DIONE: A protocol verification system built with DAFNY for I/O Automata

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Abstract. Input/Output Automata (IOA) is an expressive and popular specification framework with built-in properties for compositional reasoning. It has been shown to be effective in specifying and analyzing distributed and networked systems. The available verification engines for IOA are all based on interactive theorem provers such as Isabelle, Larch, PVS, and Coq, and are expressive but require heavy human interaction. Motivated by the advances in SMT solvers, in this work we explore a different expressivity-automation tradeoff for IOA. We present Dione, the first IOA analysis system built with DAFNY and its SMT-powered toolchain and demonstrate its effectiveness on four distributed applications. Our translator tool converts Python-esque Dione language specification of IOA and their properties to parameterized DAFNY modules (similar to theories in theorem provers). DIONE automatically generates the relevant compatibility and composition lemmas for the IOA specifications, which can then be checked with Dafny on a per module-basis. We ensure that all resulting formulas are expressed mostly in fragments solvable by SMT solvers and hence enables Bounded Model Checking and k-induction-based invariant checking using Z3. We present successful applications of Dione in verification of an asynchronous leader election algorithm, two self-stabilizing mutual exclusion algorithms, and CAN bus Arbitration: we automatically prove key invariants of all four protocols; for the last three this involves reasoning about arbitrary number of participants. These analyses are largely automatic with minimal manual inputs needed, and they demonstrate the effectiveness of this approach in analyzing networked and distributed systems.

1 Introduction

For modeling and verifying network and distributed systems, compositional approaches are considered essential in achieving modularity and scalability. A recent study on building verified industrial scale systems highlight the importance of compositional reasoning and connecting layers of abstractions, especially for integrating formal methods into software development practices [19]. The Input/Output Automata (IOA) [17] framework comes with an expressive modeling language [7], and a powerful set of simulation and substitutivity results that support reasoning about compositions, abstractions, and substitutions. The framework has been used to model and analyze a wide variety of distributed systems

ranging from high-level protocols for mutual exclusion, consensus, leader election [17], to implementation-level specifications for shared memory and group communication services [5,6,3], and communication protocols like TCP [22].

Currently available mechanized verification engines for IOA are mostly based on interactive theorem provers including Isabelle/HOL [18], Larch Shared Language [2], PVS [16], and recently CoqIOA [1]. All of these provide expressive specification languages based on higher order theories for modeling IOA, but they require a nontrivial amount of human interactions for constructing proofs¹.

In this paper, we aim to achieve higher automation for verification of distributed systems expressed in IOA, by restricting the specifications to less expressive theories with established decision procedures. One predominant option is First Order Theories supported by existing Satisfiability Modulo Theories (SMT) solvers. There has been a wave of advancements in SMT solvers, which in turn has led to the creation of widely diverse program verification engines like CBMC [4], DAFNY [14], and SEAHORN [13]. We therefore believe that theories solvable with SMT solvers can be expressive enough for a broadly useful class of IOA, and an SMT-based IOA verification engine can advance the design and analysis of distributed systems.

Specifically, we build a toolchain based on DAFNY [14], instead of directly interfacing with SMT solvers. DAFNY provides a higher level of abstraction for developing both system specifications and for proof strategies [15]. The IronFleet project [10], for instance, demonstrates how practical consensus protocols can be modeled and verified in DAFNY. The project further develops a DAFNY implementation of the model with formal conformance guarantees, and ultimately generates correct-by-construction C# source code from the implementation. These results suggest that DAFNY is a good candidate for supporting modeling and mostly-automatic verification for distributed protocols as IOA.

In this work, we propose DIONE—a modeling language and verification system built using DAFNY for IOA. The DIONE workflow is similar to that of [2,16]. First, users model the protocol and specify the desired invariant in the DIONE language. For example in Figure 1, a distributed algorithm from the textbook [8] can be faithfully modeled in DIONE. DIONE frontend then translates DIONE to DAFNY specification language. Additionally, DIONE generates DAFNY lemmas so that proving these lemmas is equivalent to Bounded Model Checking (BMC) and k-induction based invariant checking for the original IOA. There are three major contributions in this work:

- (i) We have developed a Python-like language, Dione, for naturally specifying distributed systems as compositions of parameterized IOA. We have implemented the supporting tools for (a) translating Dione specifications to Dafny, and for (b) carrying out automatic proofs for Dione specifications. Our translator uses modules in Dafny to automatically generate (nontrivial) composition and compatibility checks for parameterized automata.
- (ii) We have demonstrated that DIONE can effectively model diverse distributed protocols: self-stabilizing mutual exclusion algorithms with different network

¹ Also, none of the tools appear to be maintained for at least two years.

```
# s[0] \in {1,3}, s[n-1] \in {2, 4}, and N(i) returns neighbors of i #
program StableArray;
{program for process i, i=0 or i=n-1}
\exists \texttt{j} \in \texttt{N(i)} \colon (\texttt{s[j]} = \texttt{s[i]+1} \text{ mod } \texttt{4}) \rightarrow \texttt{s[i]} := \texttt{s[i]+2} \text{ mod } \texttt{4}
od
{program for process i, 0<i<n-1}
∃j
    \in \mathbb{N}(i): (s[j] = s[i]+1 \mod 4) \rightarrow s[i] := s[j]
od
Status: type = IntRange[0:4]
@automaton
def StableArray(N: int):
    where= 2 \le N
    class signature:
         @output
         def trans(i: int): where=(0 \leq i \leq N)
     class states:
         s: Seq[Status]
     initially = len(s) = N  and (s[0] = 1  or s[0] = 3) and (s[N-1] = 0  or s[N-1] = 2)
    class transitions:
         @output
         \texttt{@pre}((\texttt{i} \texttt{==0} \ \texttt{and} \ \texttt{s}[\texttt{i} \texttt{+}1] \texttt{==} \texttt{i} \texttt{n} \texttt{cre}(\texttt{s}[\texttt{i}])) \\ \texttt{or}(\texttt{i} \texttt{==N-1} \ \texttt{and} \ \texttt{s}[\texttt{i} \texttt{-}1] \texttt{==} \texttt{i} \texttt{n} \texttt{cre}(\texttt{s}[\texttt{i}]))) \\
         def trans(i): s[i] = incre(incre(s[i]))
         @output
         Opre(0 < i < N-1 \text{ and } s[i-1] == incre(s[i]))
         def trans(i): s[i] = s[i-1]
         Opre(0 < i < N-1 \text{ and } s[i+1] == incre(s[i]))
         def trans(i): s[i] = s[i+1]
     invariant_of = len(s) == N  and (s[0] == 1  or s[0] == 3) and (s[N-1] == 0  or s[N-1] == 2)
```

