An introduction to active matter

Active matter: what it is?

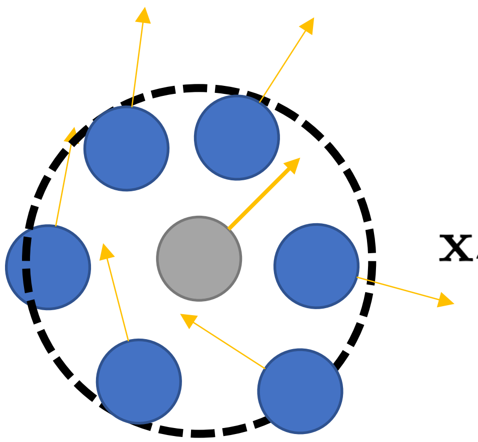
"Each active particle consumes and dissipates energy going through a cycle that fuels internal changes, generally leading to motion"

Active matter: what it is?

VOLUME 75, NUMBER 6 PHYSICAL REVIEW LETTERS 7 AUGUST 1995

Novel Type of Phase Transition in a System of Self-Driven Particles

Tamás Vicsek,[1,2] András Czirók,[1] Eshel Ben-Jacob,[3] Inon Cohen,[3] and Ofer Shochet[3]
[1]*Department of Atomic Physics, Eötvös University, Budapest, Puskin u 5-7, 1088 Hungary*
[2]*Institute for Technical Physics, Budapest, P.O.B. 76, 1325 Hungary*
[3]*School of Physics, Tel-Aviv University, 69978 Tel-Aviv, Israel*
(Received 25 April 1994)

