Lecture 1

Punit Parmananda

PH111

Email : punit@phy.iitb.ac.in

Office: Room 203, 2^{nd} Floor,
Physics Department

Outline

Attendance (80% classes, 100% Tutorials)

Attitude (Doubts)

Grading

Tentatively:

1. Tutorial Quizzes: 5 Marks
2. Regular Quiz (1 hour): 15 marks
3. End-semester Exam (2 hours): 30 marks

Thus, the maximum marks for PH111 is 50.

Content(1st Part)

- A brief review of vectors in plane polar coordinates.
- Velocity, acceleration, and kinematic equations in plane-polar coordinates.
- Newton's law of gravitation and Kepler's laws of motion.

- An Introduction to Mechanics, by Daniel Kleppner and Robert Kolenkow, Second Edition, Cambridge University Press.
- If the second edition is not available, one can buy the first edition which is published in India by McGraw Hill Education, and is not very expensive.
- For revision of concepts:
  - Fundamentals of Physics, by David Halliday, Robert Resnick, and Jearl Walker, John Wiley & Sons, 10th Edition.
  - University Physics, by Hugh D. Young and Roger A. Freedman, Pearson, 13th Edition.
  - Physics for Scientists and Engineers with Modern Physics by John W. Jewett and Raymond A. Serway, 7th Edition, Cengage India.

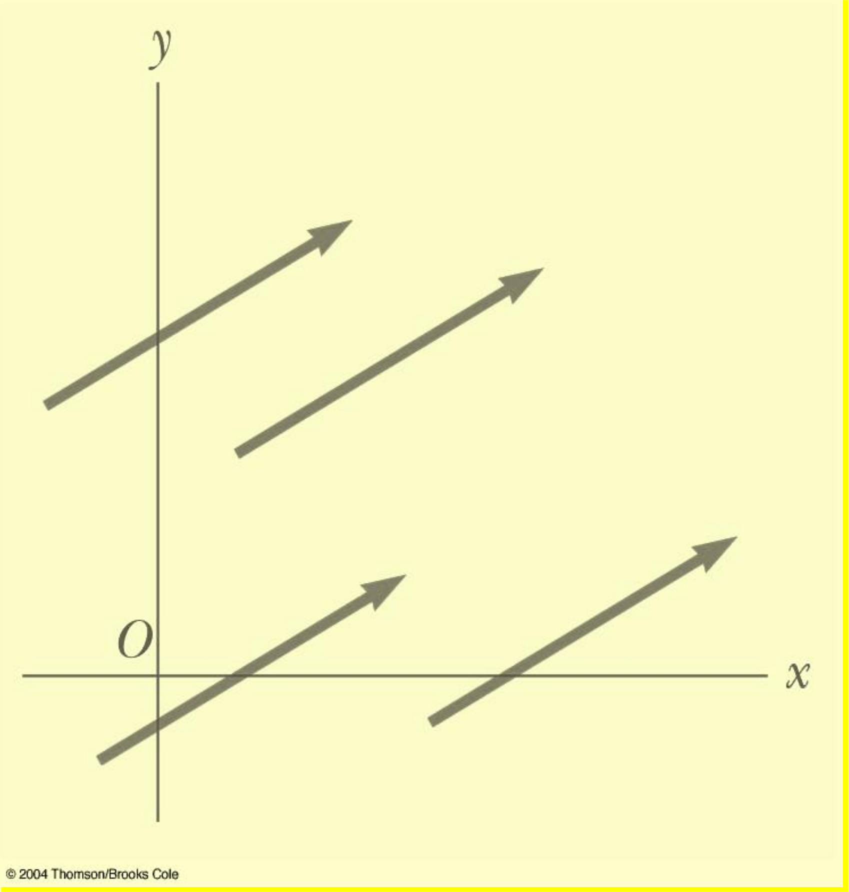
Definitions

- A ***scalar quantity*** is completely specified by a single value with an appropriate unit and has no direction.

- A ***vector quantity*** is completely described by a number and appropriate units plus a direction.

Vectors

- Two vectors are ***equal*** if they have the same magnitude and the same direction.
- **A** = **B** if *A* = *B* and they point along parallel lines.
- All the vectors shown are ***equal.***



If length of a vector is one unit, it is called a unit vector.
The unit vector associated with a vector **A** is defined as

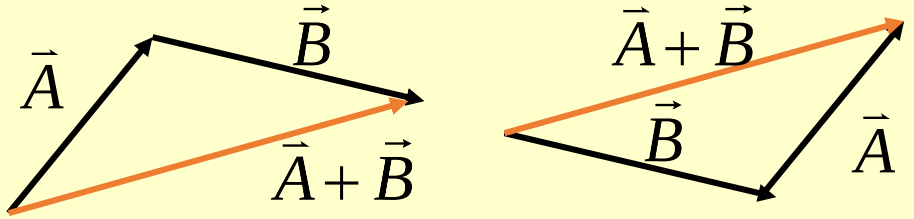
$$\widehat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|} \; ; \; \mathbf{A} = |\mathbf{A}|\widehat{\mathbf{A}}$$

where $|\boldsymbol{A}|$ is the length (magnitude) of the vector.

