

Genetic Algorithms

From evolution to solutions

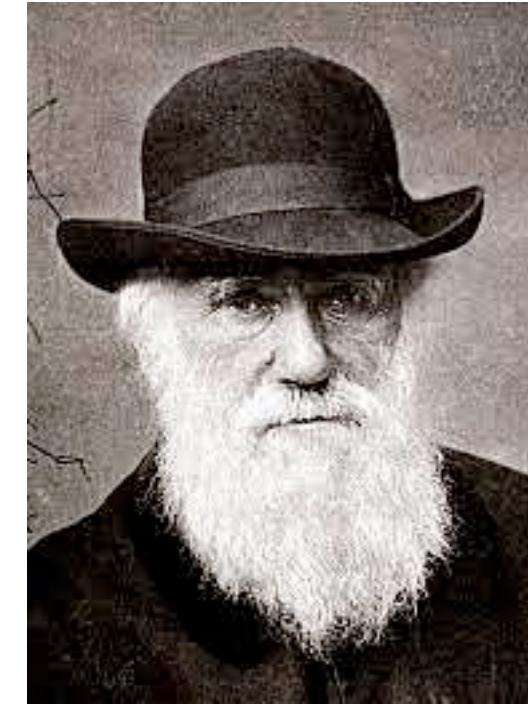
Daniel Matoz

Evolution in a nutshell



Jean-Baptiste Lamarck

all the physical changes occurring in an individual during its lifetime are inherited by its offspring



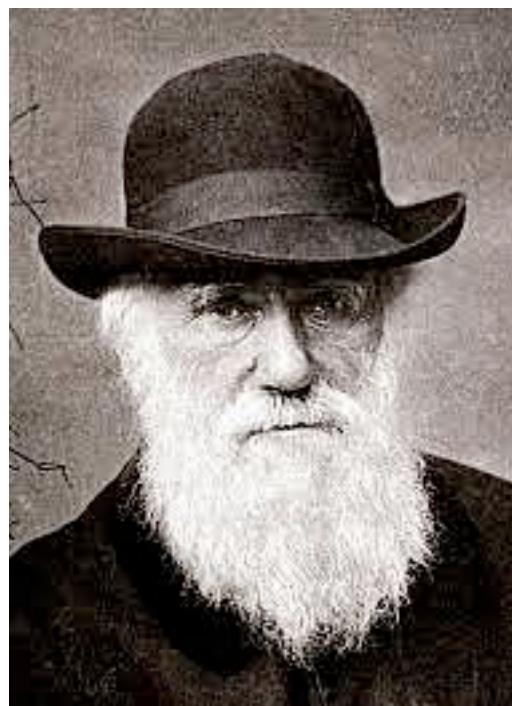
Charles Darwin

(1) variation—a liberalizing factor is present in all forms of life; (2) heredity—the conservative force that transmits similar organic form from one generation to another; and (3) the struggle for existence—which determines the variations that will confer advantages in a given environment, thus altering species through a selective reproductive rate.

