Numeri complessi

$$x_1^2 - 2x_1 + 2 = 0 \longrightarrow (1+i)^2 - 2(1+i) + 2 = 0$$

$$1 + i^2 + 2i - 2 - 2i + 2 = 0$$

$$1 - 1 = 0 \;\checkmark$$

Insieme numeri complessi $= \mathbb{C} : \left\{ \underbrace{a+ib}_{\text{numeri complessi}} : \begin{array}{l} a,b \in \mathbb{R} \\ i^2 = -1 \end{array} \right\}$

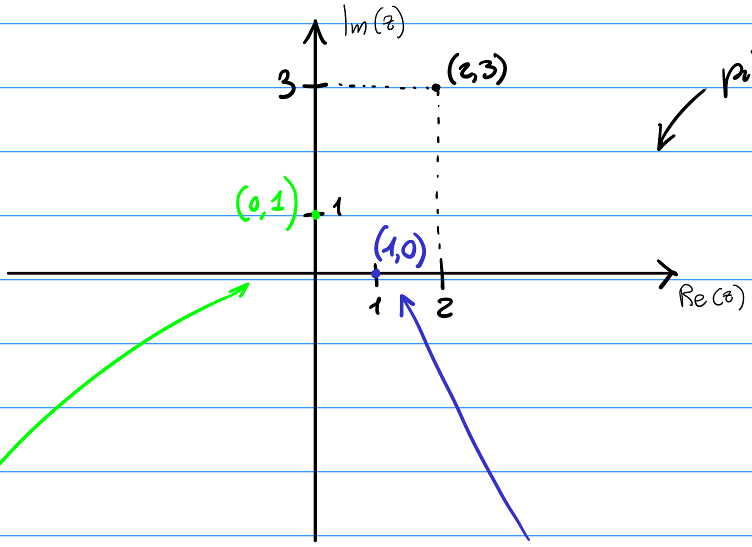
$$\mathbb{R} \subseteq \mathbb{C}$$

$z = a + ib$

$Re(z) = a$ parte reale, con $a, b \in \mathbb{R}$

$Im(z) = b$ parte immaginaria

$z(2,3) = (a,b)$



piano di Gauss

$z = i$

$Re(z) = 0 \qquad Im(z) = 1$

$z = 1$

$Re(z) = 1 \qquad Im(z) = 0$

Operazioni possibili:

• SOMMA

$z_1, z_2 \in \mathbb{C}$ $\qquad z_1 + z_2$ (immagine il piano di Gauss)

$$= (x_1+x_2)+i(y_1+y_2)$$

$z_1 = x_1 + iy_1$
$z_2 = x_2 + i y_2$

props:

commutativa ✓
associativa ✓
elem. neutro : $z_1 + 0 = 0 + z_1$ (coord $(0,0) \to$ origine)
opposto : $-z_1 = -a - ib$
$\quad \hookrightarrow z_1 + (-z_1) = 0$

• MOLTIPLICAZIONE

$z_1, z_2 \in \mathbb{C}$

$z_1 = x_1 + iy_1 \to 2+3i$
$z_2 = x_2 + iy_2 \to 1-i$

$$z_1 \cdot z_2 = (2+3i)(1-i) = 2-2i+3i-\underbrace{3i(i)}_{3} = 5+i$$

$$z_1 \cdot z_2 = (x_1+iy_1)(x_2+iy_2) = x_1x_2 + ix_1y_2 + iy_1x_2 + i^2 y_1y_2$$
$$= \underbrace{x_1x_2 - y_1y_2}_{Re(z_1 \cdot z_2)} + i\underbrace{(x_1y_2 + y_1x_2)}_{Im(z_1 \cdot z_2)}$$

props:

• commutativa ✓
• associativa ✓
• el. neutro: $1 \cdot z_1 = z_1 \cdot 1 = z_1$
• reciproco $\frac{1}{z_1} \cdot z_1 = 1$

$(\mathbb{C}, +, \cdot)$ si dice campo

Calcolare reciproco di un numero complesso!

$i \xrightarrow{rec} \frac{1}{i} = -i$ $\qquad z_1 = 1 - 5i$

$$\frac{1}{z_1} = \frac{1}{1-5i} = \frac{1}{1-5i} \cdot \frac{1+5i}{1+5i} = \frac{1+5i}{1-(5i)^2} = \frac{1+5i}{26}$$

DIVISIONE

$z_1 = 1 - 5i$

$z_2 = 1 + i$

$$\longrightarrow \frac{z_1}{z_2} = \frac{1-5i}{1+i} \cdot \frac{1-i}{1-i} = \frac{(1-5i)(1-i)}{1-(i)^2} = \frac{1-i-5i+5i^2}{2} = \frac{-4-6i}{2} = -2-3i$$