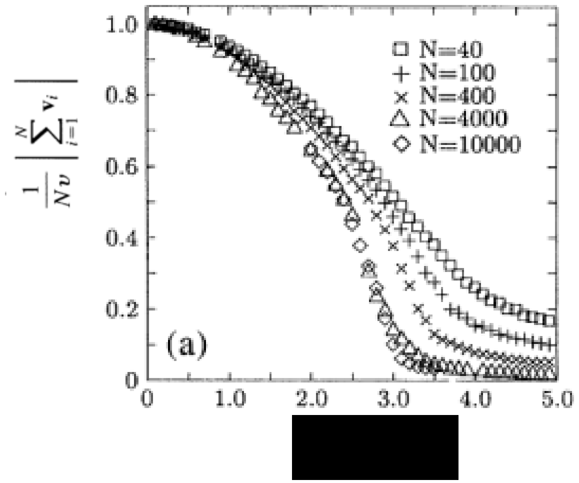


Self propulsion velocity

$$\mathbf{x}_i(t+\Delta t) = \mathbf{x}_i(t) + \mathbf{v}_i \Delta t$$

$$\theta(t+\Delta t) = \langle \theta(t) \rangle + \Delta\theta$$

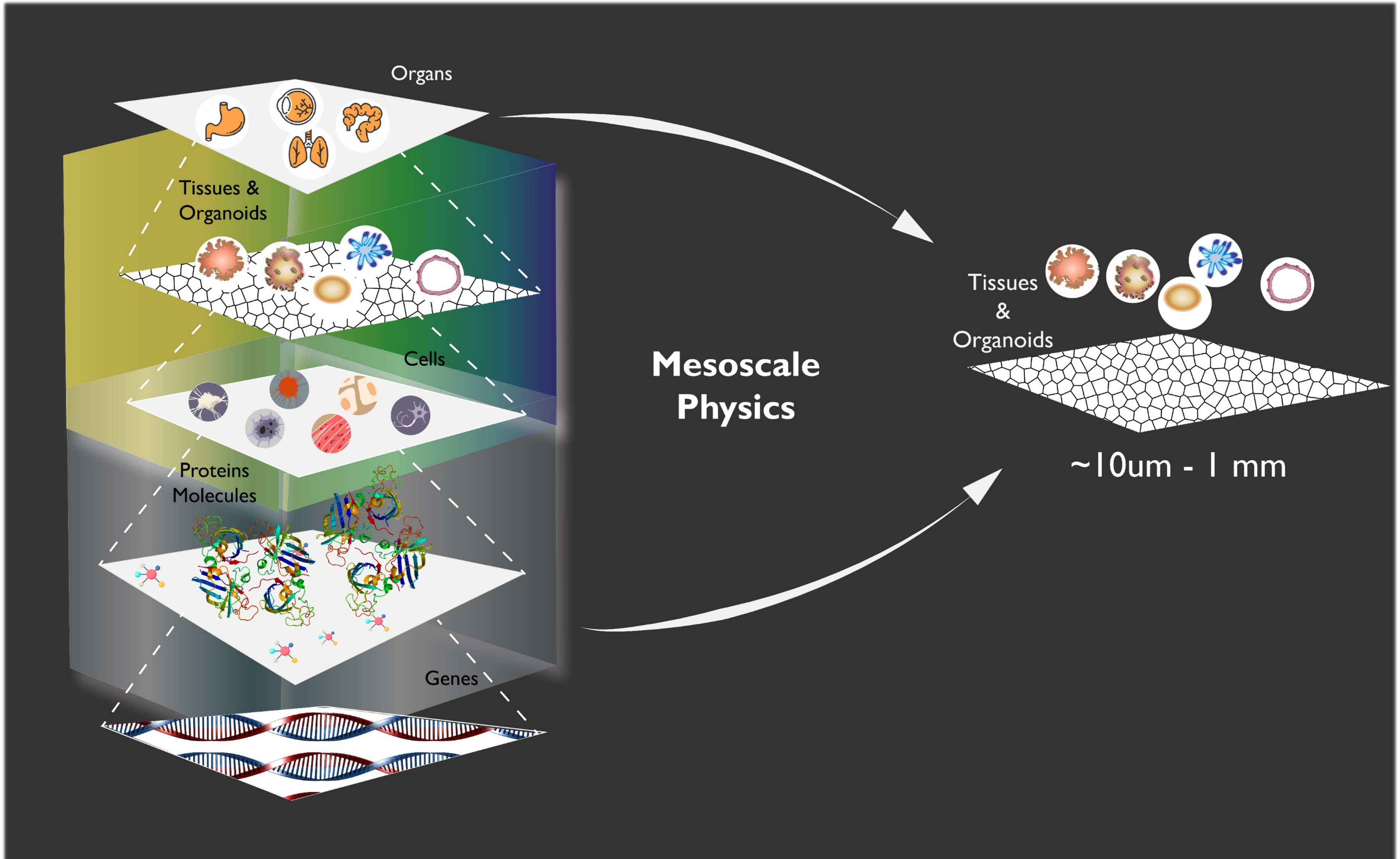
Noise



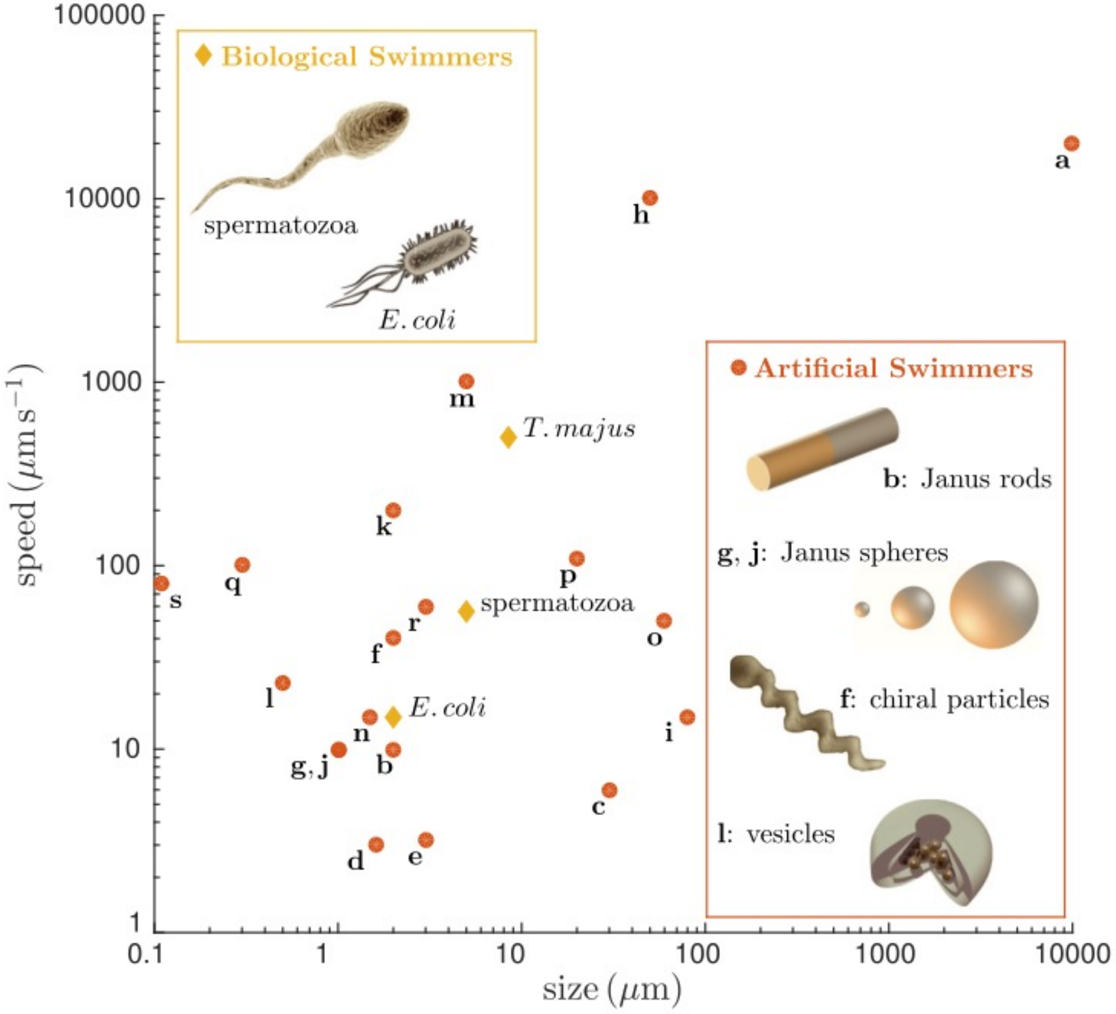
"Each active particle consumes and dissipates energy going through a cycle that fuels internal changes, generally leading to motion"

