Tut-1 Part-1

Monday, 8 May 2023 1:53 AM

2

Compton scattering

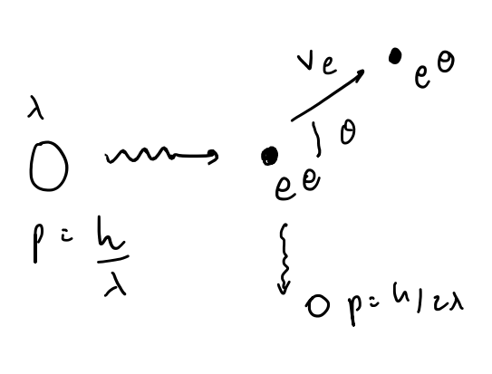
$\theta \Rightarrow$ angle of scattering of photon , $\phi \Rightarrow$ recoil angle of $e^{\ominus}$

1. * A photon of energy $h\nu$ is scattered through 90° by an electron initially at rest. The scattered photon has a wavelength twice that of the incident photon. Find the frequency of the incident photon and the recoil angle of the electron.

Q1 photon of energy $h\nu$ ($\to \lambda$), scatter 90°
$e^{\ominus}$ at rest , scattered photon $\Rightarrow 2\lambda$

Find freq and recoil angle

$\Rightarrow$ momentum conservatn



$$m_e v_e \cos\theta = \frac{h}{\lambda} \qquad \Rightarrow \tan\theta = 1/2$$

$$m_e v_e \sin\theta = \frac{h}{2\lambda} \qquad \theta = \tan^{-1}(1/2)$$

Compton formula $\Rightarrow \lambda' = \lambda_0 + \lambda_c(1-\cos\theta)$

$\theta = \pi/2,\ \lambda' = 2\lambda_0$

$\Rightarrow \boxed{\lambda_c = \lambda_0} \qquad \lambda_c = \frac{h}{m_0 c} = \frac{c}{\nu_{in}}$

$$\boxed{\nu_{in} = \frac{m_0 c^2}{h}}$$

Derivatn of compton scattering formula:

2 major eqns $\Rightarrow$ energy and momentum conservatn

Energy conservatn

$$h\nu + E_{e^{\ominus}} = h\nu' + E_e'$$

$$\frac{hc}{\lambda} + E_e = \frac{hc}{\lambda'} + E_e'$$

∂ initially at rest (only relativistic mass energy)

Now : $E = m_0 c^2$, $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$, $m = \gamma m_0$

$$m^2\left[1 - \frac{v^2}{c^2}\right] = m_0^2$$

$$m^2c^2 - m^2v^2 = m_0^2c^2$$

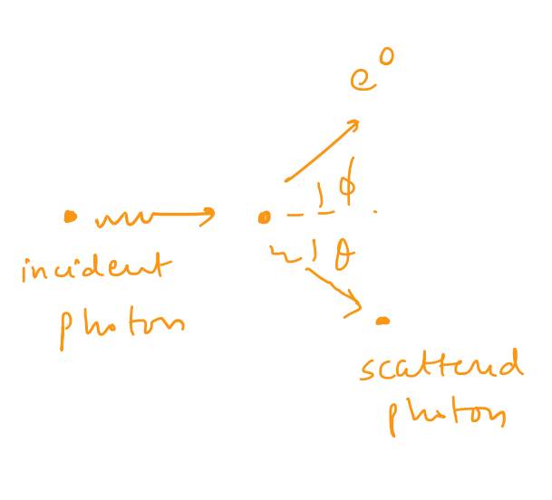
$$E^2 - p^2c^2 = m_0^2c^4$$

$$E = \sqrt{p^2c^2 + m_0^2c^4} = \underline{\gamma m_0 c^2}$$

$p = \frac{h}{\lambda} \Rightarrow \hbar|k|$ $\quad k = 2\pi/\lambda \Rightarrow$ wave vector

$$\Rightarrow \frac{hc}{\lambda} + m_0c^2 = \frac{hc}{\lambda'} + E_e'$$

$$E_e = \gamma m_0 c^2$$



Conservatn of momentum

Recoil angle $= \theta$

$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p_e\cos\phi \Rightarrow$ horizontal

