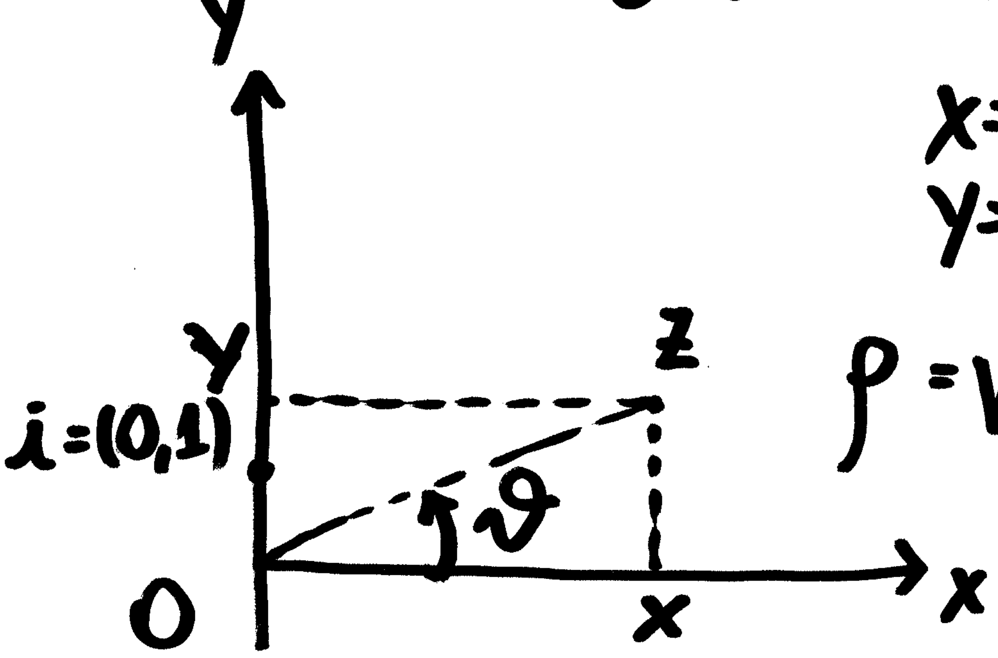
AL LEZ. 2

$Z = x + iy$ FORMA ALGEBRICA

$Z = \rho e^{i\vartheta}$ ρ = MODULO DI $Z = |Z|$

FORMA POLARE ϑ = ARGOMENTO DI Z



$x = Re(Z)$
$y = Im(Z)$

$\rho = \sqrt{x^2+y^2}$

PITAGORA

POLARE $\longrightarrow$ ALG. TEOR. TRIANG. RETT.

$$\begin{cases} x = |Z| \cos\vartheta \\ y = |Z| \sin\vartheta \end{cases}$$

$\rho = 2$ $\vartheta = 30° = \frac{\pi}{6}$

$$\begin{cases} x = 2 \cdot \underset{\uparrow \; \sqrt{3}/2}{\cos 30°} = \sqrt{3} \\ y = 2 \underset{\downarrow \; 1/2}{\sin 30°} = 1 \end{cases}$$

ALG. → POLARE

$z = \sqrt{3} + i \qquad \rho = ? \quad \vartheta = ?$

$\rho = |z| = \sqrt{x^2 + y^2} = \sqrt{3+1} = 2$

ANDATE A VEDERE IN QUALE QUADRANTE SI TROVA Z — 1° QUAD.

$$\begin{cases} x = \rho \cos\vartheta \\ y = \rho \sin\vartheta \end{cases} \qquad \begin{cases} \sqrt{3} = 2\cos\vartheta \\ 1 = 2\sin\vartheta \end{cases}$$

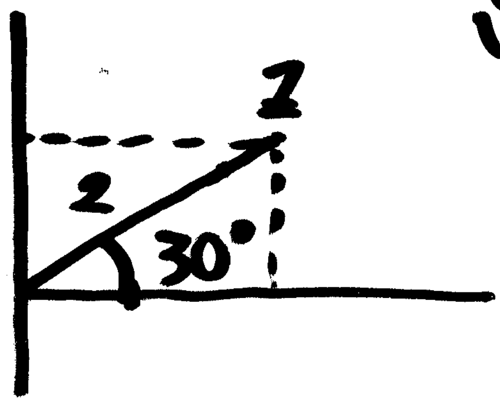
$$\begin{cases} \cos\vartheta = \frac{\sqrt{3}}{2} \\ \sin\vartheta = \frac{1}{2} \end{cases}$$

