Lecture 10

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Galilean Transformation

RECAP

Position

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t$$

Velocity

$$u'_x = \frac{dx'}{dt} = \frac{dx}{dt} - v \qquad u'_y = \frac{dy'}{dt} = \frac{dy}{dt} = u_y \qquad u'_z = \frac{dz'}{dt} = \frac{dz}{dt} = u_z \qquad dt' = dt$$

$$u'_x = u_x - v \qquad u'_y = u_y \qquad u'_z = u_z \qquad dt' = dt$$

Acceleration

$$a'_x = \frac{du'_x}{dt} = \frac{du_x}{dt} = a_x \qquad a'_y = \frac{du'_y}{dt} = \frac{du_y}{dt} = a_y \qquad a'_z = \frac{du'_z}{dt} = \frac{du_z}{dt} = a_z$$

$$a'_x = a_x \qquad a'_y = a_y \qquad a'_z = a_z$$

Houston we have a PROBLEM

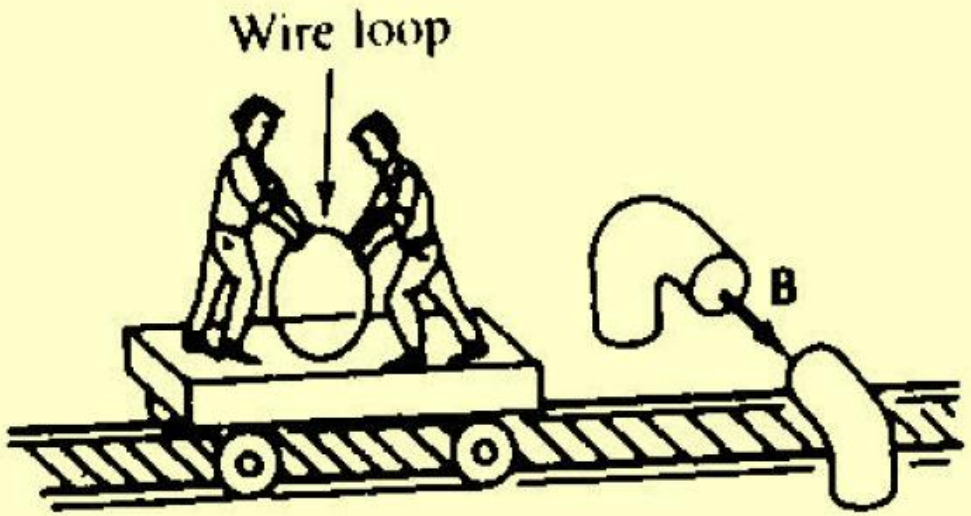
Lorentz force is a velocity dependent force and could depend on **inertial frame of reference (ifor)** used

Velocity Dependent Lorentz force:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

The correct transformation of electric and magnetic fields as we go from one **ifor** to another cannot be obtained using Galilean relativity.

Illustration



S is an observer on the ground

S’ is an observer on the train moving with a constant velocity v

Another Problem

Speed of light is related to fundamental constants.

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

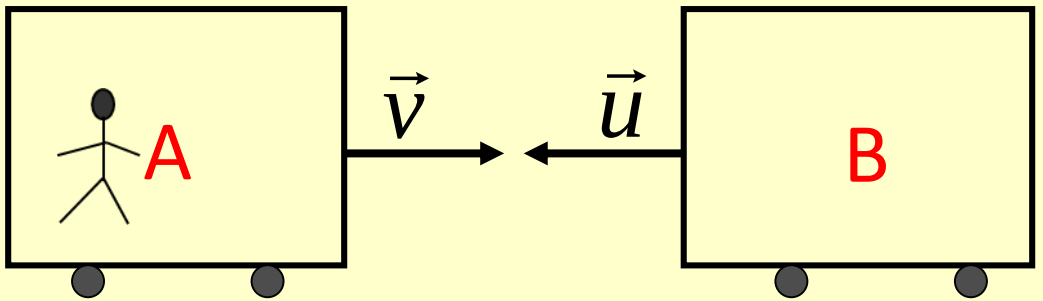
The speed of light in vaccum (c), depends on two fundamental constants, the permeability of free space μ_0 and permittivity of free space ε_0.

Why is this a problem ?

