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Лабораторна робота №4 Розв’язування задачі лінійної регресії

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Мета: навчитися працювати з простою моделлю лінійної регресії.

Варіант: 15 – dataset “Weather Dataset” (<https://www.kaggle.com/muthuj7/weather-dataset>)

Хід виконання роботи:

```
[1] import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error, r2_score
from datetime import datetime
import itertools
```

```
[2] # connecting to gdrive
from google.colab import drive
drive.mount('/content/gdrive', force_remount=True)
gdrive_path = f"/content/gdrive/MyDrive/ds/"

Mounted at /content/gdrive
```

```
[3] # reading training dataset to pandas dataframe
dataset = pd.read_csv("/content/gdrive/MyDrive/ds/weatherHistory.csv")
# show general info about the original dataframe
dataset.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 96453 entries, 0 to 96452
Data columns (total 12 columns):
#   Column                                Non-Null Count  Dtype
---  -
0   Formatted Date                        96453 non-null  object
1   Summary                              96453 non-null  object
2   Precip Type                          95936 non-null  object
3   Temperature (C)                     96453 non-null  float64
4   Apparent Temperature (C)            96453 non-null  float64
5   Humidity                            96453 non-null  float64
6   Wind Speed (km/h)                   96453 non-null  float64
7   Wind Bearing (degrees)              96453 non-null  float64
8   Visibility (km)                     96453 non-null  float64
9   Loud Cover                          96453 non-null  float64
10  Pressure (millibars)                 96453 non-null  float64
11  Daily Summary                        96453 non-null  object
dtypes: float64(8), object(4)
memory usage: 8.8+ MB
```

```
[4] # converting date strings into numeric timestamps ignoring years notation
```

```
def get_timestamp(date_str):  
    date_obj = datetime.strptime(date_str, '%Y-%m-%d %H:%M:%S.%f %z')  
    timestamp = date_obj.month*100 + date_obj.day  
    return timestamp
```

```
# generating timestamps to distinguish Seasons of the year
```

```
dataset["Season"] = dataset["Formatted Date"].apply(get_timestamp)
```

```
# sorting dataframe rows by the month-day timestamps
```

```
dataset.sort_values('Season', ascending=True, inplace=True)
```

```
[5] # selecting columns of numerical type for the linear regression analysis
```

```
data_num = dataset.drop(["Daily Summary", "Precip Type", "Summary", "Formatted Date"], axis=1)
```

```
# show general info of the resulting dataframe subset
```

```
data_num.info()
```

```
<class 'pandas.core.frame.DataFrame'>
```

```
Int64Index: 96453 entries, 90576 to 72212
```

```
Data columns (total 9 columns):
```

#	Column	Non-Null Count	Dtype
0	Temperature (C)	96453 non-null	float64
1	Apparent Temperature (C)	96453 non-null	float64
2	Humidity	96453 non-null	float64
3	Wind Speed (km/h)	96453 non-null	float64
4	Wind Bearing (degrees)	96453 non-null	float64
5	Visibility (km)	96453 non-null	float64
6	Loud Cover	96453 non-null	float64
7	Pressure (millibars)	96453 non-null	float64
8	Season	96453 non-null	int64

```
dtypes: float64(8), int64(1)
```

```
memory usage: 7.4 MB
```

```
[6] # show per-column description of numerical data
```

```
data_num.describe()
```

	Temperature (C)	Apparent Temperature (C)	Humidity	Wind Speed (km/h)	Wind Bearing (degrees)	Visibility (km)	Loud Cover	Pressure (millibars)	Season
count	96453.000000	96453.000000	96453.000000	96453.000000	96453.000000	96453.000000	96453.0	96453.000000	96453.000000
mean	11.932678	10.855029	0.734899	10.810640	187.509232	10.347325	0.0	1003.235956	668.106995
std	9.551546	10.696847	0.195473	6.913571	107.393428	4.192123	0.0	116.969906	345.054625
min	-21.822222	-27.716667	0.000000	0.000000	0.000000	0.000000	0.0	0.000000	101.000000
25%	4.688889	2.311111	0.600000	5.828200	116.000000	8.339800	0.0	1011.900000	402.000000
50%	12.000000	12.000000	0.780000	9.965900	180.000000	10.046400	0.0	1016.450000	702.000000
75%	18.838889	18.838889	0.890000	14.135800	290.000000	14.812000	0.0	1021.090000	1001.000000
max	39.905556	39.344444	1.000000	63.852600	359.000000	16.100000	0.0	1046.380000	1231.000000

