ES 1

(a) $\sin(2x-\pi) = \cos x$

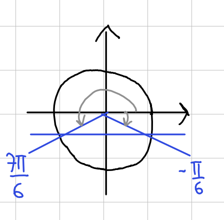
$\sin(2x)\overset{-1}{\cos(\pi)} - \cos(2x)\overset{0}{\sin\pi} = \cos x$

$-\sin(2x) = \cos x$

$-2\sin x\cos x = \cos x$

$\cos x\,(1+2\sin x) = 0$

- $\cos x = 0$; $x = \frac{\pi}{2} + k\pi$
- $1+2\sin x = 0$; $\sin x = -\frac{1}{2}$



$x = -\frac{\pi}{6} + k\pi \quad \vee \quad x = \frac{7\pi}{6} + k\pi$

$x \in \left\{ \frac{\pi}{2} + k\pi ; -\frac{\pi}{6} + k\pi ; \frac{7\pi}{6} + k\pi \right\}$

(b) $\cos x = -\sin x$

$\cos x + \sin x = 0$

$\frac{2}{\sqrt{2}}\left(\overset{\sin\frac{\pi}{4}}{\frac{\sqrt{2}}{2}}\cos x + \overset{\cos\frac{\pi}{4}}{\frac{\sqrt{2}}{2}}\sin x \right) = 0$

$\sqrt{2}\,\sin(\pi/4 + x) = 0$

$\sin(\pi/4 + x) = 0$

$\frac{\pi}{4} + x = k\pi$

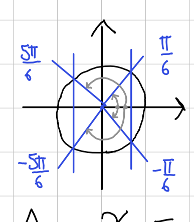
$x = -\frac{\pi}{4} + k\pi$

$x \in \left\{ -\frac{\pi}{4} + k\pi \right\}$

(c) $4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$



$x = \pm\frac{\pi}{6} + 2k\pi \quad \wedge \quad x = \pm\frac{5}{6}\pi + 2k\pi$

$x \in \left\{ \pm\frac{\pi}{6} + 2k\pi ; \pm\frac{5}{6}\pi + 2k\pi \right\}$

(d) $2\cos x + 2\sin x - (\sqrt{3}+1) = 0$

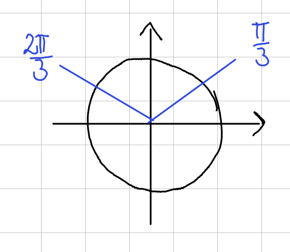
$2\cos x + 2\sin x = \sqrt{3}+1$

$\cos x + \sin x = \frac{\sqrt{3}}{2} + \frac{1}{2}$ PRIMA DI ELEVARE AL QUADRATO IMPONGO $\sin x + \cos x > 0$

$\cos^2 x + 2\sin x\cos x + \sin^2 x = \frac{3}{4} + \frac{2\sqrt{3}}{4} + \frac{1}{4}$

$\sin 2x + \cancel{1} = \cancel{1} + \frac{\sqrt{3}}{2}$

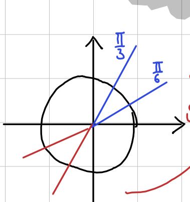
$\sin 2x = \frac{\sqrt{3}}{2}$



$2x = \frac{\pi}{3} + 2k\pi \quad \vee \quad 2x = \frac{2\pi}{3} + 2k\pi$

$x = \frac{\pi}{6} + k\pi \quad \vee \quad x = \frac{\pi}{3} + k\pi$ NOTA: ALTRA RISOLUZIONE PAG. 4

$x \in \left\{ \frac{\pi}{6} + 2k\pi ; \frac{\pi}{3} + 2k\pi \right\}$



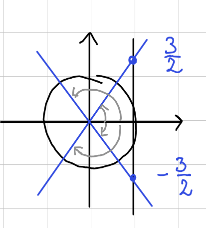
SCARTO LE SOLUZIONI CON SOMMA NEGATIVA

(e) $4\sin^2 x - 9\cos^2 x = 0$

$\frac{4\sin^2 x}{9\cos^2 x} = 1$

$\tan^2 x = \frac{9}{4}$

$\tan x = \pm\frac{3}{2}$



$x = \pm\arctan\left(\frac{3}{2}\right) + k\pi$

$x \in \left\{ \pm\arctan\left(\frac{3}{2}\right) + k\pi \right\}$

(f) $3\sin^2 x - 8\sqrt{3}\sin x\cos x + 15\cos^2 x = 0$

$\frac{3\sin^2 x}{\cos^2 x} - \frac{8\sqrt{3}\sin x\cos x}{\cos^2 x} + \frac{15\cos^2 x}{\cos^2 x} = 0$

