ES 1

(a) $\lim_{x\to 3} (2x+8) = 14$

$\forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0$

$|x-3| < \delta_\varepsilon \implies |(2x-8)-14| < \varepsilon$

$\implies |2x-6| < \varepsilon$

$\implies 2|x-3| < \varepsilon$

$\implies |x-3| < \varepsilon/2$

Usando $\delta_\varepsilon = \varepsilon/2$

(b) $\lim_{x\to -2} x^2 = 4$

$\forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0$

$|x+2| < \delta_\varepsilon \implies |x^2-4| < \varepsilon$

CENTRO IN -2

CENTRO IN 2

$|x+2-4| < \delta_\varepsilon - 4$

$|x-2| < \delta_\varepsilon - 4$

$|x^2-4| = |x-2| \cdot |x+2|$

$< (\delta_\varepsilon - 4) \cdot \delta_\varepsilon$

$< \delta_\varepsilon^2 - 4\delta_\varepsilon$

Usando $\delta_\varepsilon^2 - 4\delta_\varepsilon = \varepsilon$

$\delta_\varepsilon^2 - 4\delta_\varepsilon - \varepsilon = 0$

$\delta_\varepsilon = 2 \pm \sqrt{4+\varepsilon}$

$+ \quad 2 + \sqrt{4+\varepsilon} > 0 \quad \checkmark$

$- \quad \underbrace{2 - \underbrace{\sqrt{4+\varepsilon}}_{>2}}_{<0} \quad \times$

