This homework assignment is to be presented on exercise session on **December 3, 2019**. Students should tick in TUWEL problems they have solved and are prepared to present their detailed solutions on blackboard. The solved problems should be ticked by **23:30h on December 2, 2019**.

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(1) **Two-sample t-test**
Create two independent samples from the normal distribution. The first sample of size 10 shall be taken from the $N(0,1)$-distribution. The second sample of size 20 shall be taken from the $N(-1,1)$ distribution. Test the null hypothesis that the populations means are equal with a (two-sided) two-sample t-test on the 5%-significance level:

- (a) Calculate the t-statistic (without `t.test()`)
- (b) Compare it to the output of `t.test()`
- (c) Interpret the result of the test

(2) **Two-sample t-test using normal approximation**
Messages are frequently sent from a sender to either receiver 1 or receiver 2. For both receivers, several times for the transfer were measured (in seconds) and stored in the file `waitingtimes2.Rdata`.

- (a) Plot both data sets. Is their distribution approximately bell-shaped?
- (b) Test the null-hypothesis of equal mean transfer times for both receivers on the 1%-level with a two sample t-test (using the normal approximation).
- (c) Compare your result to the output of `t.test()`

(3) **Equivalence of test and confidence interval**
In the two-sample situation show that the null hypotheses $H_0 : d = d_0$ of the two-sided test is rejected if and only if the confidence interval does not overlap d_0.

(4) **Confidence interval (without R)**
In the situation of the two-sample t-test let for the first group the sample size be $n_y = 4$, the mean $\bar{y} = 40$, and the empirical variance $s_y^2 = 64$. For the second group let $n_x = 9$, $\bar{x} = 20$ and $s_x^2 = 81$. Further, let the null hypothesis be $H_0 : d = \mu_y - \mu_x = 10$. What is the smallest positive quantile q of the associated t-distribution for which the two-sample confidence interval does not overlap the null parameter 10? What is this confidence interval?

(5) **Which statement is correct?**
In the situation of a (two-sided) two-sample t-Test on the 5%-level assume that the null hypothesis $H_0 : d = d_0$ was rejected. Comment on the following statements.

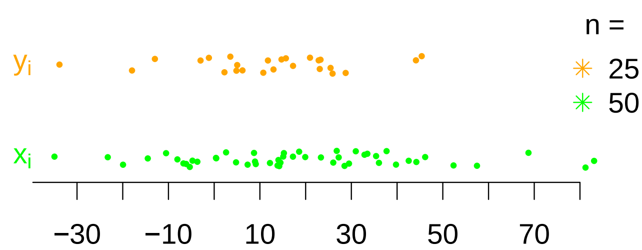
- (a) The null hypothesis was also rejected on the 7%-level.
- (b) The 99% confidence interval does not contain d_0

(c) If both sample sizes are increased by a factor 4, then the value of the t-statistic is halved

(d) If one of the sample sizes is increased, then the width of the 95%-confidence interval is increased

(e) With a probability of 5% the decision was wrong if the null hypothesis was true

(6) **Naive two-sample t-Test**
For the data depicted below, can we reject $H_0 : d = \mu_y - \mu_x = -7.5$ on the significance level of 5% in a (two-sided) two-sample t-test?



(7) **Simulation of test-power**
Simulate the test-power in the two-sample t-test: Let $X_1, \ldots, X_n, Y_1, \ldots, Y_n$ be independent random variables with $X_i \sim N(0, \sigma^2)$ and $Y_i \sim N(d, \sigma^2)$ for all $i = 1, 2, \ldots, n$. Simulate the test-power (relative frequency of rejections) for $d \in \{-5, -4.5, -4, \ldots, 5\}$ in 1000 simulations each. Use the parameters

(a) $n = 10$ and $\sigma = 3$

(b) $n = 20$ and $\sigma = 3$

(c) $n = 20$ and $\sigma = 1$

for each of which you plot the testpower against d. Comment on your graphic. Hint: You can access the p-value with `t.test()$p.value`.