Lecture 9

Punit Parmananda

PH111

Email : punit@phy.iitb.ac.in

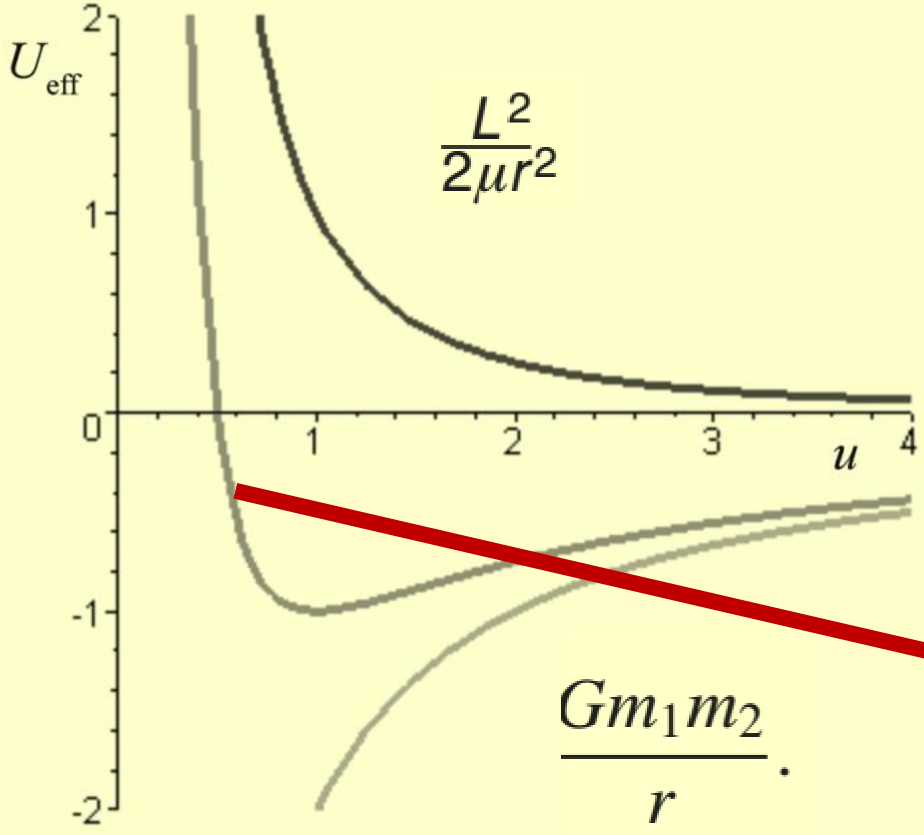
Office: Room 203, 2nd Floor,
Physics Department

