This is the second homework assignment. The problems are to be presented on exercise session on **October 22, 2019**. Students should tick in TUWEL problems they have solved and are prepared to present their detailed solutions on blackboard. The problems should be ticked by **23:30 on October 21, 2019**.

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(1) Based on its analysis of the future demand for its products, the financial department at a certain corporation has determined that there is a 0.17 probability that the company will lose 1.2 million dollars during the next year, a 0.21 probability that it will lose 0.7 million dollars, a 0.37 probability that it will make a profit of 0.9 million dollars, and a 0.25 probability that it will make a profit of 2.3 million dollars.

(a) Let X be a random variable that denotes the profit (in million dollars) earned by this corporation during the next year. Write the probability distribution of X.

(b) Find the mean and standard deviation of the probability distribution of part (a).

(c) Give a brief interpretation of the value of the mean.

(d) Compute $P(|X| \leq 1)$ and $F_X(1.5)$, where $F_X(x)$ is the cumulative distribution function (cdf) of X.

(2) **Fair six sided die**
A fair six sided die is rolled repeatedly until the sum of all obtained numbers is greater than 6. Let X be the number of times the die was rolled and let F be the cumulative distribution function for X. Compute $F(1)$, $F(2)$, $F(6)$ and $F(7)$.

(3) **Density and distribution function**
Let X a continuous random variable with the density

$$f_X(x) = \begin{cases} \frac{x^2}{a}, & x \in (0, 3) \\ 0, & \text{otherwise} \end{cases}$$

where a ia a nonzero constant.

(a) Determine the value of a.

(b) Find the cumulative distribution function of the random variable X.

(c) Compute the expectation of $Z = 2X^3 + 5$.

(4) **River floods**

A certain river floods every year. Suppose that the low-water mark is set at 1 and the high-water mark Y has cumulative distribution function (cdf)

$$F_Y(y) = P(Y \leq y) = \begin{cases} 1 - \frac{1}{y^2}, & 1 \leq y < \infty \\ 0, & y < 1 \end{cases} .$$

(a) Verify that $F_Y(y)$ is a cumulative distribution function.

(b) Compute $P(-1 \leq Y \leq \frac{1}{2})$.

(c) Find $f_Y(y)$, the probability density function (pdf) of Y.

(d) If the low-water mark is reset at 0 and we use a unit of measurement that is $\frac{1}{10}$ of that given previously, the high-water mark becomes $Z = 10(Y - 1)$.
Find the cdf $F_Z(z)$ and pdf $f_Z(z)$.

(5) **Target**

If the probability of hitting a target is $\frac{1}{5}$, and ten shots are fired independently, what is the probability of the target being hit at least twice? What is the conditional probability that the target is hit at least twice, given that it is hit at least once?

(6) **Continuous random variable**

Let X be a random variable whose cumulative distribution function (cdf) is of the form

$$F(x) = \frac{e^x}{1 + e^x}, \quad x \in \mathbb{R}.$$

(a) Determine the associated probability density function (pdf) $f(x)$.

(b) Use R function `plot()` to sketch the cdf F and pdf f.

(c) Find an expression for the p-quantile x_p and then determine the three quartiles (25%, 50% and 75% quantiles) of the distribution.

*H*int: Recall, x_p is the p-quantile if it holds $F(x_p) = p$.

(7) ***R*-functions**

(a) Using the *R* software, define a vector x that contains the values of the column `Height` from the dataset `trees`. Round the values of x to the tenth place and then output the number of each type. The commands `round()` and `table()` should be used.

Note that `trees` is the *R* Dataset Package containing Diameter, Height and Volume for 31 Black Cherry trees.

(b) Write a function in R that outputs the first n Fibonacci numbers. Calculate the sum of the first 10 and the first 20 Fibonacci numbers.

(c) Consider the curve given by the parametrization

$$t \mapsto \begin{pmatrix} \sin 2t \\ \cos 3t \end{pmatrix} \quad \text{for} \quad 0 \leq t < 2\pi .$$

Set the working directory by using *R* function `setdw()`. Use `plot()` to plot the curve and `dev.print()` to save the result as a pdf file.