Tut-3 Part-2
Tuesday, 23 May 2023 10:56 AM

Heisenberg's uncertanity principle

$\Delta y = \sqrt{\langle y^2 \rangle - \langle y \rangle^2}$ usually $\langle y \rangle = 0$ (expectatn value) $\Rightarrow \Delta y = \sqrt{\langle y^2 \rangle}$

Δy ↓ real measurable quantity

eg) $e^{\ominus}$ moving in a box
$\langle v \rangle = 0$

Q1

1. Estimate the uncertainty in the position of (a) a neutron moving at 5×10^6 m s^{-1} and (b) a 50 kg person moving at 2 m s^{-1}.

$\Delta x \, \Delta p \geq \frac{h}{4\pi}$, $\frac{h}{p} = \lambda$, $\Delta x = 6 : \frac{\lambda}{2}$

2. A lead nucleus has a radius 7×10^{-15} m. Consider a proton bound within nucleus. Using the uncertainty relation $\Delta p . \Delta r \geq \hbar/2$, estimate the root mean square speed of the proton, assuming it to be non-relativistic. (You can assume that the average value of p^2 is square of the uncertainty in momentum.)

Q2 proton is bound in nucleus of rad $\Rightarrow 7 \times 10^{-15}$ m

$\therefore \Delta x = r$

$\Delta x \, \Delta p \geq \hbar/2$ and we have to find $\sqrt{\langle v^2 \rangle}$ ↳ RMS vel

imp Since proton is effectively bound in a sphere $\langle p \rangle = 0$ due to velocities randomly in all direc^{ns}

$\Delta p^2 = \langle p^2 \rangle - \langle p \rangle^2$

$\Delta p = \sqrt{\langle p^2 \rangle} = \frac{\hbar}{2\Delta x} = 7.5416 \times 10^{-21}$

For rms vel, divide by m_p

$\sqrt{\langle v^2 \rangle} = \frac{7.5416 \times 10^{-21}}{1.67 \times 10^{-27}} = 4.5159 \times 10^{6}$ m/s

3. * A π^0 meson is an unstable particle produced in highenergy particle collisions. It has a mass-energy equivalent of about 135MeV, and it exists for an average lifetime of only 8.7×10^{-17} s before decaying into two γ rays. Using the uncertainty principle, estimate the fractional uncertainty $\Delta m/m$ in its mass determination.

Q3 $m_0 c^2 = 135$ MeV , $\Delta t = 8.7 \times 10^{-17}$

By uncertainty principle : $\Delta E \, \Delta t = \frac{h}{4\pi}$

$\Delta E = \frac{h}{4\pi \Delta t}$

$\therefore \frac{\Delta m}{m} = \frac{\Delta m c^2}{m c^2}$

$= \frac{\Delta E}{E} = \frac{3.79 \times 10^{-6}}{135} = 2.8 \times 10^{-8}$

4. * For a non-relativistic electron, using the uncertainty relation $\Delta x \Delta p_x = \hbar/2$
(a) Derive the expression for the minimum kinetic energy of the electron localized in a region of size ' a '.
(b) If the uncertainty in the location of a particle is equal to its de Broglie wavelength, show that the uncertainty in the measurement of its velocity is same as the particle velocity. upto a constant factor
(c) Using the expression in (b), calculate the uncertainty in the velocity of an electron having energy 0.2keV
(d) An electron of energy 0.2keV is passed through a circular hole of radius 10^{-6} m. What is the uncertainty introduced in the angle of emergence in radians? (Given $\tan\theta \cong \theta$)

Q4 Non relativistic $e^{\ominus}$

a) For localisat^n in a. $\Delta x = a/2$ (can also take a)

$KE = \frac{\Delta p_x^2}{2m} = \frac{\langle p_x^2 \rangle}{2m}$, $KE_{min} = \frac{\hbar^2}{2ma^2}$

$\Delta x \, m \, \Delta v = \frac{\hbar}{2} \Rightarrow \Delta v = \frac{\hbar}{am}$

If $\Delta x = a$, $KE_{min} = \frac{\hbar^2}{8ma^2}$

b) If $\Delta x = \lambda$, then $\Delta v = v$

$\lambda \, \Delta p = \frac{\hbar}{2}$ and $\Delta p = \frac{h}{\lambda \times 4\pi}$

$\Delta v_x = \frac{v}{4\pi}$ → const factor

$\Delta v_x \approx v$, order analysis

c) Energy = 0.2 keV

$\Delta v_x = \frac{v}{4\pi}$, $\Delta v = \sqrt{\frac{2E}{m}} \times \frac{1}{4\pi} = \frac{1}{4\pi}\sqrt{\frac{2 \times 0.2 \text{ keV}}{\frac{511 \text{ keV}}{c^2}}} = 6.7 \times 10^5$ m/s

$e^{\ominus}$

d) $\tan\theta \simeq \theta$ [small angle approximat^n)

rad $= 10^{-6}$ m , uncertainty in angle of emergence

$\tan\theta = \frac{v_y}{v_x} = \theta$ → uncertainty due to slit diffract^n , given $\Delta y = 2 \times 10^{-6}$ m hole dia

$= \Delta p_y / \Delta p_x$

$\Delta y \, \Delta p_y = \hbar/2$

$\theta = \frac{(\hbar/2\Delta y)}{\sqrt{2Em}} = 9.3 \times 10^{-5}$ rad

5. An atom in an excited state 1.8eV above the ground state remains in that excited state 2.0μs before moving to the ground state. Find (a) the frequency of the emitted photon, (b) its wavelength, and (c) its approximate uncertainty in energy.

Q5 $\Delta E \, \Delta t \geq h/4\pi$

Just tell verbally

6. * An electron microscope is designed to resolve objects as small as 0.14 nm. What energy electrons must be used in this instrument?

Q6 To resolve objects smaller than 0.14 nm.

$\lambda < 0.14$ for $e^{\ominus}$ microscope.

Note : $e^{\ominus}$ microscopes are relativistic

$\therefore \lambda = 0.14$ nm $\Rightarrow \frac{h}{\lambda} = p \Rightarrow \frac{6.634 \times 10^{-34}}{0.14 \times 10^{-9}} = p$

Energy req by $e^{\ominus}$ = $\sqrt{p^2c^2 + m_0^2c^4} - m_0c^2$ ↳ 0.511 MeV

$= (8.8 \times 10^{-3} + 0.26)^{1/2} - 0.511$

7. * Show that the uncertainty principle can be expressed in the form $\Delta L \Delta\theta \geq \hbar/2$, where θ is the angle and L the angular momentum. For what uncertainty in L will the angular position of a particle be completely undetermined?

Q7 Proving only for a circular case

Assumptn ⇒ rad of circular path is precisely known

$L = rp \Rightarrow \Delta L = r\Delta p$, $x = r\theta$, $\Delta x = r\Delta\theta$

$\therefore \Delta p \, \Delta x = \frac{\Delta L}{\cancel{r}} \times \cancel{r} \Delta\theta \geq \frac{\hbar}{2}$

For completely undetermined posn, when uncertainty is total path $\Rightarrow \therefore \Delta\theta = 2\pi$

$\therefore \Delta L = \frac{\hbar}{4\pi}$