The Loud Cover column should be deleted, and the Pressure column should be cleaned from zero values considered as empty cells due to the common lower limit of 950 millibars.

```
[7] # removing column containing a static zero value
data_num.drop(["loud Cover"], axis=1, inplace=True)
# removing rows containing impossible zero values
data_num = data_num.drop(data_num.loc[data_num["Pressure (millibars)"] < 900].index)
# show per-column description of numerical data
data_num.describe()
```

	Temperature (C)	Apparent Temperature (C)	Humidity	Wind Speed (km/h)	Wind Bearing (degrees)	Visibility (km)	Pressure (millibars)	Season
count	95165.000000	95165.000000	95165.000000	95165.000000	95165.000000	95165.000000	95165.000000	95165.000000
mean	11.949284	10.878331	0.734965	10.784307	187.428015	10.381889	1016.814140	667.020281
std	9.546270	10.686240	0.195589	6.892106	107.405898	4.185364	7.778356	344.388383
min	-21.822222	-27.716667	0.000000	0.000000	0.000000	0.000000	973.780000	101.000000
25%	4.733333	2.338889	0.600000	5.812100	116.000000	8.420300	1012.120000	402.000000
50%	12.038889	12.038889	0.780000	9.917600	180.000000	10.046400	1016.550000	702.000000
75%	18.838889	18.838889	0.890000	14.119700	290.000000	14.908600	1021.160000	1001.000000
max	39.905556	39.344444	1.000000	63.852600	359.000000	16.100000	1046.380000	1231.000000

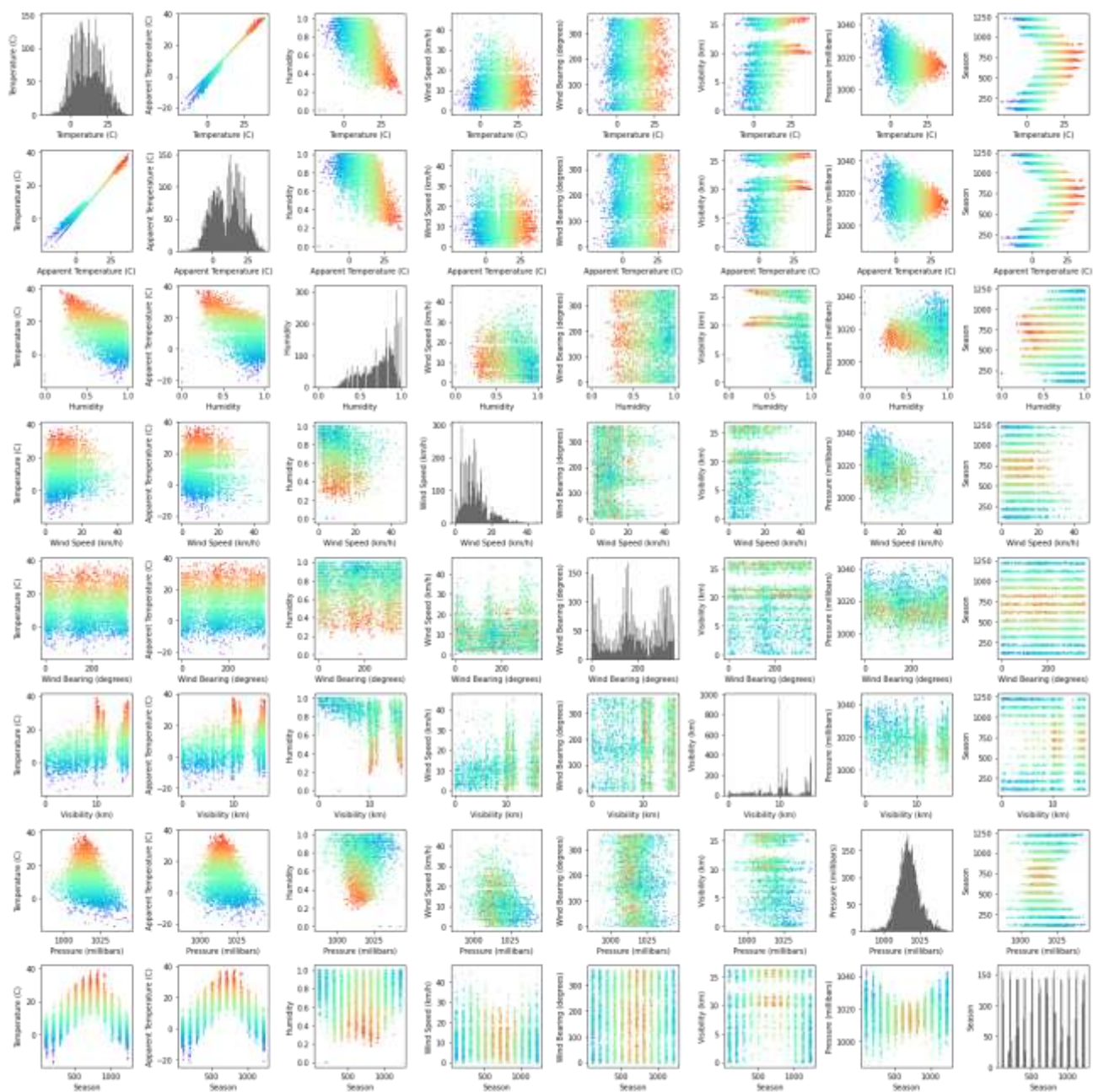
```
[8] # show first 5 rows of the selected dataframe subset
data_num.head()
```