RECAP

$$U_{\text{eff}} = \frac{L^2}{2\mu r^2} - \frac{G m_1 m_2}{r}.$$

$L^2/(2\mu r^2)$ dominates at small r

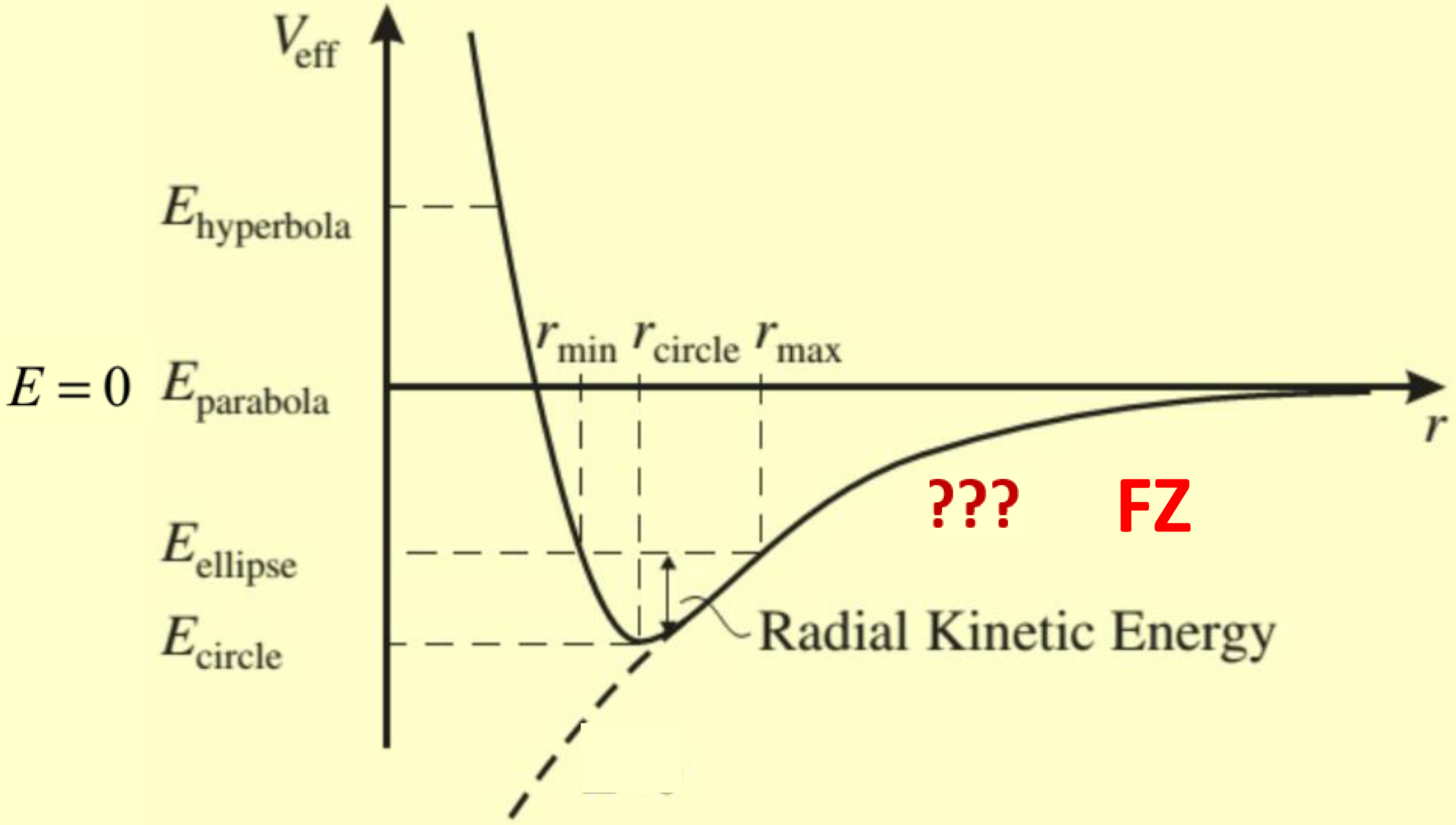
$-Gm_1m_2/r$ dominates at large r



Graph of effective potential energy

$$E = \frac{1}{2}\mu\dot{r}^2 + U_{\text{eff}}(r).$$

RECAP



RECAP

Circular Orbit $E = E_{min} < 0$ $\quad \varepsilon = 0$,

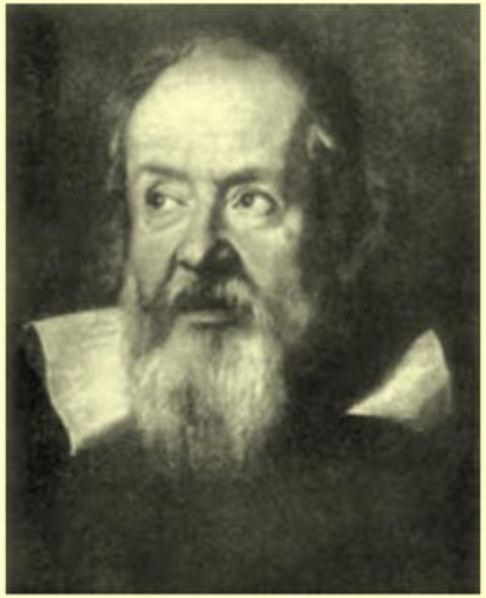
Elliptic Orbit $E_{min} < E < 0$ $\quad 0 < \varepsilon < 1$

Parabolic Orbit $E = 0$ $\quad \varepsilon = 1$

Hyperbolic Orbit $E > 0$ $\quad \varepsilon > 1$

Galilean Relativity

The fundamental laws of physics are the same in all frames of reference moving with constant velocity with respect to one another.