CONIUGATO

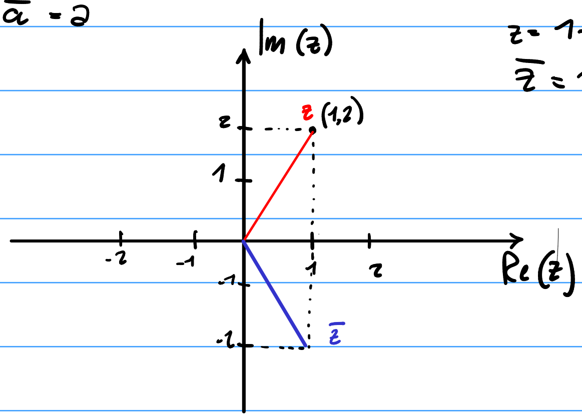
$z_1 = a + ib \; ; \quad \overline{z_1} = a - ib$

coniugato di z_1

$a \in \mathbb{R} \quad \overline{a} = a$

$z = 1 + 2i = (1,2)$

$\overline{z} = 1 - 2i = (1,-2)$

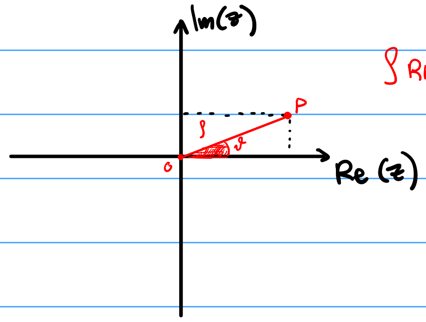


da dimostrare!

$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

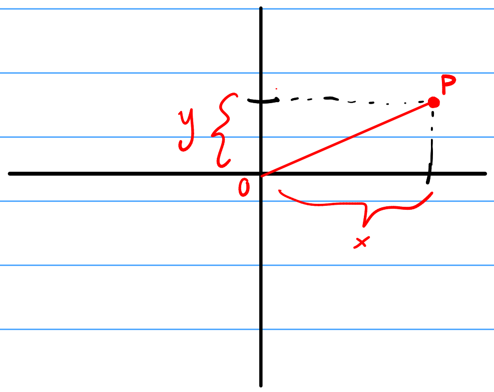
$\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

FORMA TRIGONOMETRICA (FORMA POLARE)



ρ RHO $\quad \rho = |z|$ - lunghezza segmento

ϑ theta $\rightarrow$ argomento di z $=$ angolo tra x e segmento OP



$$\rho = \sqrt{x^2+y^2}$$

$$\begin{cases} x = \rho\cos(\vartheta) \\ y = \rho\sin(\vartheta) \end{cases}$$ tutto per trovare ?

$$\begin{cases} \cos(\vartheta) = \dfrac{x}{\rho} \\ \sin(\vartheta) = \dfrac{y}{\rho} \end{cases}$$ trovare l'angolo che soddisfa le condizioni

inoltre $\operatorname{tg}(\vartheta) = \dfrac{y}{x} = \dfrac{\cancel{\rho}\sin(\vartheta)}{\cancel{\rho}\cos(\vartheta)}$

$\vartheta = \operatorname{arctg}\left(\dfrac{y}{x}\right)$ se x e $y > 0$

esempio

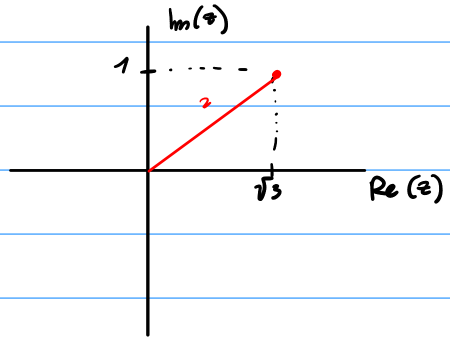
$$z = \sqrt{3} + i$$

$Re(\sqrt{3})$, $Im(1)$ $\qquad |z| = \rho = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{4} = 2$

cerchiamo

$$\begin{cases} \cos(\vartheta) = \dfrac{\sqrt{3}}{2} \\ \sin(\vartheta) = \dfrac{1}{2} \end{cases}$$

$$\vartheta = 30° = \frac{\pi}{6}$$



$0 \leq \vartheta < 2\pi$

forma trigonometrica $\sqrt{3} + i = \underbrace{2}_{\rho}\cos\left(\dfrac{\pi}{6}\right) + i\underbrace{2}_{\rho}\sin\left(\dfrac{\pi}{6}\right)$

FORMULA DI EULERO

$z = x + iy \mapsto$ rappresentazione algebrica

$= |z|\,(\cos(\vartheta) + i\sin(\vartheta))$

$= \rho\,(\cos(\vartheta) + i\sin(\vartheta))$ ↓ forma trigonometrica

$0 \leq \vartheta < 2\pi$

$\rho \geq 0$

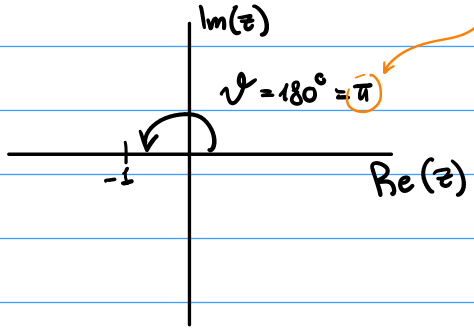
$$\begin{cases} X = \rho\cos(\vartheta) \\ Y = \rho\sin(\vartheta) \end{cases}$$

$$\rho(\cos(\vartheta)+i\sin(\vartheta)) = \underline{\underline{\rho e^{i\vartheta}}}$$

ecco cosa dice Eulero!