	Temperature (C)	Apparent Temperature (C)	Humidity	Wind Speed (km/h)	Wind Bearing (degrees)	Visibility (km)	Pressure (millibars)	Season
90576	-8.561111	-12.072222	0.88	6.4400	140.0	15.7297	1031.31	101
73039	1.116667	-1.527778	0.86	8.4042	111.0	5.4257	1023.21	101
73038	1.177778	-2.116667	0.85	10.9319	111.0	8.0017	1023.37	101
73037	-0.883333	-2.588889	0.92	5.0393	121.0	9.7566	1023.37	101
73036	-0.411111	-2.950000	0.90	7.2450	121.0	6.5091	1023.62	101

```
[9] # drawing a matrix of 2D scatter plots to visualize inter-variable dependencies
# for all columns in the selected dataframe subset
def scatter_matrix(data, color_col="Temperature (C)", palette="rainbow", save_file=None):
    num_cols = data.columns.tolist()
    fig, axes = plt.subplots(nrows=len(num_cols), ncols=len(num_cols), figsize=(20, 20))

    # Loop through each subplot and plot scatter plot
    for i, ax_i in enumerate(axes):
        for j, ax_j in enumerate(ax_i):
            if i == j:
                ax_j.hist(data[num_cols[i]], bins=100, color="dimgray")
            else:
                ax_j.scatter(data[num_cols[i]], data[num_cols[j]], s=1, c=data[color_col], cmap=palette)
                ax_j.set_xlabel(num_cols[i])
                ax_j.set_ylabel(num_cols[j])
    plt.tight_layout()
    if save_file is not None:
        plt.savefig(f"/content/gdrive/MyDrive/ds/{save_file}.jpg")
    plt.show()
```

```
[10] scatter_matrix(data_num.loc[:20, :], save_file="full_scatter_matrix")
```




```
[11] # calculating the measures of Total Correlation Strength (TCS) between each column
# and all other columns in the dataframe subset selected for the analysis
tcs = data_num.corr().abs().sum(axis=1).sort_values(ascending=False)
# reordering columns in the selected dataframe
data_num = data_num[list(tcs.index)]
# displaying columns sorted by their TCS scores reduced in the diagonal values
tcs-1
```

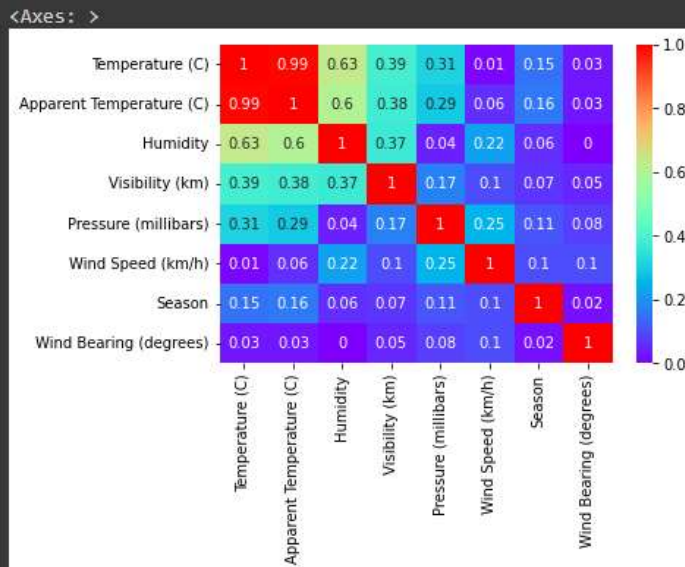
```
Temperature (C)      2.521071
Apparent Temperature (C) 2.512137
Humidity             1.928309
Visibility (km)       1.528236
Pressure (millibars)  1.251767
Wind Speed (km/h)     0.847956
Season               0.669971
Wind Bearing (degrees) 0.308454
dtype: float64
```

Variables with high total correlation strength have a stronger linear relationship with other variables in the dataset, making them good candidates to model as output variables in a linear regression model.

