

# Continuous distributions

- Suppose  $f(y)$  is a probability density function for a continuous random variable  $Y$ .
- Then  $\int_{-\infty}^{\infty} f(y)dy = 1$    A) T B) F



# Continuous distributions

- Suppose  $f(y)$  is a probability density function for a continuous random variable  $Y$ .
- Then:  $P(Y = 2) = 0$  A) T B) F



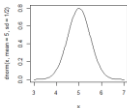
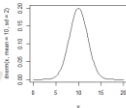
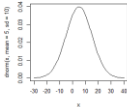
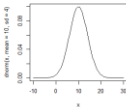
# Continuous distributions

- Suppose  $f(y)$  is a probability density function for a continuous random variable  $Y$ .
- Then:  $f(y) \geq 0$  A) T B) F



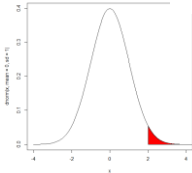
# Which line is NOT plotted?

- A) `curve(dnorm(x, mean=10, sd=4), xlim=c(-10,30))`
- B) `curve(dnorm(x, mean=10, sd=2), xlim=c(0,20))`
- C) `curve(dnorm(x, mean=5, sd=10), xlim=c(-30,40))`
- D) `curve(dnorm(x, mean=5, sd=1/2), xlim=c(4,7))`



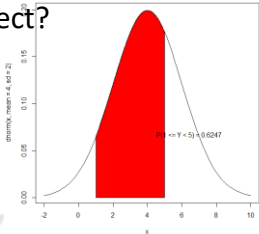
# What is the size of the area shaded?

- A) 0.5
- B) 0.7
- C)  $< 0.5$
- D)  $> 0.5$



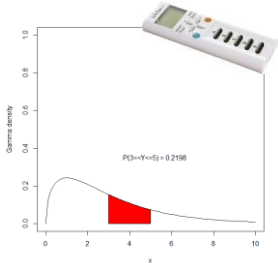
# Which line is incorrect?

- A) `curve(dnorm(x, mean=4,sd=2),xlim=c(-2,10))`
- B) `xcurve=seq(1,5,length=1000)`
- C) `ycurve=dnorm(xcurve,mean=4,sd=2)`
- D) `polygon(c(1,xcurve,6),c(0,ycurve,0),col="Red")`
- E) `text(locator(1),paste("P(1 <= Y < 5)", "=", round(pnorm(5,mean=4,sd=2)-pnorm(1,mean=4,sd=2),4)))`



# What line calculates the area?

- A) `curve(dchisq(x, df=3), xlim=c(0,10), ylim=c(0,1), ylab="Gamma density")`
- B) `xcurve=seq(3,5,length=1000)`
- C) `ycurve=dchisq(xcurve,df=3)`
- D) `polygon(c(3,xcurve,5),c(0,ycurve,0),col="Red")`
- E) `text(locator(1),paste("P(3=<Y<=5)","=",round(pchisq(5,df=3)-pchisq(3,df=3),4)))`

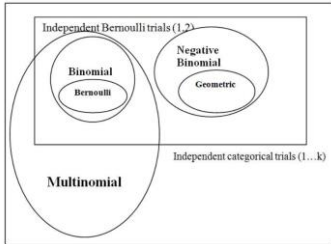


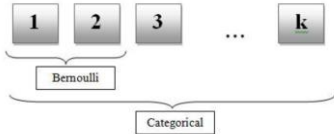
# HWK due Wed

- Read Examples 6.1,2,3  
page 212









Are there independent categorical trials?

a. Yes

i. Is the number of trials fixed?

1. Yes

a. Are there more than two categories?

i. Yes – Multinomial

ii. No

1. Are there 2 or more trials

a. Yes – Binomial

b. No – Bernoulli

2. No

a. Trials till first success?

i. Yes – Geometric

ii. No – Negative Binomial

b. No

i. Is there a constant rate?

1. Yes – Poisson

2. No – Hyper-geometric



GO TO  
Example

# Are there independent categorical trials?

- A) Yes
- B) No
- C) I don't know

25% of bottled water is just tap water packaged in a bottle.

Consider a sample of five bottled water brands. Let  $Y$  = number of these brands that use tap water.



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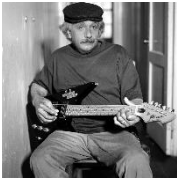
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# BINOMIAL



## *Binomial*

$$p(y) = \binom{n}{y} p^y q^{n-y}, (y = 0, 1, \dots, n)$$

$$\mu = np, \sigma^2 = npq$$

1. N identical Bernoulli trials
2. Trials are independent
3. Y is the binomial random variable and stands for the number of successes in the n trials

# Are there independent categorical trials?

- A) Yes
- B) No
- I don't know



Suppose the number  $Y$  of cracks per concrete specimen is approximately 2.5 (We are interested in the number of cracks per Block)

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# POISSON



*Poisson*

$$p(y) = \frac{\lambda^y e^{-\lambda}}{y!}, y = (0, 1, 2, \dots)$$

$$\mu = \lambda, \sigma^2 = \lambda$$

1.  $Y$  is the number of times an event occurs within a unit of measurement (like time).



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Assume hitting oil at one drilling location is independent of another and the probability of a success is 0.3. What is the probability that a driller will hit oil for the first time on the 10<sup>th</sup> drilling?



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# GEOMETRIC



## *Geometric*

$$p(y) = pq^{y-1}, (y = 1, 2, \dots)$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{q}{p^2}$$

1. A sequence of identical Bernoulli trials
2. Each trial is independent
3. The random variable of interest is the number of trials  $Y$  until the first success.



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# HYPER-GEOMETRIC



## *Hypergeometric*

$$p(y) = \frac{\binom{r}{y} \binom{N-r}{n-y}}{\binom{N}{n}}, y = \text{Maximum}\{0, n - (N - r)\} \dots$$

*Minimum*( $r, n$ )

$$\mu = \frac{nr}{N}, \sigma^2 = \frac{r(N-r)n(N-n)}{N^2(N-1)}$$

1. Randomly choose  $n$  elements from  $N$ , without replacement
2. The  $N$  objects are made up of two groups,  $r$  successes and  $(N-r)$  failures
3. Assume large sample size i.e.  $n/N > 0.05$
4. The random variable of interest is  $Y$  the number of successes in the  $n$  elements.