$z=-1 \rightarrow \rho=1 \quad \vartheta=\pi$

$-1 = e^{i\pi} \rightsquigarrow 0 = 1+e^{i\pi}$



$z=-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$ → FORMA POLARE? (PER CASA)

POTENZA

- $|z_1 \cdot z_2| = |z_1|\cdot|z_2|$ con $n \in \mathbb{N}$
- $|z_1^n| = |z_1|^n$
- $|z^n| = |z|^n e^{in\vartheta}$

$z_1 = \rho_1 e^{i\vartheta_1}$ dove $\rho_1 = |z_1|$

$z_2 = \rho_2 e^{i\vartheta_2}$ dove $\rho_2 = |z_2|$

$$z_1 z_2 = (\rho_1 e^{i\vartheta_1})(\rho_2 e^{i\vartheta_2}) = \rho_1\cdot\rho_2 e^{i\vartheta_1}\cdot e^{i\vartheta_2} = \underbrace{\rho_1\cdot\rho_2\cdot e^{i(\vartheta_1+\vartheta_2)}}$$

$|z_1 z_2| = \rho_1\cdot\rho_2 = |z_1|\cdot|z_2|$

$\mathrm{Arg}(z_1\cdot z_2) = \vartheta_1+\vartheta_2$ (angoli si sommano!)

- $|z^n| = |z|^n$

dimostrazioni formule potenza

$|z \cdot z| = |z^2| = |z|^2$ $\quad n \in \mathbb{N}$

- **FORMULA DI DE MOIVRE** (importante)

$z \in \mathbb{C}$ $\quad \rho = |z|$; $\vartheta = Arg(z)$

$$z^n = \rho^n \cdot e^{in\vartheta}$$

forma algebrica $\begin{cases} x = \rho\cos(\vartheta) \\ y = \rho\sin(\vartheta) \end{cases}$

$$z^n = \rho^n \cos(n\vartheta) + i\rho^n \sin(n\vartheta) = \rho^n(\cos(n\vartheta) + i\sin(n\vartheta))$$

DIMOSTRAZIONE

$z = \rho e^{i\vartheta}$ $\qquad z^n = (\rho e^{i\vartheta})^n = \rho^n e^{n i\vartheta}$

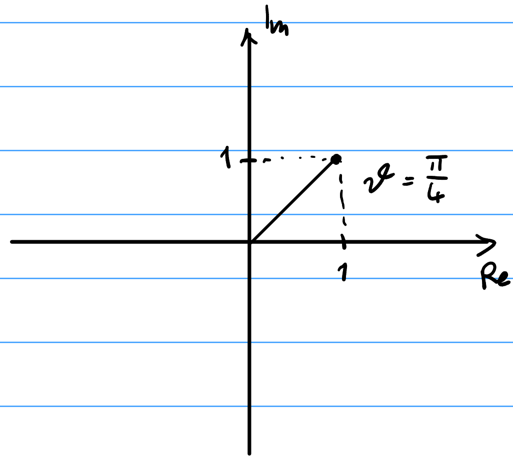
ESERCIZIO

$z = 1 + i$ $\qquad z^7 = (1+i)^7$

soluzione

$|z| = \sqrt{2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$Arg(z)$ $\qquad \begin{cases} x = \rho\cos(\vartheta) \\ y = \rho\sin(\vartheta) \end{cases}$



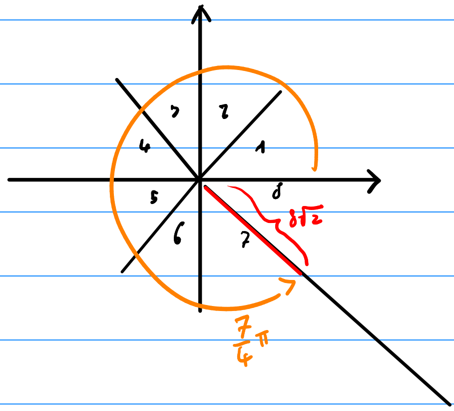
$$\begin{cases} \sqrt{2} \cdot \cos\left(\frac{\pi}{4}\right) = \sqrt{2} \cdot \frac{\sqrt{2}}{2} = 1 \\ \sqrt{2} \cdot \sin\left(\frac{\pi}{4}\right) = \sqrt{2} \cdot \frac{\sqrt{2}}{2} = 1 \end{cases}$$

