Shortest path optimizations

From graphs to ants

Daniel Matoz 2022

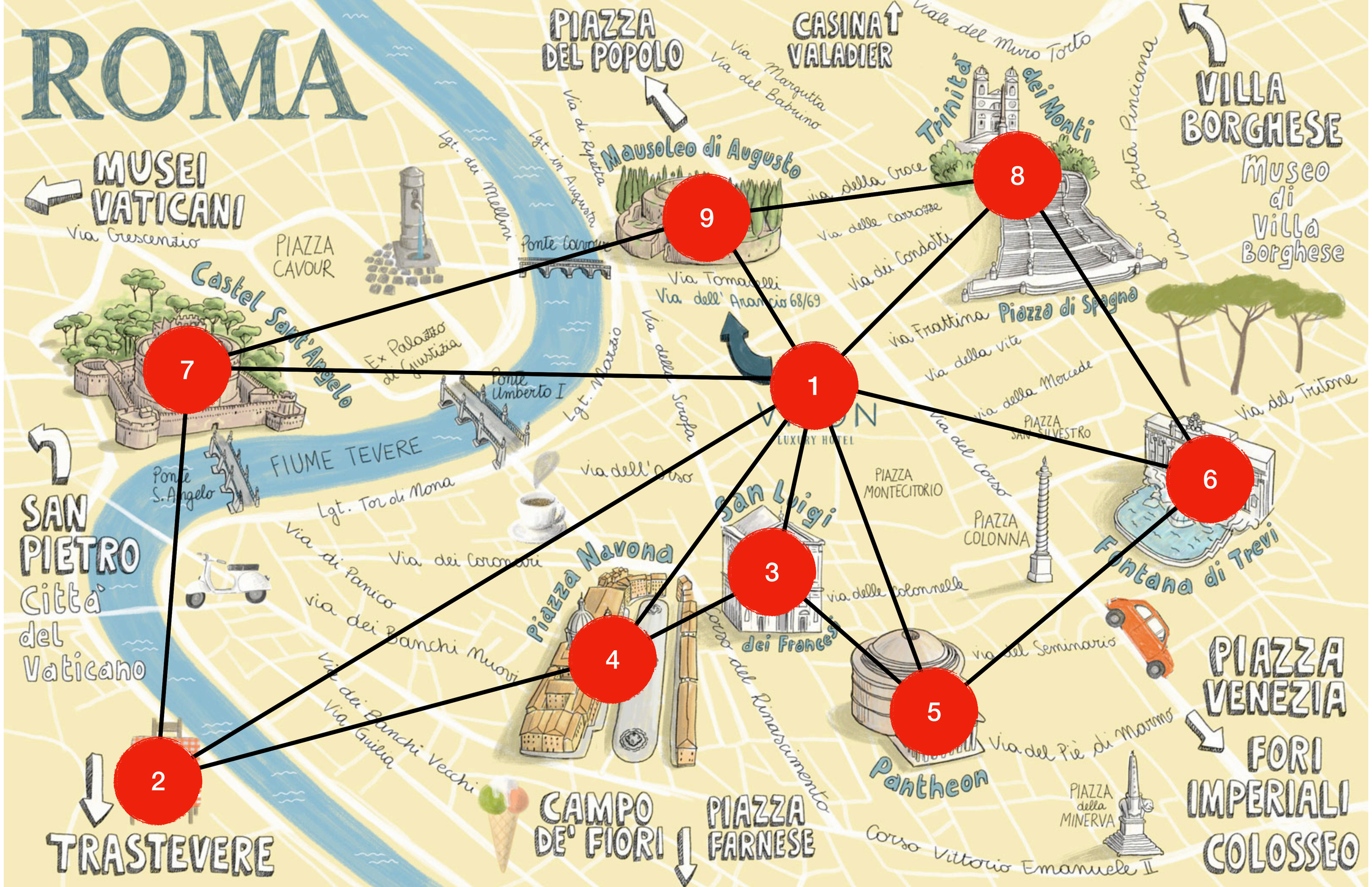
Problem

- I’m visiting Rome for only a few days, and I want to make the most of it. How I can plan my sightseeing in a way that minimizes the walking between attractions?

Problem


ROMA
PIAZZA DEL POPOLO
CASINA VALADIER
Viale del Muro Torto
Via Margutta
Via del Babuino
Via di Ripetta
Mausoleo di Augusto
Trinità dei Monti
VILLA BORGHESE
Museo di Villa Borghese
Porta Pinciana
Via Veneto
MUSEI VATICANI
Via Cresenzio
Lgt. dei Mellini
Lgt. in Augusta
PIAZZA CAVOUR
Ponte Cavour
Via della Croce
Via delle Carrozze
Via dei Condotti
Via Tomacelli
Via dell' Arancio 68/69
Via Frattina
Piazza di Spagna
Via della Vite
Via della Mercede
Via del Tritone
Castel Sant'Angelo
Ex Palazzo di Giustizia
Ponte Umberto I
Lgt. Marzio
Via della Scrofa
VILON
LUXURY HOTEL
PIAZZA SAN SILVESTRO
Via del Corso
PIAZZA MONTECITORIO
PIAZZA COLONNA
Fontana di Trevi
FIUME TEVERE
Ponte S. Angelo
Lgt. Tor di Nona
via dell' Orso
San Luigi dei Francesi
via delle Colonnelle
SAN PIETRO Città del Vaticano
Via dei Coronari
Via di Panico
via dei Banchi Nuovi
Via dei Banchi Vecchi
Via Giulia
Piazza Navona
Corso del Rinascimento
via del Seminario
Pantheon
Via del Pie di Marmo
PIAZZA VENEZIA
FORI IMPERIALI COLOSSEO
PIAZZA della MINERVA
Corso Vittorio Emanuele II
TRASTEVERE
CAMPO DE' FIORI
PIAZZA FARNESE

Problem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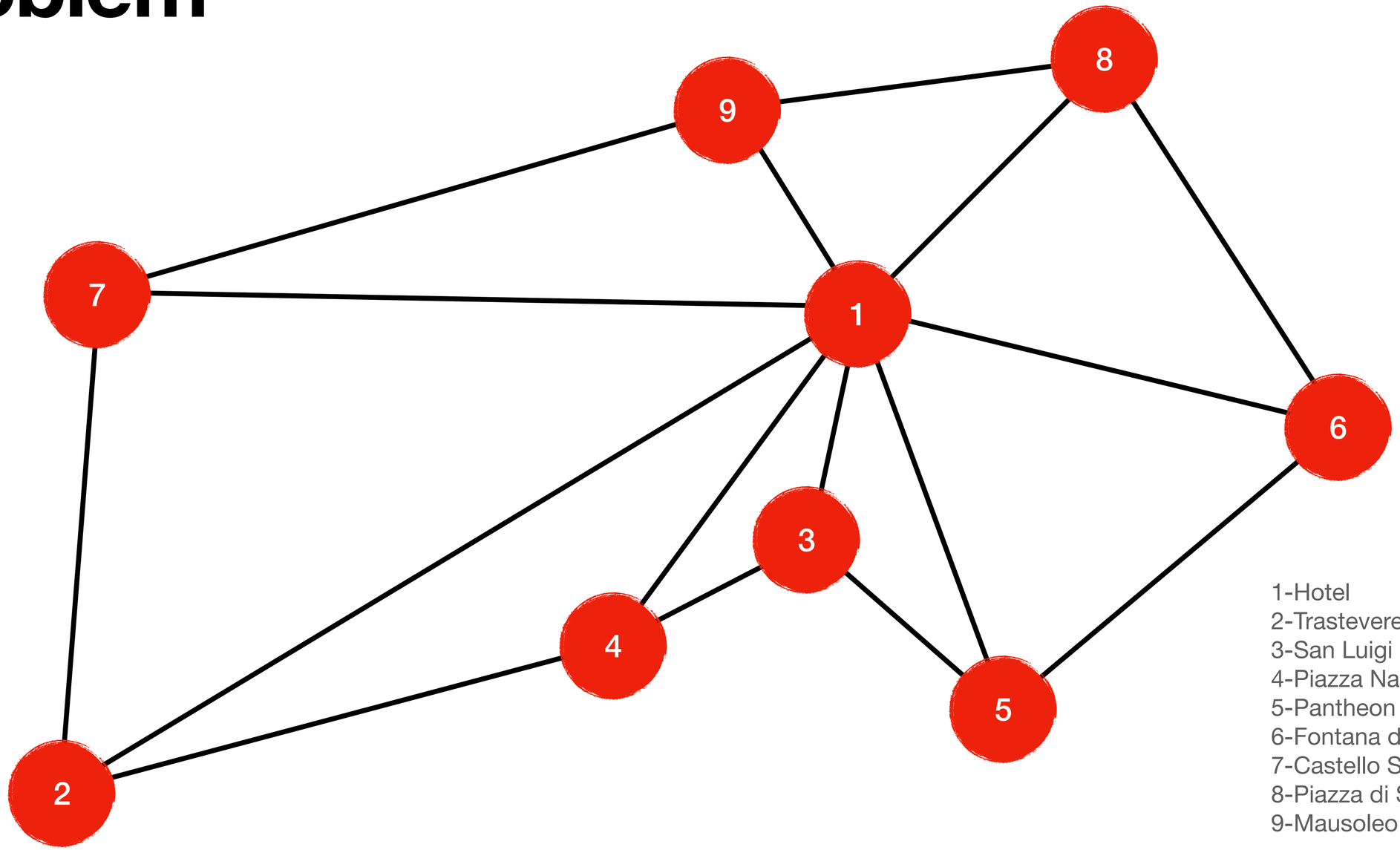
Problem



1-Hotel
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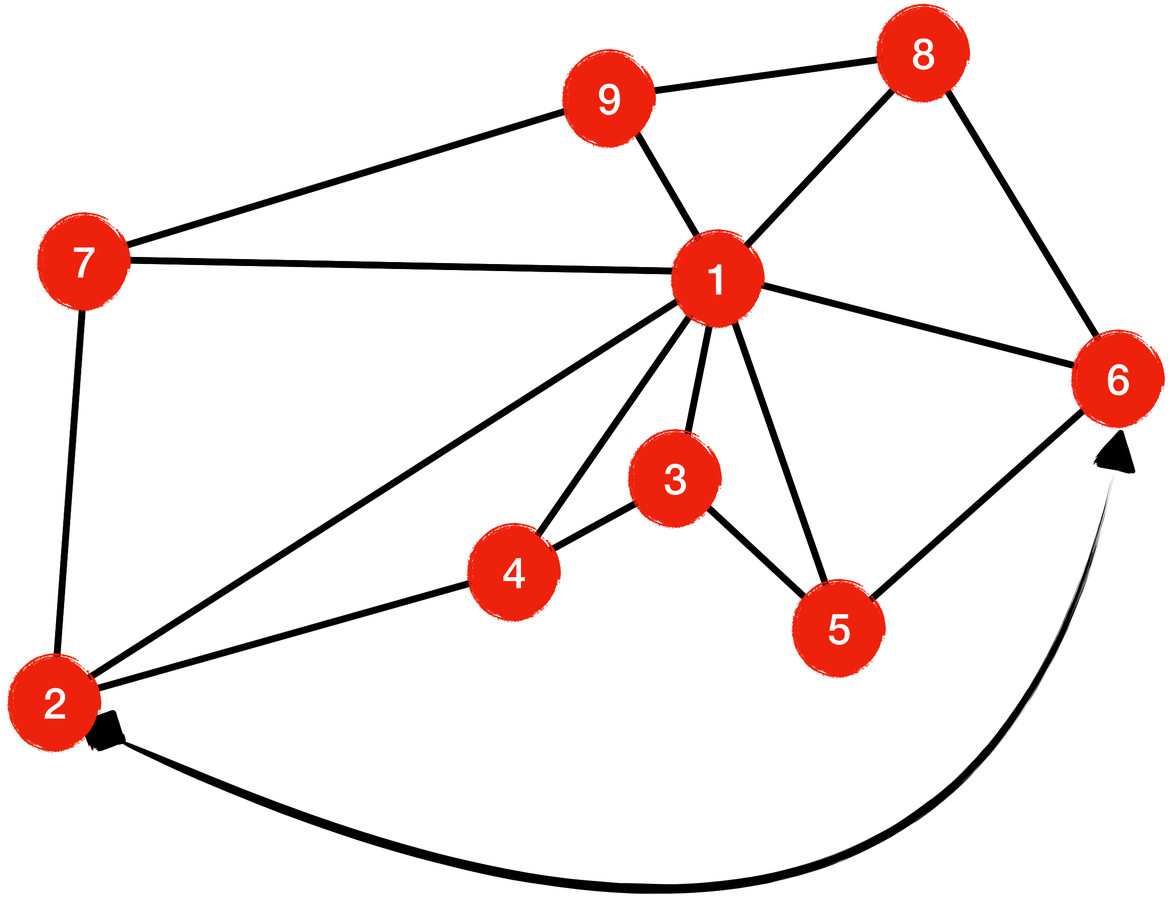
Problem



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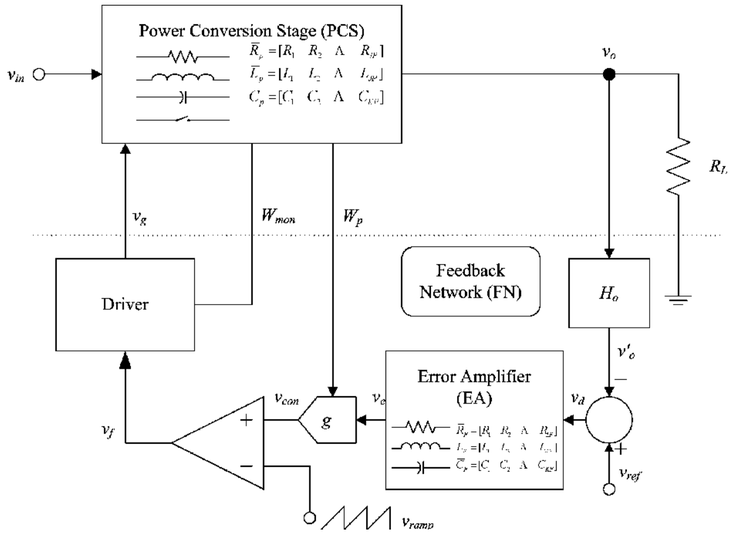
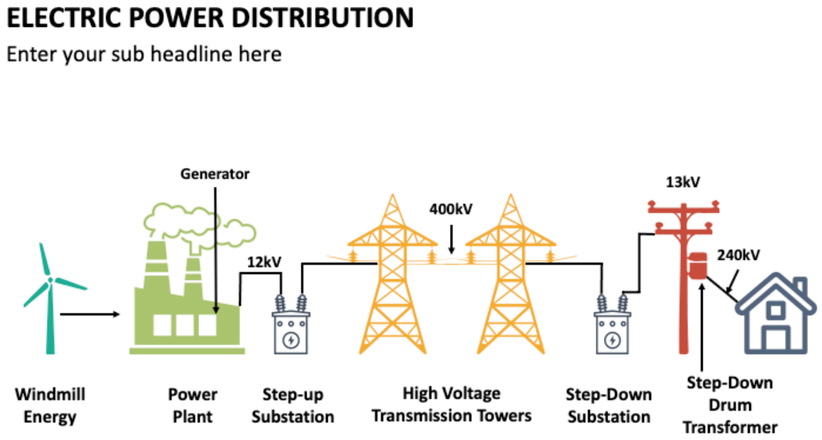
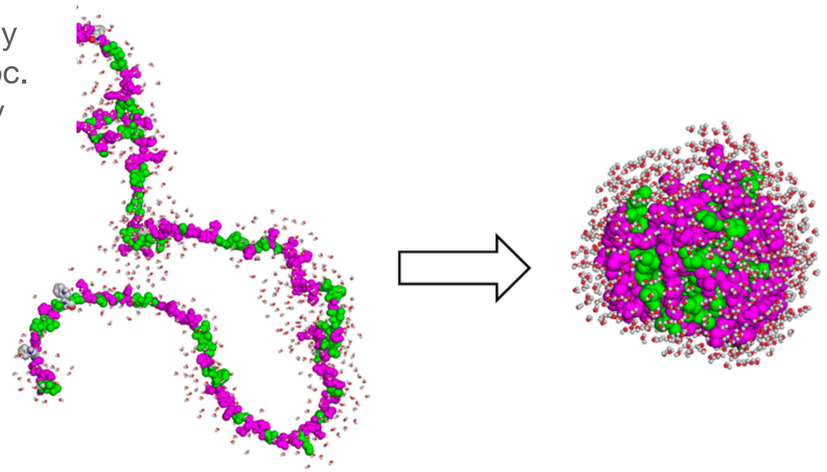
Why this problem matters to you?