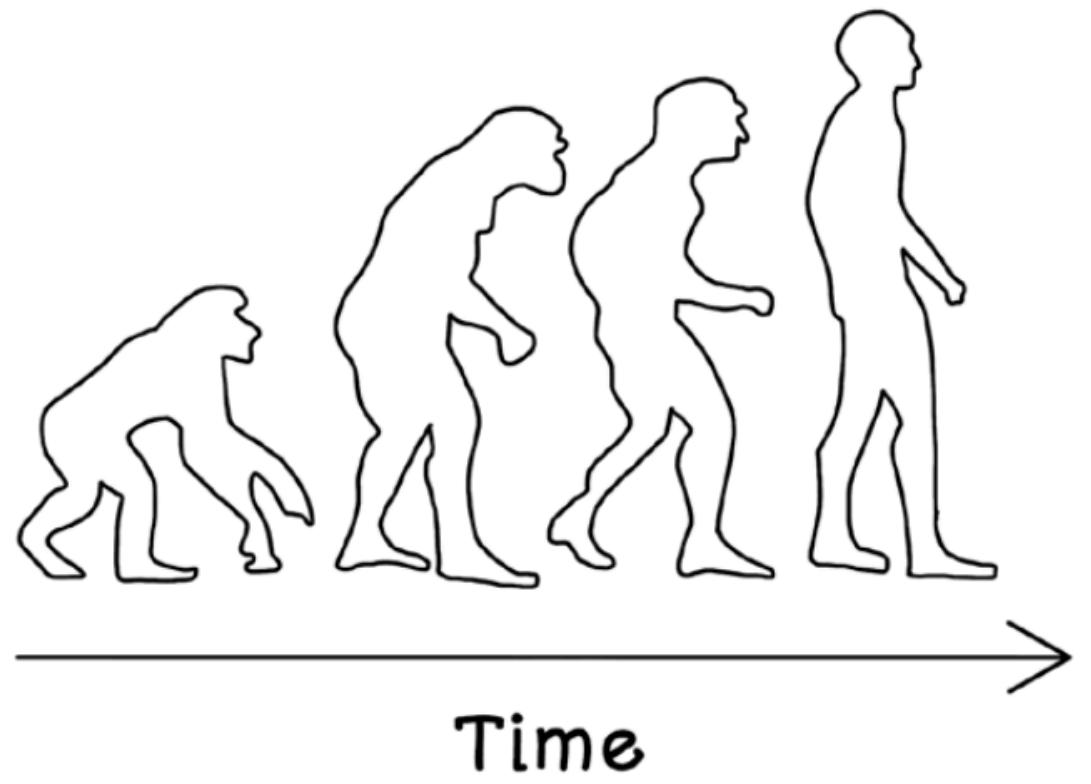
Evolution in a nutshell



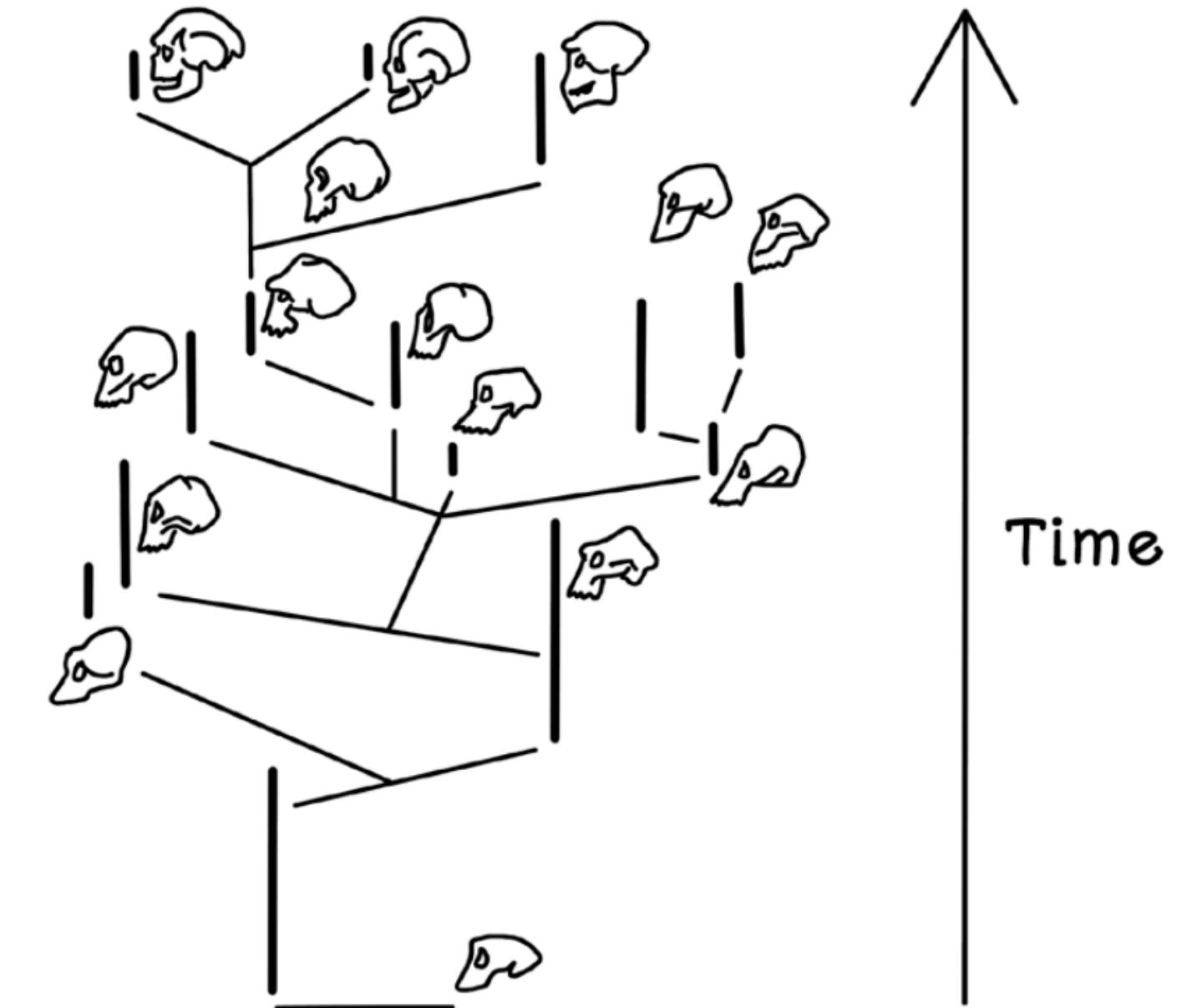
Jean-Baptiste Lamarck



Charles Darwin

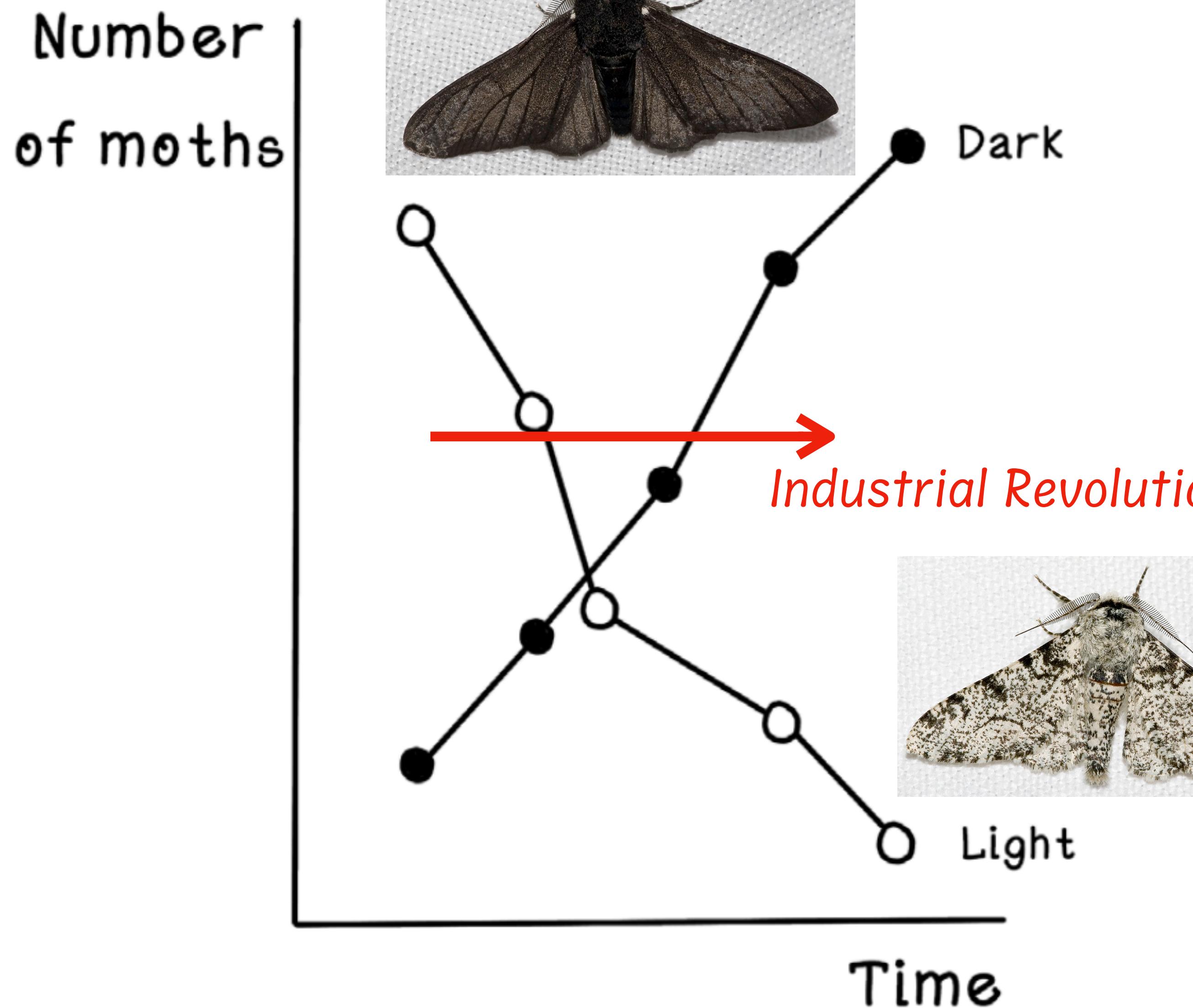


Perceived evolution



Actual evolution

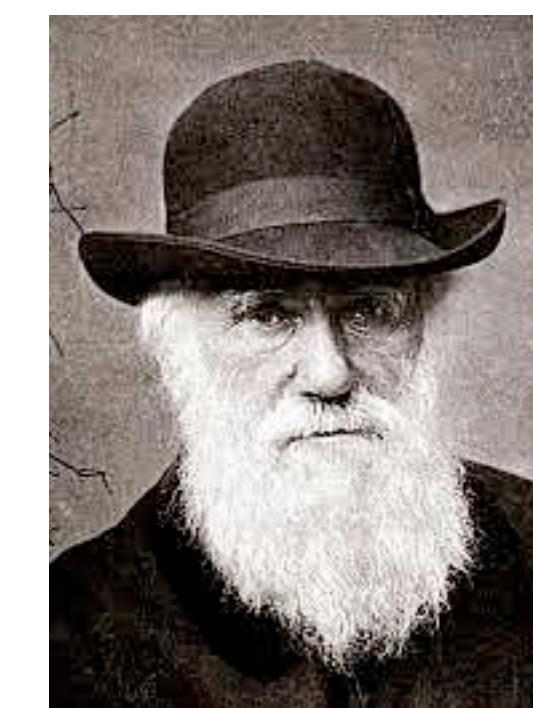
Evolution for adaptation



effective camouflage!!