Galileo Galilei
1564 - 1642

DIALOGO
DI
GALILEO GALILEI LINCEO
MATEMATICO SOPRAORDINARIO
DELLO STVDIO DI PISA.
E Filosofo, e Matematico primario del
SERENISSIMO
GR.DVCA DI TOSCANA.
Doue ne i congressi di quattro giornate si discorre
sopra i due
MASSIMI SISTEMI DEL MONDO
TOLEMAICO, E COPERNICANO;
Proponendo indeterminatamente le ragioni Filosofiche, e Naturali
tanto per l'una, quanto per l'altra parte.
CON PRI VILEGI.
IN FIORENZA, Per Gio: Batista Landini MDCXXXII.
CON LICENZA DE' SVPERIORI.

Dialogue Concerning the Two Chief World Systems

Copernican system vs Ptolemaic system

Heliocentrism vs Geocentrism

Galileo's Ship

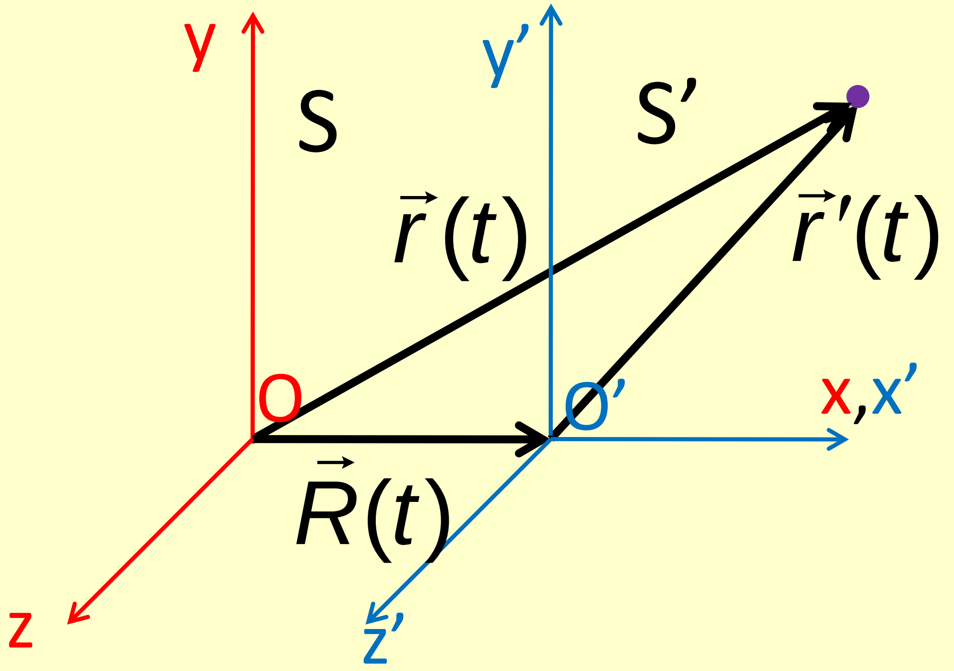
Ship traveling at constant speed on a smooth sea. Any observer doing experiments (playing billiard) under deck would not be able to tell if ship was moving or stationary.

Today we can make the same observation on a plane.

Inertial Frame of Reference

"Isolated Objects move with constant Velocity"

Two Inertial Frames



$$\boxed{\vec{r}(t) = \vec{R}(t) + \vec{r}'(t)}$$

What about the velocity ?

From Newton’s law velocity can be evaluated in any frame, if needed. **We must specify the frame to know the velocity.**

Velocity and Acceleration

$$\vec{r} = \vec{R} + \vec{r}\,'$$

$$\vec{v} = \vec{v}_o + \vec{v}\,'$$

$$\vec{a} = \vec{a}\,'$$

Newton’s Second law

- Velocity is a frame dependent quantity.
- Acceleration is same in all inertial frames.
- Newton’s second law depends on acceleration and Force.