Zhang J, Chung H S-H, Lo A W-L and Huang T 2009 Extended Ant Colony Optimization Algorithm for Power Electronic Circuit Design IEEE Trans. Power Electron. 24 147–62

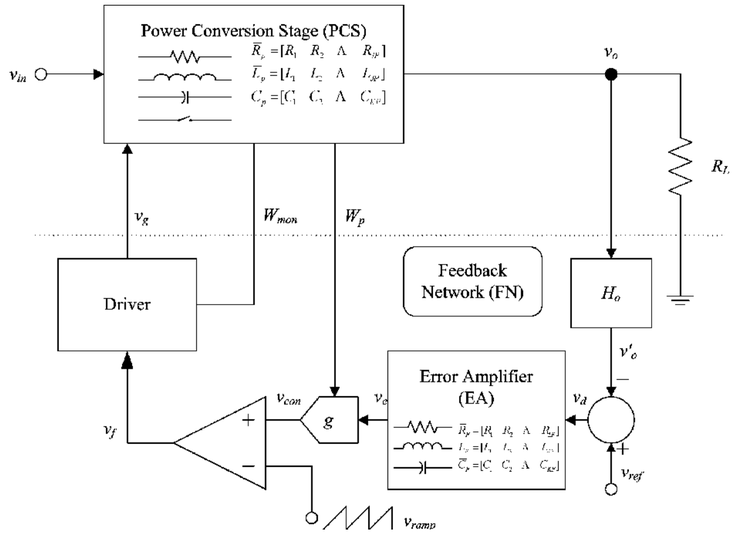


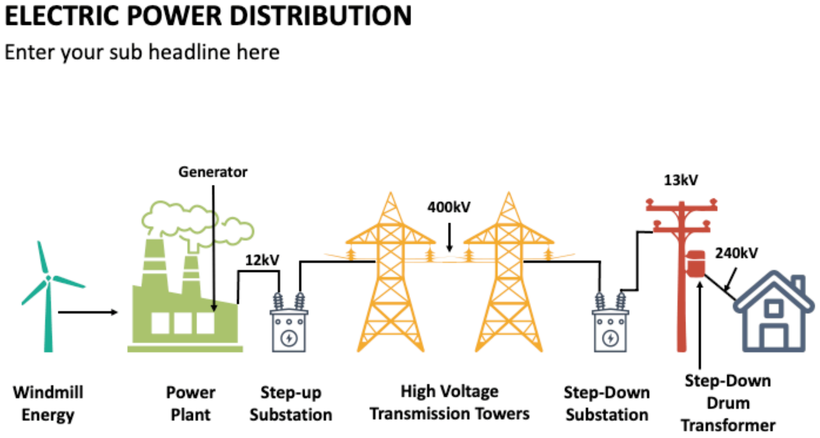
Warner L and Vogel U 2008 Optimization of energy supply networks using ant colony optimization Proc. Environmental Informatics and Industrial Ecology

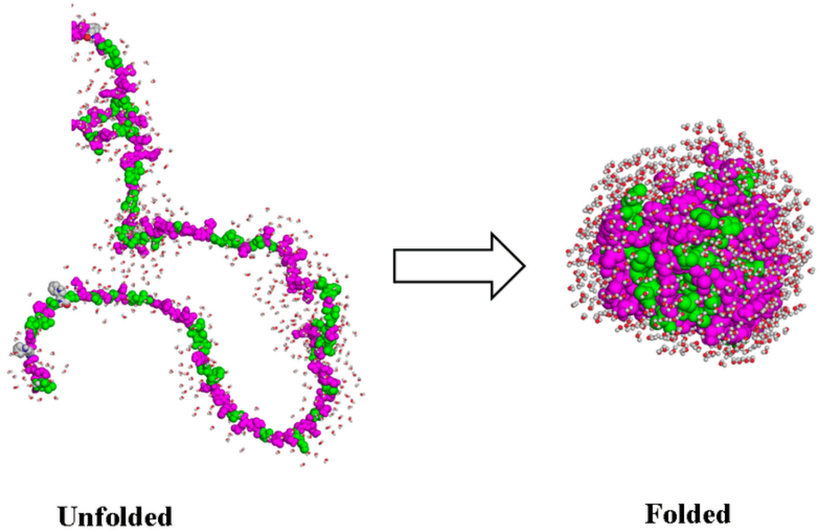


Hu X, Zhang J, Xiao J and Li Y 2008 Protein folding in hydrophobic-polar lattice model: a flexible ant-colony optimization approach Protein Pept. Lett. 15 469–77

Why this problem matters to you?

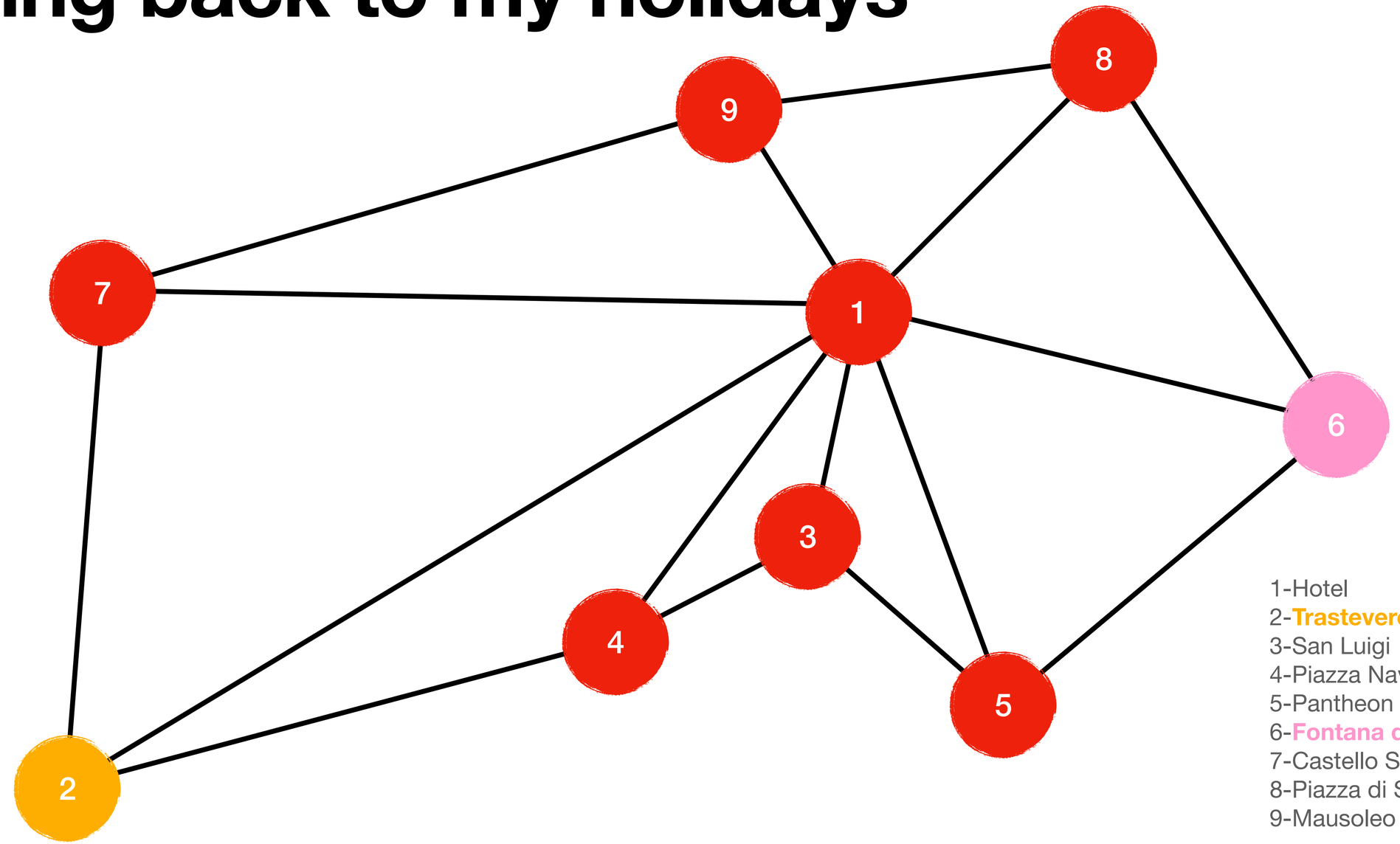






Optimization problems!

Going back to my holidays



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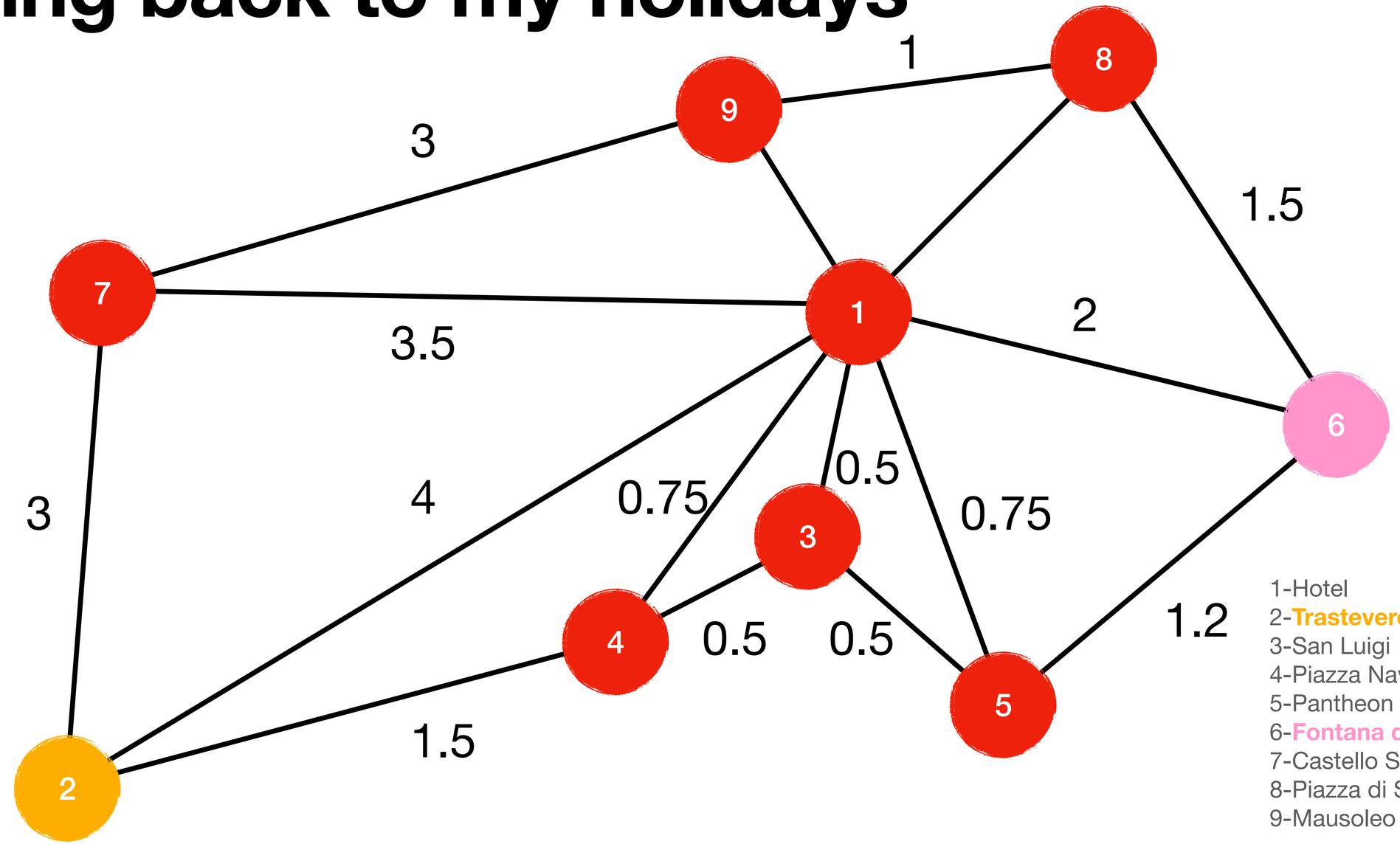
Going back to my holidays



