ES 1

Siano A, B, C tre insiemi. Dimostrare che

$$\underbrace{(A\cap B = A\cap C)}_{\mathrm{I}} \wedge \underbrace{(A\cup B = A\cup C)}_{\mathrm{II}} \Longrightarrow B=C$$

$\underline{\text{TH}|}$ $C=B$, che richiede $\underline{B\subseteq C}$ e $B\supseteq C$,

$\Longrightarrow$ $\underline{B\subseteq C}$ ossia $\forall x\in B \Longrightarrow x\in C$

- $\forall x\in B \Longrightarrow x\in A\cup B$, poiché $A\cup B = \{x \mid x\in A \vee x\in B\}$
- $\forall x\in B \Longrightarrow x\in A\cup C$, per ipotesi II

quindi $\forall x\in B \Longrightarrow \underbrace{x\in A}_{(b)} \vee \underbrace{x\in C}_{(a)}$

(a) $\forall x\in B \Longrightarrow x\in C$ ✓

(b) $\forall x\in B \Longrightarrow x\in A$

$\Longrightarrow x\in A\cap B$ $\quad A\cap B = \{x \mid x\in A \wedge x\in B\}$

$\Longrightarrow x\in A\cap C$ per ipotesi I

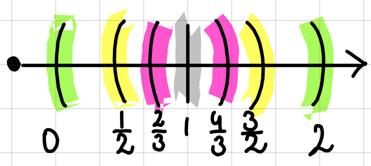
$\Longrightarrow x\in C$ ✓

$\Longrightarrow$ $\underline{B\supseteq C}$ stesso ragionamento poiché intersezione e unione sono commutative e la ipotesi é simmetrica rispetto a B e C ✓

ES 2

(a) $$\bigcup_{n=1}^{+\infty} \left[1 - \overset{\frac{n-1}{n}}{\frac{1}{n}} \, , \, 1 + \overset{\frac{n+1}{n}}{\frac{1}{n}} \right] =$$

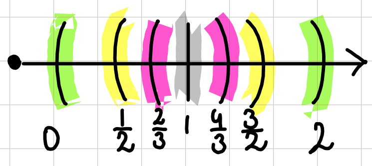
$$= \underbrace{[0,2]}_{n=1} \cup \underbrace{\left[\frac{1}{2},\frac{3}{2}\right]}_{n=2} \cup \underbrace{\left[\frac{2}{3},\frac{4}{3}\right]}_{n=3} \cup \cdots \cup \underbrace{[1^-,1^+]}_{n=+\infty}$$



$$= [0,2]$$

(b) $$\bigcap_{n=1}^{+\infty} \left[1 - \overset{\frac{n-1}{n}}{\frac{1}{n}} \, , \, 1 + \overset{\frac{n+1}{n}}{\frac{1}{n}} \right] =$$

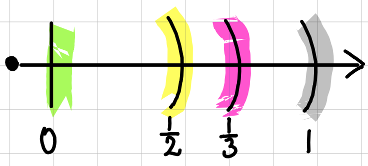
$$= \underbrace{[0,2]}_{n=1} \cap \underbrace{\left[\frac{1}{2},\frac{3}{2}\right]}_{n=2} \cap \underbrace{\left[\frac{2}{3},\frac{4}{3}\right]}_{n=3} \cap \cdots \cap \underbrace{[1^-,1^+]}_{n=+\infty}$$



$$= \{1\}$$ TEOREMA DI CARNOT

(c) $$\bigcup_{n=1}^{+\infty} \left[0 \, , \, 1 - \overset{\frac{n-1}{n}}{\frac{1}{n}} \right] =$$

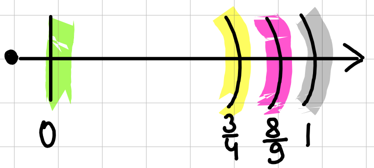
$$= \underbrace{[0,0]}_{n=1} \cup \underbrace{\left[0,\frac{1}{2}\right]}_{n=2} \cup \underbrace{\left[0,\frac{2}{3}\right]}_{n=3} \cup \cdots \cup \underbrace{[0,1^-]}_{n=+\infty}$$



$$= [0,1[$$

(d) $$\bigcup_{n=1}^{+\infty} \left[0 \, , \, 1 - \overset{\frac{n^2-1}{n^2}}{\frac{1}{n^2}} \right] =$$

$$= \underbrace{[0,0]}_{n=1} \cup \underbrace{\left[0,\frac{3}{4}\right]}_{n=2} \cup \underbrace{\left[0,\frac{8}{9}\right]}_{n=3} \cup \cdots \cup \underbrace{[0,1^-]}_{n=+\infty}$$



simile a (c) ma cresce più velocemente verso 1

$$= [0,1[$$

ES 3

(a) $E = \left\{ \frac{n}{n+1}, n \in \mathbb{N} \right\} = \left\{ 1 - \frac{1}{n+1}, n \in \mathbb{N} \right\}$ $\quad \frac{n+1-1}{n+1}$

$= \left\{ 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, 1^- \right\}$

- inf e sup $\quad \underset{b}{\overset{\min}{\max}} \{ b \in \mathbb{R} \mid \forall x \in E : b \le x \}$ $\quad x \le b$

  0 $\quad$ 1

- min e max $\quad b \in E \mid \forall x \in E : b \le x$ $\quad x \le b$

  0 $\quad \nexists b \in E$

- punti interni $\quad \{ x_0 \in E \mid \exists \delta > 0 : B(x_0, \delta[\in E \}$

  $\mathring{E} = \emptyset$ $\quad$ nessun punto interno

- punti di accumulazione $\quad x_0 \in \mathbb{R} \mid$ ogni intorno di x_0 cadono infiniti punti di E

  1

- punti isolati $\quad$ punti non di accumulazione

  E $\quad$ tutti i punti

- punti di frontiera $\quad x_0 \mid x_0$ né INTERNO né ESTERNO

  $\mathcal{F}(E) = E \cup \{1\}$ $\quad$ NON E' INTERNO AD $\mathbb{R} \setminus E$

- aperto o chiuso $\quad$ APERTO $\quad A = A^\circ$

  CHIUSO $\quad A^c$ è APERTO

  NE' APERTO NE' CHIUSO

(b) $E =]-1, 5]$

- inf e sup

  $-1 \quad 5$

- min e max

  $\nexists x \in E \quad 5$

- punti interni

  $\mathring{E} =]-1, 5[$

- punti di accumulazione

  E tutti

- punti isolati

  $\nexists x \in E$

- punti di frontiera

  $\mathcal{F}(E) = \{5, -1\}$

- aperto o chiuso

  NE' APERTO NE' CHIUSO