$$\vec{F} = m\vec{a}$$

Newton’s Law in Different Frames

$$\vec{F} = m\vec{a}$$

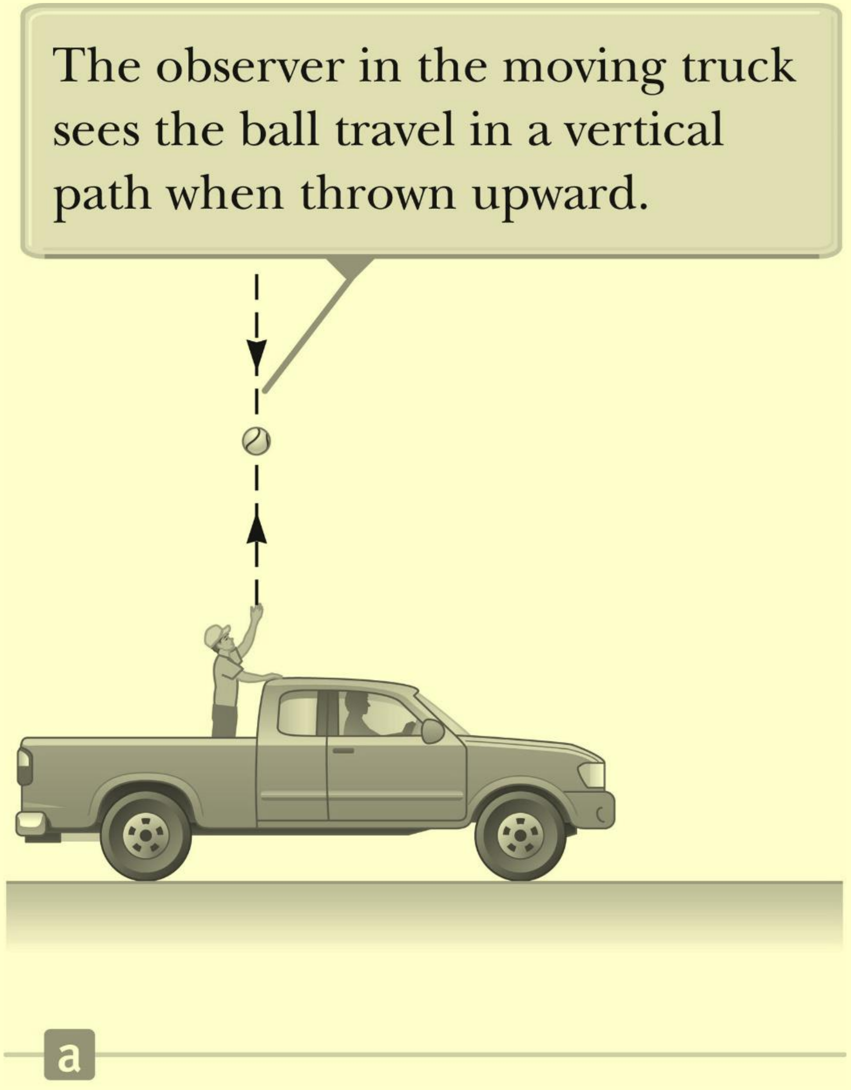
“Newton’s law is valid in all inertial frames”

