ES 2

(a) f, g crescenti $\quad \forall x_1, x_2 \in D_f \;\; x_1 \le x_2 \Longrightarrow f(x_1) \le f(x_2)$

Dimostrare $f \circ g$ é CRESCENTE

$\forall x_1, x_2 \in D_g \quad x_1 \le x_2 \Longrightarrow g(x_1) \le g(x_2)$

$\forall y_1, y_2 \in D_f \quad y_1 \le y_2 \Longrightarrow f(y_1) \le f(y_2)$

Siano $y_1 = g(x_1)$, $y_2 = g(x_2)$

$$\forall x_1, x_2 \in D_g \quad x_1 \le x_2 \Rightarrow \underbrace{f(g(x_1))}_{f\circ g(x_1)} \le \underbrace{f(g(x_2))}_{f\circ g(x_2)}$$

(b) f, g crescenti

Dimostrare $f+g$ é CRESCENTE

$$\forall x_1, x_2 : \quad x_1 \le x_2 \Longrightarrow \quad f(x_1) \le f(x_2)$$

$x \le y$
$x+a \le x+a$

$$f(x_1)+g(x_1) \le f(x_2)+g(x_1)$$

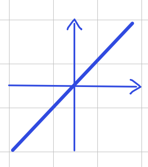
$g(x_1) \le g(x_2)$
CRESCENTE

$$f(x_1)+g(x_1) \le f(x_2)+g(x_2)$$

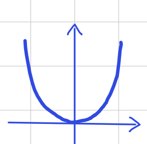
(c) f, g crescenti

esempio $f \cdot g$ NON crescente

$f(x) = g(x) = x$



$h(x) = f(x) \cdot g(x) = x^2$



(d) f e g periodiche di $T \quad f(x) = f(x+T)$

– $\underline{h_1 = f+g}$

$$\begin{aligned} h_1(x) &= f(x) + g(x) \\ &= f(x+T) + g(x+T) \\ &= h_1(x+T) \end{aligned}$$

– $\underline{h_2 = f \cdot g}$

$$\begin{aligned} h_2(x) &= f(x) \cdot g(x) \\ &= f(x+T) \cdot g(x+T) \\ &= h_2(x+T) \end{aligned}$$

E' sempre periodica ma non é garantito il periodo sia T. Può dipendere dalla specifica funzione
Indaghiamo se può esistere periodo $T' < T$

- $f(x) = \sin(x)$; PERIODO 2π

  $g(x) = \dfrac{1}{\sin(x)}$, PERIODO 2π

  $h_2(x) = 1$ funzione costante

- $f(x) = \sin(x)$; PERIODO 2π

  $g(x) = \cos(x)$; PERIODO 2π

  $$\begin{aligned} h_2(x) &= \sin(x)\cos(x) \\ &= \frac{\sin(2x)}{2} \quad \text{PERIODO } \pi \end{aligned}$$

  Trovato $T' < T$

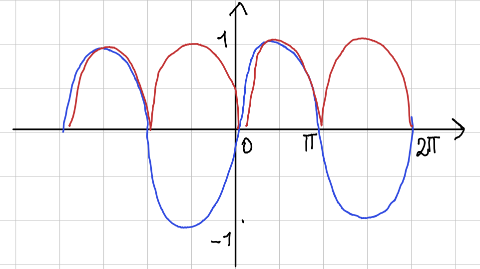
– $\underline{h_3 = |f|}$

$$\begin{aligned} h_3(x) &= |f(x)| \\ &= |f(x+T)| \\ &= h_3(x+T) \end{aligned}$$

Indaghiamo come per h_2 se può esistere un periodo $T' < T$

- $f(x) = \sin(x)$ PERIODO 2π

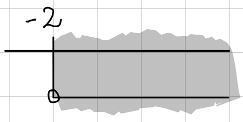
  $h_3(x) = |\sin(x)|$ PERIODO π



ES 3

(a) $f(x) = \log_2(x^2+1) + \log_5(x+2)$

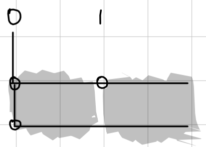
$$D_f: \begin{cases} x^2+1>0 \\ x+2>0 \end{cases} = \begin{cases} \forall x \in \mathbb{R} \\ x>-2 \end{cases}$$



$x \in \,]-2, +\infty[$

(b) $f(x) = \log_x(x)$

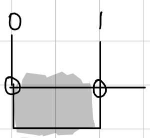
$$D_f: \begin{cases} x>0 \ \wedge \ x \neq 1 \\ x>0 \end{cases}$$

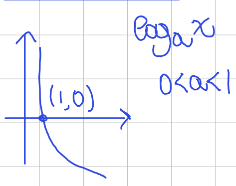


$x \in \,]0,1[\, \cup \,]1,+\infty[$

(c) $f(x) = \log_2\left(\log_{\frac{1}{2}} x\right)$

$$D_f: \begin{cases} x>0 \\ \log_{\frac{1}{2}} x > 0 \end{cases} = \begin{cases} x>0 \\ \underline{0<x<1} \end{cases}$$





$x \in \,]0,1[$

(d) $f(x) = \log_a\left(\frac{x}{x-1}\right)$

D_f: $\frac{x}{x-1} > 0$ $\quad$ $N>0: \ x>0$ $\quad$ $D>0: \ x>1$

	0		1	
−		+		+
−		−		+
+		−		+

$x \in \,]-\infty, 0[\, \cup \,]1+\infty[$

per qualsiasi $a \in \,]0,1[\, \cup \,]1,+\infty[$

ES 4

(a) $\log_a b \cdot \log_b c \cdot \log_b c$

Ciascun a, b, c appare come $\begin{cases} \text{BASE} & a \in]0,1[\cup]1,+\infty[\\ \text{ARGOMENTO} & a \in]0,+\infty[\end{cases}$

Unendo le due abbiamo $a, b, c \in]0,1[\cup]1,+\infty[$

RICORDIAMO REGOLA DEL CAMBIO BASE $\qquad \log_a b = \dfrac{\log_c b}{\log_c a}$

$$\log_a b \cdot \log_b c \cdot \log_c a =$$

$$= \cancel{\log_a b} \cdot \frac{\cancel{\log_a c}}{\cancel{\log_a b}} \cdot \frac{\log_a a}{\cancel{\log_a c}}$$

$$= 1$$

(b) $\forall b > 0 \qquad \log_a b = \log_{\sqrt{a}} b + \log_{\frac{1}{a}} b$

$$= \frac{\log_a b}{\log_a \sqrt{a}} + \frac{\log_a b}{\log_a \frac{1}{a}}$$

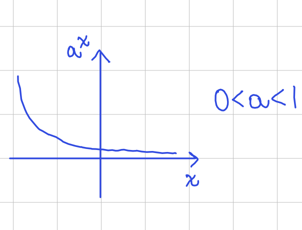
$$= \frac{\log_a b}{1/2} + \frac{\log_a b}{-1}$$

$$= 2 \log_a b - \log_a b$$

$$= \log_a b$$

ES 5

(a) $\left(\frac{1}{4}\right)^x < 64$



$\left(\frac{1}{4}\right)^x < \left(\frac{1}{4}\right)^{-3}$

$x > -3$

$x \in]-3, +\infty[$

(b) $\frac{35}{2}\left(\frac{1}{5}\right)^{2x} > 0.7 \cdot 5^x$

$\frac{35}{2} 5^{-2x} > \frac{7}{10} 5^x$

$\frac{\cancel{35}^{5}}{\cancel{2}_{1}} \frac{\cancel{10}^{5}}{\cancel{7}_{1}} 5^{-2x} > 5^x$