$3\tan^2 x - 8\sqrt{3}\tan x + 15 = 0$

$t = \tan x$

$3t^2 - 8\sqrt{3}t + 15 = 0$

$t_1, t_2 = \frac{8\sqrt{3} \pm \overset{\sqrt{12} = 2\sqrt{3}}{\sqrt{192-180}}}{6} = \begin{cases} + \ \frac{10\sqrt{3}}{6} = \frac{5\sqrt{3}}{3} \\ - \ \frac{6\sqrt{3}}{6} = \sqrt{3} \end{cases}$

$\tan x = \sqrt{3}$; $x = \frac{\pi}{3} + k\pi$

$\tan x = \frac{5\sqrt{3}}{6}$; $x = \arctan\left(\frac{5\sqrt{3}}{6}\right) + k\pi$

$x \in \left\{ \frac{\pi}{3} + k\pi ; \arctan\left(\frac{5\sqrt{3}}{6}\right) + k\pi \right\}$

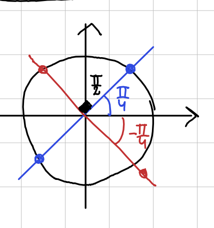
(g) $4\sin^2(x)\cos^2(x) - 4\cos^4(x) = 0$

$4\cos^2(x)\left[\sin^2(x) - \cos^2(x)\right] = 0$

$4\cos^2(x)\,(\sin(x) - \cos(x))\,(\sin(x) + \cos(x)) = 0$

$\cos x = 0$; $\sin(x - \pi/4) = 0$; $\sin(x + \pi/4) = 0$

$x = \frac{\pi}{2} + k\pi$; $x - \pi/4 = \pi$, $x = \pi/4 + \pi$; $x + \pi/4 = \pi$, $x = -\pi/4 + \pi$



$x = \pi/4 + k\pi/2$

$x \in \left\{ \frac{\pi}{2} + k\pi ; \pi/4 + k\pi/2 \right\}$

ES 2

(a) DOMINIO di $f(x) = \arctan(\arcsin(x))$

$$\begin{cases} -1 \le x \le 1 \\ \forall x \in \mathbb{R} \end{cases} \qquad -1 \le x \le 1$$

(b) DOMINIO di $f(x) = \log_3 |\arctan(x)|$

$$|\arctan(x)| > 0$$

$$\arctan(x) \neq 0$$

$$x \neq \tan(0)$$

$$x \neq 0$$

(c) VERIFICARE $\forall x \in [-1, 1]$

$$\arcsin(x) + \arccos(x) = \frac{\pi}{2}$$

Sia

$$A = \arcsin(x) \in [-\pi/2, \pi/2] \longrightarrow \sin(A) = x$$

$$B = \arccos(x) \in [0, \pi/2] \longrightarrow \cos(B) = x$$

$$\sin(A) = \cos(B)$$

$$\sin(A) = \sin(\pi/2 - B)$$

NOTA: vale poiché ragioniamo nel dominio ristretto

$$A = \pi/2 - B$$

$$A + B = \pi/2$$

$$\arcsin(x) + \arccos(x) = \pi/2$$

(d) VERIFICARE $\forall x \in \mathbb{R}$: $\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$