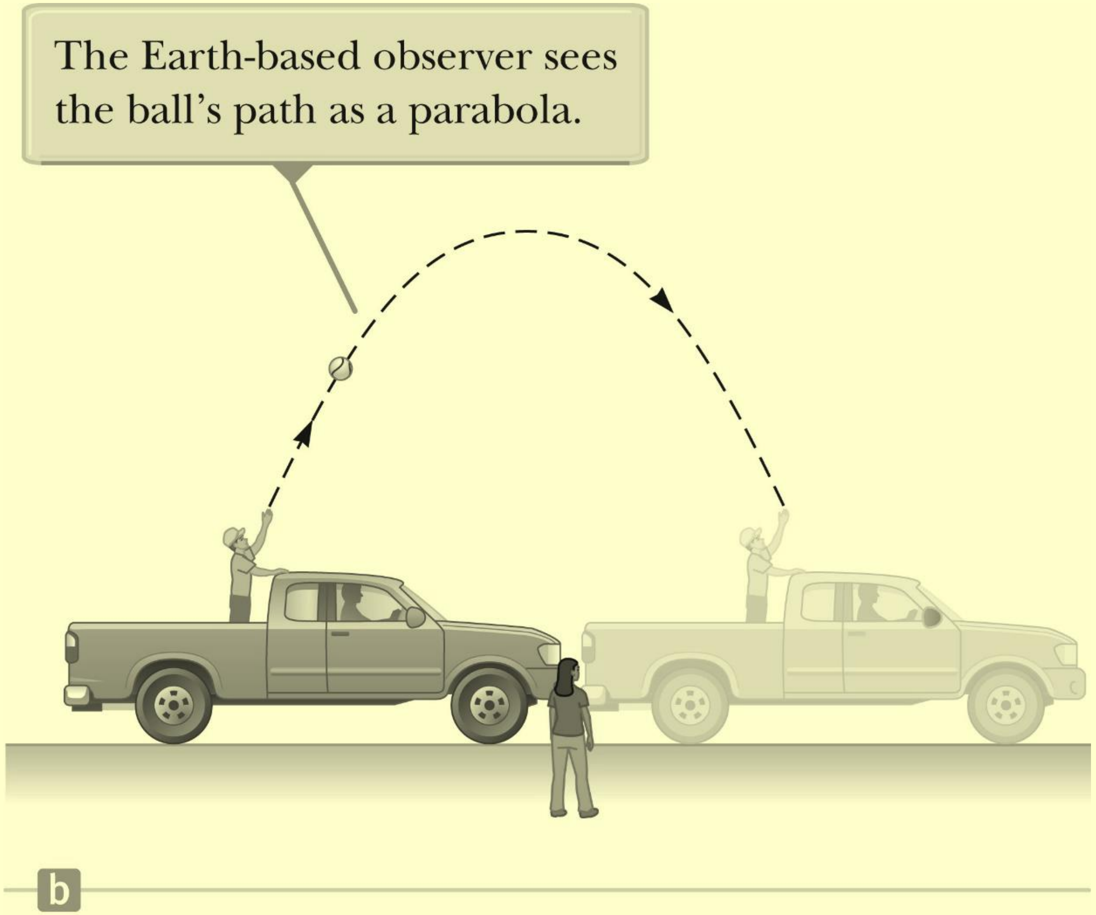
provided

force is same in all frames.

Galilean Relativity – Example

- The truck moves with a constant velocity with respect to the ground.
- The observer in the truck throws a ball straight up.
    - It appears to move in a vertical path.
    - The law of gravity and equations of motion under uniform acceleration are obeyed.





- There is a stationary observer on the ground.
  - Views the path of the ball thrown to be a parabola
  - The ball has a velocity to the right equal to the velocity of the truck.

COMPARISON

- The two observers disagree on the shape of the ball's path.
- **Both agree that the motion obeys the law of gravity and Newton's laws of motion.**
- **Both agree on how long the ball was in the air**.
- Conclusion: ***There is no preferred frame of reference for describing the laws of mechanics***.

Equivalence of Frames

Galilean Transformation

Galilean Transformation

Event: is an occurrence at a point in space and an instant of ***time***

Described by four measurements; namely three space and one time in a particular reference frame