$5^{-2x+2} > 5^x$

$-2x+2 > x$

$-3x > -2$

$x < 2/3$

$x \in]-\infty, 2/3]$

(c) $9\left(\frac{2}{3}\right)^x + 2 + 4\left(\frac{2}{3}\right)^{-x} \leq 0$

Somma di tre numeri positivi non può essere negativa o nulla

$\nexists x \in \mathbb{R}$

(d) $\log(x+5) - \log(4-x) + \log(3x-1) >$
$> \log(3x-1) - \log(x+4)$

$D: \begin{cases} x+5>0 \\ 4-x>0 \\ 3x-1>0 \\ x+4>0 \end{cases} \quad \begin{cases} x>-5 \\ x<4 \\ x>1/3 \\ x>-4 \end{cases}$

$-5 \quad -4 \quad 1/3 \quad 4$

$1/3 < x < 4$

$\log(x+5) - \log(4-x) + \log(x+4) > 0$

$\log\left(\frac{(x+5)(x+4)}{4-x}\right) > \log 1$

$\frac{(x+5)(x+4)}{4-x} > 1$

$\frac{x^2+9x+20}{4-x} - 1 > 0$

$\frac{x^2+9x+20-4+x}{4-x} > 0$

$\frac{x^2+10x+16}{4-x} > 0$

$N>0 \quad x^2+10x+16>0$

$x_1, x_2 = -5 \pm \sqrt{25-16}^{\,3} = \begin{cases} + & -2 \\ - & -8 \end{cases}$

$-8 \quad -2$

$x<-8 \lor x>-2$

$D>0 \quad 4-x>0$
$x<4$

-8	-2	4	
+	−	+	+
+	+	+	−
+	−	+	−

$x<-8 \lor -2<x<4$

$D \begin{cases} x<-8 \lor -2<x<4 \\ 1/3 < x < 4 \end{cases}$

$-8 \quad -2 \quad 1/3 \quad 4$

$1/3<x<4$

$x \in]1/3, 4[$

(e) $\frac{1}{2}\log(-x^2+2x) < \log x$

$0 \quad 2$

$D: \begin{cases} -x^2+2x>0 \\ x>0 \end{cases} \quad \begin{cases} x(-x+2)>0 \\ x>0 \end{cases} \quad \begin{cases} 0<x<2 \\ x>0 \end{cases}$

$0<x<2$

$\log\sqrt{-x^2+2x} < \log x$

$\sqrt{-x^2+2x} < x$

$-x^2+2x<x^2$

$-2x^2+2x<0$

$-x^2+x<0$

$x(-x+1)<0$

$0 \quad 1$

$x<0 \lor x>1$

$D: \begin{cases} x<0 \lor x>1 \\ 0<x<2 \end{cases}$

$0 \quad 1 \quad 2$

$1<x<2$

$x \in]1, 2[$

(f) $\log\left(2+\frac{1}{x}\right) - \log\left(2-\frac{1}{x}\right) <$
$< \log(2x+1) - \log(1-2x)$

$D. \begin{cases} 2+1/x>0 \\ 2-1/x>0 \\ 2x+1>0 \\ 1-2x>0 \end{cases} \quad \begin{cases} \frac{2x+1}{x}>0 \\ \frac{2x-1}{x}>0 \\ x>-1/2 \\ x<1/2 \end{cases} \quad \begin{cases} x<-1/2 \lor x>0 \\ x<0 \lor x>1/2 \\ x>-1/2 \\ x<1/2 \end{cases}$

$-\frac{1}{2} \quad 0$

$0 \quad \frac{1}{2}$

$-1/2 \quad 0 \quad 1/2$

$\nexists x \in \mathbb{R}$

Il dominio è l'insieme vuoto, quindi anche la soluzione

(g) $2\log_{\sqrt{3}}(1-x) - \log_{\sqrt{3}}(3-x) < 2$

$D: \begin{cases} 1-x>0 \\ 3-x>0 \end{cases} \quad \begin{cases} x<1 \\ x<3 \end{cases} \quad x<1$

$\log_{\sqrt{3}}\left(\frac{(1-x)^2}{3-x}\right) < \log_{\sqrt{3}} 3 \leftarrow 2$
$= 2 \cdot 1$
$= 2 \cdot \log_{\sqrt{3}} \sqrt{3}$
$= \log_{\sqrt{3}} \sqrt{3}^2$
$= \log_{\sqrt{3}} 3$

$\frac{(1-x)^2}{3-x} < 3$

$\frac{1-2x+x^2-9+3x}{3-x} < 0$

$\frac{x^2+x-8}{3-x} < 0$

$N>0 \quad x^2+x-8>0 \quad x_1, x_2 = \frac{-1 \pm \sqrt{1+32}}{2}$

$\frac{-1-\sqrt{33}}{2} \quad \frac{-1+\sqrt{33}}{2}$

$-3{,}37 \qquad 2{,}37$

$x < \frac{-1-\sqrt{33}}{2} \lor x > \frac{-1+\sqrt{33}}{2}$

$D>0 \quad 3-x>0 \quad x<3$

	$\frac{-1-\sqrt{33}}{2}$	$\frac{-1+\sqrt{33}}{2}$	3
+	+	+	−
+	−	+	+
+	−	+	−

$\frac{-1-\sqrt{33}}{2} < x < \frac{-1+\sqrt{33}}{2} \lor x>3$

$D \begin{cases} \frac{-1-\sqrt{33}}{2} < x < \frac{-1+\sqrt{33}}{2} \lor x>3 \\ x<1 \end{cases}$

$\frac{-1-\sqrt{33}}{2} \quad 1 \quad \frac{-1+\sqrt{33}}{2} \quad 3$

$x \in \left]\frac{-1-\sqrt{33}}{2}, 1\right[$

(h) $3\log_5(x-4) > \frac{6}{\log_5(x-4)+1}$

$D \begin{cases} x-4>0 \\ \log_5(x-4)+1 \neq 0 \end{cases} \quad \begin{cases} x>4 \\ x \neq 21/5 \end{cases} \quad x>4 \land x \neq \frac{21}{5}$

$\log_5(x-4) \neq -1$

$\log_5(x-4) \neq \log_5 1/5$

$x-4 \neq 1/5$

$x \neq 21/5$

$\log_5(x-4) = t$

$3t > \frac{6}{t+1}$

$\frac{3t^2+3t-6}{t+1} > 0$

$\frac{t^2+t-2}{t+1} > 0$

$N>0 \quad t^2+t-2>0 \quad t_1, t_2 = \frac{-1 \pm \sqrt{1+8}^{\,3}}{2} = \begin{cases} + & 1 \\ - & -2 \end{cases}$