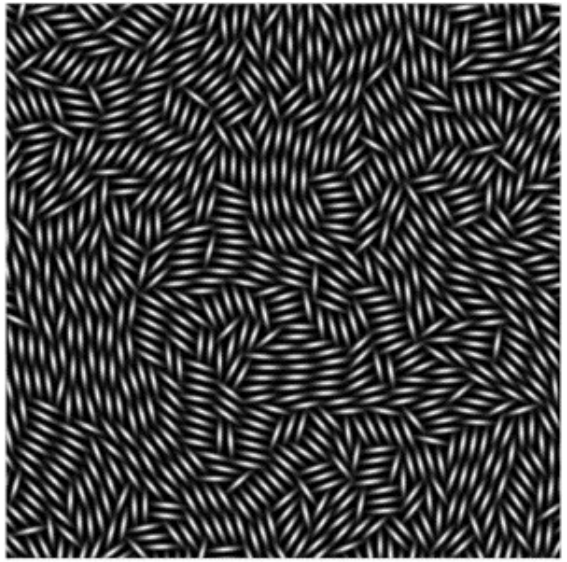
Growth, form and active mechanics in biology

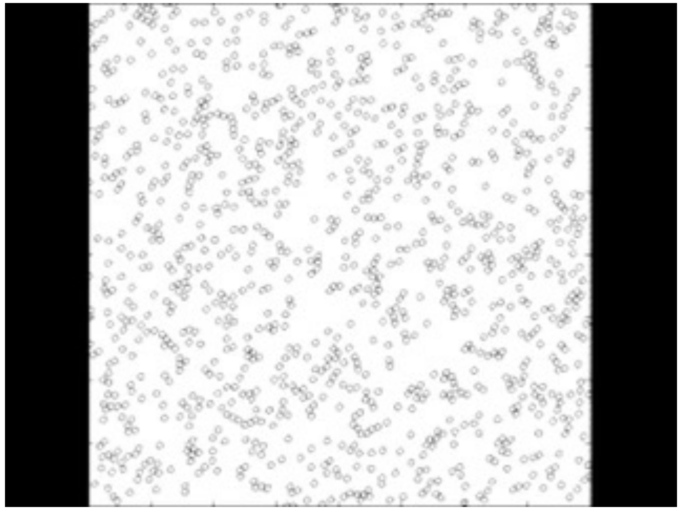
Soft Matter Physics: mesoscale phenomenon



Soft Matter Physics: mesoscale phenomenon

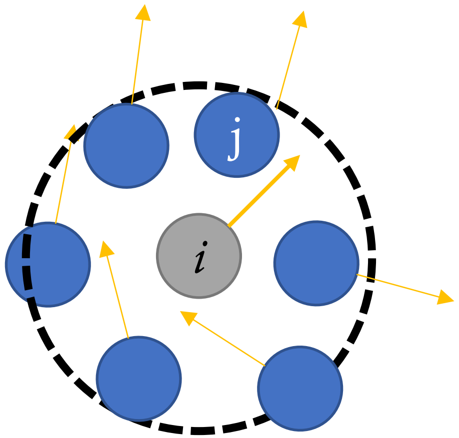






What it makes active matter different?