$$z = \rho e^{i\vartheta} = \sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$z = \left(\sqrt{2} \cdot e^{i\frac{\pi}{4}}\right)^7 = \sqrt{2}^7 \cdot e^{i\frac{7}{4}\pi}$$

$$= \sqrt{2 \cdot 2^6} \cdot \text{"}$$

$$= 8\sqrt{2}\; e^{i\frac{7}{4}\pi}$$



$x, y?$ $\begin{cases} x = \rho\cos(\vartheta) \\ y = \rho\sin(\vartheta) \end{cases}$

$$x = 8\sqrt{2} \cdot \cos\left(\tfrac{7}{4}\pi\right) = 8\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 8$$

$$y = 8\sqrt{2} \cdot \sin\left(\tfrac{7}{4}\pi\right) = 8\sqrt{2} \cdot -\frac{\sqrt{2}}{2} = -8$$

quindi $(1+i)^7 = 8 - 8i$

Trova la soluzione a:

forma algebrica $\underbrace{1 + i - \frac{i}{1-2i}}_{\text{no pot. trova subito Re e Im}}$ e $\underbrace{(1-i)^{11}}_{\text{potenza, meglio forma polare}}$

RADICI N-ESIME DI $z \in \mathbb{C}$

dato $w \in \mathbb{C}$ diremo che $z \in \mathbb{C}$ è radice n-esima di w se $z^n = w$

e scriveremo $z = \sqrt[n]{w}$ $(n \in \mathbb{N})$

$w = i$ $\qquad$ $z^2 = i$ $\qquad$ $z = \sqrt{i}$

① Approccio geometrico

$z = x + iy$; $z^2 = i$ $\qquad$ $x, y \in \mathbb{R}$

$$(x+iy)^2 = i$$

stessa parte imm. che reale per essere uguali

$$x^2 - y^2 + 2xiy = i$$

$$\begin{cases} x^2 - y^2 = 0 \\ 2xy = 1 \end{cases}$$

$$(x-y)(x+y) = 0$$

① $(x-y) = 0 \to x = y$

② $(x+y) = 0 \to x = -y$

$$\begin{cases} x = y \\ 2xy = 1 \end{cases} \qquad \begin{cases} x = \frac{1}{\sqrt{2}} \\ y = \frac{1}{\sqrt{2}} \end{cases} \vee \begin{cases} x = -\frac{1}{\sqrt{2}} \\ y = -\frac{1}{\sqrt{2}} \end{cases}$$

$z_1 = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ $\qquad$ $z_2 = -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$ soluzioni!!

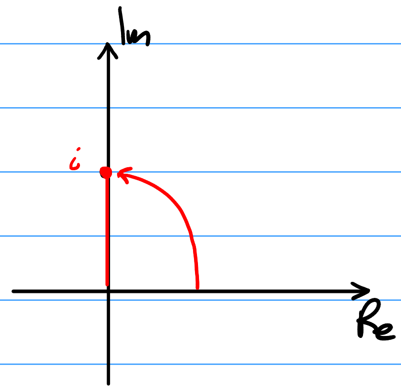
perché

$$\begin{cases} x = -y \\ 2xy = 1 \end{cases} \qquad \begin{cases} x = -y \\ y^2 = -\frac{1}{2} \end{cases}$$ (non esiste soluzione!!)

② Soluzione in forma polare

$z = \sqrt{i}$; $z^2 = i$

$z = \rho e^{i\theta}$ $\qquad$ $i = 1 e^{i\frac{\pi}{2}}$



Incognite

$$\rho \geq 0 \quad , \quad \vartheta \in [0, 2\pi[$$

$$z^2 = i$$

$$(\rho e^{i\vartheta})^2 = e^{i\frac{\pi}{2}} \rightarrow \rho^2 e^{i2\vartheta} = 1 e^{i\frac{\pi}{2}}$$

- $\rho^2 = 1 \rightarrow \rho = \pm 1 \rightarrow \rho = 1$
- $2\vartheta = \frac{\pi}{2} + 2k\pi$ (importante la periodicità) con $k \in \mathbb{N}$

$$\vartheta = \frac{\pi}{4} + k\pi$$

$k = 0 \rightarrow \vartheta = \frac{\pi}{4}$ 1 soluzione $\hookrightarrow z_1 = e^{i\frac{\pi}{4}}$

$k = 1 \rightarrow \vartheta = \frac{5\pi}{4}$ 2 soluzione $\hookrightarrow z_2 = e^{i\frac{5}{4}\pi}$