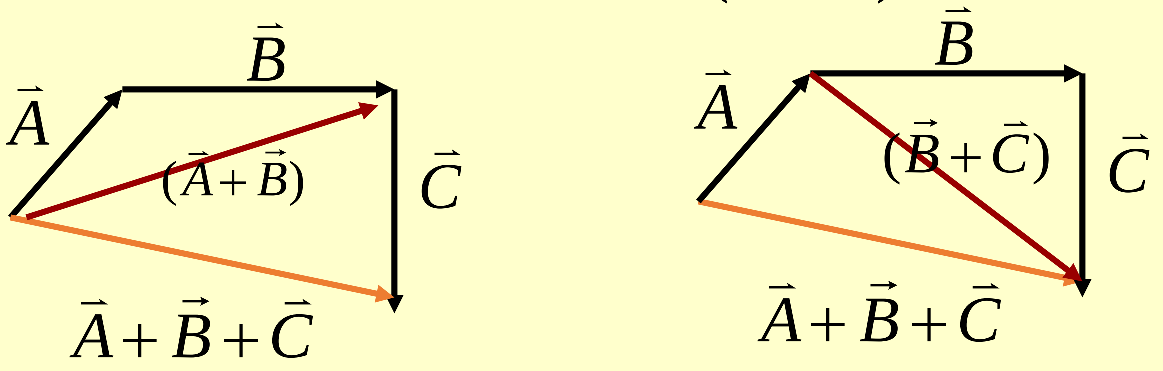
Vector Operations

- Properties:

Vector addition is commutative: $\vec{A}+\vec{B}=\vec{B}+\vec{A}$



Vector addition is associative: $(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$



Vector Operations

- Distributive law also holds

$$c(\mathbf{A}+\mathbf{B}) = c\mathbf{A}+ c\mathbf{B}$$

$$(c + d)\mathbf{A} = c\mathbf{A}+ d\mathbf{A}$$

above *c* and *d* are scalars.

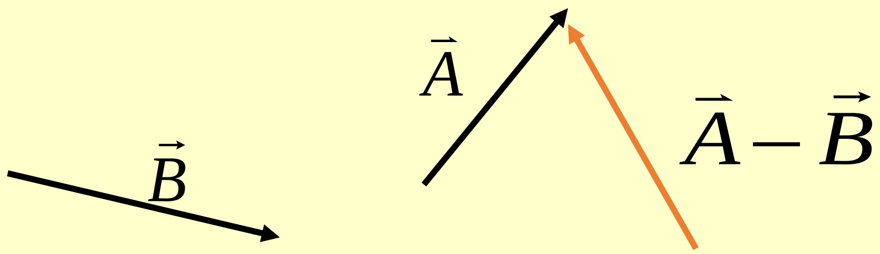
Vector Operations

- Properties:

  Order of addition and multiplication: $m(\vec{A}+\vec{B}) = m\vec{B} + m\vec{A}$

- Definitions:

  Difference between Vectors: (Graphical representation of subtraction) $\vec{A}-\vec{B} = ?$

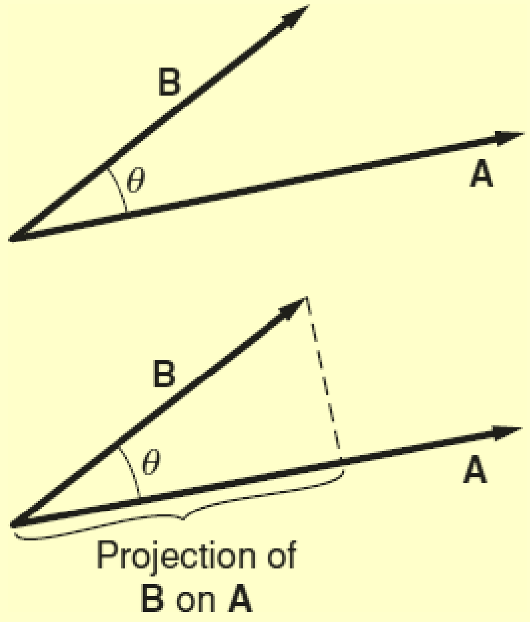


1. **Redraw arrows "tail to tail" (keep same direction and length)**
2. **Draw new arrow from tail of second arrow to tip of first arrow.**
3. **This arrow represents the vector difference.**

Multiplication of vectors

- Can one also multiply two vectors? Yes, and in two possible ways!
- In one case, the end result is a scalar, so the product is called "scalar product" or "dot product".
- In the other case, the end result is a vector, and the product is called "cross product".