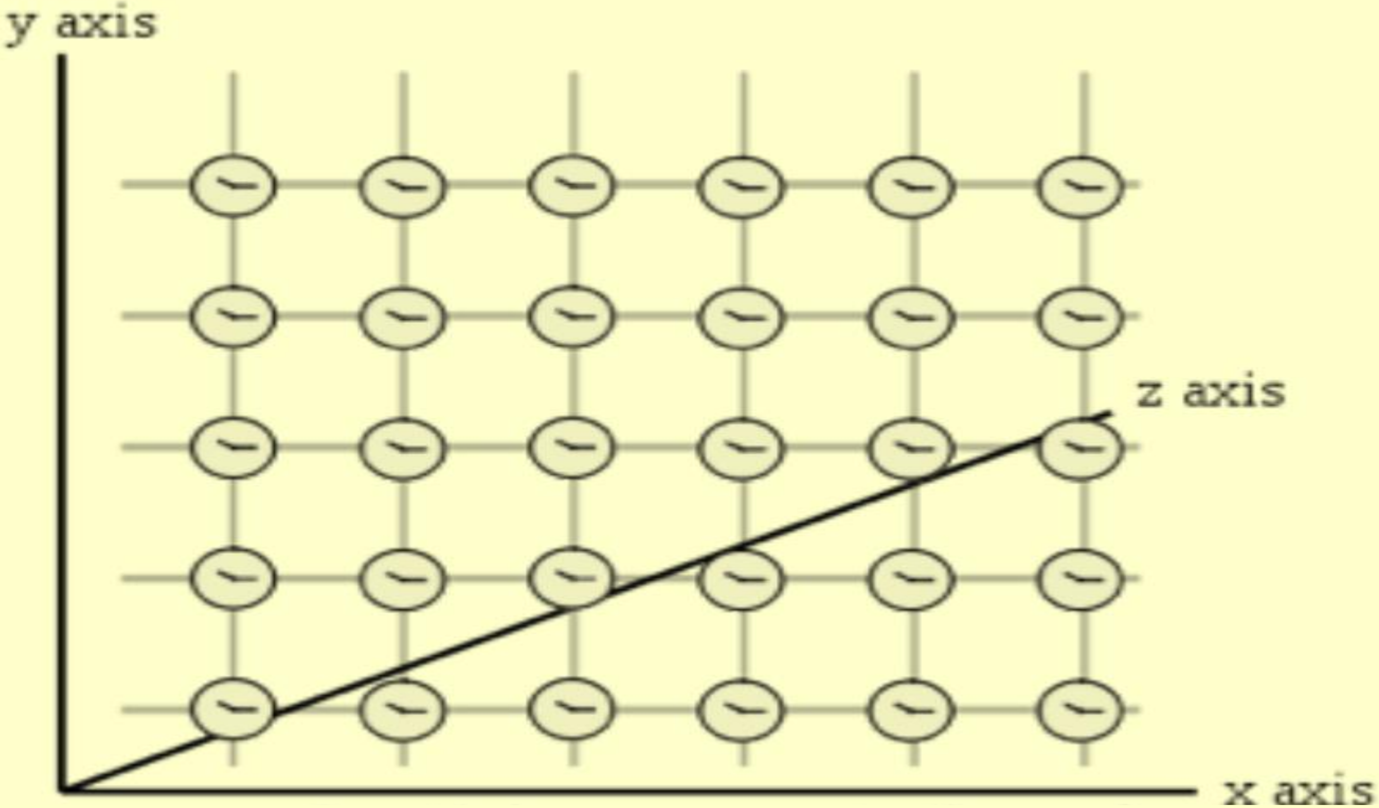
Ex: Lightning at x=1000m, y=2000m, z=3000m and at time t = 4 sec from your balcony

Same event (lightning), from a train, could have different set of numbers

To describe events one needs to establish a reference frame

Event; E(x,y,z,t)

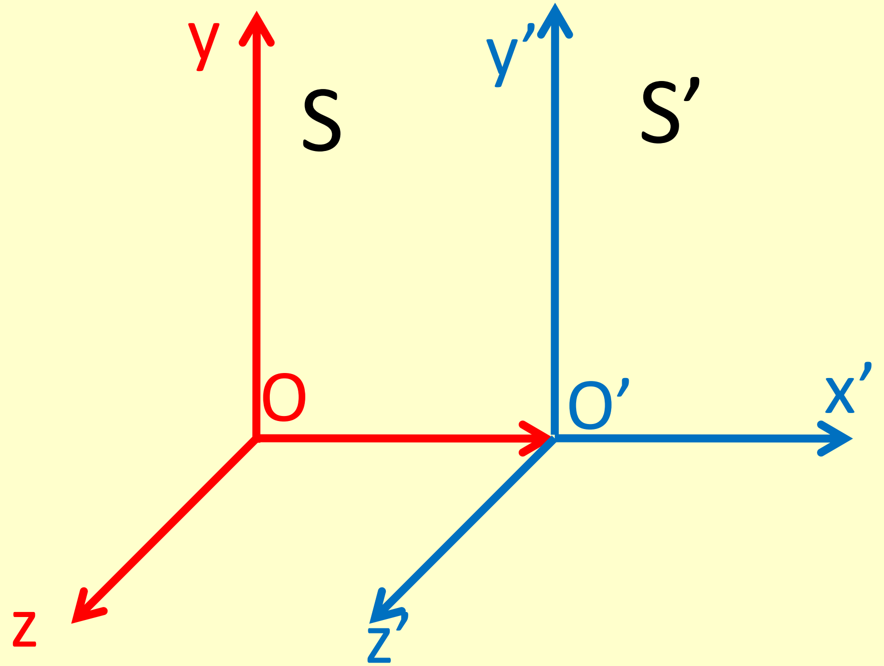
Pick spatial coordinate frame (origin, coordinate axes, unit length).



