MA 108 Differential Equations-I,
End Semester Examination : June 28, 2022

9.30 am - 11.30 am Total marks: 35

Name :	*RollNo.* :
Division and Tutorial :	Invig. sign :

General Instructions: Question paper (QP) has two parts. Part A has 7 questions of one mark each. You should write only the final answer in the space provided in the QP itself. Part B has 7 subjective type questions of 4 marks each. Answers without proper justification carry no marks for Part B questions. You should write the answers of Part B in the answer booklet provided.

PART A

1. First order ODE, which has tangents to the curve $y = \sin 2x, -\frac{\pi}{4} < x < \frac{\pi}{4}$ as a family of solutions is, given by

$y = xy' + \sin\left(\cos^{-1}\left(\frac{y'}{2}\right)\right) - \frac{y'}{2}\cos^{-1}\left(\frac{y'}{2}\right)$
or $y = xy' + \sqrt{1 - \frac{(y')^2}{4}} - \frac{y'}{2}\cos^{-1}(y'/2)$

2. Let $y = y(x), x \in \mathbb{R}$ be the solution of the initial value problem $y' + y = 1 + x$, $y(0) = a$. The value of a for which $y = y(x), x \in \mathbb{R}$ touches but does not cross the x-axis is $1/e$

3. Consider the ODE $(5x + 4y^2)\,dx + 2xy\,dy = 0$. Value of n for which $\mu(x) = x^n$ is an integrating factor for the ODE is 3

4. Value of α such that the family of curves $y = cx^2 + 2\alpha, c \in \mathbb{R}$ are orthogonal to the family of ellipses $x^2 + 2y^2 = y + c, c + \frac{1}{8} > 0$ is $1/8$

5. Let y_1, y_2 be two linearly independent solutions of a second order linear homogeneous ODE with $W(y_1, y_2)(x) = e^x(x^2 - x + 1), x \in \mathbb{R}$. If $y_1(x) = x, x \in \mathbb{R}$, then the ODE is given by

CANCELLED due to some ambiguity

6. A particular solution of $x^3y''' - x^2y'' + xy' - y = \ln x, x > 0$ is

$y_p(x) = -4 - \ln x$

7. Let $f : [0, \infty) \to \mathbb{R}$ be the left continuous function which is an inverse Laplace transform of $(e^{-\frac{\pi s}{2\omega}} - e^{-\frac{3\pi s}{2\omega}})\dfrac{s}{s^2+\omega^2}$, $s > 0$. Then f is

$$\boxed{f(t) = \begin{cases} 0 & 0 \leq t \leq \pi/2\omega \\ \sin \omega t & \pi/2\omega < t \leq 3\pi/2\omega \\ 2\sin\omega t & t \geq 3\pi/2\omega \end{cases}}$$

PART B

8. Find the general solution of $(x+1)^2y'' + (x+1)y' - y = 2\ln(x+1) + x - 1$. [4]

9. Let p, q be continuous functions defined on $\mathbb{R}$ such that $p(x) \neq 0$ for all $x \in \mathbb{R}$. Also let y_1, y_2 be linearly independent solutions of the ODE

$$q(x)y'' + p(x)y' + 2p(x)y = 0$$

satisfying $y_1''(x_0) = y_2''(x_0) = 0$ for some $x_0 \in \mathbb{R}$. Show that $q(x_0) = 0$. [4]

10. Let p be a continuous function defined on $\mathbb{R}$ satisfying $p(x) \leq 0$ for $x \geq 0$. Consider the ODE

$$y'' + (p(x) - 3)y' - 3p(x)y \;=\; 0, x \geq 0.$$

Show that the ODE has a linearly independent set $\mathcal{S}$ of solutions such that $\mathcal{S}$ has two elements and both elements of $\mathcal{S}$ are convex functions on $(0, \infty)$. [4]

11. Using the method of variation of parameters, solve

$(x^2 + x)y'' + (2 - x^2)y' - (2 + x)y = x(x+1)^2, x > 0.$ [4]

12. Find the general solution of $y'' - 5y' + 4y = (3x+2)e^{-2x}, x \in \mathbb{R}$. [4]

13. Using Laplace transform technique, solve the initial value problem

$$y'' + y \;=\; \begin{cases} \sin t & 0 \leq t < \pi \\ 0 & t \geq \pi, \end{cases}$$

$y(0) = y'(0) = 0.$ [4]

14. Using Laplace transform technique, solve the ODE

$$ty'' + (1-t)y' + ny = 0, t \geq 0,$$

where n is a positive integer. [4]