$k = 2 \rightarrow \vartheta = \frac{\pi}{4} + 2\pi$!! non va bene, perché

$z_1 = e^{i\frac{\pi}{4}} \quad z_2 = e^{i\frac{5}{4}\pi}$

$\hookrightarrow z_1 = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

$z_2 = -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

TEOREMA

data l'equazione $z^n = w$, $w \in \mathbb{C}$, $n \in \mathbb{N}$ con $n > 1$

ci sono esattamente n soluzioni distinte in $\mathbb{C}$, $z_0, \dots, z_{n-1} \in \mathbb{C}$

$$z_n = \sqrt[n]{\rho}\, e^{i\vartheta_k} \text{ con } k = 0, \dots, n-1$$

$|z_k| = \sqrt[n]{\rho}$ $\quad \hookrightarrow \operatorname{Arg}(z_k) = \vartheta_k = \frac{\vartheta}{n} + \frac{2k\pi}{n}$ dove $|w| = \rho$ $\operatorname{Arg}(w) = \vartheta$

ESEMPI (PARTE SOPRA)

1) $z^4 - 1 = 0 \longrightarrow z^4 = 1$

$w = 1$; $\rho = |w| = 1$; $\vartheta = \mathrm{Arg}(w) = 0$

$w = 1\, e^{i \cdot 0} = 1$

$z_k = \sqrt[k]{\rho}\; e^{i \vartheta_k}$ $\quad k = 0, \dots, \overset{n-1}{\overset{\uparrow}{3}}$

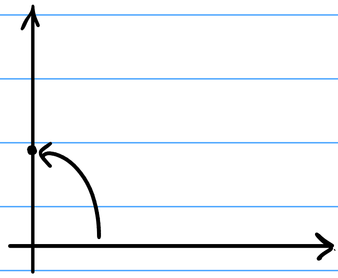
$\vartheta_k = \frac{\vartheta}{4} + \frac{2k\pi}{4} = \frac{\pi}{2} \cdot k \quad k = 0, \dots, 3$

$z_1 = 1 \cdot e^{i0} = 1$

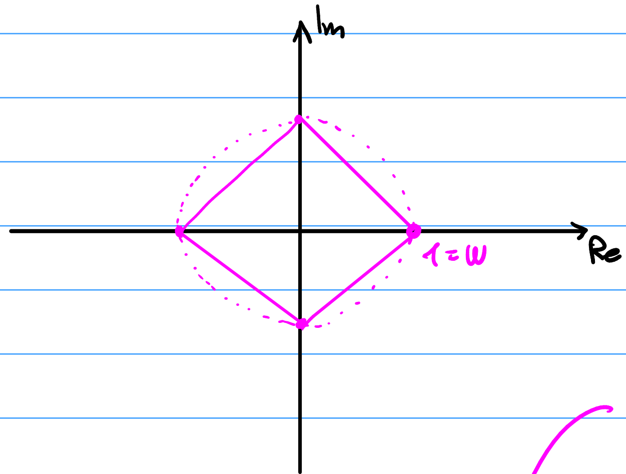
$z_2 = 1 \cdot e^{i\frac{\pi}{2}} = i$

$z_3 = 1 \cdot e^{i2\frac{\pi}{2}} = e^{i\pi} = -1$

$z_4 = 1 \cdot e^{i3\frac{\pi}{2}} = e^{i\frac{3}{2}\pi} = -i$



$z^4 - 1 = 0 \xrightarrow{\text{sol.}} 1, i, -1, -i \qquad (-i)^4 = \left((-i)^2\right)^2 = (-1)^2 = 1$

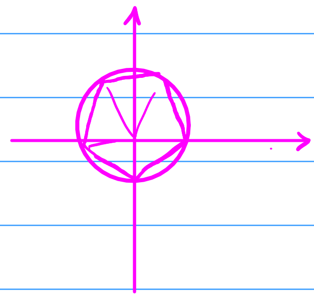


$|z_k| = \sqrt[n]{\rho}$

$\vartheta_k = \frac{\vartheta}{n} + \frac{2k\pi}{n}$

$\vartheta_0 = \frac{\vartheta}{n}$

$\vartheta_1 = \frac{\vartheta}{n} + \frac{2\pi}{n}$



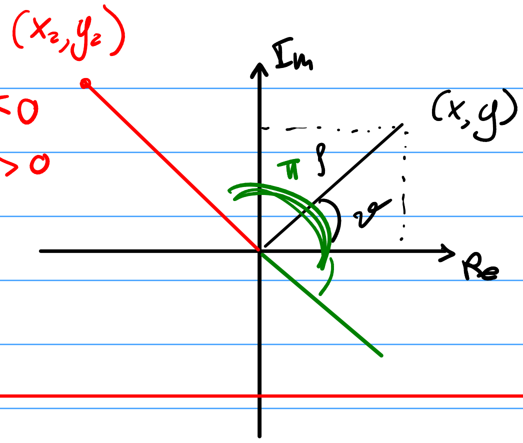
$x_2 < 0$

$y_2 > 0$

ϑ_2 ?