- Microscopy reversibility

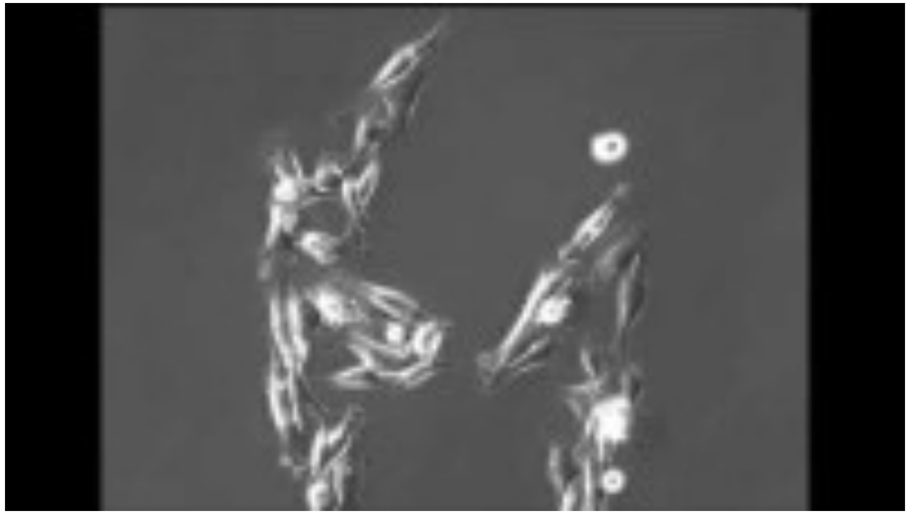


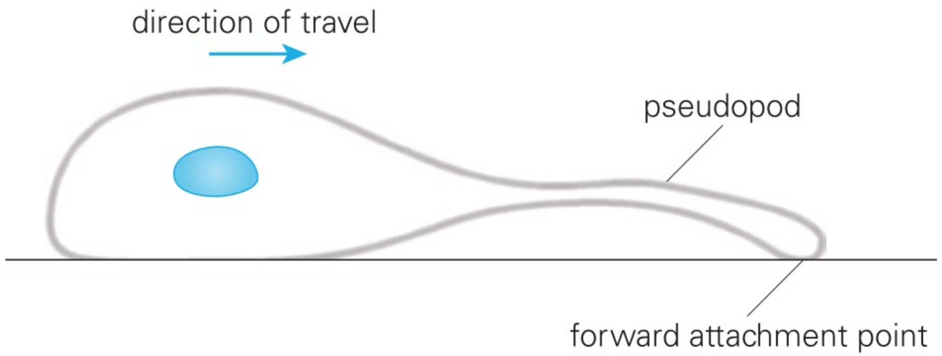
$$m\frac{\partial^2 \mathbf{r}_i}{\partial t^2} = \sum_j \mathbf{F}_{ij}$$

$$\mathbf{v}_i = \frac{\partial \mathbf{r}_i}{\partial t}$$

$$\begin{aligned} t &\to -t \\ \{\mathbf{r}_i\} &\to \{\mathbf{r}_i\} \\ \{\mathbf{v}_i\} &\to \{-\mathbf{v}_i\}\,, \end{aligned}$$

Active matter and Microscopic Reversibility





$$m\frac{\partial^2 \mathbf{r}_i}{\partial t^2} = -\sum_j \nabla\phi_{ij} + \mathbf{F}_{NC}$$

$$\begin{aligned} t &\rightarrow -t \\ \{\mathbf{r}_i\} &\rightarrow \{\mathbf{r}_i\} \\ \{\mathbf{v}_i\} &\rightarrow \{-\mathbf{v}_i\}\,, \end{aligned}$$