$$A = \arctan(x) \in [-\pi/2, \pi/2] \longrightarrow \tan(A) = x$$

$$x = \tan(A)$$

$$x = \frac{\sin(A)}{\cos(A)}$$

$$x^2 = \frac{\sin^2(A)}{\cos^2(A)}$$

$$x^2 = \frac{\sin^2(A)}{1 - \sin^2(A)}$$

$$\sin^2(A) = x^2(1 - \sin^2(A))$$

$$\sin^2(A) - x^2 + x^2 \sin^2(A) = 0$$

$$\sin^2(A)(1 + x^2) = x^2$$

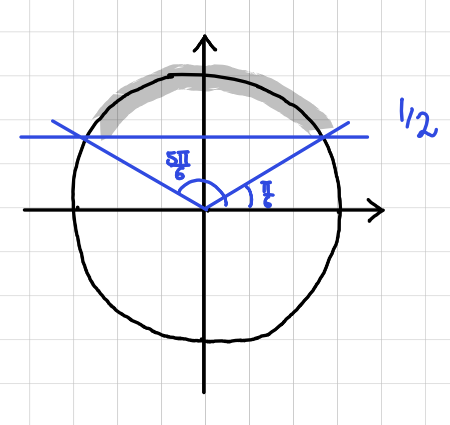
$$\sin^2(A) = \frac{x^2}{1+x^2}$$

$$\sin(A) = \sqrt{\frac{x^2}{1+x^2}}$$

$$\sin(\arctan(x)) = \sqrt{\frac{x^2}{1+x^2}}$$

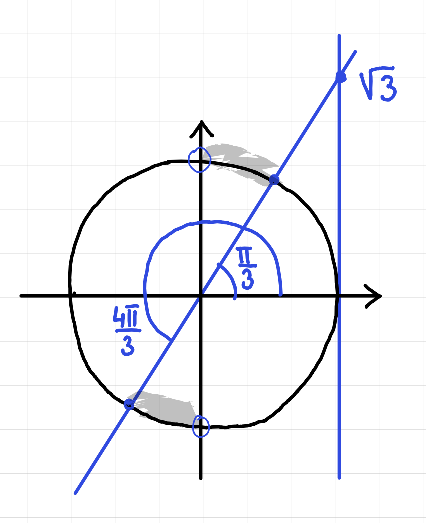
ES 3

(a) $\sin x > \frac{1}{2}$



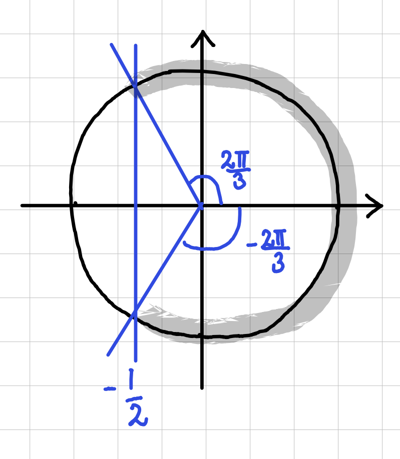
$x \in \left] \frac{\pi}{6} + 2k\pi, \frac{5}{6}\pi + 2k\pi \right[$

(b) $\tan x \geq \sqrt{3}$



$x \in \left[\frac{\pi}{3} + k\pi, \frac{\pi}{2} + k\pi \right[$

(c) $\cos x > -\frac{1}{2}$



$x \in \left] -\frac{2\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi \right[$

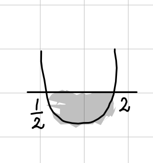
(d) $2\sin^2 x + 5\cos x - 4 > 0$

$2(1-\cos^2 x) + 5\cos x - 4 > 0$

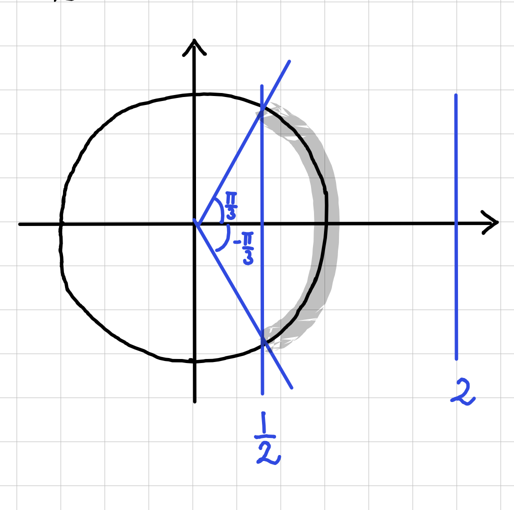
$-2\cos^2 x + 5\cos x - 2 > 0$

$2\cos^2 x - 5\cos x + 2 < 0$

$t_1, t_2 = \frac{5 \pm \sqrt{25-16}}{4} \begin{matrix} \nearrow 2 \\ \searrow \frac{1}{2} \end{matrix}$