On the other hand, variables with low total correlation strength have weaker linear relationships with each other, making them good predictors for a linear regression model.

However, Wind Bearing variable is considered insignificant and ought to be removed from the selected dataset columns.

```
[20] # creating a heatmap displaying the correlation matrix
sns.heatmap(data_num.corr().abs().round(2), annot=True, cmap="rainbow")
```



Humidity, Temperature, Visibility, and Pressure columns demonstrate the most significant influence and strong connections among each other.

While Season and Wind Speed columns show more shallow linear relationship between them and any other columns.

There is a notable multicollinearity between Apparent Temperature and Temperature columns, which should be accounted for in the linear regression model.

```
[13] # removing insignificant columns and multicollinearity in the selected dataframe
data = data_num.drop(["Apparent Temperature (C)", "Wind Bearing (degrees)"], axis=1)
data_cols = list(data.columns)

# creating a list of possible output labels
labels = data_cols[:4]

# generating new correlation matrix
data_corr = data.corr().abs().round(2)
data_corr
```

	Temperature (C)	Humidity	Visibility (km)	Pressure (millibars)	Wind Speed (km/h)	Season
Temperature (C)	1.00	0.63	0.39	0.31	0.01	0.15
Humidity	0.63	1.00	0.37	0.04	0.22	0.06
Visibility (km)	0.39	0.37	1.00	0.17	0.10	0.07
Pressure (millibars)	0.31	0.04	0.17	1.00	0.25	0.11
Wind Speed (km/h)	0.01	0.22	0.10	0.25	1.00	0.10
Season	0.15	0.06	0.07	0.11	0.10	1.00

```
[14] # defining a lower threshold of the correlation matrix values
lower_threshold = 0.1

# forming a list of relevant factors for the selected labels
plan = {}
for label in labels:
    plan[label] = data_corr.columns[(data_corr.columns != label)
    & (data_corr.loc[label, :] >= lower_threshold)].tolist()

# showing label and its factors selected by the criteria of correlation matrix analysis
for label, factors in plan.items():
    print(f"{label} - {factors}")

Temperature (C) - ['Humidity', 'Visibility (km)', 'Pressure (millibars)', 'Season']
Humidity - ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)']
Visibility (km) - ['Temperature (C)', 'Humidity', 'Pressure (millibars)', 'Wind Speed (km/h)']
Pressure (millibars) - ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)', 'Season']
```

```
[15] # calculating performance scores test for the suggested label-factors models

result_comb, result_model = plan.copy(), plan.copy()
# loop through the selected labels
for label_col in plan.keys():
    factor_cols = plan[label_col]

    rmse_train_best, rmse_val_best, r2_val_best = [], [], []
    combos_best, models_best = [], []
    iter = 0

    # converting dataframe column into its NumPy array representation
    # shape of the resulting array is matching the original form of table column
    Y = data_num[label_col].values.reshape(-1, 1)

    # loop through the number of utilized factor variables
    for x in range(len(factor_cols)):
        # loop through the set of unique factor combinations
        for subset in itertools.combinations(factor_cols, x+1):
            factors = list(subset)
            combos_best.append(factors)
```

```
[15] # converting dataframe columns into their NumPy arrays representations
# shape of the resulting array is matching the original form of table columns
X = data_num[factors].values.reshape(-1, len(factors))

# splitting 2 array representations of data values into training (70%) and
# validation (30%) sets reproducible with a defined sampling seed
X_train, X_val, Y_train, Y_val = train_test_split(X, Y, test_size=0.3, random_state=1337)

# setting linear regression model implemented in Scikit-learn
LR_model = LinearRegression()
# training the model to predict Y outputs based on the X inputs
LR_model.fit(X_train, Y_train)
models_best.append(LR_model)

# estimating Y outputs based on the training X inputs using the model
Y_pred = LR_model.predict(X_train)
# calculating MSE between the genuine and the predicted Y coordinate values
mse = mean_squared_error(Y_train, Y_pred)
# calculating root MSE to measure performance of the trained regression model
rmse_t = np.sqrt(mse)
rmse_train_best.append(rmse_t)
# printing new performance records
if round(rmse_t, 2) <= round(min(rmse_train_best), 2):
    print(f"{iter}\tRMSE train:\t{rmse_t}")
    print(f"Label:\t{label_col}\tFactors:\t{factors}")

# predicting Y outputs using the trained model based on the validation X inputs
Y_pred = LR_model.predict(X_val)
# calculating MSE between the genuine and the predicted Y coordinate values
mse = mean_squared_error(Y_val, Y_pred)
# calculating RMSE to measure model performance on the cross-validation dataset
rmse_v = np.sqrt(mse)
rmse_val_best.append(rmse_v)