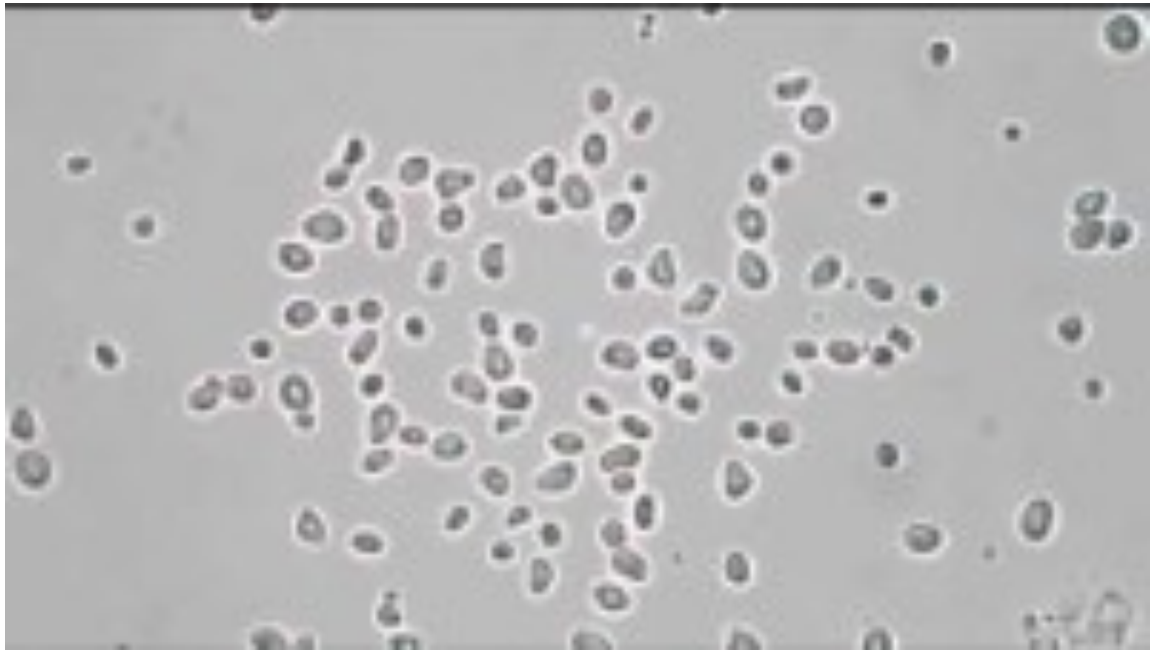
Active matter breaks local breaking of detailed balance

$$p\left[\{\mathbf{r}_i(0)\},\{\mathbf{v}_i(0)\};\left(\left\{\mathbf{r}_i^{'}(t)\right\},\left\{\mathbf{v}_i^{'}(t)\right\}\right)\right]=p\left[\left\{\mathbf{r}_i^{'}(t)\right\},\left\{-\mathbf{v}_i^{'}(t)\right\};\{\mathbf{r}_i(0)\},\{-\mathbf{v}_i(0)\}\right]$$

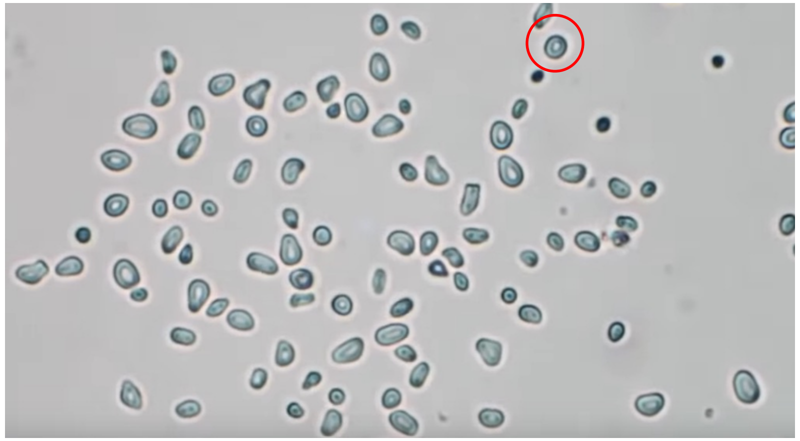
Probability of going for A to B is the same that going from B to A

Modelling Dry Active Matter

- **Brownian Motion**
  Brownian motion, which originally referred to the irregular movement of small pollen grains in water reported by Robert Brown (1828), is the prototypical stochastic mechanism for describing equilibrium and non-equilibrium processes.



