PH 112: Quantum Physics and Applications

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Week 01, Lecture3: Wave Particle Duality and Matter Waves

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Compton Scattering: Recap

- The scattered radiation has a longer wavelength than the wavelength of the incident radiation.

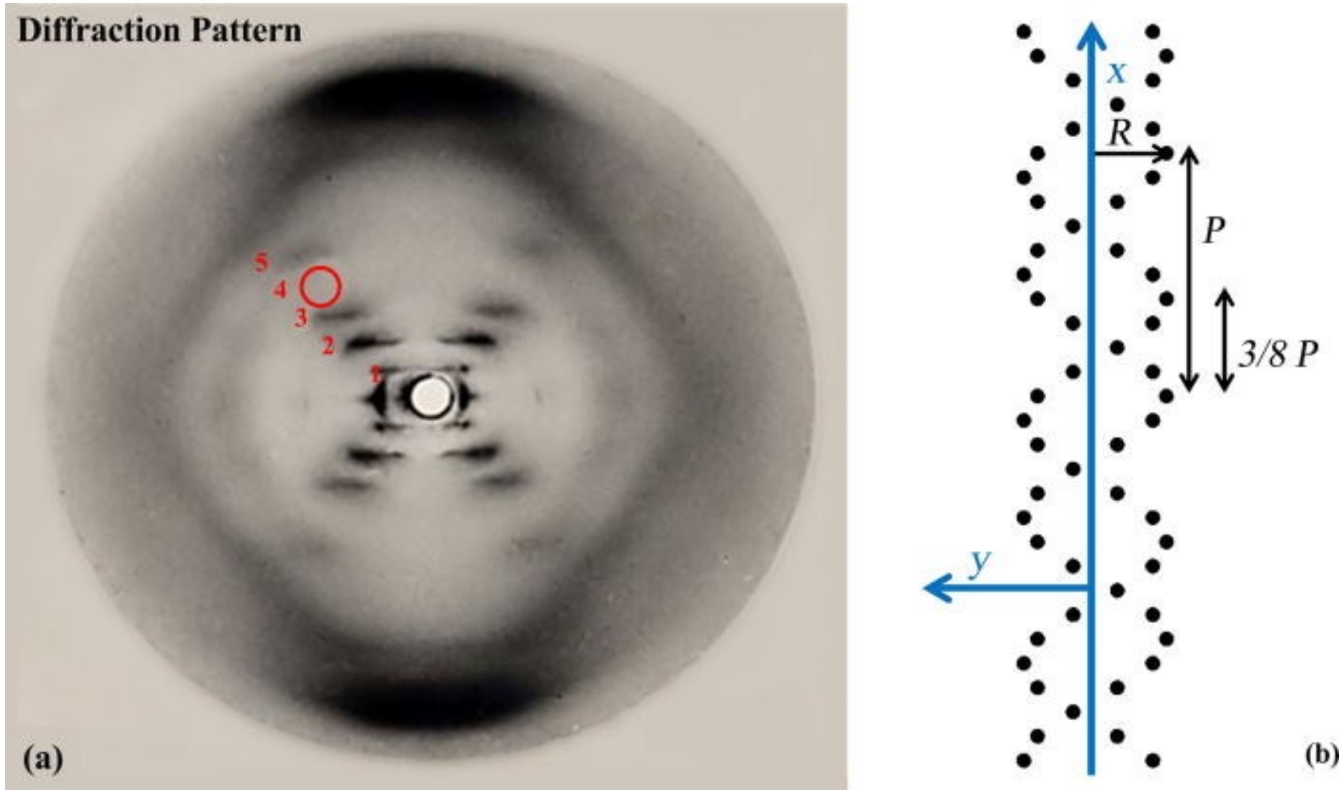
- In today's usage, the term Compton scattering is used for the inelastic scattering of photons by free, charged particles.

- In Compton scattering, treating photons as particles with momenta that can be transferred to charged particles provides the theoretical background to explain the wavelength shifts measured in experiments; this is the evidence that radiation consists of photons.

X-ray diffraction

X-Rays Again!

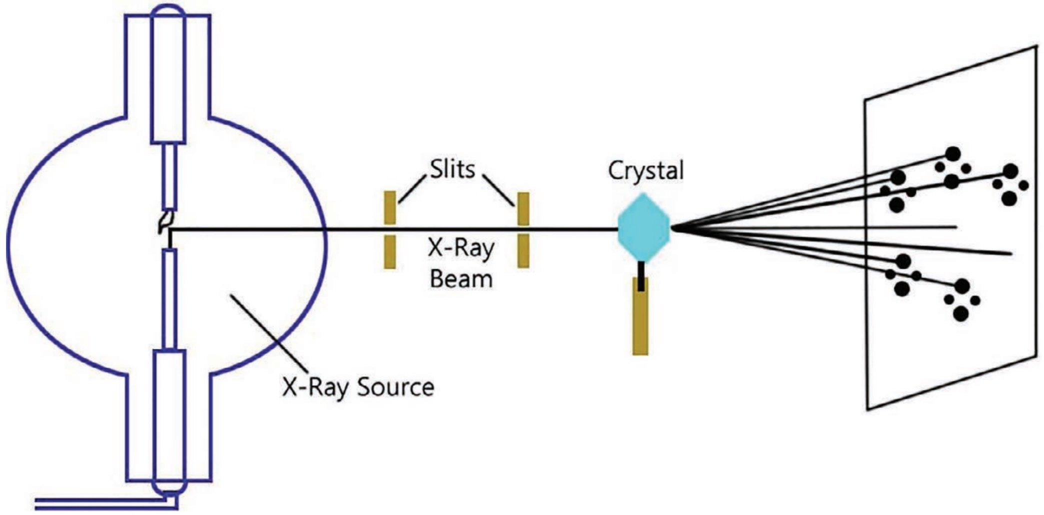
- X-ray photons are very energetic: $\lambda \sim 10^{-8}\ to\ 10^{-12}\ meters$.
- Typical X-ray photons act like rays when they encounter macroscopic objects, like teeth, and produce sharp shadows.
- However, they behave differently when they encounter smaller objects like atoms.
- For objects of size 0.1 nm, X-rays can be used to detect the location, shape, and size of atoms and molecules. The process is called **X-ray diffraction**
- The most famous example of X-ray diffraction is the discovery of the double-helical structure of **DNA** in 1953.

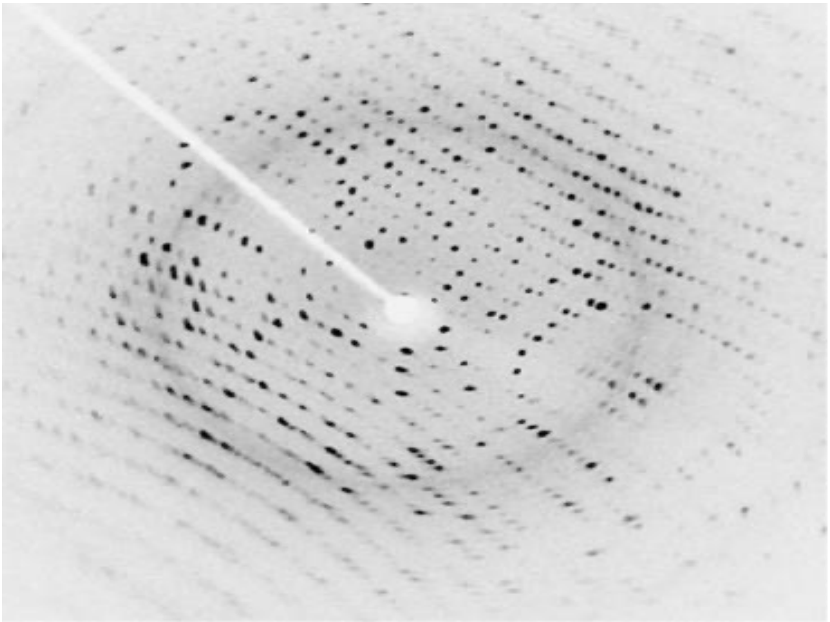


The well-known Photo 51, the diffraction pattern from DNA in its so-called B configuration. The dimensions of DNA are: pitch P = 3.4 nm, radius R = 1 nm, and a phase difference between the two helices (sine waves) of ΔP = 3P/8. Several important features include the characteristic X-shape or distorted rhombus, the ten diffracted orders per X, and the missing fourth order. (b) A two-dimensional projection of the phosphate molecules in the DNA backbone. The projection outlines two sine waves.

X-Ray Scattering

- Max von Laue suggested that if x-rays were a form of electromagnetic radiation, interference effects should be observed.
- Crystals act as three-dimensional gratings, scattering the waves and producing observable interference effects.