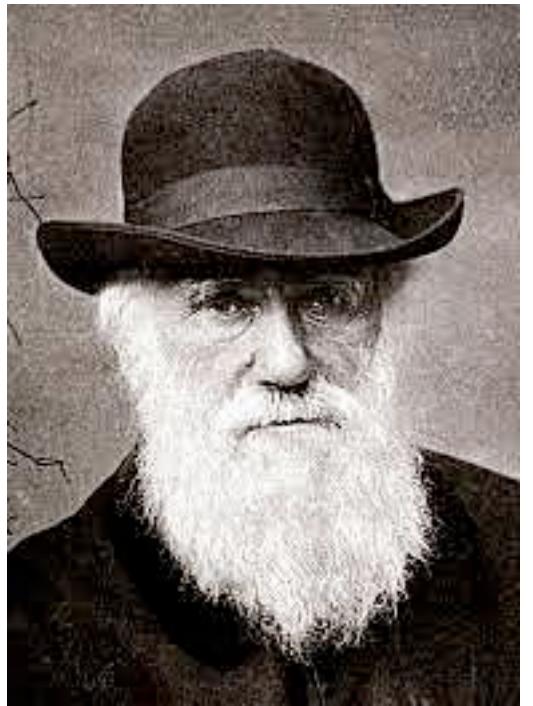
Bernard Kettlewell



Charles Darwin

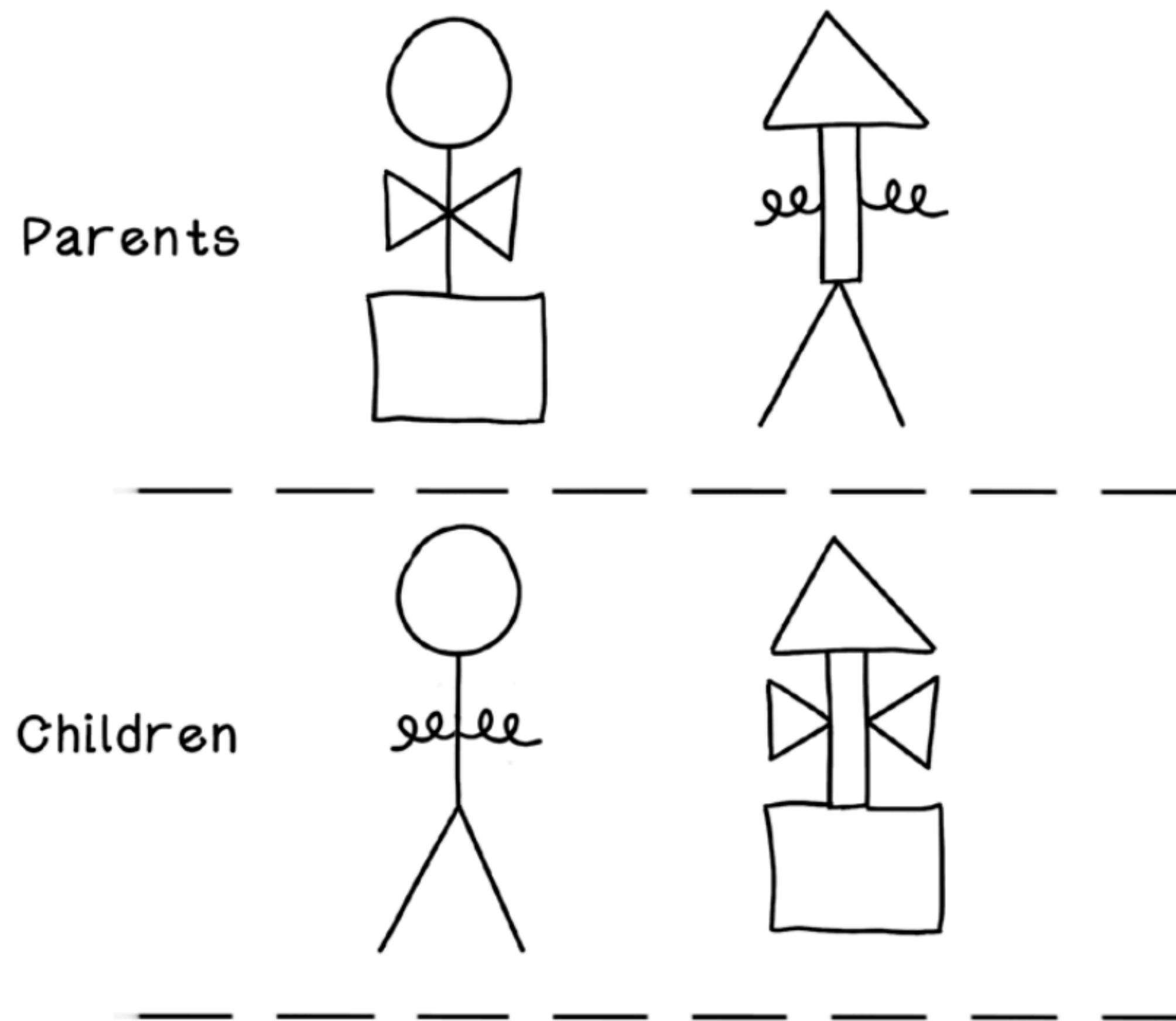
Darwin: (3) the struggle for existence—which determines the variations that will confer advantages in a given environment, thus altering species through a selective reproductive rate.

Reproduction and mutation



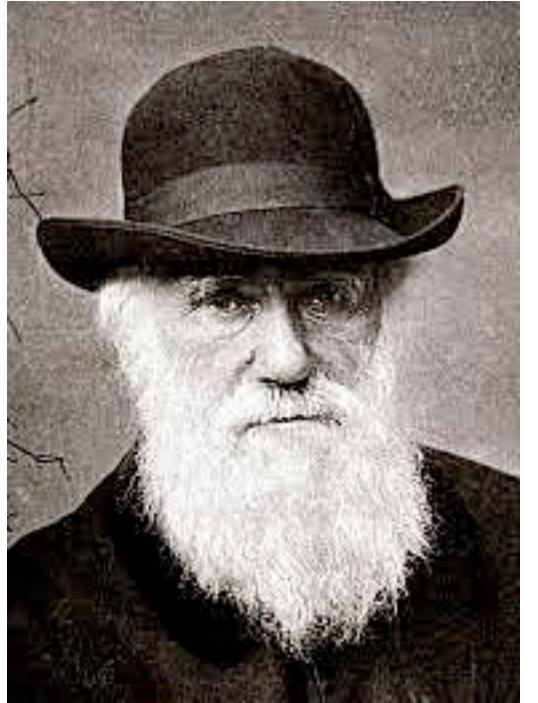
Charles Darwin

(2) heredity—the conservative force that transmits similar organic form from one generation to another; and



Darwinian evolution theory

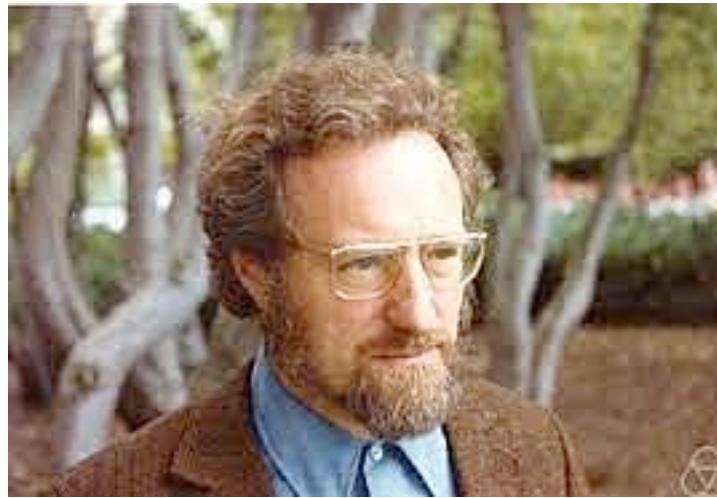
A population has the following attributes:



Charles Darwin

- **Variety** – Individuals in the population have different genetic traits.
- **Heredity** – A child inherits genetic properties from its parents.
- **Selection** – A mechanism that measures the fitness of individuals. Stronger individuals have the highest likelihood of survival (survival of the fittest).

The origin of evolutionary algorithms



Alex S. Fraser



Hans-Joachin Bremermann

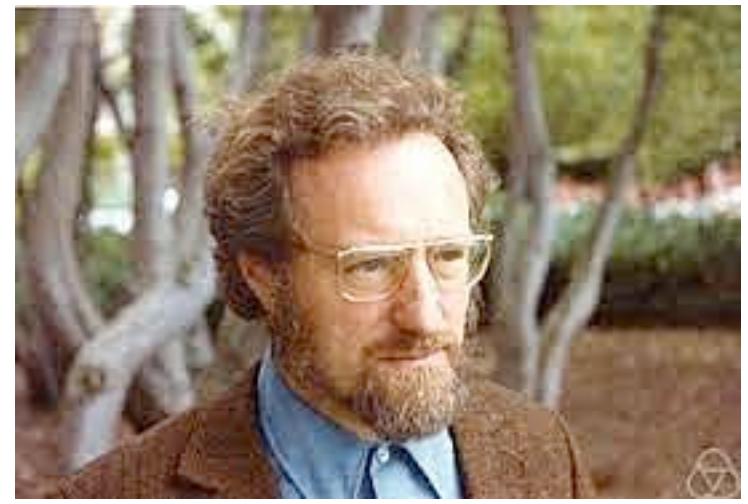


John Holland

- Variety
 - Hereditary
 - Selection
- This makes perfect sense to use this as an optimization technique!