Pictorially, the dot product can be shown as



Multiplication of vectors

- Mathematically it is defined as

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta,$$

where θ is the angle between two vectors. Which can also be stated as

$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &= \text{projection of } \mathbf{A} \text{ on } \mathbf{B} \\ &= \text{projection of } \mathbf{B} \text{ on } \mathbf{A} \end{aligned}$$

- Naturally

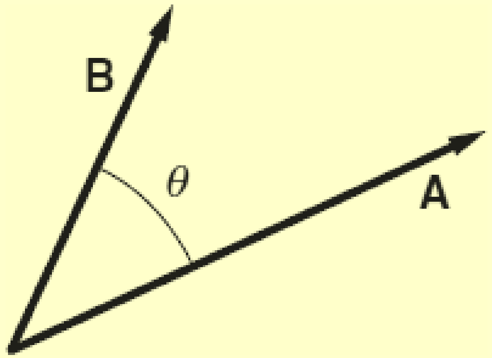
$$\mathbf{A} \cdot \mathbf{A} = AA\cos\theta \ = A^2 = |\mathbf{A}|^2$$

- This helps as define $|\mathbf{A}|$ as

$$A = |\mathbf{A}| = \mathbf{A} \cdot \mathbf{A}$$

Vector Cross Product

- Consider two vectors **A** and **B**, with an angle θ between them, as shown below



- The cross product of the two vectors yields a third vector **C** (say), and the operation is mathematically denoted as

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}.$$

- The magnitude of **C** is given by

$$C = AB\sin\theta.$$

And the direction of **C** is perpendicular to both **A** and **B**, given by the right-hand rule

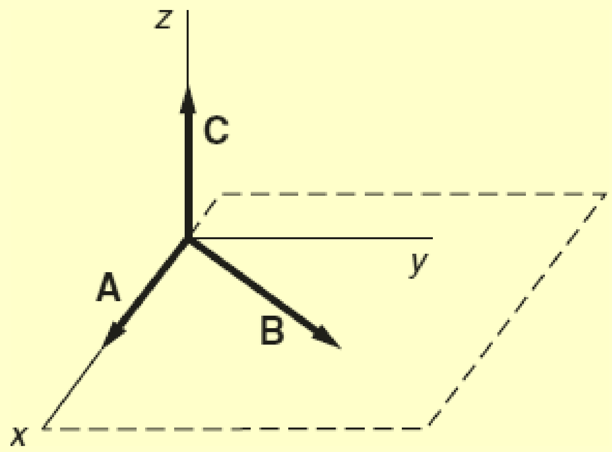
- This θ is taken to be the angle which is less than π

Vector Cross Product

- Easy to verify, that the cross product of a vector with itself is null vector

$$\mathbf{A} \times \mathbf{A} = \mathbf{0}$$

- As a matter of fact, cross product between any two parallel ($\theta = 0$) and anti-parallel ($\theta = \pi$), will always be zero.
- The direction of the cross product can be understood from the following figure



- A consequence of right-hand rule is

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

Dot and Cross products in Physics

- Work done W, due to a force **F**, causing displacement **d**, is given by

$$W = \mathbf{F} \cdot \mathbf{d}$$

- Torque $\boldsymbol{\tau}$, due to a force **F**, applied at a point whose position vector with respect to the reference point is **r**, is given by

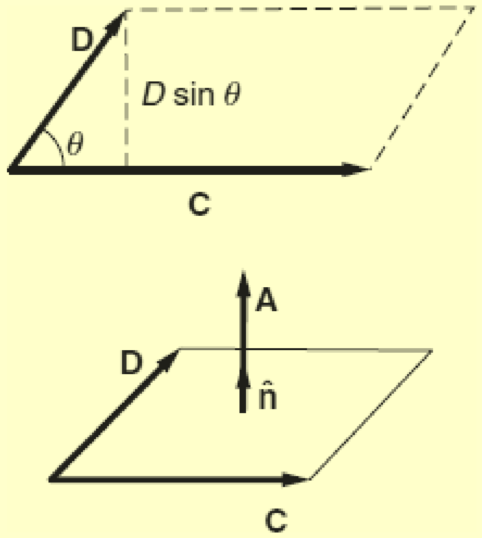
$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

- Force **F** acting on a charged particle with charge q, moving with velocity **v**, exposed to a magnetic field **B**, is given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}.$$