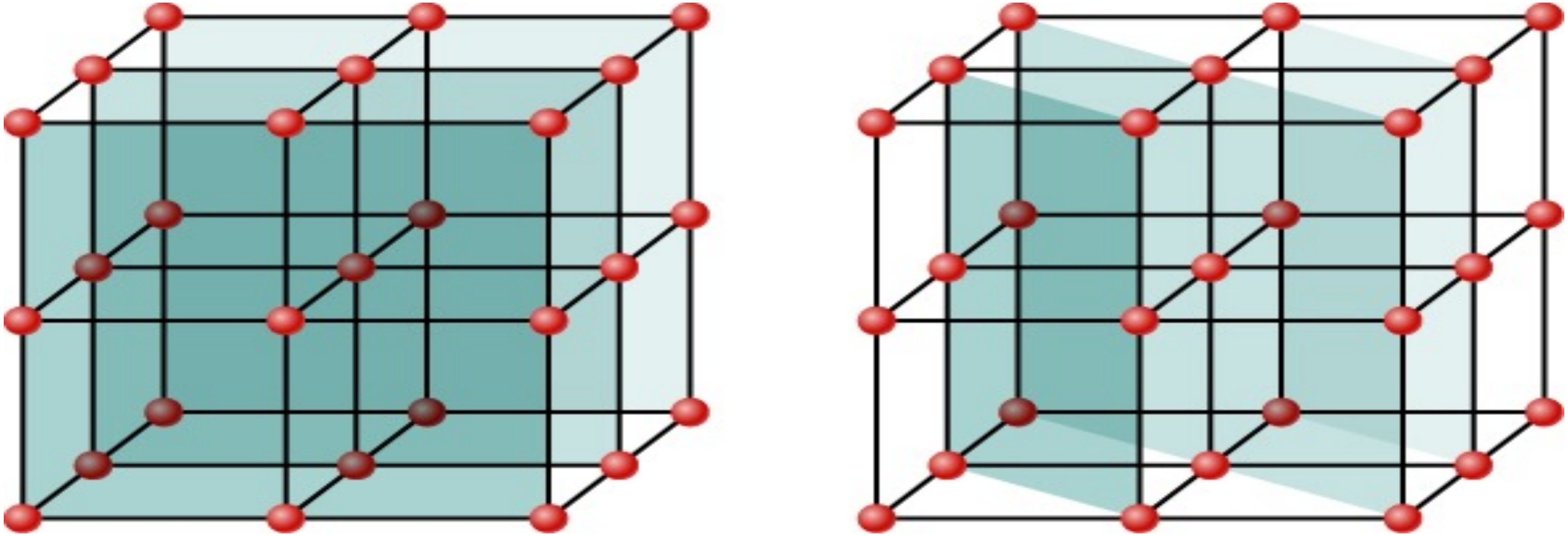




X-ray diffraction from the crystal of a protein (hen egg lysozyme) produced this interference pattern. Analysis of the pattern yields information about the structure of the protein. (credit: Wikimedia)

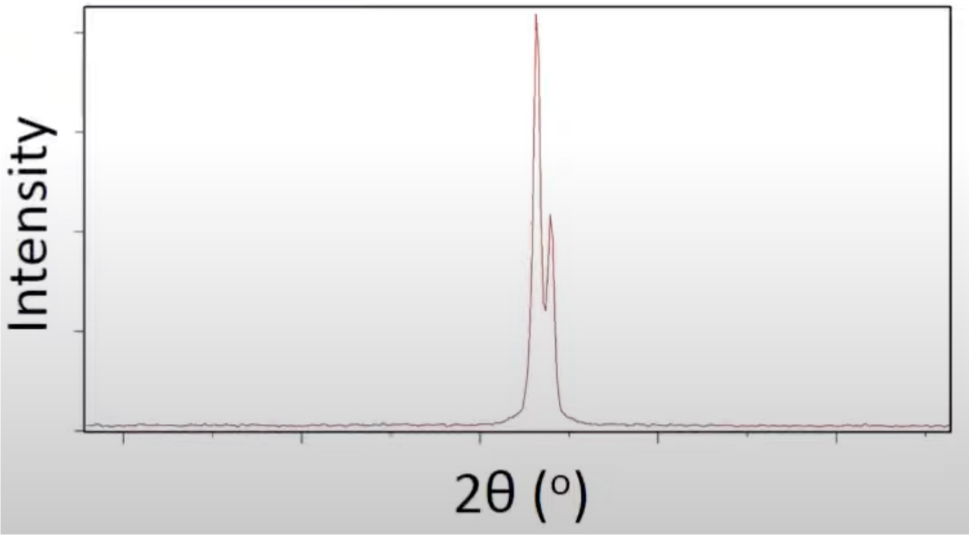
X-ray scattering from Crystals

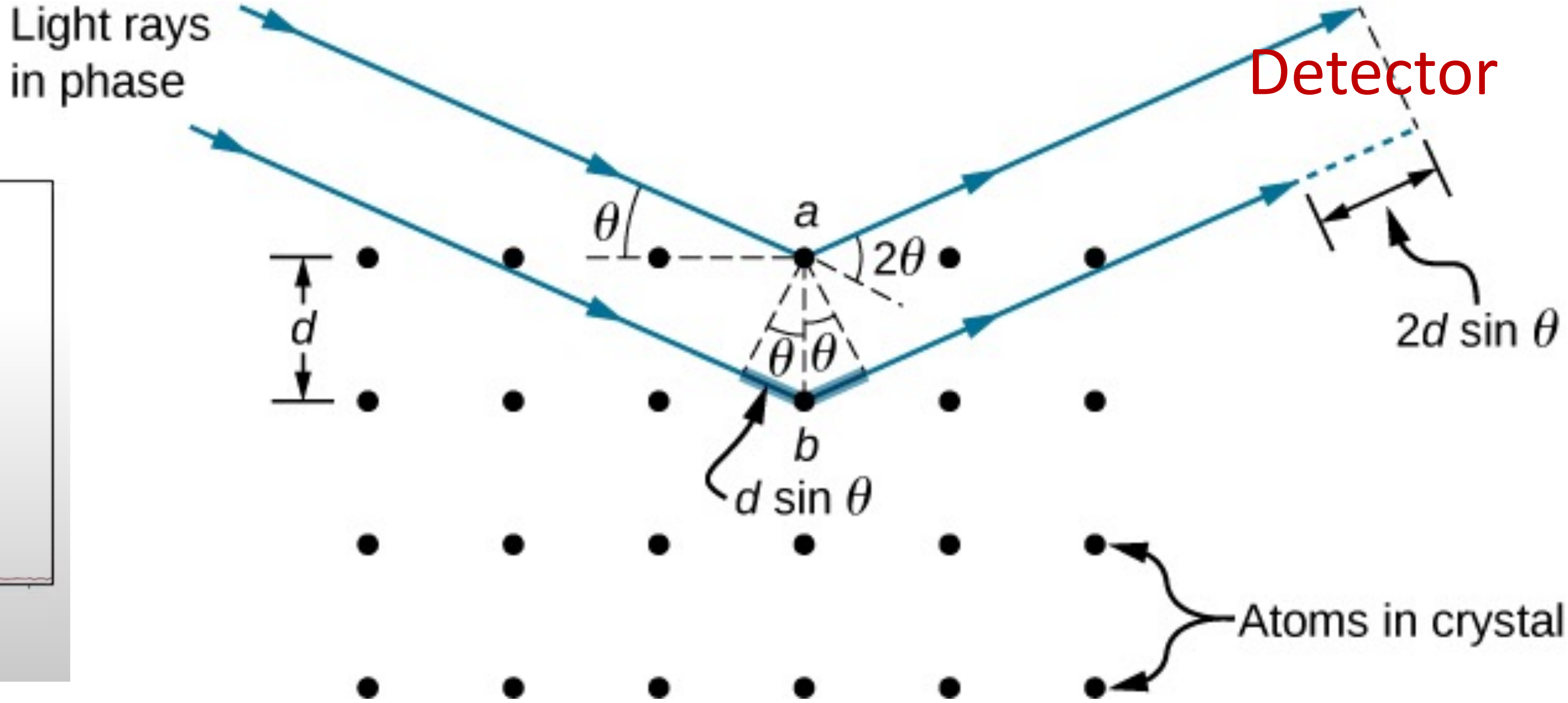
- William Lawrence Bragg interpreted the x-ray scattering as the reflection of the incident x-ray beam from a unique set of planes of atoms within the crystal.



Because of the regularity that makes a crystal structure, one crystal can have many families of planes within its geometry, each one giving rise to X-ray diffraction.