# printing new performance records
if round(rmse_v, 2) <= round(min(rmse_val_best), 2):
    print(f"{iter}\tRMSE cross-val:\t{rmse_v}")
    print(f"Label:\t{label_col}\tFactors:\t{factors}")

# calculating the R-squared (coefficient of determination) score for the
# trained linear regression model on the cross-validation dataset
score = r2_score(Y_val, Y_pred)
r2_val_best.append(score)
# printing new performance records
if round(score, 4) >= round(max(r2_val_best), 4):
    print(f"{iter}\tR2 cross-val:\t{score}")
    print(f"Label:\t{label_col}\tFactors:\t{factors}")

iter += 1
```



```
[15] print(f"Total amount of processed factor combinations:\t{iter}")

# getting the best performance score indexes
ind_t = rmse_train_best.index(min(rmse_train_best))
ind_v = rmse_val_best.index(min(rmse_val_best))
ind_r = r2_val_best.index(max(r2_val_best))

# getting index of the shortest best factor combination for a final per-label result
result_comb[label_col] = min([combos_best[i] for i in [ind_t, ind_v, ind_r]], key=len)
ind = combos_best.index(result_comb[label_col])
# setting the corresponding model for the label in the resulting dictionary
result_model[label_col] = models_best[ind]

print(f"{label_col} - best model cross-validation performance:")
print(f"RMSE\t{rmse_val_best[ind]}\nR2\t{r2_val_best[ind]}")

0      RMSE train:      7.398019314246577
Label: Temperature (C) Factors:      ['Humidity']
0      RMSE cross-val: 7.3778437180304035
Label: Temperature (C) Factors:      ['Humidity']
0      R2 cross-val:    0.4002220682856359
Label: Temperature (C) Factors:      ['Humidity']
4      RMSE train:      7.21834008290361
Label: Temperature (C) Factors:      ['Humidity', 'Visibility (km)']
4      RMSE cross-val: 7.1963873038558095
Label: Temperature (C) Factors:      ['Humidity', 'Visibility (km)']
4      R2 cross-val:    0.4293620686618933
Label: Temperature (C) Factors:      ['Humidity', 'Visibility (km)']
5      RMSE train:      6.873008448670756
Label: Temperature (C) Factors:      ['Humidity', 'Pressure (millibars)']
5      RMSE cross-val: 6.862804515360536
Label: Temperature (C) Factors:      ['Humidity', 'Pressure (millibars)']
5      R2 cross-val:    0.4810388636013384
Label: Temperature (C) Factors:      ['Humidity', 'Pressure (millibars)']
10     RMSE train:      6.76966527282062
Label: Temperature (C) Factors:      ['Humidity', 'Visibility (km)', 'Pressure (millibars)']
10     RMSE cross-val: 6.756554599720952
Label: Temperature (C) Factors:      ['Humidity', 'Visibility (km)', 'Pressure (millibars)']
10     R2 cross-val:    0.4969835816741712
Label: Temperature (C) Factors:      ['Humidity', 'Visibility (km)', 'Pressure (millibars)']
12     RMSE train:      6.545292433557022
Label: Temperature (C) Factors:      ['Humidity', 'Pressure (millibars)', 'Season']
12     RMSE cross-val: 6.5166946789979985
Label: Temperature (C) Factors:      ['Humidity', 'Pressure (millibars)', 'Season']
12     R2 cross-val:    0.5320641415369975
Label: Temperature (C) Factors:      ['Humidity', 'Pressure (millibars)', 'Season']
14     RMSE train:      6.423837438097231
Label: Temperature (C) Factors:      ['Humidity', 'Visibility (km)', 'Pressure (millibars)', 'Season']
14     RMSE cross-val: 6.390850487522191
Label: Temperature (C) Factors:      ['Humidity', 'Visibility (km)', 'Pressure (millibars)', 'Season']
14     R2 cross-val:    0.5499623024184881
Label: Temperature (C) Factors:      ['Humidity', 'Visibility (km)', 'Pressure (millibars)', 'Season']
```

```

5 RMSE cross-val: 7.171761617618323
[15] Label: Pressure (millibars) Factors: ['Temperature (C)', 'Wind Speed (km/h)']
5 R2 cross-val: 0.15581275616235757
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Wind Speed (km/h)']
10 RMSE train: 7.113403159088038
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)']
10 RMSE cross-val: 7.170527464015245
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)']
10 R2 cross-val: 0.15610327532352586
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)']
12 RMSE train: 7.036380404049737
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Wind Speed (km/h)', 'Season']
12 RMSE cross-val: 7.0823102459266964
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Wind Speed (km/h)', 'Season']
12 R2 cross-val: 0.17674004858526615
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Wind Speed (km/h)', 'Season']
14 RMSE train: 7.0362511813172075
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)', 'Season']
14 RMSE cross-val: 7.0824878352913165
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)', 'Season']
14 R2 cross-val: 0.17669876147989327
Label: Pressure (millibars) Factors: ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)', 'Season']
Total amount of processed factor combinations: 15
Pressure (millibars) - best model cross-validation performance:
RMSE 7.0823102459266964
R2 0.17674004858526615