Fig. 1. Textbook description of a self-stabilizing mutual exclusion algorithm [8, Section 17.3.2] (top) and specification in DIONE (bottom).

topologies, leader election algorithms, and the CAN bus Arbitration. These showcase how protocols and requirements for an arbitrary number of processes can be modeled with parallel composition and collection types.

(iii) Finally, we have demonstrated that Dione can achieve a promising level of proof automation. The ratio lengths of Dione specifications to Dafny models plus proofs is about 1:3 to 1:5. In the worst case only two out of five lemmas require user specified proof strategies. In particular, (a) for self-stabilization algorithms, our verifier is able to prove invariants for an arbitrary number of processes by k-induction with one or two manually added basic facts about a generic sequence. (b) For the leader election algorithm and CAN bus Arbitration, we show that auxiliary invariants or over-approximation of transitions can be easily integrated to fully automate verification, and over-approximation itself can be automatically validated by the verifier.

Related Work. We have mentioned most related works in previous paragraphs. The translation by DIONE resembles the translation to Larch [2] as both are using First Order Theories. DIONE benefits from the advancement in SMT solving and program verification. With better understanding on the efficiency and decidability of SMT solving, DIONE is designed to achieve higher automation

using features in DAFNY. In return, it is difficult to specify user-defined sorts and theories not supported by DAFNY compared with [2] and all other works.

The design decision to model distributed protocols in DAFNY is inspired by IronFleet [10]. IronFleet focused on connecting layers of abstractions from specification to implementation, and their proofs of refinement based on Temporal Logic of Actions (TLA) are mostly manually written. Besides the differences between TLA and IOA, our work focuses on automatic proof generation.

More broadly, Tuttle and Goel [23] demonstrated how to use the SMT-based checker, DVF, to verify a consensus protocol. Ivy [20] recently explored using EPR, a decidable fragment of FOL, to model and verify infinite state transition systems. However, the DVF model for the protocol explicitly describes the global state transition system instead of the local state of a participant of the protocol, and Ivy defines the execution of a protocol as one RML program within only one while loop; it is likely cumbersome to cleanly model parallel composition in both DVF and Ivy. Moreover, it is unclear how to specify which automaton has control over certain actions. On the other hand, our IOA-based language provides compositions that list the components and **input** and **output** keywords to differentiate sets of actions. For our purpose of verifying IOA, we believe DVF and Ivy can be used as an alternative back-end for DIONE.

2 Background: Brief overview of IOA and DAFNY

Mathematical Notations. Let X be a finite set of variable names. To model variables with static types, we assume a function type(x) to return the set of possible values for x. A valuation, s, is a mapping from $x \in X$ to a value $s(x) \in type(x)$, and val(X) denotes the set of all possible valuations of X. For a valuation $s \in val(X)$ and a subset of variable names $Y \subseteq X$, we use $s \mid Y$ to denote the restriction of s to Y.

Input/Output Automata. An IOA [17], $\mathcal{A} = (\Sigma, X, Q, \Theta, \delta)$, is a tuple where (i) $\Sigma = \Sigma^I \dot{\cup} \Sigma^O \dot{\cup} \Sigma^H$ is the set of all actions partitioned into Σ^I , Σ^O , and Σ^H representing **input**, **output**, and **internal** actions, respectively. (ii) X is a finite set of state variable names. (iii) $Q \subseteq val(X)$ is the set of states (iv) $\Theta \subseteq Q$ is the set of initial states. (v) $\delta \subseteq Q \times \Sigma \times Q$ is the transition relation. An important requirement for an IOA \mathcal{A} is that it is input enabled. That is, it cannot block input actions. Formally,

$$\forall s \in Q, a \in \Sigma^I, \exists s' \in Q, (s, a, s') \in \delta$$

A pair of IOA \mathcal{A}_1 and \mathcal{A}_2 are *compatible* if and only if the following holds: (i) Output actions are disjoint, i.e., $\Sigma_1^O \cap \Sigma_2^O = \emptyset$; (ii) Internal actions are only used by itself, i.e., $\Sigma_1^H \cap \Sigma_2 = \Sigma_2^H \cap \Sigma_1 = \emptyset$; and (iii) Variable names are disjoint, i.e., $X_1 \cap X_2 = \emptyset$. This ensures that a composition of compatible automata (defined next) is also an IOA.

