Numeri complessi

$$x_1^2 - 2x_1 + 2 = 0 \longrightarrow (1+i)^2 - 2(1+i) + 2 = 0$$

$$\| $$

$$1 + i^2 + 2i - 2 - 2i + 2 = 0$$

$$\|$$

$$1 - 1 = 0 \quad \checkmark$$

Insieme numeri complessi $= \mathbb{C} : \left\{ \underbrace{a+ib}_{\text{numeri complessi}} : \begin{array}{l} a,b \in \mathbb{R} \\ i^2 = -1 \end{array} \right\}$

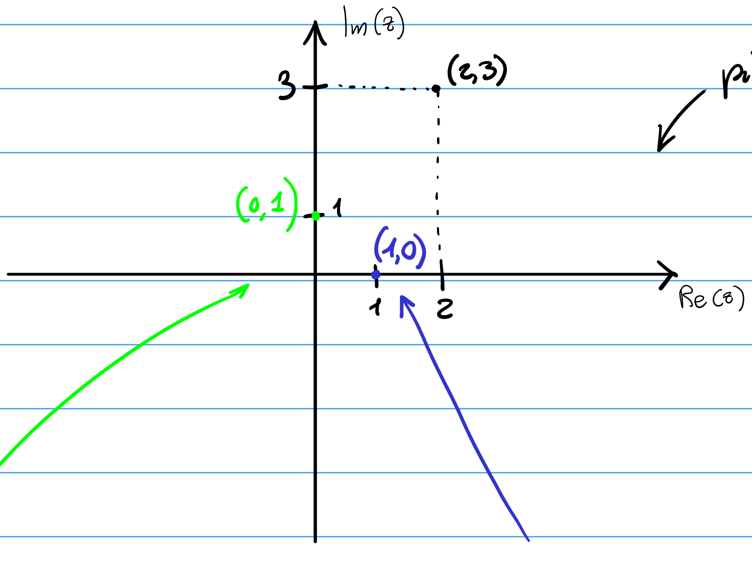
$$\mathbb{R} \subset \mathbb{C}$$

$z = a + ib$

$Re(z) = a$ parte reale — con $a,b \in \mathbb{R}$

$Im(z) = b$ parte immaginaria

$z(2,3) = (a,b)$



piano di Gauss

$z = i$

$Re(z) = 0 \qquad Im(z) = 1$

$z = 1$

$Re(z) = 1 \qquad Im(z) = 0$

Operazioni possibili:

- SOMMA

$z_1, z_2 \in \mathbb{C}$ $\qquad z_1 + z_2$ (immagine il piano di Gauss)

$z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

$= (x_1 + x_2) + i(y_1 + y_2)$

props:

commutativa ✓

associativa ✓

elem. neutro : $z_1 + 0 = 0 + z_1$ (coord $(0,0)$ -> origine)

opposto : $-z_1 = -a - ib$

↳ $z_1 + (-z_1) = 0$

- MOLTIPLICAZIONE

$z_1, z_2 \in \mathbb{C}$

$z_1 = x_1 + iy_1 \rightarrow 2+3i$

$z_2 = x_2 + iy_2 \rightarrow 1-i$

$z_1 \cdot z_2 = (2+3i)(1-i) = 2 - 2i + 3i \underbrace{-3i(i)}_{3}$

$= 5 + i$

$z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1x_2 + ix_1y_2 + iy_1x_2 + i^2 y_1y_2$

$= \underbrace{x_1x_2 - y_1y_2}_{Re(z_1 \cdot z_2)} + i\underbrace{(x_1y_2 + y_1x_2)}_{Im(z_1 \cdot z_2)}$

props:

- commutativa ✓
- associativa ✓
- el. neutro: $1 \cdot z_1 = z_1 \cdot 1 = z_1$
- reciproco $\frac{1}{z_1} \cdot z_1 = 1$

$(\mathbb{C}, +, \cdot)$ si dice campo

Calcolare reciproco di un numero complesso!

$i \xrightarrow{rec} \frac{1}{i} = -i$ $z_1 = 1 - 5i$

$$\frac{1}{z_1} = \frac{1}{1-5i} = \frac{1}{1-5i} \cdot \frac{1+5i}{1+5i} = \frac{1+5i}{1-(5i)^2} = \frac{1+5i}{26}$$

DIVISIONE

$z_1 = 1 - 5i$

$z_2 = 1 + i$

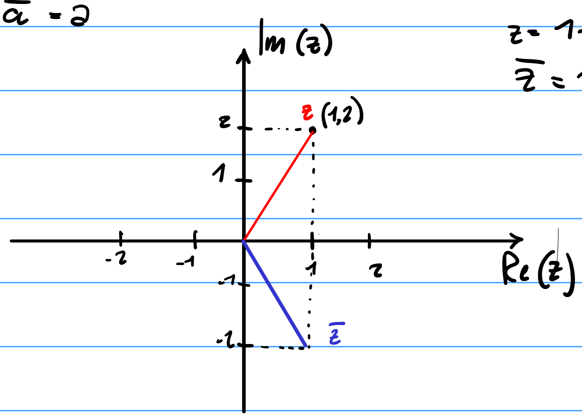
$$\longrightarrow \frac{z_1}{z_2} = \frac{1-5i}{1+i} \cdot \frac{1-i}{1-i} = \frac{(1-5i)(1-i)}{1-(i)^2} = \frac{1-i-5i+5i^2}{2} = \frac{-4-6i}{2} = -2-3i$$