Area as a cross product

- Even the surface area can be defined as a vector, in terms of a cross product
- Consider the parallelogram shown below



- Its area can be written as

$$\begin{aligned} A &= \text{base} \times \text{height} \\ &= CD \sin\theta \\ &= |\mathbf{C} \times \mathbf{D}| \end{aligned}$$

Area as Cross product

- The direction is chosen to be one of the outward drawn normals $\hat{\mathbf{n}}$, so that

$$\mathbf{A} = |\mathbf{C} \times \mathbf{D}|\ \hat{\mathbf{n}}$$

- There is an ambiguity in the choice of $\hat{\mathbf{n}}$, because there are two possibilities
- Choice doesn't matter as long as we are consistent with it

Coordinate Systems

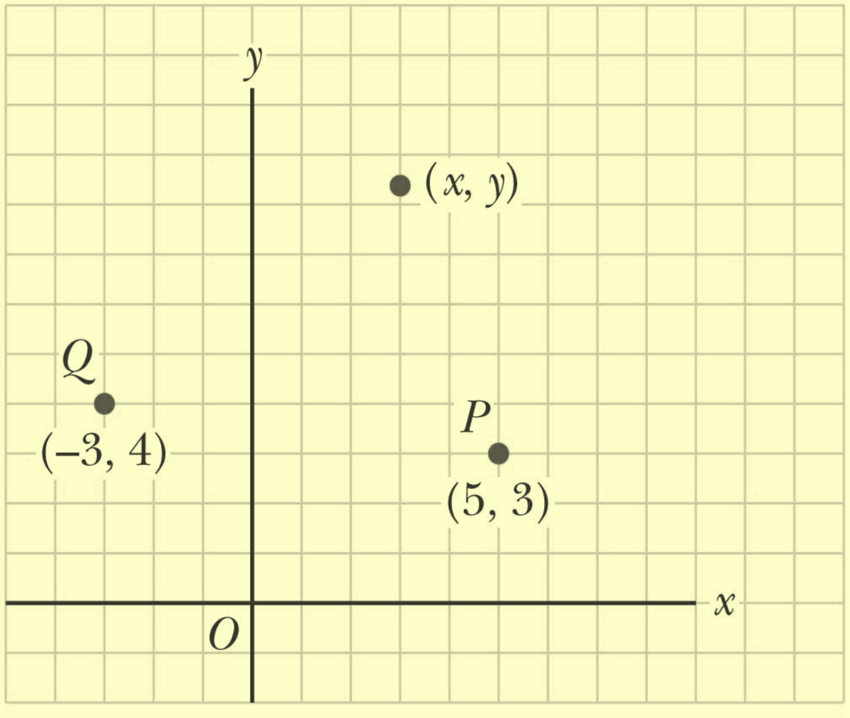
- Used to describe the position of a point in space

- Coordinate system consists of

1. A fixed reference point called the origin
2. specific axes with scales and labels
3. Scaling that enables one to label a point relative to the origin and the axes

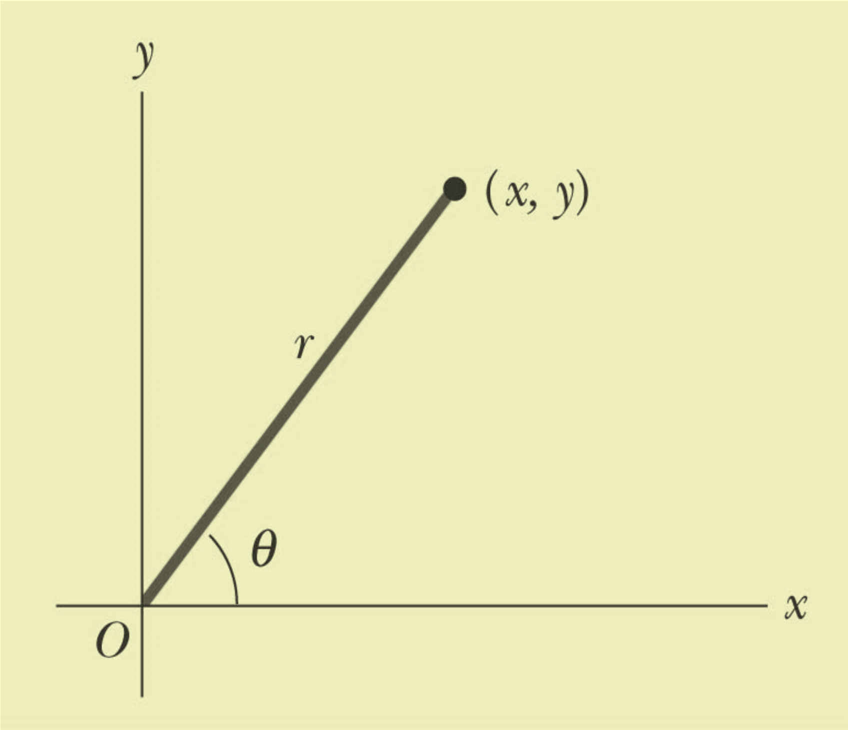
Cartesian Coordinate System

- Also called rectangular coordinate system
- *x*- and *y*- axes intersect at the origin
- Points are labelled (*x*,*y*)