$90 < \vartheta_2 < 180$

$\frac{\pi}{2} < \vartheta_2 < \pi$



$\rho \geq 0$

$0 \leq \vartheta < 2\pi$

$\vartheta_2 = \mathrm{arctg}\left(\frac{y}{x}\right)$

no, non funzionerebbe

$\vartheta = \mathrm{arctg}\left(\frac{y}{x}\right) \longrightarrow 0 < \vartheta < 90° \left(\frac{\pi}{2}\right)$

$\mathrm{arctg} : \mathbb{R} \longrightarrow \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

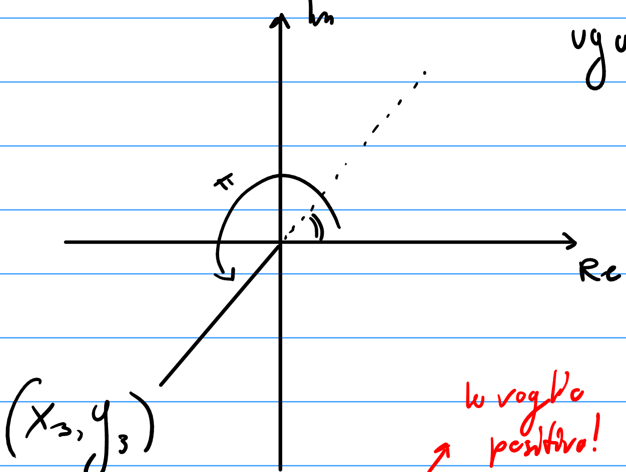
$k \longrightarrow \vartheta = \mathrm{arctg}(k) : \mathrm{tg}(\vartheta) = k$

se $x_2 < 0$

aggiungo π

$\mathrm{arctg}\left(\frac{y}{x}\right) + \pi$

" uguali "



$\vartheta_3 = \mathrm{arctg}\left(\frac{y_3}{x_3}\right) + \pi$

$y_3 < 0$, $x_3 < 0$

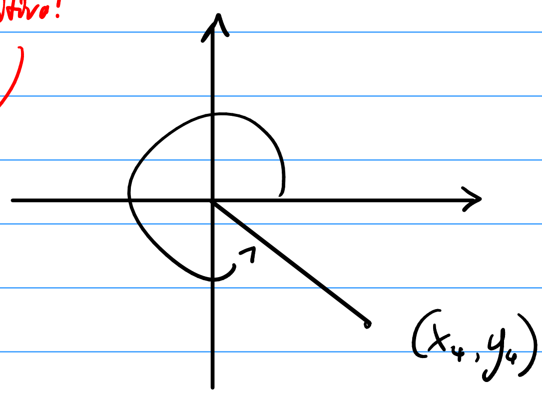
lo voglio positivo!

$\left]-\frac{\pi}{2}, 0\right[$

< 0

$\vartheta_4 = \mathrm{arctg}\left(\frac{y_4}{x_4}\right) + 2\pi$

$x_4 > 0$ e $y_4 < 0$



complesso

$Im = 0 \quad Re > 0 \quad (0)$ $\qquad$ $Im > 0 \quad Re = 0 \quad \left(\frac{\pi}{2}\right)$

$Im = 0 \quad Re < 0 \quad (\pi)$ $\qquad$ $Im < 0 \quad Re = 0 \quad \left(\frac{3\pi}{2}\right)$

ESERCIZIO

$w = 1 + i$

$z \in \mathbb{C} \qquad z^3 = 1 + i \qquad z = \sqrt[3]{1+i}$

$z^n = w \longrightarrow n$ soluzioni $, \; z_0, z_1, \dots, z_{n-1}$

$|z_k| = \sqrt[n]{|w|}$

$$\vartheta_k = \frac{Arg(w)}{n} + \frac{2\pi}{n}k \qquad k = 0, \dots, n-1$$

$|w| = \sqrt{1+1} = \sqrt{2}$

$$\vartheta_w : \begin{cases} x = \rho \cos(\vartheta) \\ y = \rho \sin(\vartheta) \end{cases} \quad \begin{cases} \cos(\vartheta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin(\vartheta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases}$$

$= \vartheta_w = \frac{\pi}{4}$

$$\vartheta_n = \frac{\vartheta_w}{3} + \frac{2\pi}{3}k \qquad k = 0, 1, 2$$

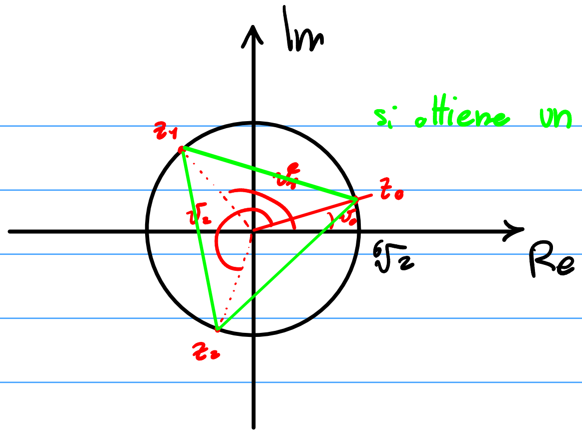
$\vartheta_0 = \frac{\pi}{12} = 15°$

$\vartheta_1 = \frac{\pi}{12} + \frac{2}{3}\pi = \frac{3}{4}\pi$

$\vartheta_2 = \frac{\pi}{12} + \frac{4}{3}\pi$

radici cubiche

$$|z_i| = \sqrt[3]{|w|} = \sqrt[3]{\sqrt{2}} = \sqrt[6]{2} = 2^{\frac{1}{6}}$$



si ottiene un triangolo equilatero

Teorema fondamentale dell'algebra

un eq polinomiale di grado n in campo complesso ha esattamente n soluzioni contate con la loro molteplicità