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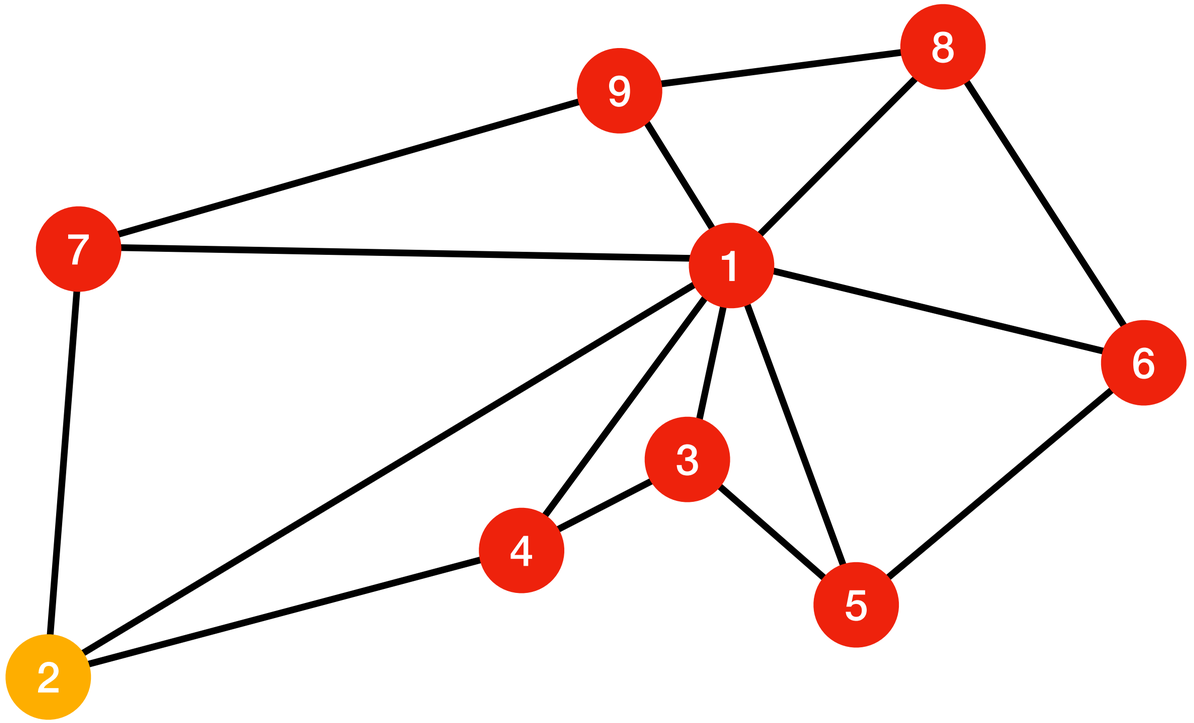
Brute force

- Starting at the node 2 we can calculate the shortest path to the next node connected to it. Then re run the procedure for the next until we find a solution, this is called *Breadth-first search.*

Brute force



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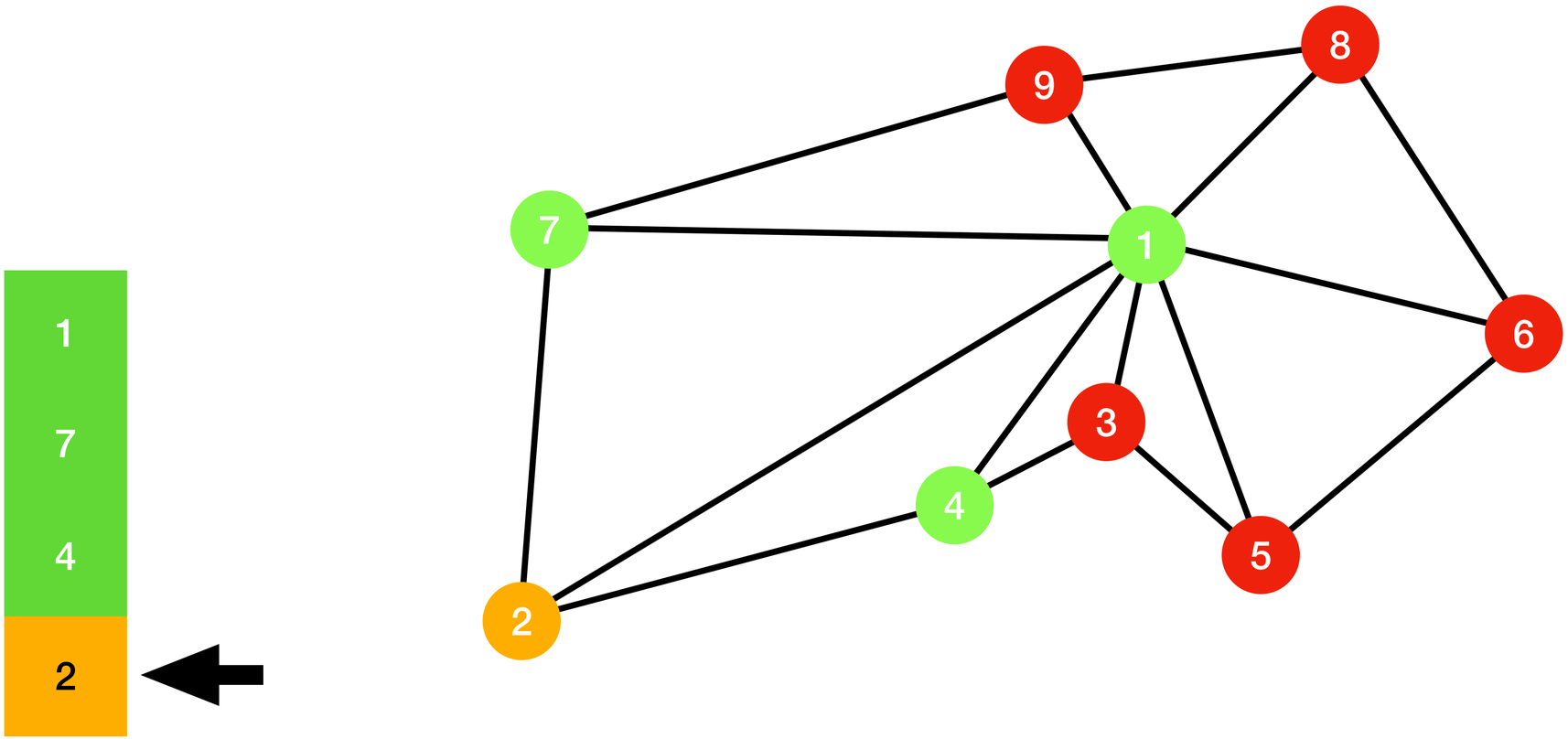
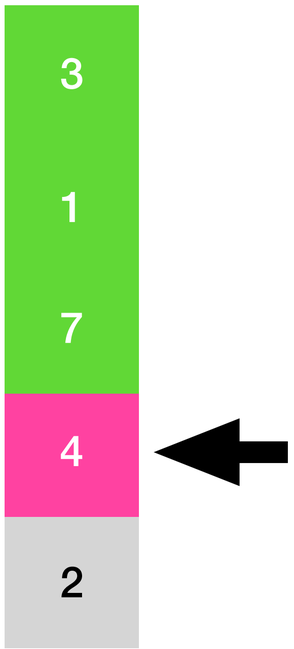
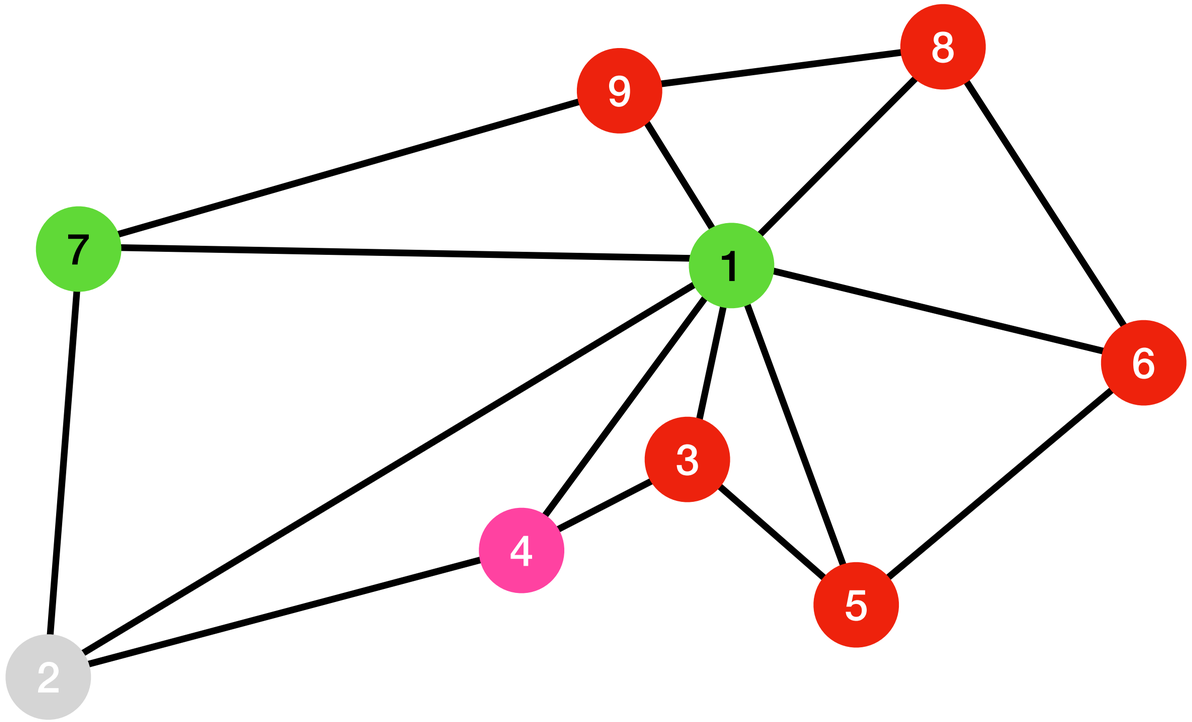
Brute force



Brute force





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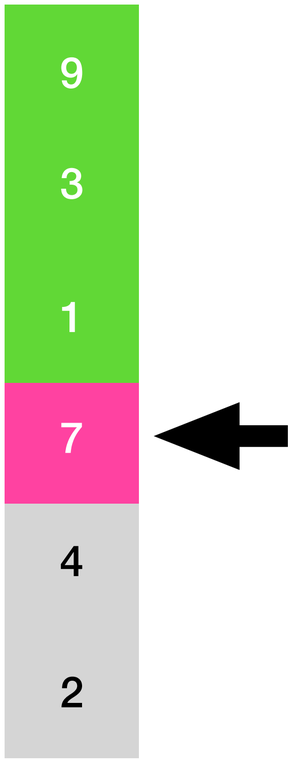

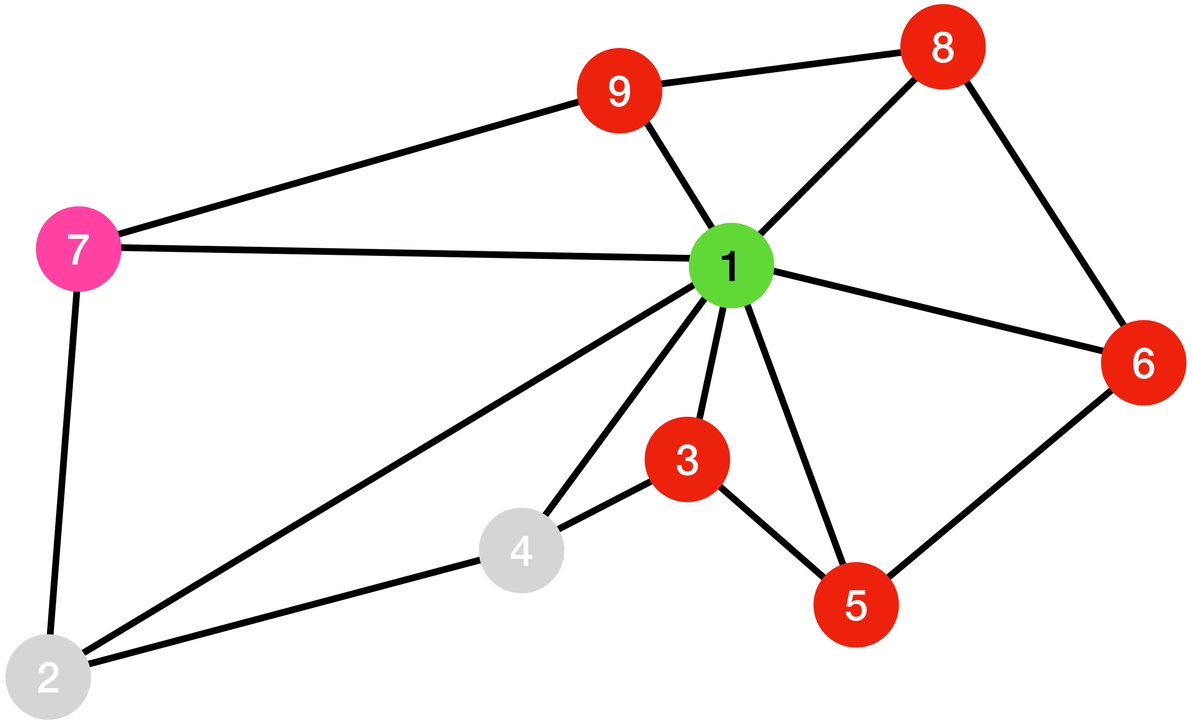

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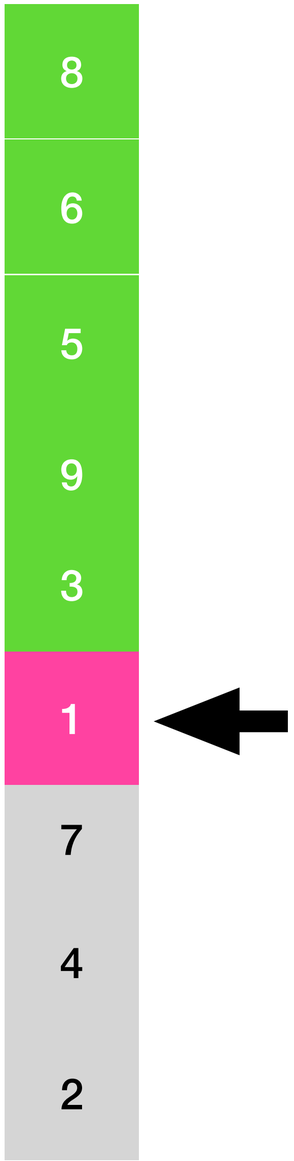

