PH111: Tutorial Sheet 5

This tutorial sheet contains problems related to the central force motion

1. In the lectures, we argued that the effective potential for the central force problem is
$$V_{eff}(r) = \frac{L^2}{2\mu r^2} + V(r),$$
where $V(r)$ is the potential energy corresponding to the central force, and L is the angular momentum. Consider the case of gravitational motion so that $V(r) = -\frac{C}{r}$, with $C > 0$. Plot the effective potential as a function of r, and argue based upon the plot that for $E \geq 0$, orbits will be unbound, while for $E < 0$, we will obtain bound orbits, where E is the total energy of the system.

2. Suppose a satellite is moving around a planet in a circular orbit of radius r_0. Due to a collision with another object, satellite's orbit gets perturbed. Show that the radial position of the satellite will execute simple harmonic motion with $\omega = \frac{L}{mr_0^2}$, where L is the initial angular momentum of the satellite.

3. In this problem we will explore an alternative way of obtaining the equation of the curve corresponding to the central force orbits.

   (a) Make a change of variable $u = \frac{1}{r}$ and show that the $u - \theta$ differential equation for a central force $\mathbf{F}(\mathbf{r}) = f(r)\hat{\mathbf{r}}$ is
   $$\frac{d^2u}{d\theta^2} + u = -\frac{\mu}{u^2L^2}f(\frac{1}{u})$$

   (b) Integrate this differential equation for the case of gravitational force ($f(r) = -\frac{C}{r^2}$), and show that it leads to the same orbit as obtained in the lectures
   $$r = \frac{r_0}{1 - \epsilon\cos\theta}$$

4. A particle of mass m is moving under the influence of a central force $\mathbf{F}(\mathbf{r}) = -\frac{C}{r^3}\hat{\mathbf{r}}$, with $C > 0$. Find the nonzero values of angular momentum L for which the particle will move in a circular orbit.

5. A geostationary orbit is one in which a satellite moves in a circular orbit at the given height in the equatorial plane, so that its angular velocity of rotation around earth is same as earth's angular velocity, thereby, making it look stationary when seen from a point on equator. Assuming that the earth's rotational velocity, and radius, respectively, are $\Omega_e = \frac{2\pi}{86400}$ rad/s, and$R_e = 6400$ km, calculate the altitude of the satellite, and its orbital velocity.

6. A space company wants to launch a satellite of mass $m = 2000$ kg, in an elliptical orbit around earth, so that the altitude of the satellite above earth at perigee is 1100 kms, and at apogee it is 35,850 kms. Assuming that the launch takes place at the equator,

calculate: (a) energy of the satellite in the elliptical orbit, (b) energy required to launch the satellite, (c) eccentricity of the orbit, (d) angular momentum of the satellite, and (e) speeds of the satellite at apogee and perigee. Use the values of R_e and Ω_e specified in the previous problem.

7. The ultimate aim of the space company of the previous problem is to put the satellite in a geostationary orbit. Therefore, after launching it in the elliptical orbit, the company wants to transfer it in a geostationary orbit by firing rockets at the apogee to increase its speed to the required one. How much change in speed is needed to put the satellite in the geostationary orbit, and how much energy will be required to achieve that change?