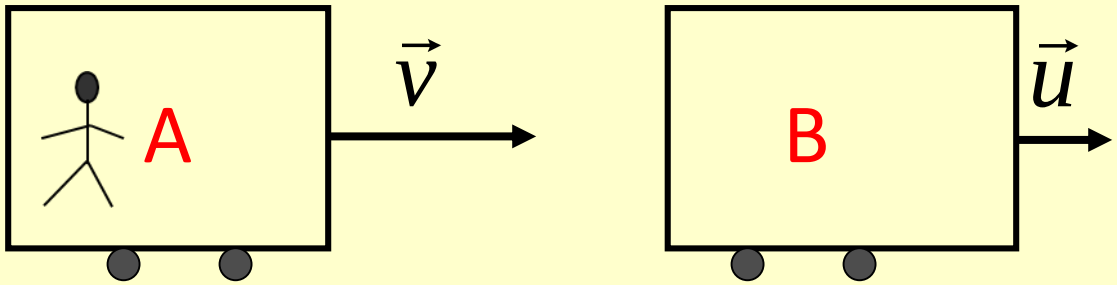
One Slide Digression: Definition

Permittivity and permeability are two different measures used in electromagnetism. Permittivity measures the ability of a material to store energy within the material. Permeability is a measure of the ability of a material to support the formation of a magnetic field within the material.

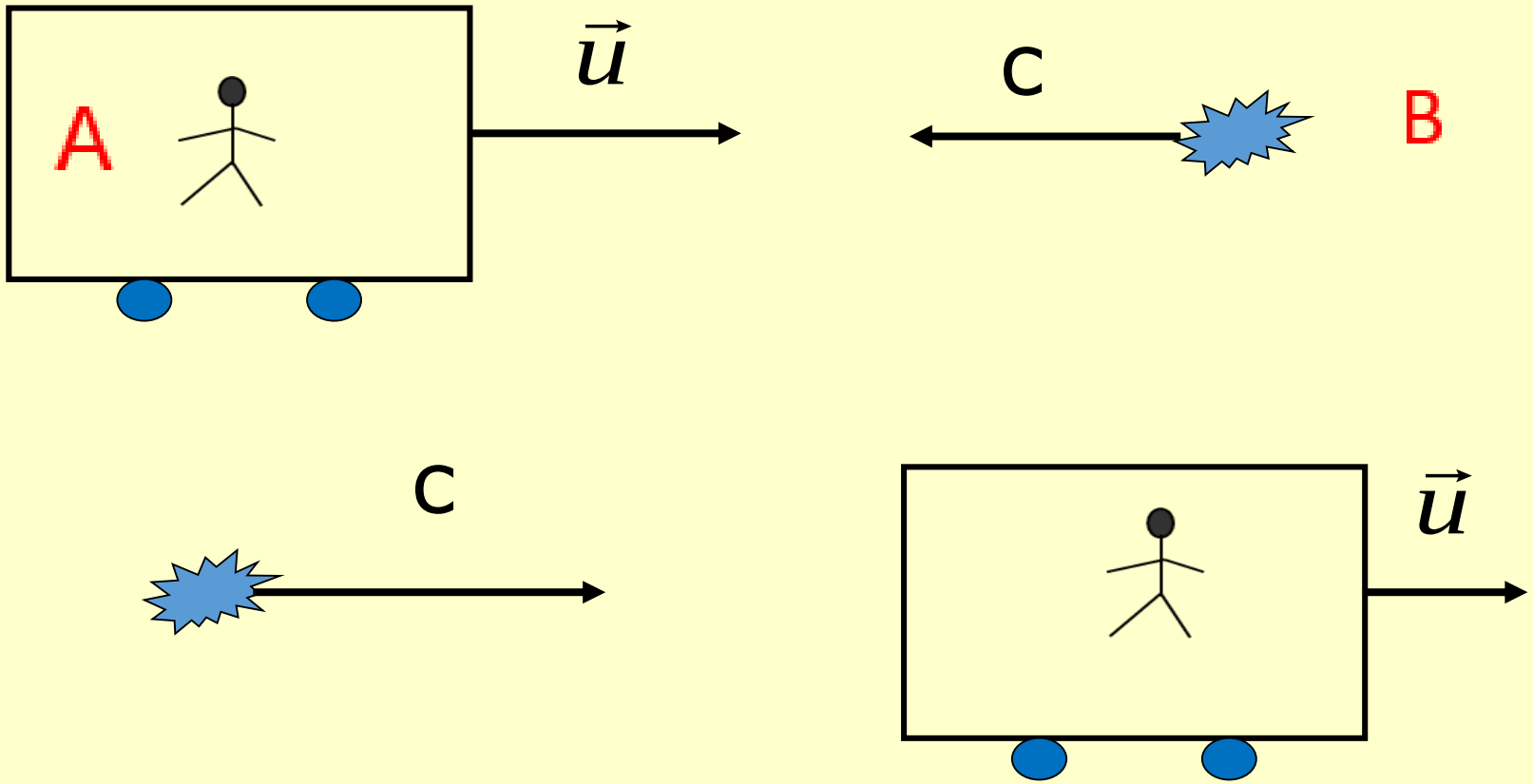
Why is this a problem ?