Polar Coordinate System

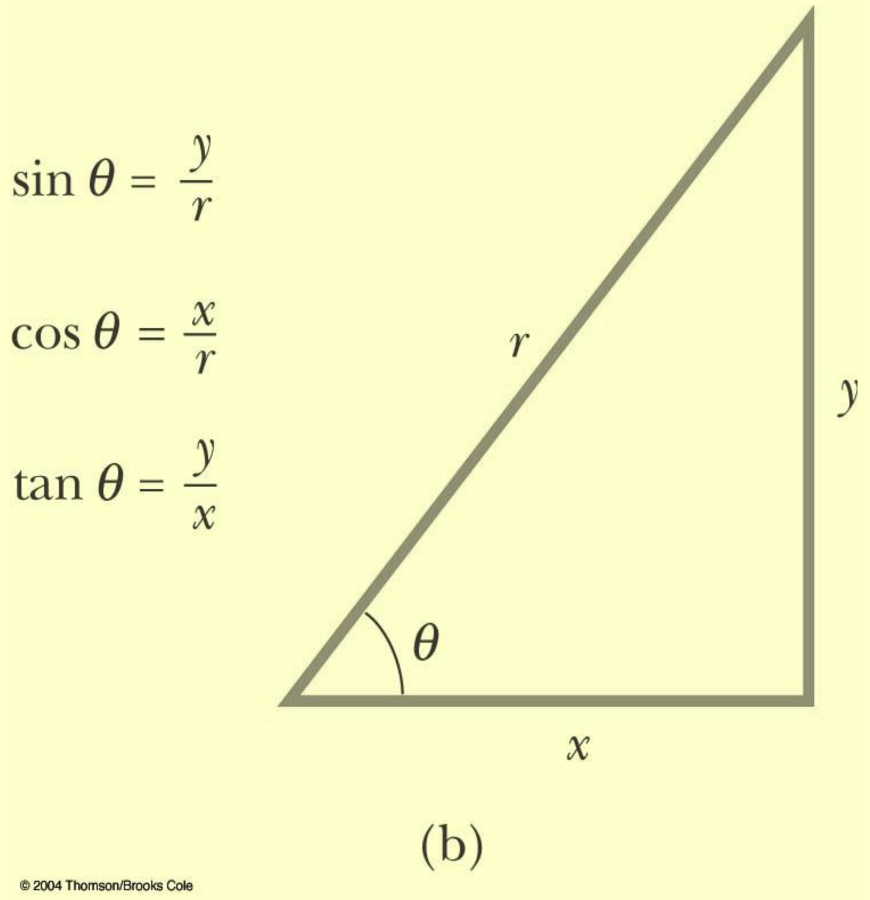
- Origin and reference line are noted
- Point is distance *r* from the origin in the direction of angle θ, ***counter clockwise*** from reference line
- Points are labelled (r,θ)



(a)

Polar to Cartesian Coordinates

- Based on forming a right triangle from *r* and θ
- $x = r\cos\theta$
- $y = r\sin\theta$



