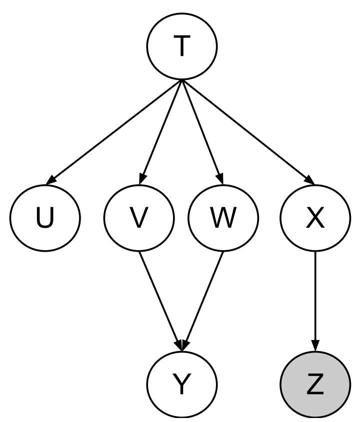
Name_1: ______________________

ID_1#: ______________________

Name_2: ______________________

ID_2#: ______________________

For the Baye's net below, we are given the query $P(Y| + z)$. All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering: X, T, U, V, W.



Complete the following description of the factors generated in this process: After inserting evidence, we have the following factors to start out with: $P(T), P(U|T), P(V|T), P(W|T), P(X|T)$, $P(Y|V, W), P(+z|X)$

1. When eliminating X we generate a new factor f_1 as follows:

2. This leaves us with the factors:

3. When eliminating T we generate a new factor f_2 as follows:

4. This leaves us with the factors:

5. When eliminating U we generate a new factor f_3 as follows:

6. This leaves us with the factors:

7. When eliminating V we generate a new factor f_4 as follows:

8. This leaves us with the factors:

9. When eliminating W we generate a new factor f_5 as follows:

10. This leaves us with the factors:

11. How would you obtain $P(Y| + z)$ from the factors left above:

12. What is the size of the largest factor that gets generated during the above process?

13. Does there exist a better elimination ordering (one which generates smaller largest factors)? If yes, which ordering?

(a) When eliminating X we generate a new factor f_1 as follows:

$$f_1(T, +z) = \sum_x P(x|T)P(+z|x)$$

(b) This leaves us with the factors:

$$P(T), P(U|T), P(V|T), P(W|T), P(Y|V, W), f_1(T, +z)$$

(c) When eliminating T we generate a new factor f_2 as follows:

$$f_2(U, V, W, +z) = \sum_t P(t)P(U|t)P(V|t)P(W|t)f_1(t, +z).$$

(d) This leaves us with the factors:

$$P(Y|V, W), f_2(U, V, W, +z)$$

(e) When eliminating U we generate a new factor f_3 as follows:

$$f_3(V, W, +z) = \sum_u f_2(u, V, W, +z)$$

(f) This leaves us with the factors:

$$P(Y|V, W), f_3(V, W, +z)$$

(g) When eliminating V we generate a new factor f_4 as follows:

$$f_4(W, Y, +z) = \sum_v f_3(v, W, +z)P(Y|v, W)$$

(h) This leaves us with the factors:

$$f_4(W, Y, +z)$$

(i) When eliminating W we generate a new factor f_5 as follows:

$$f_5(Y, +z) = \sum_w f_4(w, Y, +z)$$

(j) This leaves us with the factors:

$$f_5(Y, +z)$$

(k) How would you obtain $P(Y \mid +z)$ from the factors left above:
Simply renormalize $f_5(Y, +z)$ to obtain $P(Y \mid +z)$. Concretely,

$$P(y \mid +z) = \frac{f_5(y, +z)}{\sum_{y'} f_5(y', +z)}$$

(l) What is the size of the largest factor that gets generated during the above process?
$f_2(U, V, W, +z)$, of size 3.

(m) Does there exist a better elimination ordering (one which generates smaller largest factors)?
Yes, elimination ordering of X, U, T, V, W generates only factors of size 2.