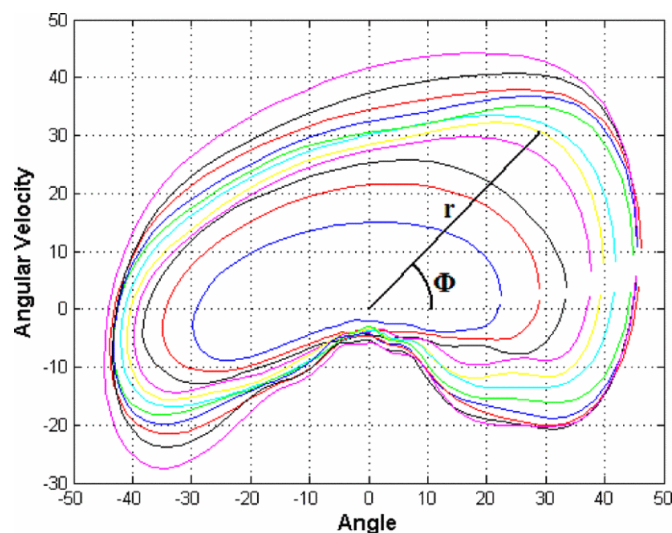


Studies using phase portraits

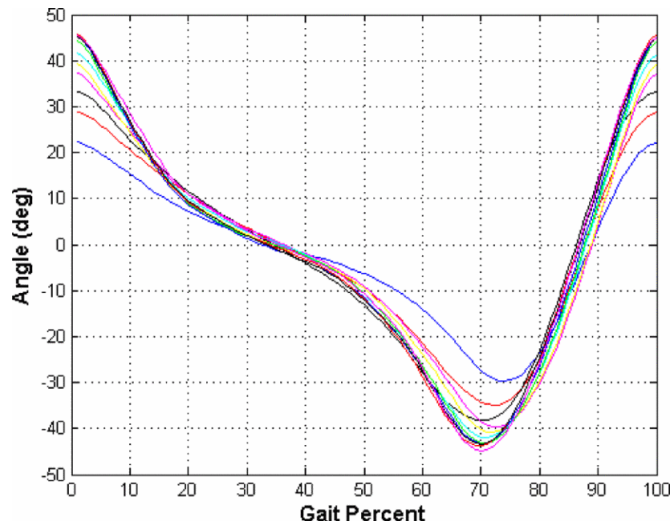
Authors and year		Task
Holgate et al. 1 2009	Tibia angle & tibia scaled angular velocity	Continuous gait phase estimation
Quintero et al. 2 ,2017	Thigh angle & its derivative(estimated thigh velocity)	Continuous gait phase estimation; gait speed estimation(as a byproduct)
Quintero et al. 3 2018	Thigh angle & its integral	Continuous gait phase estimation

Holgate et al.,2009

Holgate et al.: The polar angle between the **tibia angle** and its **scaled angular velocity** has an invertible relationship with the gait phase and is not subject-dependent.



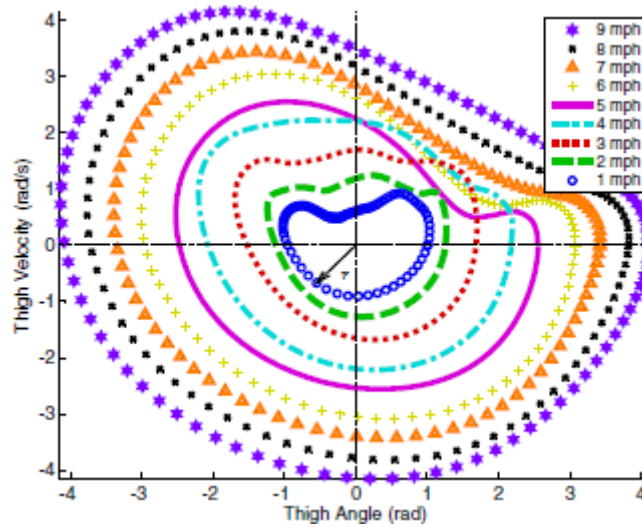
Tibia angular velocity multiplied by a scaling factor versus tibia angle. The closer the curve is to the origin, the shorter the stride length. Polar angle Φ represents the progression around the curve based on gait percent. r is the polar radius and is related to the stride length of the particular curve.