Given two compatible automata A_1 and A_2 , we can construct the parallel composition automaton $A_1 \parallel A_2 = (\Sigma, X, Q, \Theta, \delta)$ where

- $\text{(i)} \ \ \varSigma^I = (\varSigma_1^I \cup \varSigma_2^I) \setminus (\varSigma_1^O \cup \varSigma_2^O); \quad \ \varSigma^O = \varSigma_1^O \cup \varSigma_2^O; \quad \ \varSigma^H = \varSigma_1^H \cup \varSigma_2^H$
- (ii) $X = X_1 \cup X_2$
- $(iii) \ Q = \{ s \in val(X) \mid s \mid X_1 \in Q_1 \land s \mid X_2 \in Q_2 \}$
- (iv) $\Theta = \{ s \in Q \mid s \mid X_1 \in \Theta_1 \land s \mid X_2 \in \Theta_2 \}$
- (v) The transition δ is defined as follows with shorthand expressions $s_1 = s \upharpoonright X_1$, $s_2 = s \upharpoonright X_2$, $s_1' = s' \upharpoonright X_1$, and $s_2' = s' \upharpoonright X_2$:

$$\{(s, a, s') \in Q \times \Sigma \times Q \mid (a \in \Sigma_1 \Rightarrow (s_1, a, s'_1) \in \delta_1) \land (a \notin \Sigma_1 \Rightarrow s_1 = s'_1) \land (a \in \Sigma_2 \Rightarrow (s_2, a, s'_2) \in \delta_2) \land (a \notin \Sigma_2 \Rightarrow s_2 = s'_2)\}$$

It can be checked that the composed automaton $A_1 \parallel A_2$ is indeed an IOA. The theory of IOA also defines executions, traces, invariance, abstractions, refinements, forward-backward simulation relations, and substitutivity properties. We refer the interested reader to [17,7] for more complete overview.

IOA with Parameters. To concisely model a distributed protocol, a standard practice is to use parameterized automaton specifications. Formal parameters may appear in names of automata, actions, and in all the predicate and transition definitions. This interdependence of parameters and variables make compatibility and composition nontrivial to implement.

For example, it is standard to assume that each participating process is distinguished by a unique address ip, and that messages are tagged with source and destination ips. We can specify the address as a parameter ip of Addr type and define a family of automata $\{A_{ip}\}_{ip\in Addr}$. The complete behavior of the protocol is then modeled by composing automata instantiated with distinct ips. For specifying one **output** action sending a message, we can use a tuple (src, dst, ...) to represent a tagged message. However, in order to satisfy compatibility criteria, the set of **output** actions Σ_{ip}^O for sending messages has to be constrained by ip, e.g., $\Sigma_{ip}^O = \{(src, dst, ...) | src, dst \in Addr \land src = ip\}$, or else the action sets for different automaton instances may intersect. Consequently, it is not obvious whether the compatibility criteria are satisfied when the constraints are more complex, or when a composite automaton is used in another composition. Hence, it is important to have an automated check of the compatibility criteria.

3 Overview of Dione

In this section, we discuss the design and features of DIONE as well as the translation from DIONE language to DAFNY for model checking.

3.1 DIONE Language

To provide users a familiar modeling language for IOA while allowing DIONE to be extensible, DIONE language borrows the syntax from one of the most popular programming languages, PYTHON. This design decision allows us to potentially embed DIONE into normal PYTHON programs and use existing PYTHON packages such as Z3Py for verification. Further, our tools benefit from existing and

```
1 Loc: type = nat
                                          1 \text{ type } \mathsf{Loc} = \mathsf{nat}
2 Addr: type = Enum[A, B, C]
                                         _2 datatype Addr = A \mid B \mid C
3 Msg: type = NamedTuple[src: Addr, 3 datatype Msg = Msg(src: Addr,
                            dst: Addr, 4
                                                                 dst: Addr,
                            val: int]
                                                                 val: int)
6 @automaton
                                         6 datatype Parameter = Parameter(ip: Addr,
7 def Proc(ip: Addr, b: int):
                                                                               b: int)
                                         8 datatype State = State(pc: Loc, x: int)
      where = b > = 10
      class states:
                                         9 predicate aut_where(p: Parameter)
9
10
          pc: Loc
                                         10 { p.b \ge 10 }
                                         11 predicate initially (s: State, p: Parameter)
11
           x: int
      initially = (pc==1 and 0<=x<b)12 { s.pc = 1 \land 0 \le s.x < p.b
12
```

Fig. 2. DIONE types and states.

Fig. 3. Translated DAFNY from Figure 2.

advanced compiler/interpreter infrastructure of Python, and in the future an interpreter for Dione programs could be built on top of Python.

The syntax of Dione is a strict subset of Python 3.7 syntax. We interpret Dione code as defining an IOA as in [7]. At the top level of a Dione program, one can only define either a *type* or an *automaton*.

Data Types. A type definition is simply an assignment with type hints. The left hand side is annotated with the built-in base meta-type, type, to indicate the identifier represents a type. On the right hand side, one can provide an existing type to create a type synonym, or use a built-in type constructor to build new types, for example Enum and NamedTuple in Figure 2. The list of built-in types for DIONE can be found in our project website.

Automata. An automaton definition uses a PYTHON function definition syntax decorated with either @automaton or @composition to define a primitive or a composite automaton. The function parameters naturally serve as the parameters of the automaton. One can optionally add an assignment to the where variable to bound the parameter values for the automaton. For a primitive automaton, four DIONE program constructs, namely states, initially, signature, and transitions, are required to specify state variables, initial states, the signature, and the transition relation as discussed in Section 2. For a composition, components is required to specify the component automata. For either primitive or composite automaton, one can optionally assign an invariant of variable to specify the invariant.

States. The states class is to specify variables X and consists of only variables annotated with types as shown in Figure 2. The initially variable should be assigned with a Boolean expression over state variables and/or automaton parameter. An allowed initial state is when initially is evaluated to true.