$\frac{h\nu'}{c}\sin\theta = p_e\sin\phi \Rightarrow$ vertical

Square and add (eliminate ϕ)

$$p_e^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - \frac{2h^2\nu\nu'\cos\theta}{c^2}$$

substitute

$$(h\nu - h\nu' + m_0c^2)^2 = E_e^2 = m_0^2c^4 + p_e^2c^2$$

$$\therefore (h\nu)(\cancel{h}\nu')(1-\cos\theta) = \cancel{h}(\nu-\nu')m_0c^2$$

scattered

$$\boxed{\underset{\Delta\lambda}{\lambda' - \lambda} = \frac{h}{m_0 c}(1-\cos\theta)} \qquad \lambda_c = 2.43 \times 10^{-12}\ m$$

$$\cancel{2}h\nu m_0c^2 - \cancel{2}h\nu' m_0c^2 - \cancel{2}h^2\nu\nu' = -\cancel{2}h^2\nu\nu'\cos\theta$$

$$h(1-\cos\theta) = m_0c^2\left(\frac{\nu'-\nu}{\nu\nu'}\right)$$

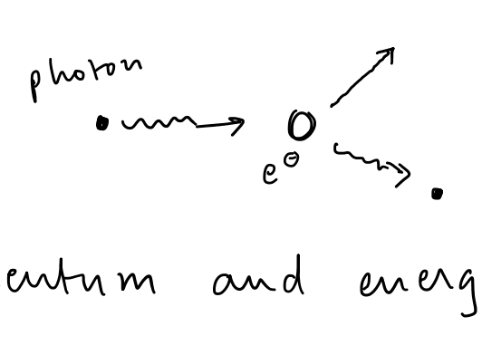
$$\Rightarrow \boxed{\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)}$$

2. Derive the relation for the recoil kinetic energy of the electron and its recoil angle ϕ in Compton scattering. Show that

$$\text{K.E. (electron)} = \frac{\Delta\lambda/\lambda}{1+(\Delta\lambda/\lambda)}hf$$

$$✓\cot\phi = \left(1 + \frac{hf}{mc^2}\right)\tan\frac{\theta}{2}$$

Q2



momentum and energy conservatn

KE of $e^{\ominus} \Rightarrow \frac{m_e v_e^2}{2}$

Second relatn

$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p_e\cos\phi \Rightarrow \frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta = p_e\cos\phi$

$\frac{h\nu'}{c}\sin\theta = p_e\sin\phi$

$$\Rightarrow \cot\phi = \frac{\nu - \nu'\cos\theta}{\nu'\sin\theta} = \left(\frac{\nu}{\nu'} - \cos\theta\right)\frac{1}{\sin\theta}$$

compton : $\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{mc^2}\left[2\cos^2\theta/2\right]$$

$$\frac{\nu - \nu'}{\nu'} = \frac{hf}{mc^2}\left[2\cos^2\theta/2\right]$$

$$\frac{\nu}{\nu'} = 1 + \frac{hf}{mc^2}(1-\cos\theta)$$

$$\cot\phi = 1 + \frac{hf}{mc^2} - \left(1 + \frac{hf}{mc^2}\right)\cos\theta$$

$$= \left(1 + \frac{hf}{mc^2}\right)\left[\frac{1-\cos\theta}{\sin\theta}\right] = \left(1 + \frac{hf}{mc^2}\right)\tan\frac{\theta}{2}$$

KE $e^{\ominus} \Rightarrow p_e^2/2m_e$

$$KE = \frac{\Delta\lambda/\lambda}{1 + \Delta\lambda/\lambda}(hf)$$

$$= \frac{\Delta\lambda}{\lambda'}\frac{hf\lambda}{\lambda} = \frac{\Delta\lambda\, hc}{\lambda\lambda'}$$

$$= \left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)hc$$

$$= (\nu - \nu')h$$

Energy conservatn :

$$h\nu + E_{e^{\ominus}} = h\nu' + E_e'$$

Note in compton scattering $e^{\ominus}$ is stationary initially

$$h(\nu - \nu') = \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2$$

$$= m_0c^2[\gamma - 1]$$

3. Show that a free electron cannot absorb a photon so that a photoelectron requires bound electron. However, the electron can be free in Compton Effect. Why?