An observer sitting on *A* would notice the speed of *B* as *(v+u).*



Speed of light frame dependent?



B is stationary but is emitting light which is traveling with a speed of *c* in the same direction as the original direction of *B*. According to ***classical relative velocity formula*** the speed of light as seen by an observer on *A* would be *(c+v).*

But Wait a minute, if it is

If the following expression for the speed of light is valid in all frames of reference.

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

Fundamental constants ε_o and μ_o become frame dependent?

That means basic electric and magnetic forces are also frame dependent

The other option

- The following expression valid only in some specific frame?

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

The Second Option implies

- All the inertial frames, though equivalent from mechanical point of view are not equivalent from electromagnetic point of view.
- A special inertial frame may then exist, which can be identified as absolute rest..

Tough choice

Do you allow fundamental constants to become frame dependent?

Or

You allow a special inertial frame?

Ideally None?

But if there is no other choice given, one is likely to prefer the lesser evil, which is the second choice.

Ether

- Earlier ideas favored the concept of a special frame.
- It was imagined that the universe is filled with ocean of ether.
- All planets, stars, galaxies float in ether.

The speed of light is ‘c’ only in this medium as given by the expression involving fundamental constants.

In other frames the speed of light could be different from ‘c’. One can determine the speed of the frame by measuring the speed of light in that frame

Light needs this medium to travel.

Ether can be thought as signifying absolute rest.

Going by the speed of sound in various media (eg: 330 m/s in air) , the ether should be extremely rigid to support such a large value for c, yet allowing the earth and other objects to move freely.

Does it make sense ??

That nature has made mechanical processes equivalent in inertial frames but not the electromagnetic processes?

Does it make sense ??

The situation then seems to be as follows.* The fact that the Galilean relativity principle *does* apply to the Newtonian laws of mechanics but *not* to Maxwell's laws of electromagnetism requires us to choose the correct consequences from amongst the following possibilities.

MOTIVATION TO LOOK FOR ETHER

1. A relativity principle exists for mechanics, but *not* for electrodynamics; in electrodynamics there *is* a preferred inertial frame; that is, the ether frame. Should this alternative be correct the Galilean transformations would apply and we would be able to locate the ether frame experimentally.

2. A relativity principle exists *both* for mechanics and for electrodynamics, but the laws of electrodynamics as given by Maxwell are *not* correct. If this alternative were correct, we ought to be able to perform experiments that show deviations from Maxwell's electrodynamics and reformulate the electromagnetic laws. The Galilean transformations would apply here also.

3. A relativity principle exists *both* for mechanics and for electrodynamics, but the laws of mechanics as given by Newton are *not* correct. If this alternative is the correct one, we should be able to perform experiments which show deviations from Newtonian mechanics and reformulate the mechanical laws. In that event, the correct transformation laws would not be the Galilean ones (for they are inconsistent with the invariance of Maxwell's equations) but some other ones which are consistent with classical electromagnetism and the new mechanics.

Newtonian mechanics fails when applied to high-speed particles

It needs to be generalized to yield correct results at high speeds while still maintaining its excellent agreement with experiment at low speeds.

REITERATE

The Need for Aether

- The wave nature of light seemed to require a propagation medium. It was called the **luminiferous ether** or just **ether (or aether)**.

  – Aether had to have such a low density that the planets could move through it without loss of energy.

  – It had to have an elasticity to support the high velocity of light waves.