$\cos^{-1}$

$\boxed{\cos}$

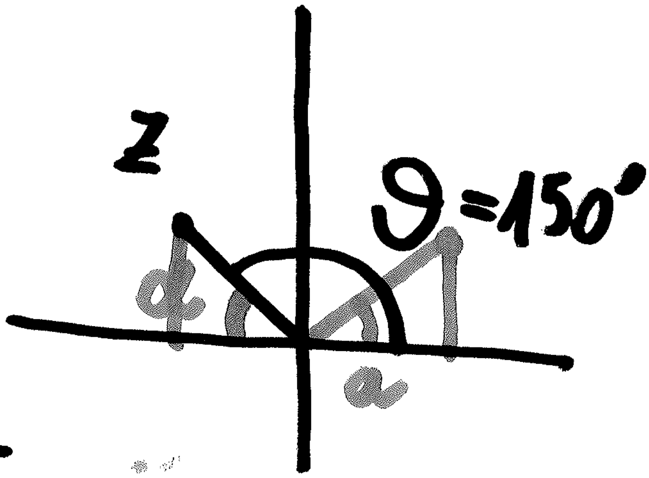
$\cos^{-1}(\sqrt{3} \div 2) = 30°$

$\vartheta = 30°$



$z = -\sqrt{3} + i \qquad \rho = 2$

$$\begin{cases} -\sqrt{3} = 2\cos\theta \\ 1 = 2\sin\theta \end{cases}$$



$\alpha = 180° - \theta \leftarrow$

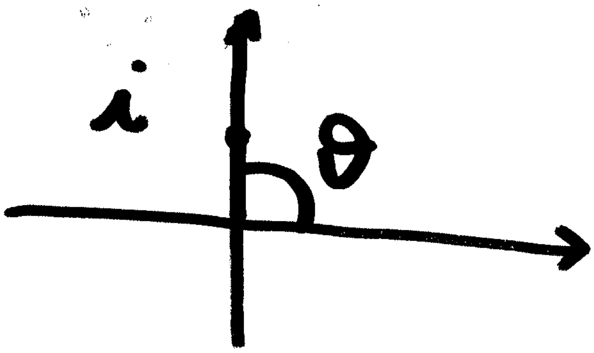
$$\begin{cases} \sqrt{3} = 2\cos\alpha \\ 1 = 2\sin\alpha \end{cases}$$ 1° QUAD

$\alpha = 30°$

$\theta = 180 - \alpha = 180 - 30 = \boxed{150°}$

TRIG. → POL.

$i = (0, 1)$



CASI PARTICOLARI

"A OCCHIO" TROVATE θ

$\rho = \sqrt{x^2 + y^2} = 1 \qquad \theta = \pi/2 = 90°$

$z_1, z_2 \in \mathbb{C}$

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2| \quad \text{PROP.}$$

FORMULA DI DE MOIVRE

$$z^n = |z|^n (\cos(n\theta) + i\sin(n\theta))$$

$$z = |z| e^{i\theta} = \cancel{|z|(\cos\theta + i s}$$
$$= |z|(\cos\theta + i\sin\theta)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

ESPONENZIALE IN CAMPO COMPLESSO

EX $(1+i)^7 \rightarrow$ IN FORMA ALGEBRICA

~~DE MOIVRE ⇝ FOR~~

1° PASSO TRASF. IL MIO NUMERO DI BASE $\rightarrow 1+i$ IN FORMA POLARE

2° PASSO APPLICO LA POTENZA CON DE MOIVRE

3° PASSO

RITRASFORMATE IL NUMERO OTTENUTO DA POLARE AD ALG.

$z^7 \qquad z = 1+i$

ALG. → POL. $\quad |z| = \rho = \sqrt{1+1} = \sqrt{2}$

NEL 1° QUADR.

$$\begin{cases} 1 = \sqrt{2}\cos\theta \\ 1 = \sqrt{2}\sin\theta \end{cases} \rightarrow \theta = 45° = \frac{\pi}{4}$$

$$z = \sqrt{2}\, e^{i\pi/4}$$

DE MOIVRE $\quad n = 7$

$$z^7 = (\sqrt{2})^7 \cdot \left(\cos\left(\frac{7}{4}\pi\right) + i\sin\left(\frac{7}{4}\pi\right)\right)$$

$$= 2^{7/2} \cdot \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$$

$$\cos\left(\frac{7}{4}\pi\right) = \cos\left(2\pi - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$\sin\left(\frac{7}{4}\pi\right) = \sin\left(2\pi - \frac{\pi}{4}\right) = -\sin\frac{\pi}{4}$$

(6)

$$z = \frac{2^{7/2}}{\sqrt{2}} - i\,\frac{2^{7/2}}{\sqrt{2}} =$$

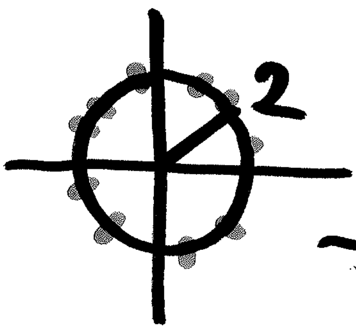
$$= 2^3 - i\,2^3 = 8 - 8i$$

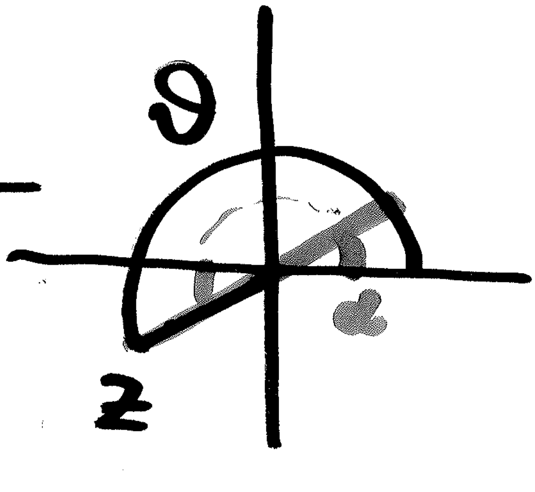
X CASA

- DET. IN FORMA ALG.

