Tut 2 - part 2 (Use phone for this)
Tuesday, 16 May 2023 11:24 PM

Fourier transform

1. * If $\phi(k) = A(a - |k|), |k| \leq a$, and 0 elsewhere. Where a is a positive parameter and A is a normalization factor to be found.
(a) Find the Fourier transform for $\phi(k)$
(b) Calculate the uncertainties Δx and Δp and check whether they satisfy the uncertainty principle.

Boundary condn ⇒ Normalizatn (prob of particle in whole space is 1)

a1 $$\phi(k) = \begin{cases} A(a - |k|) & -a < k < a \\ 0 & \text{elsewhere} \end{cases}$$

a) Open in phone and explain first ques

b) $\Delta x = \frac{4\pi}{a}$ plot graph

$\frac{1-\cos x}{x^2} \Rightarrow 1$ at $x = 0$

$p = \hbar k \Rightarrow \Delta p = \hbar \Delta k$, $\Delta k = 2a$

$\Delta x \Delta k = \frac{\Delta x \Delta p}{\hbar} = 8\pi$

2. A wave packet is of the form $f(x) = \cos^2\left(\frac{x}{2}\right)$ (for $-\pi \leq x \leq \pi$) and $f(x) = 0$ elsewhere
(a) Plot $f(x)$ versus x.
(b) Calculate the Fourier transform of $f(x)$, i.e. $g(k) = \int_{-\infty}^{+\infty} f(x)e^{-ikx}dx$?
(c) At what value of k, $|g(k)|$ attains its maximum value?
(d) Calculate the value(s) of k where the function $g(k)$ has its first zero.
(e) Considering the first zero(s) of both the functions $f(x)$ and $g(k)$ to define their spreads (i.e. Δx and Δk), calculate the uncertainty product $\Delta x . \Delta k$.

4. A wave packet is of the form $f(x) = \exp(-\alpha|x|) \cdot \exp(ik_0 x)$ (for $-\infty \leq x \leq \infty$) where α, k_0 are positive constants.
(a) Plot $|f(x)|$ versus x.
(b) At what values of x does $|f(x)|$ attain half of its maximum value? Consider the full width at half maxima (FWHM) as a measure of the spread (uncertainty) in x, find Δx
(c) Calculate the Fourier transform of $f(x)$, i.e. $g(k) = \int_{-\infty}^{+\infty} f(x)e^{ikx}dx$
(d) Plot $g(k)$ versus k.
(e) Find the values of k at which $g(k)$ attains half its maximum value? Using the same concept of FWHM as in part (b), calculate Δk ? Hence calculate the product $\Delta x . \Delta k$
[Given : $\int_0^{\infty} e^{-(\alpha - ik)x}dx = \frac{1}{\alpha - ik}$]

Q4

$$\int_0^{\infty} e^{-(\alpha - ik)x}\, dx = \frac{1}{\alpha - ik}$$

$$\int_0^{-\infty} e^{-(\alpha - ik)(-t)}\, -dt$$