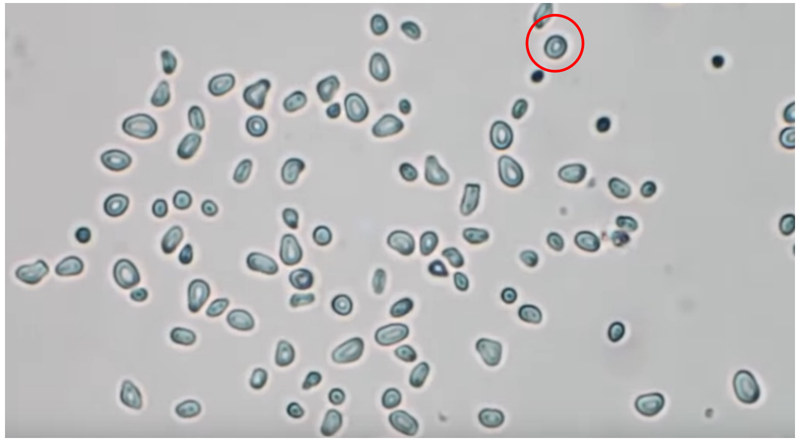
Equation of motion for a tagged particle in a fluid



Consider the case where the **solute** particle is large ~ 1 μm compared with the **fluid.**

The solute particle is constantly being hit by the fluid particles and as a result of constant collisions with the solvent atoms, the particle is experiencing random motion. Although each of those collisions is governed by the deterministic equations of motion, the sheer number of collisions means that over timescales comparable with the relaxation time the system has no memory of the history of collisions, and this is essentially a random process

Equation of motion for a tagged particle in a fluid



Given that the forces exerted by the fluid into the particle is random then we can write (1D):

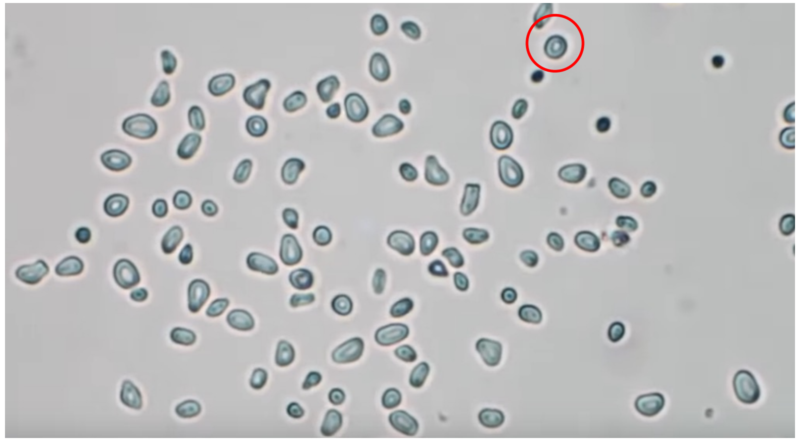
$$m\dot{v} = F(t)$$

$$m\dot{v} = \int_0^t dt' F(t'),$$

Given that *F(t)* is a random variable with $\langle F(t) \cdot F(t') \rangle = \alpha \delta(t - t')$

$$\boxed{\langle m\dot{v} \rangle = \left\langle \int_0^t dt' F(t') \right\rangle = \int_0^t dt' \langle F(t') \rangle = 0}$$

Equation of motion for a tagged particle in a fluid



Kinetic Energy

$$\left\langle \frac{1}{2} m v^2 \right\rangle = \tfrac{1}{2} \int_0^t dt' \int_0^t dt'' \left\langle F(t') F(t'') \right\rangle$$

$$= \tfrac{\alpha}{2} \int_0^t dt' \int_0^t dt'' \delta(t' - t'')$$

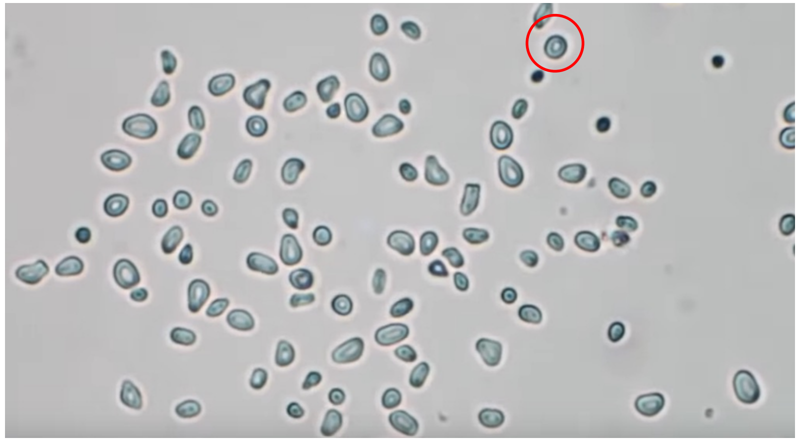
$$= \tfrac{\alpha}{2} \int_0^t dt'$$

$$= \tfrac{\alpha}{2} t.$$

Nonsense! The kinetic energy increase with time

What are we missing?