Evolutionary algorithms

The life cycle of a genetic algorithm



Alex S. Fraser



Hans-Joachin Bremermann

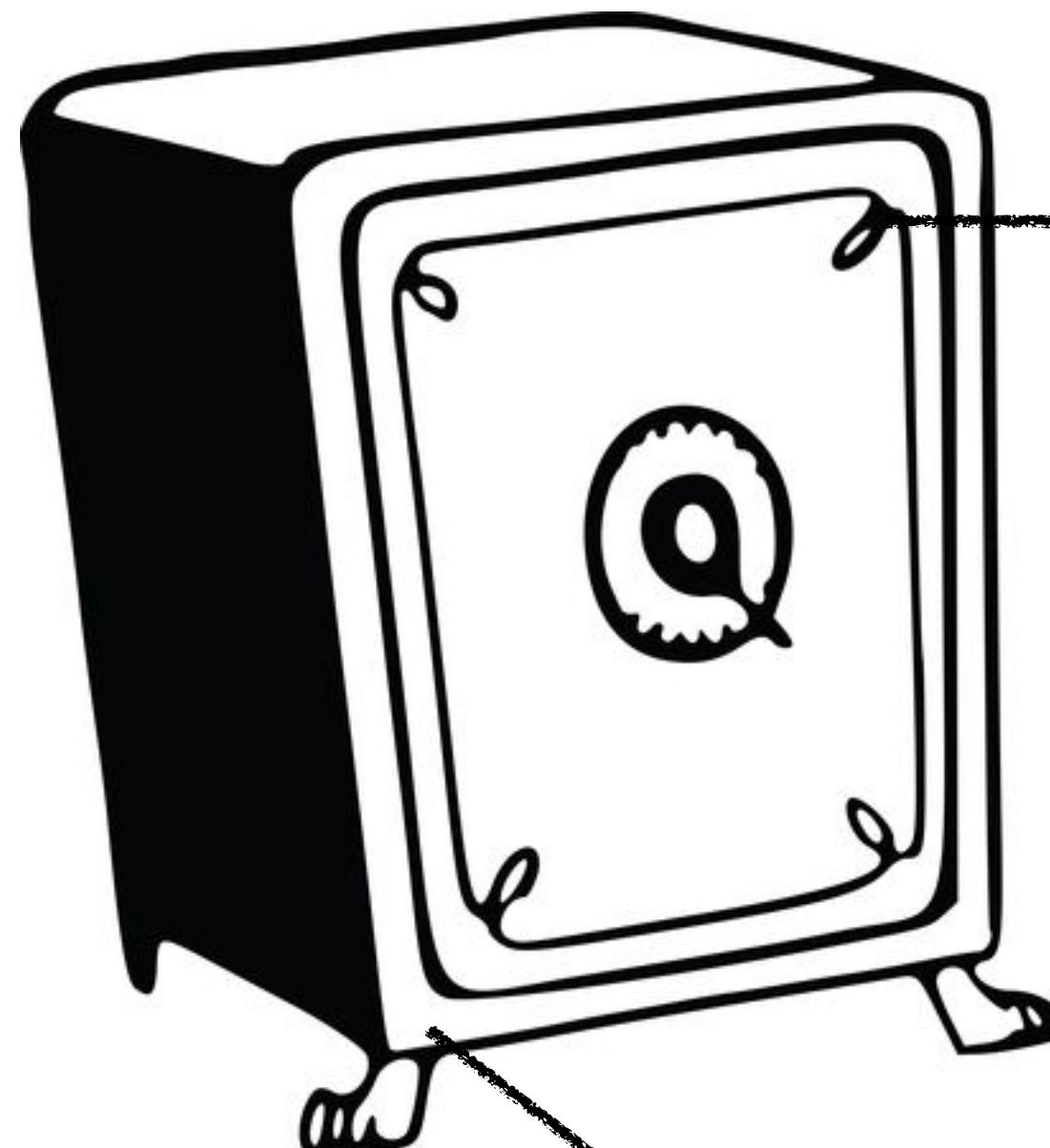


John Holland

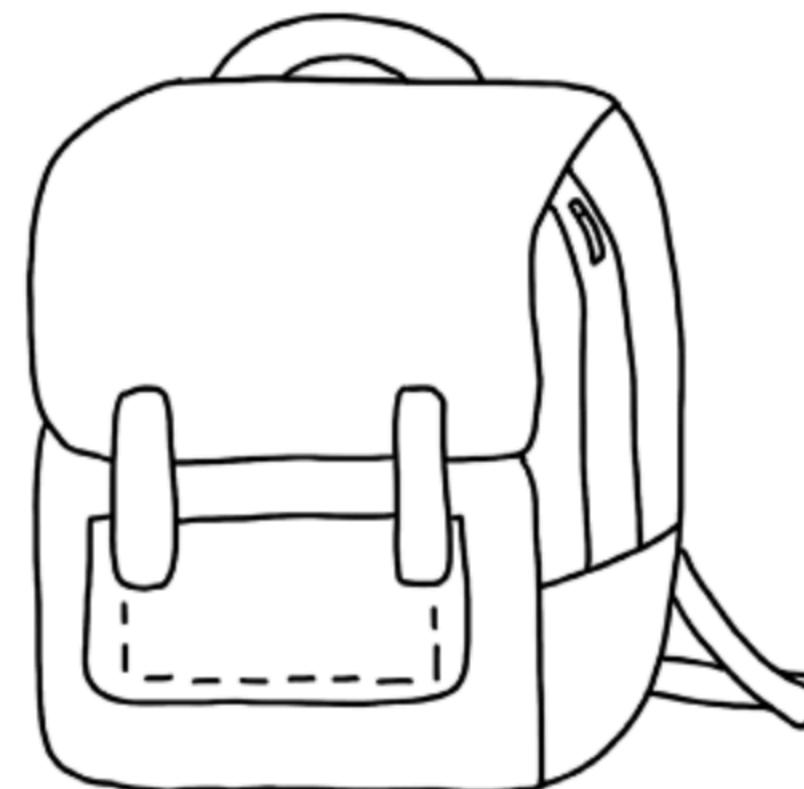
1. Creating a population—Creating a random population of potential solutions.
1. Measuring the fitness of individuals in the population—Determining how good a specific solution is. This task is accomplished by using a fitness function that scores solutions to determine how good they are.
2. Selecting parents based on their fitness—Selecting pairs of parents that will reproduce offspring.
3. Reproducing individuals from parents—Creating offspring from their parents by mixing genetic information and applying slight mutations to the offspring.
4. Populating the next generation—Selecting individuals and offspring from the population that will survive to the next generation.