$$1 + i - \frac{i}{1-2i} \longrightarrow A + iB$$

- $$(1-i)^{11} \longrightarrow A + iB$$





$$\begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases}$$

$$\begin{cases} -\frac{1}{\sqrt{2}} = \cos\theta \\ -\frac{1}{\sqrt{2}} = \sin\theta \end{cases}$$

ρ = RHO

RADICI N-ESIME DI UN NUMERO COMPLESSO

DEF. DATO $W \in \mathbb{C}$ $(n \in \mathbb{N})$

Z È RADICE n-ESIMA DI W

$$Z = \sqrt[n]{W}$$

SE

$$Z^n = W$$

ESEMPIO $\sqrt{i} = ?$ $W = i$, $n = 2$

Z È UNA RADICE QUADRATA DI i SE

$$Z^2 = i \qquad Z = X + iY$$

SCRIVETE Z IN FORMA ALGEBRICA

$$(X + iY)^2 = i$$

$(x+iy)^2 = i$

$x^2 + (iy)^2 + 2ixy = i$

$\underbrace{x^2 - y^2} + i\,\underbrace{2xy} = i$

2 METODI

$\begin{cases} x^2 - y^2 = 0 \\ 2xy = 1 \end{cases}$

SOSTITUZIONE

RIDUZ.

$\downarrow$

POSSONO ESSERE VALIDI

$\begin{cases} 2x - y = 1 \\ x + 3y = 5 \end{cases}$

SIST. LINEARE

$(x+y)\cdot(x-y) = 0$

$y^2 = x^2$

$y = |x|$

$x - y = 0 \quad y = x \quad \boxed{\text{Sol. 1}}$

$x + y = 0 \quad y = -x \quad \boxed{\text{Sol. 2}}$

$y = \pm x$

$x = y$ 1

$2x^2 = 1 \quad x^2 = 1/2 \rightarrow x = \pm \frac{1}{\sqrt{2}}$

$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = z_1 = -\frac{1}{\sqrt{2}} + i\,\frac{-1}{\sqrt{2}}$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = z_2 = \frac{1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}}$

$z = x + iy \quad x, y \in \mathbb{R}$

2 $y = -x$

$-2x^2 = 1 \quad x^2 = -\frac{1}{2}$ NON HA SOL.

IMPOSSIBILE

$z_1^2 \longrightarrow$ VERIFICATE CHE VENGA i

$z_2^2 \longrightarrow$

METODO ALT. → USIAMO SOLO LA FORMA POLARE

$$z^2 = i$$

$$z = \rho e^{i\vartheta} \qquad i = e^{i\pi/2}$$

$$(\rho e^{i\vartheta})^2 = e^{i\pi/2} \qquad (a^2)^3 = a^6$$

$$\rho^2 (e^{i\vartheta})^2 = e^{i\pi/2}$$

$$\rho^2 e^{2i\vartheta} = e^{i\pi/2}$$

$$\vartheta \in [0, 2\pi[$$

$$\begin{cases} \rho^2 = 1 \\ 2\vartheta = \frac{\pi}{2} + 2k\pi \quad k = 0, 1, \dots\dots \end{cases}$$

$$e^{i\vartheta} = \cos\vartheta + i\sin\vartheta$$

$$e^{i(\vartheta + 4\pi)} = e^{i\vartheta} \cdot e^{i4\pi} = e^{i\vartheta}$$

$$e^{i4\pi} = \underset{\substack{\| \\ 1}}{\cos 4\pi} + i\underset{\substack{\| \\ 0}}{\sin 4\pi}$$

$$e^{i(\theta + 2k\pi)} = e^{i\theta} \qquad k = 0, 1, 2, 3, \ldots$$

$$\boxed{\theta \in [0, 4\pi[}$$

$$\begin{cases} \rho^2 = 1 \rightarrow \rho = 1 \\ 2\theta = \dfrac{\pi}{2} + 2k\pi \end{cases}$$

$k=0 \qquad 2\theta = \dfrac{\pi}{2} \rightarrow \theta_1 = \dfrac{\pi}{4}$ ACCETT.

$k=1 \qquad 2\theta = \dfrac{\pi}{2} + 2\pi \rightarrow \theta = \dfrac{\pi}{4} + \pi = \dfrac{5}{4}\pi$

ACCETT.

$k=2 \qquad 2\theta = \dfrac{\pi}{2} + 4\pi$

$\rightarrow \theta = \dfrac{\pi}{4} + 2\pi > 2\pi$

NON ACC.

$$z_1 = e^{i\pi/4}$$

$$z_2 = e^{i\frac{5}{4}\pi}$$