$-2 \quad 1$

$x<-2 \lor x>1$

$D>0 \quad t+1>0$

$t>-1$

-2	-1	1	
+	−	−	+
−	−	+	+
−	+	−	+

$-2<t<-1 \lor t>1$

$\begin{cases} -2 < \log_5(x-4) \\ \log_5(x-4) < -1 \end{cases} \lor \log_5(x-4) > 1$

$\begin{cases} \log_5 \frac{1}{25} < \log_5(x-4) \\ \log_5(x-4) < \log_5 \frac{1}{5} \end{cases} \lor \log_5(x-4) > \log_5(5)$

$\begin{cases} \frac{1}{25} < x-4 \\ x-4 < 1/5 \end{cases} \lor x-4>5$

$\begin{cases} x > \frac{101}{25} \\ x < 21/5 \end{cases} \lor x>9$

$\frac{101}{25} < x < \frac{21}{5} \lor x>9$

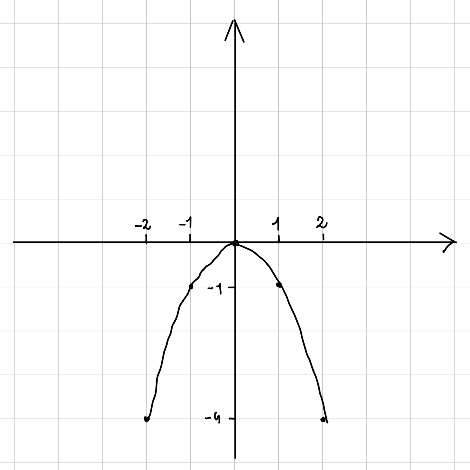
$D \begin{cases} \frac{101}{25} < x < \frac{21}{5} \lor x>9 \\ x>4 \land x \neq \frac{21}{5} \end{cases}$

$4 \quad \frac{101}{25} \quad \frac{21}{5} \quad 9$

$x \in \left]\frac{101}{25}, \frac{21}{5}\right[\cup]9, +\infty[$

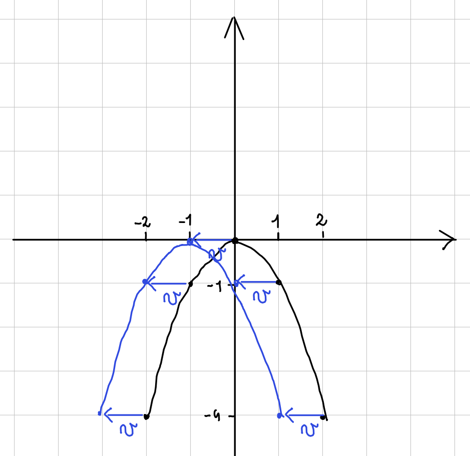
ES 1 (dalle slides) - ESERCIZIO 1

$f(x) = -x^2$

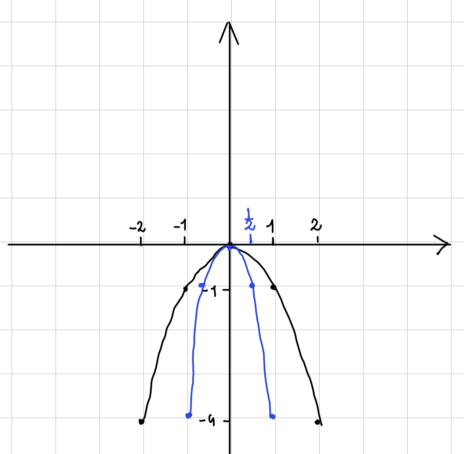


x	y
0	0
-1	-1
1	-1
2	-4
-2	-4

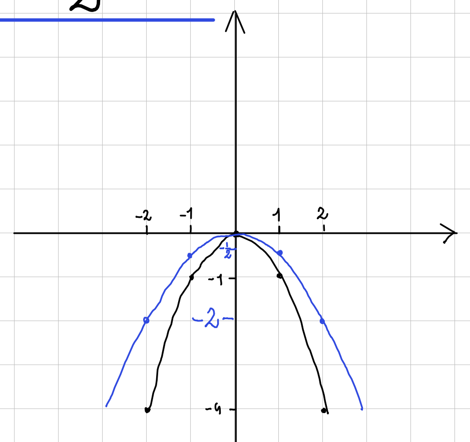
$y = f(x+1)$ TRASLAZIONE $\vec{v}(-1, 0)$



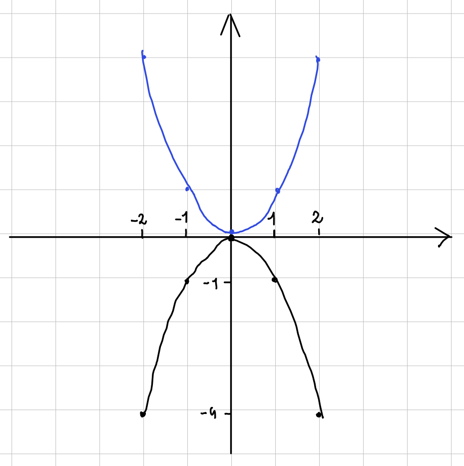
$y = f(2x)$ CONTRAZIONE ASSE X DI 2



$y = \frac{1}{2} f(x)$ CONTRAZIONE ASSE Y DI 2

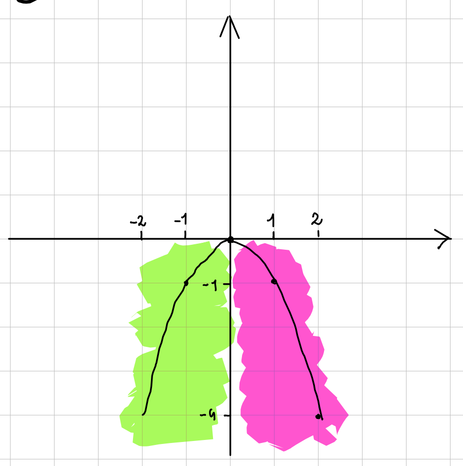


$y = -f(x)$ SIMMETRIA ASSE X



$y = f^{-1}(x)$

$f: \mathbb{R} \to \mathbb{R}$
$x \mapsto f(x) = -x^2$



f non è

né iniettiva $f(x) = f(-x)$

né suriettiva $f(\mathbb{R}) = \mathbb{R}^- \cup \{0\}$

Dobbiamo restringere il dominio e codominio
Vediamo entrambe le possibili risoluzioni