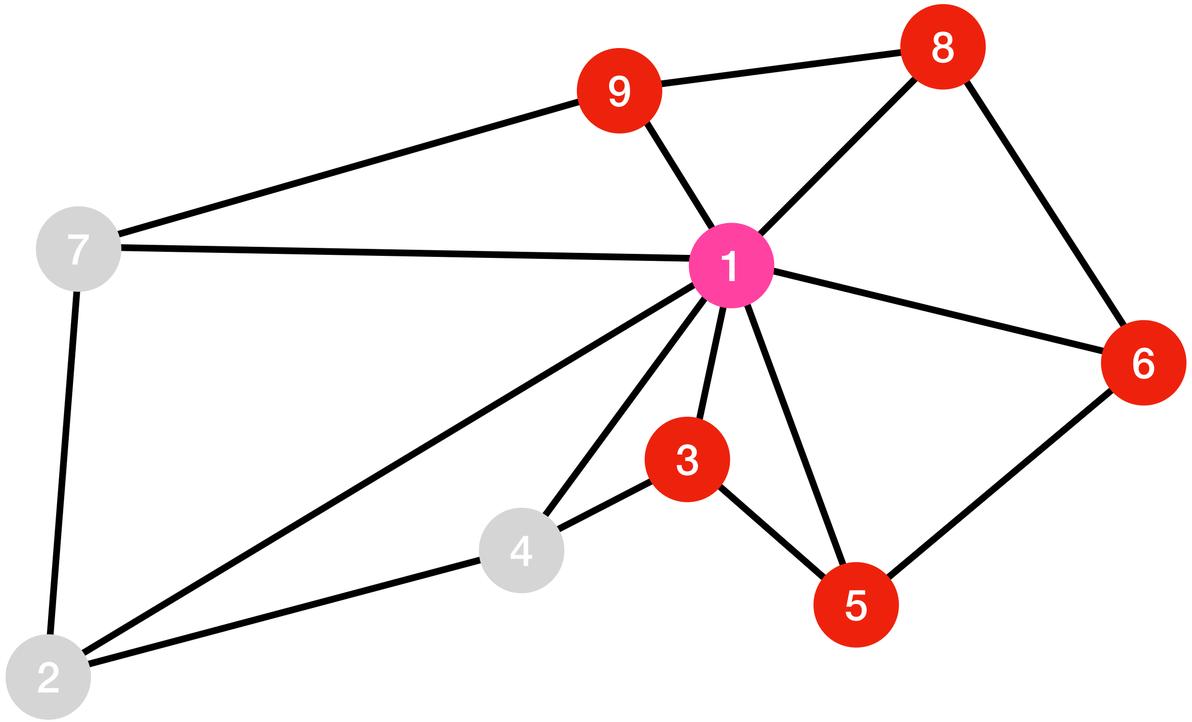

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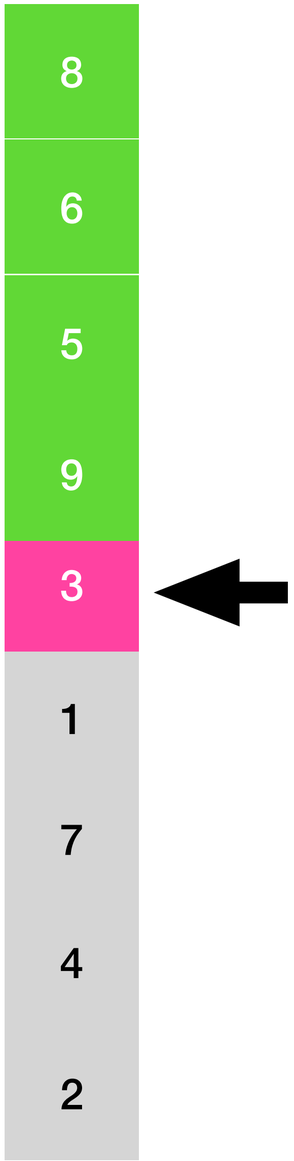

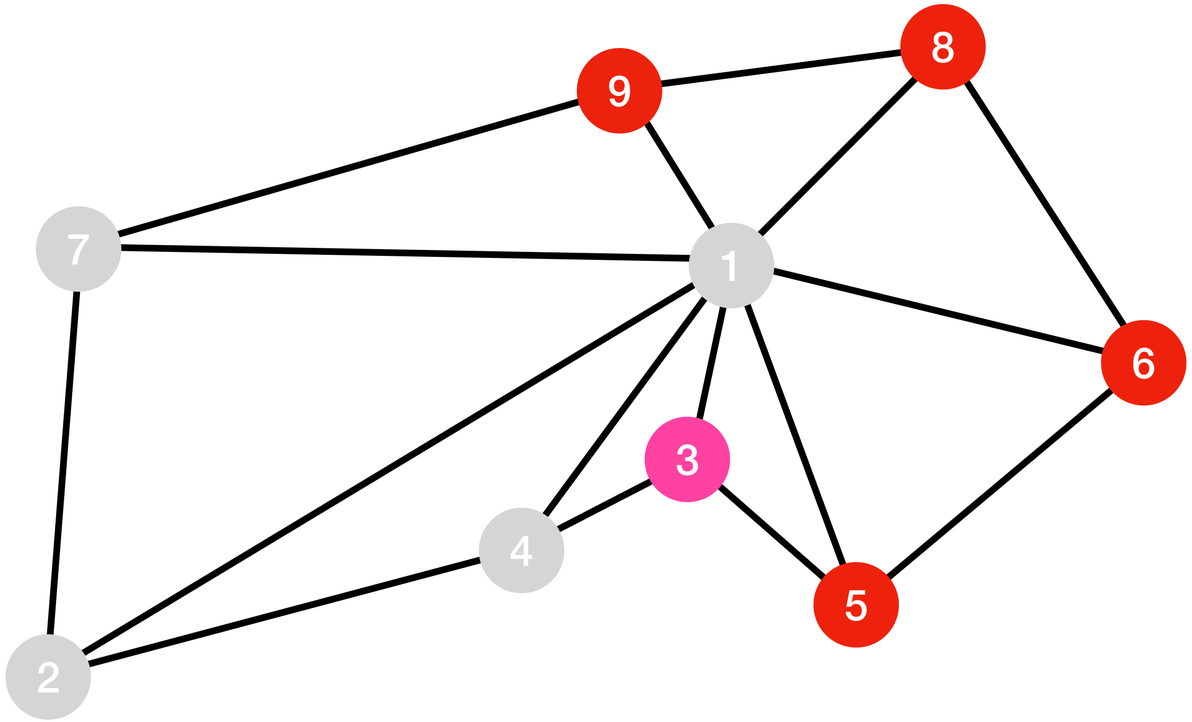

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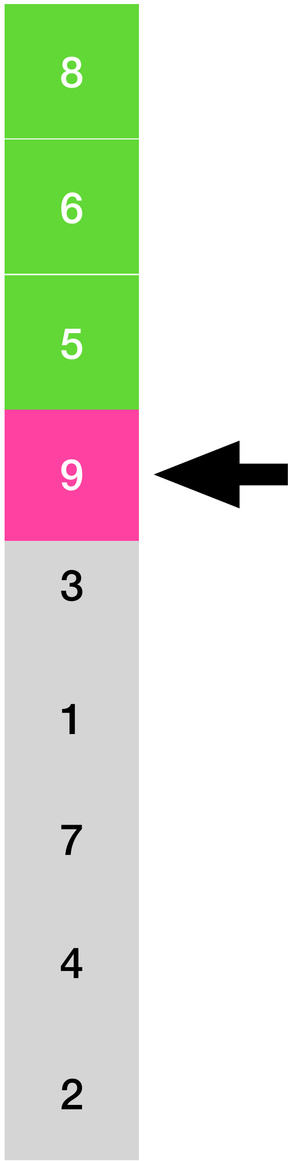

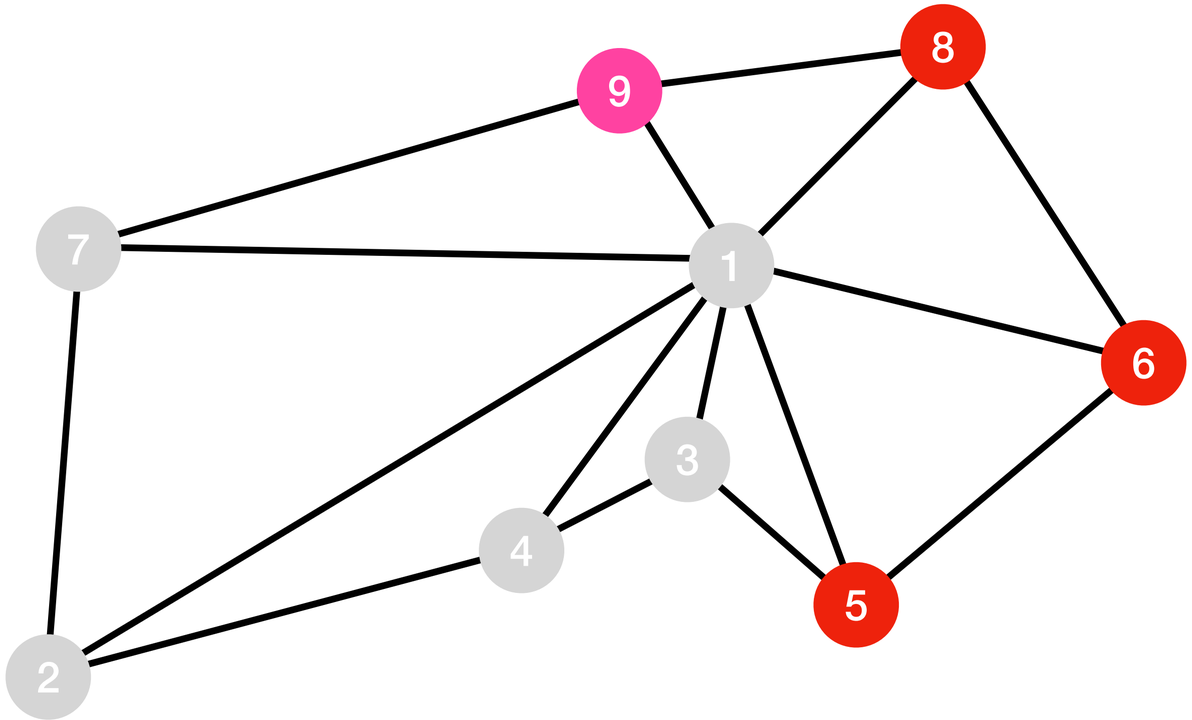

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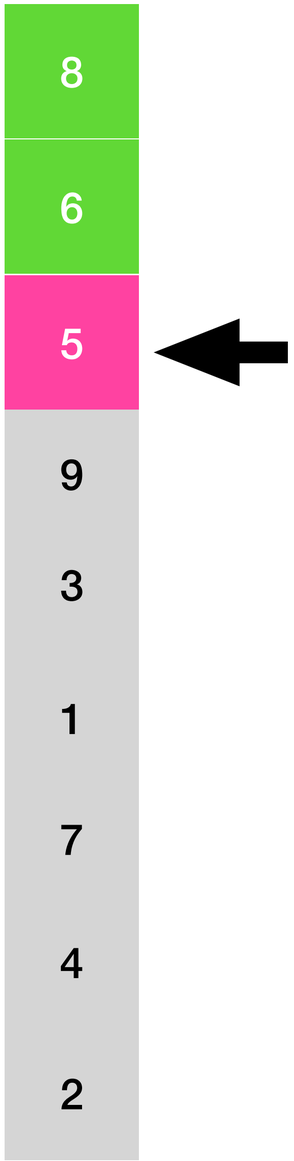

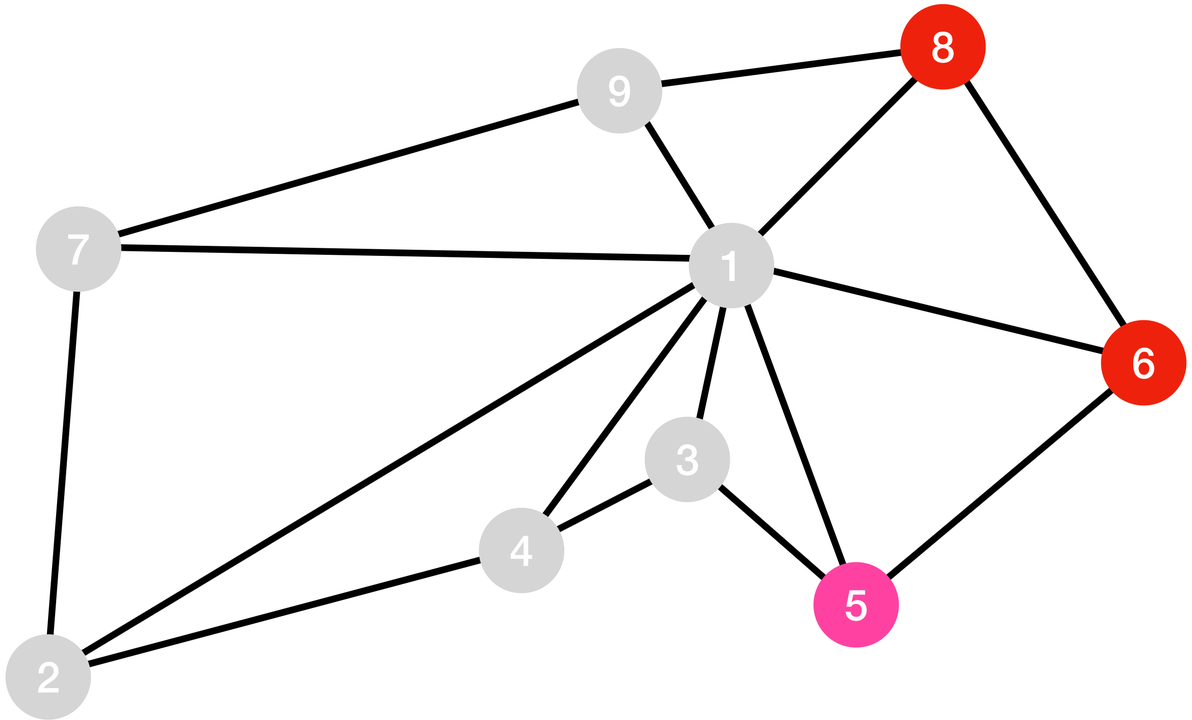

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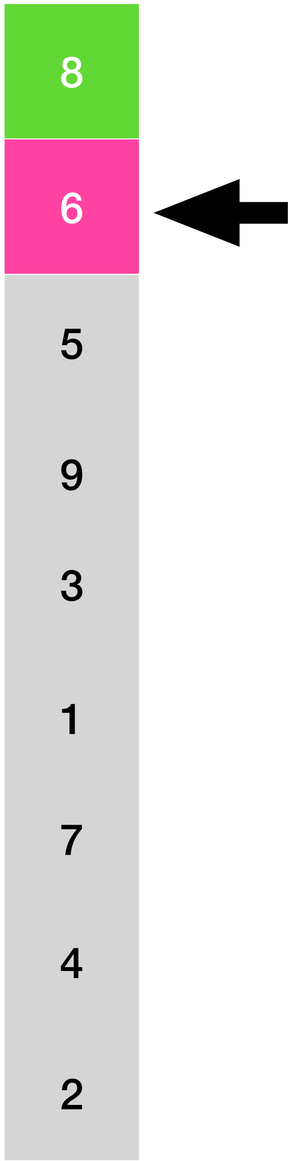