Bragg's Law





Two conditions for constructive interference of scattered X-rays:

1. The angle of incidence must equal the angle of reflection of the outgoing wave.

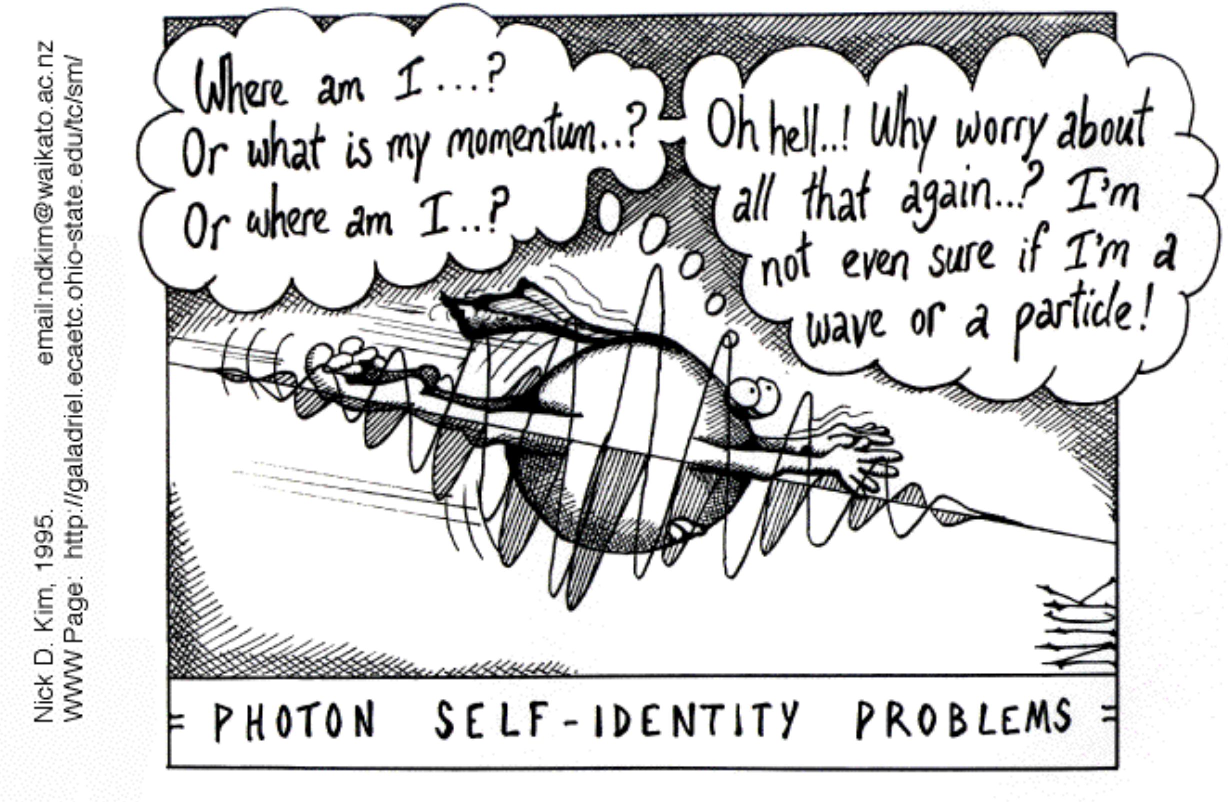
2. The difference in path lengths must be an integral number of wavelengths.

X-ray diffraction with a crystal. Two incident waves reflect off two planes of a crystal. The difference in path lengths is indicated by the dashed line.

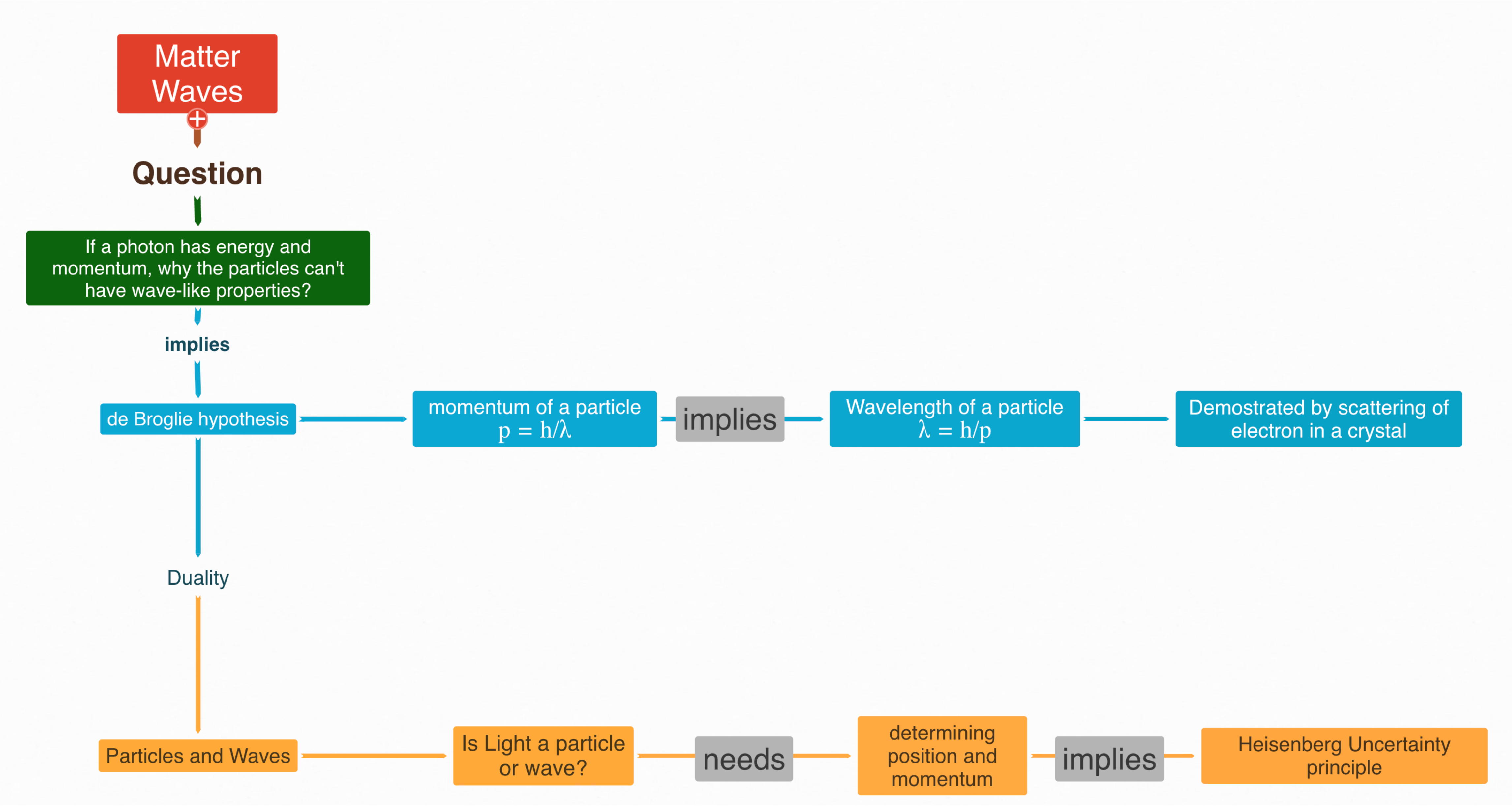
Bragg's Law: $n\,\lambda = 2\,d\,sin(\theta)$
(n = integer)

Electromagnetic waves or photons?

- We knew these as waves and now, we know they are particles. However, they are dual descriptions!
- These have wave attributes and particle attributes.
- Wave attributes because it interferes and diffracts.
- Particle attributes as it has a definite amount of energy, it comes in packets, and cannot be broken into other things.



de Broglie hypothesis and consequences


Matter Waves
Question
If a photon has energy and momentum, why the particles can't have wave-like properties?
implies
de Broglie hypothesis
momentum of a particle $p = h/\lambda$
implies
Wavelength of a particle $\lambda = h/p$
Demostrated by scattering of electron in a crystal
Duality
Particles and Waves
Is Light a particle or wave?
needs
determining position and momentum
implies
Heisenberg Uncertainty principle

de Broglie: If waves can mimic particles, then perhaps particles can mimic waves

Louis de Broglie 1892-1987
Nobel Prize 1929
"for his discovery of the wave
nature of electrons"

- According to Planck
  Light $E = h\,\nu$

  According to Einstein
  Mass $E = m\,c^2$

- de Broglie combined the two energy relations: One can treat the energy of light to have an equivalent mass!

$$h\,\nu = h\frac{c}{\lambda} = m\,c^2 \implies \lambda = \frac{h}{m\,c} = \frac{h}{p}$$