$A = \mathbb{R}^- \cup \{0\}$

$B = A$

$g: A \to B$

$x \mapsto g(x) = -x^2$

$y = -x^2$

$x^2 = -y$

$x = \pm\sqrt{-y}$

$x = -\sqrt{-y}$ scelgo − poiché $x \in A$

$f^{-1}: B \to A$

$x \to f^{-1}(x) = -\sqrt{-x}$

$f(f^{-1}(x)) =$

$= -(-\sqrt{-x})^2$

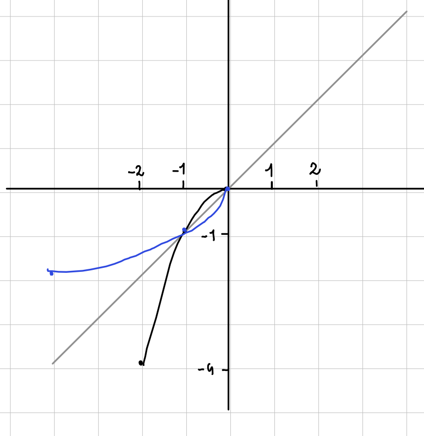
$= -(-x)$

$= x$ ✓

x (∈A)	$f(x)$ (∈B)	x (∈B)	$f^{-1}(x)$ (∈A)
0	0	0	0
-1	-1	-1	-1
-2	-4	-4	-2

✓

SIMMETRIA RISPETTO ALLA BISETTRICE DEL 1° E 3° QUADRANTE ($y = x$)



$A = \mathbb{R}^+ \cup \{0\}$

$B = \mathbb{R}^- \cup \{0\}$

$g: A \to B$

$x \mapsto g(x) = -x^2$

$y = -x^2$

$x^2 = -y$

$x = \pm\sqrt{-y}$

$x = +\sqrt{-y}$ scelgo + poiché $x \in A$

$f^{-1}: B \to A$

$x \to f^{-1}(x) = \sqrt{-x}$

$f(f^{-1}(x)) =$

$= -(\sqrt{-x})^2$

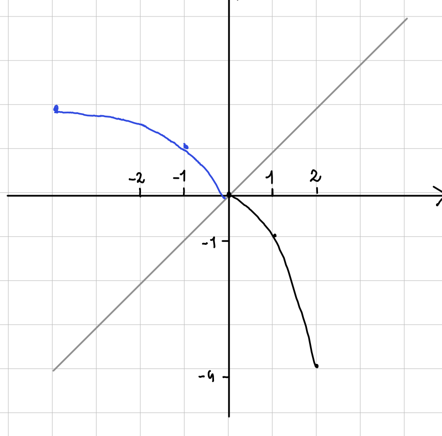
$= -(-x)$

$= x$ ✓

x (∈A)	$f(x)$ (∈B)	x (∈B)	$f^{-1}(x)$ (∈A)
0	0	0	0
1	-1	-1	1
2	-4	-4	2

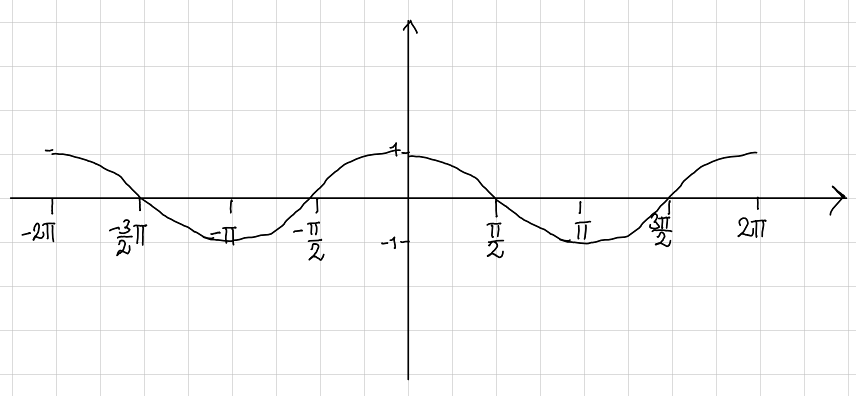
✓

SIMMETRIA RISPETTO ALLA BISETTRICE DEL 1° E 3° QUADRANTE ($y = x$)

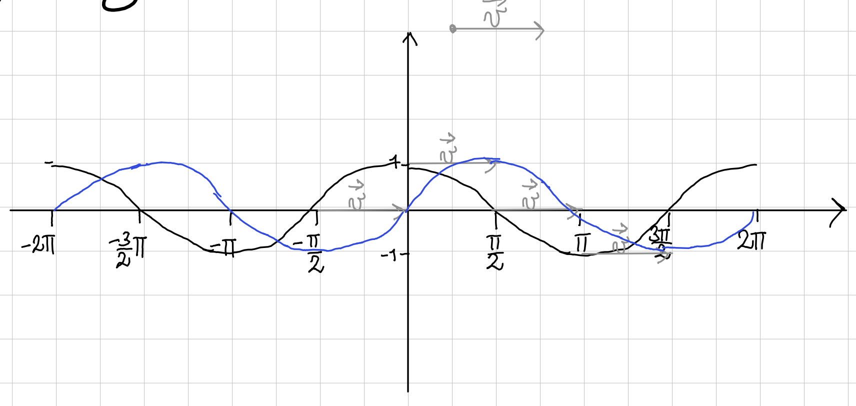


ES 1 (dalle slides) - ESERCIZIO 2

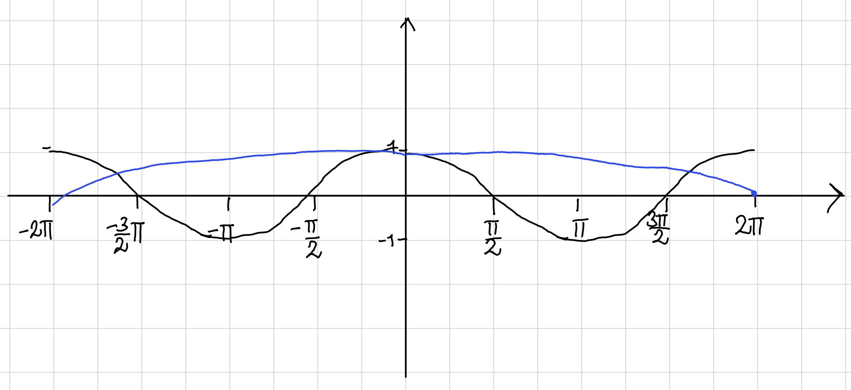
$g(x) = \cos(x)$



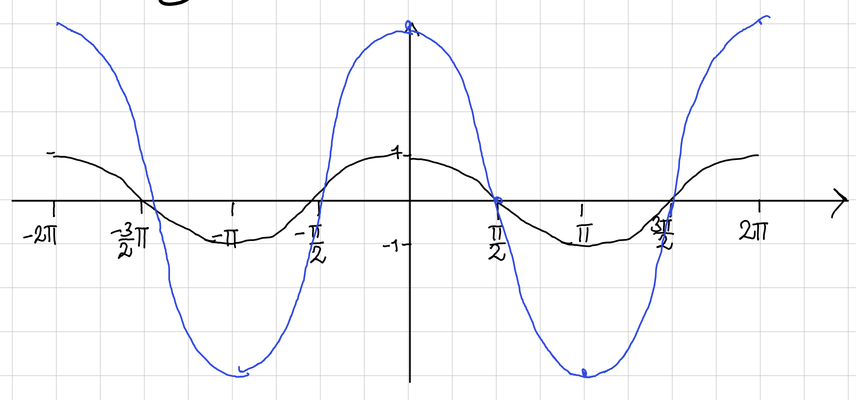
$y = g(x - \pi/2)$ TRASLAZIONE $\vec{v}\,(\pi/2, 0)$



