usage: teamwork for week 5

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```
clear all; close all; clc
```

1. 证明方程 $e^x + 10x - 2 = 0$ 存在唯一实根，用二分法求此根，要求误差不超过 $\frac{1}{2} \times 10^{-2}$ 。

解：

设 $f(x) = e^x + 10x - 2$，使用matlab符号运算求导：

```
syms x
f_1(x) = exp(x) + 10*x - 2
```

f_1(x) = $10\,x + e^x - 2$

```
df(x) = diff(f_1(x))% the first-order derivative of f(x)
```

df(x) = $e^x + 10$

```
f_1 = @(x)(exp(x) + 10*x - 2)
```

```
f_1 = 包含以下值的 function_handle:
    @(x)(exp(x)+10*x-2)
```

```
greater_than_0 = f_1(1)% f(1) greater than 0
```

```
greater_than_0 =
  10.718281828459045
```

```
less_than_0 = f_1(0)% f(0) less than 0
```

```
less_than_0 =
    -1
```

得到 $\frac{df(x)}{dx} = e^x + 10$，易得 $(\forall x)(\frac{df(x)}{dx} > 0)$，即 $f(x)$ 在整个实数上单调增，且 $f(x)$ 在整个实数上连续，且由于 $f(1) > 0, f(0) < 0$，满足零点存在定理，故函数 $f(x)$ 存在唯一零点，即方程 $e^x + 10x - 2 = 0$ 存在唯一实根。

```
[x_1, ~] = dichotomy(f_1, 1, 0, 0.5e-2)
```

```
dichotomy:
the root of f(x)=0 is x = 0.089844, the number of iterations is 7
x_1 =
   0.089843750000000
```

由二分法得到近似解 $x \approx 0.0898$。

2. 用定点迭代法求解方程$x = e^{-x}$在$x = 0.5$附近的一个根， 要求：事先确定有根区间并判断迭代公式的收敛性，误差上限为$\epsilon = 10^{-5}$.

解：

```
sample = 0:0.01:1
```

```
sample = 1×101
                0   0.010000000000000   0.020000000000000   0.030000000000000 ···
```

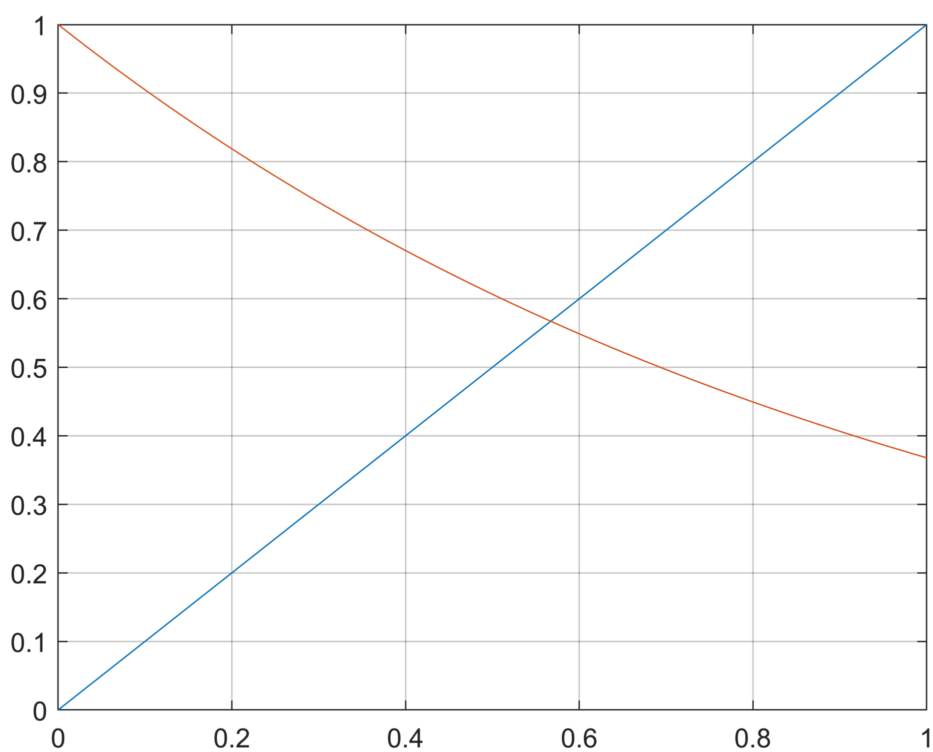
```
f_2_1 = @(x)x
```

```
f_2_1 = 包含以下值的 function_handle:
    @(x)x
```

```
f_2_2 = @(x)(exp(-x))
```

```
f_2_2 = 包含以下值的 function_handle:
    @(x)(exp(-x))
```

```
plot(sample, f_2_1(sample), sample, f_2_2(sample))
grid on
```



设$g(x) = e^{-x}$，求导得到$|g'(x)| = |-e^{-x}| = e^{-x}$，易得当$x > 0$时，满足$|g'(x)| < 1$，从图中容易看出，根在$(0, 1)$的区间内，且满足定点迭代公式的收敛性，故取$x = 0.5$为初始迭代点$x_0$。

```
format long
[x_2, ~] = fixedPoint(f_2_2, 0.5, 1e-5, 100000)
```

```
fixedPoint:
the root of g(x)=x is x = 0.567148, the number of iterations is 17
x_2 =
   0.567147746330625
```

由定点迭代法得到近似解$x \approx 0.56714$。

3. 用定点迭代法和定点迭代法的Aitken(埃特金)加速方法，计算$x = \ln(x+2)$在$x = 0.5$附近的近似根，要求$|x^* - x_k| < 10^{-4}$，并比较收敛的快慢。

解:

```
f_3 = @(x)log(x+2)
```

```
f_3 = 包含以下值的 function_handle:
    @(x)log(x+2)
```

```
[x_3_1, flag_3_1] = fixedPoint(f_3, 0.5, 1e-4, 10000)
```

```
fixedPoint:
the root of g(x)=x is x = 1.146114, the number of iterations is 8
x_3_1 =
   1.146113571453652
flag_3_1 =
     8
```