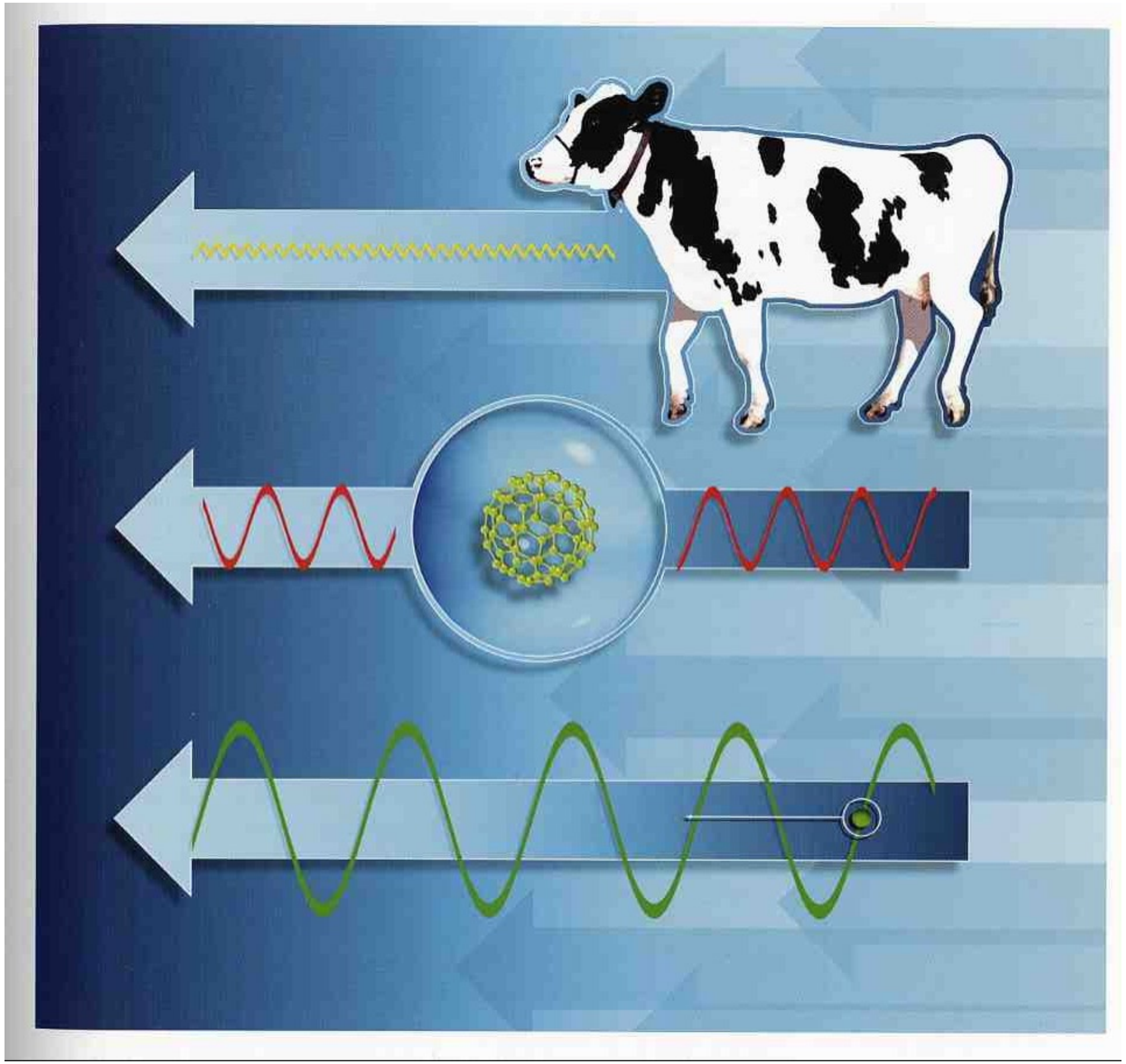
Same relation which Compton obtained!

- Light is not special. Such a relation should be true for all particles.

$$\lambda = \frac{h}{p} = \frac{h}{m\,v}$$

Two seemingly incompatible conceptions can each represent an aspect of the truth ... They may serve in turn to represent the facts without ever entering into direct conflict. ***de Broglie, Dialectica***

de Broglie waves in everyday life!



$$\lambda = \frac{h}{m\,v} = \frac{h}{p}$$ h = 6.6 x 10^{-34} Js

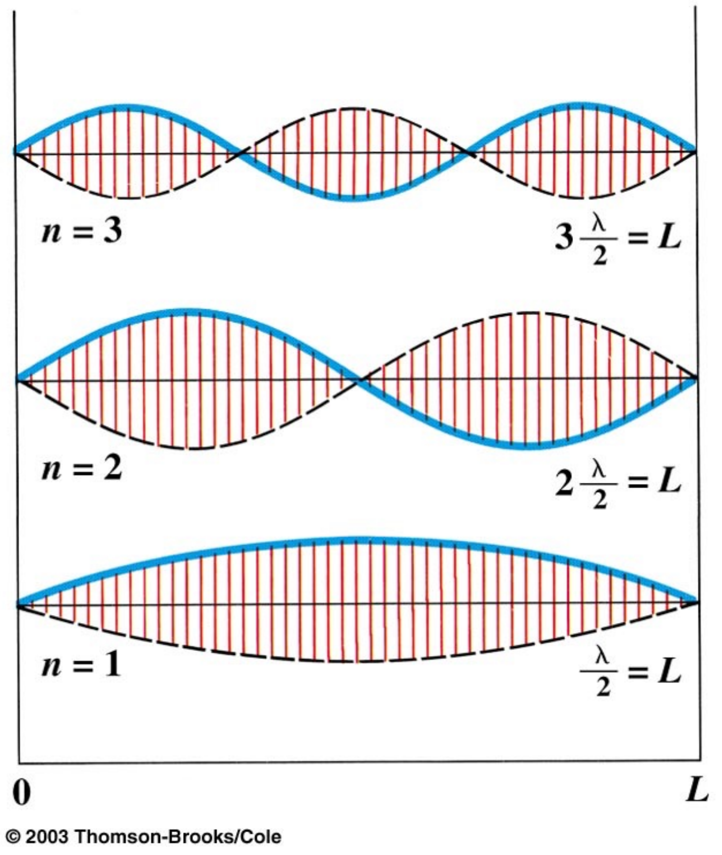
Consider two cases:

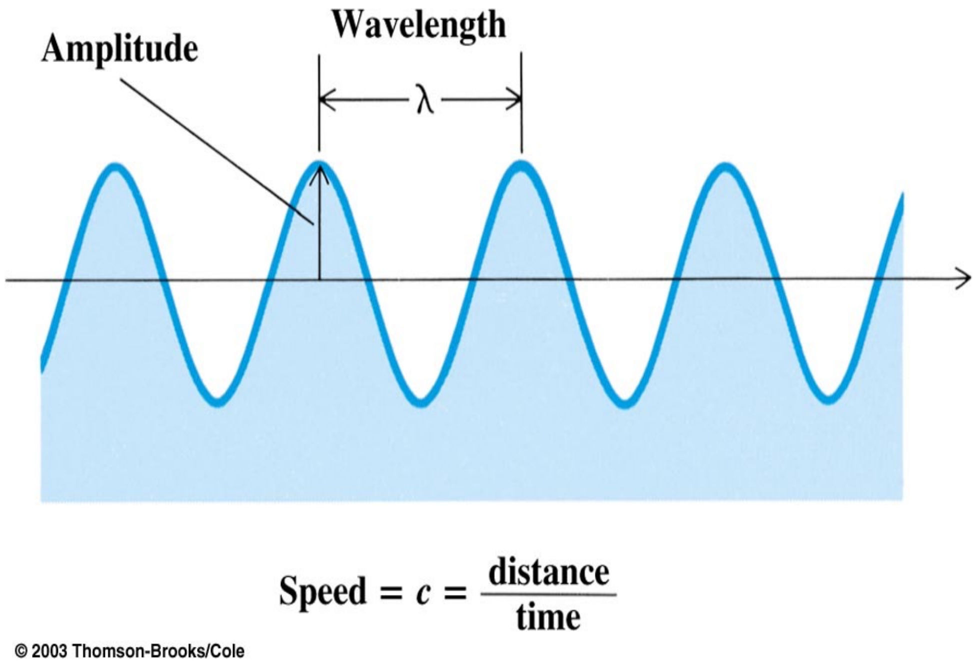
$$\lambda_{Cow} = \frac{10^{-34}}{10^{6} \times 10^{2}} \sim 10^{-42}\ m$$

$$\lambda_{electron} = \frac{10^{-34}}{10^{-31} \times 10^{2}} \sim 10^{-5}\ m$$

The wave properties of matter are only apparent for very small masses of matter.
Expt showing intereference for 420 atoms.