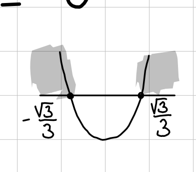
$\frac{1}{2} < \cos x < 2$



$x \in \left] -\frac{\pi}{3} + 2k\pi, \frac{\pi}{3} + 2k\pi \right[$

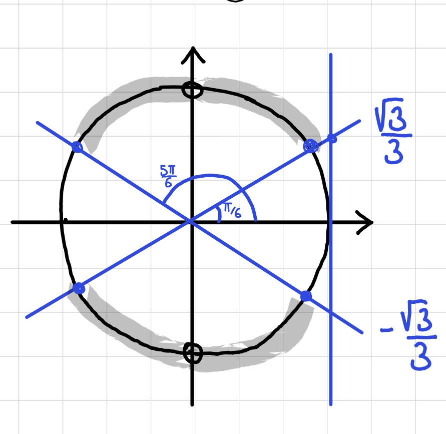
(e) $3\tan^2 x - 1 \geq 0$

$\tan^2 x \geq \frac{1}{3}$



$\sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot 1 = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan x \leq -\frac{\sqrt{3}}{3} \lor \tan x \geq \frac{\sqrt{3}}{3}$



$x \in \left[\frac{\pi}{6} + k\pi, \frac{\pi}{2} + k\pi \right[\cup \left] \frac{\pi}{2} + k\pi, \frac{5\pi}{6} + k\pi \right]$

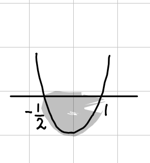
(f) $\cos(2x) + \sin x \geq 0$

$1 - 2\sin^2 x + \sin x \geq 0$

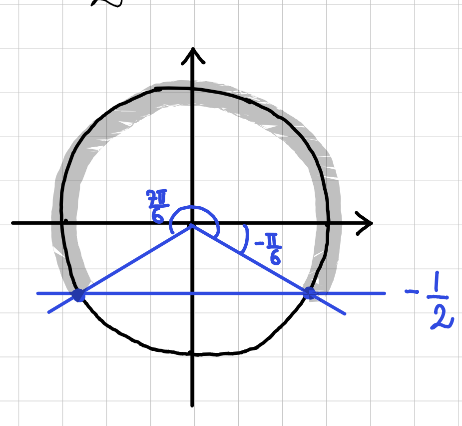
$-2\sin^2 x + \sin x + 1 \geq 0$

$2\sin^2 x - \sin x - 1 \leq 0$

$t_1, t_2 = \frac{1 \pm \sqrt{1+8}}{4} \begin{matrix} \nearrow 1 \\ \searrow -\frac{1}{2} \end{matrix}$



$-\frac{1}{2} \leq \sin x \leq 1$



$x \in \left[-\frac{\pi}{6} + 2k\pi, \frac{7\pi}{6} + 2k\pi \right]$

(g) $\cos^2 x + (\sqrt{3}-1)\sin(x)\cos(x) - \sqrt{3}\sin^2 x > 0$

$\frac{-\sqrt{3}\sin^2 x}{\cos^2 x} + \frac{(\sqrt{3}-1)\sin(x)\cos(x)}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} > 0$

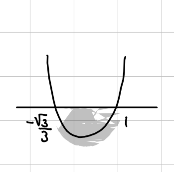
$-\sqrt{3}\tan^2 x + (\sqrt{3}-1)\tan x + 1 > 0$

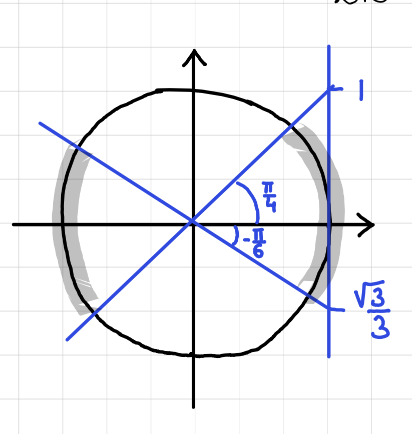
$\sqrt{3}\tan^2 x - (\sqrt{3}-1)\tan x - 1 < 0$

$t_1, t_2 = \frac{\sqrt{3}-1 \pm \sqrt{3-2\sqrt{3}+1+4\sqrt{3}}}{2\sqrt{3}}$

$= \frac{\sqrt{3}-1 \pm \sqrt{(1+\sqrt{3})^2}}{2\sqrt{3}}$

$= \frac{\sqrt{3}-1 \pm (1+\sqrt{3})}{2\sqrt{3}} \begin{matrix} \frac{\sqrt{3}-1+1+\sqrt{3}}{2\sqrt{3}} = 1 \\ \frac{\sqrt{3}-1-1-\sqrt{3}}{2\sqrt{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \end{matrix}$