```
[x_3_2, flag_3_2] = fixedPointWithAitken(f_3, 0.5, 1e-4, 10000)
```

```
fixedPoint with Aitken:
the root of g(x)=x is x = 1.146198, the number of iterations is 6
x_3_2 =
   1.146198178947666
flag_3_2 =
     6
```

用定点迭代法和Aitken加速方法都得到近似解$x \approx 1.1461$，定点迭代法经过8次迭代得到结果，而Aitken加速方法仅经过6次迭代就得到了结果，Aitken加速方法收敛速度比普通的定点迭代法更快。

4. 割线法是跟牛顿法相似的数值方法，但它需要两个接近真实根的初值x_0和x_1，由此得到函数$f(x)$上两个点$(x_0, y_0 = f(x_0))$和$(x_1, y_1 = f(x_1))$，连接这两点得到一条直线，称之为割线:

$$y - y_1 = \frac{y_1 - y_0}{x_1 - x_0}(x - x_1)$$

我们用这条割线近似函数本身，因此设$y = 0$，由上式解出割线与x-轴的截点，作为下次迭代的初值。这个过程一直进行下去，利用如下迭代关系:

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{y_n - y_{n-1}} y_n$$

分别用割线法和牛顿法求解下列方程的根，比较两者的收敛速度:

(1) $xe^x - 1 = 0$;

(2) $\ln x + x - 2 = 0$.

解:

(1)设$f_1(x) = xe^x - 1$，易得$f'_1(x) = e^x + xe^x$

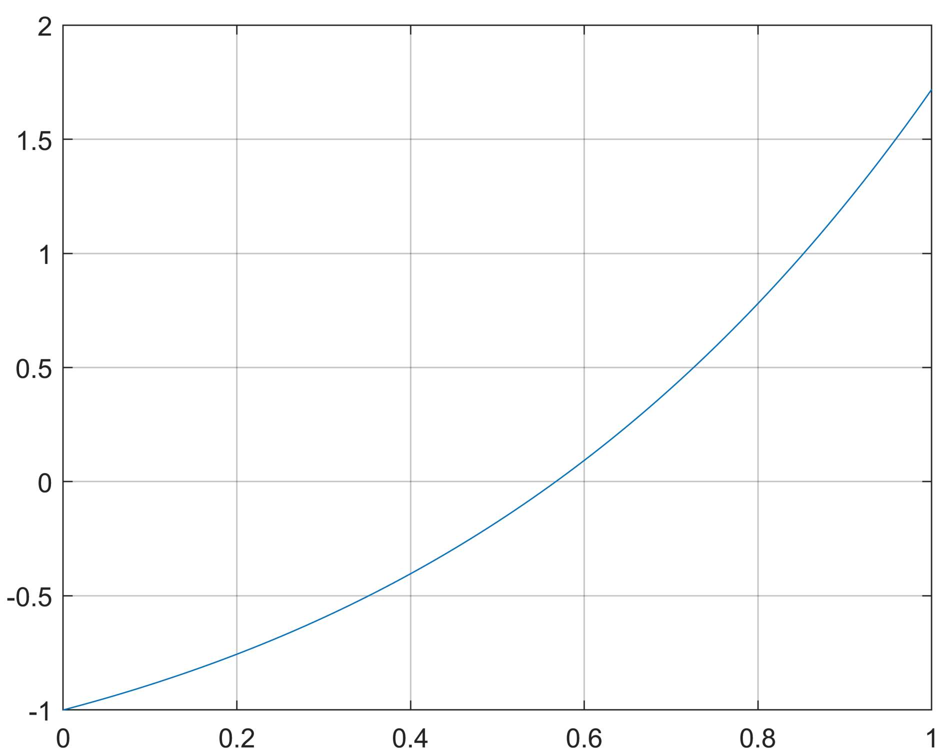
```
f_4_1 = @(x)(x.*exp(x) - 1)
```

```
f_4_1 = 包含以下值的 function_handle:
    @(x)(x.*exp(x)-1)
```

```
df_4_1 = @(x)(exp(x) + x.*exp(x))
```

```
df_4_1 = 包含以下值的 function_handle:
    @(x)(exp(x)+x.*exp(x))
```

```
sample1 = 0:0.01:1;
plot(sample1, f_4_1(sample1))
grid on
```



由图解法可以得到根在区间$(0, 1)$内，取0.5为牛顿-拉夫森方法的初始估计值，$x_0 = 0.5, x_1 = 0.6$为割线法的两个初值。

****

我们组写了两种牛顿-拉夫森方法，区别在于一种需要将导数作为参数传入，另一种在函数内部计算导数，前者考虑到存在函数无法使用***matlabFunction***函数转换为可计算函数，故给出参数传入接口；后者对于一些复杂函数方便了牛顿法的使用。

在之后的***Halley***函数同理，我们也分为了需要提供导数和内置导数计算两个版本。

****

```
[x_4_1_Netwon, flag_4_1_Netwon] = Netwon(f_4_1, df_4_1, 0.5, 1e-5, 10000, 1, 0)
```

```
Newton's method:
the root of f(x)=0 is x = 0.567143, the number of iterations is 4
x_4_1_Netwon =
   0.567143290409784
flag_4_1_Netwon =
     4
```

```
[x_4_1_Netwon_without_df, flag_4_1_Netwon_without_df] = Netwon_without_df(f_4_1, 0.5, 1e-5, 100
```

```
Newton's method without df:
the root of f(x)=0 is x = 0.567143, the number of iterations is 4
x_4_1_Netwon_without_df =
   0.567143290409784
flag_4_1_Netwon_without_df =
     4
```

```
[x_4_1_secant, flag_4_1_secant] = secant(f_4_1, 0.5, 0.6, 1e-5, 10000)
```

```
secant method:
the root of f(x)=0 is x = 0.567143, the number of iterations is 4
x_4_1_secant =
   0.567143290406878
flag_4_1_secant =
     4
```

对题（1）$xe^x - 1 = 0$，牛顿-拉夫森方法和割线法均得到近似解$x \approx 0.56714$，而且两者均为4次迭代，但事实上割线法是否收敛以及收敛速度也取决于两个初值的选择，无法直接判断两种方法的收敛速度。

(2)设$f_2(x) = ln(x) + x - 2$，易得$f'_2(x) = \frac{1}{x} + 1$

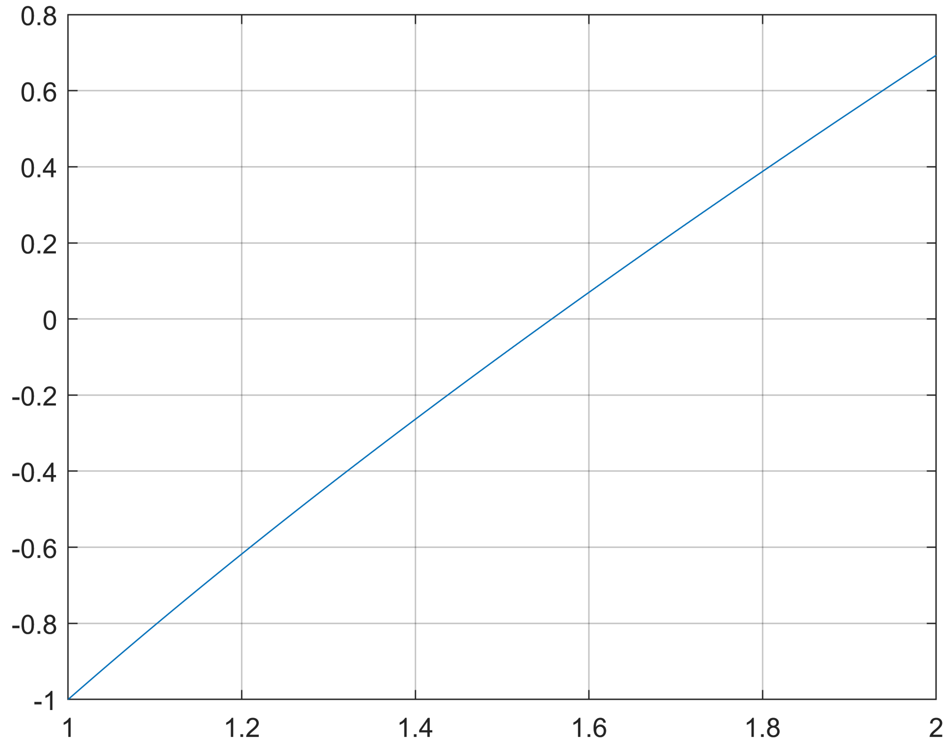
```
f_4_2 = @(x)(log(x)+x-2)
```

```
f_4_2 = 包含以下值的 function_handle:
    @(x)(log(x)+x-2)
```

```
df_4_2 = @(x)(ones(size(x))./x+1)
```

```
df_4_2 = 包含以下值的 function_handle:
    @(x)(ones(size(x))./x+1)
```

```
sample2 = 1:0.01:2;
plot(sample2, f_4_2(sample2))
grid on
```



由图解法可以得到根在区间$(1,2)$内，取1.5为牛顿-拉夫森方法的初始估计值，

$x_0 = 1.5, x_1 = 1.6$为割线法的两个初值。