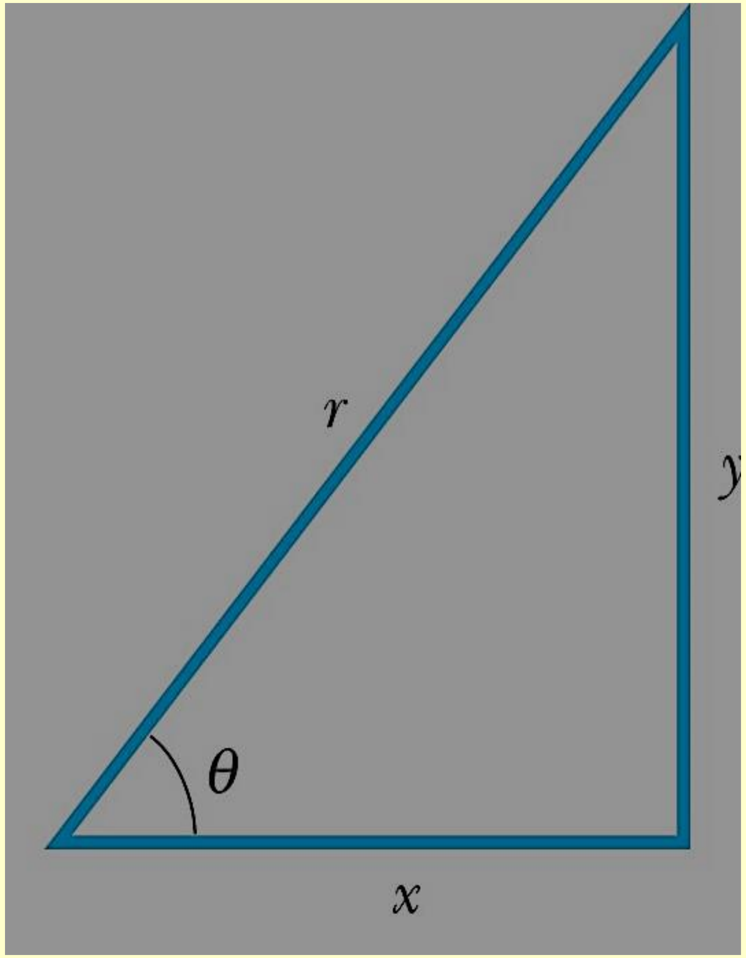
Cartesian to Polar Coordinates

- r is the hypotenuse and θ an angle

$$\tan\theta = \frac{y}{x}\ ; \qquad \theta = \tan^{-1}\frac{y}{x}$$

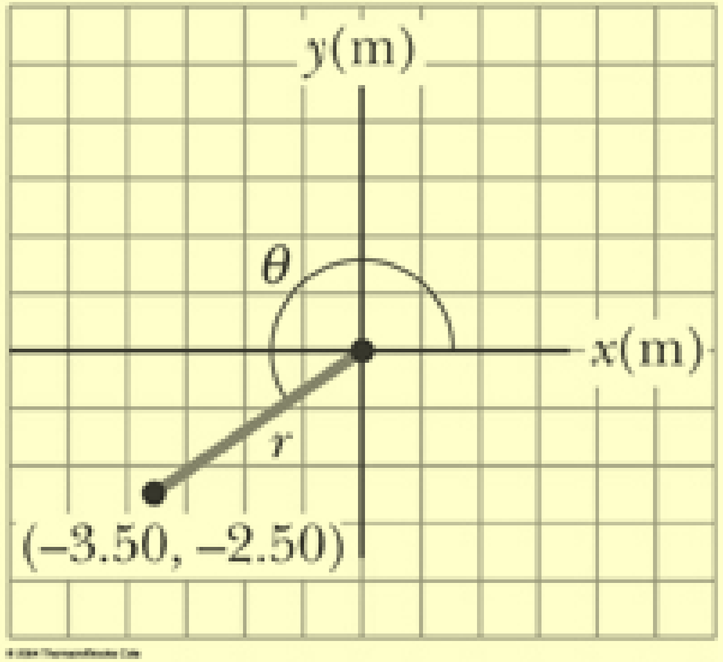
$$r = \sqrt{x^2 + y^2}$$

θ must be ***counter clockwise*** from positive x axis for these equations to be valid



Example

The Cartesian coordinates of a point in the *x*-*y* plane are (*x* , *y*) = (-3.50, -2.50) m, as shown in the figure. Find the polar coordinates of this point.



$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\ \text{m})^2 + (-2.50\ \text{m})^2} = 4.30\ \text{m}$$

$$\tan\theta = \frac{y}{x} = \frac{-2.50\ \text{m}}{-3.50\ \text{m}} = 0.714$$

$$\theta = 216°$$

Unit Vectors

- A ***unit vector*** is a dimensionless vector with a magnitude of exactly 1.

- ***Unit vectors*** are used to specify a direction and have no other physical significance