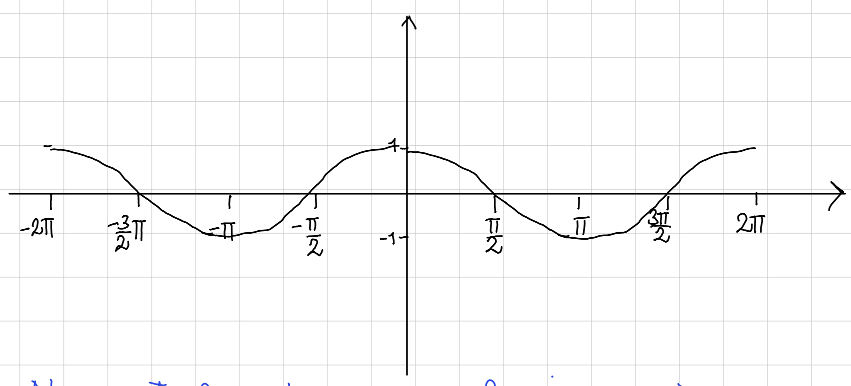
$y = g(x/4)$ DILATAZIONE ASSE X DI 4



$y = 4g(x)$ DILATAZIONE ASSE Y DI 4

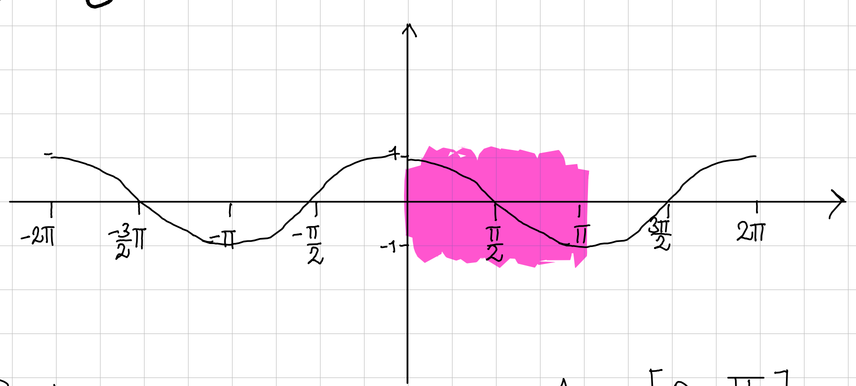


$y = g(-x)$ SIMMETRIA RISPETTO ALL' ASSE Y



Nessuna trasformazione: coseno funzione pari (cioè $f(x)=f(-x)$) e quindi simmetrica sull'asse y

$y = g^{-1}(x)$



Restrizione di dominio $A = [0, \pi]$

$B = [-1, 1]$

$h : A \to B$

$x \mapsto h(x) : \cos(x)$

$y = \cos(x)$

$x = \arccos(y)$

$g^{-1} : B \to A$

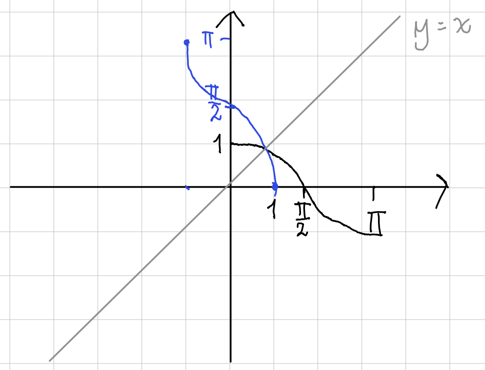
$x \mapsto g^{-1}(x) = \arccos(x)$

$g(g^{-1}(x)) = \cos(\arccos(x)) = x$ ✓

$\in A$ x	$\in B$ $g(x)$
0	1
$\pi/6$	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$
$\pi/3$	$1/2$
$\pi/2$	0
$2\pi/3$	$-1/2$
$3\pi/4$	$-\sqrt{2}/2$
$5\pi/6$	$-\sqrt{3}/2$
π	-1

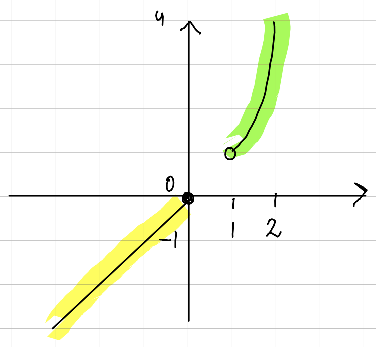
$\in B$ x	$\in A$ $g^{-1}(x)$
1	0
$\sqrt{3}/2$	$\pi/6$
$\sqrt{2}/2$	$\pi/4$
$1/2$	$\pi/3$
0	$\pi/2$
$-1/2$	$2\pi/3$
$-\sqrt{2}/2$	$3\pi/4$
$-\sqrt{3}/2$	$5\pi/6$
-1	π

SIMMETRIA RISPETTO ALLA BISETTRICE DEL 1° E 3° QUADRANTE ($y=x$)



ES 1 (dalle slides) - ESERCIZIO 3

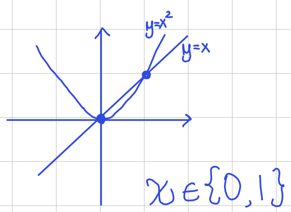
$$f(x) = \begin{cases} x & \text{se } x \le 0; \\ x^2 & \text{se } x > 1; \end{cases}$$



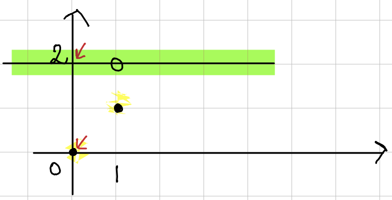
NON DEFINITA PER INTERVALLO $]0,1]$ X

$x=0 \lor x=1$

$$g(x) = \begin{cases} x & \text{se } x^2 = x; \\ 2 & \text{se } x \ne 1; \end{cases}$$



$x \in \{0,1\}$

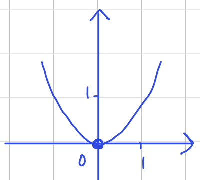


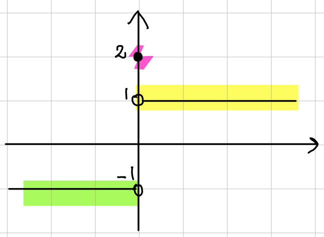
$g(0) = 0$ non é una funzione

$g(0) = 2$ X

$$h(x) = \begin{cases} 1 & \text{se } x > 0; \\ -1 & \text{se } x < 0; \\ 2 & \text{se } x^2 = 0; \end{cases}$$