```
[x_4_2_Netwon, flag_4_2_Netwon] = Netwon(f_4_2, df_4_2, 1.5, 1e-5, 10000, 2, 1)
```

```
Newton's method:
the root of f(x)=0 is x = 1.557146, the number of iterations is 3
x_4_2_Netwon =
   1.557145598997611
flag_4_2_Netwon =
     3
```

```
[x_4_2_Netwon_without_df, flag_4_2_Netwon_without_df] = Netwon_without_df(f_4_2, 1.5, 1e-5, 100
```

```
Newton's method without df:
the root of f(x)=0 is x = 1.557146, the number of iterations is 3
x_4_2_Netwon_without_df =
   1.557145598997611
flag_4_2_Netwon_without_df =
     3
```

```
[x_4_2_secant, flag_4_2_secant] = secant(f_4_2, 1.5, 1.6, 1e-5, 10000)
```

```
secant method:
the root of f(x)=0 is x = 1.557146, the number of iterations is 3
x_4_2_secant =
   1.557145599061281
flag_4_2_secant =
     3
```

对题（2）$\ln x + x - 2 = 0$，牛顿-拉夫森方法和割线法均得到近似解$x \approx 1.55714$，而且两者均为3次迭代。

5. Halley方法是求解非线性方程的一个数值方法， 它采用的递推公式是

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2(f'(x_n))^2 - f(x_n)f''(x_n)}$$

(1) 编写程序实现**Halley**算法， 并求解$f(x) = 5x^7 + 2x - 1$和$g(x) = 1/x^3 - 10$.

(2) 请根据你的结果判断**Halley**方法的收敛速度是否比牛顿方法更快? **Halley**方法是线性收敛， 二次收敛，还是三次收敛?

解:

(1)求解$f(x) = 5x^7 + 2x - 1$，易得$f'(x) = 35x^6 + 2$， $f''(x) = 210x^5$

```
f_5_1 = @(x)(5*x.^7 + 2*x - 1)
```

```
f_5_1 = 包含以下值的 function_handle:
    @(x)(5*x.^7+2*x-1)
```

```
df_5_1 = @(x)(35*x^6+2)
```

```
df_5_1 = 包含以下值的 function_handle:
    @(x)(35*x^6+2)
```

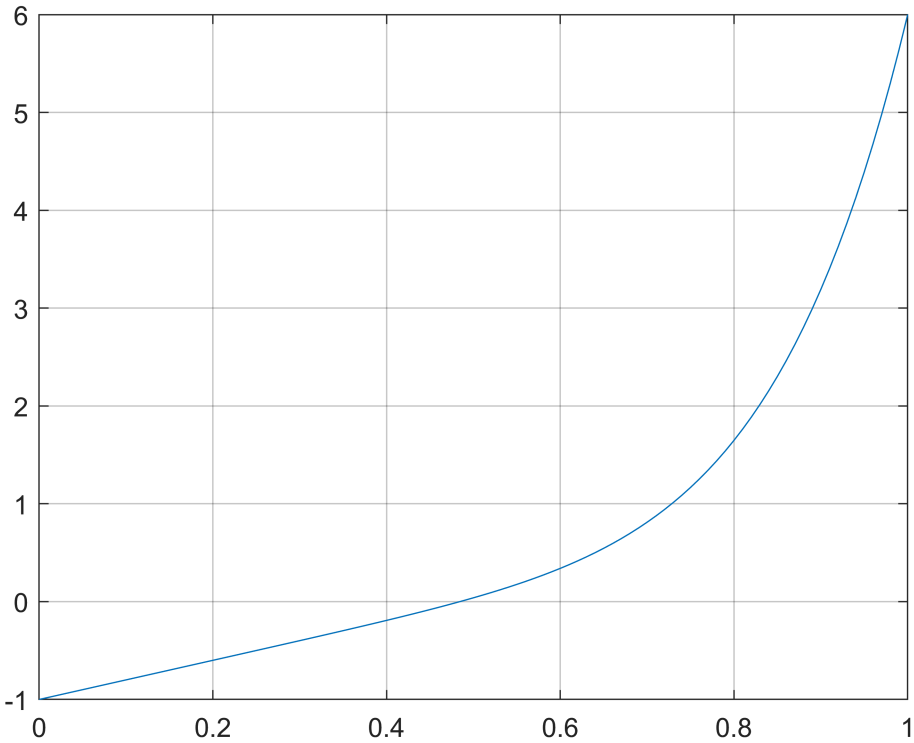
```
ddf_5_1 = @(x)(210*x^5)
```

```
ddf_5_1 = 包含以下值的 function_handle:
    @(x)(210*x^5)
```

```
sample5_1 = 0:0.01:1
```

```
sample5_1 = 1×101
                   0   0.010000000000000   0.020000000000000   0.030000000000000 ···
```

```
plot(sample5_1, f_5_1(sample5_1))
grid on
```



由图解法可以得到根在区间$(0,1)$内，取0.4为Halley算法和牛顿-拉夫森法的初始迭代点，$(0,1)$为二分法上下界。

```
[x_5_1_Netwon, flag_5_1_Netwon] = Netwon(f_5_1, df_5_1, 0.4, 1e-5, 10000, 1, 0)
```

```
Newton's method:
the root of f(x)=0 is x = 0.484363, the number of iterations is 4
x_5_1_Netwon =
   0.484363490583416
flag_5_1_Netwon =
     4
```

```
[x_5_1_Halley, flag_5_1_Halley] = Halley(f_5_1, df_5_1, ddf_5_1, 0.4, 1e-5, 10000, 1, 0)
```

```
Halley method:
the root of f(x)=0 is x = 0.484363, the number of iterations is 3
x_5_1_Halley =
   0.484363490583416
flag_5_1_Halley =
     3
```

```
[x_5_1_Halley_without_df_ddf, flag_5_1_Halley_without_df_ddf] = Halley_without_df_ddf(f_5_1, 0.
```

```
Halley method without df and ddf:
the root of f(x)=0 is x = 0.484363, the number of iterations is 3
x_5_1_Halley_without_df_ddf =
   0.484363490583416
flag_5_1_Halley_without_df_ddf =
     3
```

对$5x^7+2x-1=0$，Halley算法和牛顿-拉夫森方法均得到近似解$x\approx 0.48436$，Halley算法为3次迭代，牛顿-拉夫森方法为4次迭代。

求解$g(x)=1/x^3-10$，易得$g'(x)=-\dfrac{3}{x^4}$，$g''(x)=\dfrac{12}{x^5}$