```

```

[16] # comparing initial choice and calculated by metrics result
print(f"\tBased on the correlation matrix analysis:")
for label, factors in plan.items():
    print(f"{label} - {factors}")

print(f"\tBased on the linear regression model performance:")
for label, factors in result_comb.items():
    print(f"{label} - {factors}")

```

```

Based on the correlation matrix analysis:
Temperature (C) - ['Humidity', 'Visibility (km)', 'Pressure (millibars)', 'Season']
Humidity - ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)']
Visibility (km) - ['Temperature (C)', 'Humidity', 'Pressure (millibars)', 'Wind Speed (km/h)']
Pressure (millibars) - ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)', 'Season']
Based on the linear regression model performance:
Temperature (C) - ['Humidity', 'Visibility (km)', 'Pressure (millibars)', 'Season']
Humidity - ['Temperature (C)', 'Visibility (km)', 'Wind Speed (km/h)']
Visibility (km) - ['Temperature (C)', 'Humidity', 'Pressure (millibars)', 'Wind Speed (km/h)']
Pressure (millibars) - ['Temperature (C)', 'Wind Speed (km/h)', 'Season']

```

The combinations of factors per label are identical, thus the initial analytical suggestions have been confirmed by the computational metric tests (train and valid RMSE, R2) done on all possible combinations of factors relevant to the selected labels to identify the best set of label-specific input variables.

The **R-squared** score is a measure of how well the regression line (or curve) fits the actual data points, and represents the proportion of the variance in y_{test} that is predictable from x_{test} .

It's calculated by comparing the variation of y_{test} from their mean (total sum of squares, or TSS) to the variation of y_{pred} from y_{test} (residual sum of squares, or RSS).

The formula for R2:

