Markowitz (1959) mean-variance optimization (MVO), has been the standard to determine efficient portfolios. MVO works by selecting assets which their volatilities are anti-correlated to each other, but their returns are correlated, to create the optimal portfolio that lowers the overall risk. Markowitz MVO algorithm provides the correct method to invest in a series of assets given that all the inputs are 100% correct. However, the problem that limits the performance of the MVO algorithm is the uncertainty in the data. In other words, MVO is highly affected by estimation errors. As result of this, MVO assigns a set of unrealistic weightages to the assets in the portfolio (since it doesn't account for possible future data variance) which results in underperformance of the portfolio when used with future data. To rectify this problem, one can introduce resampling techniques such as Monte Carlo Simulation (used in this assignment) or bootstrapping methods into the MVO problem. By doing so, you will have a better understanding of the uncertainty within the data. By resampling the data, we can simulate new dataset that will represent what the future data might be (account for possible variances that the new data may have) which is typically more stable and realistic compared to the original data.

Below in Figure 1, we can see the classical frontier (not resample data – blue line) and the resampled efficient frontier (red line). The resampled efficient frontier is the average of all the 100 simulated data (yellow dashed line) from the original dataset (consisting of 25 assets). By inspecting Figure 1, we observe a significant dispersion of the resampled frontier. The volatility/standard deviation of the classical efficient frontier ranges approximately between 0.03 and 0.58, with a max return of 0.1, whereas the volatility/standard deviation of the resampled efficient frontier ranges approximately between 0.03 and 0.27 with a max return of 0.091. This means the resampled efficient frontier has less than half of the risk range of the classical efficient frontier while providing almost identical max returns. This is why investors should stray away from using MVO without any resampling as it typically yields higher risk for the same returns. If the investor is a risk hater, the minimum variance portfolio is the optimal portfolio for any simulated efficient frontier. In contrast, if an investor is a risk lover, the maximum return portfolio is the optimal portfolio for any simulated efficient frontier.

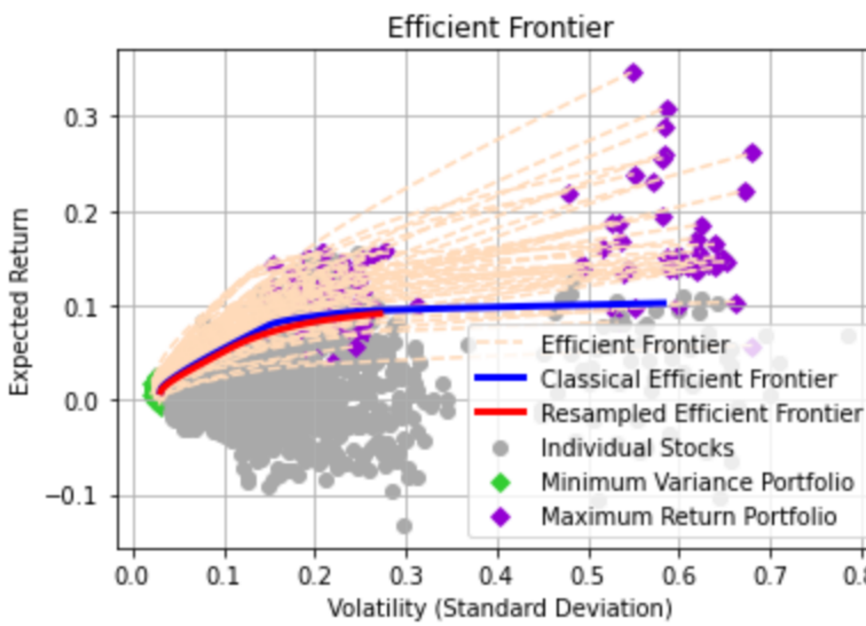

Figure 1: Classical, Resampled, and All Simulated Efficient Frontier Plots

Figure 2 displays the portfolio composition maps for both the classical and resampled efficient frontier. Every color in the composition map figure represents a unique stock. A vertical slice of the chart demonstrates the weights of each stock in the portfolio at that level of risk. When we compare the figure on the left (Classical Efficient Frontier) with the figure on the right (Resampled Efficient Frontier) we can clearly observe that the resampled composition map has much more smoother transitions from one risk level to the other. In addition, in the classical composition map, the weights are disproportional, meaning the portfolios are made of primarily one stock and small portions of another stock (e.g., at risk level 46 of the classical composition map, the portfolio is primarily made of purple and a bit of cyan). In addition, in the left figure, some of the stocks are not included over the entire spectrum of risk such as BABA, JPM, FB, etc., which leads to the portfolio being riskier as it's not as diverse (less stocks in the portfolio for a given risk level). Whereas the right figure includes all the assets which leads to the portfolios being more diverse. This is why investors tend to not use Classical Efficient Frontier without any resampling techniques.