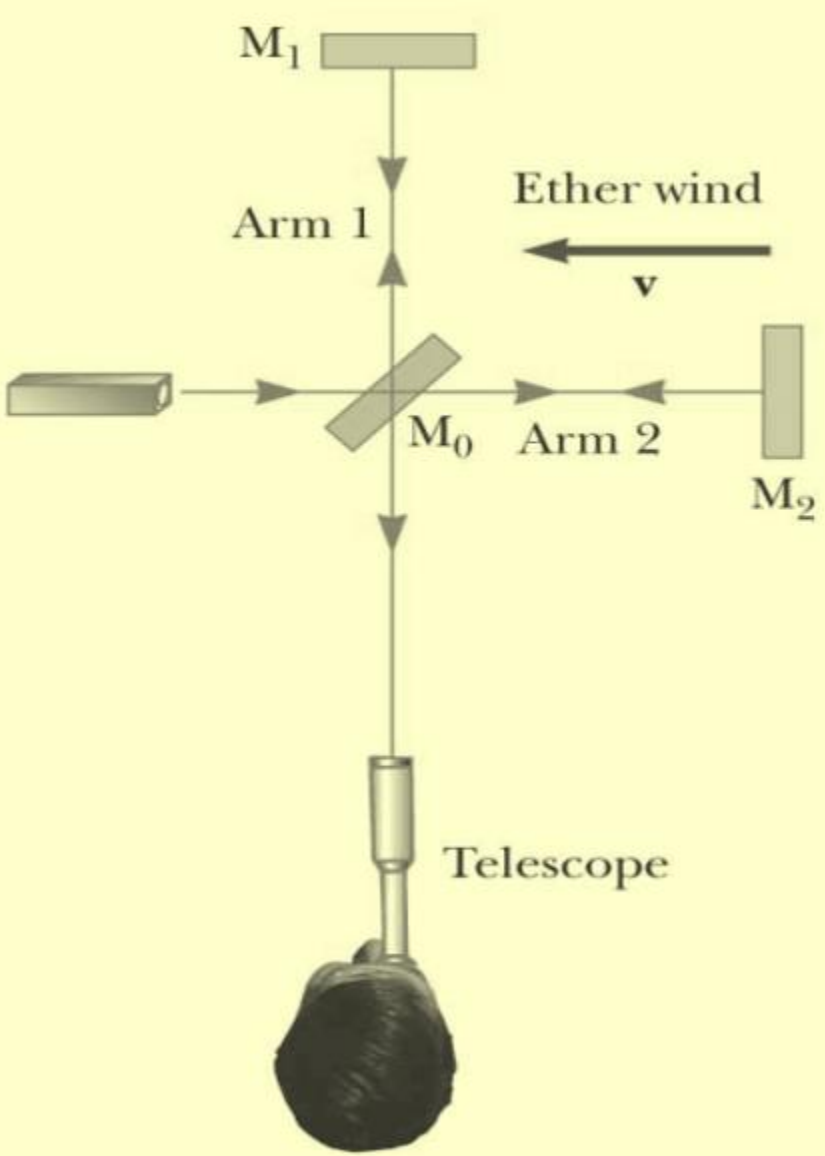
By this point the mechanical qualities of the aether had become more and more magical: it had to be a fluid in order to fill space, but one that was millions of times more rigid than steel in order to support the high frequencies of light waves. It also had to be massless and without viscosity, otherwise it would visibly affect the orbits of planets. Additionally it appeared it had to be completely transparent, non-dispersive, incompressible, and continuous at a very small scale. Maxwell wrote in *Encyclopædia Britannica*:

