$e^{-ikx} = (e^{-ikx})(-ik)(i)$
$= ke^{-ikx}$
$\frac{e^{-ikx}}{\partial x^2} = (-ik)(ik)e^{-ikx}$
$= k^2 e^{-ikx}$ (k^2+k)

CH 107 Quiz

October 11, 2019 **Time: 80 Minutes** **Full Marks: 18**

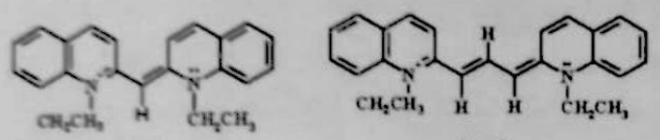
Answer all the parts of the same question **together**. Use **only Pen** to write your answers (**including sketches**) and answers in pencil will not be graded.

Question 1

(a)	What is a node?	[1]
(b)	Find an eigen-function and the corresponding eigen-value of the operator $i\frac{\partial}{\partial x}+\frac{1}{2}\frac{\partial^2}{\partial x^2}$	[2]
(c)	The Schrodinger equation for a particle of mass m is given by $-\frac{\hbar^2}{2I}\frac{d^2}{d\theta^2}\Psi_n(\theta)=E_n\Psi_n(\theta)$, where $I$is a constant and θ (with range 0 – 2π) is the angle that describes the position of the particle. Assuming the general solution is of the form $\Psi_n(\theta)=Ae^{in\theta}$, obtain the expression for the eigenvalues E_n. Using proper boundary conditions, find out the permissible values of n, and assuming $\Psi_n(\theta)$ is well-behaved evaluate the normalization constant A.	[3]

Question 2

(a) Among the two dyes (A and B) given below, identify which of them will absorb at shorter and longer wavelengths. [1]



Dye A Dye B

(b)	If repeated measurements of energy on an atom give the values -2.0, -5.3, and -9.7 eV with probabilities 0.3, 0.5 and 0.2, respectively, find the average and the most probable energy.	[1]
(c)	For a 2D box with lengths $L_x=L$, and $L_y=2L$, find all the doubly degenerate states for $n_x\le 4$ and $n_y\le 4$.	[2]
(d)	A normalized wavefunction is given by $\Psi=\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}e^{-(Zr/a_0)}$, where Z and a_0 correspond to nuclear charge and Bohr radius, respectively. Consider a hypothetical hydrogen like atom with nuclear charge of 2. In such a scenario plot Ψas a function of r on the same graph for the hydrogen atom and the hydrogen like atom with enhanced nuclear charge. Bring out the differences between the two functions clearly.	[2]

Question 3

(a)	The wavefunctions for a particle in a 1D box of length 'L'are orthonormal to each other, i.e. $\int_{-\infty}^{\infty}dx\psi_i^*(x)\psi_j(x)=$ 0for i≠j, and $\int_{-\infty}^{\infty}dx\psi_i^*(x)\psi_i(x)=1$ for i = j. Verify this for i = 2, j = 1, 2. Givensin $\theta\cos\phi=0.5(\cos(\theta-\phi)-\cos(\theta+\phi))$.	[2]
(b)	The magnitude of the force acting between two charges is given by $\lvert\vec{F}\rvert=\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$. Evaluate the average (expectation) value of the magnitude of force (in terms of ϵ_0, a_0, Z and e) when the normalized wavefunctionfor this system is given by $\Psi=\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}e^{-(Zr/a_0)}$.	[2]
(C)	In addition to Columbic potential between electron and nucleus, let us consider gravitation potential within the hydrogen atom given by $V=-\frac{Gm_am_b}{r_{ab}}$ and the gravitational potential of the earth is given by $V=mgh$, where h is measured along the z-direction. (i) Write the new complete Hamiltonian in the Cartesian coordinates, (ii) separate the Hamiltonian in center of mass $\vec{R}$and relative frame$\vec{r}$assuming that the kinetic energy separates out (iii) comment on how electronic energy changes.	[2]

Turn over for equation sheet