Q3 Photo $e^{\ominus}$ requires bound $e^{\ominus}$
photoelectric effect is not possible with free $e^{\ominus}$.

Assume initial momentum of $e^{\ominus} = 0$

$\Rightarrow \frac{h}{\lambda} = p$ $\quad$ [photoelectric effect $\Rightarrow$ light generates $e^{\ominus}$]

Energy conservatn

$$\frac{hc}{\lambda} + m_0c^2 = \sqrt{m_0^2c^4 + p^2c^2}$$

substitute $p \Rightarrow \overset{a}{\frac{hc}{\lambda}} + \overset{b}{m_0c^2} = \left[\overset{a^2}{m_0^2c^4} + \overset{b^2}{\left(\frac{hc}{\lambda}\right)^2}\right]^{1/2}$

$$a + b = \sqrt{(a+b)^2 - 2ab}$$

$\therefore 2ab = 0 \Rightarrow \frac{2m_0c^2hc}{\lambda} = 0$

$\therefore$ either $m_e = 0$ or $\frac{h}{\lambda} = 0$, $\therefore$ photoelectric effect is not possible

Compton effect:

$$\frac{h}{\lambda} = p\cos\theta + \frac{h}{\lambda'}\cos\phi \ , \quad p\sin\theta = \frac{h}{\lambda'}\sin\phi$$

Energy conservatn $\Rightarrow$ no contradictn

compton for free $e^{\ominus}$ is possible $\Rightarrow$ photon absorptn and reemission happens

Q4

4. Two Compton scattering experiments were performed using x-rays (incident energies E_1 and $E_2 = E_1/2$). In the first experiment, the increase in wavelength of the scattered x-ray, when measured at an angle $\theta = 45°$, is $7{\times}10^{-14}$ m. In the second experiment, the wavelength of the scattered x-ray, when measured at an angle $\theta = 60°$, is $9.9{\times}10^{-12}$ m.

(a) Calculate the Compton wavelength and the mass (m) of the scatterer.

(b) Find the wavelengths of the incident x-rays in the two experiments.

a) For the first experiment, $\Delta\lambda = 7 \times 10^{-14}$m and $\theta = 45°$

$$7 \times 10^{-14} = \lambda_c\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\lambda_c = 2.4 \times 10^{-13}m \quad \text{(Compton Wavelength)}$$

$$m_0 = 0.92 \times 10^{-29}Kg \quad \text{(Mass of scatterer)}$$

b) For the second experiment, $\lambda' = 9.9 \times 10^{-12}$m and $\theta = 60°$, and let λ_2 be the incident wavelength

$$9.9 \times 10^{-12} - \lambda_2 = 2.4 \times 10^{-13}\left(1 - \frac{1}{2}\right)$$

$$\lambda_2 = 9.8 \times 10^{-12}m$$

$$\lambda_1 = 4.9 \times 10^{-12}m$$

since $E_2 = E_1/2 \implies \lambda_2 = 2 \times \lambda_1$.

5. Find the smallest energy that a photon can have and still transfer 50% of its energy to an electron initially at rest.

Q5 Transfer 50% energy to $e^{\ominus}$ initially at rest

$\frac{hc}{2\lambda} = E_e' \cdot m_0c^2$ $\quad$ initial energy $\Rightarrow hc/\lambda$

$$\lambda' = \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\lambda_{max} = \frac{2h}{m_0 c}$$

$$E_{min} = \frac{hc}{\lambda_{max}} = \frac{m_0c^2}{2}$$

$$= 0.255\ MeV$$

6. *γ-rays are scattered from electrons initially at rest. Assume the it is back-scattered and its energy is much larger than the electron's rest-mass energy, $E \gg m_ec^2$.
(a) Calculate the wavelength shift
(b) Show that the energy of the scattered beam is half the rest mass energy of the electron, regardless of the energy of the incident beam
(c) Calculate the electron's recoil kinetic energy if the energy of the incident radiation is 150MeV

Q6 γ rays $\Rightarrow e^{\ominus}$ initially at rest
back scattered $\Rightarrow \theta = 180°$, $E \gg m_ec^2$