Signature and Transitions. The signature class is for declaring the set of actions used in the automaton. It should contain only functions decorated with

@input, @output, or @internal, to declare an action in Σ^I , Σ^O , or Σ^H . Similarly, an assignment to where can be added to bound the parameters of the action. The where clause is crucial for sharing a same action name across multiple automata. For instance, the **output** action send defined in Figure 4 is constrained with messages sent from its own ip. Without the constraint, there can be no compatible automata using send as an **output** action because their output sets of actions are not disjoint. With the constraint, other automata using send as an **output** action are still compatible if the source address is not ip. To define transition relation for the automaton in transitions class, we follow [7] to specify preconditions and effects for actions. As show in Figure 4, the action is decorated with @pre as the precondition, and the effect of this action is specified within the function body. Currently, DIONE allows only some commonly used Python statements and expressions in the effect. An updated list of supported statements and expressions is available on our project website.

Composition. The components class is required for specifying a composite automaton. It is a list of component names annotated by the automaton instantiated with actual parameter values. For Figure 6 as an example, the Sys automaton is composed of three components, p1, p2, and env. Both p1 and p2 are instances of automaton Proc, and env is an instance of Env.

3.2 Translating DIONE to DAFNY

In this section, we informally describe the translation of program constructs from DIONE to DAFNY in our implementation. In particular, we illustrate the insights into systematically translating from notions of sets, subsets, and transitions in Section 2 to data types and predicates expressible in DAFNY. We first describe translating all constructs for one automaton with parameters and then we provide the composition of automata with compatibility checks by DAFNY.

Data Types. Benefitting from the rich types available in DAFNY, a broad class of types are supported in DIONE. Primitive types such as bool, int, str, nat, real, and bvN (bit-vectors with N bits) are translated to equivalent types in DAFNY. Python type float however is not supported due to the limitation of DAFNY. Collection types including Mapping, Sequence, Set, and Counter are available as map, seq, set, and multiset respectively. NamedTuple, Enum, and Union are directly modeled with inductive data types in DAFNY. Currently, we do not allow other kinds of user defined types for simplicity.

States and Automaton Parameters. The states class is used to declare variables of an automaton, or formally val(X). Instead of directly using a function or map in DAFNY for mapping from variable names X to values, we model the state with a record type, a special case of inductive data types with only one constructor. Each field of the record type then corresponds to a variable of the automaton. Similarly, we introduce a new record type Parameter to model the parameter space of the automaton. For the example in Figure 3, we simply

```
1 datatype Action=send (m: Msg) | recv (m: Msg)
                                                                                                                                                                   | hide (k: int)
                                                                                                            3 predicate Output (act: Action, p: Parameter)
       class signature:
                                                                                                             4 { act.send? ∧ act.m.src = p.ip }
                     @output
                     def send(m: Msg):
                                                                                                            5 predicate Input (act: Action, p: Parameter)
                                 where = (m.src == ip)6 { act.recv? }
                                                                                                            7 predicate Internal (act: Action, p: Parameter)
                     def recv(m: Msg): pass
                                                                                                            8 { act.hide? }
                                                                                                            9 predicate Signature (act: Action, p: Parameter)
                                                                                                          10 {Output(act,p) \left\rightarrow\left(act,p) \left\right\right\right) \left\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\righta
                     def hide(k: int): pass
 9
                                                                                                          11
10 class transitions:
                                                                                                          12 predicate pre_send(act: Action, s: State,
                                                                                                                                                                                    p: Parameter)
11
                     @output
                                                                                                          13
                     @pre(m==Msg(ip, B, 10)
                                                                                                         14 { act.send?\landact.m=Msg(p.ip,B,10)\land s.pc=1 }
12
                                     and pc == 1)
                                                                                                          15 function eff_send(act: Action, s: State,
13
                     def send(m):
                                                                                                                                                                                 p: Parameter): State
14
                                                                                                          16
15
                                 pc = pc + 1
                                                                                                          17
                                                                                                                             requires pre_send(act,s) {
                                   x = x + m.val
                                                                                                                             var s: State := s.(pc := s.pc+1);
                                                                                                          18
16
                                                                                                                             var s: State := s.(x := s.x+act.m.val); s
                                                                                                          19
```

Fig. 4. DIONE signature and transitions.

Fig. 5. Translated DAFNY from Figure 4.

define a Parameter type (line 6) and a State type (line 8). The bound over the parameter space is translated into the aut_where predicate (line 9). Initial set of states specified through (pc==1 and 0<=x<b) is translated to the initially predicate below:

Notice that initially refers to the parameter and bound the value of s.x with p.b. This translation ensures that checking $s \in \Theta_p$ is equivalent to asking initially (s,p).

Actions and Transitions. To model the set of actions for a network of automata, we collect all actions declared in the signature class from all automata. We then specify an algebraic data type Action with each action as an individual constructor. For example, we specify Action with two constructors send and recv at line 1 in Figure 5 representing sending and receiving messages where the message m is a parameter of the action. For each automaton, the three predicates input, output, and internal symbolically represent Σ_p^I, Σ_p^O , and Σ_p^H . Similarly, the translation makes sure $act \in \Sigma_p^O$ is equivalent to output (act,p) and so on. For example, output predicate (line 3) constrains the source m.src of sent messages with its own ip.