$\delta_\varepsilon = 2 + \sqrt{4+\varepsilon}$

(c) $\lim_{x\to 1} (x^2+x+6) = 8$

$\forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0$

$|x-1| < \delta_\varepsilon \implies |(x^2+x+6)-8| < \varepsilon$

$\implies |x^2+x-2| < \varepsilon$

$\implies |x-1| \cdot |x+2| < \varepsilon$

$+3$

$|x+2| < \delta_\varepsilon + 3$

$|x-1| \cdot |x+2| < \delta_\varepsilon (\delta_\varepsilon + 3)$

$< \delta_\varepsilon^2 + 3\delta_\varepsilon$

Usando $\delta_\varepsilon^2 + 3\delta_\varepsilon = \varepsilon$

$\delta_\varepsilon^2 + 3\delta_\varepsilon - \varepsilon = 0$

$\delta_\varepsilon = \dfrac{-3 \pm \sqrt{9+4\varepsilon}}{2}$ $\quad > 3$

$+ \quad \dfrac{-3+\sqrt{9+4\varepsilon}}{2} > 0 \quad \checkmark$

$- \quad \dfrac{-3-\sqrt{9+4\varepsilon}}{2} < 0 \quad \times$

$\delta_\varepsilon = \dfrac{-3+\sqrt{9+4\varepsilon}}{2}$

(d) $\lim_{x\to -2} (x^2+3x-1) = -3$

$\forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0$

$|x+2| < \delta_\varepsilon \implies |(x^2+3x-1)+3| < \varepsilon$

$\implies |x^2+3x+2| < \varepsilon$

$\implies |x+1| \cdot |x+2| < \varepsilon$

-1

$|x+1| < \delta_\varepsilon - 1$

$|x+1| \cdot |x+2| = (\delta_\varepsilon - 1)\,\delta_\varepsilon$

$= \delta_\varepsilon^2 - \delta_\varepsilon$

Usando $\delta_\varepsilon^2 - \delta_\varepsilon = \varepsilon$

$\delta_\varepsilon^2 - \delta_\varepsilon - \varepsilon = 0$

$\delta_\varepsilon = \dfrac{1 + \sqrt{1+4\varepsilon}}{2}$ $\quad > 1$

$+ \quad \dfrac{1+\sqrt{1+4\varepsilon}}{2} > 0 \quad \checkmark$

$- \quad \dfrac{1-\sqrt{1+4\varepsilon}}{2} < 0 \quad \times$

$\delta_\varepsilon = \dfrac{1+\sqrt{1+4\varepsilon}}{2}$

(e) $\lim_{x\to -6^+} \dfrac{1}{(x+6)^2} = +\infty$

$\forall M > 0 \quad \exists \delta_M > 0$

$-6 < x < -6 + \delta_M \implies \dfrac{1}{(x+6)^2} > M$

$\implies (x+6)^{-2} > M$

$0 < x+6 < \delta_M$

$(x+6)^{-2} > \delta_M^{-2}$

Usando $\delta_M^{-2} = M$

$\delta_M = M^{-1/2}$

$\delta_M = \dfrac{1}{\sqrt{M}}$

(f) $\lim_{x\to 0} -\dfrac{1}{x^2} = -\infty$

Studiamo i due casi I e II

I $\quad \lim_{x\to 0^-} -\dfrac{1}{x^2} = -\infty$

$\forall M < 0 \quad \exists \delta_M > 0$

$-\delta_M < x < 0 \implies -\dfrac{1}{x^2} < M$

$\implies -x^{-2} < M$

$\implies x^{-2} > -M$

$\implies x^2 < -\dfrac{1}{M}$

$\implies \pm x < \sqrt{-\dfrac{1}{M}}$

$\implies \pm x < \sqrt{-\dfrac{1}{M}}$

SCELGO - PERCHE' STO STUDIANDO 0^-

$\implies -x < \sqrt{-\dfrac{1}{M}}$

$\implies x > -\sqrt{-\dfrac{1}{M}}$

Usando $-\delta_M = -\sqrt{-\dfrac{1}{M}}$

$\delta_M = \sqrt{-\dfrac{1}{M}}$

II $\quad \lim_{x\to 0^+} -\dfrac{1}{x^2} = -\infty$

$\forall M < 0 \quad \exists \delta_M > 0$

$0 < x < \delta_M \implies -\dfrac{1}{x^2} < M$

$\implies -x^{-2} < M$

$\implies x^{-2} > -M$

$\implies x^2 < -\dfrac{1}{M}$

$\implies \pm x < \sqrt{-\dfrac{1}{M}}$

$\implies \pm x < \sqrt{-\dfrac{1}{M}}$

SCELGO + PERCHE' STO STUDIANDO 0^+

$\implies +x < \sqrt{-\dfrac{1}{M}}$

$\implies x < \sqrt{-\dfrac{1}{M}}$

Usando $\delta_M = \sqrt{-\dfrac{1}{M}}$

(g) $\lim_{x\to -\infty} \dfrac{1}{x^3} = 0$

$\forall \varepsilon > 0 \quad \exists M < 0$

$x < M \implies \left|\left(\dfrac{1}{x^3}\right) - 0\right| < \varepsilon$

$\implies \left|\dfrac{1}{x^3}\right| < \varepsilon$

$\implies |x^{-3}| < \varepsilon$

$\implies -\varepsilon < x^{-3} < \varepsilon$

$x^3 < M^3$

$x^{-3} > M^{-3}$

Usando $M^{-3} = -\varepsilon$

$M = \sqrt[3]{\dfrac{1}{-\varepsilon}}$

ESAME 30/01/2023

$$\lim_{x \to \frac{3}{8}^-} \frac{8}{(8x-3)^3} = -\infty$$

$\forall M < 0 \qquad \exists \delta_M > 0$

$$\frac{3}{8} - \delta_M < x < \frac{3}{8} \implies \underbrace{\frac{8}{(8x-3)^3}}_{\substack{<0 \\ \text{INTORNO SINISTRO}}} < \underbrace{M}_{<0}$$

$$\implies \frac{8}{M} < (8x-3)^3$$

$$\implies 8x-3 > \sqrt[3]{\frac{M}{8}}$$

$$\implies x > \frac{\sqrt[3]{M/8}}{8} + \frac{3}{8}$$

Usiamo $\cancel{\frac{3}{8}} - \delta_M = \frac{\sqrt[3]{M/8}}{8} + \cancel{\frac{3}{8}}$

$$\delta_M = -\frac{\sqrt[3]{M/8}}{8}$$

ESAME 09/06/2022

$$\lim_{x \to -\infty} \frac{-3x+1}{2x+3} = -\frac{3}{2}$$

$$\forall \varepsilon > 0 \qquad \exists M < 0$$

$$x < M \implies \left| \frac{-3x+1}{2x+3} + \frac{3}{2} \right| < \varepsilon$$

$$\implies \left| \frac{11}{2(2x+3)} \right| < \varepsilon$$

$$\implies \frac{1}{|2x+3|} < \frac{2\varepsilon}{11}$$

$$\implies |2x+3| > \frac{11}{2\varepsilon}$$

$$\implies 2x+3 < -\frac{11}{2\varepsilon} \lor 2x+3 > \frac{11}{2\varepsilon}$$

$$\implies 2x < -\frac{11}{2\varepsilon} - 3 \lor 2x > \frac{11}{2\varepsilon} - 3$$

$$\implies x < -\frac{11}{4\varepsilon} - \frac{3}{2} \lor x > \frac{11}{4\varepsilon} - \frac{3}{2}$$

Usiamo $M = -\frac{11}{4\varepsilon} - \frac{3}{2}$