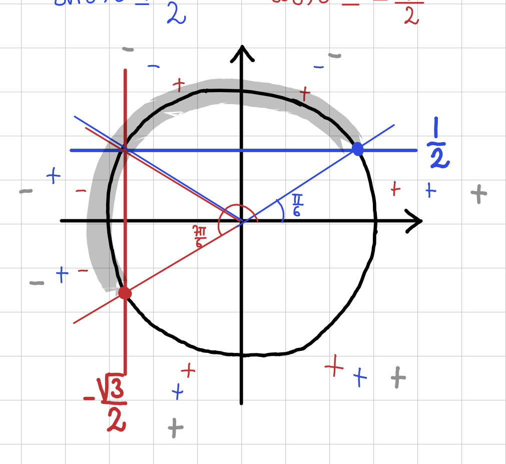




$x \in \left] -\frac{\pi}{6} + k\pi, \frac{\pi}{4} + k\pi \right[$

(h) $\underbrace{(1-2\sin x)}_{\geq 0}\underbrace{(2\cos x + \sqrt{3})}_{\geq 0} \leq 0$

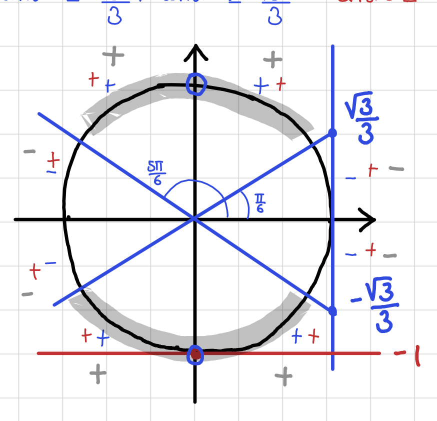
$\sin x \leq \frac{1}{2}$ $\quad$ $\cos x \geq -\frac{\sqrt{3}}{2}$



$x \in \left[\frac{\pi}{6} + 2k\pi, \frac{7\pi}{6} + 2k\pi \right]$

(i) $\underbrace{(3\tan^2 x - 1)}_{\geq 0}\underbrace{(\sin x + 1)}_{\geq 0} \geq 0$

$\tan x \leq -\frac{\sqrt{3}}{3} \lor \tan x \geq -\frac{\sqrt{3}}{3}$ $\quad$ $\sin x \geq -1$

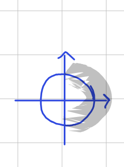


$x \in \left[\frac{\pi}{6} + k\pi, \frac{\pi}{2} + k\pi \right[\cup \left] \frac{\pi}{2} + k\pi, \frac{5\pi}{6} + k\pi \right]$

(j) $\frac{\cos^2 x - \sin^2 x}{\sqrt{3}\tan(x) + 1} \leq 0$

$\frac{\cos 2x}{\sqrt{3}\tan x + 1} \leq 0$

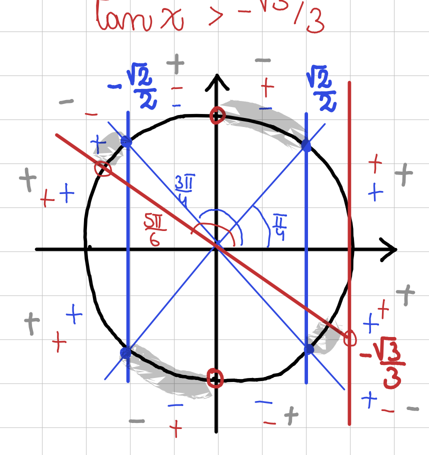
$\cos 2x \geq 0$



$-\pi/2 + 2k\pi \leq 2x \leq \pi/2 + 2k\pi$

$-\pi/4 + k\pi \leq x \leq \pi/4 + 2k\pi$

$\tan x > -\sqrt{3}/3$



$x \in \left[\frac{\pi}{4} + k\pi, \frac{\pi}{2} + k\pi \right[\cup \left[\frac{3\pi}{4} + 2k\pi, \frac{5\pi}{6} + 2k\pi \right[$