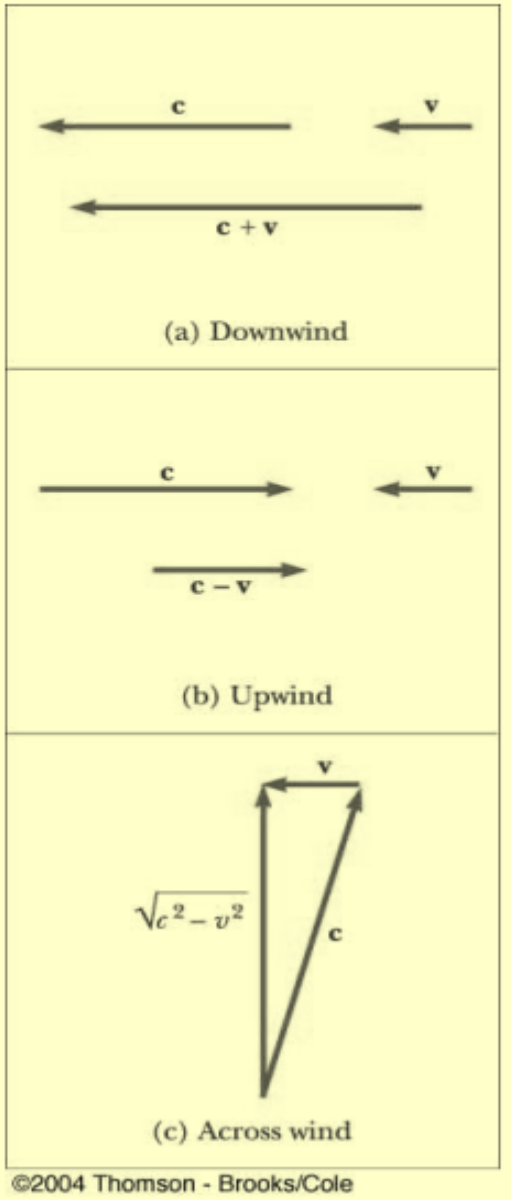
The Michelson-Morley Experiment

- If light propagates through the ether, then the Earth also passes through it
  - Like a ship creating a wind on deck, some direction is into the wind, and another is across the wind
  - Try to measure the difference in the speed of light in two different directions
  - Since the speed of the Earth in its orbit is about 3×10^4 m/s, need to measure fractional differences in the speed of light of about 10^{-4}!
  - Have to use an interferometer!

Ether was modeled as a fluid. When a body such as the Earth moved through the ether, it was hypothesized, that it should create a wind called the “ether wind” (Michelson & Morley 1887).

The Michelson-Morley Experiment

- 



First consider the beam traveling parallel to the direction of the ether wind

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2Lc}{c^2-v^2} = \frac{2L}{c}\left(1-\frac{v^2}{c^2}\right)^{-1}$$

Now consider the light beam traveling perpendicular to the wind,

$$t_2 = \frac{2L}{(c^2-v^2)^{1/2}} = \frac{2L}{c}\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

Thus, the time difference between the light beam traveling horizontally and the beam traveling vertically is

$$\Delta t = t_1 - t_2 = \frac{2L}{c}\left[\left(1 - \frac{v^2}{c^2}\right)^{-1} - \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\right]$$

Because $v^2/c^2 \ll 1$, this expression can be simplified by using the following binomial expansion after dropping all terms higher than second order:

$$(1 - x)^n \approx 1 - nx \quad (\text{for } x \ll 1)$$

In our case, $x = v^2/c^2$, and we find

$$\Delta t = t_1 - t_2 \approx \frac{Lv^2}{c^3}$$

The two light beams start out in phase and return to form an interference pattern. Let us assume that the interferometer is adjusted for parallel fringes and that a telescope is focused on one of these fringes. The time difference between the two light beams gives rise to a phase difference between the beams, producing the interference fringe pattern when they combine at the position of the telescope.

By rotating the interferometer through 90 degrees
results in a net time difference of 2 X $\frac{Lv^2}{c^3}$

The path difference corresponding to this time difference

$$\Delta d = c(2\Delta t) = \frac{2Lv^2}{c^2}$$

The corresponding fringe shift is equal to this path difference divided by the wavelength of light, λ, because a change in path of 1 wavelength corresponds to a shift of 1 fringe.

$$\text{Shift} = \frac{2Lv^2}{\lambda c^2}$$

In the experiments by Michelson and Morley, each light beam was reflected by mirrors many times to give an increased effective path length L of about 11 m. Using this value, and taking v to be equal to 3×10^4 m/s, the speed of the Earth about the Sun, gives a path difference of

$$\Delta d = \frac{2(11\text{ m})(3 \times 10^4\text{ m/s})^2}{(3 \times 10^8\text{ m/s})^2} = 2.2 \times 10^{-7}\text{ m}$$

This extra distance of travel should produce a noticeable shift in the fringe pattern. Specifically, using light of wavelength 500 nm, we find a fringe shift of