Equation of motion for a tagged particle in a fluid



Fluid drag….

A particle moving through the fluid will experience drag that will slow it down so that drag is proportional to the velocity of the particle and acts in the opposite direction of the velocity

$$\boxed{m\ddot{\mathbf{r}} = -\gamma\dot{\mathbf{r}} + \mathbf{F}\left(t\right)}$$

The Langevin equation

The Langevin equation

Consider the Langevin equation of motion for 1D

$$m\ddot{x} = -\gamma\dot{x} + F(t),$$

or

$$v = \dot{x}$$

$$m\dot{v} = -\gamma v + F(t).$$

We can write the last equation as

$$\frac{dv}{dt} = -\frac{\gamma}{m}v + \frac{1}{m}F(t).$$

The Langevin equation

We now introduce substitution a function $y(t)$ such that

$$v(t) = e^{-\frac{\gamma}{m}t} y(t) \,.$$

This gives us

$$\frac{dv}{dt} = \frac{d}{dt}\left(e^{-\frac{\gamma}{m}t} y(t)\right) = -\frac{\gamma}{m} e^{-\frac{\gamma}{m}t} y(t) + e^{-\frac{\gamma}{m}t}\frac{dy}{dt},$$

or

$$\cancel{-\frac{\gamma}{m} e^{-\frac{\gamma}{m}t} y(t)} + e^{-\frac{\gamma}{m}t}\frac{dy}{dt} = \cancel{-\frac{\gamma}{m} e^{-\frac{\gamma}{m}t} y(t)} + \frac{1}{m}F(t)$$

or

$$\frac{dy}{dt} = \frac{1}{m} e^{\frac{\gamma}{m}t} F(t) \,.$$

The Langevin equation

Nothing that $y(0) = v(0)$ we can formally integrate the last expression to obtain

$$y(t) = v(0) + \frac{1}{m}\int_0^t dt' e^{\frac{\gamma}{m}t'} F(t'),$$

or

$$v(t) = v(0)\, e^{-\frac{\gamma}{m}t} + \frac{1}{m}\int_0^t dt' e^{-\frac{\gamma}{m}(t-t')} F(t').$$

The variance in the displacement can be obtained by integrating the previous equation

$$\left\langle (x(t) - x(0))^2 \right\rangle = \frac{m^2}{\gamma^2}\left(v^2(0) - \frac{\alpha}{2m\gamma}\right)\left(1 - e^{-\frac{\gamma}{m}t}\right)^2 + \frac{\alpha}{\gamma^2}\left[t - \frac{m}{\gamma}\right]\left(1 - e^{-\frac{\gamma}{m}t}\right),$$

for longer time $t \to \infty$ we can write,

$$\boxed{\left\langle (x(t) - x(0))^2 \right\rangle \sim \frac{\alpha}{\gamma^2} t},$$

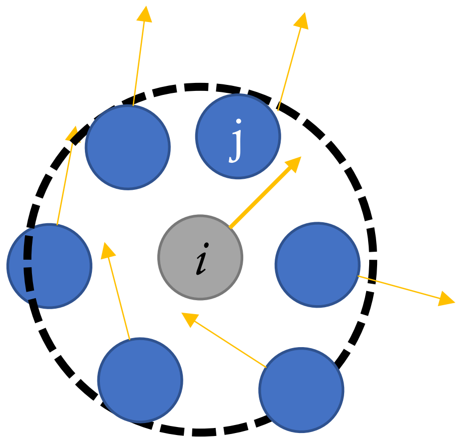
Mean square displacement

Viscous limit: Overdamped limit

$$Re = \frac{\text{inertial forces}}{\text{friction forces}} = \frac{\rho u L}{\mu} \approx 10^{-5}$$