$$R^2 = 1 - \frac{RSS}{TSS}$$

$$TSS = \sum_{i=1}^n (y_i - \bar{y})^2$$

$$RSS = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

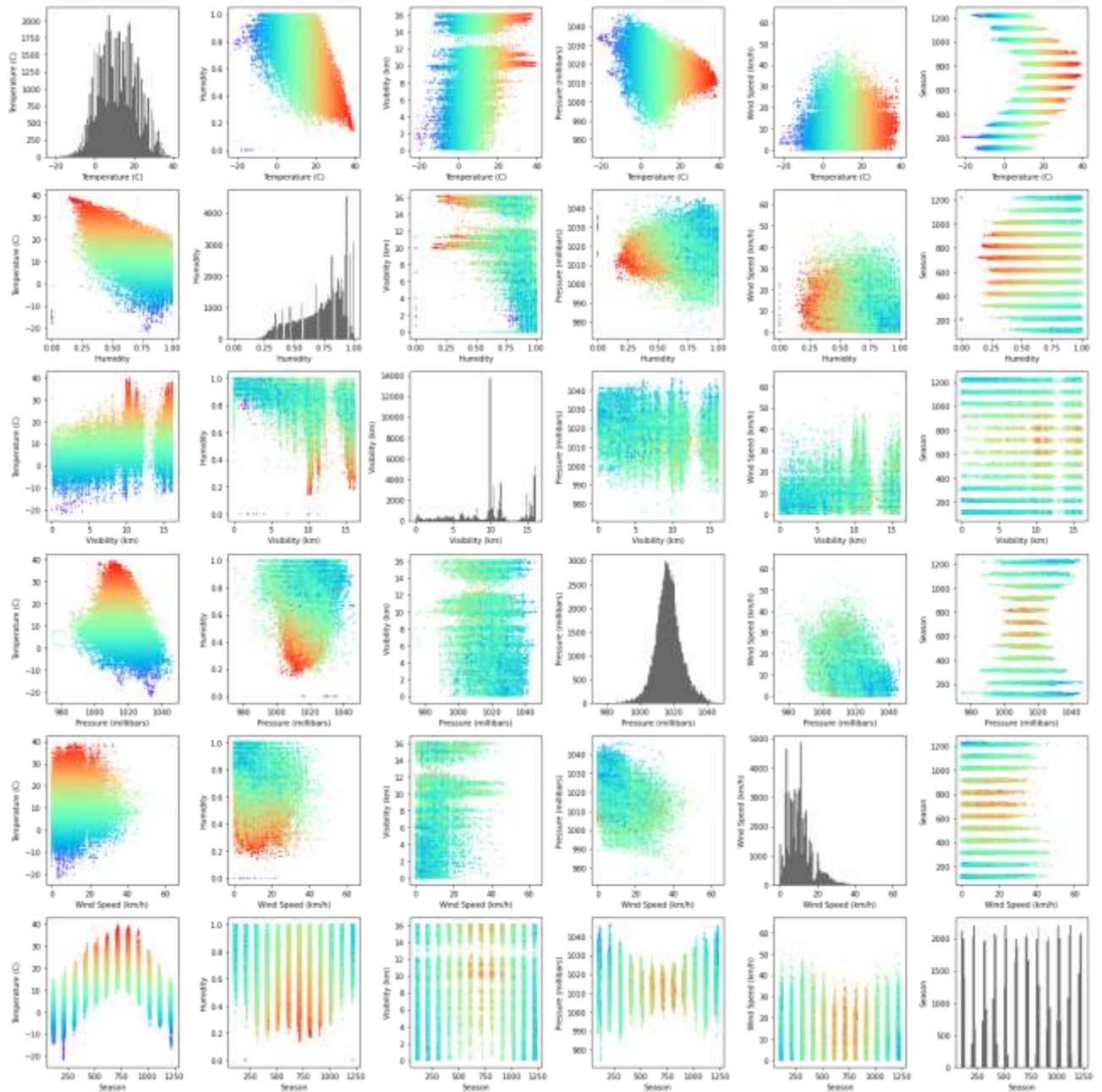
where y_i is the actual output value, \bar{y} is the mean of the actual output values y_{test} , and \hat{y}_i is the predicted output value.

The R^2 score ranges from 0 to 1:

0 - the model is no better than predicting the mean of y_{test} ;