Two familiar macroscopic waves: Standing wave and a traveling wave

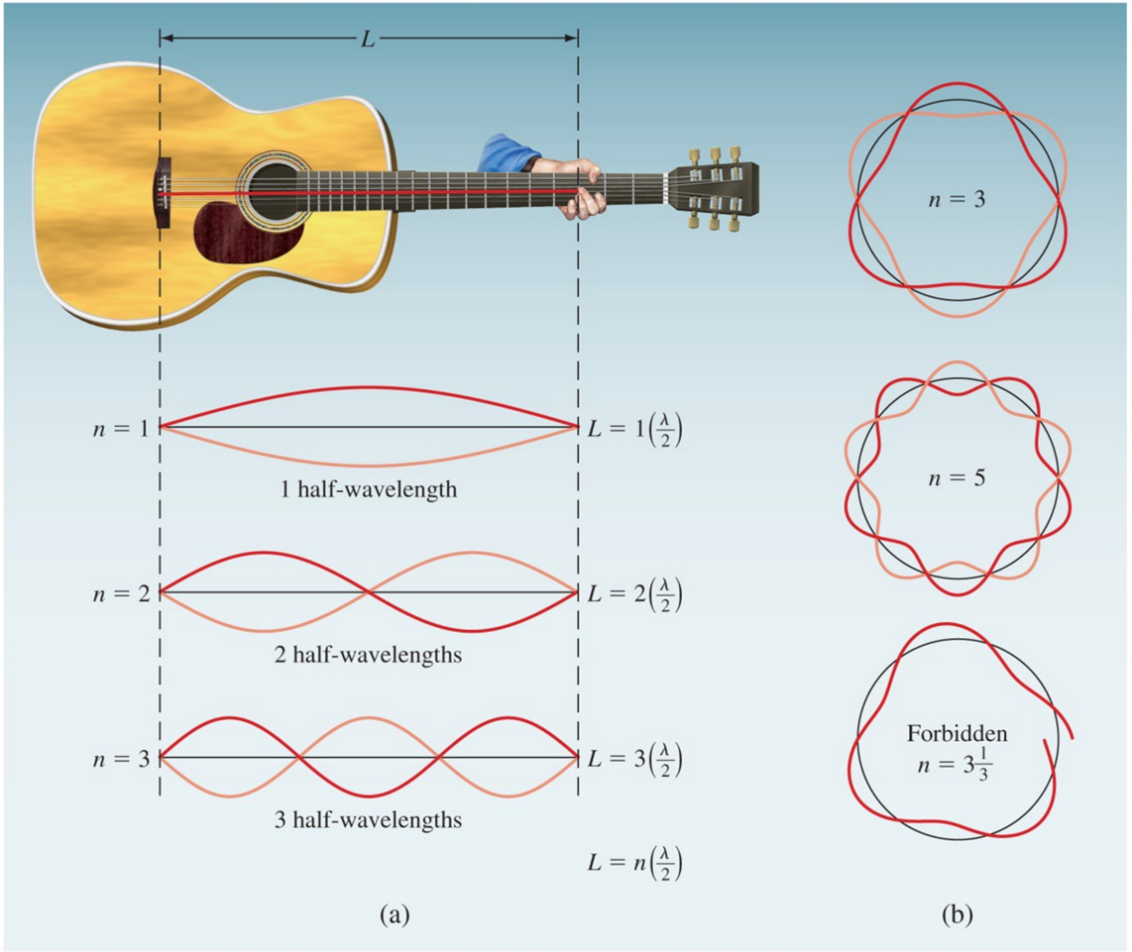




Harmonics (stable standing waves) of a linear string fixed at both ends is characterized by its wavelength (frequency) and its amplitude.

A traveling water wave is characterized by its speed, wavelength (frequency) and its amplitude.

de Broglie hypothesis consequences: Wave properties of matter



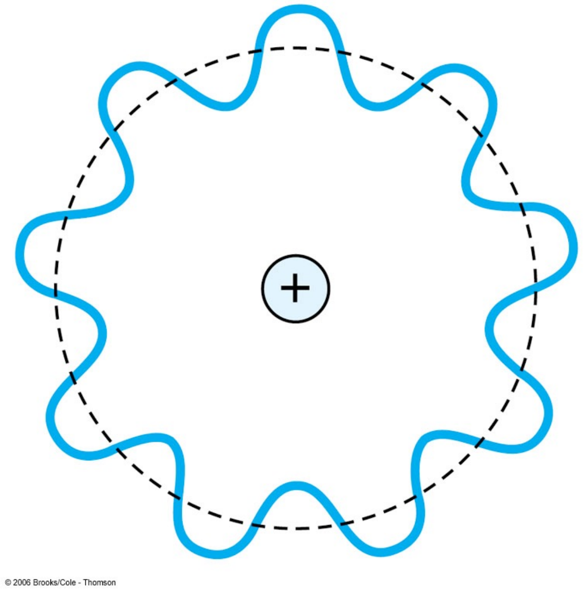
- De Broglie proposed mathematical solution to Bohr's atom by answering two questions:
  - Why electrons have angular momentum?
  - Why electrons have discrete energy levels?
- He said, if an electron has a wavelength then it's going to form a standing wave around the nucleus.
- There are some points called nodes (where the wave exhibits no motion at all).

de Broglie hypothesis : Bohr's Quantization Condition

- If the perimeter is not a whole number multiple of λ, the next time the wave completes a revolution it will not coincide with the previous wave. The waves will superimpose and over a period of time eventually cancel itself.

- The waves must perfectly coincide to exist. Hence the allowed orbits have a radius which satisfies

$$2\pi r = n\lambda = n\frac{h}{p}$$



- The angular momentum becomes:

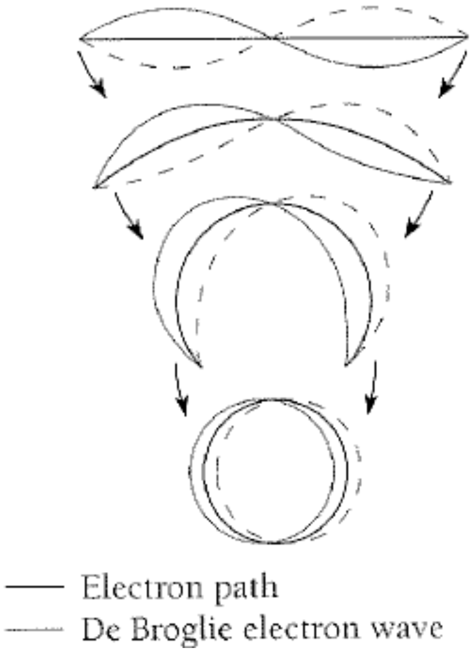
$$L = rp = \frac{nh}{2\pi} = n\hbar$$

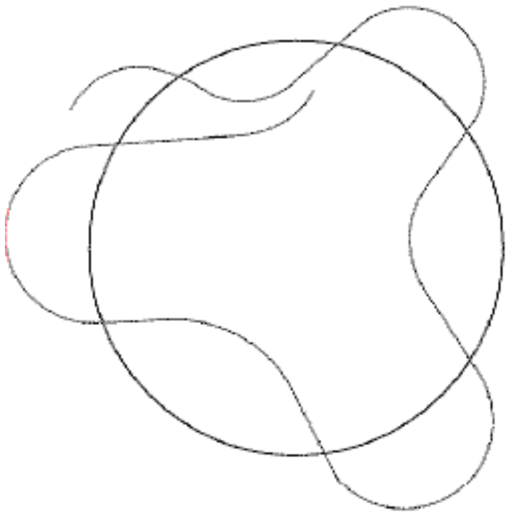
- A standing wave is the reason why the electron around the Bohr atom do not release energy.

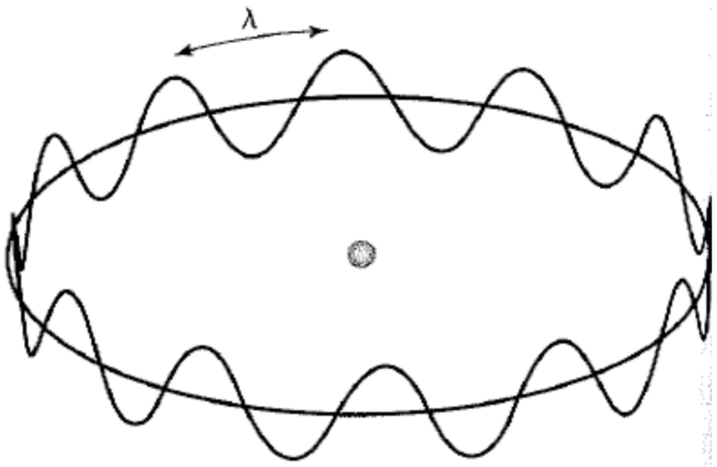
de Broglie waves

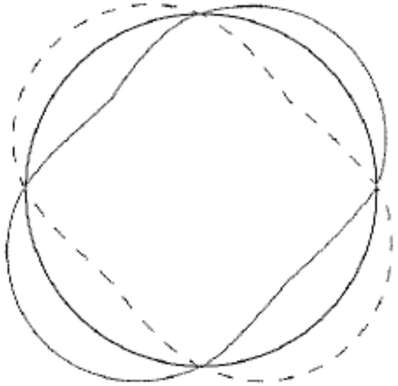
Here are different ways of representing de broglie standing waves. His conjecture is the amount of standing waves is set so the number of wavelengths that we always have

$$2\pi r = n\lambda \qquad L = n\hbar$$

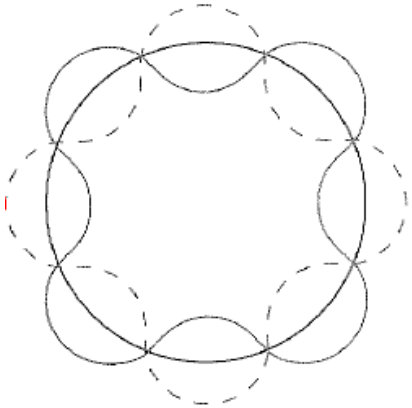




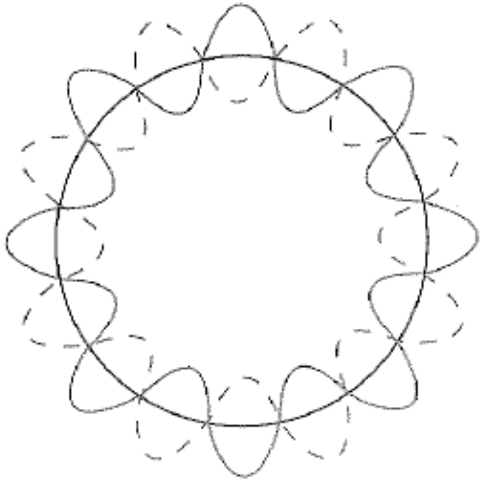




Circumference = 2 wavelengths



Circumference = 4 wavelengths



Circumference = 8 wavelengths

de Broglie waves: Experimental verification

- It seems to be a natural conjecture, but is it true?
- What is the experimental evidence for such a hypothesis?
- If so, can we observe electron waves?
- What would you need to see to believe that this is actually true?

