This is the third homework assignment. The problems are to be presented on exercise session on **October 29, 2019**. Students should tick in TUWEL problems they have solved and are prepared to present their detailed solutions on blackboard. The problems should be ticked by **23:30 on October 28, 2019**.

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(1) **Basketball free throws**
Two professional basketball players, Tom and John, each throw ten free throws with a basketball. Tom makes 80% of the free throws he tries, while John makes 85% of the free throws he tries.

(a) What is the probability that the number of free throws that Tom will make is exactly 7?

(b) What is the probability that the number of free throws that John will make is at least 8?

(c) Player who achives the highest score wins the game. It is assumed that the two players do not influence each other when throwing. What is the probability that neither Tom or John will win the game?

Hint: Use R-function `dbinom()` to calculate the probability mass functions.

(2) **Train**
Suppose a train is delayed by approximately 60 minutes. Assuming that a uniform distributionon $(0, 60)$ was used to model the delayed time (in minutes), compute the probability that train will reach its final destination in the interval between 57 and 60 minutes. Calculate the expectation and standard deviation of the delayed time.

(3) **Hurricane insurance**
An insurance company needs to asses the risk associated with providing hurricane insurance. During 22 years from 1990 through 2011, Florida was hit by 27 major hurricanes (level 3 and above). The insurance company assumed Poisson distribution for modeling number of hurricanes.

(a) If hurricanes are independent and the expectation has not changed, what is the probability of having a year in Florida with each of the following?

(1) No hits.
(2) Exactly one hit.
(3) More than two hits.

(b) Use R to estimate the number of hurricane hits that will occur with the probability 99.5%.

(4) **Time to failure**

Let X be an exponential random variable $X \sim \exp(\lambda)$, i.e. its probability density function is given by

$$f_X(x) = \begin{cases} \lambda\, e^{-\lambda x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

and its mean equals $E(X) = \frac{1}{\lambda}$.

(a) Compute $P(X > x)$.

(b) Suppose we are testing two different brands of light bulbs B_1 and B_2 whose lifetimes are exponential random variables with mean $\frac{1}{3}$ and $\frac{1}{6}$ years respectively. Assuming that the lifetime of the two light bulbs are independent, find the expected time before one of the bulbs fails.

(5) **Uniform-exponential relationship**

Let X be a random variable, uniformlly distributed on $(0, 1)$.

(a) Find the cumulative distribution function of X.

(b) Find the distribution of $Z = -\log X$.

(6) **Tire company**

A tire manufacturer believes that the tread life of its snow tires can be distributed by a Normal model with a mean (expectation) of 32 000 miles and a standard deviation of 2 500 miles.

(a) If you buy a set of these tires, would it be reasonable for you to hope that they will last 40 000 miles? Explain your answer.

(b) Approximately what fraction of these tires can be expected to last less than 30 000 miles?

(c) Approximately what fraction of these tires can be expected to last between 30 000 and 35 000 miles?

(d) Calculate the interquartile range of this distribution.

Recall, the interquartile range is the diference between upper and lower quartile, i.e.

$$IQR = x_{0.75} - x_{0.25}.$$

(e) In a marketing strategy, a local tire dealer wants to offer a refund to any customer whose tires fail to last a certain number of miles. However, the dealer does not want to take too big risk. If the dealer is willing to give refunds to no more than one of every 25 customers, for what milage can he guarantee these tires to last?

Note: Table of standard Normal distribution should be used for all computations.

(7) **Drug company**

Manufacturing and selling drugs that claim to reduce an individual's cholesterol level is big business. A company would like to market their drug to women if their cholesterol is in the top 15%. Assume the cholesterol levels of adult American women can be desribed by a Normal model with a mean of 188 mg/dL and a standard deviation of 24. Use *R* to answer the following questions.

(a) Draw and label the Normal model.

(b) What percent of adult women do you expect to have cholesterol levels over 200 mg/dL?

(c) What percent of adult women do you expect to have cholesterol levels between 150 and 170 mg/dL?

(d) Calculate the interquartile range of the cholesterol levels.

(e) Above what value are the highest 15% of women's cholesterol levels?

Hint: R commands `pnorm()`, `qnorm()` and `dnorm()` are useful.