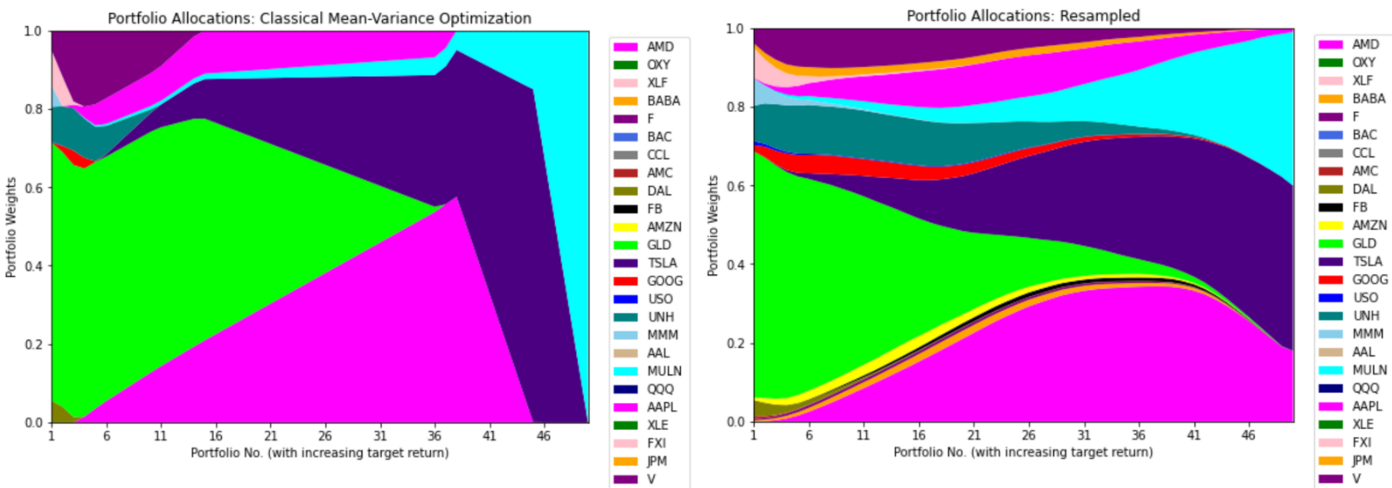

Figure 2: Classical and Resampled Portfolio Allocations Plots

The performance of all 6 portfolios can be seen below:

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On the Classical Efficient Frontier, the minimum variance portfolio returns $ [28.22424688]
On the Classical Efficient Frontier, the maximum return portfolio returns $ [-584.59094047]
On the Classical Efficient Frontier, the optimal Sharpe ratio portfolio returns $ [279.5883226]

On the Resampled Efficient Frontier, the minimum variance portfolio returns $ [59.06599267]
On the Resampled Efficient Frontier, the maximum return portfolio returns $ [37.33403948]
On the Resampled Efficient Frontier, the optimal Sharpe ratio portfolio returns $ [327.28394219]
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Figure 3: Portfolios Performance Results

As seen in Figure 3, the Resampled Efficient Frontier outperforms the Classical Efficient Frontier on all three performance metrics (minimizing risk/variance, maximizing return, and maximizing the risk adjusted return (Sharpe ratio)). In particular, we can see the maximum return portfolio is significantly improved from the Classical to the Resampled Efficient Frontier. The maximum return portfolio value changed from the investor losing $584.6 and increased to a value of positive $37.3 (gain). Also, we can see that the sharp ratio in both the classical and resampled provide that best possible proportion of the assets between the different methods (minimum variance and maximum return). This makes sense as the sharp ratio is a direct method to measure the reward to risk ratio and will inform the investor how much more risk they are taking on for additional return and essentially provides the best portfolio in the efficient frontier. The sharp ratio in this assignment was calculated using a risk-free rate of 0.002 which saves the investor from risky investments. Also, the maximum return portfolio performs the worst in both the classical and resampled efficient frontiers because it takes on the most risk and has the highest variance. Intuitively, we might think the maximum return leads to the highest return, however, this comes with the cost of the highest risk (most variance). If investor is a risk lover and wants the highest possible return regardless of the risk, they should use the maximum return portfolio in the classical method (most variance), however, this is not recommended for most investors.