The thief problem

Maximizing the profit



Capacity limit : 9 Kg



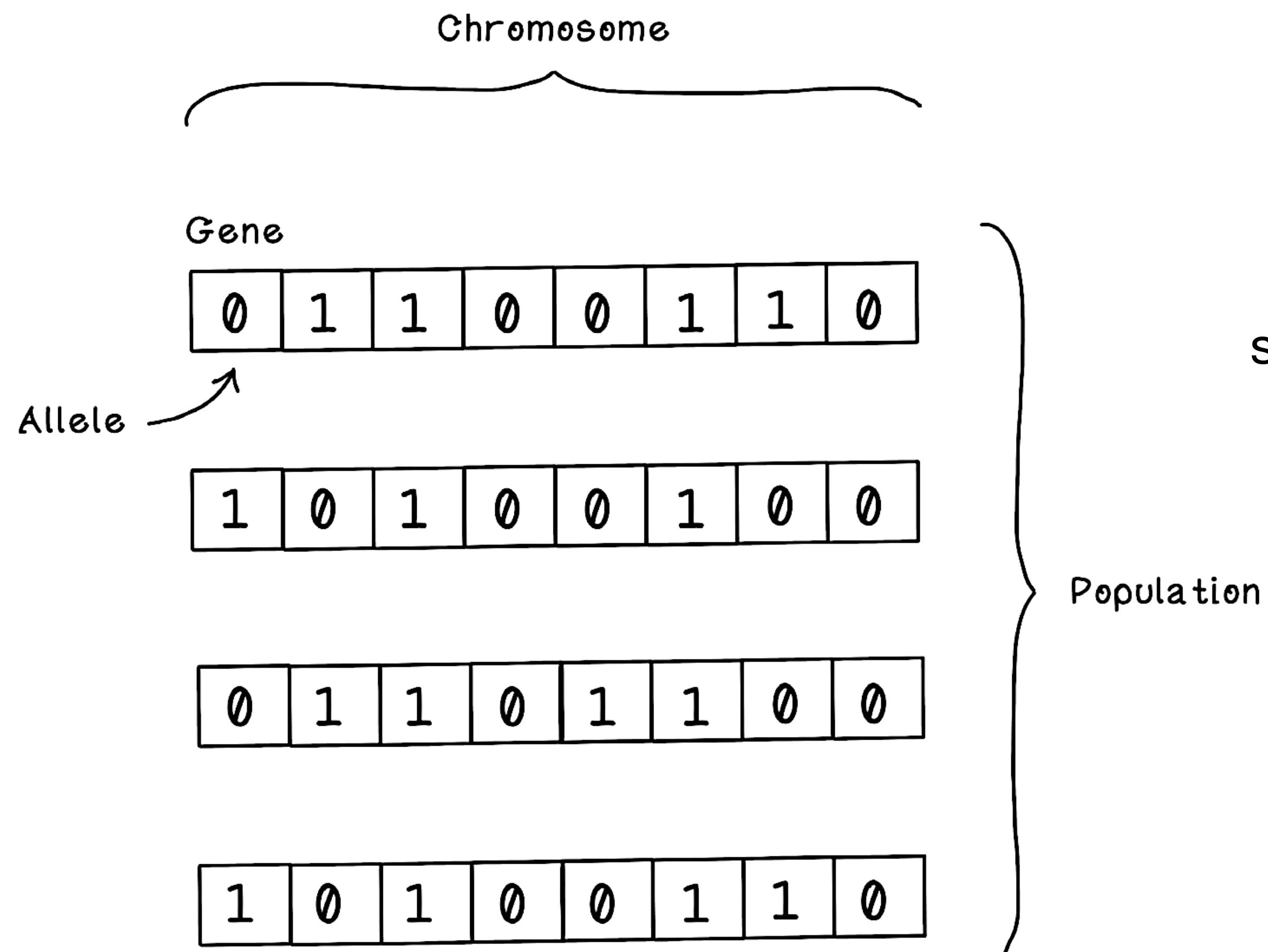
1	bracelet	3 Kg	\$4
2	gold bar	7 Kg	\$7
3	crown	4 Kg	\$5
4	coin	1 Kg	\$1
5	sabre	5 Kg	\$4
6	sword	4 Kg	\$3
7	diamond	2 Kg	\$5
8	chalice	3 Kg	\$1

The goal is to fit as many items into the sack as possible so that the total value is maximized and the total weight does not exceed the limit



Evolutionary algorithms

0-Creating a population: *encoding*



A **chromosome** is made up of genes. The **gene** is the logical type for the unit, and the **allele** is the actual value stored in that unit. A **genotype** is a representation of a solution, and a **phenotype** is a unique solution itself. Each chromosome always has the same number of genes. A collection of **chromosomes** forms a population

The thief problem

0-Creating a population: *encoding*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
0	1	1	0	0	1	0	0	0	1	0	1	1	0	0	0	1	1	1	0	0	0	1	0	0	1

A binary string of length 26 representing a solution to the thief problem. The items are indexed from 1 to 26. A value of 1 at index i indicates that item i is taken, while 0 indicates it is left behind.

Legend:

- Axe (Axe icon)
- Bronze Coin (Crown icon)
- Crown (Crown icon)
- Diamond Statue (Diamond icon)
- Emerald Belt (Belt icon)
- Fossil (Fossil icon)
- Gold Coin (Coin icon)
- Helmet (Helmet icon)
- Ink (Ink icon)
- Jewel Box (Jewel Box icon)
- Knife (Knife icon)
- Long Sword (Sword icon)
- Mask (Mask icon)
- Necklace (Necklace icon)
- Opal Badge (Opal Badge icon)
- Pearls (Pearls icon)
- Quiver (Quiver icon)
- Ruby Ring (Ruby Ring icon)
- Silver Bracelet (Silver Bracelet icon)
- Timepiece (Timepiece icon)
- Uniform (Uniform icon)
- Venom Potion (Potion icon)
- Wool Scarf (Scarf icon)
- Crossbow (Crossbow icon)
- Yesterday Book (Book icon)
- Zink Cup (Zink Cup icon)

The thief problem

1-Create an initial (valid) population

0 1 1 0 0 1 0 0 0 1 0 1 1 0 0 0 1 1 1 0 0 0 1 0 0 1

0 0 1 1 0 1 0 1 0 0 0 1 0 0 0 1 1 0 1 0 0 0 1 0 0 0

1 1 1 0 0 1 0 0 1 1 1 0 1 1 0 0 0 1 0 0 1 0 1 1 0 1

0 0 0 0 1 0 0 0 0 1 0 0 1 0 1 0 1 1 0 1 0 0 1 0 0 1

0 1 1 0 0 1 0 1 0 1 0 1 1 0 0 0 1 0 1 0 0 0 1 0 0 0

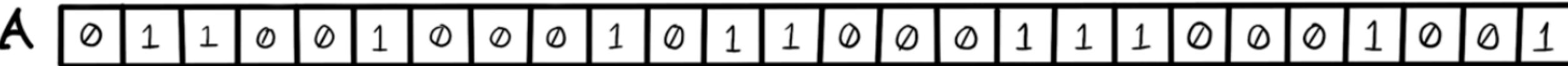
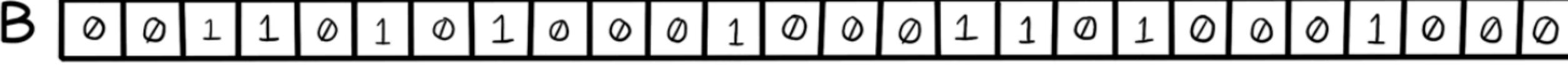
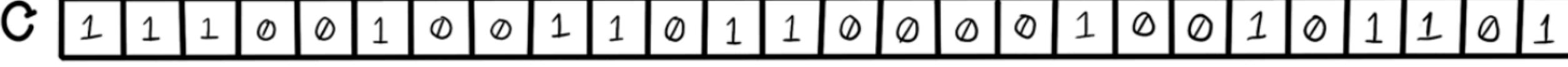
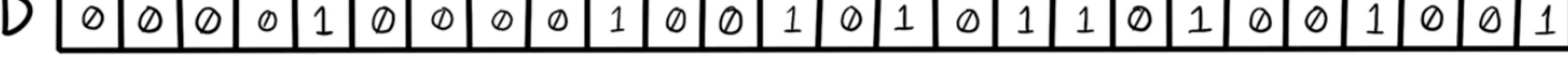
⋮
⋮
⋮

Population size

Which solutions are performing well?