The precondition @pre for each transition is represented by a predicate, and the body of each transition is rewritten to an effect function using LET expressions ("var ...;" in DAFNY). The translation for the limited kinds of statements in DIONE, such as assignments and if-conditions, to a function producing a new state is not specific to DAFNY. It has been discussed thoroughly in other works such as [16] for PVS. We recommend interested readers to refer to [16] for details. The only major difference is that, we further specify the

```
{\scriptstyle 1} \ \textbf{module} \ \mathsf{Type} \ \{ \ \textbf{type} \ \mathsf{Msg} = \ldots; \ \textbf{type} \ \mathsf{Action} = \ldots \ \}
2 def Proc(ip:Addr, b: int): 2 module Proc { import Type
                                     type State = ...; datatype Parameter = ...
4 /* All predicates and functions */ }
                                     5 module Env { import Type; type State =
5 @automaton
                                      6 /* All predicates and functions */ }
6 def Env():
                                     7 module Sys { import Env; import Proc
                                      8 datatype State = State (env: Env. State,
                                                                     p1: Proc. State,
9 @composition
                                                                      p2: Proc. State)
10 def Sys():
                                     10
                                    11 predicate output(act: Action) {
       class components:
11
           env: Env()
                                    12 Env. output (act)
            p1: Proc(A,10)
p2: Proc(B,20)
                                    13 V Proc.output(act, Proc. Parameter(A,10))
13
                                     14 V Proc. output (act, Proc. Parameter (B, 20)) }
14
                                     15 // Similar for input, output, internal, transitions
15
                                     16 predicate invariant_of(s: State)
       invariant_of = \
16
                                    17 { 0 \le s \cdot p1 \cdot x < 10
18 } // End Sys
            0 <= p1.x < 10
```

Fig. 6. DIONE composition.

Fig. 7. Translated DAFNY from Figure 6.

precondition of the effect function with requires clause. This instructs DAFNY verifier to only consider states and actions satisfying the precondition.

Lastly, given all pairs of translated (pre_i , eff_i) for the automaton, the whole transition relation $\delta \subseteq Q \times \Sigma \times Q$ is modeled by the transitions predicate over current state s, a given action act, and next state s' as follows.

```
predicate transitions(s:State, act:Action, s':State)
{ (pre_1(act,s) \( \Lambda \) s'=eff_1(act,s)) \( \Lambda \) (pre_2(act,s) \( \Lambda \) s'=eff_2(act,s))\( \Lambda \)... }
```

Primitive Automaton with Parameters. A primitive automaton is translated into a DAFNY module to group Parameter and State types as well as functions and predicates defining actions and transitions. Action type and other types shared across multiple automata are declared as a separate module and imported to each automaton module. An example layout is shown in Figure 7. Note that both automata can define its own Parameter and State types without causing naming collision.

Composition and Invariants. Given all component automata and their corresponding modules, we define another module and import those component modules. We then can define State for the composition with State from each component module. The State essentially represents the Cartesian product of each component state space. With the new State type, we can then define the necessary predicates, namely initially, input, output, internal, and transitions, according to the composition operation defined in Section 2. For Figure 7 as an example, we can implement output predicate (line 11) using the disjunction of output from each component to represent $\Sigma_{Sys}^O = \Sigma_{Env}^O \cup \Sigma_{Proc(A,10)}^O \cup \Sigma_{Proc(B,20)}^O$. Do notice that the parameter value for instantiating each component automaton needs to be passed down to the predicates from sub-modules. Finally, translating an invariant of an au-

tomaton becomes defining an invariant_of predicate with current state as the argument and is similar to defining initially.

Checking Compatibility Axioms for IOA. In specifying systems in IOA, several simple mistakes are easily overlooked when designing larger and more complicated systems. We list three required axioms when specifying IOA and provide translated lemmas below to detect simple violations with DAFNY. For simplicity, we do not include parameters here. The first lemma simply states Σ^I , Σ^O , and Σ^H are mutually disjoint. The second lemma checks the IOA is input-enabled, i.e., $\forall s \in Q.a \in \Sigma^I. \exists s' \in Q.(s,a,s') \in \delta$. The third states that two component IOAs assigned with concrete parameters are compatible when $\Sigma^O_1 \cap \Sigma^O_2 = \emptyset$ and $\Sigma^H_1 \cap \Sigma_2 = \Sigma^H_2 \cap \Sigma_1 = \emptyset$.

3.3 Bounded Model Checking and k-induction with DAFNY

With the system automaton with invariant specification translated to DAFNY, we now discuss how to preform Bounded Model Checking (BMC) to detect violations. Given a bound k, BMC intends to prove that the invariant holds at any state reachable from initial states in k transitions. Formally, given a candidate invariant φ , we check the validity of following proposition:

$$\forall s_0, a_1, s_1, ..., a_k, s_k, (s_0 \in \Theta \land \bigwedge_{i=0}^{i < k} \delta(s_i, a_{i+1}, s_{i+1})) \implies \bigwedge_{i=0}^{i < k+1} \varphi(s_i) \quad (1)$$

We can ask DAFNY to prove the following equivalent lemma:

The underlying engine of DAFNY translates the lemma further into formulas in First Order Theories supported by SMT solver, Microsoft Z3. If the lemma is proven, then it proves no violation within k transitions. Otherwise, Z3 returns a valuation disproving the formula, and we can use the debugger in the DAFNY

² For readers unfamiliar with DAFNY, question mark symbol '?' and prime symbol '', and prime symbol '', '' can be used in identifiers.

tool chain to find the valuation of state and action variables. That is, we can reconstruct the execution from a counterexample of Proposition 1.

Similarly, to prove invariant φ by k-induction, we prove Proposition 1 to ensure the states reachable in k-steps are within the invariant, and then prove Proposition 2 to show the invariant is inductive at (k+1)-th step:

$$\forall s_0, a_1, s_1, ..., a_{k+1}, s_{k+1}, (\bigwedge_{i=0}^{i < k+1} \varphi(s_i) \wedge \delta(s_i, a_{i+1}, s_{i+1})) \implies \varphi(s_{k+1})$$
 (2)

Or in Dafny:

```
lemma induction_proof?(s0: State,
  a1: Action, s1: State,...,ak: Action, sk: State,a{k+1}: Action,s{k+1}: State)
requires invariant_of(s0) \( \Lambda \) transitions(s0,a1,s1) \( \Lambda \)...
  \( \Lambda \) invariant_of(sk) \( \Lambda \) transitions(sk,a{k+1},s{k+1})
ensures invariant_of(s{k+1})
```

4 Case Studies with DIONE

Table 1. Protocols verified with DIONE. #A is the number of primitive and composite automata. #Ln (resp., #Ty, #Fn, #Lem) is the total number of lines (resp., custom types, functions, lemmas) in DIONE or DAFNY code. Numbers inside parentheses indicate manually added items for proving. #k is the number of transitions for BMC and induction proof. #PO is the number of proof obligations reported by DAFNY. Time and Mem shows the time and peak memory usage to prove with DAFNY.