$$z^3=0 \quad = (3 \text{ soluzioni})$$

$$\underbrace{z\cdot 0\cdot z \qquad 0\cdot z\cdot z \qquad z\cdot z\cdot 0}$$

$$z^2-(4+i)z+4+2i=0$$

1) $$z_{1,2}=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{4+i\pm\sqrt{(4+i)^2-(4+2i)\cdot 4}}{2}$$

2) $z=x+iy \qquad x,y\in\mathbb{R}$

$(x+iy)^2-(4+i)(x+iy)+4+2i=0$

$x^2-y^2+2xyi-4x-4iy-ix+y+4+2i=0$

$x^2-y^2-4x+y+4+i(2yx-4y-x+2)=0$

$$\begin{cases} x^2-y^2-4x+y+4=0 \\ 2yx-4y-x+2=0 \end{cases}$$

$2y(x-2)-(x-2)=(x-2)(2y-1)=0$

1) $x-2=0 \quad x=2$ $4-y^2-8+y+4=0$	2) $y=\frac{1}{2}$	una sola dà soluzioni!!

EQ. Algebriche (polinomiali)

EQ. NON Algebriche

problema

$$z^2 + i\,\mathrm{Im}(z) + 2\bar{z} = 0$$

$$z = x + iy \qquad x, y \in \mathbb{R}$$

$$(x+iy)^2 + iy + 2(x - iy) = 0$$

$$x^2 - y^2 + 2xyi + iy + 2x - 2iy = 0$$

$$x^2 - y^2 + 2x + i(2xy + y - 2y) = 0$$

$$\begin{cases} x^2 - y^2 + 2x = 0 \\ 2xy - y = 0 \end{cases} \qquad y(2x-1) = 0$$

① $y = 0$ $\vee$ (oppure) ② $x = \frac{1}{2}$

①	②
$x^2 + 2x = 0$	$\frac{1}{4} - y^2 + 1 = 0$
$x(x+2) = 0$	$\frac{5}{4} - y^2 = 0$
$x = 0$	$y = \pm \frac{\sqrt{5}}{2}$
$x = -2$	

1.1 $z_1 = 0$

1.2 $z_1 = -2$

2.1 $= \frac{1}{2} + \frac{\sqrt{5}}{2} i$

2.2 $= \frac{1}{2} - \frac{\sqrt{5}}{2} i$

Vettori geometrici e sistemi lineari

$w \in \mathbb{C}$; $z^n = w$ → n radici complesse distinte

$z_0, \dots, z_{n-1}$

$\vartheta = Arg(w)$; $\rho = |w|$

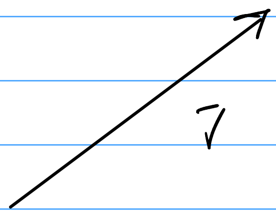
$|z_k| = \sqrt[n]{|w|}$ $\quad k = 0, \dots, n-1$

$Arg(z_k) = \vartheta_k = \frac{\vartheta}{n} + \frac{2k\pi}{n}$

Vettore geometrico

NOTAZIONI: $\vec{v}$; $\bar{v}$; $\mathbf{v}$; $\underline{v}$

(→ SCALARE): $\alpha, a, b, \dots\dots$

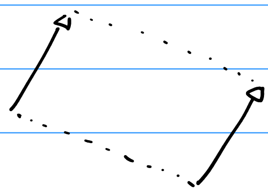


- intensità (o modulo del vettore = lunghezza) ≥ 0 sempre
- direzione (tramite il verso)

VETTORI EQUIPOLLENTI

↓

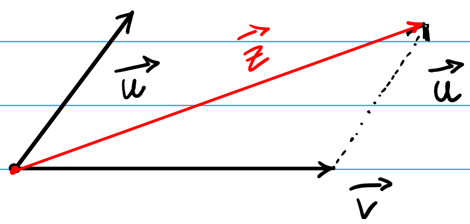
VETTORE GEOMETRICO (rappresenta tutti i vettori a lui equipollenti)



stesse caratteristiche ma diversa applicazione

Operazioni tra vettori geometrici (sul piano euclideo, 2D) (A_2)