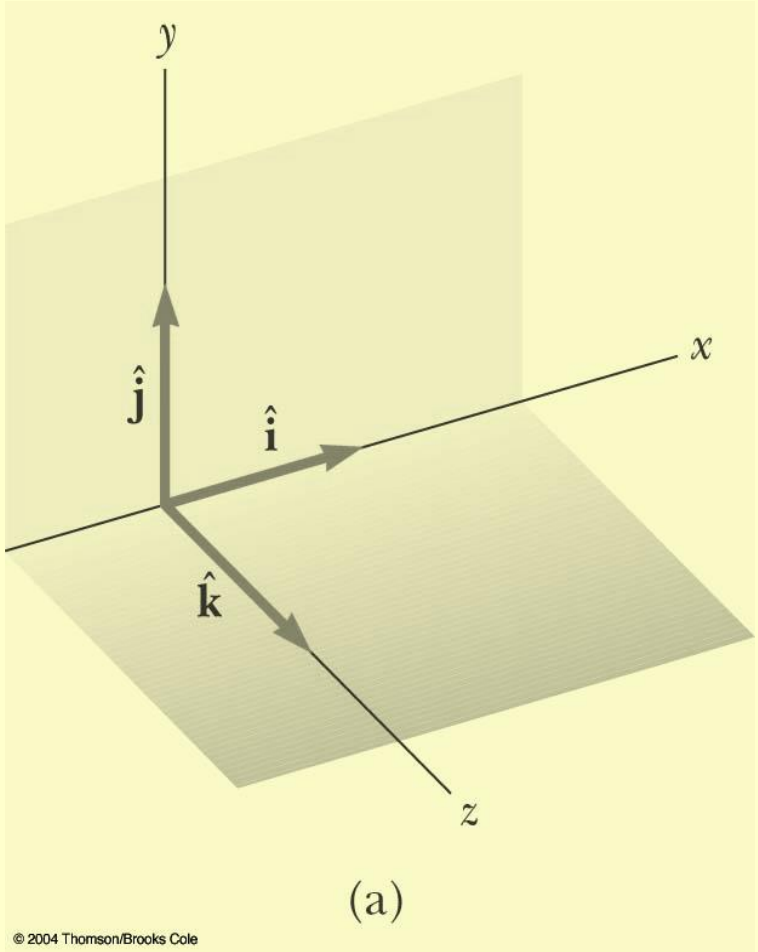
Unit vectors in Cartesian Coordinates

- The symbols

$$\hat{\mathrm{i}}, \hat{\mathrm{j}}, \text{and } \hat{\mathrm{k}}$$

represent unit vectors

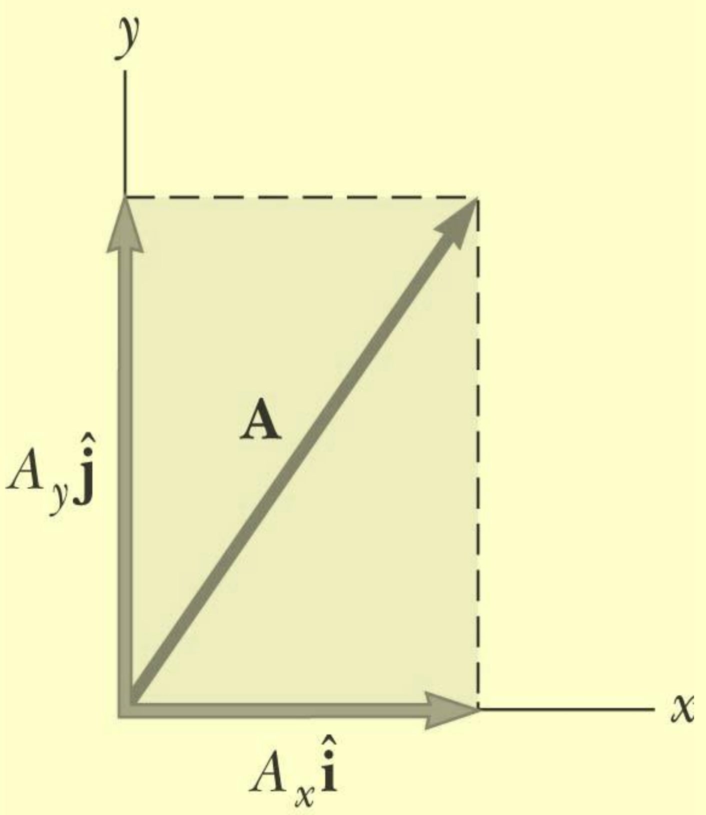
- They form a set of mutually perpendicular vectors



Unit Vectors in Vector Notation (2D)

- $\mathbf{A_x}$ is the same as A_x and $\mathbf{A_y}$ is the same as A_y etc.
- The complete vector can be expressed as

$$\mathbf{A} = \mathbf{A_x}\ \hat{\mathbf{i}} + \mathbf{A_y}\ \hat{\mathbf{j}}$$

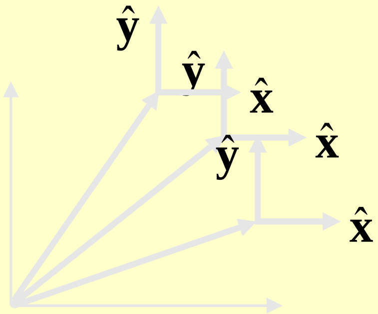


(b)

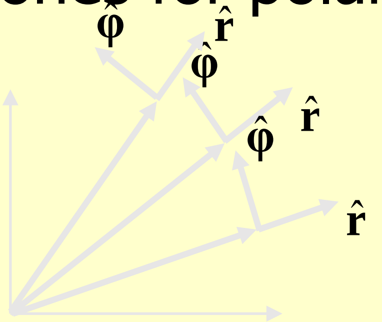
Polar Coordinate Unit Vectors

- One way to construct a unit vector is to take any vector $\mathbf{r}$ and divide by its length $|\mathbf{r}|$. Clearly, such a unit vector is in the direction of $\mathbf{r}$ but has unit length: $\hat{r} = \dfrac{\mathbf{r}}{|r|}$

- There is a major difference between the behavior of the Cartesian unit vectors and the corresponding ones for polar coordinates.