	DIONE			Dafny						Time	Mem
Protocol	#A	# T y	#Ln	#Ту	#Fn	#Lem	#Ln	#k	#PO	(s)	(MB)
StableArray	1	1	24	4	17(0)	4(1)	115	0	32	4	84.5
StableRing	1	1	20	3	13(0)	5(2)	115	0	66	166	377.6
AsyncLCR	2	2	47	7	26(2)	5(0)	159	0	51	3	100.3
CANArb	3	2	62	6	38(5)	6(0)	333	24	60	287	169.7

We analyze four interesting distributed protocols with very different structure and properties using DIONE: The first two, StableArray and StableRing, are self-stabilizing mutual exclusion algorithms on an array and a ring network, AsyncLCR is a classic leader election algorithm on a ring, and CANArb is the protocol that arbitrates the access to CAN bus. For each case study, we first describe our DIONE models and invariants, then explain necessary auxiliary lemmas and functions added in the translated DAFNY code for verification, and finally discuss verification results based on BMC and k-induction using the DAFNY verifier. All experiments are conducted with DAFNY 2.2.0 on Ubuntu 18.04 LTS running on Intel Xeon CPU E3-1240 v3 at 3.40GHz with 4 cores and 8 GB RAM. To obtain

the verification results, all invariants for one system are conjuncted together as one big invariant predicate. DAFNY would verify all lemmas specified in a case study including disjointness of actions for primitive automata, compatibility in compositions, BMC, and k-induction for invariant checking. Each reported time and memory usage is the average of running DAFNY three times. All our code, the DIONE translator, the input specifications for the examples in DIONE language, their DAFNY translations, and the proofs are available at our repository.

4.1 Self-stabilization Protocol on a Bidirectional Array

StableArray in Figure 1 is the self-stabilizing algorithm for mutual exclusion on a bidirectional array topology from [8, Section 17.3.2]. The system consists of an array of N processes with at least two processes, i.e., $N \geq 2$. All processes except for process 0 and N-1 can remain in any of $\{0,1,2,3\}$ states. Process 0 should stay in $\{1,3\}$, and process N-1 should stay in $\{0,2\}$. We model the system with a global state variable, s, of sequence type with the length |s|=N, and each process state s[i] is of an enumeration type from 0 to 3, and we also specify that s[0] should be 1 or 3 and s[N-1] be 0 or 2. Formally, the following invariant should trivially hold for StableArray:

Invariant 1.
$$|s| = N \land (s[0] = 1 \lor s[0] = 3) \land (s[N-1] = 0 \lor s[N-1] = 2)$$

A process i is considered holding a token if any of its neighbor process j satisfies s[j] = incre(s[i]) where incre finds the next value in the enumeration type, equivalently, incre(n) = (n+1)%4. A process i holding a token except for process 0 and N-1 can initiate a transition to copy the state from the above mentioned neighbor j. If process 0 or N-1 is holding a token, it increments twice, i.e., s[0] = incre(incre(s[0])). For achieving mutual exclusion, a legal configuration of the system is when the number of processes holding tokens is exactly one. A desired invariant is that, once in a legal configuration, the system continues to be in the legal configuration. Here we prove a relaxed invariant that the number of processes holding a token is at most one. Formally,

$$Invariant \ 2. \ \left| \left\{ i \mid 0 \leq i < N \land \begin{pmatrix} i \neq 0 \land s[i-1] = incre(s[i]) \lor \\ i \neq N-1 \land s[i+1] = incre(s[i]) \end{pmatrix} \right\} \right| \leq 1$$

We can specify the conjunction of Invariant 1 and 2 in Dione as below:

Proof Strategy. As shown in Table 1, we are able to automatically prove that the conjunction of Invariant 1 and 2 is an inductive invariant for arbitrarily many processes (N) in 4 seconds. We however needed to manually introduce one auxiliary fact (lemma) in the translated DAFNY code to make the proof go through by induction. The intuition behind this lemma is that, when some process i makes a transition, only processes i-1, i, and i+1 can either lose or

gain tokens, and all other processes should remain the same. Therefore, the set of processes holding tokens before and after a transition could only differ for these three processes. This extra lemma simply enumerates all eight cases where the size of the set may either increase, decrease, or remain the same, and DAFNY is able to prove this lemma automatically. We then use this lemma over the two states before and after a transition in specifying the proof strategy, DAFNY verifier further infers that those cases where the size increases are impossible and successfully proves that the invariant is inductive.