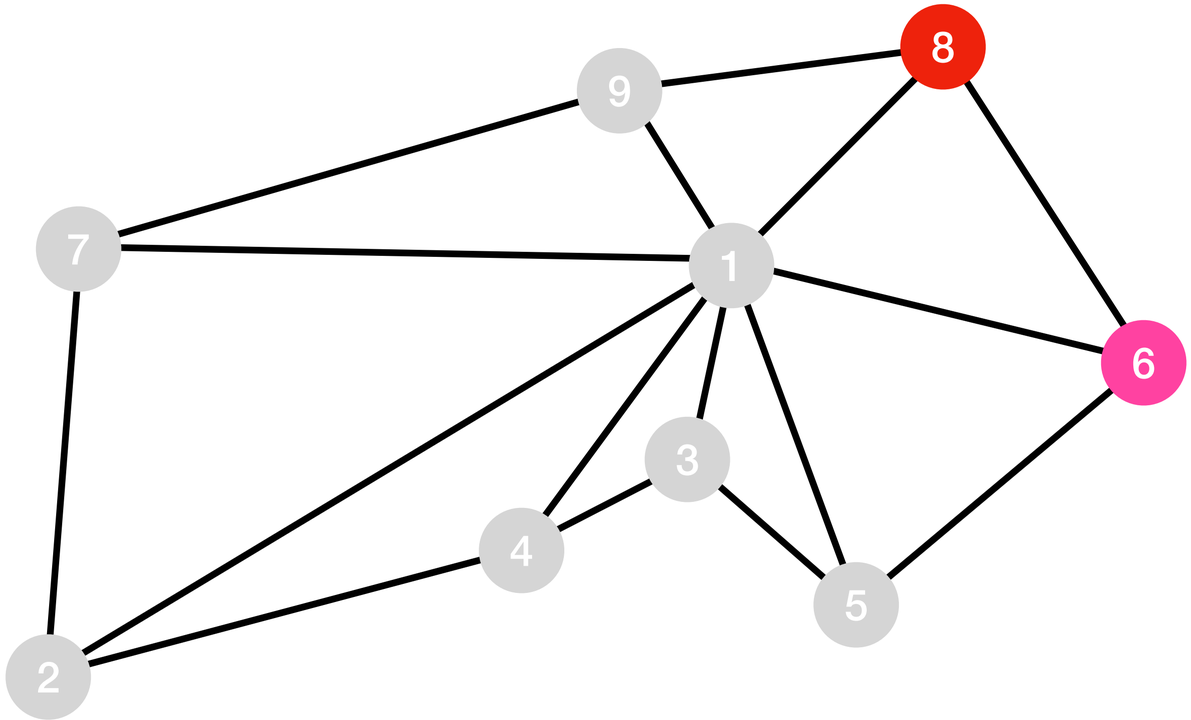

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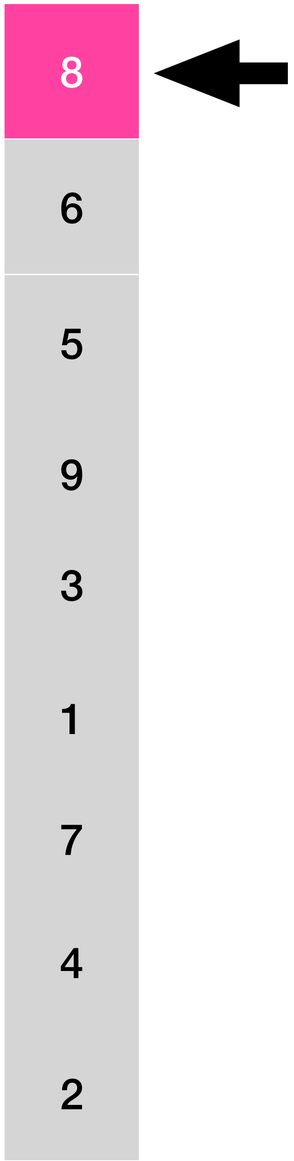
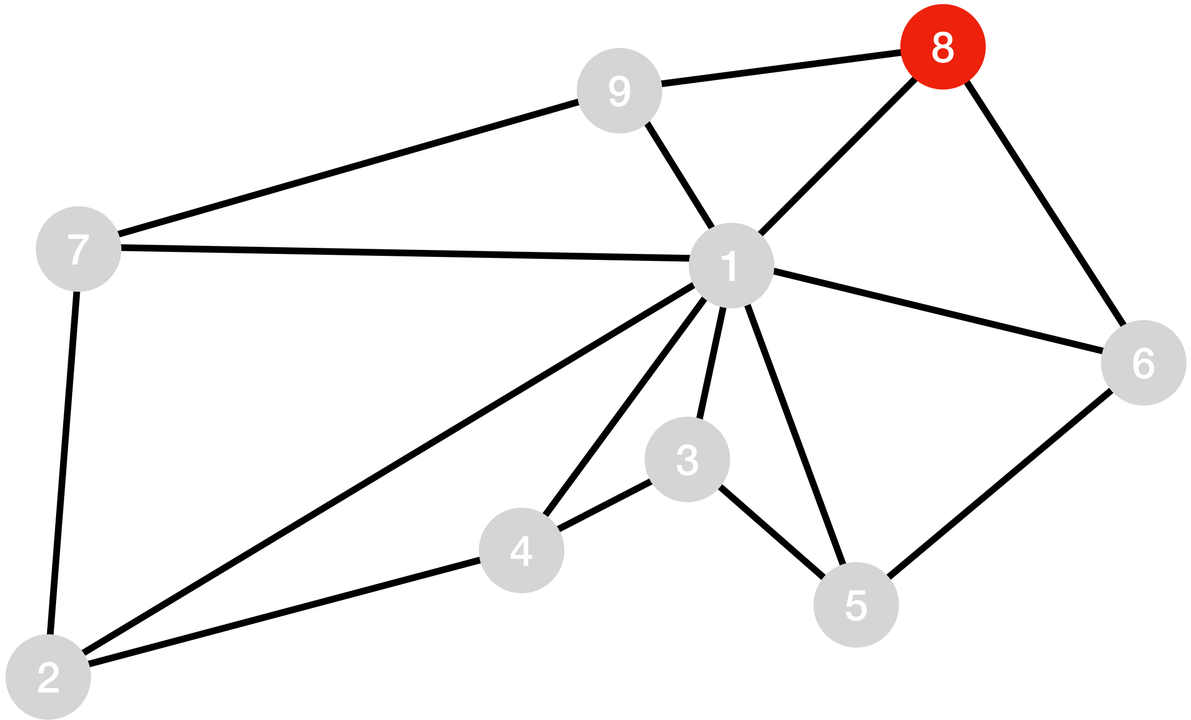
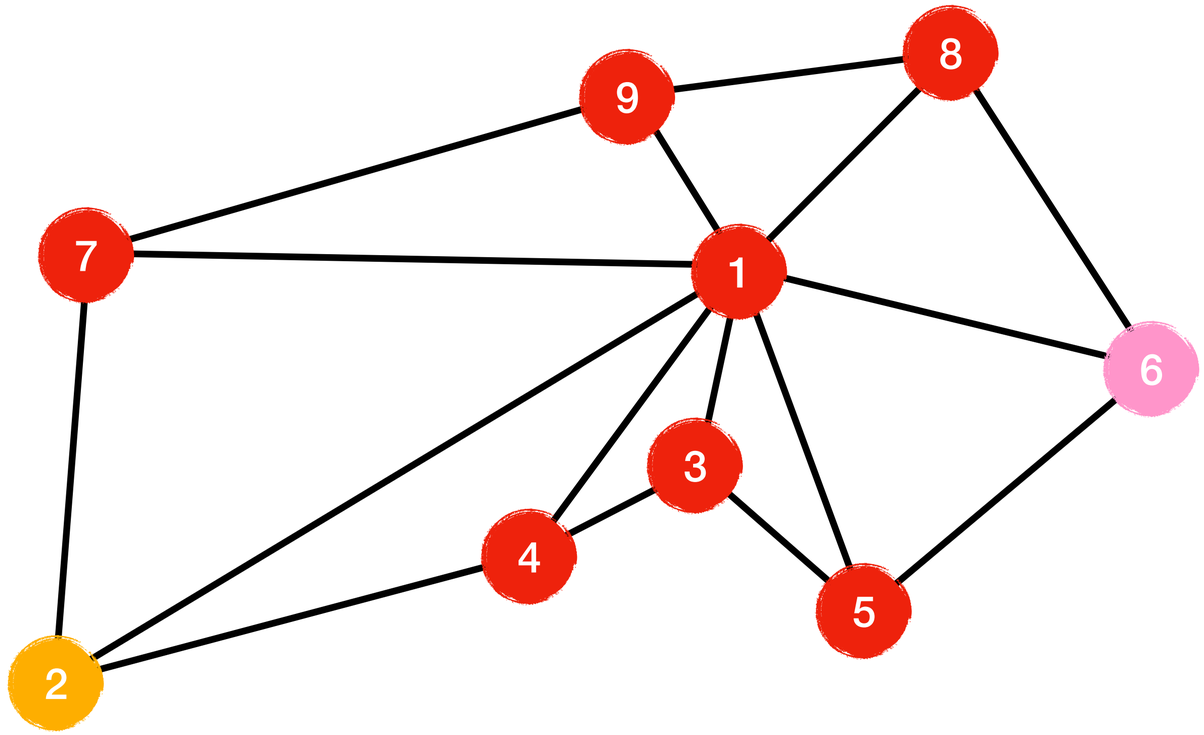




Brute force

Adjacency matrix

	1	2	3	4	5	6	7	8	9
1	0	1	1	1	1	1	1	1	1
2	1	0	-1	1	-1	-1	1	-1	-1
3	1	-1	0	1	1	-1	-1	-1	-1
4	1	1	1	0	-1	-1	-1	-1	-1
5	1	-1	1	-1	0	1	-1	-1	-1
6	1	-1	-1	-1	1	0	-1	1	-1
7	1	1	-1	-1	-1	-1	0	-1	1
8	1	-1	-1	-1	-1	1	-1	0	1
9	1	-1	-1	-1	-1	-1	1	1	0

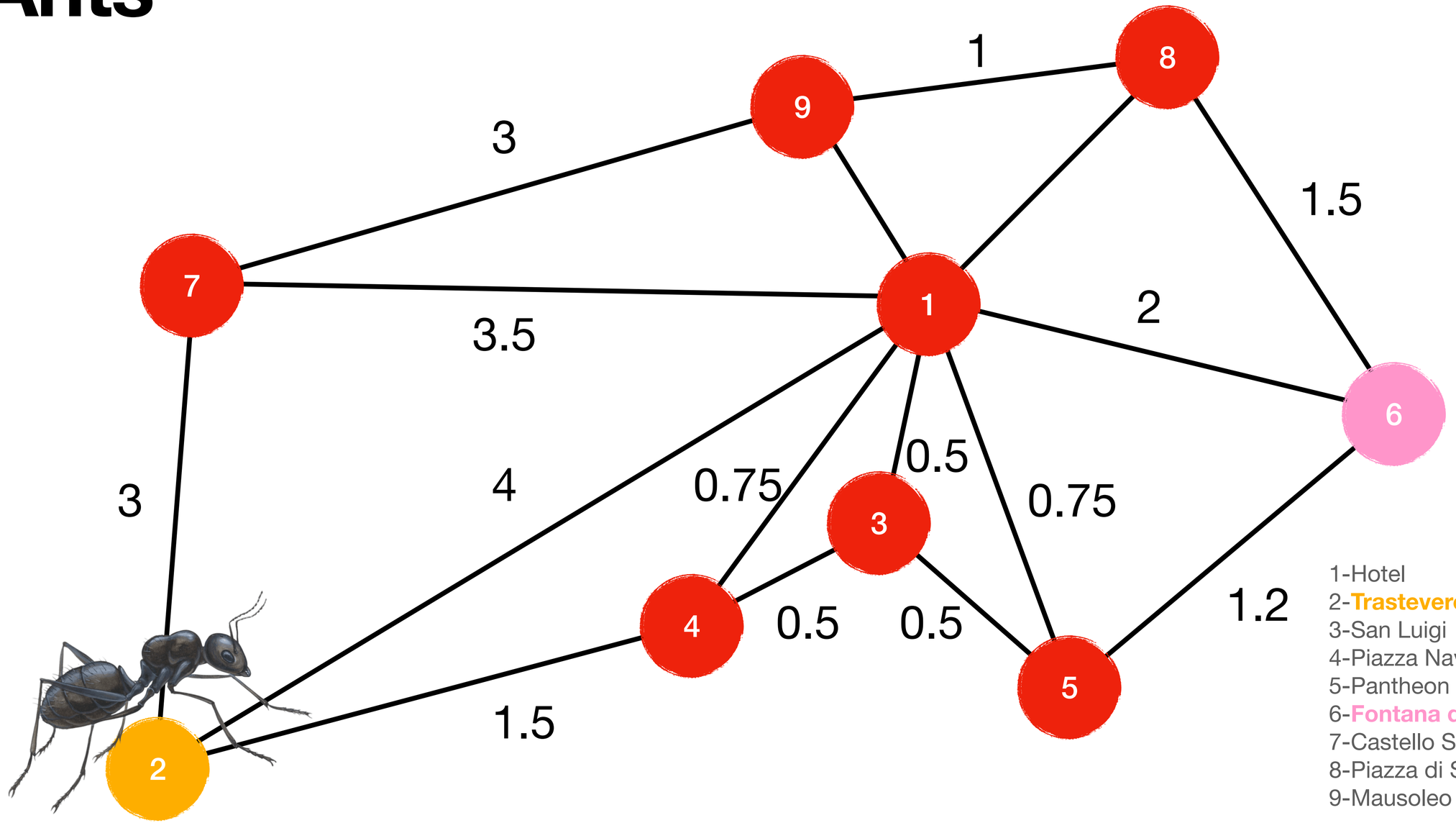


Ants

Swarm Intelligence

Source: Pexels: Andre Moura.

