Following Youtube Video

☆ PD controller - Step 1

- damping ratio with 5% OS

$$\zeta = 0.69$$

- finding ω_n with $T_s = 0.1$

$$T_s = \frac{4}{\omega_n \zeta} \rightarrow 0.1 = \frac{4}{\omega_n \cdot 0.69} \rightarrow \omega_n = 57.97 \text{ rad/s}$$

- Desired poles:

$$-\omega_n \zeta \pm j\omega_n \sqrt{1-\zeta^2} \rightarrow -40 \pm j42$$

- Open loop transfer function

$$G(s) = \frac{14273\, G_c}{s^2 + 59.67s + 483.5}$$

where $G_c = \frac{K(s+a)(s+b)}{s}$

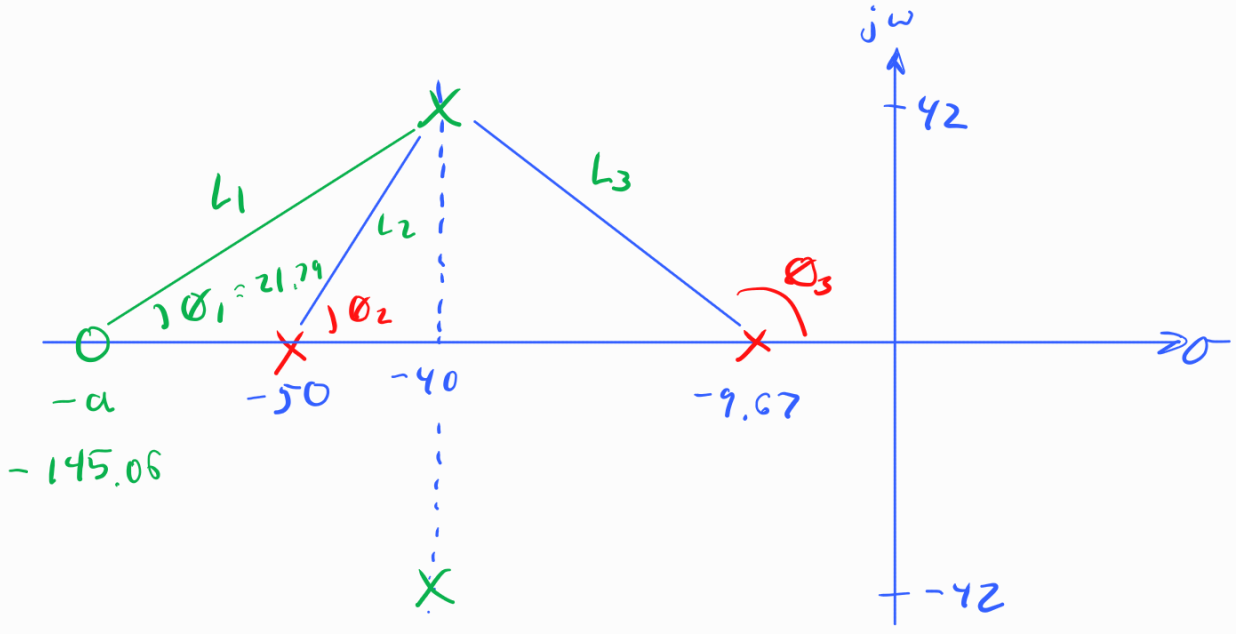
$$G(s) = \frac{14273\, K (s+a)(s+b)}{s(s^2 + 59.67s + 483.5)}$$

↑ Type 1 system

PD controller - Step 2:

Open loop poles: $s = -9.67$, $s = -50$

Desired poles: $-40 \pm j42$



* Desired poles

$G_{PD} = K(s + a)$ → pole at $-a$

K: proportional; (s + a): derivative

- $\sum \theta_z - \sum \theta_p = -180$

$$\theta_1 - (\theta_2 + \theta_3) = -180$$

where $\theta_2 = \tan^{-1}\left(\frac{42}{10}\right) \rightarrow \theta_2 = 75.96°$

$$\theta_3 = 180 - \tan^{-1}\left(\frac{42}{30.33}\right) \rightarrow \theta_3 = 125.83°$$

$$\theta_1 - (75.96 + 125.83) = -180$$

$\theta_1 - 201.79 = -180$

$\boxed{\theta_1 = 21.79°}$ angle of pole

$\theta_1 = \tan^{-1}\left(\frac{42}{a-40}\right)$

$\tan^{-1}\left(\frac{42}{a-40}\right) = 21.79$

$\frac{42}{a-40} = \tan(21.79)$

$\frac{42}{\tan(21.79)} = a - 40 \rightarrow \boxed{a = 145.06}$

- Finding gain for pole

$$K_{overall} = \frac{\pi L_p}{\pi L_z}$$

$K_{overall} = \frac{L_2 \cdot L_3}{L_1}$ where $L_2 = \sqrt{10^2 + 42^2} \rightarrow L_2 = 43.17$

$L_3 = \sqrt{30.33^2 + 42^2} \rightarrow L_3 = 51.81$

$L_1 = \sqrt{95.06^2 + 42^2} \rightarrow L_1 = 103.92$

$K_{overall} = \frac{(43.17)(51.81)}{103.92}$

$K_{overall} = 21.5$ gain for the controller

$14273\,K = 21.5 \rightarrow \boxed{K = 0.0015}$ gain

$$G_{PD} = \underbrace{0.0015}_{K}\,(s + \underbrace{145.06}_{a})$$

Adding PI to eliminate steady state error

Step 4:

$$G_c = \overbrace{\frac{K(s+a)(s+b)}{s}}^{\text{PD controller}}$$

zero at $-b$

pole at origin

let $b = -0.1$ so,

$$\boxed{G_c = \frac{0.0015\,(s + 145.06)(s+0.1)}{s}}$$

PID controller

0.001