Electron diffraction

Electron Diffraction

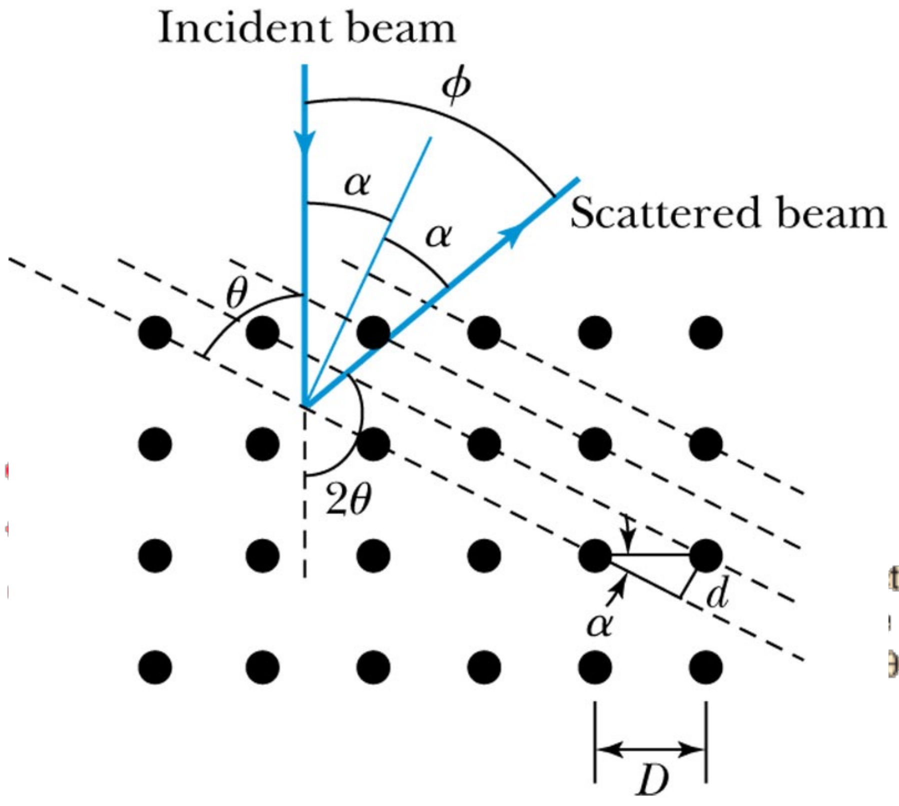
- Accelerated electrons (in a potential around 150 ev) have wavelength of order 1 Angstrom $= 10^{-10}\ m$.

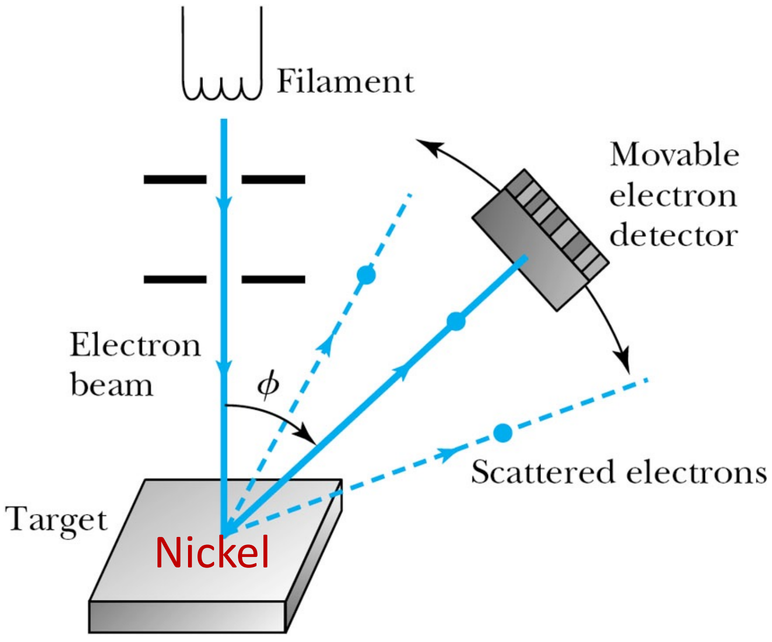
- This is of the same order as atomic spacing!

- If de Broglie's hypothesis was right, electrons (like x–ray photons) will undergo diffraction at atomic surfaces if the atoms are lined up in planes, like a crystal.

Davisson-Germer experiment

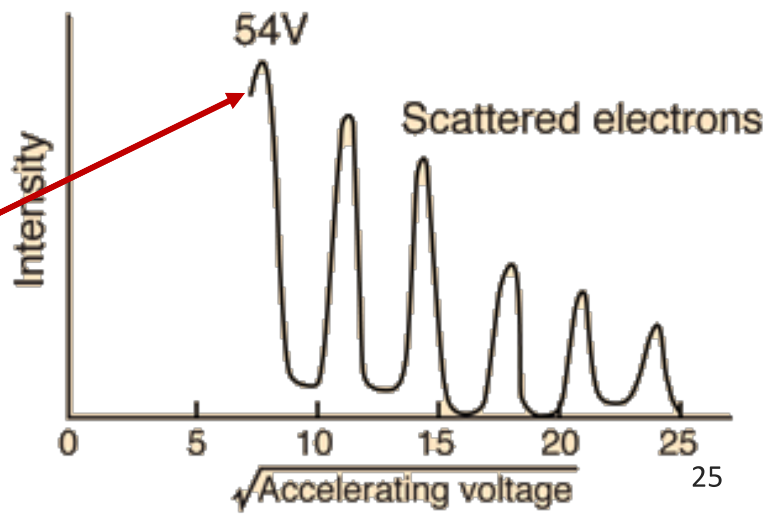
- The electron gun produces beam of electrons.
- High tension battery accelerates the electron beam.
- Electron passed through the pinhole to strike target normally.
- Electrons were scattered in all directions which were detected by the movable detector.





$$\lambda = \frac{D \sin \phi}{n}$$

Maximum intensity was due to constructive interference a phenomenon confined only to waves



Proof of de Broglie relation

- Accelerated electrons have energy $\frac{1}{2}\, m\, v^2 = e\, V \Longrightarrow v = \sqrt{\frac{2\, e\, V}{m}}$

- From de Broglie, the electrons will have: $\lambda = \frac{h}{m\, v} = \frac{h}{p} = \frac{h}{\sqrt{2m\, e\, V}} = 1.67\ \text{Å}$

- Davisson-Germer found lattice spacing: $\lambda = d \sin\theta = 1.65\ \text{Å}$

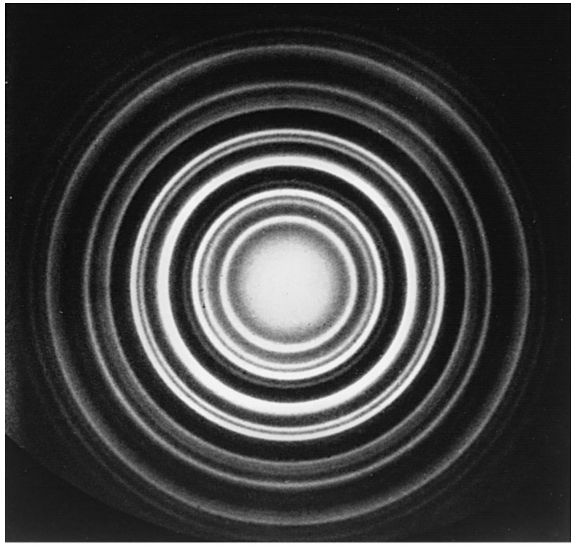
Excellent agreement between theory and experiment!

- Experimental proof of de Broglie waves.

Electron diffraction through transmission

- George P. Thomson (1892–1975), son of J. J. Thomson, reported seeing the effects of electron diffraction in transmission experiments.
- The first target was celluloid, and soon after that gold, aluminum, and platinum were used.
- The randomly oriented polycrystalline sample of SnO_2 produces rings as shown in the figure at the bottom.

(a)



(b)

Neutron diffraction

How does this work at all?

- Davisson-Germer experiment showed that electrons could interfere.
- This means that other microscopic particles, like Neutrons, should have wavelike behavior with a wavelength $\lambda = h/p$.
- Again the structures are at atomic-bond resolution level (1Å = 10^{-10} m)
- Therefore we need neutrons with momentum

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ Js}{10^{-10}\ m} = 6.626 \times 10^{-24}\ Kg\ m\ s^{-1}$$

- If these Neutrons are nonrelativistic $v = \frac{p}{m} = \frac{6.626 \times 10^{-24}}{1.675 \times 10^{-27}\ m} = 3960$ m/s

What is the Kinetic energy of these neutrons?