$x = 0$





√

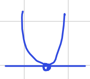
$x = \pm 1$

$$e(x) = \begin{cases} x^2 & \text{se } (x^2-1)^2 \le 0; \\ x & \text{se } |x| \ne 1; \end{cases}$$

$x \ne \pm 1$

perché $t^2 \le 0$

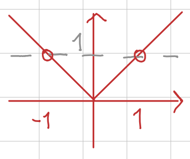
$t = 0$



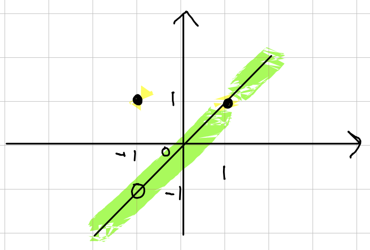
$(x^2-1)^2 = 0$

$x^2 - 1 = 0$

$x = \pm 1$



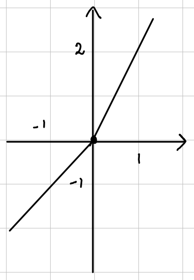
$x \ne \pm 1$



√

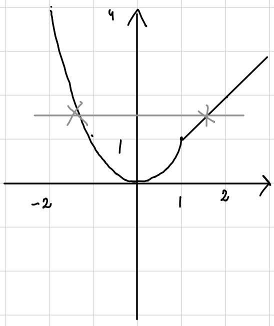
ES 1 (dalle slides) - ESERCIZIO 4

$$f(x) = \begin{cases} x & se \ x \leq 0; \\ 2x & se \ x > 0; \end{cases}$$



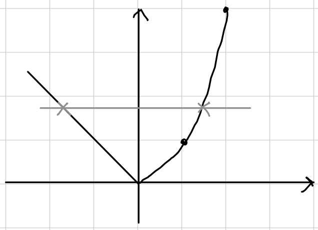
INIETTIVA ✓
INVERTIBILE ✓

$$g(x) = \begin{cases} x^2 & se \ x \leq 1; \\ x & se \ x > 1; \end{cases}$$



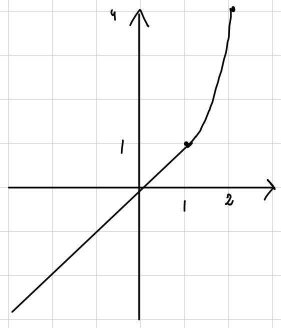
INIETTIVA X
INVERTIBILE X

$$h(x) = \begin{cases} x^2 & se \ x \geq 0; \\ -x & se \ x < 0; \end{cases}$$



INIETTIVA X
INVERTIBILE X

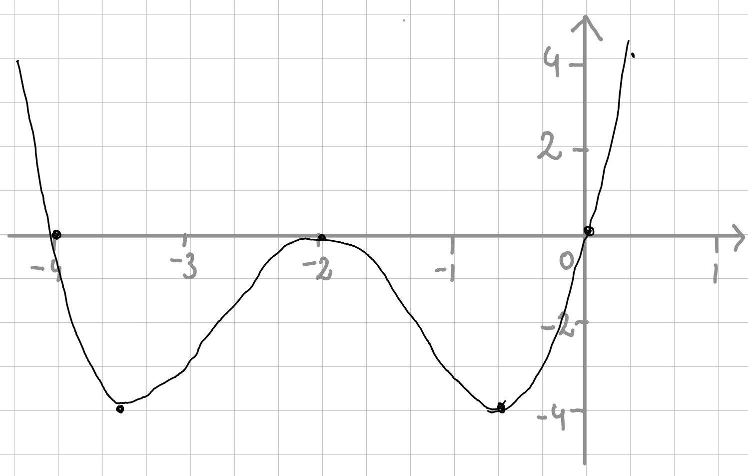
$$e(x) = \begin{cases} x^2 & se \ x \geq 1 \\ x & se \ x < 1 \end{cases}$$