2-measuring the fitness of individuals in the population

Bag Capacity: 223 kg

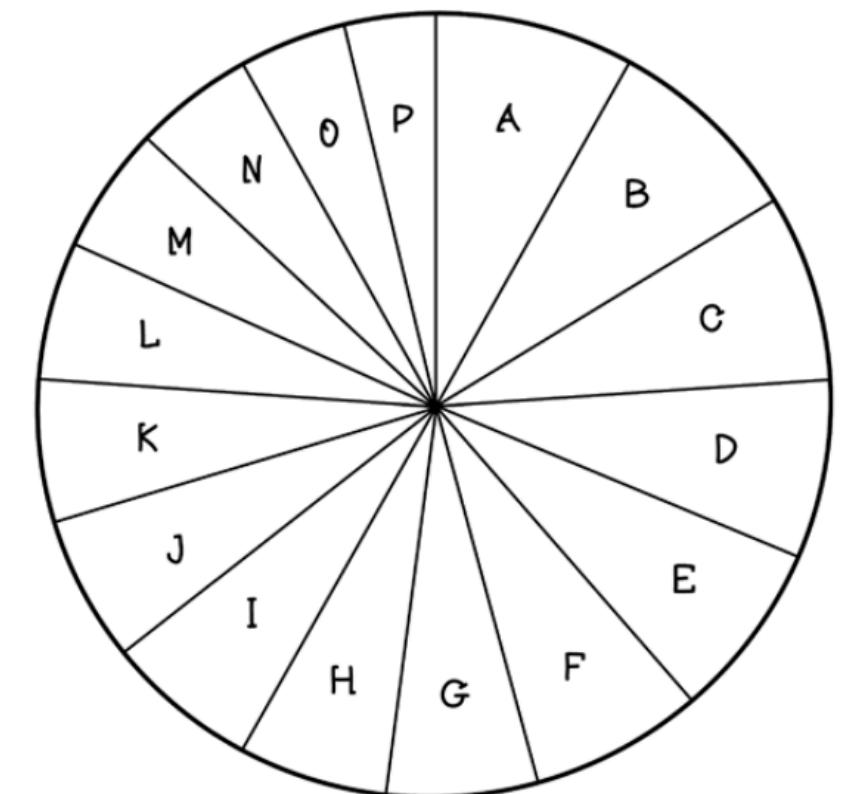
	<i>Collection of items</i>	<i>Street Value</i>
A		11,393,360
B		10,866,684
C		0 (Overweight)
D		10,715,475

Generating more solutions

3-Selecting parents based on their fitness

Bag Capacity: 223 kg

A	1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 0 1 0 0 0 1	13,107,019
B	1 1 0 0 0 1 0 0 1 1 1 1 1 1 1 1 0 1 0 0 0 1 0 0 0	12,965,145
C	0 0 1 1 0 1 1 0 1 0 0 1 1 0 0 0 0 1 0 1 0 1 1 0 0 0	12,344,873
D	0 0 1 1 1 1 1 0 0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 0 0 0	11,739,363
E	1 1 0 0 0 1 0 0 1 1 1 1 1 1 0 1 1 1 0 1 0 0 0 1 0 0 0	11,711,159
F	1 1 0 0 0 1 0 0 1 1 1 1 1 0 1 0 1 1 0 1 0 0 0 1 0 0 0	11,611,967
G	1 0 1 0 0 1 1 1 0 0 0 0 1 0 0 1 0 1 1 0 0 0 0 0 1 0	10,042,441
H	1 1 0 0 0 1 0 0 1 1 1 1 1 1 0 1 1 1 0 1 0 0 0 0 0 0 0	9,883,682
I	1 1 0 0 0 1 0 0 1 1 1 1 1 1 0 0 1 0 1 0 0 0 0 1 0 0 0	9,857,597
J	0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0 1	9,670,184
K	0 0 0 0 1 1 0 1 1 1 0 1 0 1 0 0 0 1 0 1 0 0 0 0 0 0 0	9,277,580
L	1 0 0 0 0 1 0 0 1 0 0 0 0 1 0 0 1 1 0 0 0 1 0 1 0 0	8,931,719
M	0 1 0 0 0 0 0 1 1 1 0 1 1 1 1 0 0 0 1 0 0 0 0 0 0 0 0	8,324,936
N	1 1 1 0 0 1 0 0 0 1 0 1 0 0 0 0 0 1 1 0 1 0 0 0 0 0	8,018,760
O	0 0 0 1 1 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 1	6,900,314
P	0 0 0 1 1 0 0 0 0 0 1 1 0 1 0 0 0 1 0 0 0 0 1 0 0 0	6,056,664