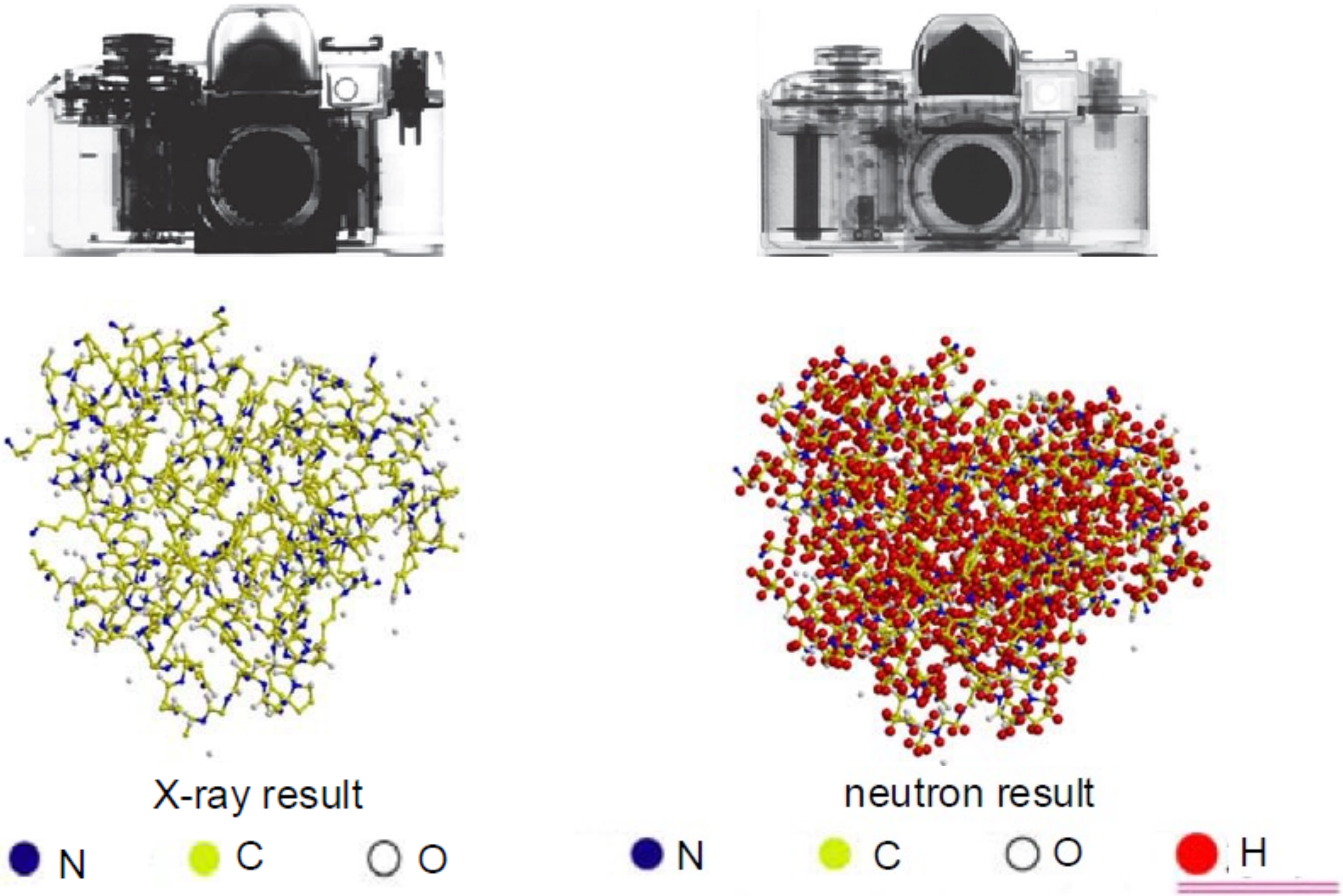
- Kinetic energy of these neutrons is

$$KE = \frac{p^2}{2\,m} = \frac{(6.626 \times 10^{-24})^2}{2 \times 1.675 \times 10^{-27}} = 1.313 \times 10^{-20} J = 820\ eV$$

- Energy of relativistic neutron is 2 MeV.
- Energy of thermal neutrons $\frac{3}{2}\ k_B T \sim 0.025\ eV$

- KE of the neutrons for diffraction is less than thermal neutrons, however, greater than relativistic neutron.

X-ray vs. Neutron



Summary

- If waves can mimic particles, then particles can mimic waves --- de Broglie

$$\lambda = \frac{h}{m\,v} = \frac{h}{p}$$

- Experimental verification by electron diffraction.

  Davisson and Germer technique is still used in **Low-energy electron diffraction** (LEED).

- We have now observed Neutron diffraction.

Wave-Particle Duality

Particle-like wave behavior (example, photoelectric effect)

Wave-like particle behavior (example, Davisson-Germer experiment)

$$\underbrace{\qquad\qquad\qquad\qquad}$$

Wave-particle duality

Mathematical descriptions:

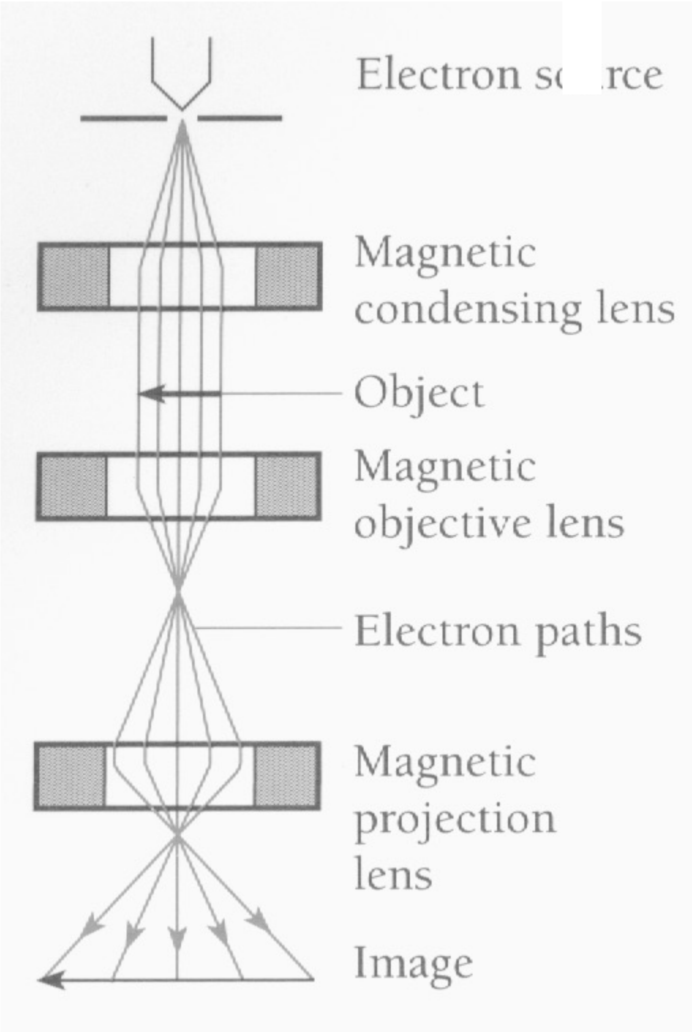
The momentum of a photon is: $$p = \frac{h}{\lambda}$$

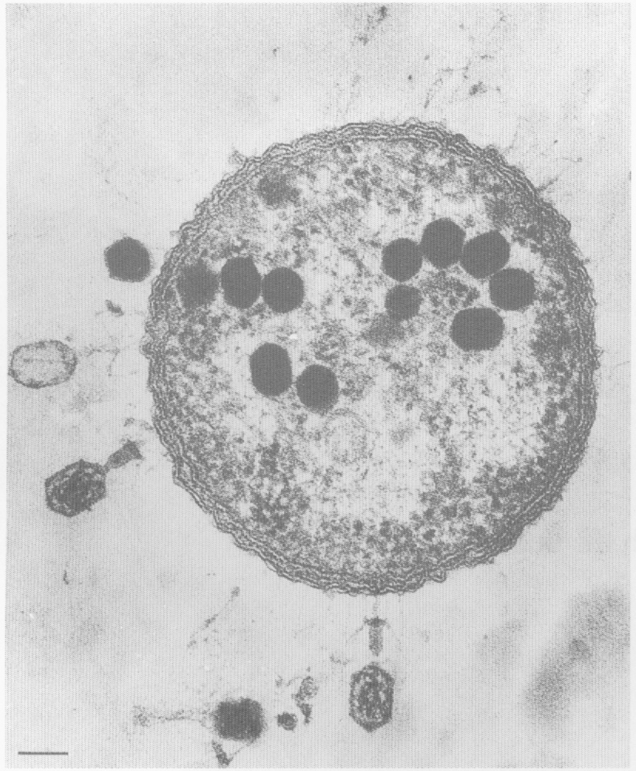
The wavelength of a particle is: $$\lambda = \frac{h}{p}$$