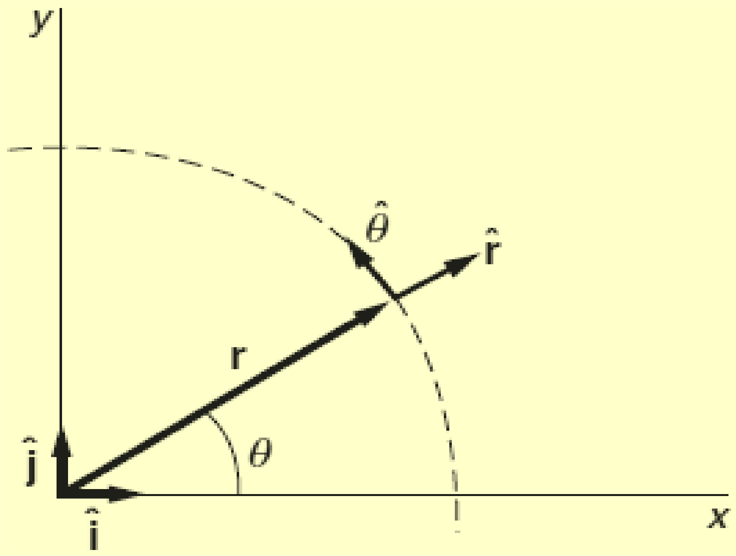


Cartesian unit vectors are constant



*Polar coordinate unit vectors change (direction) with time (**Time dependent)***

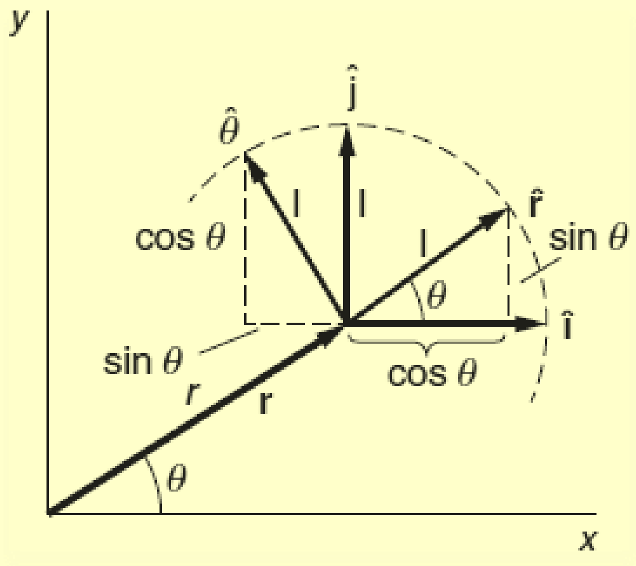
Polar Coordinate Unit Vectors



- Direction of $\hat{r}$ is the one in which r increases, but θ is held fixed.
- Similarly $\hat{\boldsymbol{\theta}}$ is in the direction in which θ increases, but *r* is held fixed.
- Yet $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ are mutually perpendicular ***(radial and tangential),*** just like $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$. .
- Also note that unlike Cartesian coordinates (*r*, θ) have different dimensions.
- *r* has dimensions of length, while θ is dimensionless.

Relation between plane polar and Cartesian unit vectors

- Consider the figure below



- From above, it is easy to derive the relationship between two sets of unit vectors

$$\hat{\mathbf{r}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$$
$$\hat{\boldsymbol{\theta}} = -\sin\theta\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}}$$

Polar-Cartesian Relationship

- And, the inverse relationship

$$\hat{\mathbf{i}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

$$\hat{\mathbf{j}} = \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\boldsymbol{\theta}}$$

Polar-Cartesian Comparison

- Position vector of an arbitrary point P in two coordinate systems is given by

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$\mathbf{r} = r\hat{\mathbf{r}}$$

We require two coordinates to specify a point in two dimension. In the second equation the unit vector itself is dependent on θ, which provides the second coordinate

- Infinitesimal displacement **dr** is given by

$$\mathbf{dr} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}$$

$$\mathbf{dr} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}} \Rightarrow$$ ***(Will Revisit)***