Questions to Alex

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Matrix H has the form:

$$H = \begin{bmatrix} 0 & 0 & v & we^{-ik} \\ 0 & 0 & w & v \\ v & w & 0 & 0 \\ we^{ik} & v & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & h^\dagger \\ h & 0 \end{bmatrix}$$

The h matrix has the form:

$$h = \begin{pmatrix} v & w \\ \mathrm{we}^{ik} & v \end{pmatrix}$$

For our calculations we want to verify the equality of two winding number formulas:

$$\mathcal{W} = \oint \frac{\mathrm{d}k}{2\pi i} \operatorname{tr}\left[h^{-1}\partial_k h\right] = \sum_{j=1}^{\mathcal{D}/2} \oint \frac{\mathrm{d}k}{2\pi i} \partial_k \log h_j$$

Hence, let's first calculate quantity $\operatorname{tr}\left[h^{-1}\partial_k h\right]$. The components are as follow:

$$h^{-1} = \frac{1}{v^2 - w^2 e^{ik}} \begin{bmatrix} v & -w \\ -we^(ik) & v \end{bmatrix}, \quad \partial_k = \begin{bmatrix} 0 & 0 \\ ie^{ik}w & 0 \end{bmatrix}$$

Therefore:

$$h^{-1}\partial_k h = \begin{pmatrix} -\frac{ie^{ik}w^2}{v^2 - e^{ik}w^2} & 0 \\ \frac{ie^{ik}vw}{v^2 - e^{ik}w^2} & 0 \end{pmatrix} \implies \operatorname{tr}\begin{bmatrix} -\frac{ie^{ik}w^2}{v^2 - e^{ik}w^2} & 0 \\ \frac{ie^{ik}vw}{v^2 - e^{ik}w^2} & 0 \end{bmatrix} = -\frac{iw^2 e^{ik}}{v^2 - e^{ik}w^2}$$

And the second component is:

$$\partial_k \log h = \frac{iwe^{ik}}{v + e^{ik}w}$$