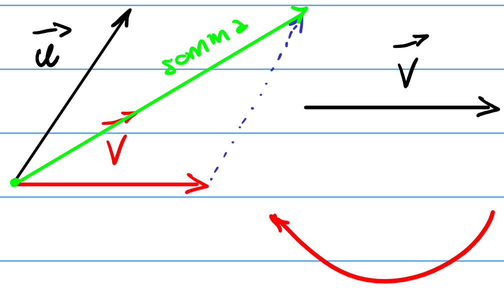
1) SOMMA



$\vec{z} = \vec{u} + \vec{v}$

(metodo punta-coda o parallelogramma)

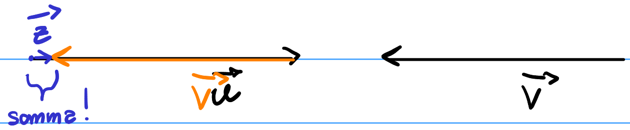
oppure



somma?

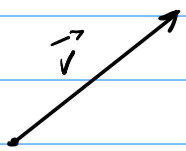
si trasla dove vogliamo!

oppure



MOLTIPLICAZIONE (SCALARE-VETTORE)

$\vec{v}$; $\lambda \in \mathbb{R}$ scalare



$\vec{z} = \lambda \vec{v}$

• $\vec{z}$ ha stessa direzione di $\vec{v}$

modulo sempre >0

• $|\vec{z}| = |\lambda| \cdot |\vec{v}|$

quindi

$\lambda > 1$: dilatazione

$0 < \lambda < 1$: contrazione

PROPRIETÁ SOMMA

• COMMUTATIVA ✓

• ASSOCIATIVA ✓ (difficile da dimostrare)

• EL. NEUTRO $\vec{v} = \vec{0}$ / vettore nullo

• OPPOSTO $\vec{u} \rightarrow -\vec{u}$

PROPRIETÁ SCALARE

• EL. NEUTRO $\lambda = 1$ $(\bar{z} = 1 \cdot \bar{v} = \bar{v})$

• COMMUTATIVA

• ASSOCIATIVA $\alpha, \beta \in \mathbb{R}$

$\bar{z} = (\alpha\beta) \cdot \bar{v}$

$= \alpha \cdot (\bar{v}\beta)$

.... (difficile da dimostrare)

• verso cambia se $\lambda < 0$

se $\lambda > 0$ rimane lo stesso

• se $\lambda = 0$ $= \vec{0}$ (vettore nullo)

(intensità nulla e verso indefinito)

ENTRAMBE HANNO PROPRIETÁ DISTRIBUTIVA

① $\lambda \cdot (\bar{u} + \bar{v}) = \lambda \bar{u} + \lambda \bar{v}$ $\forall \lambda \in \mathbb{R}$

② $(\lambda \mu) \bar{v} = \lambda \bar{v} + \lambda \bar{v}$ $\forall \lambda, \mu \in \mathbb{R}$

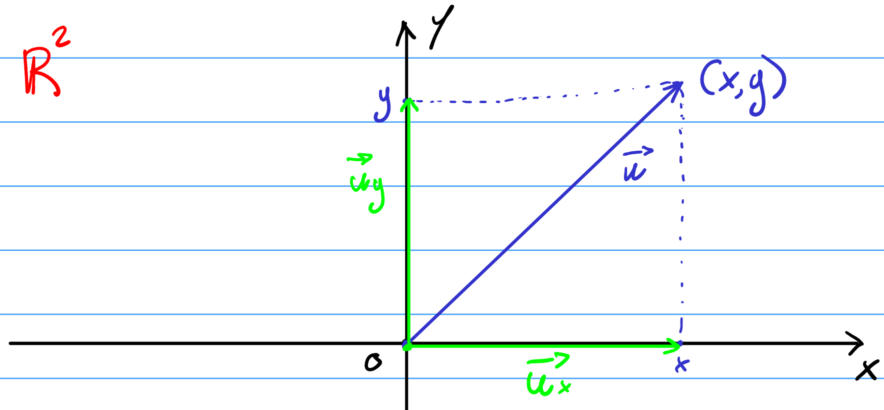
mu

non sono facili da dimostrare

$\forall \bar{u}, \bar{v}$ vettori geometrici

Piano cartesiano $\mathbb{R}^2$

O = origine



$$\mathbb{R}^2 = \{(x,y) : \forall x,y \in \mathbb{R}\}$$

corrispondenza tra spazio vettori geometrici e il piano cartesiano $\mathbb{R}^2$

$$\vec{V} = (V_x, V_y) \; ; \; \vec{u} = (u_x, u_y)$$

somma $\quad \vec{z} = \vec{V} + \vec{u} = (V_x + u_x \, , \, V_y + u_y)$

scalare $\quad \lambda \cdot \vec{V} = |\lambda| \cdot |\vec{V}| = (\lambda V_x, \lambda V_y) \qquad \vec{o} = (0,0)$

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2}$$

SISTEMI LINEARI