4.2 Self-stabilization Protocol on a Ring

The second case study StableRing is the Dijkstra's famous self-stabilizing algorithm for mutual exclusion on a ring [8, Section 17.3.1]. Our DIONE model can be found at this link. The system is parameterized with an arbitrary number of N processes and K states where N < K. Each process always stays in one of $\{0, ..., K-1\}$ states. Similarly, we model the system with a global sequence s as the state variable with |s| = N and $0 \le s[i] < K$ for each process i; the following invariant should be maintained (we skip DIONE versions from now on):

Invariant 3.
$$|s| = N \land \forall i (0 \le i < N \implies 0 <= s[i] < K)$$

Process i is considered holding a token by checking its predecessors i-1 and see if it's in one of two conditions: (1) $i=0 \land s[i]=s[N-1]$ or (2) $i \neq 0 \land s[i] \neq s[i-1]$. Any process except process 0 holding a token can initiate a transition to copy the state from its predecessor in the ring, i.e., s[i]=s[i-1]. If process 0 is holding a token, it can assign itself the value from process N-1 plus one, i.e., s[i]=(s[N-1]+1)%K. The definition of legal configuration is the same that only one process holding a token. For StableRing, we prove the desired invariant that, once in a legal configuration, the system continues to be in the legal configuration. Formally, the following invariant should hold:

$$Invariant \ 4. \ \left| \left\{ i \mid 0 \leq i < N \land \begin{pmatrix} i = 0 \land s[i] = s[N-1] \lor \\ i \neq 0 \land s[i] \neq s[i-1] \end{pmatrix} \right\} \right| = 1$$

Proof Strategy. As reported in Table 1, two of five lemmas are manually written to prove Invariant 4. Our first lemma is to establish the axiom that, if an element i is in a set I, then $|I| = 1 \iff I = \{i\}$. This helps DAFNY infer from inductive hypothesis that only one process i was holding a token in the prestate s before a transition. Notice that after transition, process i' = (i+1)%N will replace process i to hold the token. This leads to only two possible scenarios in the state after transition s': (1) i' = 0 and every process state is the same as its predecessor OR (2) $i' \neq 0$ and only process state s'[i'] is different from its predecessor. Otherwise, at least two processes will be different from their predecessors and hence holding tokens. The difficulty for DAFNY here is to infer s'[0] = s'[N-1] in scenario 1 to show process 0 is holding a token and s'[j] = s'[j-1] in scenario 2 so that every other process j does not hold a token. Based on this observation, the second simple lemma we add asserts that, if every element is equal to its predecessor in

a sequence, then all elements are identical. This lemma can be proven by DAFNY by induction on the length of the sequence. Our proof strategy thus simply splits into two cases i=N-1 and $i\neq N-1$ and applies the lemma over appropriate (sub-)sequences of s'. DAFNY can then infer the intermediate result mentioned above and prove the invariant inductively.

4.3 Asynchronous Leader Election on a Ring

Our third case study explores the possibility to use DIONE to model parameterized systems via composition and check the correctness. AsyncLCR is a simplified version of the leader election algorithm from [17, Section 15.1]. The DIONE model can be found at this link. In this system, each process is instantiated with a unique $u \in UID$ for voting. The main algorithm flow is the following: Each process maintains a queue q of votes to be sent to its successor and a status variable initialized as UNKNOWN. When a vote from its predecessor is delivered, only the vote greater than u is added into the queue. The leader is decided when a process receives a vote v where v = u and sets its status to CHOSEN. A process with status = CHOSEN can then report itself as the leader and set its status to REPORTED. The algorithm guarantees that the reported leader process should have the maximum UID over all processes. For simplicity, our model first assumes the vote is delivered instantaneously and removes the need of channel automata by merging send and receive into one send recv action. Second, although it is straightforward to model an arbitrary number of processes in DIONE, we only consider three processes to reduce the verification effort in this study. Thus, our model for the system is a composition of three automata $\{P_0, P_1, P_2\}$. Each P_i is an instance of the same automaton design assigned with actual parameters, the index i and the UID value u_i . We prove the key invariant described in [17] that no process other than the process with the max UID can report itself as the leader. Formally,

$$\mathit{Invariant 5.} \ \bigwedge_{i=0}^2 \left(u_i \neq \max(u0,u1,u2) \ \Longrightarrow \ P_i.status = \mathtt{UNKNOWN} \right).$$

Proof Strategy. In Table 1, we reported that two functions and no extra lemmas are needed. These two functions, needed for an auxiliary invariant, are currently not expressible in DIONE. The first required function i_{max} takes all u_i and returns the index i with the greatest value, and the second function between(lo, i, hi) checks if lo must pass thru i to reach hi in the ring topology where $lo \neq i$ and $lo \neq hi$. We then use both functions to build the invariant modified from [17]:

Invariant 6.
$$\bigwedge_{i=0}^{2} \bigwedge_{j=0 \land j \neq i}^{2} (i \neq i_{max} \land between(i, i_{max}, j)) \implies u_i \notin P_j.q.$$

This invariant is based on that i must reach i_{max} before reaching j, so the vote u_i would always be dropped before reaching j. DAFNY is able to prove the conjunction of Invariant 5 and 6 to be inductive without any additional lemma.

4.4 CAN bus Arbitration

Our last case, we consider a vastly different communication protocol, the arbitration protocol for Controller Area Network (CAN bus). CAN bus is widely used in automotive for communications between Electronic Control Units (ECUs). CAN bus Arbitration is a bit-level arbitration protocol to resolve collision between ECUs according to the priority of CAN data frames being sent. Due to the lack of space, we only provide a sketch of the arbitration protocol and one invariant property. Interested readers can find detailed descriptions in Appendix A.1

At a high level, the protocol assumes all ECUs are synchronized with a global clock. In each clock cycle, all ECUs attempting to send a data frame simultaneously transmit the 11-bit arbitration field of the frames bit by bit. This 11-bit field represents the priority and should be different for competing frames. Each ECU also monitors the bus to see if it loses the arbitration and stops transmitting. After all 11 bits are transmitted, only the winner continues to transmit the rest of the frame. A major property of CAN specified in [11, Section 6.3] states the following: "The transmitter with the frame of highest priority shall gain the bus access". We can paraphrase the statement to be more like an invariant as follows: "The ECU with the frame of highest priority shall keep transmitting in every cycle".

