CH-107-2021 **Quiz paper -12 Marks** **Time – 40 mins**

Q.1 Which of the following wavefunction(s) is/are eigen-function(s) of the operator $i\frac{d}{dx}$

OPTIONS=

A. x^3
B. e^{-9ix}
C. $tan(ax)$
D. $e^{-ix} + e^{-i7x}$
E. e^{-ix^2}
F. This question is incorrect

Q.2 True or false: Operator $\hat{A}$ is a non-linear operator, where $\hat{A}$ is given by

$$\hat{A} = \frac{d^2}{dx^2} + x$$

OPTIONS=

A. True
B. False
C. This question is incorrect

Q.3 Consider the following wavefunction for a particle in 1-D:

$\phi(x) = 0$ for $x \leq 0$

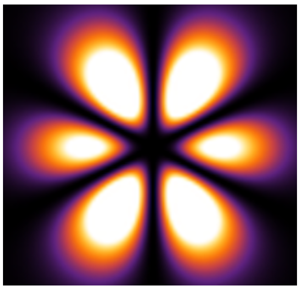
$\phi(x) = Ce^{-4x}(1 - e^{-4x})$ for $x \geq 0$

Where C = normalisation constant. The most probable position (x) for the particle is

OPTIONS=

A. $x = 0$
B. $x = \frac{1}{4}ln(2)$
C. $x = \frac{1}{4}ln(6)$
D. $x = \frac{1}{4}ln(10)$
E. This question is incorrect

Q.4 For the following xz-projection of a hydrogenic orbital, the values of n and l are (no radial nodes beyond the region shown):



OPTIONS=

A. n= 3, l = 2
B. n = 4, l = 3
C. n = 7, l = 6
D. n = 5, l = 3
E. This question is incorrect

Q.5 For a particle in a 1-D box (box length = L), ψ_n denotes the acceptable eigenfunctions in the form of $\psi_n = \sqrt{\frac{2}{L}} sin \frac{n\pi x}{L}$ and $E_n = \frac{n^2h^2}{8mL^2}$, where n can be 1,2,3 ... A new wavefunction ϕ is constructed as $\phi = c_1\psi_3 + c_2\psi_4 + c_3\psi_5$ where c_1, c_2 and c_3 are real, $c_1^2 + c_2^2 + c_3^2 = 1$ and $c_3 > c_2 > c_1$.

Identify ***all*** the correct ***statement(s)***:

OPTIONS=

A. ψ_2 is orthogonal to ϕ.
B. ϕ is an eigenfunction of the particle in a 1-D box Hamiltonian operator.
C. ϕ is normalised.
D. E_4 will be the most probable value of energy that will be obtained for a large number of measurements with the state ϕ.
E. This question is incorrect

Q.6 A 1.0 gm particle is constrained inside an one dimensional box of length L = 100.0 cm. What is the closest quantum number (*n*) if the energy of the particle is 10^{-3}Joules.

Given: h = $6.6 \times 10^{-34} kgm^2s^{-1}$

OPTIONS=

A. 4×10^{33}

B. 4×10^{30}

C. 4×10^{16}

D. 4×10^{0}

E. This question is incorrect

Q.7 Which of the following ***is/are*** acceptable ***wavefunction(s)*** for a particle confined on a ring:

OPTIONS=

A. $\frac{1}{\sqrt{2\pi}} e^{-5i\phi}$

B. $\frac{1}{\sqrt{2\pi}} e^{-5\phi}$

C. $\frac{1}{\sqrt{2\pi}} e^{-i\phi/2}$

D. $\frac{1}{\sqrt{2\pi}}$

E. This question is incorrect.

Q.8 Consider the spherical harmonics as given below; $Y(\theta, \phi) = \frac{1}{8}\sqrt{\frac{21}{\pi}} sin\theta(1 - 5cos^2\theta)e^{i\phi}$

Determine the ***angle*** (***in degrees***) that the angular momentum vector makes with z-axis.

Ans:

Q.9 For Li^{+2}, the energy difference between the ground state

($n_1 = 1, l_1 = 0, m_1 = 0$) and an excited state $(n_2 = 7, l_2 = 4, m_2 = 2)$ depends on

OPTIONS=

A. only n_1 and n_2

B. only l_1 and l_2

C. n_1, l_1 and n_2, l_2

D. n_1, l_1, m_1 and n_2, l_2, m_2

E. This question is incorrect.

Q.10 What is the integral $\int \psi^*_{(2,1,1)} \psi_{(3,0,0)} d\tau$?