Ants



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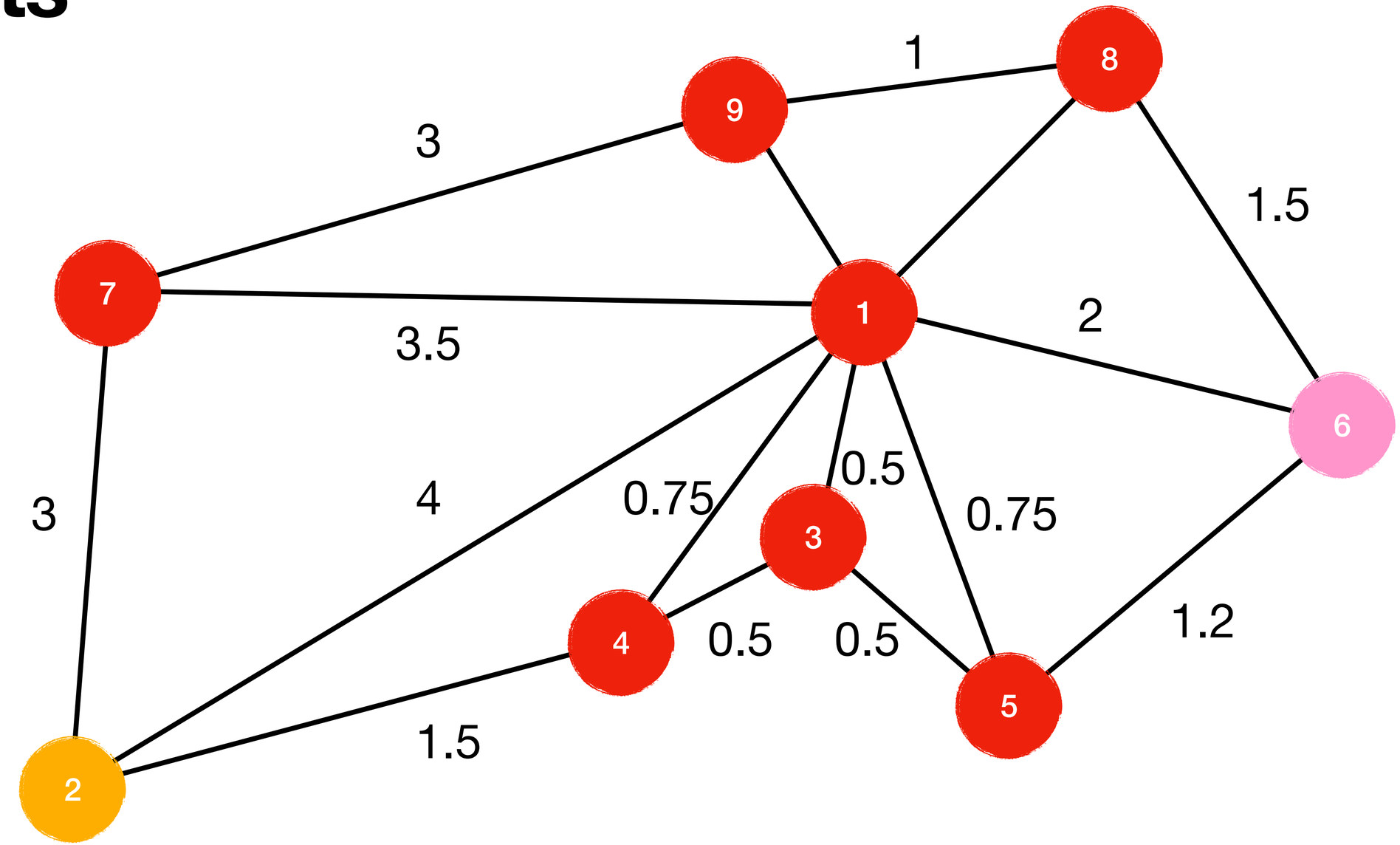
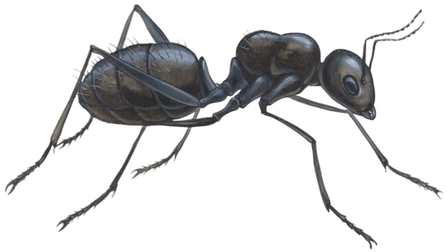
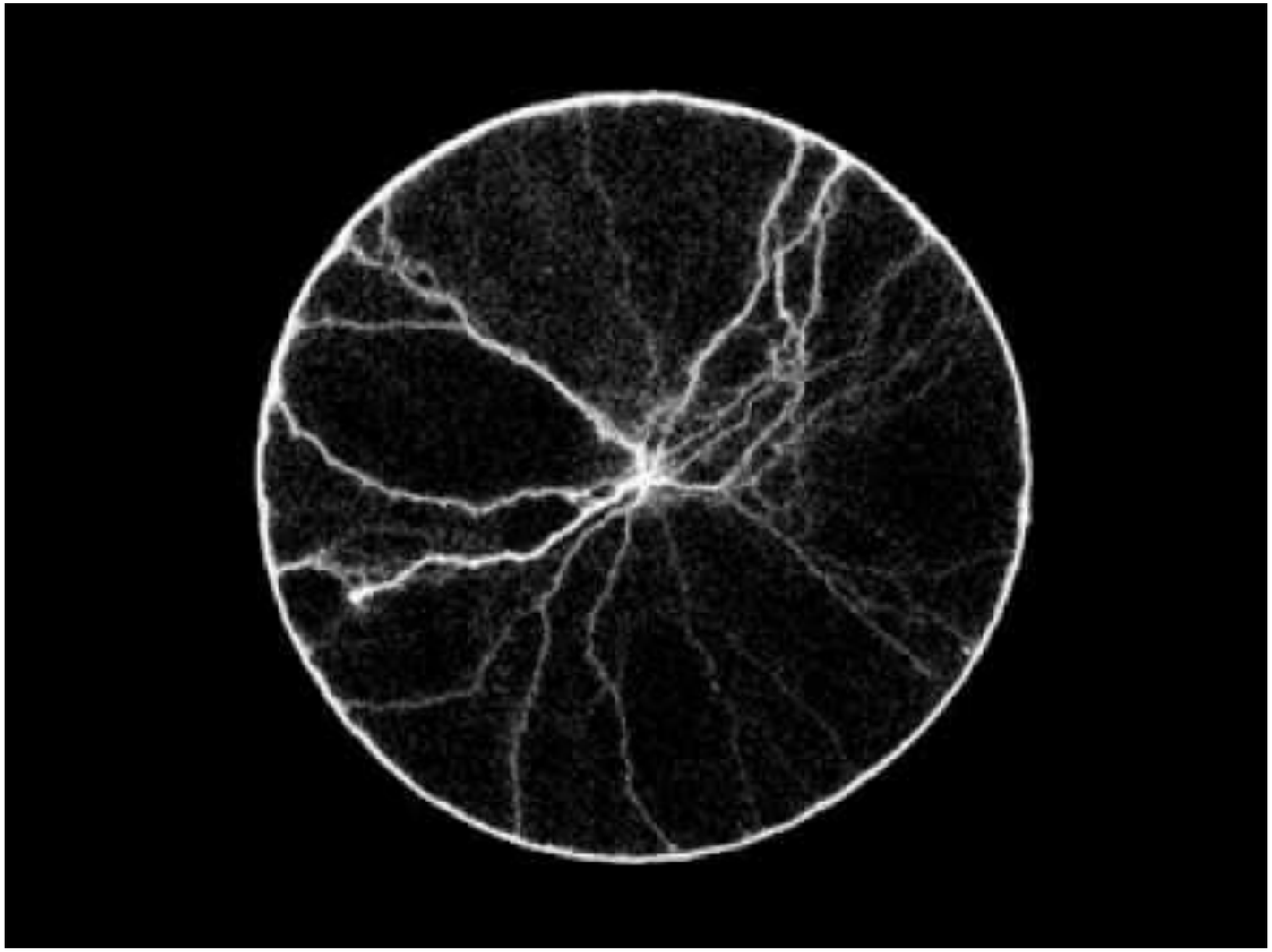
Ants



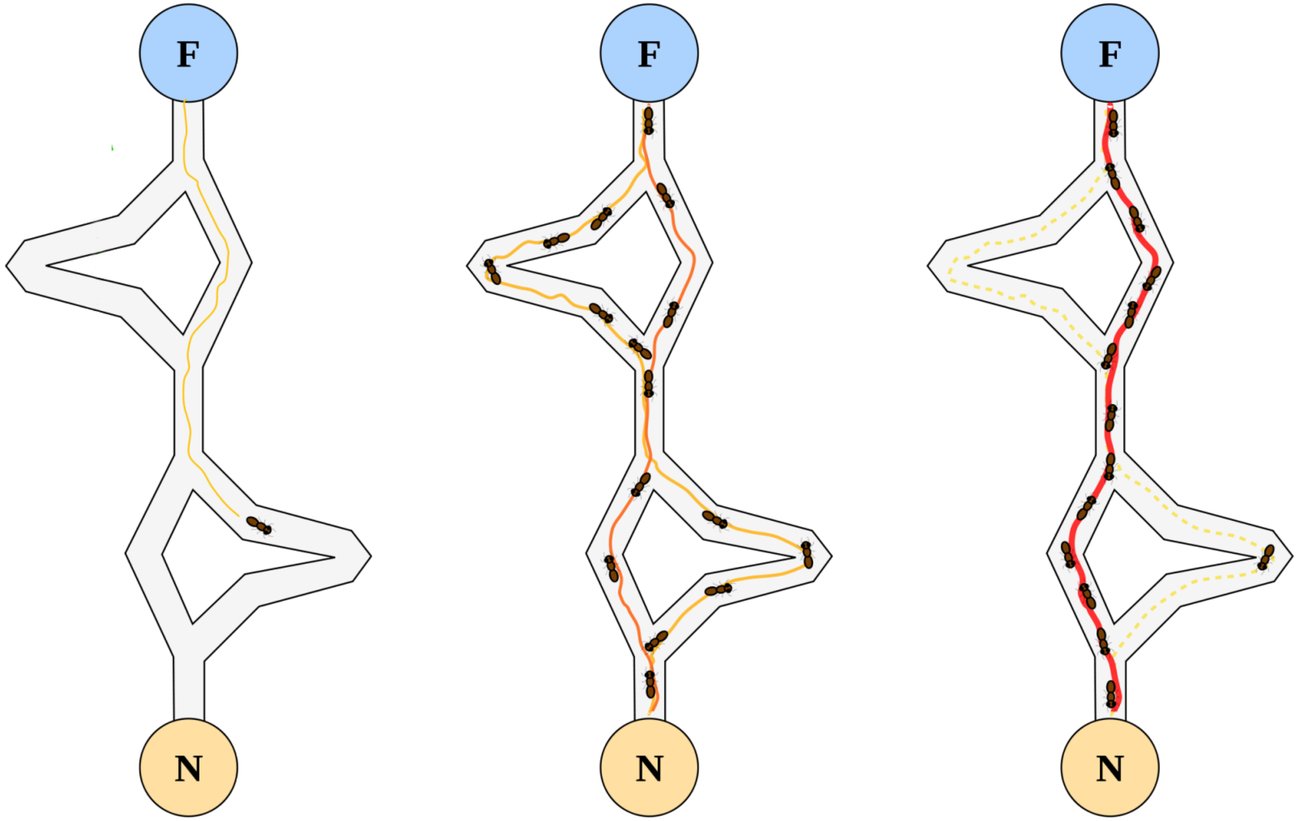
Pheromone trails





Modelling ant “social” behaviour

… with artificial ants

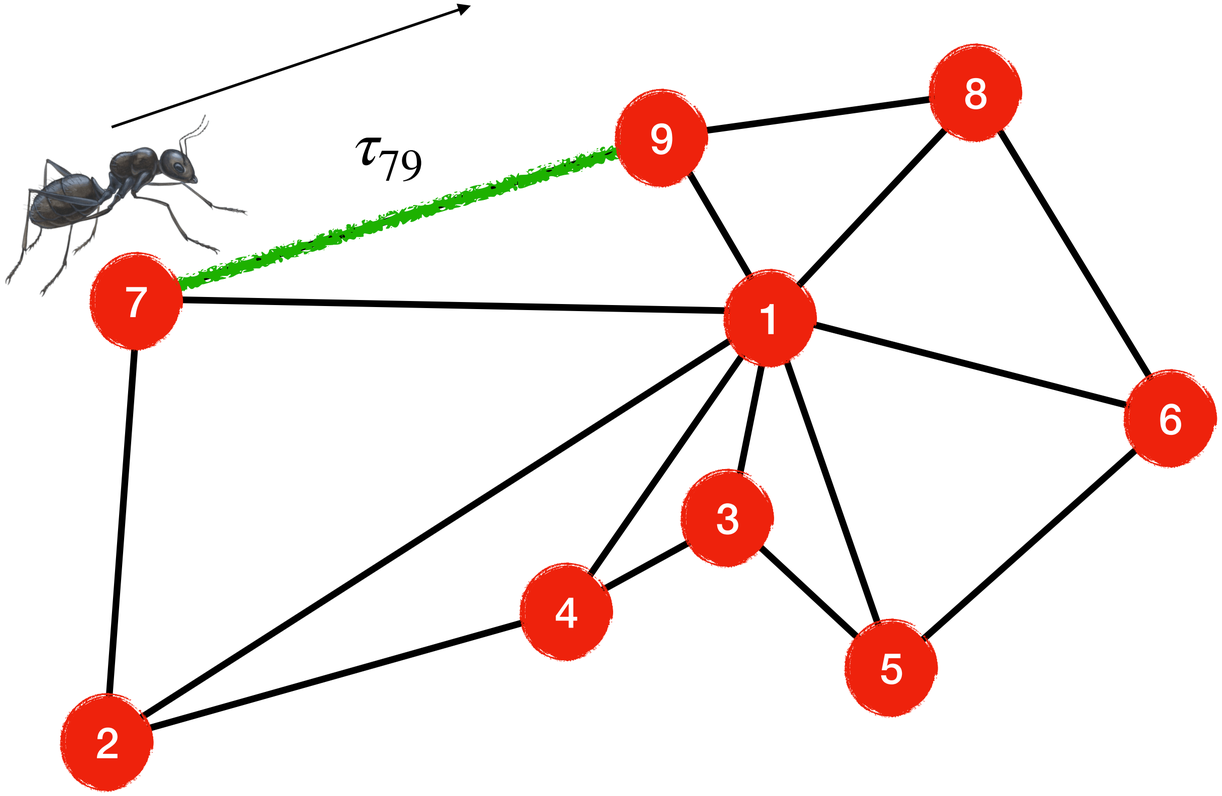


“The shortest path contains more pheromones due high traffic”

Modelling ant “social” behaviour

… with artificial ants

$$p_{ij} = \frac{w_{ij}^{\alpha}\tau_{ij}^{\beta}}{\sum_{<i,k>} w_{ik}^{\alpha}\tau_{ik}^{\beta}} \text{ with } 0 < \alpha < 1 \text{ and } \beta \geq 1$$



τ_{ij} pheromones laid at ij edge

w_{ij} weight between ij edge

Modelling ant “social” behaviour

… with artificial ants

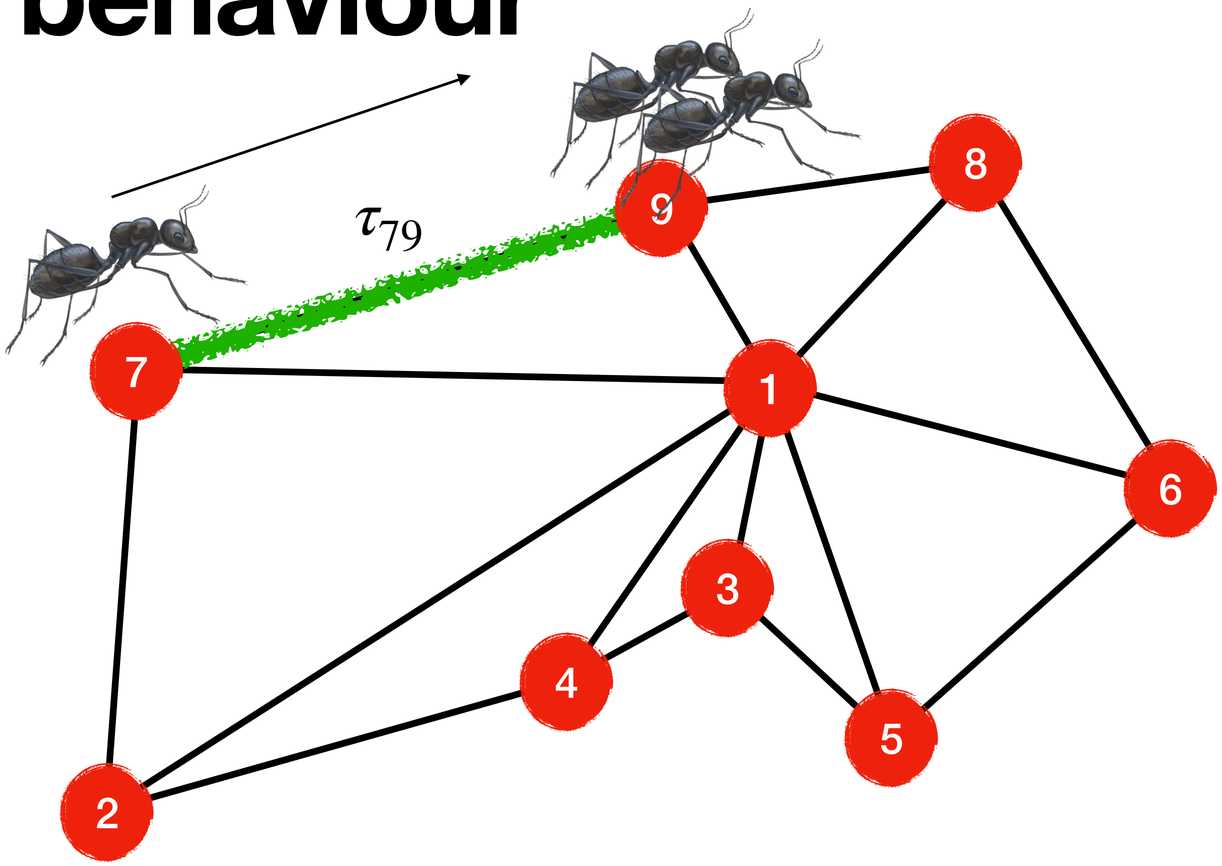
$$p_{ij} = \frac{w_{ij}^{\alpha}\tau_{ij}^{\beta}}{\sum_{<i,k>} w_{ik}^{\alpha}\tau_{ik}^{\beta}} \text{ with } 0 < \alpha < 1 \text{ and } \beta \geq 1$$