Our DIONE model available at this link is designed following specifications in [11]. Detail description of the model is in Appendix A.2. In short, the model contains a NodeSeq automaton that uses a sequence of states to model an arbitrary number of participating ECUs similar to our first two case studies. A unique design choice in the NodeSeq automaton is to use a list comprehension expression to update the state of an individual node. This eliminates the need for a for-loop and later on reduces the effort for verification. Additionally, a Bus automaton is deigned to model the bus. Notice that this Bus automaton can be considered as a broadcast channel where all ECUs send to and recv from the Bus automaton. The system is naturally the composition of these two automata, and the invariant is formulated accordingly.

Proof Strategy. In order to prove the invariant for this case study, we have to manually introduce the following code in DAFNY. First, to reflect the synchronized communication, only the executions with actions alternating between send and recv should be considered as one send and one recv action compose a cycle. Second, four auxiliary functions are added to support the aforementioned list comprehension expression. However, these functions are for generic type of lists and can be reused in more cases. Finally, we have to manually figure an over-approximation of one effect of the recv action, and use this approximation in the transition relation instead. Fortunately, we can instruct DAFNY to automatically check whether the approximation is indeed an over-approximation.

With above mentioned manual efforts, DAFNY is able to prove both BMC and induction with k=24. k=24 is simply because there are only 11 bits in arbitration field, and hence the states should stay the same ever after 12 cycles, i.e., 24 steps. This showcases the potential of DIONE in verifying parameterized

system composed with different environment models, and this can be achieved within a manageable amount of manual effort.

5 Conclusion

We presented DIONE, a formal framework for analyzing distributed systems with specification language based on I/O automata and verification methods powered by DAFNY. The key compatibility conditions for IOA models are encoded as lemmas that can be automatically discharged by DAFNY in all of our cases. Our case studies show that a range of different distributed protocols can be naturally modeled in DIONE for Bounded Model Checking and k-induction invariant checking. The translated DAFNY specifications in the case studies were analyzed with DAFNY verifier automatically, with little extra manual annotations. These results are encouraging, and suggest several exciting future directions. (1) In the spirit of [10], IOA specifications could be translated to synthesizable DAFNY code, and hence, to correct-by-construction C# implementations for protocols. (2) Uniform or parameterized verification for distributed systems using small model properties [21] or theory of arrays [9]. Finally, (3) DIONE could be extended to support timed, hybrid, and probabilistic I/O Automata [12].

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A CAN bus Arbitration

CAN bus is a long lasting and extremely popular communication protocol for Electronic Control Units (ECUs) in automotive. Specifically, the data link layer of CAN bus implements Carrier-sense multiple access with collision detection (CSMA/CD) to decide which ECU has the access to CAN bus at a certain time. In this section, we first describe the arbitration protocol handling collisions and then give our DIONE model for the protocol.

A.1 Protocol Description

According to [11, Section 10.4], it is assumed that all nodes are synchronized for bit transmission, and every node implements the same mechanism to serially transmit and receive MAC data frame as shown in Figure 8. The frame format starts with a 11-bit arbitration field (ID bits) representing the priority of this frame, and smaller value of ID bits represents a higher priority. Arbitration happens only when multiple nodes simultaneously send start of frame (SOF) bit and subsequently each of ID bits. Figure 8 shows an arbitration between three nodes, Node 2, 5, and 14, sending CAN frames with priority 0010101010, 00000100110, and 00000110100. Bus row represents the bit values monitored on bus. When the bits sent by different nodes differs, nodes sending 0 (DOMINANT)

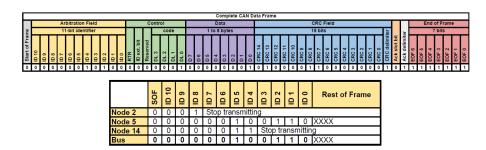


Fig. 8. Complete CAN data frame structure (Top). Example arbitration with three nodes and CAN Data observed on bus (Bottom).

win over those sending 1 (RECESSIVE) such as at ID8 and ID4. Nodes who lost the arbitration then stop transmitting and only send logic 1 ever after. Conceptually, CAN bus is a logic AND gate of an arbitrary number of inputs so that logic 0 wins at every cycle, and the frame with most leading 0s wins, such as in Figure 8 that node 5 wins and transmits the rest of frame.

A.2 DIONE model

The system is composed of a NodeSeq automaton and a Bus. The Bus automaton computes the logical conjunction via universal quantification on the sequence of bits msgs, stores at the bus variable at send, and returns the value of bus through recv action. The NodeSeq automaton is an abstraction of an arbitrary number of nodes. Its state consists of a pos to indicate which ID bit is currently transmitted and a sequence of NodeStates for individual nodes. Each node has a transmit variable to denote if this node is transmitting and an arb variable for the 11-bit arbitration field. Initially, pos starts at 10 and transmit is true to model that all nodes start to send at the same time. When NodeSeq output a send action to Bus, msgs is the sequence of bits sent by all nodes. Each node either extracts the bit from arb via bv_extract at current pos or sends RECESSIVE if it already stopped transmitting.

Upon receiving msg from Bus in recv action, each node then compares if the received bit is what it transmitted. If a node observes unequal bit values, then this node will set its transmit to False to stop transmitting. Notice that this requires an iteration through all nodes. Here, we choose to use the list comprehension to create a new sequence of NodeState with each node updated using if-then-else expression. The state variable nodes is then updated with this new list. Also pos is decremented until less than 0.

Finally, the invariant described in [11, Section 6.3] states the following: "The transmitter with the frame of highest priority shall gain the bus access", and explicitly assumes that the priority of each frame is distinct. Our invariant is therefore relaxed to allow multiple frames of the same highest priority. To express this invariant, we assume a function <code>i_min</code> returning the index of any node with the smallest <code>arb</code> field value. The invariant is then formally specified by saying that <code>i_min</code> node should stay transmitting during the whole arbitration.