Introduce clocks to measure time of an event. Imagine a clock at each position in space, all clocks synchronized,

Scenario

- Take two inertial frames S and S'. Choose x and x' axes along the direction of relative velocity of the frames.
- Assume y and z axes to be parallel to y' and z' axes respectively.



- Frame S’ moves with respect to S along x-axis with a velocity $\vec{v}$
- Frame S moves with respect to S’ along x-axis with a velocity $-\vec{v}$
- Clock of each observer is set to zero when their origins coincide.

Galilean Transformations

Direct Transformation

$$x' = x - vt, \quad y' = y, \quad z' = z$$

$$\boxed{t' = t}$$

Inverse Transformation

$$x = x' + vt', \quad y = y', \quad z = z'$$

$$\boxed{t = t'}$$

Classical Velocity Transformation

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v,$$

$$\frac{dy'}{dt'} = \frac{dy}{dt}$$

$$\frac{dz'}{dt'} = \frac{dz}{dt}$$

$$u'_x = u_x - v, \quad u'_y = u_y \quad u'_z = u_z$$

The inverse velocity transformation can be similarly derived

$$u_x = u'_x + v, \quad u_y = u'_y \quad u_z = u'_z$$

$$\boxed{t' = t}$$

Galilean transformations formally expresses the ideas that time is absolute

Furthermore, two events that are ***simultaneous*** in a given frame will be seen to be ***simultaneous*** in another frame. That means, ***time is a frame independent quantity hence absolute!***

Galilean transformations imply

length, mass, and time—the three basic quantities in mechanics are all independent of the relative motion of the measurer (or observer).

$\boxed{t' = t}$ **(Final Statement and Prelude)**

That is, *in classical mechanics, all clocks run at the same rate regardless of their velocity*, so that the time at which an event occurs for an observer in S is the same as the time for the same event in S′. Consequently, the time interval between two successive events should be the same for both observers. Although this assumption may seem obvious, it turns out to be *incorrect* when treating situations in which v is comparable to the speed of light. In fact, this point represents one of the most profound differences between Newtonian concepts and the ideas contained in Einstein's theory of relativity.

In Newton's defense

In the macroscopic world of our ordinary experiences, the speed u of moving objects or mechanical waves with respect to any observer is always less than c. For example, an artificial satellite circling the earth may move at 18,000 mph with respect to the earth; here $u/c = 0.000027$. Sound waves in air at room temperature move at 332 m/sec through the air so that $u/c = 0.0000010$. It is in this ever-present, but limited, macroscopic environment that our ideas about space and time are first formulated and in which Newton developed his system of mechanics.

Galilean Transformation (Summary)

Position

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t$$

Velocity

$$u'_x = \frac{dx'}{dt} = \frac{dx}{dt} - v \qquad u'_y = \frac{dy'}{dt} = \frac{dy}{dt} = u_y \qquad u'_z = \frac{dz'}{dt} = \frac{dz}{dt} = u_z \qquad dt' = dt$$

$$u'_x = u_x - v \qquad u'_y = u_y \qquad u'_z = u_z \qquad dt' = dt$$

Acceleration

$$a'_x = \frac{du'_x}{dt} = \frac{du_x}{dt} = a_x \qquad a'_y = \frac{du'_y}{dt} = \frac{du_y}{dt} = a_y \qquad a'_z = \frac{du'_z}{dt} = \frac{du_z}{dt} = a_z$$

$$a'_x = a_x \qquad a'_y = a_y \qquad a'_z = a_z$$

Absolute space and absolute time:

Suppose that you are on a plane. At 12:00, you leave your seat to talk to a friend seated a few rows in front of you. At 12:15, you return to your seat. You might say that: at 12:15, you were at the same point in space where you were at 12:00.

However, what would a ground-based person claim? If the plane were going 600 mi/hr, that person might say: "at 12:15, you were at a point in space 150 miles away from where you were at 12:00."