Pheromones updates

$$\Delta\tau_{ij,n} = \begin{cases} \frac{Q}{L(P_n)} & \text{if an ant } n \text{ crossed } ij \\ 0 & \text{otherwise} \end{cases}$$

$P_n = \{s, i_1, i_2, \ldots, i_k, f\}$ vertex sequence

$L(P_n) = \sum_{j \in P_n} d_{j,j+1}$ path distance



τ_{ij} pheromones laid at ij edge

w_{ij} weight between ij edge

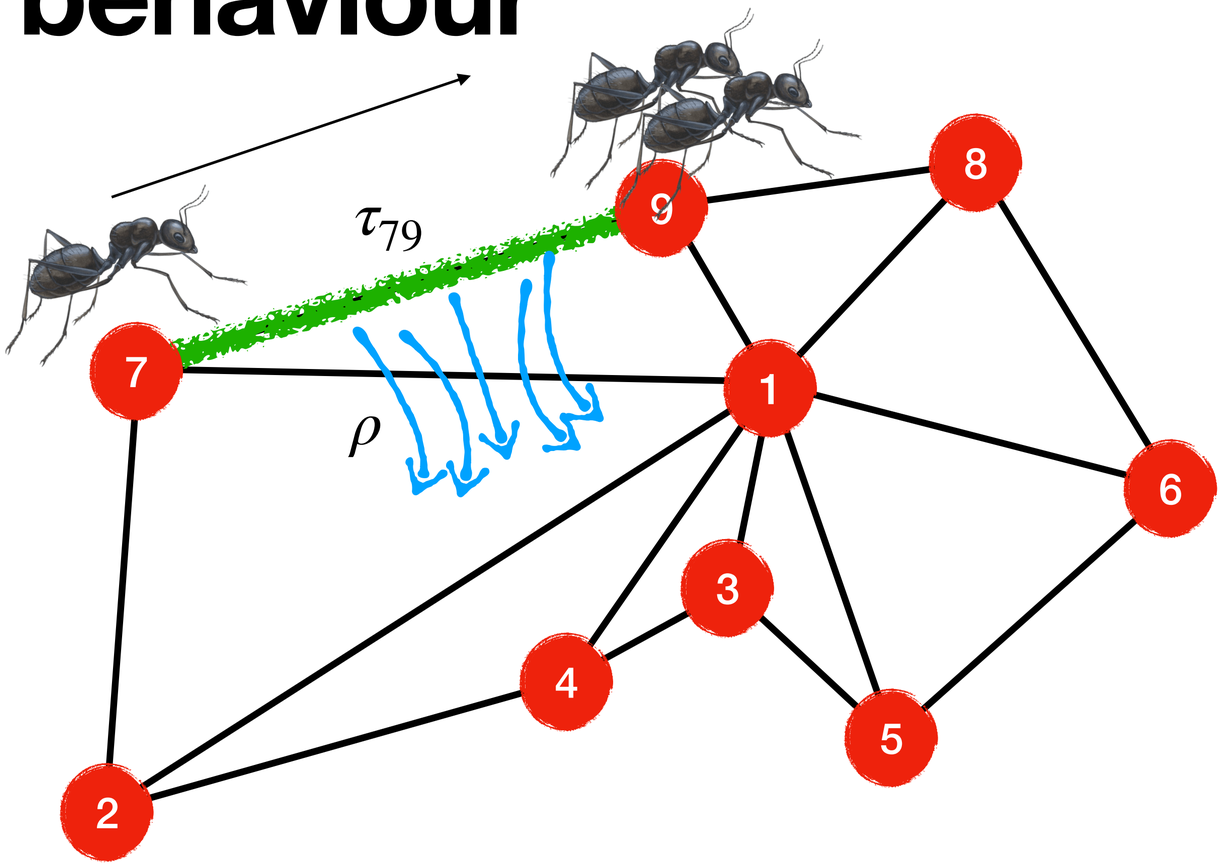
Modelling ant “social” behaviour

… with artificial ants

Pheromones updates

$$\tau_{ij} = (1-\rho)\tau_{ij} + \bar{\sum_{n \in ij}} \Delta\tau_{ij,n}$$

$\bar{\sum}$ sum over all the ants that reached f



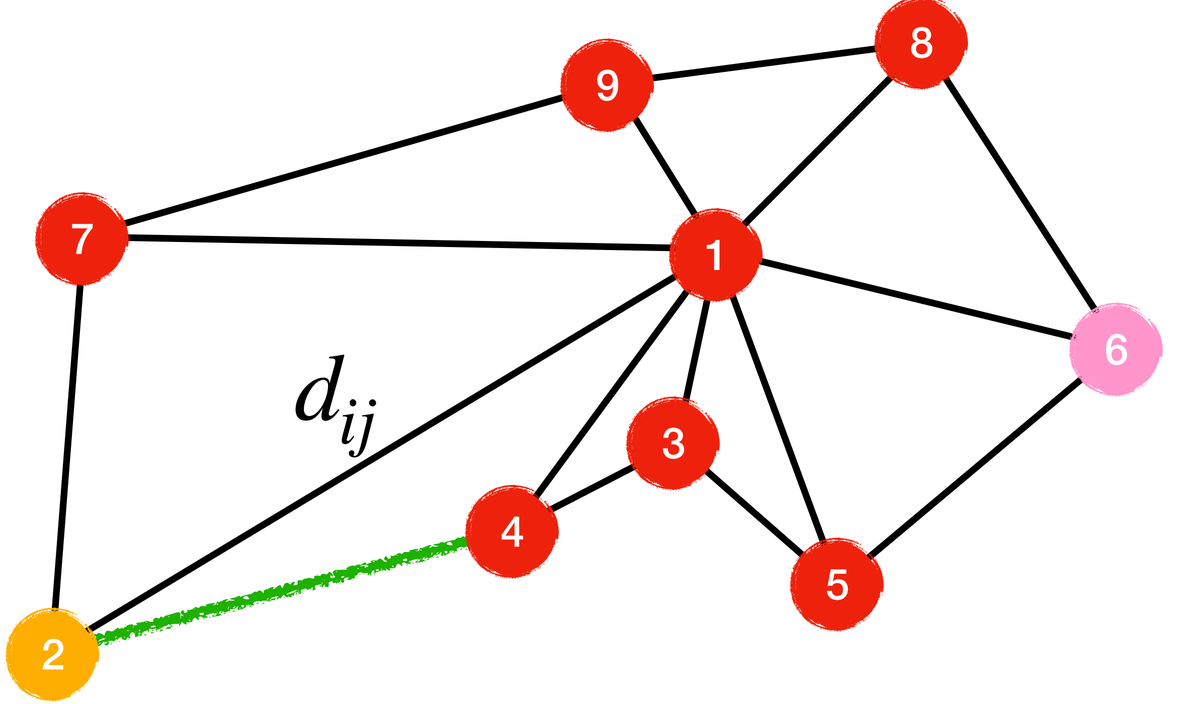
τ_{ij} pheromones laid at ij edge

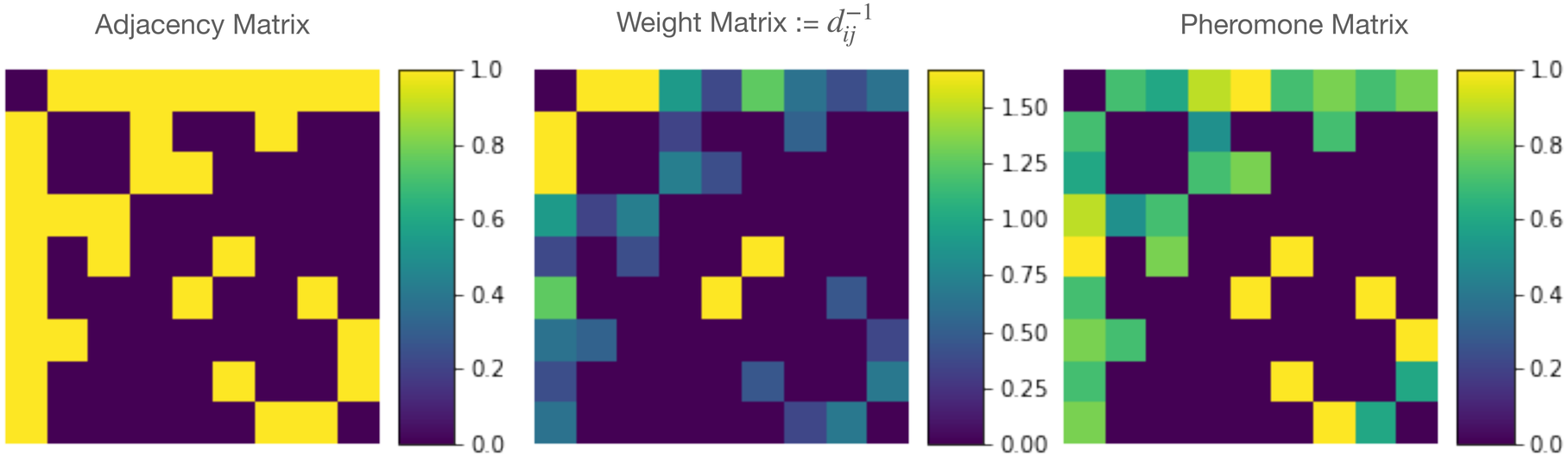
w_{ij} weight between ij edge

ρ evaporation rate

Ants in action

- Adjacency Matrix
- Weight Matrix
- Pheromone Matrix

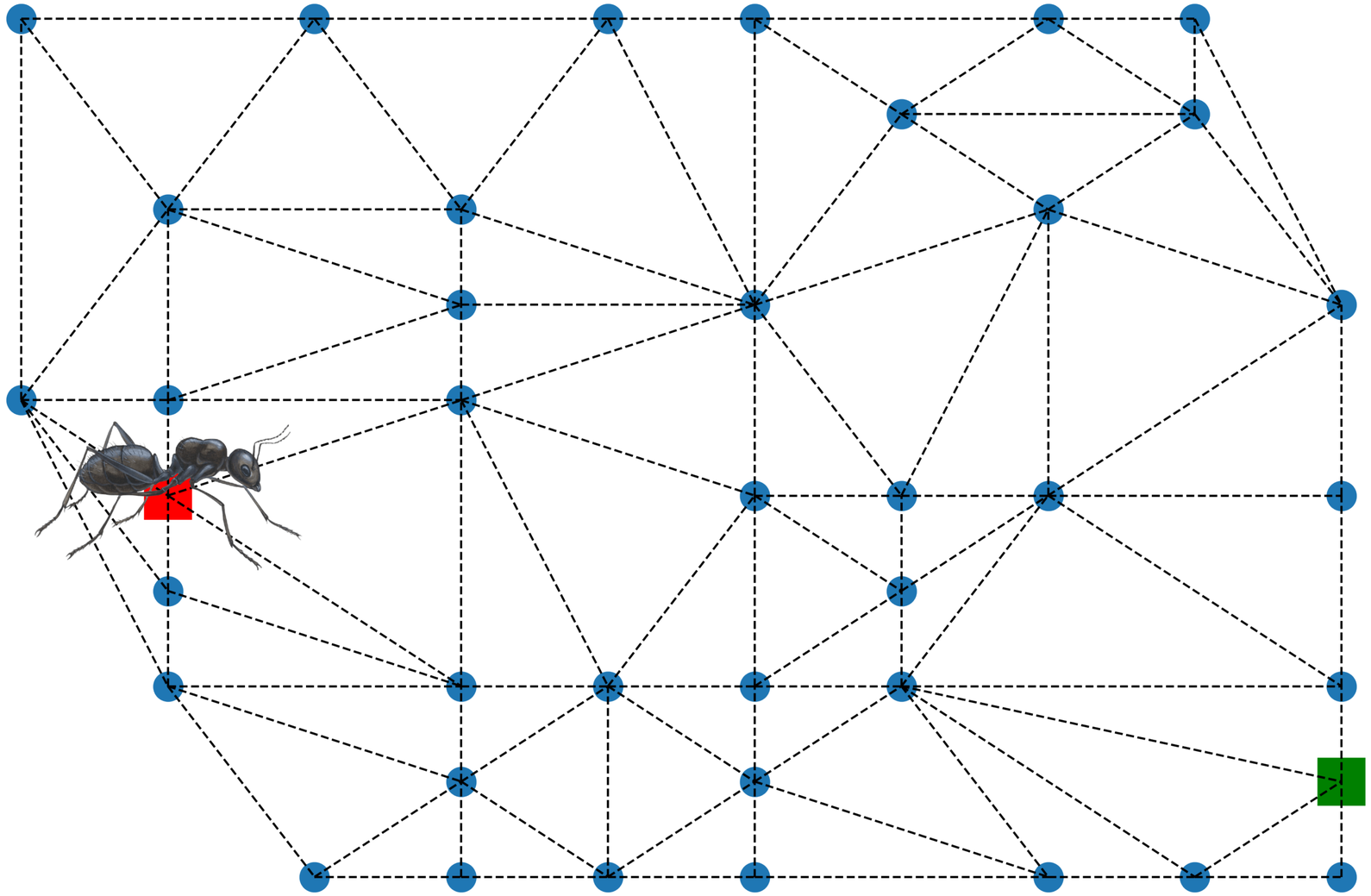




Ants in action

$$p_{ij} = \frac{w_{ij}^{\alpha}\tau_{ij}^{\beta}}{\sum_{<i,k>} w_{ik}^{\alpha}\tau_{ik}^{\beta}} \text{ with } 0 < \alpha < 1 \text{ and } \beta \geq 1$$