Here $d\tau$ is the volume element. $\psi_{(n,l,m)}$ is the hydrogenic eigenfunction with quantum numbers n, l, m.

Given $\psi_{(3,0,0)} = \frac{1}{243}\sqrt{\frac{3}{\pi a_0^3}}(27 - 18\frac{r}{a_0} + 2(\frac{r}{a_0})^2)e^{-(r/3a_0)}$ and $\psi_{(2,1,1)} = \frac{1}{8}\sqrt{\frac{1}{\pi a_0^3}}(\frac{r}{a_0})e^{-(r/2a_0)}sin\theta e^{i\phi}$

OPTIONS=

A. $\frac{1}{\pi}$
B. $\frac{1}{2\pi}$
C. π
D. 0
E. This question is incorrect.

Q.11 For a quantum particle on a ring, how many distinct transitions are possible if you consider m = 0,± 1, ± 2 and if all transitions are allowed?

OPTIONS=

A. 8
B. 2
C. 3
D. 5
E. This question is incorrect.

Q.12 Which of the following pairs of hydrogenic atomic orbitals have the ***same radial distribution functions***?

OPTIONS=

A. $2p_z$ and $2p_x$
B. $2p_z$ and $3d_{z^2}$
C. $2s$ and $3s$
D. $3d_{z^2}$ and $3d_{x^2-y^2}$
E. The question is incorrect

Q1. operator $i\frac{d}{dx}$

A. x^3 $\quad i\frac{d}{dx}(x^3) = i(3x^2)$.

$\boxed{\therefore \text{NO}}$

B. e^{-9ix} $\quad i\frac{d}{dx}\left(e^{-9ix}\right) = -9i(i)\,e^{-9ix}$

$+9e^{-9ix}$

$\boxed{\therefore \text{YES}}$

eigen value = 9.

C. $\tan(\alpha x)$ $\quad i\frac{d}{dx}(\tan\alpha x) = i\sec^2(\alpha x)$.

$\boxed{\therefore \text{No.}}$

D. $e^{-ix} + e^{-i7x}$

$$i\frac{d}{dx}\left[e^{-ix} + e^{-i7x}\right] = e^{-ix} + 7e^{-i7x}$$

$\boxed{\therefore \text{No}}$

E. e^{-ix^2} $\quad i\frac{d}{dx}\left[e^{-ix^2}\right] = 2x \cdot e^{-ix^2}$

Eigen value not Constant

$\boxed{\therefore \text{NO.}}$

Q2 For a operator $\hat{A}$ to be linear it should satisfy

$$\hat{A}(u+v) = \hat{A}(u) + \hat{A}(v) \quad -(1)$$

&

$$\hat{A}(cu) = c\,\hat{A}(u) \quad -(2)$$

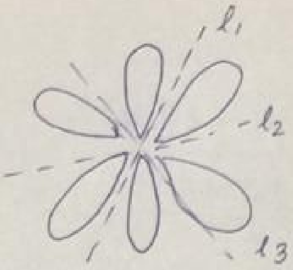
where u & v are eigen functions.

& the operator $\left[\frac{d^2}{dx^2} + x\right]$ satisfies (1) $\therefore$ it is linear

Q.4

$n-l-1=0$ No radial nodes

$l=3$



$n=4$

$\boxed{B}$, $n=4, l=3$

Q3 $\psi = Ce^{-4x}(1-e^{-4x})$

Most probable x $\frac{dP}{dx}=0$ $\frac{d(\psi^2 dx)}{dx}=0$

Here maximing ψ to get most probable x

$$\frac{d(e^{-4x}(1-e^{-4x}))}{dx}=0$$

$$\frac{d(e^{-4x}-e^{-8x})}{dx}=0$$

$$-4e^{-4x}+8e^{-8x}=0$$

$$-4e^{-4x}[1-2e^{-4x}]=0$$

$$e^{-4x}=\frac{1}{2} \quad \text{or} \quad e^{4x}=2$$

Taking ln both sides

$$4x=\ln(2)$$

$$\boxed{x=\frac{1}{4}\ln(2)}$$

5) $\phi = c_1\psi_3 + c_2\psi_4 + c_3\psi_5$

A. $\langle\psi_2|\phi\rangle = c_1\cancelto{0}{\langle\psi_2|\psi_3\rangle} + c_2\cancelto{0}{\langle\psi_2|\psi_4\rangle} + c_3\cancelto{0}{\langle\psi_2|\psi_5\rangle}$