1 - perfect fit of the model to the data, where all the variation in y_{test} is explained by y_{pred} .

```
[17] # plotting matrix of 2D scatter plots for the training dataset formed from
# the final dataframe subset of raw labels and factors
I = data.index.values.reshape(-1, 1)
Y = data[labels].values.reshape(-1, len(labels))
I_train, I_val, Y_train, Y_val = train_test_split(I, Y, test_size=0.3,
                                                    random_state=1337)
scatter_matrix(data[data.index.isin(I_train.flatten())], save_file="raw_scatter_matrix")
```



```
[18] # drawing a matrix of 2D scatter plots to visualize inter-variable dependencies
# for the label columns in the final dataframe subset, and show predicted values
num_cols = data.columns.tolist()
fig, axes = plt.subplots(nrows=len(num_cols), ncols=len(num_cols), figsize=(20, 20))

# Loop through each subplot and plot scatter plot
for i, ax_i in enumerate(axes):
    i_data = data[num_cols[i]].values.reshape(-1, 1)
    for j, ax_j in enumerate(ax_i):
        if i == j:
            ax_j.hist(data[num_cols[i]], bins=100, color="dimgray")
        else:
            yax_col_label = num_cols[j]
            # plotting original data on the scatter plot
            j_data = data_num[yax_col_label].values.reshape(-1, 1)
            i_train, i_val, j_train, j_val = train_test_split(i_data, j_data,
                                                            test_size=0.3,
                                                            random_state=1337)

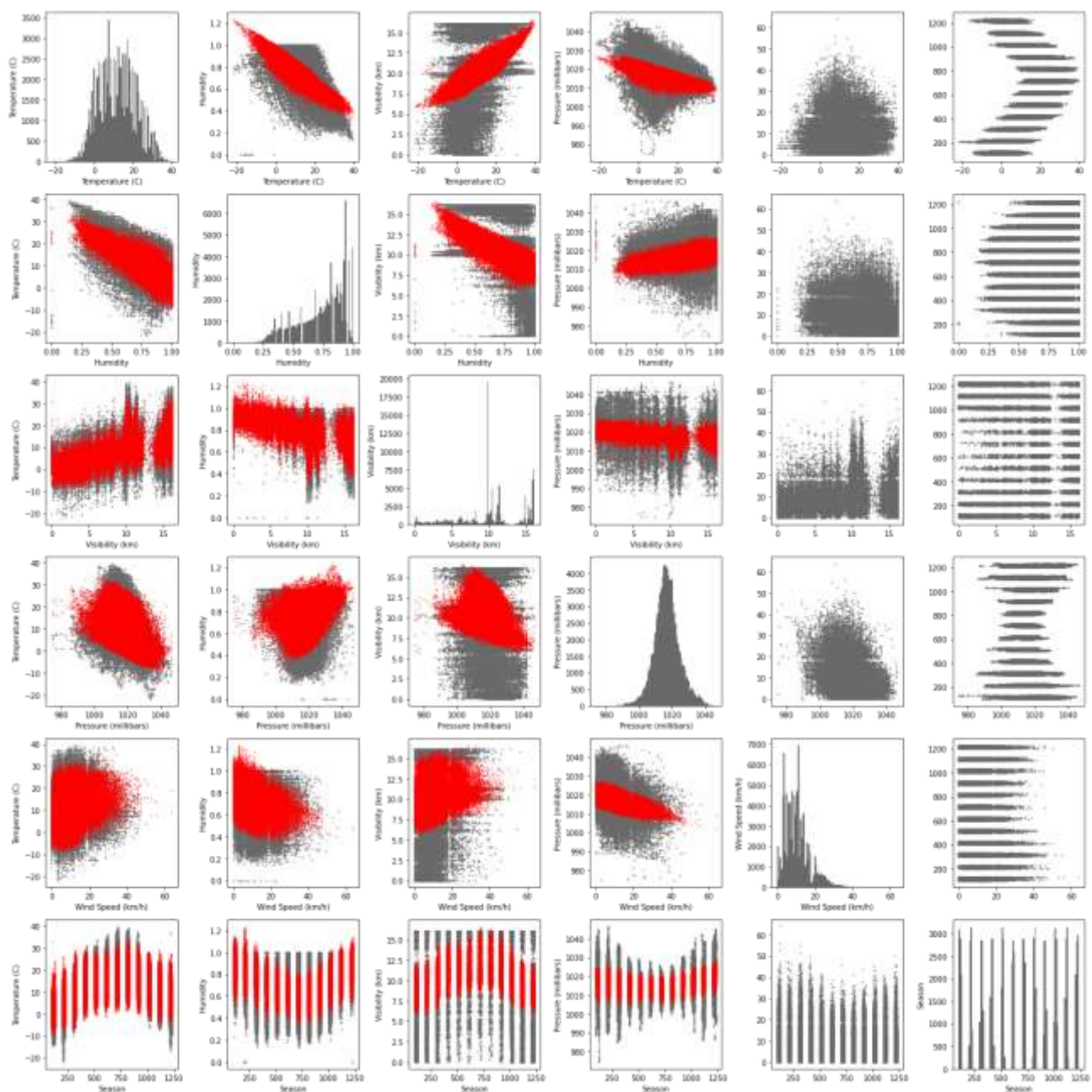
            ax_j.scatter(i_train, j_train, s=1, color="dimgray")

            # skip for labels without models
            if not yax_col_label in result_comb.keys():
                continue

            # plotting points predicted on the training sequence of all factors
            # into separate per-factor subplot
            input_col_labels = result_comb[yax_col_label]
            Y = data_num[yax_col_label].values.reshape(-1, 1)
            X = data_num[input_col_labels].values.reshape(-1, len(input_col_labels))
            LR_model = result_model[yax_col_label]

            X_train, X_val, Y_train, Y_val = train_test_split(X, Y, test_size=0.3,
                                                            random_state=1337)

            Y_pred = LR_model.predict(X_train)
            ax_j.scatter(i_train, Y_pred, color="red", s=1, alpha=0.5)
            ax_j.set_xlabel(num_cols[i])
            ax_j.set_ylabel(num_cols[j])
plt.tight_layout()
plt.savefig("/content/gdrive/MyDrive/ds/final_scatter_matrix.jpg")
plt.show()
```

Вихідний код у jupyter notebook:

<https://colab.research.google.com/drive/1KhsVVI-x4DoUkneuV4Pf4JJxINcs9C7P?usp=sharing>

Висновки: було розглянуто основні методи мови Python для опрацювання простої моделі лінійної регресії числових даних, з використанням структур даних та інструментів бібліотек Pandas та Sci-kit learn.