PH111: Tutorial Sheet 3

This tutorial sheet contains problems related to work-energy theorem, conservative force, and potential energy.

1. Mass m rotates on a frictionless table, held to circular path by a string which passes through a hole in the table. The string is slowly pulled through the hole so that the radius of the circle changes from R_0 to R_1. Show that the work done in pulling the string equals the increase in kinetic energy of the mass.

2. A particle of mass m moves in one dimension along the positive x axis. It is acted on by a constant force directed towards the origin with the magnitude B, and and inverse law repulsive force of magnitude A/x^2.

   (a) Find the potential energy function $V(x)$

   (b) Plot the potential energy as a function of x, and the total energy of the system, assuming that the maximum kinetic energy is $K_0 = \frac{1}{2}mv_0^2$.

   (c) What is the point of equilibrium, i.e., the point where net force acting on the particle is zero.

3. A particle of mass M is held fixed at the origin. The gravitational potential energy of another particle of m, in the field of the first mass, is given by

$$V(\mathbf{r}) = -\frac{GMm}{r},$$

   where G is the gravitational constant, and r is the distance of mass m from the origin.

   (a) What is the force acting on the particle of mass m?

   (b) Calculate the curl of this force.

4. Consider a 2D force field $\mathbf{F} = A(y^2\hat{\mathbf{i}} + 2x^2\hat{\mathbf{j}})$. Calculate the work done by this force in going around a closed path which is a square made up of sides of length a, lying in the xy-plane, with two of its vertices located at the origin, and point (a, a). Find the answer by doing the line integral, as well as by using the Stokes' theorem. The path is traversed in a counter clockwise manner.

5. Find the forces for the following potential energies

   (a) $V(x, y, z) = Ax^2 + By^2 + Cz^2$

   (b) $V(x, y, z) = A\ln(x^2 + y^2 + z^2)$

   (c) $V(r, \theta) = A\cos\theta/r^2$ (r and θ are plane polar coordinates)

   Above, A, B, and, C are constants.

6. Determine whether each of the following forces is conservative. Find the potential energy function if it exists. A, α, β are constants.

(a) $\mathbf{F} = A(3\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

(b) $\mathbf{F} = Axyz(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

(c) $F_x = A\sin(\alpha y)\cos(\beta z)$, $F_y = -Ax\alpha\cos(\alpha y)\cos(\beta z)$, $F_z = Ax\sin(\alpha y)\sin(\beta z)$