CONIUGATO

$z_1 = a + ib$; $\bar{z}_1 = a - ib$

coniugato di z_1

$a \in \mathbb{R}$ $\bar{a} = a$



$z = 1 + 2i = (1,2)$

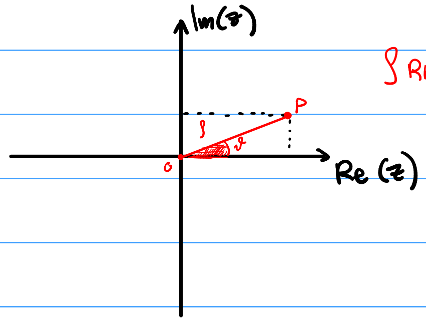
$\bar{z} = 1 - 2i = (1,-2)$

da dimostrare!

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

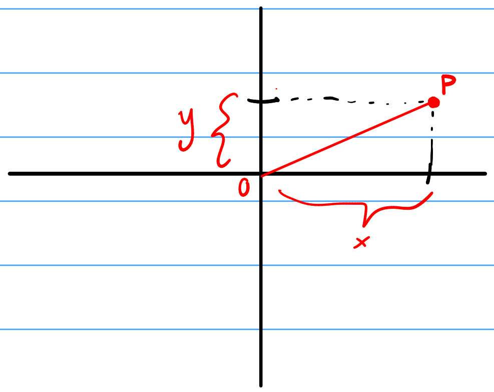
$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$

FORMA TRIGONOMETRICA (FORMA POLARE)



ρ RHO $\rho = |z|$ - lunghezza segmento

ϑ theta → argomento di z = angolo tra x e segmento OP



$$\rho = \sqrt{x^2+y^2}$$

$$\begin{cases} x = \rho\cos(\vartheta) \\ y = \rho\sin(\vartheta) \end{cases}$$ tutto per trovare?

$$\begin{cases} \cos(\vartheta) = \dfrac{x}{\rho} \\ \sin(\vartheta) = \dfrac{y}{\rho} \end{cases}$$ trovare l'angolo che soddisfa le condizioni

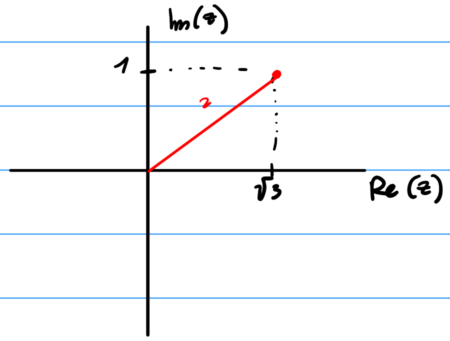
esempio

$$z = \sqrt{3} + i$$

$\mathrm{Re}(\sqrt{3})$, $\mathrm{Im}(1)$ $\qquad |z| = \rho = \sqrt{(\sqrt{3})^2+(1)^2} = \sqrt{4} = 2$

cerchiamo

$$\begin{cases} \cos(\vartheta) = \dfrac{\sqrt{3}}{2} \\ \sin(\vartheta) = \dfrac{1}{2} \end{cases}$$



$$\vartheta = 30° = \frac{\pi}{6}$$

$$0 \le \vartheta < 2\pi$$

forma trigonometrica $\sqrt{3} + i = \underbrace{2}_{\rho}\cos\left(\frac{\pi}{6}\right) + i\,\underbrace{2}_{\rho}\sin\left(\frac{\pi}{6}\right)$

FORMULA DI EULERO

$z = x + iy \mapsto$ rappresentazione algebrica

$$= |z|\left(\cos(\vartheta) + i\sin(\vartheta)\right)$$
$$= \rho\left(\cos(\vartheta) + i\sin(\vartheta)\right)$$

$\downarrow$ forma trigonometrica

$$0 \le \vartheta < 2\pi$$
$$\rho \ge 0$$

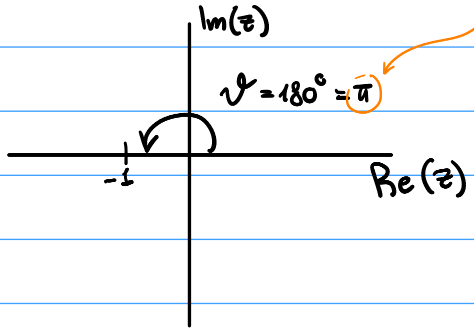
$$\begin{cases} x = \rho\cos(\vartheta) \\ y = \rho\sin(\vartheta) \end{cases}$$

$$\rho(\cos(\vartheta)+i\sin(\vartheta)) = \rho e^{i\vartheta}$$

ecco cosa dice Eulero!

$z=-1 \rightarrow \rho=1$ $\quad \vartheta=\pi$

$$-1 = e^{i\pi} \rightsquigarrow 0 = 1+e^{i\pi}$$



$z=-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$ → FORMA POLARE? (PER CASA)

POTENZA

- $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$ $\qquad$ con $n \in \mathbb{N}$
- $|z_1^n| = |z_1|^n$
- $|z^n| = |z|^n e^{in\vartheta}$