λ is called the *de Broglie wavelength*

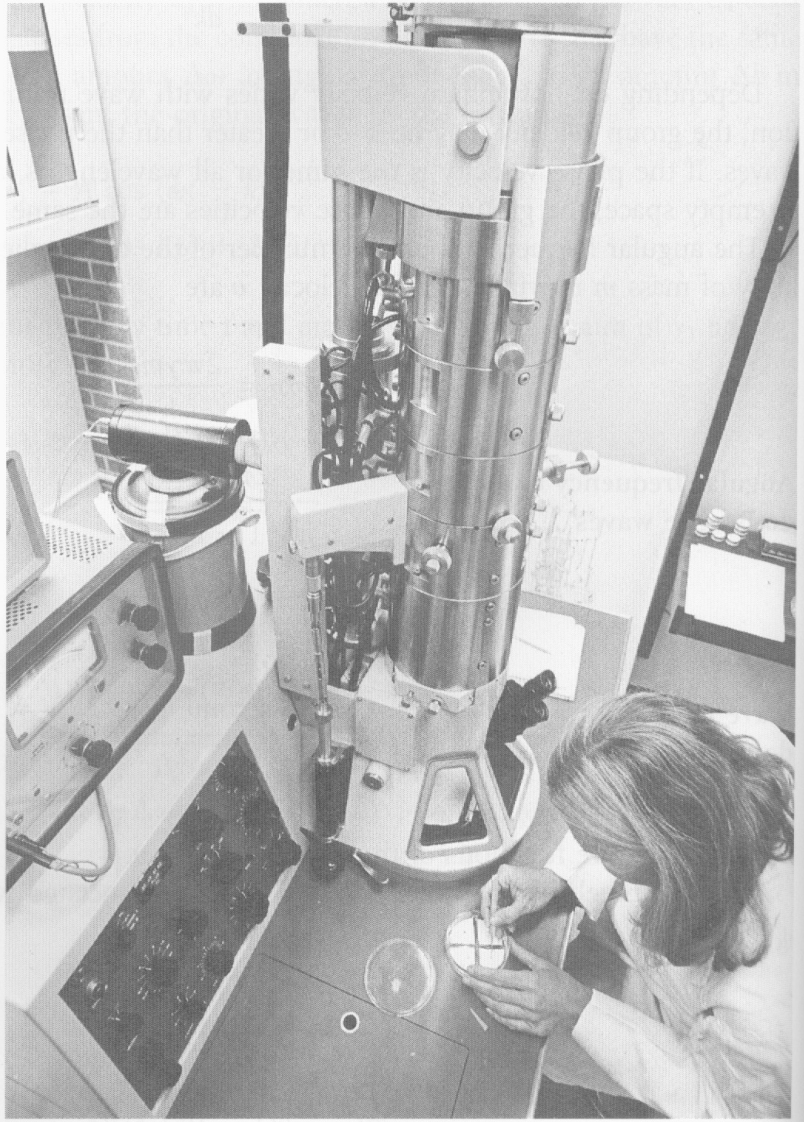
Electron microscope

optical microscope, the elec-
ı microscope can produce
rp images at higher magnifi-
ons. The electron beam in an
tron microscope is focused
magnetic fields.





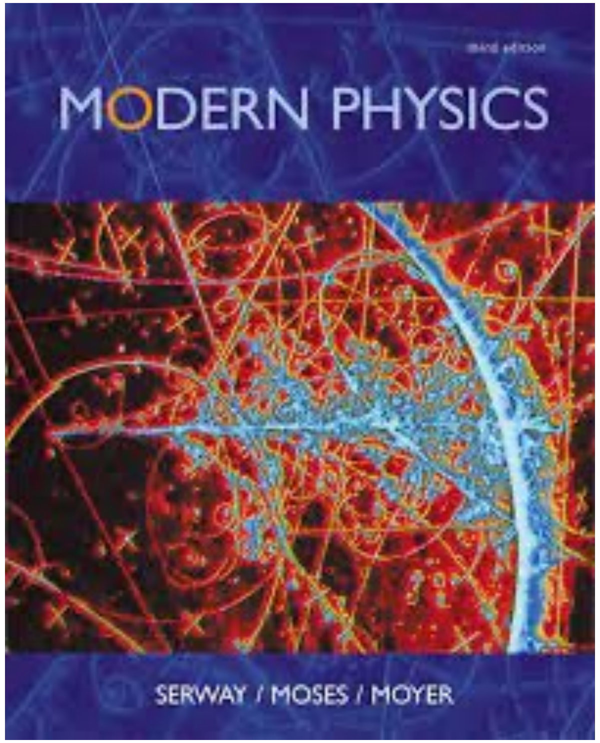
Electron micrograph showing bacteriophage viruses in an *Escherichia coli* bacterium. The bacterium is approximately 1 μm across.



An electron microscope.

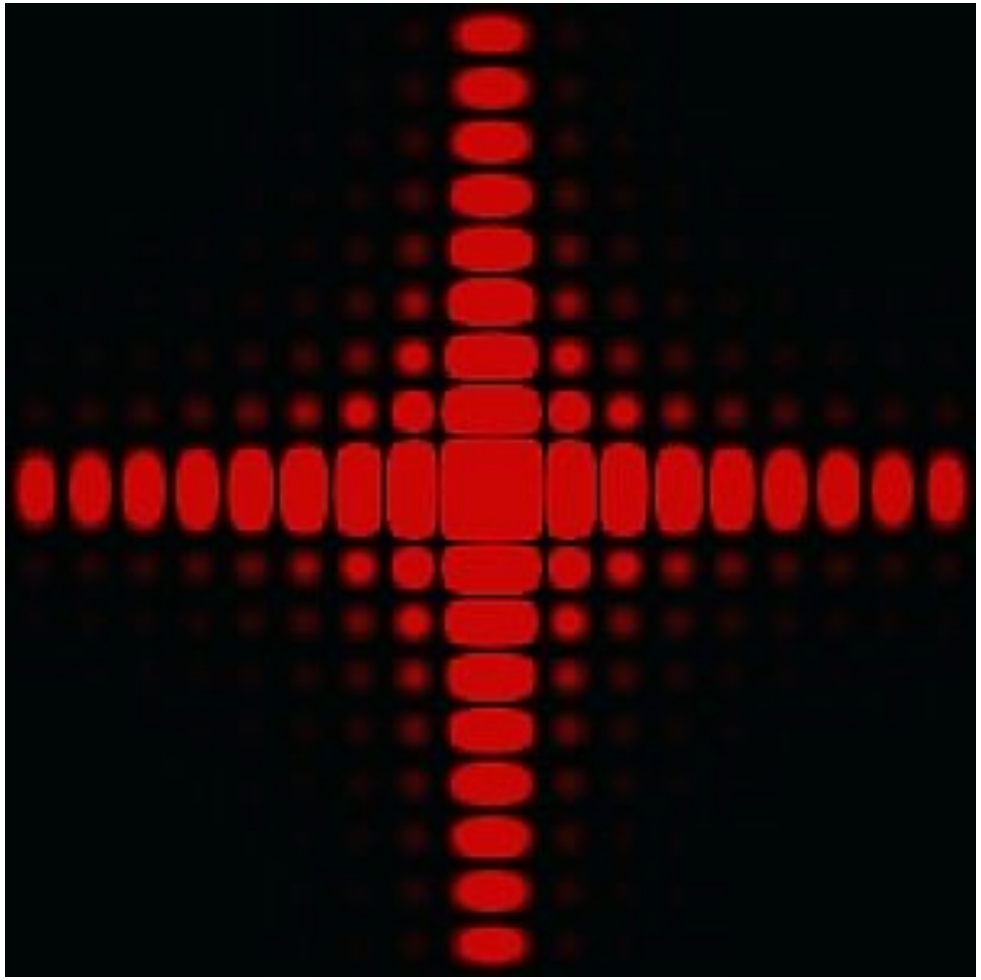
Recommended Reading

Matter wave, sections 5.1 and 5.2 in page 152 and 154.



Diffraction of Light

Diffraction



1. Diffraction was coined by Francesco Grimaldi, who was the first to characterize it.

2. James Gregory observed the diffraction patterns from a bird feather—the first diffraction grating to be discovered.

3. Augustin-Jean Fresnel did more definitive studies and calculations of diffraction thus supporting wave theory of light.

Diffraction of water waves

1. Diffraction occurs for all waves.
2. Diffraction shows that the waves do not always travel in straight line.
3. Waves do bend around objects. The extend of bending is referred to as Diffraction.

Fraunhofer Diffraction from a Slit

Fraunhofer Diffraction from a slit is the Fourier transform of a rectangular function. [We will see this when we do Fourier transform.]

Diffraction is a sinc function. The intensity is then sinc^2.

$$t(x) = \mathrm{rect}(x/w)$$

$$E(k_x) \propto \mathscr{F}\{t(x)\}$$

$$\boxed{\begin{aligned} E(k_x) &\propto \mathrm{sinc}(wk_x/2) \\ E(x') &\propto \mathrm{sinc}(wkx'/2z) \end{aligned}}$$

$$\boxed{\begin{aligned} I(k_x) &\propto \mathrm{sinc}^2(wk_x/2) \\ I(x') &\propto \mathrm{sinc}^2(wkx'/2z) \end{aligned}}$$