Tibia angle profile for able-bodied human gait. Each curve represents a different stride length. The closer the curve is to the zero degree axis, the shorter the stride length.

The hip was chosen based on a more extensive study (although offline) carried out by [77], which reports that the phase angle obtained from the phase portrait of the hip is linearly and monotonically increasing, and bounded, even under perturbations. The phase portrait was scaled by a factor estimated by the ratio of difference in maximum phase angle and minimum phase angle to the difference in the first derivative of the same, so as to improve the monotonicity and linearity.

Quintero et al.,2017



Quintero et al.,2018

Quintero et al.: **thigh angular position** and its corresponding **integral** to form a well-defined thigh orbit.

A phase angle $\vartheta(t)$ is computed by utilizing thigh angular position $\phi(t)$ and its integral

$\Phi(t) = \int_0^t \phi(\tau) d\tau$ in the following way:

$$\vartheta(t) = \text{atan2}((\Phi(t) + \Gamma)z, (\phi(t) + \gamma))$$

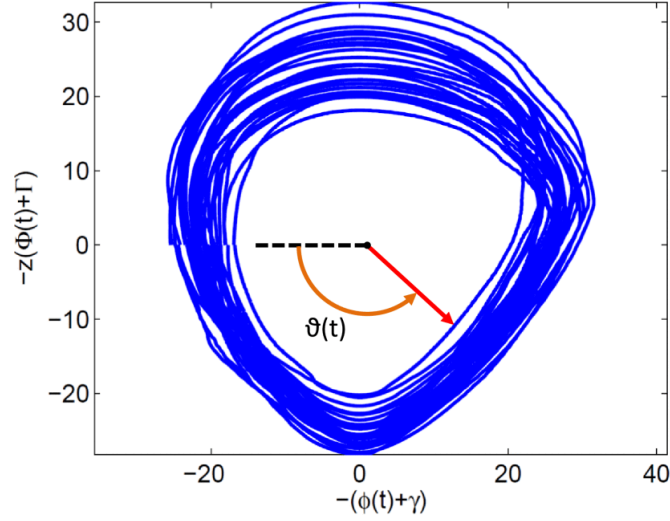
where the scale factor z , the thigh angle shift γ , and the thigh integral shift Γ are given by

$$z = \frac{|\phi_{\max} - \phi_{\min}|}{|\Phi_{\max} - \Phi_{\min}|},$$

$$\gamma = -\left(\frac{\phi_{\max} + \phi_{\min}}{2}\right), \quad \Gamma = -\left(\frac{\Phi_{\max} + \Phi_{\min}}{2}\right).$$

Fig. 3.

Phase plane of the thigh angle $\phi(t)$ vs. its integral $\Phi(t)$ during prosthetic leg experiments (see Section IV). The phase plane has been scaled by z and shifted by (γ, Γ) to achieve a circular orbit across the stride, which improves the linearity of the phase variable $\vartheta(t)$.



1. M. A. Holgate, T. G. Sugar, and A. W. Bohler, "A novel control algorithm for wearable robotics using phase plane invariants," in 2009 IEEE International Conference on Robotics and Automation, pp. 3845–3850, Kobe, Japan, 2009. [↵](#)
2. Quintero, D.; Lambert, D.J.; Villarreal, D.J.; Gregg, R.D. Real-Time continuous gait phase and speed estimation from a single sensor. In Proceedings of the 2017 IEEE Conference on Control Technology and Applications (CCTA), Maui, HI, USA, 27–30 August 2017; pp. 847–852. [\[Google Scholar\]](#) [↵](#)
3. D. Quintero, D. J. Villarreal, D. J. Lambert, S. Kapp, and R. D. Gregg, "Continuous-phase control of a powered knee-ankle prosthesis: amputee experiments across speeds and inclines," IEEE Transactions on Robotics, vol. 34, no. 3, pp. 686–701, 2018. [↵](#)