$$\tau_{ij}|_{t=0} = 1$$

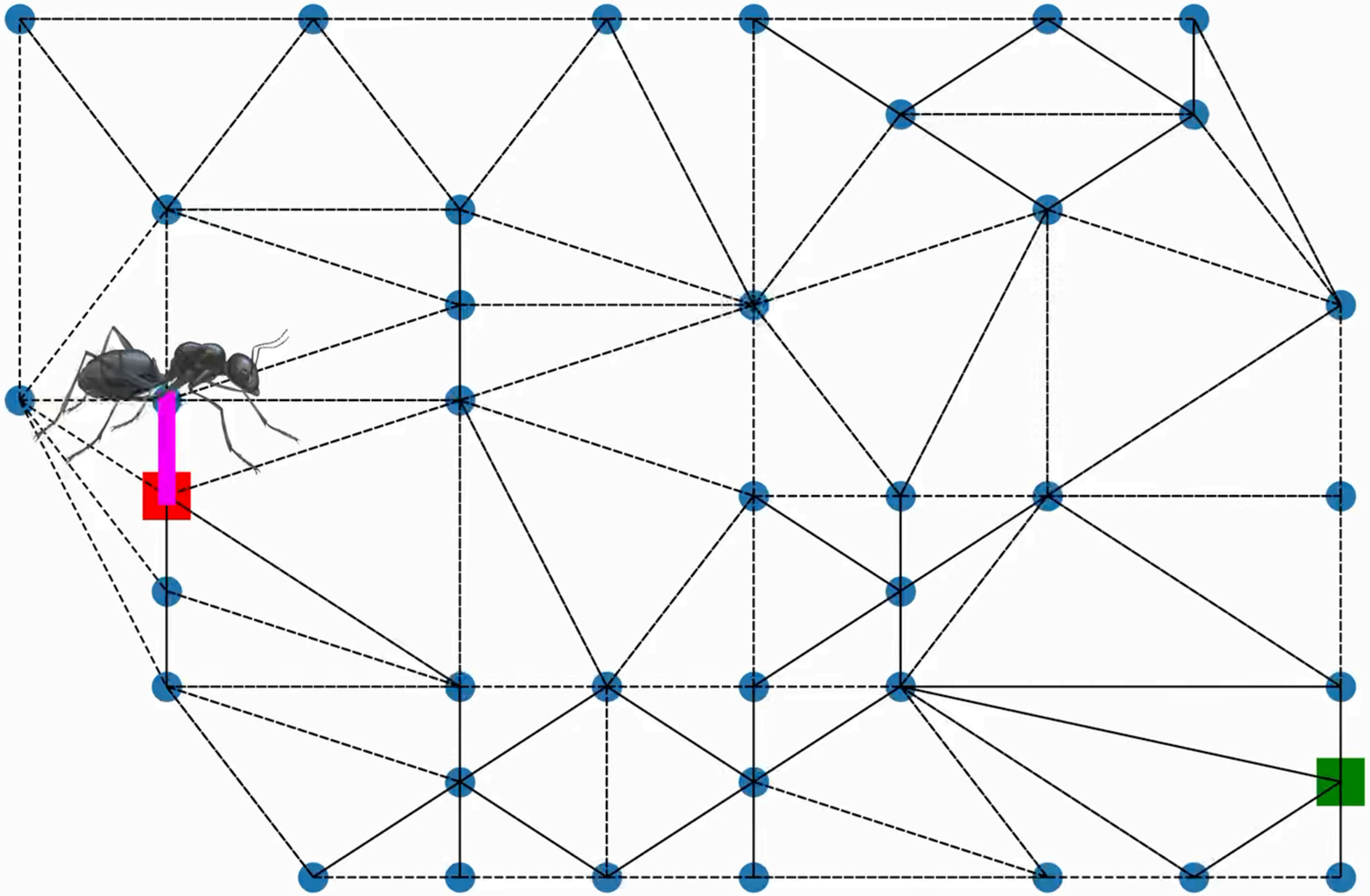


Iteration = 0

Ants in action

$$p_{ij} = \frac{w_{ij}^{\alpha}\tau_{ij}^{\beta}}{\sum_{<i,k>} w_{ik}^{\alpha}\tau_{ik}^{\beta}} \text{ with } 0 < \alpha < 1 \text{ and } \beta \geq 1$$

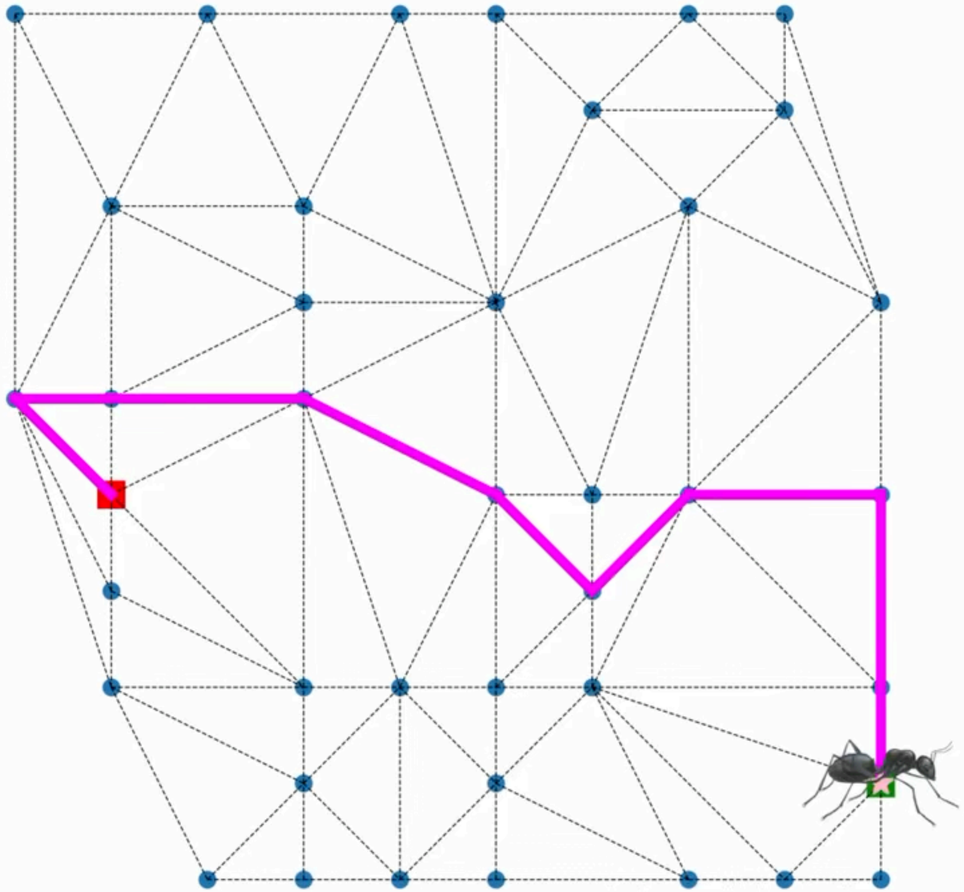
$$\tau_{ij}|_{t=0} = 1$$

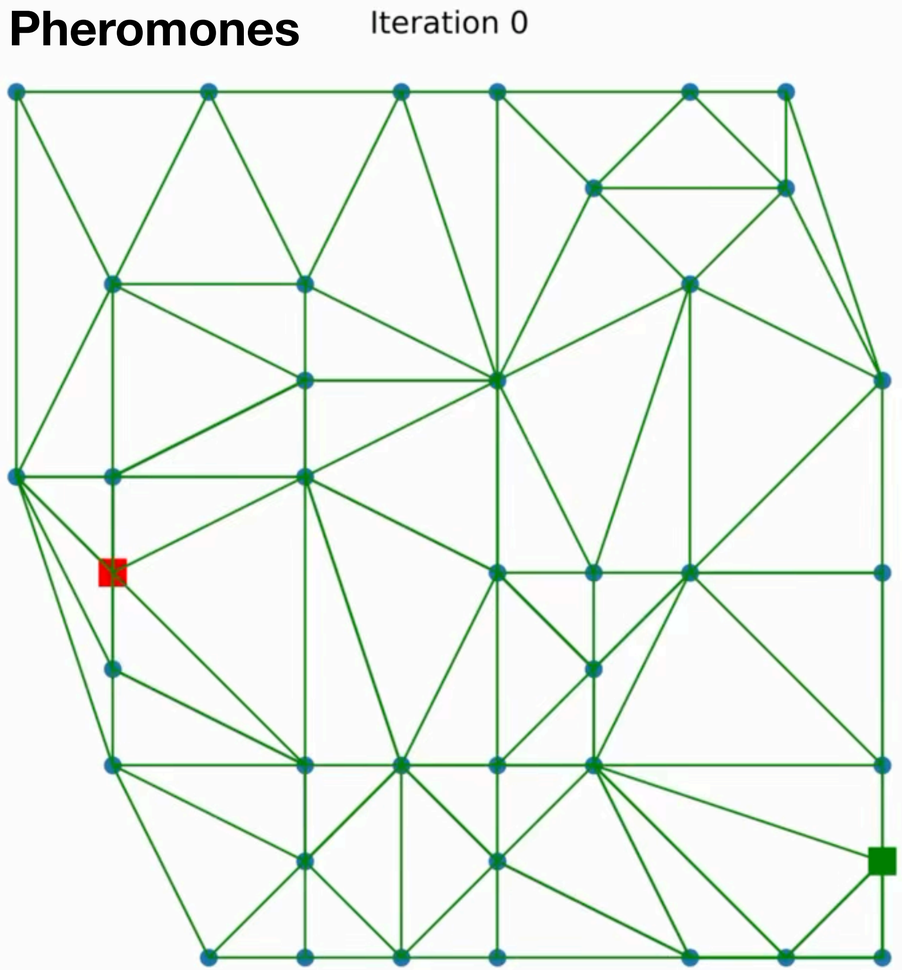


Iteration = 0

Ants in action

$$\tau_{ij} = (1-\rho)\tau_{ij} + \sum_{n \in ij}^{\bar{}} \Delta\tau_{ij,n}$$





$$\tau_{ij}|_{t=0} = 1$$

Ants Lab

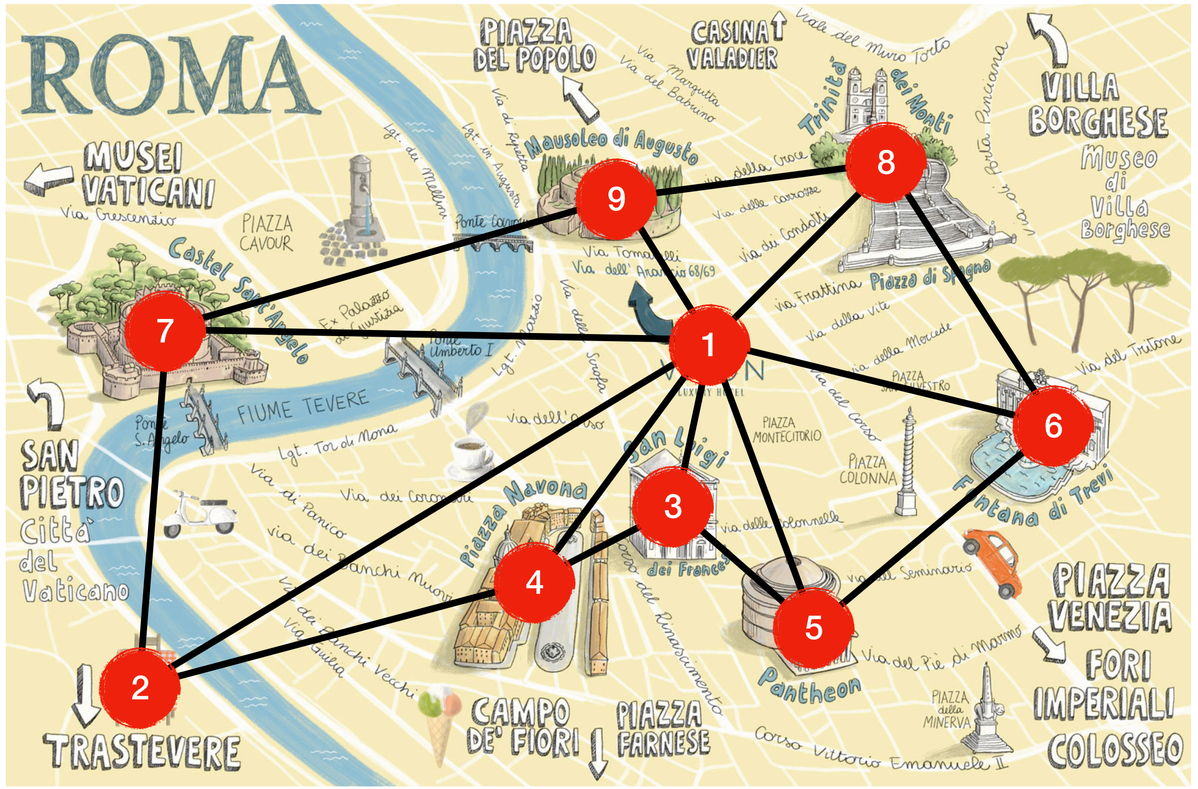
Code Ant colony optimization for any graph. Provided to points in the map you must return the shortest path between these points. As a reference use the following parameters:

$\alpha = 1, \beta = 1, \rho = 0.5$

First try with small number of points (no more than 40) so you can test your algorithm.

Extra

Suppose that I want to from 2 to 6 but I want to make a stop at 3 and 9. Generalize the ant-colony optimization to the goal of reaching multiple targets in sequence; for example, the case in which you seek the shortest path passing first through point A, then B, then C, and so on.



Ants Lab

A first ant-colony optimization roadmap:

1- With the code provided to generate graphs, generate the connection matrix, weight matrix, paths, and distance matrix

2-Generate the pheromone matrix.

3-Write a function that, given a path P, calculates the length of the path by summing the distances covered at each step, according to $L(P_n) = \sum_{j \in P_n} d_{j,j+1}$

4-Write a function that, given a sequence of vertices, simplifies it (i.e., whenever the sequence has a repeated vertex, the subsequence between the two instances of the repeated vertex is removed together with one occurrence of the repeated vertex). For example, 1,2,3,2,4 should be simplified to 1,2,4, eliminating the jump from 2 to 3 and back. Another example: 1,2,3,5,6,2,4,5,3 can become either 1,2,4,5,3 by elimination of the subsequence between the repeated 2 or 1,2,3 by elimination of the subsequence between the repeated 3.

5-Code a function that implements the branch decision rule given by the equation $p_{ij} = \frac{w_{ij}^{\alpha} \tau_{ij}^{\beta}}{\sum_{<i,k>} w_{ik}^{\alpha} \tau_{ik}^{\beta}}$ with $0 < \alpha < 1$ and $\beta \geq 1$

6. Write a function that updates τ_{ij} according to the rules $\Delta \tau_{ij,n}$