```
f_5_2 = @(x)(ones(size(x))./(x.^3) - 10)
```

```
f_5_2 = 包含以下值的 function_handle:
    @(x)(ones(size(x))./(x.^3)-10)
```

```
df_5_2 = @(x)(-3*ones(size(x))./(x.^4))
```

```
df_5_2 = 包含以下值的 function_handle:
    @(x)(-3*ones(size(x))./(x.^4))
```

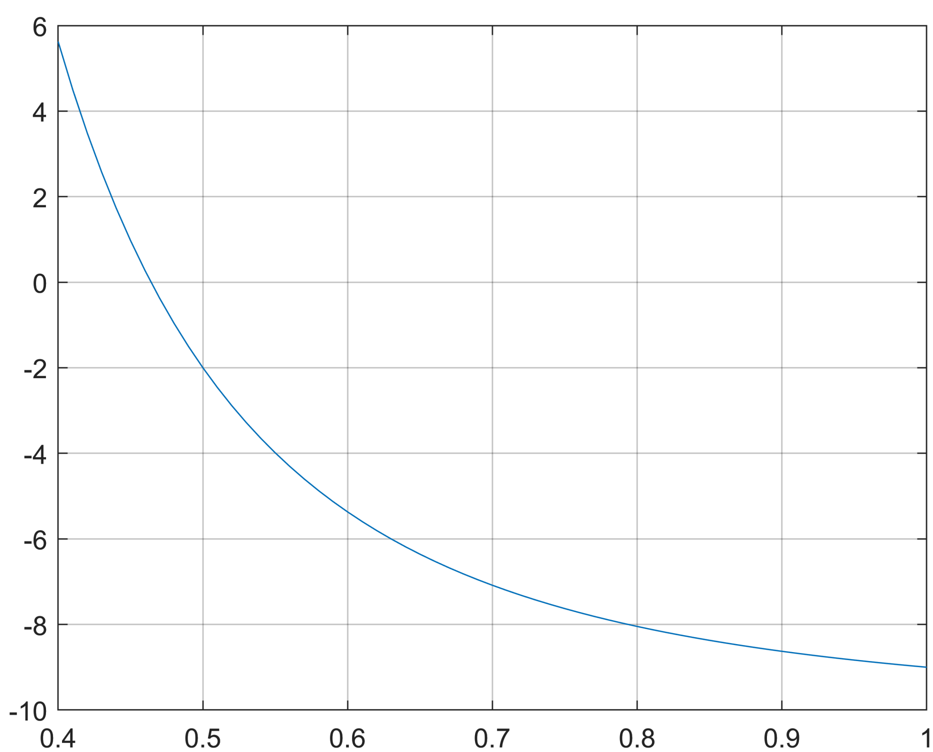
```
ddf_5_2 = @(x)(12*ones(size(x))./(x.^5))
```

```
ddf_5_2 = 包含以下值的 function_handle:
    @(x)(12*ones(size(x))./(x.^5))
```

```
sample5_2 = 0.4:0.01:1
```

```
sample5_2 = 1×61
   0.400000000000000   0.410000000000000   0.420000000000000   0.430000000000000 ···
```

```
plot(sample5_2, f_5_2(sample5_2))
grid on
```



由图解法可以得到根在区间$(0.4, 0.5)$内，取0.4为Halley算法和牛顿-拉夫森法的初始迭代点，$(0.4, 0.5)$为二分法上下界。

```
[x_5_2_Netwon, flag_5_2_Netwon] = Netwon(f_5_2, df_5_2, 0.4, 1e-5, 10000, 0.5, 0.4)
```

```
Newton's method:
the root of f(x)=0 is x = 0.464159, the number of iterations is 4
x_5_2_Netwon =
   0.464158883244858
flag_5_2_Netwon =
     4
```

```
[x_5_2_Halley, flag_5_2_Halley] = Halley(f_5_2, df_5_2, ddf_5_2, 0.4, 1e-5, 10000, 0.5, 0.4)
```

```
Halley method:
the root of f(x)=0 is x = 0.464159, the number of iterations is 3
x_5_2_Halley =
   0.464158883361278
flag_5_2_Halley =
     3
```

```
[x_5_2_Halley_without_df_ddf, flag_5_2_Halley_without_df_ddf] = Halley_without_df_ddf(f_5_2, 0.
```

```
Halley method without df and ddf:
the root of f(x)=0 is x = 0.464159, the number of iterations is 3
x_5_2_Halley_without_df_ddf =
   0.464158883361278
flag_5_2_Halley_without_df_ddf =
     3
```

对$1/x^3 - 10 = 0$，Halley算法和牛顿-拉夫森法均得到近似解$x \approx 0.46415$，Halley算法为3次迭代，牛顿-拉夫森方法为4次迭代。

(2)根据结果明显Halley方法的收敛速度比牛顿方法更快，事实上牛顿方法为二次收敛，而Halley方法是三次收敛。

$proof:$

$x_r - the\ real\ result\ of\ f(x)=0,\ we\ have\ x_{n+1}=x_n-\dfrac{2f(x_n)f'(x_n)}{2(f'(x_n))^2-f(x_n)f''(x_n)}$ ——(1)

$Expand\ f(x_r)\ Taylor\ to\ the\ quadratic\ term\ at\ x_n:$

$$f(x_r)\approx f(x_n)+f'(x_n)(x_r-x_n)+\frac{f''(x_n)}{2!}(x_r-x_n)^2=0 \text{ ——(2)}$$

$$from(1)\ we\ have\ (x_n-x_{n+1})\left[2(f'(x_n))^2-f(x_n)f''(x_n)\right]=2f(x_n)f'(x_n) \text{ ——(3)}$$

$$from(2)\ we\ have\ 2f(x_n)f'(x_n)+2(f'(x_n))^2(x_r-x_n)+f''(x_n)f'(x_n)(x_r-x_n)^2=0$$

$$and\ f'(x_n)(x_r-x_n)=-f(x_n)-\frac{f''(x_n)}{2!}(x_r-x_n)^2$$

$$\Rightarrow 2f(x_n)f'(x_n)+\left[2(f'(x_n))^2-f''(x_n)f(x_n)\right](x_r-x_n)-\frac{(f''(x_n))^2}{2}(x_r-x_n)^3=0 \text{ ——(4)}$$

$$from(3)\ and\ (4)\ we\ have\ \left[2(f'(x_n))^2-f''(x_n)f(x_n)\right](x_r-x_{n+1})-\frac{(f''(x_n))^2}{2}(x_r-x_n)^3=0$$

$$actually,\ x_r-x_{n+1}=E_{n+1},\ x_r-x_n=E_n$$

$$\therefore E_{n+1}=\frac{(f''(x_n))^2}{2\left[2(f'(x_n))^2-f''(x_n)f(x_n)\right]}E_n^3$$

$end\ proof.$

6. 已知一个小球做斜抛运动， 它的运动轨道由下式给出

$$y=(\tan\theta_0)x-\frac{g}{2v_0^2\cos^2\theta_0}x^2+y_0$$

若初始速率$v_0=30$**m/s,** 发球处的高度为**1.8m,** 接球处的高度为1m。 求投射的初始角度θ_0**,** 使得发球处和接球处的水平距离为**90m.** $g=9.81$**m/s**^{2}**.**

解:

将题目中变量代入得方程$1=(tan\theta_0)\times 90-\dfrac{9.81}{2\times 30^2cos^2\theta_0}\times 90^2+1.8$， 其中$\theta_0$为弧度制。

化简后求$f(\theta_0)=90(tan\theta_0)-\dfrac{9.81\times 90^2}{2\times 30^2cos^2\theta_0}+0.8$的零点。

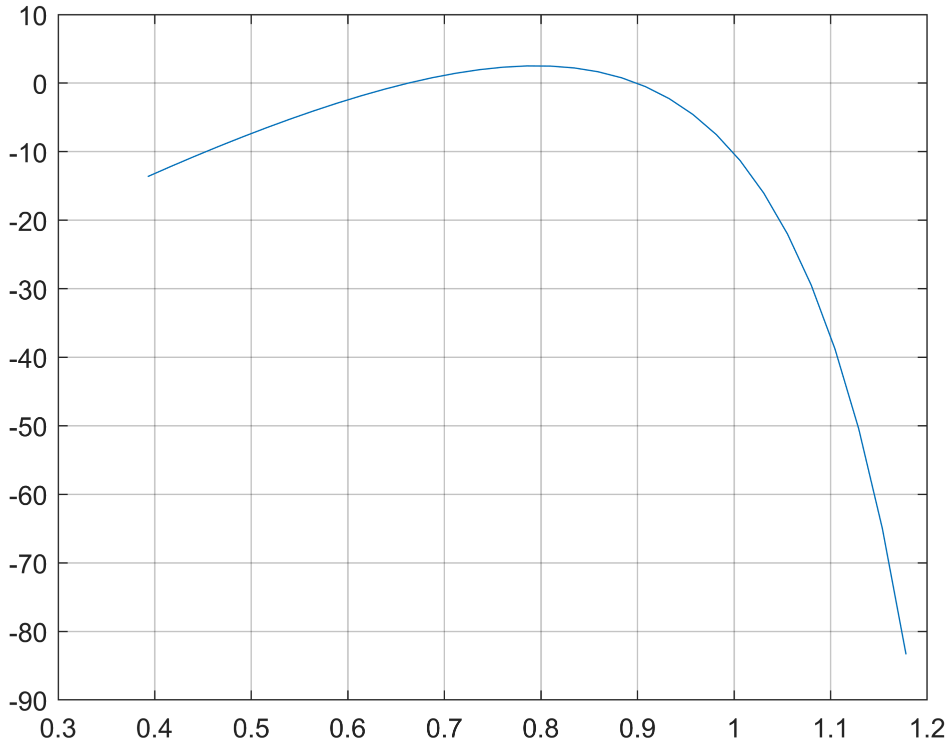
```
sample_6 = pi/8:pi/128:(pi/2-pi/8)
```