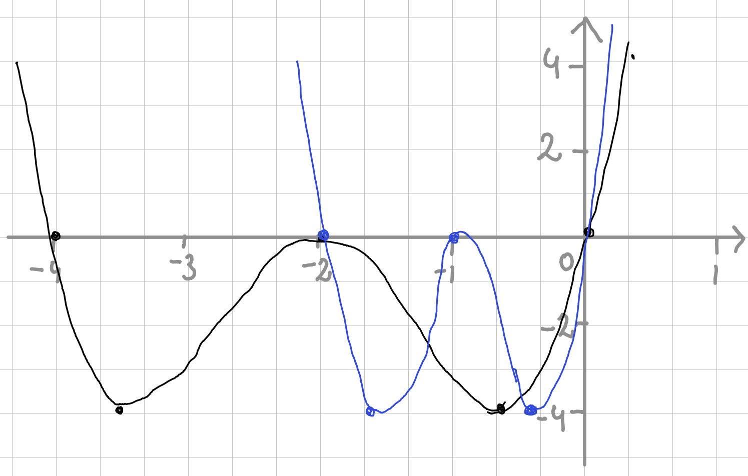
INIETTIVA ✓
INVERTIBILE ✓

ES 1 (dalle slides) - ESERCIZIO 5

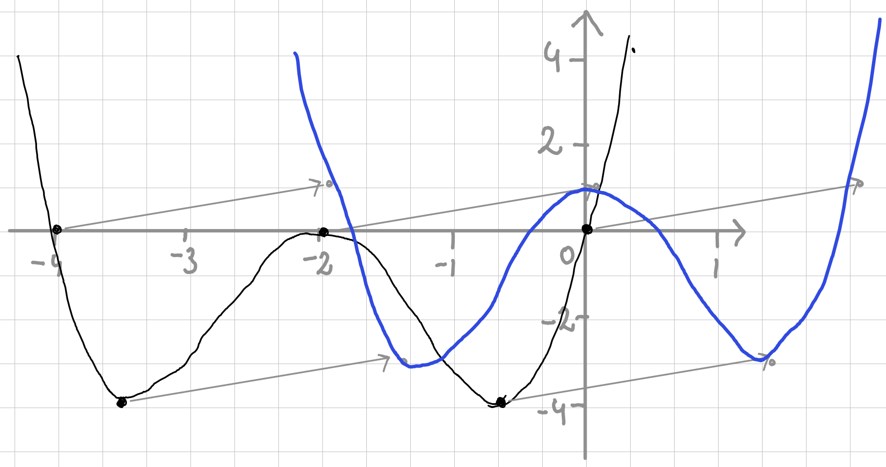
$y = f(x)$



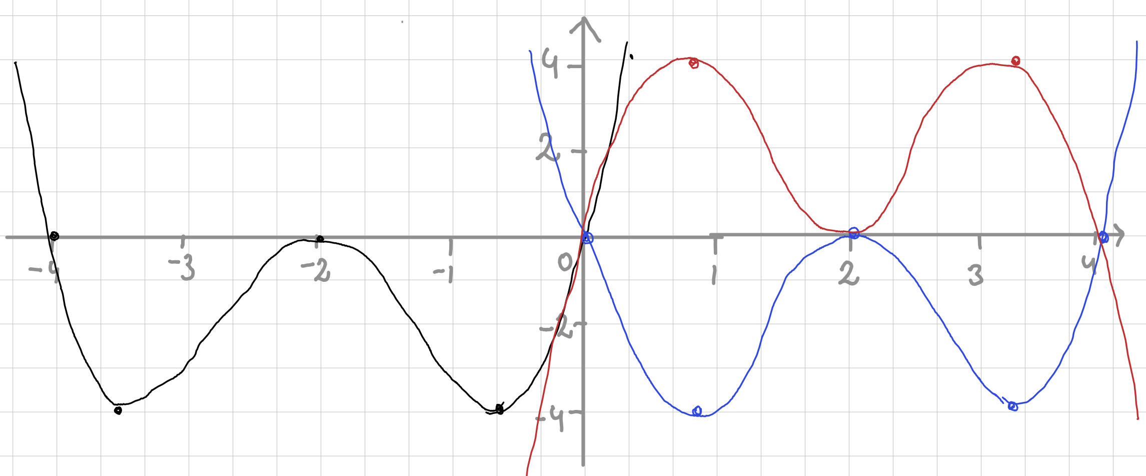
$y = f(2x)$ CONTRAZIONE ASSE X DI 2



$y = f(x-2)+1$ TRASLAZIONE DI $\vec{v}\,(2,1)$

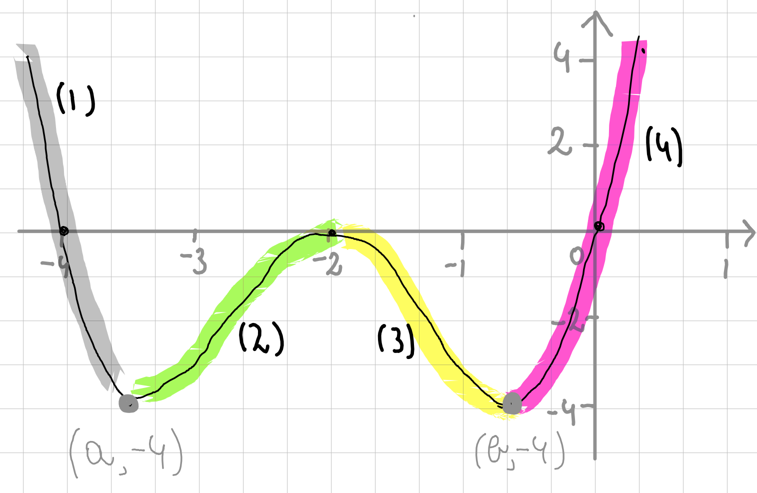


$y = -f(-x)$ I SIMMETRIA RISPETTO ALL'ASSE Y $f(-x)$
II SIMMETRIA RISPETTO ALL'ASSE X



$y = f^{-1}(x)$ SIMMETRIA RISPETTO ALLA BISETTRICE DEL 1° E 3° QUADRANTE $(y=x)$

Dobbiamo restringere il dominio a quattro possibili intervalli



CASO 4

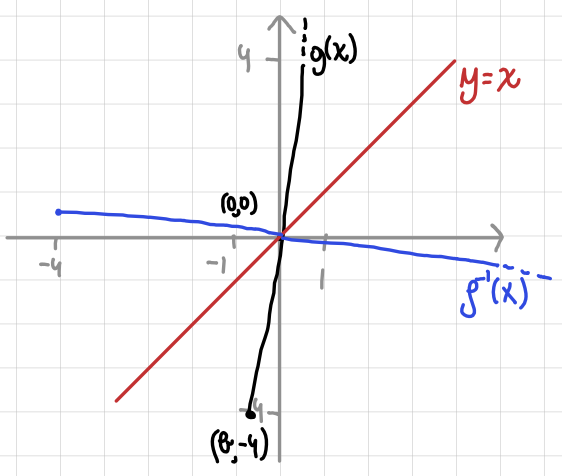
$g: [b, +\infty[\ \to [-4, +\infty[$

$x \mapsto g(x) = f(x)$

$f^{-1}: [-4, +\infty[\ \to [b, +\infty[$

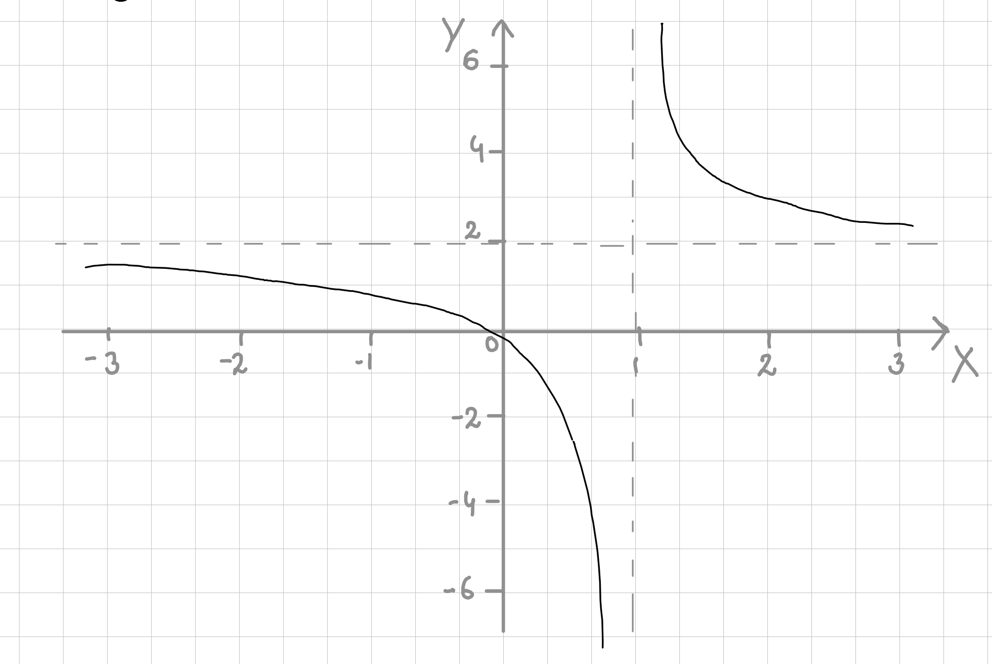
$x \mapsto f^{-1}(x)$

Usiamo le stesse unità per i due assi



ES 1 (dalle slides) - ESERCIZIO 6

