) \\ \rightarrow (0, 0) \end{array}$$

Linear map

$$f: \mathbb{R}^2 \rightarrow \mathbb{R} \longleftrightarrow \underline{1 \times 2 \text{-matrix}}$$

$$f((x_1, y_1) + (x_2, y_2))$$

$$= f(x_1, y_1) + f(x_2, y_2)$$

$$f(\alpha x_1, \alpha y_1) = \alpha \cdot f(x_1, y_1)$$

$$\begin{array}{c} \textcircled{[\alpha]} \longleftrightarrow \alpha \\ \left(\mathbb{R}^2 \xrightarrow{f(h)} \mathbb{R} \right) \longleftrightarrow \underline{f(x_0)} \end{array}$$

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