```
sample_6 = 1×33
   0.392699081698724    0.417242774304894    0.441786466911065    0.466330159517235 ···
```

```
f_6 = @(x)(90*tan(x)-9.81*90^2*ones(size(x))./(2*30^2*(cos(x)).^2))+0.8
```

```
f_6 = 包含以下值的 function_handle:
    @(x)(90*tan(x)-9.81*90^2*ones(size(x))./(2*30^2*(cos(x)).^2))+0.8
```

```
plot(sample_6, f_6(sample_6))
grid on
```



理论上$\theta_0 \in [0, \frac{\pi}{2}]$，由图解法可以得到根在$(0.6, 0.7)$和$(0.8, 1)$的区间内，由于方程难以化为$x = g(x)$，而且导数较为复杂，故使用割线法求解，选取$x_0 = 0.6, x_1 = 0.7$为第一个根的初值、$x_0 = 0.8, x_1 = 1.0$为第二个根的初值。

```
[x_6_1, ~] = secant(f_6, 0.6, 0.7, 1e-5, 10000)
```

```
secant method:
the root of f(x)=0 is x = 0.662509, the number of iterations is 4
x_6_1 =
   0.662509230665580
```

```
x_6_1_check = fzero(f_6, [0.6, 0.7])
```

```
x_6_1_check =
   0.662509214398280
```

```
e_r_6_1 = relative_e(x_6_1, x_6_1_check)
```

```
相对误差为: 2.455407e-08
e_r_6_1 =
     2.455407401293869e-08
```

```
[x_6_2, ~] = secant(f_6, 0.8, 1.0, 1e-5, 10000)
```

```
secant method:
the root of f(x)=0 is x = 0.899398, the number of iterations is 8
x_6_2 =
   0.899398457591826
```

```
x_6_2_check = fzero(f_6, [0.8, 1.0])
```

```
x_6_2_check =
   0.899398457607284
```

```
e_r_6_2 = relative_e(x_6_2, x_6_2_check)
```

```
相对误差为: -1.718676e-11
e_r_6_2 =
    -1.718676084377725e-11
```

```
theta_0_1 = rad2deg(x_6_1)
```

```
theta_0_1 =
  37.958982805596868
```

```
theta_0_2 = rad2deg(x_6_2)
```

```
theta_0_2 =
  51.531735720587577
```

由割线法求解得到$\theta_0 \approx 0.66251 = 37.959^\circ$或$\theta_0 \approx 0.89939 = 51.531^\circ$。（并通过fzero函数进行验证正确），即当投射的初始角度为37.959°或51.531°时，发球处和接球处的水平距离为90m。

7. 一个质量为*m*的物块从高处*h*静止释放后落到一个非线性弹簧上， 使得弹簧收缩*d*, 此时物块所受的恢复力为

$F = -(k_1 d + k_2 d^{3/2})$

忽略空气阻力等因素， 可以证明下式满足

$$0 = \frac{2k_2 d^{5/2}}{5} + \frac{1}{2}k_1 d^2 - mgd - mgh$$

若已知这些参数$k_1 = 40,000g/s^2, k_2 = 40g/(s^2m^{1/2}), m = 95g, g = 9.81m/s^2, h = 0.43m$，求*d*.

解:

将参数代入方程得$\frac{2\times 40d^{5/2}}{5} + \frac{1}{2}\times 40000d^2 - 95\times 9.81d - 95\times 9.81\times 0.43 = 0$，

令$f(x) = \frac{2\times 40d^{5/2}}{5} + \frac{1}{2}\times 40000d^2 - 95\times 9.81d - 95\times 9.81\times 0.43$，求其零点。

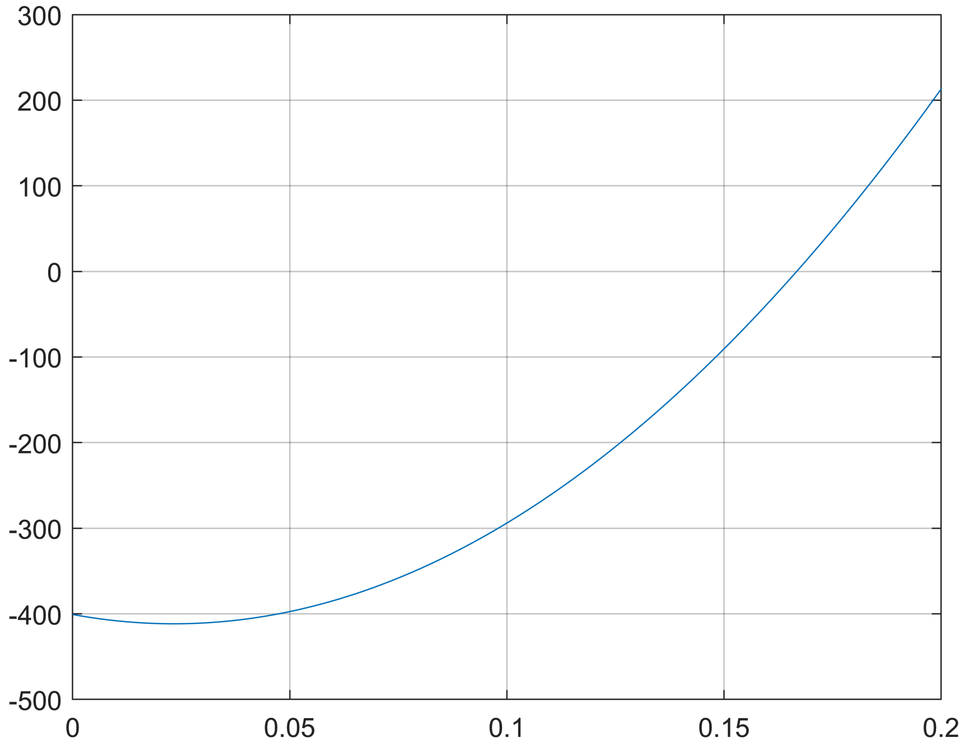
```
sample_7 = 0:0.001:0.2
```

