D. Pasetto

Esercizi del corso

Analisi Matematica - Mod. 1

Primo semestre 2024/2025

Foglio 3: Funzioni trigonometriche

Esercizio 1 (Equazioni trigonometriche) ....

Risolvere le seguenti equazioni ($k \in \mathbb{Z}$).

(a) $\sin(2x - \pi) = \cos x$ S: $x \in \left\{\frac{\pi}{2} + k\pi; -\frac{\pi}{6} + 2k\pi; \frac{7}{6}\pi + 2k\pi\right\}$

(b) $\cos x = -\sin x$ S: $x \in \left\{-\frac{\pi}{4} + k\pi\right\}$

(c) $4\cos^2 x - 3 = 0$ S: $x \in \left\{\pm\frac{\pi}{6} + 2k\pi; \pm\frac{5}{6}\pi + 2k\pi\right\}$

(d) $2\cos x + 2\sin x - (\sqrt{3} + 1) = 0$ S: $x \in \left\{\frac{\pi}{3} + 2k\pi; \frac{\pi}{6} + 2k\pi\right\}$

(e) $4\sin^2 x - 9\cos^2 x = 0$ S: $x \in \left\{\pm\mathrm{arctg}(\frac{3}{2}) + k\pi\right\}$

(f) $3\sin^2 x - 8\sqrt{3}\sin x\cos x + 15\cos^2 x = 0$ S: $x \in \left\{\frac{\pi}{3} + k\pi; \mathrm{arctg}(\frac{5\sqrt{3}}{3}) + k\pi\right\}$

(g) $4\sin^2(x)\cos^2(x) - 4\cos^4(x) = 0$ S: $x \in \left\{\frac{\pi}{2} + k\pi; \frac{\pi}{4} + k\frac{\pi}{2}\right\}$

Esercizio 2 (Funzioni trigonometriche inverse) ....

(a) Trovare il dominio di $f(x) = \mathrm{arctg}(\arcsin(x))$

(b) Trovare il dominio di $f(x) = \log_3|\mathrm{arctg}(x)|$

(c) Veriifcare che $\arcsin(x) + \arccos(x) = \frac{\pi}{2} \quad \forall x \in [-1, 1]$

(d) Verificare che $\forall x \in \mathbb{R}$ si ha: $\sin(\mathrm{arctg}(x)) = \frac{x}{\sqrt{1+x^2}}$ (idea: porre $\alpha = \mathrm{arctg}(x)$, cioè $\mathrm{tg}\alpha = x$).

Esercizio 3 (Disequazioni) ....

Risolvere le seguenti disequazioni ($k \in \mathbb{Z}$).

(a) $\sin x > \frac{1}{2}$ S: $x \in \left]\frac{\pi}{6} + 2k\pi, \frac{5}{6}\pi + 2k\pi\right[$

(b) $\tan x \geq \sqrt{3}$ S: $x \in \left[\frac{\pi}{3} + k\pi, \frac{\pi}{2} + k\pi\right[$

(c) $\cos x > -\frac{1}{2}$ S: $x \in \left]-\frac{2}{3}\pi + 2k\pi, \frac{2}{3}\pi + 2k\pi\right[$

(d) $2\sin^2 x + 5\cos x - 4 > 0$ S: $x \in \left]-\frac{\pi}{3} + 2k\pi, \frac{\pi}{3} + 2k\pi\right[$

(e) $3\tan^2 x - 1 \geq 0$ S: $x \in \left[\frac{\pi}{6} + k\pi, \frac{\pi}{2} + k\pi\right[\cup \left]\frac{\pi}{2} + k\pi, \frac{5}{6}\pi + k\pi\right]$

(f) $\cos(2x) + \sin x \geq 0$ S: $x \in \left[-\frac{\pi}{6} + 2k\pi, \frac{7}{6}\pi + 2k\pi\right]$

(g) $\cos^2 x + \left(\sqrt{3} - 1\right)\sin(x)\cos(x) - \sqrt{3}\sin^2 x > 0$ S: $x \in \left]-\frac{\pi}{6} + k\pi, \frac{\pi}{4} + k\pi\right[$

(h) $(1 - 2\sin x)(2\cos x + \sqrt{3}) \leq 0$ S: $x \in \left[\frac{\pi}{6} + 2k\pi, \frac{7}{6}\pi + 2k\pi\right]$

(i) $(3\tan^2 x - 1)(\sin x + 1) \geq 0$ S: $x \in \left[\frac{\pi}{6} + k\pi, \frac{\pi}{2} + k\pi\right[\cup \left]\frac{\pi}{2} + k\pi, \frac{5}{6}\pi + k\pi\right]$

(j) $\frac{\cos^2 x - \sin^2 x}{\sqrt{3}\ \tan(x) + 1} \leq 0$ S: $x \in \left[\frac{\pi}{4} + k\pi, \frac{\pi}{2} + k\pi\right[\cup \left[\frac{3}{4}\pi + k\pi, \frac{5}{6}\pi + k\pi\right[$

(k) $\frac{\sqrt{3}\sin x - \cos x}{1 - \sin^2 x} \geq 0$ S: $x \in \left[\frac{\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi\right[\cup \left]\frac{\pi}{2} + 2k\pi, \frac{7}{6}\pi + 2k\pi\right]$