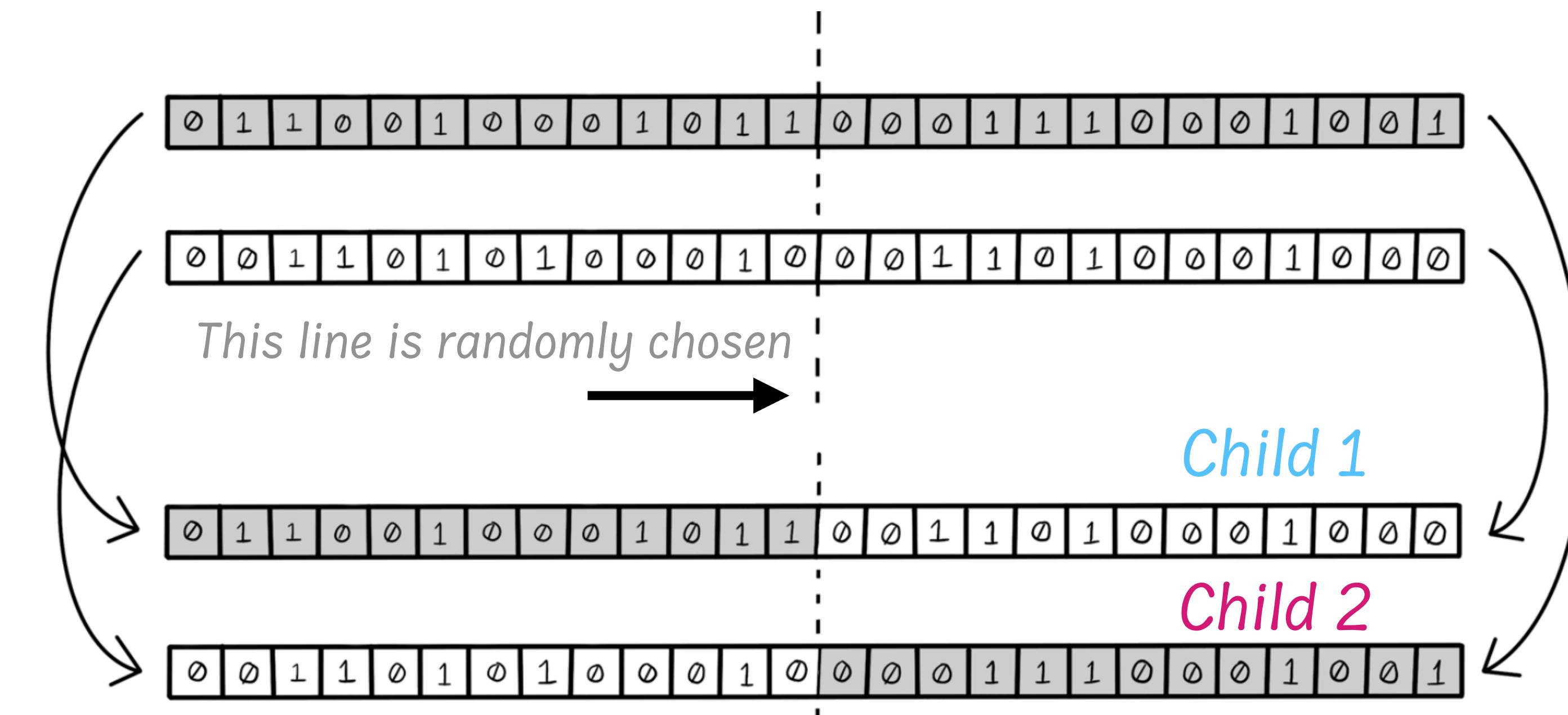
Probability of being chosen

$$p_i = \frac{value_i}{\sum_j^N value_j}$$

Generating more solutions

4-Reproducing individuals from parents: *crossover*

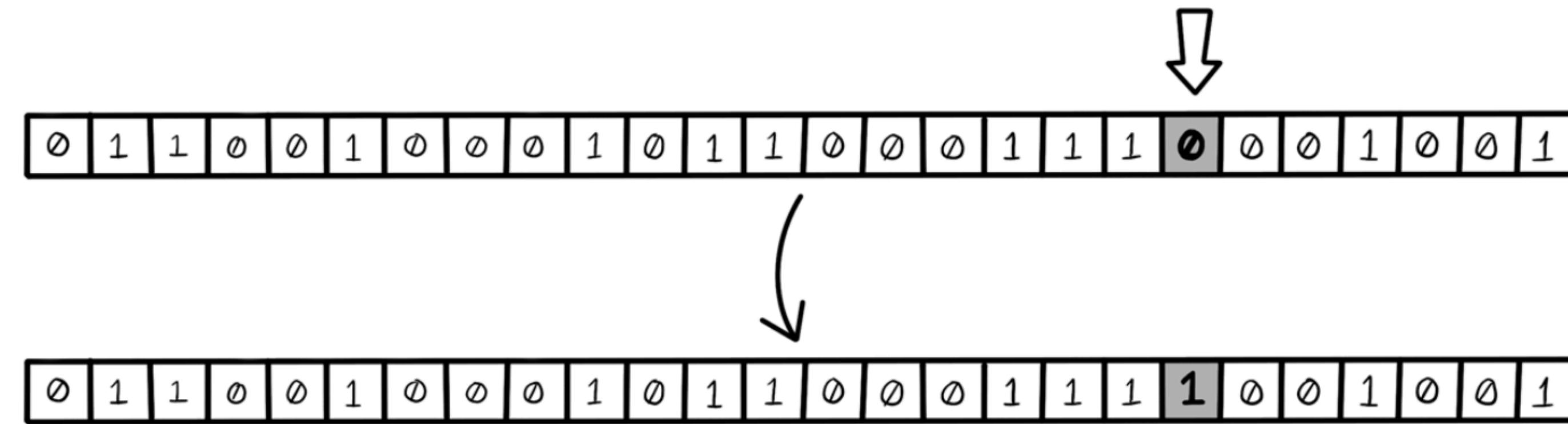
Single-point crossover: Inheriting one part from each parent



Generating more solutions

4-Reproducing individuals from parents: *mutations*

Mutation: Bit-string mutation for binary encoding



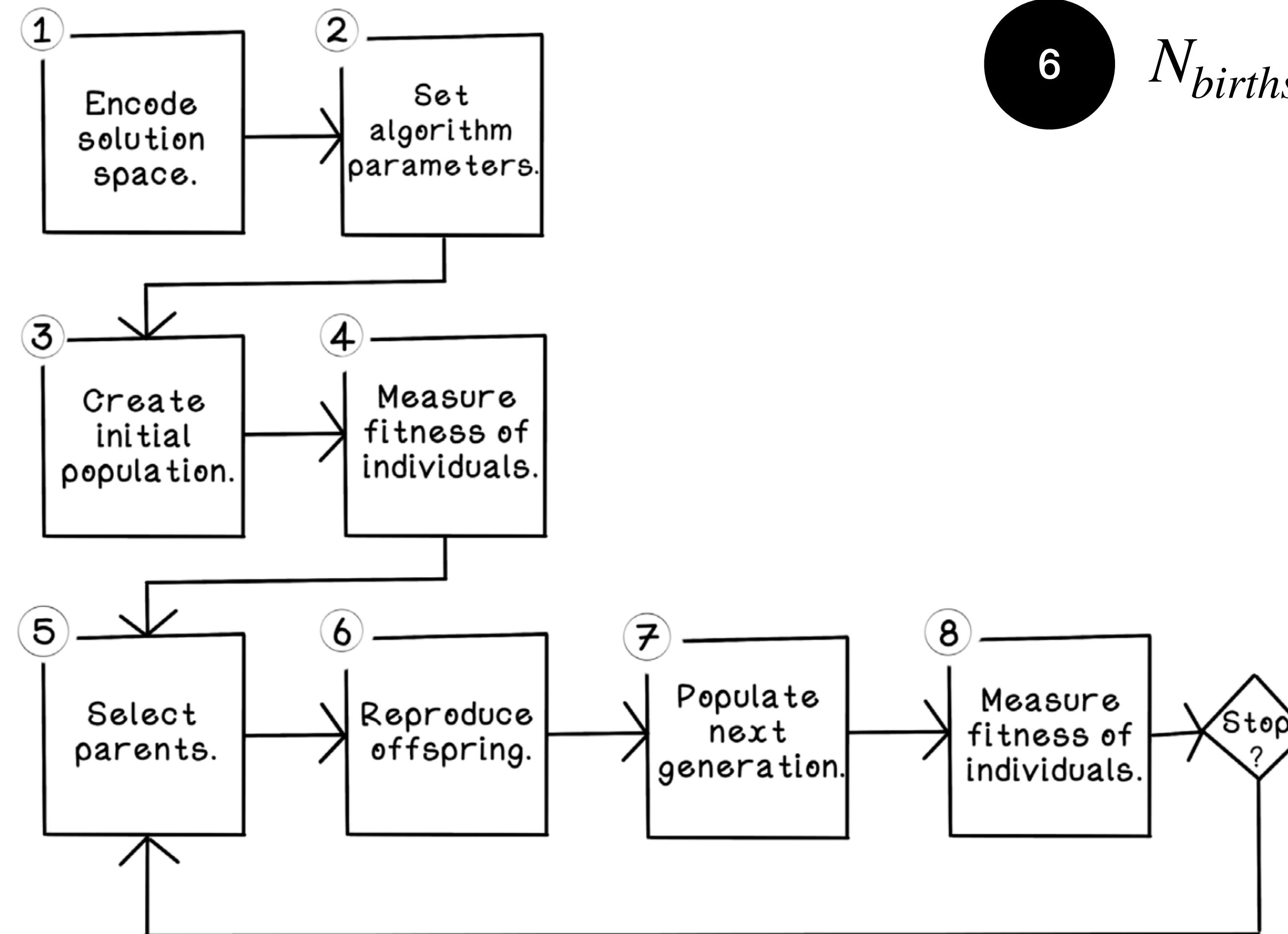
Generating more solutions

4-Populating the next generation

The size of the population is usually fixed, and because more individuals have been introduced through reproduction, some individuals must die off and be removed from the population.