```
sample_7 = 1×201
                   0   0.001000000000000   0.002000000000000   0.003000000000000 ···
```

```
f_7 = @(x)(2*40*x.^(2.5)./5 + 0.5*40000*x.^2 - 95*9.81*x - 95*9.81*0.43)
```

```
f_7 = 包含以下值的 function_handle:
    @(x)(2*40*x.^(2.5)./5+0.5*40000*x.^2-95*9.81*x-95*9.81*0.43)
```

```
plot(sample_7, f_7(sample_7))
grid on
```



由图解法可以得到根在$(0, 0.2)$的区间内，使用牛顿-拉夫森方法，其中$f'(x) = 40d^{3/2} + 40000d - 95 \times 9.81$，初始估计值为0.16，$(0, 0.2)$为二分法区间。

```
[x_7, ~] = Netwon_without_df(f_7, 0.16, 1e-5, 10000, 0.2, 0)
```

```
Newton's method without df:
the root of f(x)=0 is x = 0.166724, the number of iterations is 3
x_7 =
   0.166723562437816
```

```
x_7_check = fzero(f_7, [0.2, 0])
```

```
x_7_check =
   0.166723562437785
```

```
e_r_7 = relative_e(x_7, x_7_check)
```

```
相对误差为: 1.896169e-13
e_r_7 =
     1.896168733678443e-13
```

由牛顿-拉夫森方法得到近似解$d \approx 0.16672$m。（并通过fzero函数进行验证正确）

8. 一根质量均匀分布的缆线两端固定被悬挂起来，通过受力分析，发现缆线的高度y与水平位置x的关系由下式决定

$$y = \frac{T_A}{w}\cosh\left(\frac{w}{T_A}x\right) + y_0 - \frac{T_A}{w}$$

其中T_A是$x=0$处的张力，w是缆线单位长度的重量。

(1) 若$w=10$ **N/m,** y_0**=5 m,** 并且$x=50$**m**处， $y=15$**m,** 求T_A**.**

(2) 请在区间$[-50,100]$上绘制$y(x)$曲线。

解:

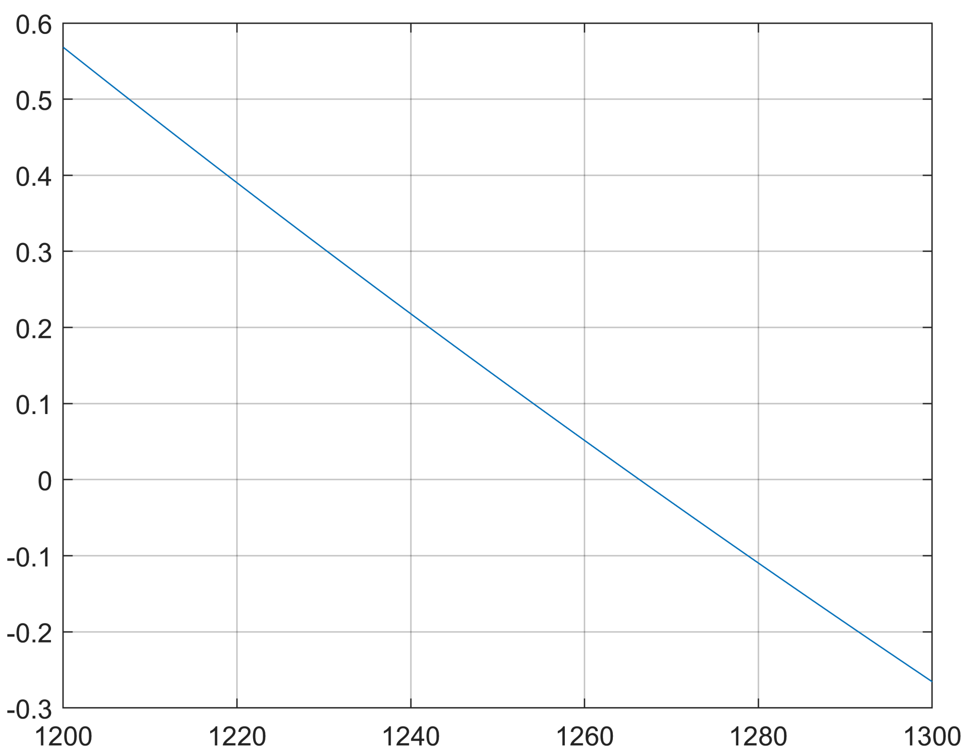
（1）将参数代入方程得$15=\frac{T_A}{10}cosh(\frac{10}{T_A}\times 50)+5-\frac{T_A}{10}$,

令$f(x)=\frac{T_A}{10}cosh(\frac{10}{T_A}\times 50)-10-\frac{T_A}{10}$，求其零点。

```
sample_8_1 = 1200:1:1300;
f_8 = @(x)(x./10.*cosh(10*50*ones(size(x))./x)- 10 - x./10)
```

```
f_8 = 包含以下值的 function_handle:
    @(x)(x./10.*cosh(10*50*ones(size(x))./x)-10-x./10)
```

```
plot(sample_8_1, f_8(sample_8_1))
grid on
```



由图解法可以得到根在$(1200,1300)$的区间内，使用割线法选取$x_0=1260, x_1=1270$为初值。

```
T_A = secant(f_8, 1260, 1270, 1e-5, 10000)
```

```
secant method:
the root of f(x)=0 is x = 1266.324360, the number of iterations is 4
T_A =
    1.266324360399889e+03
```

```
T_A_check = fzero(f_8, [1260, 1270])
```

```
T_A_check =
    1.266324360399887e+03
```

```
e_r_8 = relative_e(T_A, T_A_check)
```

```
相对误差为: 1.615987e-15
e_r_8 =
    1.615986506287281e-15
```

使用割线法得到近似解$T_A \approx 1266.32436$N。（并通过fzero函数进行验证正确）

（2）将T_A代入$y(x)$得$y(x) = \frac{1266.32436}{10}cosh(\frac{10x}{1266.32436}) + 5 - \frac{1266.32436}{10}$

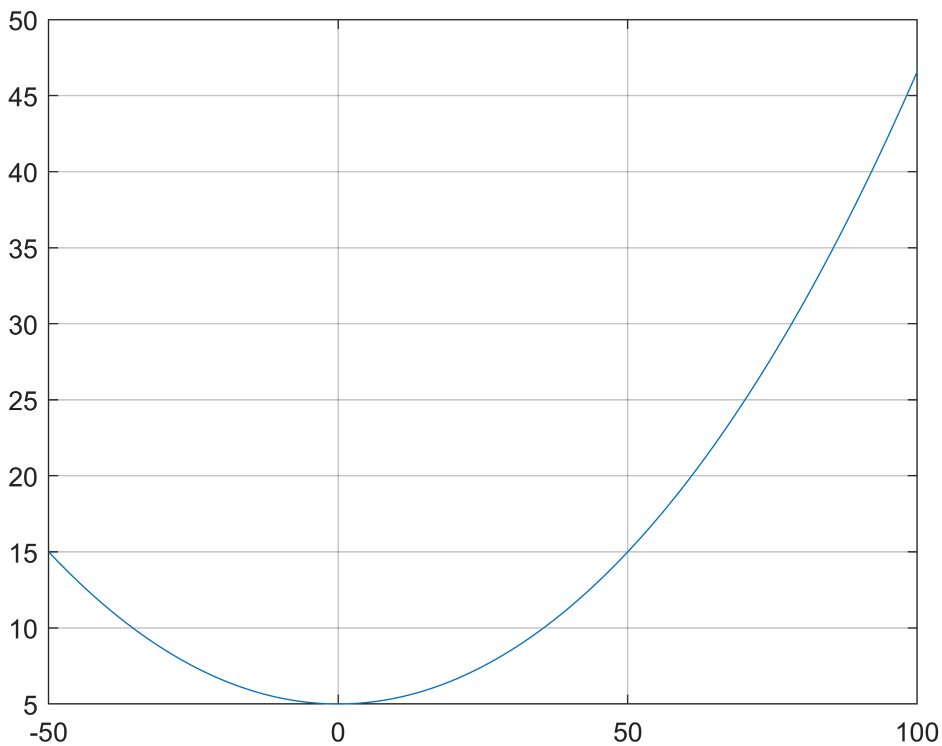
```
sample_8_2 = -50:100
```