$= 0$

Correct ✓

A, C

B. $\hat{H}\phi = c_1E_3\psi_3 + c_2E_4\psi_4 + c_3E_5\psi_5$

$\neq E[c_1\psi_3 + c_2\psi_4 + c_3\psi_5]$

Incorrect

C. $\langle\phi|\phi\rangle = c_1^2 + c_2^2 + c_3^2$

$= 1$

Correct

D. Most probable energy $\equiv \phi_i$ with maximum $|c_i|^2$

Since $c_3 > c_2 > c_1$, $\therefore$ $\underline{E_5}$ is most probable energy

Incorrect

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6) 8

$E = \frac{n^2h^2}{8mL^2}$

$n = \sqrt{\frac{8mL^2E}{h^2}}$

$= \sqrt{\frac{8\times(10^{-3}\,kg)\times(10^{-1}\,m)^2\times10^{-3}\,J}{(6.6\times10^{-34}\,kg\,m^2/s)^2}}$

$\cong 4\times10^{30}$

B

7) (A, D)

A. $\frac{1}{\sqrt{2\pi}} e^{-5i\phi}$ ✓ ($n = -5$)

B. $\frac{1}{\sqrt{2\pi}} e^{-5\phi}$ ✗ (needs i)

C. $\frac{1}{\sqrt{2\pi}} e^{-i\phi/2}$ ✗ ($n = 1/2$, not integer)

D. $\frac{1}{\sqrt{2\pi}}$ ✓ ($n = 0$)

Acceptable wavefunction

$\sim \frac{1}{\sqrt{2\pi}} e^{in\phi}$ $\quad n \in 0, \pm 1, \pm 2, \ldots$

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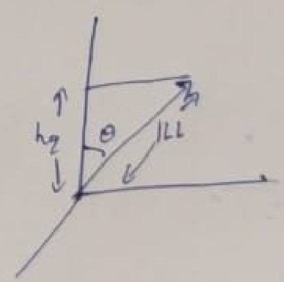
8) $Y = \frac{1}{8}\sqrt{\frac{21}{\pi}} \underbrace{\sin\theta\,[1 - 5\cos^2\theta]}_{\text{cubic} \Rightarrow l = 3} \underbrace{e^{i\phi}}_{m = 1}$



$L_z = m\hbar = \hbar$

$|L| = \sqrt{l(l+1)}\ \hbar = \sqrt{12}\ \hbar$

$\theta = \cos^{-1}\left[\frac{L_z}{|L|}\right]$

$= \cos^{-1}\left[\frac{\hbar}{\sqrt{12}\,\hbar}\right]$

$\boxed{\theta = 73.2^\circ}$

Q.9 The energy difference between the ground state ($n_1 = 1, l_1 = 0, m_1 = 0$) and any excited state (like $n_2 = 7$, $l_2 = 2$, $m_2 = 0$) ~~with~~ for Li^{2+} atom will only depend on $\boxed{(A)\ n_1 \text{ and } n_2}$ ← Ans

This is true for an isolated Li^{2+} in absence of any external field.

Q.10 The integral $\boxed{\int \psi^*(2,1,1)\, \psi(3,0,0)\, d\tau = 0}$ ← Ans

where $d\tau$ is the volume element, $\psi(n, l, m)$ is the hydrogenic eigenfunction with quantum number n, l, m.

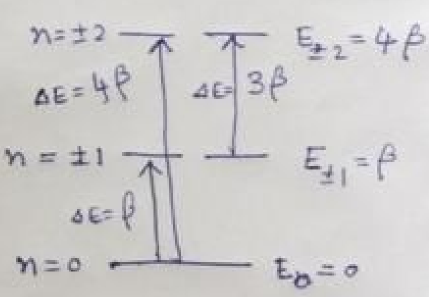
"These hydrogenic eigenfunction must be orthogonal to each other."

Q.11

For a quantum particle on a ring, the energy is given as $E_n = \frac{n^2 \hbar^2}{2I}$

where $\beta = \frac{\hbar^2}{2I}$ $\quad = n^2 \beta$

Where $n = 0, \pm 1, \pm 2 \ldots$



Total 3 distinct transitions are possible.

Q.12 Hydrogenic orbitals with same principal Q. no (n) and (l) values will have the same radial distribution functions.

For example : (A) $2p_z$ and $2p_x$ ✓

(D) $3d_{z^2}$ and $3d_{x^2-y^2}$ ✓

but (B) $2p_z$ and $3d_{z^2}$ ✗

(C) $2s$ and $3s$ ✗