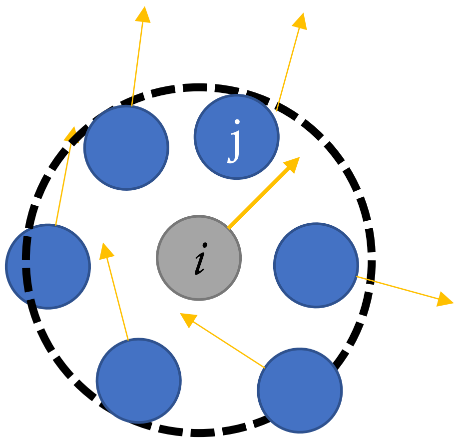
$$\cancel{m\ddot{\mathbf{r}}} = -\gamma\dot{\mathbf{r}} + \mathbf{F}\left(t\right)$$

$$\gamma\dot{\mathbf{r}} = \mathbf{F}\left(t\right)$$



$$\gamma\dot{\mathbf{r}} = -\sum_{j} \nabla\phi_{ij} + \mathbf{F}_{NC} + \mathbf{F}\left(t\right)$$

Overdamped limit Numerical Integration (Ito)



$$\gamma \dot{\mathbf{r}} = -\sum_j \nabla \phi_{ij} + \mathbf{F}_{NC} + \mathbf{F}\left(t\right)$$

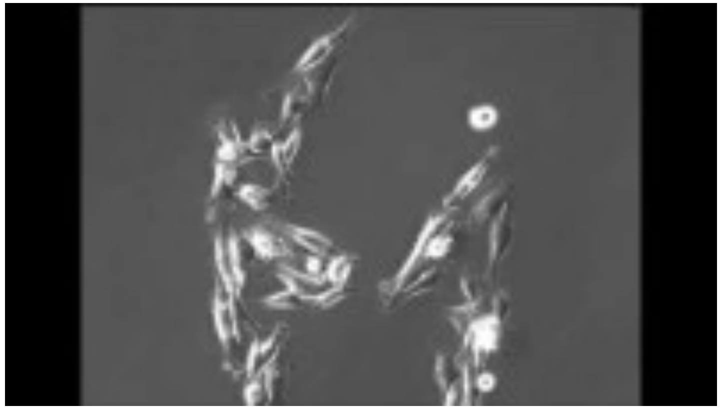
Requires knowledge if stochastic differential equation not covered in this class

Random gaussian vectors with unit variance and mean zero

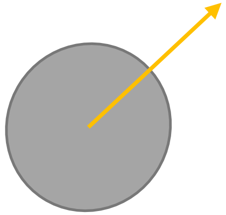
$$\vec{v}_i(t+\Delta t) = \sqrt{\alpha}\vec{\eta}_1$$

$$\vec{r}_i(t+\Delta t) = \vec{r}_i(t) - \frac{\Delta t}{\gamma}\sum_j \nabla \phi_{ij} + \frac{\Delta t}{\gamma}\vec{F}_{i,NC} + \sqrt{\Delta t \alpha}\vec{\eta}_2$$

Modelling Dry Active Matter



The **movement** of an active particle in 2d can be decomposed in a translational and rotational (director) movement.



$$\mathbf{r}_i = < x_i , y_i >$$

$$\mathbf{n}_i = < \cos\theta_i , \sin\theta_i >$$

Translational part:

$$\mathbf{F}_{i,active} = c\,\mathbf{n}_i$$

Rotational part:

$$\frac{\partial \theta_i}{\partial t} = \sqrt{\nu}\eta_{i,3}$$

Active matter and Microscopic Reversibility

$$m\frac{\partial^2 \mathbf{r}_i}{\partial t^2} = -\sum_j \nabla\phi_{ij} + \mathbf{F}_{NC}$$

$$\begin{aligned} t &\to -t \\ \{\mathbf{r}_i\} &\to \{\mathbf{r}_i\} \\ \{\mathbf{v}_i\} &\to \{-\mathbf{v}_i\}\,, \end{aligned}$$

Active matter breaks detailed balance

$$p\left[\{\mathbf{r}_i(0)\},\{\mathbf{v}_i(0)\};\left(\left\{\mathbf{r}_i^{'}(t)\right\},\left\{\mathbf{v}_i^{'}(t)\right\}\right)\right] = p\left[\left\{\mathbf{r}_i^{'}(t)\right\},\left\{-\mathbf{v}_i^{'}(t)\right\};\{\mathbf{r}_i(0)\},\{-\mathbf{v}_i(0)\}\right].$$