```
sample_8_2 = 1×151
   -50   -49   -48   -47   -46   -45   -44   -43   -42   -41   -40   -39   -38 ···
```

```
y = @(x)(1266.32436/10*cosh(10*x/1266.32436) + 5 - 1266.32436/10)
```

```
y = 包含以下值的 function_handle:
    @(x)(1266.32436/10*cosh(10*x/1266.32436)+5-1266.32436/10)
```

```
plot(sample_8_2, y(sample_8_2))
grid on
```



$y(x)$在区间$[-50,100]$的曲线如图所示。

functions*

dichotomy：二分法

$mid=(high+low)/2$

args：

f：寻根的函数

a：区间上界

b：区间下界

epsilon：误差范围

output:

当$flag=-2$，表示初始区间不存在根，$answer=NaN$；

其他情况下，$flag$表示迭代次数，$answer$为寻得的根。

```
function [answer, flag] = dichotomy(f, a, b, epsilon)
    fprintf('dichotomy:\n');
    high = a;
    low = b;
```

```
    count = 0;
    if(f(high)*f(low) > 0)
        fprintf('此区间不存在根!');
    elseif(f(high) == 0)
        answer = high;
        fprintf('the root of f(x)=0 is x = %f', answer);
    elseif(f(low) == 0)
        answer = low;
        fprintf('the root of f(x)=0 is x = %f', answer);
    else
        mid = (high + low) / 2;
        while(f(mid) ~= 0 && (high - low) > 2*epsilon)
            count = count + 1;
            if(f(high)*f(mid) < 0)
                low = mid;
            else
                high = mid;
            end
            mid = (high + low) / 2;
        end
    end
    switch count
        case 0
            flag = -2; answer = NaN;
        otherwise
            flag = count; answer = mid;
            fprintf('the root of f(x)=0 is x = %f, the number of iterations is %d', answer, cou
    end
end
```

fixedPoint：定点迭代法

$x_{n+1} = g(x_n)$

args:

g：寻根的函数

x0：初始估计值

epsilon：误差上限

max_n：最多迭代次数（防止发散）

output:

当 $flag = -1$，表示可能不收敛或最多迭代次数不足以达到精度，$answer = NaN$；

其他情况下，$flag$ 表示迭代次数，$answer$ 为寻得的根。

```
function [answer, flag] = fixedPoint(g, x0, epsilon, max_n)
    fprintf('fixedPoint:\n');
    count = 0;
    fPoint = x0;
```

```
    while(count < max_n && abs(fPoint - g(fPoint)) > epsilon)
        fPoint = g(fPoint);
        count = count + 1;
    end

    if(count == max_n)
        flag = -1; answer = NaN;
        fprintf('it may not converge or the iterations are not enough!')
    else
        flag = count; answer = fPoint;
        fprintf('the root of g(x)=x is x = %f, the number of iterations is %d', answer, count);
    end
end
```

fixedPointWithAitken：使用Aitken加速方法的定点迭代方法

$$\begin{aligned}
&Algorithm - Aitken's\ \ Method\\
&Choose\ \ initial\ \ approximation\ \ x_0\\
&Do\\
&\quad Calculate\ \ x_{3i+1}\ \ and\ \ x_{3i+2}\ \ from\ \ x_{3i}\ \ using\ \ any\ \ linear\ \ iterative\ \ method\\
&\quad Modify\ \ x_{3i+3}\ \ using\\
&\quad x_{3i+3} = x_{3i+2} - \frac{(x_{3i+2} - x_{3i+1})^2}{x_{3i} - 2x_{3i+1} + x_{2i+2}} \quad i = 0, 1, 2, \ldots\\
&while(none\ \ of\ \ the\ \ convergence\ \ criterion\ \ is\ \ met)
\end{aligned}$$

args：

g：寻根的函数

x0：初始估计值

epsilon：误差上限

max_n：最多迭代次数（防止发散）

output:

当 $flag = -1$，表示可能不收敛或最多迭代次数不足以达到精度，$answer = NaN$；

其他情况下，$flag$ 表示迭代次数，$answer$ 为寻得的根。

```
function [answer, flag] = fixedPointWithAitken(g, x0, epsilon, max_n)
    fprintf('fixedPoint with Aitken:\n');
    count = 0;
    fPoint0 = x0;
    while(count < max_n && abs(fPoint0 - g(fPoint0)) > epsilon)
        fPoint1 = g(fPoint0);
        fPoint2 = g(fPoint1);
        fPoint0 = fPoint2 - (fPoint2 - fPoint1)^2/(fPoint0 - 2*fPoint1 + fPoint2);
        count = count + 3;
    end

    if(count == max_n)
```

```
        flag = -1; answer = NaN;
        fprintf('it may not converge or the iterations are not enough!')
    else
        flag = count; answer = fPoint0;
        fprintf('the root of g(x)=x is x = %f, the number of iterations is %d', answer, count);
    end
end
```

Netwon：使用牛顿-拉夫森方法，需要将导数以参数形式传入，并用二分法区间修正发散解

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

args：

f：寻根的函数

df：寻根函数的导数

x0：初始估计值x_0

epslion：误差上限

max_n：最多计算步数

high：二分法区间上限

low：二分法区间下限

output:

当 $flag = -1$，表示可能不收敛或最多迭代次数不足以达到精度，$answer = NaN$；

当 $flag = -2$，表示初始区间不存在根，$answer = NaN$；

其他情况下，$flag$ 表示迭代次数，$answer$ 为寻得的根。

```
function [answer, flag] = Netwon(f, df, x0, epsilon, max_n, high, low)
    fprintf('Newton''s method:\n');
    count = 0;
    initialP = x0;
    if(f(high)*f(low) > 0)
        fprintf('二分法区间不存在根！');

    else
        % 假如初始估计值在二分法区间外，则用(high + low) / 2代替
        if(x0 > high || x0 < low)
            initialP = (high + low) / 2;
        end
        while(count < max_n)
            count = count + 1;
            tmp = initialP; %判断是否停
            initialP = initialP - f(initialP)/df(initialP);
            %如果迭代结果在二分范围外则换为(high + low)/2，并根据迭代结果更新high和low
            if(initialP > high || initialP < low)
```

```
                initialP = (high + low) / 2;
            end
            if(f(high)*f(initialP) < 0)
                low = initialP;
            else
                high = initialP;
            end
            if(abs(tmp - initialP) < epsilon)
                break;
            end
        end
    end
    switch count
        case max_n
            flag = -1; answer = NaN;
            fprintf('it may not converge or the iterations are not enough!')
        case 0
            flag = -2; answer = NaN;
        otherwise
            flag = count; answer = initialP;
            fprintf('the root of f(x)=0 is x = %f, the number of iterations is %d', answer, cou
    end
end
```

Netwon_without_df：使用牛顿-拉夫森方法，不需要将导数以参数形式传入，并用二分法区间修正发散解（author：鲁潇阳）

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

args：

f：寻根的函数

x0：初始估计值x_0

epslion：误差上限

max_n：最多计算步数

high：二分法区间上限

low：二分法区间下限

output:

当 $flag = -1$，表示可能不收敛或最多迭代次数不足以达到精度，$answer = NaN$；

当 $flag = -2$，表示初始区间不存在根，$answer = NaN$；

其他情况下，$flag$ 表示迭代次数，$answer$ 为寻得的根。

