PH111: Tutorial Sheet 4

This tutorial sheet contains problems related to vector nature of angular velocity, non-inertial frames of reference, and pseudo forces.

1. A particle is rotating in the xy-plane, along a circular path in counter-clockwise direction, with angular speed ω, about the z-axis.

   (a) Write down the angular velocity of the particle in the vector form, i.e., in terms of components and unit vectors.

   (b) If the particle is moving along a circle of radius a, write down its position vector $\mathbf{r}(t)$, as a function of time, assuming that $\mathbf{r}(0) = a\hat{\mathbf{i}}$

   (c) Express its velocity both in Cartesian, and plane polar coordinates

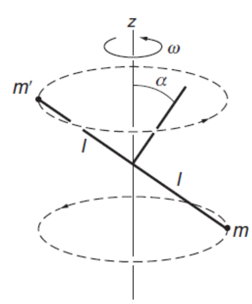
   (d) Compute the acceleration of the particle both in Cartesian, and plane polar coordinates

2. A vector $\mathbf{A}$ of magnitude a is rotating in the yz plane in a counter-clockwise manner, with a uniform angular velocity ω. It is given that $\mathbf{A}(t = 0) = a\hat{\mathbf{j}}$.

   (a) Obtain $\mathbf{A}(t)$, as a function of time.

   (b) Show that $\frac{d\mathbf{A}}{dt}$ calculated directly, and computed using $\boldsymbol{\omega} \times \mathbf{A}$, are the same.

3. Consider a simple rigid body consisting of two particles of mass m separated by a massless rod of length $2l$. The midpoint of the rod is attached to a vertical axis that rotates at angular speed ω around the z axis. The rod is skewed at angle α, as shown in the figure.



   (a) Calculate the angular momentum $\mathbf{L}(t)$ of the system, in Cartesian coordinates.

   (b) Verify that $\frac{d\mathbf{L}}{dt}$ is same as $\boldsymbol{\omega} \times \mathbf{L}$.

4. A cylinder of mass M and radius R rolls without slipping on a plank which is moving with an acceleration $\mathbf{A}$. Calculate the acceleration of the cylinder by analyzing the problem both in the inertial frame and the non-inertial frames. You can use the fact that moment of inertial of a cylinder about its axis is $\frac{1}{2}MR^2$.

5. A bead of mass m slides without friction on a horizontal rigid wire rotating at constant angular speed ω about the z axis.

(a) Find the distance of the bead from the axis of rotation $r(t)$, as a function of time given that $r(0) = 0$, and $\dot{r}(0) = v_0$.

(b) What is the force exerted on the bead by the wire.