(a) $\Delta\lambda = \lambda_c(1-\cos\theta) = 2\lambda_c$

(b) $\lambda' = \lambda + 2\lambda_c$
Divide throughout by hc

$$\Rightarrow 1/E' = 1/E + 2/E_c$$

$$= 1/E + \frac{2h}{m_0 c\, hc} = \frac{1}{E} + \frac{2}{m_0c^2}$$

$$\because E \gg m_ec^2 \Rightarrow 1/E \ll 1/m_0c^2$$

$$\therefore 1/E' = 2/m_0c^2 \Rightarrow \boxed{E' = m_ec^2/2}$$

(c) $e^{\ominus}$ recoil KE : , incident energy $= 150$ MeV

$\overset{\to 150\ MeV}{E} + \underset{\hookrightarrow 0.511\ MeV}{m_ec^2} = E' + E_{e^{\ominus}}$

$$E_{e^{\ominus}} = E - \frac{m_ec^2}{2} = 149.7445\ MeV$$

$\to$ Wrong ques

7. In Compton Scattering, show that the maximum energy of the scattered photon will be $2m_0c^2$, irrespective of the energy of the incident photon. Find the value of θ_0, the angle at which the maximum energy occurs.

Q7 Compton scattering :

$$\lambda' = \lambda + \frac{h}{m_0 c}(1-\cos\theta)$$

$$\Rightarrow E = \frac{hc}{\lambda}$$

$\therefore E$ is max when λ is min

$$E_{scattered} = \frac{hc}{\lambda + \frac{h}{m_0 c}(1-\cos\theta)}$$

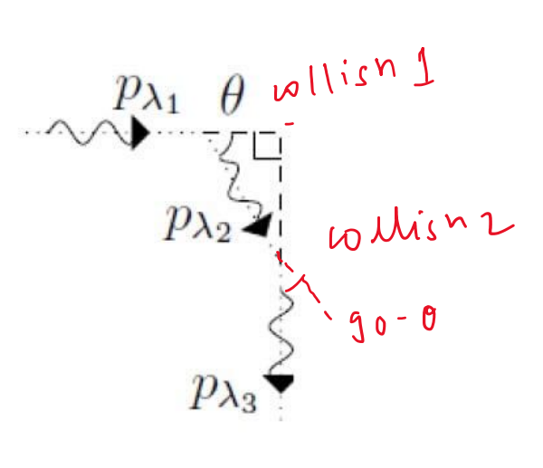
To show : $E_{scattered} \leq 2m_0c^2$

$$\therefore \frac{hc}{\lambda + \frac{h}{m_0 c}(1-\cos\theta)} < \frac{hc}{\frac{h}{m_0 c}(1-\cos\theta)} = \frac{m_0c^2}{(1-\cos\theta)}$$

$$2m_0c^2 = \frac{m_0c^2}{1-\cos\theta} \ , \quad \theta_0 = 60°$$

Just explain what's wrong if anyone asks, can read out soln as well.

8. * In a Compton scattering experiment (see figure), X-rays scattered off a free electron initially at rest at an angle $\theta(> \pi/4)$), gets re-scattered by another free electron, also initially at rest.



(a) If $\lambda_3 - \lambda_1 = 1.538 \times 10^{-12}$ m, find the value of θ.

(b) If $\lambda_2 = 68 \times 10^{-12}$ m , find the angle at which the first electron recoils due to the collision.

Q8 free $e^{\ominus}$ initially at rest

a) $\lambda_2 - \lambda_1 = \lambda_c(1-\cos\theta)$

$$\lambda_3 - \lambda_2 = \lambda_c(1-\sin\theta)$$

$$\Rightarrow \lambda_3 - \lambda_1 = \lambda_c(2 - \sin\theta - \cos\theta)$$

$$\Rightarrow 1.538 \times \cancel{10^{-12}} = 2.43 \times \cancel{10^{-12}}(2 - \cos\theta - \sin\theta)$$

$$1.367 = \cos\theta + \sin\theta$$

Square both sides.

$$1.869 = 1 + \sin 2\theta$$

$$\boxed{\theta = 30.17°}$$

b) $\lambda_2 = 68 \times 10^{-12}$ m , can use above derived formula directly do momentum conservatn.