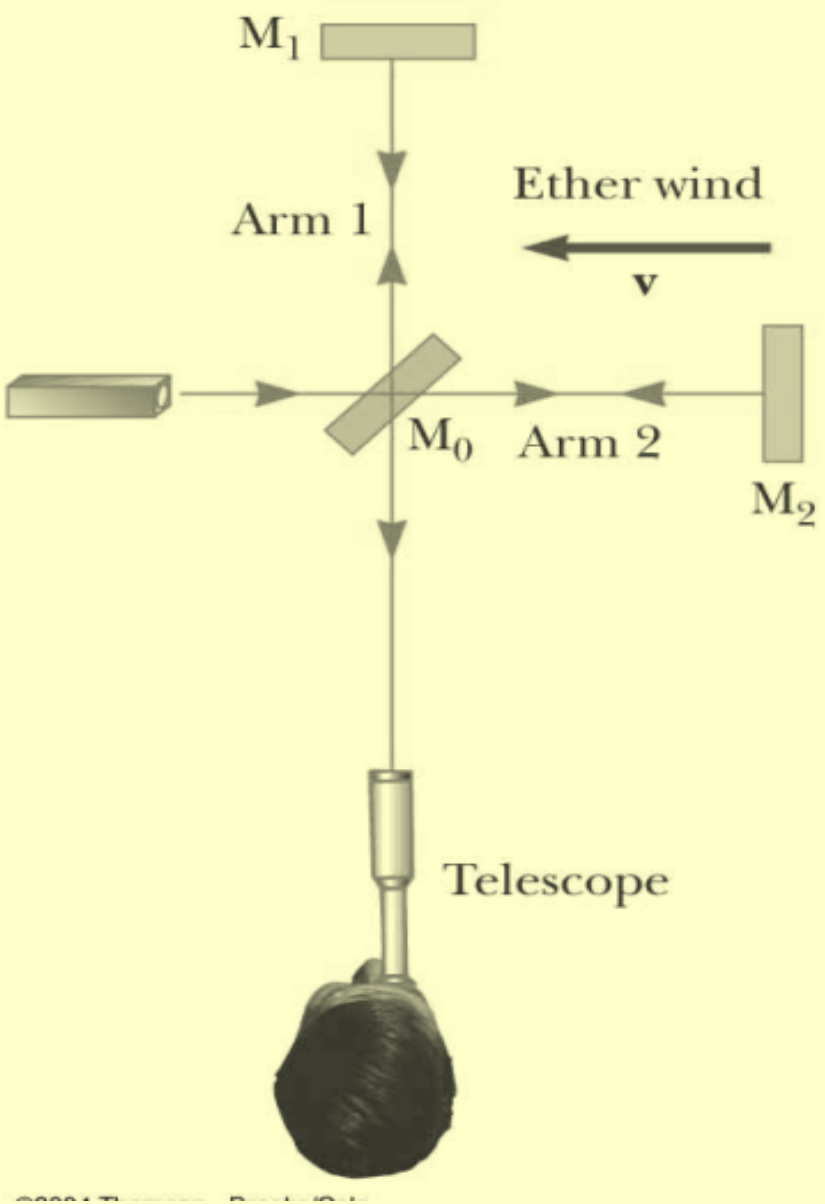
$$\text{Shift} = \frac{\Delta d}{\lambda} = \frac{2.2 \times 10^{-7}\text{ m}}{5.0 \times 10^{-7}\text{ m}} \approx 0.40 \quad \text{for } \frac{v}{c} \approx 10^{-4}$$

NO SHIFT was observed

The precision instrument designed by Michelson and Morley had the capability of detecting a shift in the fringe pattern as small as 0.01 fringe. However, *they detected no shift in the fringe pattern*. Since then, the experiment has been repeated many times by various scientists under various conditions, and no fringe shift has ever been detected. Thus, it was concluded that one cannot detect the motion of the Earth with respect to the ether.

The Michelson-Morley Experiment

- Michelson and Morley did this experiment, but saw absolutely no shift in the fringes!
- Have to add a postulate to Galilean Relativity



Michelson's Conclusion

- In several repeats and refinements with assistance from Edward Morley, he always saw *a null result.*
- He concluded that the hypothesis of the stationary aether must be incorrect.
- ***Thus, aether seems not to exist!***

Albert Michelson
(1852-1931)

Edward Morley
(1838-1923)

Another way to interpret the null result of MM experiment is this that we simply conclude that the speed of light, i.e. c, is the same in all directions and in all the inertial frames.