(k) $\frac{\sqrt{3}\sin x - \cos x}{1 - \sin^2 x} \geq 0$

$\sqrt{3}\sin x - \cos x \geq 0$

$\sqrt{3}/2 \sin x - 1/2 \cos x \geq 0$

$\cos \pi/6 \sin x - \sin \pi/6 \cos x \geq 0$

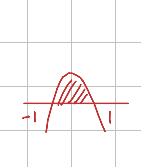
$\sin(x - \pi/6) \geq 0$

$2k\pi \leq x - \pi/6 \leq \pi + 2k\pi$

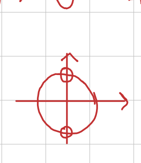
$2k\pi + \pi/6 \leq x \leq 7\pi/6 + 2k\pi$

$1 - \sin^2 x > 0$

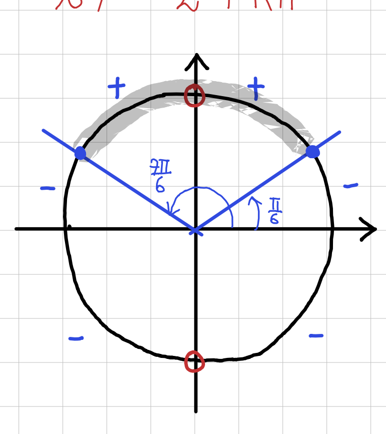
$(1 - \sin x)(1 + \sin x) > 0$



$-1 < \sin x < 1$



$x \neq \pi/2 + k\pi$



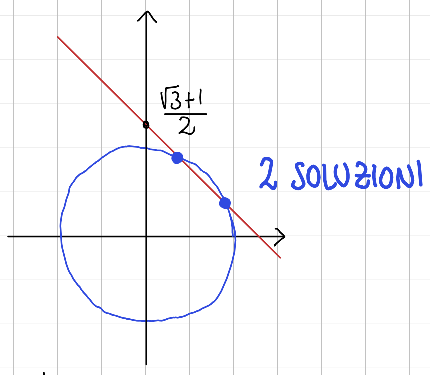
$x \in \left[\frac{\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi \right[\cup \left] \frac{\pi}{2} + 2k\pi, \frac{7\pi}{6} + 2k\pi \right]$

RISOLUZIONI USANDO SISTEMA LINEARE

1.d $2\cos x + 2\sin x - (\sqrt{3}+1) = 0$

$$2X + 2Y = \sqrt{3}+1$$

$$\begin{cases} Y = -X + \dfrac{\sqrt{3}+1}{2} \; // \\ X^2 + Y^2 = 1 \; // \end{cases}$$



$$X^2 + \left(-X + \frac{\sqrt{3}+1}{2}\right)^2 = 1$$

$$X^2 + X^2 - (\sqrt{3}+1)X + \frac{3+2\sqrt{3}+1}{4} = 1$$

$$2X^2 - (\sqrt{3}+1)X + \cancel{1} + \frac{\sqrt{3}}{2} = \cancel{1}$$

$$X_1, X_2 = \frac{\sqrt{3}+1 \pm \sqrt{3+2\sqrt{3}+1-4\sqrt{3}}}{4}$$

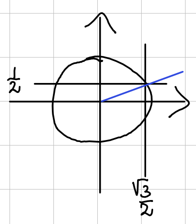
$\sqrt{3-2\sqrt{3}+1}$
$\sqrt{(\sqrt{3}-1)^2}$
$(\sqrt{3}-1)$

$$= \frac{\sqrt{3}+1 \pm (\sqrt{3}-1)}{4}$$

$$+ \quad \frac{\sqrt{3}+1+\sqrt{3}-1}{4} = \frac{\sqrt{3}}{2}$$

$$\begin{cases} X = \sqrt{3}/2 \\ Y = -\sqrt{3}/2 + \dfrac{\sqrt{3}+1}{2} \end{cases}$$

$$\begin{cases} X = \sqrt{3}/2 \\ Y = 1/2 \end{cases}$$

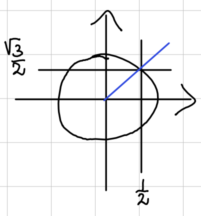


$$x = \frac{\pi}{6} + 2k\pi$$

$$- \quad \frac{\sqrt{3}+1-\sqrt{3}+1}{4} = \frac{1}{2}$$

$$\begin{cases} X = 1/2 \\ Y = -\sqrt{3}/2 + \dfrac{\sqrt{3}+1}{2} \end{cases}$$

$$\begin{cases} X = 1/2 \\ Y = \sqrt{3}/2 \end{cases}$$



$$x = \frac{\pi}{3} + 2k\pi$$