Galilean/Newtonian Relativity holds ?

It may be noted that, though earth undergoes a rotational and orbital motion, for all practical purposes, any set of axes fixed on the earth is treated as inertial frames of reference.

Example 3. A particle of mass $m_1 = 3$ kg, moving at a velocity of $u_1 = +4$ m/sec along the x-axis of frame S, approaches a second particle of mass $m_2 = 1$ kg, moving at a velocity $u_2 = -3$ m/sec along this axis. After a head-on collision, it is found that m_2 has a velocity $U_2 = +3$ m/sec along the x-axis.

(*a*) Calculate the expected velocity U_1 of m_1, after the collision.

(*b*) Discuss the collision as seen by observer S' who has a velocity v of $+2$ m/sec relative to S along the x-axis. **Is the momentum still conserved ?**

We use the law of conservation of momentum.

Before the collision the momentum of the system of two particles is

$$P = m_1u_1 + m_2u_2 = (3\text{ kg})(+4\text{ m/sec}) + 1\text{ kg }(-3\text{ m/sec})$$
$$= +9\text{ kg-m/sec.}$$

After the collision the momentum of the system,

$$P = m_1U_1 + m_2U_2,$$

is also $+9$ kg-m/sec, so that

$$+9\text{ kg-m/sec} = (3\text{ kg})(U_1) + 1\text{ kg }(+3\text{ m/sec})$$

or

$$U_1 = +2\text{ m/sec along the } x\text{-axis.}$$

(*b*) Discuss the collision as seen by observer S' who has a velocity **v** of $+2$ m/sec relative to S along the x-axis.

$$u_1' = u_1 - v = +4 \text{ m/sec} - 2 \text{ m/sec} = 2 \text{ m/sec},$$
$$u_2' = u_2 - v = -3 \text{ m/sec} - 2 \text{ m/sec} = -5 \text{ m/sec},$$
$$U_1' = U_1 - v = +2 \text{ m/sec} - 2 \text{ m/sec} = 0,$$
$$U_2' = U_2 - v = +3 \text{ m/sec} - 2 \text{ m/sec} = 1 \text{ m/sec}.$$

The system momentum in S' is

$$\begin{aligned} P' &= m_1 u_1' + m_2 u_2' = (3 \text{ kg})(2 \text{ m/sec}) + (1 \text{ kg})(-5 \text{ m/sec}) \\ &= +1 \text{ kg-m/sec} \end{aligned}$$

before the collision, and

$$\begin{aligned} P' &= m_1 U_1' + m_2 U_2' = (3 \text{ kg})(0) + (1 \text{ kg})(1 \text{ m/sec}) \\ &= +1 \text{ kg-m/sec} \end{aligned}$$

after the collision.

Hence, although the velocities and momenta have different numerical values in the two frames, S and S', when momentum is conserved in S it is also conserved in S'. ◀

No inertial frame is preferred over any other, for the laws of mechanics are the same in all. Hence, there is no physically definable absolute rest frame. We say that all inertial frames are equivalent as far as mechanics is concerned.

A 0.6 m rod flies with constant velocity along its length with a speed of 30 m/s. A particle moving in opposite direction to the rod with a speed of 30 m/s passes the rod. How much time will the particle take to cross the rod?

In particle frame, speed of rod $u_x' = 30 - (-30) = 60 \text{ m/s}$

$\therefore t = \frac{0.6}{60} = 10^{-2} \text{ s}$

According to an observer on the ground, a lightening occurs at x= 1.3 km, and the second lightening occurs at x= 0. 5 km after fifty seconds. According to an observer sitting in a car, the first lightening occurs at x’ =0.5 km and the second one at x’=1.3 km. Find the speed of the car.

E1 (event of first lightening)

E2 (event of second lightening)

$x_1 = 1300\,m$, $t = t_0$; $x_1' = 500$, $t = t_0$

$x_2 = 500\,m$, $t = t_0 + 50$; $x_2' = 1300$, $t = t_0 + 50$

$\therefore\ 500 = 1300 - u\,t_0$, $1300 = 500 - u(t_0 + 50)$

$\Rightarrow\ -800 = 800 + u \times 50 \ \Rightarrow u = -\dfrac{1600}{50} = -32\ m/s$