This is the first homework assignment. The problems are to be presented on exercise session on **October 15, 2019**. Students should tick in TUWEL problems they have solved and are prepared to present their detailed solutions on blackboard. The problems should be ticked by **23:30 on October 14, 2019**.

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(1) I **Car licence plates**

A certain state's car licence plates have three letters of the alphabet followed by a three-digit number.

(a) How many different licence plates are possible if all three-letter sequences are permitted and any number from 000 to 999 is allowed?

(b) Mary withnessed a hit-and-run accident. She knows that the first letter on the licence plate of the offender's car was a B, that the second letter was an O or Q, and that the last number was a 5. How many state's licence plates fit this description?

II **Symphony orchestra program**

A symphony orchestra has in its repertoire 30 Haydn symphonies, 15 modern works, and 9 Beethoven symphonies. Its program always consists of a Haydn symphony followed by a modern work, and then a Beethoven symphony.

(a) How many different programs can it play?

(b) How many different programs are there if the three pieces can be played in any order?

(c) How many different three-piece programs are there if more than one piece from the same category can be played and they can be played in any order?

(2) **Poker game**

A deck of 52 cards has 13 ranks $(2, 3, 4, 5, 6, 7, 8, 9, 10, \text{J}, \text{Q}, \text{K}, \text{A})$ and 4 suits ($\heartsuit$, $\spadesuit$, $\diamondsuit$, $\clubsuit$). A poker hand is a set of 5 cards randomly chosen from a deck of 52 cards.

(a) A full house in poker is a hand where three cards share one rank and two cards share another rank. How many ways are there to get a full-house? What is the probability of getting a full-house?

(b) A royal flush in poker is a hand with ten, jack, queen, king, ace in a single suit. What is the probability of getting a royal flush?

(3) **Independence**

Let A and B be independent events.

(a) Prove that A^c and B^c are also independent.

(b) If we additionally know that $P(A|B) = 0.6$ and $P(B|A) = 0.3$, compute the probabilities of the following events

(i) at most one of A or B (ii) either A or B but not both.

(4) A random sample of 400 college students was asked if college athletes should be payed. The following table gives a two-way classification of the responses.

	Should be paid	Should not be paid
Student athlete	90	10
Student nonathlete	210	90

(a) If one student is randomly selected from these 400 students, find the probability that this student

i. is in favor of paying college athletes

ii. favors paying college athletes given that the student selected is a nonathlete

iii. is an athlete and favors paying student athletes

iv. is a nonathlete or is against paying students athletes

(b) Are the events *student athlete* and *should be paid* independent? Are they mutually exclusive? Justify your answer.

(5) **Coin game**

Two players, A and B, alternately and independently flip a coin and the first player to obtain a head wins. Assume player A flips first.

(a) If the coin is fair, what is the probability that A wins?

(b) Suppose that $P(\text{head}) = p$, not necessarily $\frac{1}{2}$. What is the probability that A wins?

(6) **Computer reliability**

A campus bookstore sells two types of computers: laptops and desctops. In the last semester it sold 56% laptops and 44% desktops. Reliability rates for the two types of machines are quite different. In the first year, 5% of desktops require service, while 15% of laptops have problems requiring service.

(a) Sketch a probability tree for this situation.

(b) What percentage of computers sold by the bookstore last semester required service?

(c) Given that a computer required service, what is the probability that it was a laptop?

(7) **Coding**

When coded messages are sent, there are sometimes errors in transmission. In particular, Morse code uses ”dots” and ”dashes”, which are known to occur in proportion 3 : 4. Suppose there is interference on the transmission line, and with probability $\frac{1}{8}$ a ”dot” is mistakenly received as a ”dash” and vice versa. If we receive a ”dot”, what is the probability that a ”dot” was sent?