$y = f(x)$ $\quad f: \mathbb{R} - \{1\} \to \mathbb{R}$

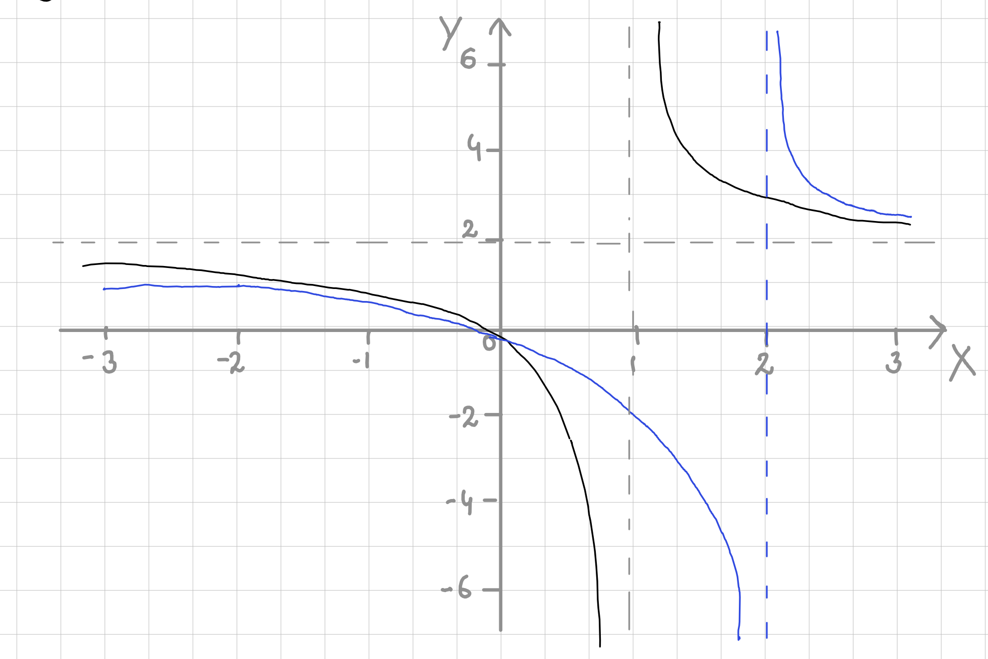


Anche se l'esercizio può essere risolto senza funzione analitica, dati gli asintoti $x=1$, $y=2$ e il punto $(0,0)$ possiamo risalire a

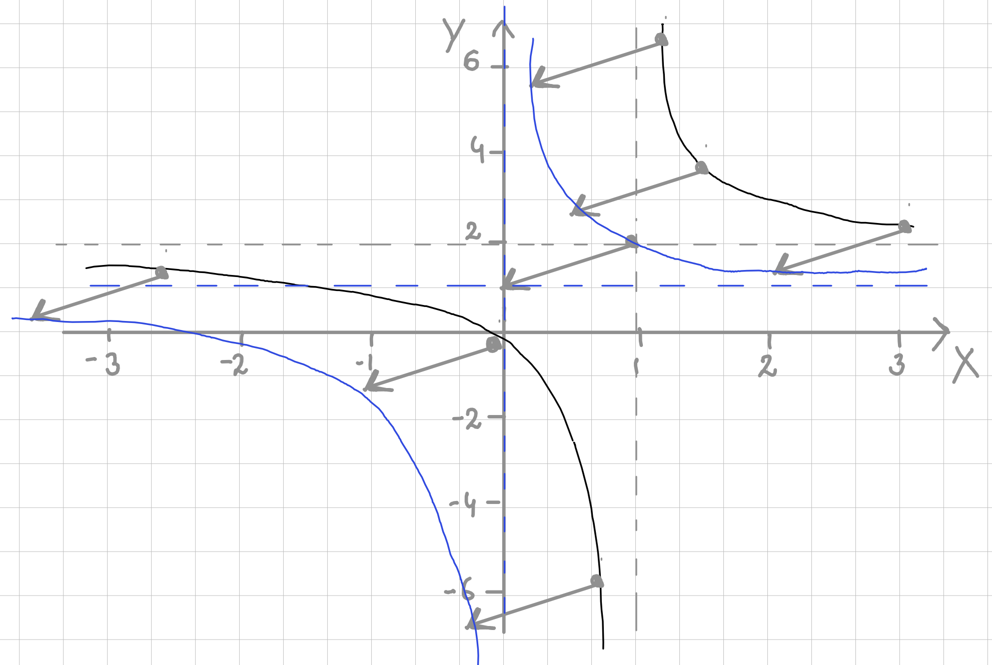
$$f(x) = \frac{2x}{x-1} \quad \text{(iperbole equilatera)}$$

da usare come controllo

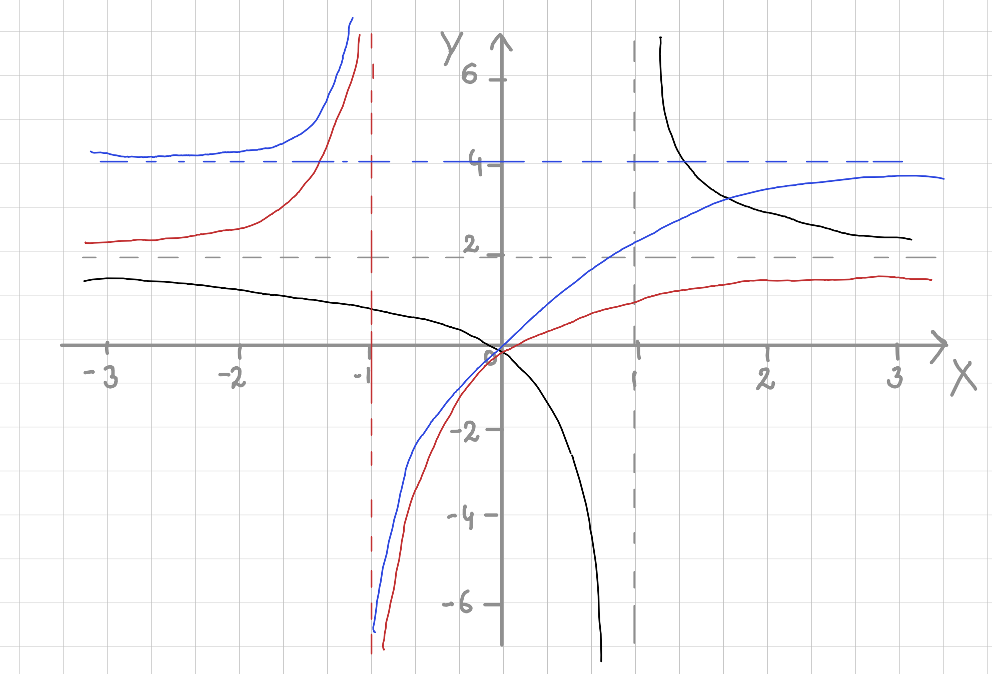
$y = f(x/2)$ DILATAZIONE ASSE X DI 2



$y = f(x+1) - 1$ TRASLAZIONE DI $\vec{v}(-1,-1)$



$y = 2f(-x)$ SIMMETRIA RISPETTO ALL'ASSE Y
DILATAZIONE DELL'ASSE Y DI 2



$y = f^{-1}(x)$ Indaghiamo se è
INIETTIVA ✓
SURIETTIVA ✗ restringiamo il codominio

$g: \,]-\infty, 1[\cup]1, +\infty[\to]-\infty, 2[\cup]2, +\infty[$

$x \mapsto g(x): f(x)$

$f^{-1}: \,]-\infty, 2[\cup]2, +\infty[\to]-\infty, 1[\cup]1, +\infty[$

$x \mapsto f^{-1}(x)$