```
function [answer, flag] = Netwon_without_df(f, x0, epsilon, max_n, high, low)
    fprintf('Newton''s method without df:\n');
    % 计算一阶导数
```

```
    syms x;
    fun = f(x);
    df = matlabFunction(diff(fun)); % 通过符号函数求导，并转换为可计算的函数
    % 求根
    count = 0;
    initialP = x0;
    if(f(high)*f(low) > 0)
        fprintf('二分法区间不存在根！');

    else
        % 假如初始估计值在二分法区间外，则用(high + low) / 2代替
        if(x0 > high || x0 < low)
            initialP = (high + low) / 2;
        end
        while(count < max_n)
            count = count + 1;
            tmp = initialP; % 保存前一次计算值，用于判断是否停
            initialP = initialP - f(initialP)/df(initialP);

            % 如果迭代结果在二分范围外则换为(high + low)/2，并根据迭代结果更新high和low
            if(initialP > high || initialP < low)
                initialP = (high + low) / 2;
            end
            if(f(high)*f(initialP) < 0)
                low = initialP;
            else
                high = initialP;
            end
            if(abs(tmp - initialP) < epsilon)
                break;
            end
        end
    end
    switch count
        case max_n
            flag = -1; answer = NaN;
            fprintf('it may not converge or the iterations are not enough!')
        case 0
            flag = -2; answer = NaN;
        otherwise
            flag = count; answer = initialP;
            fprintf('the root of f(x)=0 is x = %f, the number of iterations is %d', answer, cou
    end
end
```

secant：割线法

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

args：

f：寻根的函数

x0：初值x_0

x1：初值x_1

epsilon：误差上限

max_n：最多计算次数

output:

当 $flag = -1$，表示可能不收敛或最多迭代次数不足以达到精度，$answer = NaN$；

其他情况下，$flag$ 表示迭代次数，$answer$ 为寻得的根。

```
function [answer, flag] = secant(f, x0, x1, epsilon, max_n)
    fprintf('secant method:\n');
    count = 0;
    p1 = x0;
    p2 = x1;
    while(count < max_n && abs(p1 - p2) > epsilon)
        count = count + 1;
        %临时存放p2，作为下次迭代的p1
        tmp = p2;
        p2 = p2 - f(p2)*(p2 - p1)/(f(p2) - f(p1));
        p1 = tmp;
    end

    if(count == max_n)
        flag = -1; answer = NaN;
        fprintf('it may not converge or the iterations are not enough!')
    else
        flag = count; answer = p2;
        fprintf('the root of f(x)=0 is x = %f, the number of iterations is %d', answer, count);
    end
end
```

Halley：Halley方法，需要将函数的一阶和二阶导数以参数形式传入，并使用二分法区间修正发散解

args：

f：寻根的函数

df：寻根函数的一阶导数

ddf：寻根函数的二阶导数

x0：初始的估计值

epsilon：误差上限

max_n：最多计算的次数

high：二分法区间上限

low：二分法区间下限

output:

当 $flag = -1$，表示可能不收敛或最多迭代次数不足以达到精度，$answer = NaN$；

当 $flag = -2$，表示初始区间不存在根，$answer = NaN$；

其他情况下，$flag$ 表示迭代次数，$answer$ 为寻得的根。

```
function [answer, flag] = Halley(f, df, ddf, x0, epsilon, max_n, high, low)
    fprintf('Halley method:\n');
    count = 0;
    initialP = x0;
    if(f(high)*f(low) > 0)
        fprintf('二分法区间不存在根！');

    else
        % 假如初始估计值在二分法区间外，则用(high + low) / 2代替
        if(x0 > high || x0 < low)
            initialP = (high + low) / 2;
        end
        while(count < max_n)
            count = count + 1;
            tmp = initialP;
            initialP = initialP - 2*f(initialP)*df(initialP)/(2*(df(initialP))^2-f(initialP)*dd
            %如果迭代结果在二分范围外则换为(high + low)/2，并根据迭代结果更新high和low
            if(initialP > high || initialP < low)
                initialP = (high + low) / 2;
            end
            if(f(high)*f(initialP) < 0)
                low = initialP;
            else
                high = initialP;
            end
            if(abs(tmp - initialP) < epsilon)
                break;
            end
        end
    end
    switch count
        case max_n
            flag = -1; answer = NaN;
            fprintf('it may not converge or the iterations are not enough!')
        case 0
            flag = -2; answer = NaN;
        otherwise
            flag = count; answer = initialP;
            fprintf('the root of f(x)=0 is x = %f, the number of iterations is %d', answer, cou
    end
end
```

Halley_without_df_ddf：Halley方法，不需要将函数的一阶和二阶导数以参数形式传入，并使用二分法区间修正发散解（author：鲁潇阳）

args：

f：寻根的函数

x0：初始的估计值

epsilon：误差上限

max_n：最多计算的次数

high：二分法区间上限

low：二分法区间下限

output:

当 $flag = -1$，表示可能不收敛或最多迭代次数不足以达到精度，$answer = NaN$；

当 $flag = -2$，表示初始区间不存在根，$answer = NaN$；

其他情况下，$flag$ 表示迭代次数，$answer$ 为寻得的根。

```
function [answer, flag] = Halley_without_df_ddf(f, x0, epsilon, max_n, high, low)
    fprintf('Halley method without df and ddf:\n');
    % 计算一阶和二阶导数
    syms x;
    fun = f(x);
    df = matlabFunction(diff(fun, x, 1)); % 通过符号函数求导，并转换为可计算的函数
    ddf = matlabFunction(diff(fun, x, 2));
    % 求根
    count = 0;
    initialP = x0;
    if(f(high)*f(low) > 0)
        fprintf('二分法区间不存在根！');

    else
        % 假如初始估计值在二分法区间外，则用(high + low) / 2代替
        if(x0 > high || x0 < low)
            initialP = (high + low) / 2;
        end
        while(count < max_n)
            count = count + 1;
            tmp = initialP;
            initialP = initialP - 2*f(initialP)*df(initialP)/(2*(df(initialP))^2-f(initialP)*d
            % 如果迭代结果在二分范围外则换为(high + low)/2，并根据迭代结果更新high和low
            if(initialP > high || initialP < low)
                initialP = (high + low) / 2;
            end
            if(f(high)*f(initialP) < 0)
                low = initialP;
            else
                high = initialP;
            end
            if(abs(tmp - initialP) < epsilon)
                break;
```

```
            end
        end
    end
    switch count
        case max_n
            flag = -1; answer = NaN;
            fprintf('it may not converge or the iterations are not enough!')
        case 0
            flag = -2; answer = NaN;
        otherwise
            flag = count; answer = initialP;
            fprintf('the root of f(x)=0 is x = %f, the number of iterations is %d', answer, cou
    end
end
```

计算相对误差的函数，并显示

args:

x_approx：估计值

x_real：真值

```
function output = relative_e(x_approx, x_real)
    output = (x_approx - x_real .* ones(size(x_approx)))./x_real;
    fprintf("相对误差为: %e" , output);
end
```