The idea is to use the probability of the fittest to *remove* individuals that are not fit enough

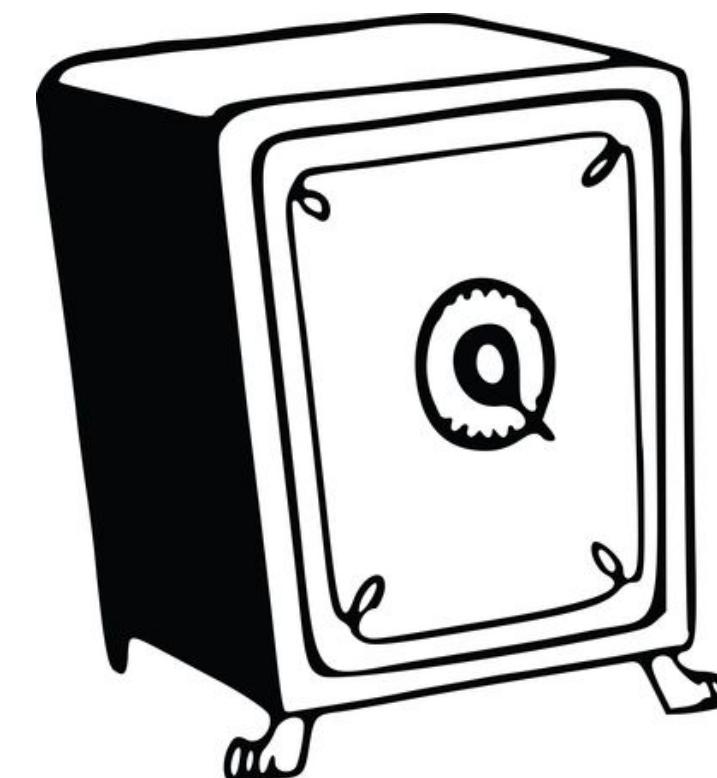
Genetic algorithm life cycle



$$N_{births} < N_{population}$$

Example:

Item Code	Item	Weight	Value
1	Axe	32.252	68674
2	Bronze coin	225.79	471010
3	Crown	46.8164	944620
4	Diamond statue	48.9494	962094
5	Emerald belt	35.384	78344
6	Fossil	265.59	579152
7	Gold coin	99.5822	902698
8	Helmet	8.00493	1686515
9	Ink	0.823576	1688691
10	Jewel box	55.2202	1056157
11	Knife	3.23618	677562
12	Long sword	38.2846	833132
13	Mask	44.676	99192
14	Necklace	1.69738	376418
15	Opal badge	6.10876	1253986
16	Pearls	8.5419	1853562
17	Quiver	6.71123	1320297
18	Ruby ring	0.69818	1301637
19	Silver bracelet	0.446517	859835
20	Timepiece	0.90962	1677534
21	Uniform	9.04818	1910501
22	Venom potion	0.730061	1528646
23	Wool scarf	9.31932	1827477
24	Crossbow	9.5236	2068204
25	Yesteryear book	2.3150575	1746556
26	Zinc cup	0.978724	2100851



```
[Crown      ', 'Diamond statue  ',
'Helmet    ', 'Ink          ', 'Jewel
box       ', 'Knife        ', 'Necklace
', 'Opal badge   ', 'Pearls       ',
'Quiver     ', 'Ruby ring   ', 'Silver
bracelet  ', 'Timepiece   ', 'Uniform
', 'Venom potion ', 'Wool scarf  ',
'Crossbow   ', 'Yesteryear book',
'Zinc cup   ']
```

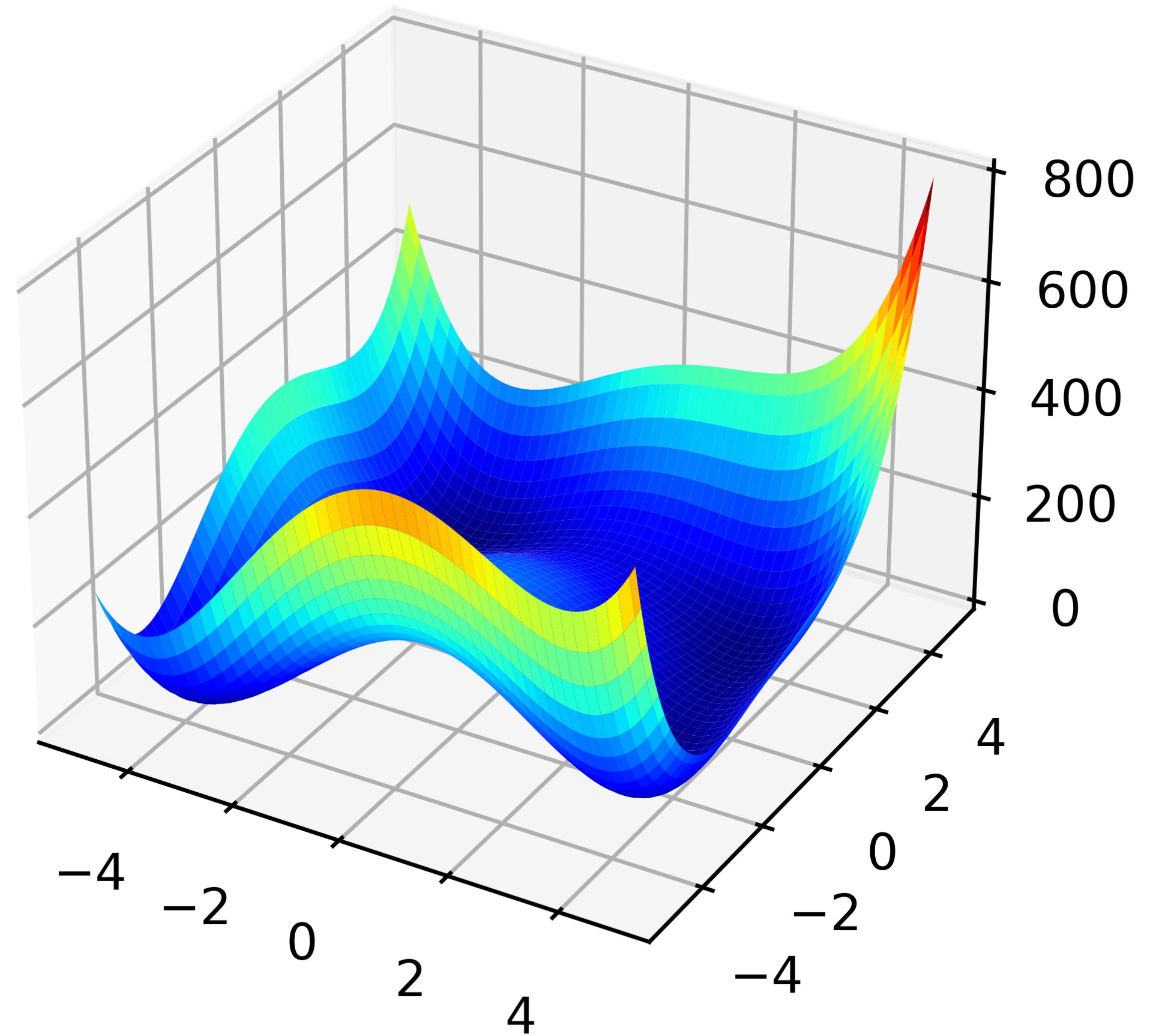
Total Value: 26841143

Lab

Use genetic algorithms to minimize the following function:

- $f(3.0, 2.0) = 0.0,$
- $f(-2.805118, 3.131312) = 0.0,$
- $f(-3.779310, -3.283186) = 0.0,$
- $f(3.584428, -1.848126) = 0.0.$

$$f(x, y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2$$

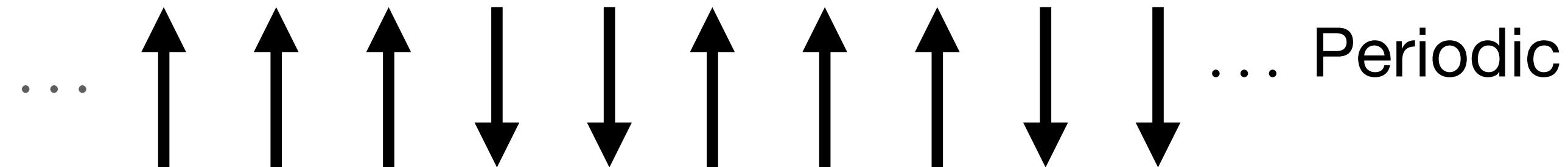


Extra

Use genetic algorithms to find the ground states of the following Hamiltonian:

$$H = - \sum_{\langle i,j \rangle} J_{ij} s_i s_j \quad s_i \in \{-1, 1\}$$

J_{ij} are Gaussian distributed $P(J_{ij}) = \sqrt{\frac{N}{2\pi J^2}} \exp \left\{ -\frac{N}{2J^2} \left(J_{ij} - \frac{J_0}{N} \right)^2 \right\}$



$$N = 10^2 - 10^4$$

$$J_0 = 0$$

$$J^2 = 1$$