Tut 1 - part 2
Monday, 8 May 2023   2:08 AM

de Broglie wavelength

General formulae ⇒ $\lambda = \frac{hc}{E\,(\text{in J})}$ (from $E = h\nu$) $= \frac{12400}{E\,(\text{in eV})}$ Å $= \frac{1240}{E\,(\text{eV})}$ nm

$\lambda_{dB} = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\sqrt{2mEc^2}} = \frac{12400}{\sqrt{2(\text{rest mass E})(E \text{ in eV})}}$ Å

rest mass, KE of particle.

1. Calculate the wavelength of the matter waves associated with the following:

(a) A 2000 kg car moving with a speed of 100 km/h.
(b) A 0.28 kg cricket ball moving with a speed of 40 m/s.
(c) An electron moving with a speed of 10^7 m/s.

Compare in each case the result with the respective dimension of the object. In which case will it be possible to observe the wave nature.

Q1 Easy. can do. No dbts I suppose.
Ask ques about possibility to observe wave nature

a) b) ⇒ $\lambda = h/p$

c) ⇒ $e^{\ominus}$ ⇒ relativistic momentum

$\lambda = \frac{h}{\gamma m v}$ $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, $v \simeq c$ ⇒ relativistic momentum, $\frac{v}{c} = \frac{1}{30}$

Wave nature will be observed only when dimensn of the object is comparable to the wavelength

M1 size of $e^{\ominus}$ ⇒ 10 fm $[10^{-14}\text{ m}]$

$\lambda = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{7}} \times \frac{\sqrt{899}}{30} = 728.6$ Å

M2 $\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\frac{mvc}{\sqrt{1-v^2/c^2}}} = \frac{hc}{\frac{mc^2\beta}{\sqrt{1-\beta^2}}} = \frac{12400 \text{ Å} \sqrt{1-1/900}}{(0.511 \text{ MeV})(1/30)}$

rest mass E

$= 728.8$ Å

2. Show that the Bohr's angular momentum quantization leads to the formation of standing waves by the electrons along the orbital circumference in hydrogen atom.

Q2 Bohr's angular momentum quantisatn condn

$mvr = \frac{nh}{2\pi} = n\hbar$

⇒ $2\pi r = \frac{nh}{p}$ ⇒ $\boxed{2\pi r = n\lambda}$ standing waves

integer multiple of wavelength ⇒ standing waves

You may be wondering why the circumference has to be $n\lambda$ and not $n\lambda/2$. The problem with the odd multiples of $\lambda/2$ is that it causes "*destructive interference*". For stability, the "*wave function*" of the electron must match itself after completing a 2π cycle. Don't worry if you are unable to understand the above statement right now because you will learn stuff like this in detail in your quantum chemistry course!

3. Determine the de Broglie wavelength of a particle of mass m and kinetic energy K. Do this for both (a) a relativistic and (b) a non-relativistic particle.

Q3 (a). relativistic particle

For a relativistic particle:

$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$, $K = \frac{1}{2}mv^2$ ⇒ $v = \sqrt{\frac{2K}{m}}$

(b) non relativistic particle:

Simply, $K = \frac{p^2}{2m}$

⇒ $p = \sqrt{2mK}$

$\lambda_{dB} = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

4. *Thermal kinetic energy of a hydrogen atom is $\sim k_BT$ and the radius is $\sim r_1$ (= 0.53 Å, radius of the $n = 1$ Bohr orbit). Find the temperature at which its de Broglie wavelength has a value of $2r_1$. Take the mass of the hydrogen atom to be that of a proton.

Q4 $\lambda = 2 \times 0.53$ Å $= 1.06$ Å

M1 $\lambda = \frac{h}{p}$, $KE = K_BT = \frac{p^2}{2m}$

$K_BT = \frac{(h/\lambda)^2}{2m_p}$ proton mass

$K_B = R/N_A$, $R = 8.314$ J
$N_A = 6.023 \times 10^{23}$

⇒ $T \simeq 850$ K

M2 $\lambda = \frac{h}{\sqrt{2m(KE)}}$ ⇒ $KE = K_BT$