CSCE 310

Data Structures + Algorithms

(Computer Science 3)

Administrivia

Review

Brute Force Algorithms

- 2+ programming language
- ~~Search~~/~~Sorting~~

linear

binary

bubble sort : $\Theta(n^2)$

selection sort

insertion sort

$\approx \frac{1}{4} n^2$

$n \rightarrow \underline{\log(n)}$

$2^{\log(n)}$

~~quick sort~~

merge sort

tim sort

$\Theta(n\log(n))$

Algorithms + Algo. Analysis

→ pseudocode

→ 5 step process:

1) Identify the input

2) Identify the input size

3) Identify the elementary operation

4) Analyze the algo. w.r.t. the input size

$$f : n \longrightarrow t(n)$$

input size   resource

5) Provide a Big-O, Θ characterization.

Basic Data Structures:

Stacks, Queues, variations: Deque,
PQueue (priority)

Heap

Trees

Graphs

Basic Discrete Math
· proofs + logic

$\omega, o, \Theta, O, \Omega$

<u>Master Theorem</u>

Problem: Closest Pair Problem

Given: A collection of points in $\mathbb{R}^2$

$$A = \{(x_1, y_1), (x_2, y_2), \ldots (x_n, y_n)\}$$

Output: The 2 closest points p_a, p_b in A

Input: A collection of points $A = \{p_1, p_2 \cdots p_n\}$

Output: 2 closest points in A

$d_{min} \leftarrow +\infty$

for each pair of points $p_a, p_b \in A$:

    $d \leftarrow \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$

    if $(d < d_{min})$

        $d_{min} \leftarrow d$

        $p \leftarrow p_a$

        $q \leftarrow p_b$

output p, q

1) Input: Collection A

2) Size: n

3) Elementary operation

comparison, like 4

4) How many times is the 4 executed

w.r.t. input size n?

$$\binom{n}{2} = \frac{n(n-1)}{2} = \frac{1}{2}n^2$$

5) $\Theta(n^2)$

$p_i \quad n=5$

~~for {~~

```
for (int i=0; i<n-1; i++) {
    for (int j=i+1; j<n; j++) {
        //TODO (op)
```

0 1	1 2	2 3	3 4
0 2	1 3	2 4	
0 3	1 4	1 }	
0 4	}	2 }	
4	3		

$$\boxed{\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1}$$

$$\sum_{i=0}^{n-2}\sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2}(n-1-i)$$

$$= \sum_{i=0}^{n-2} n - \sum_{i=0}^{n-2} 1 - \sum_{i=0}^{n-2} i$$

$$= n(n-1) - (n-1) - \frac{(n-2)(n-1)}{2}$$

$$= (n-1)(n-1) - \frac{(n-2)(n-1)}{2}$$

$$= (n-1) - \frac{2(n-1)-(n-2)}{2}$$

$$= \frac{n(n-1)}{2}$$

$\Theta(n^2)$

$$\begin{tikzcd}
\text{input} & \text{\# of operations} \\
n \arrow[r] \arrow[d] \arrow[dd, bend right] & n^2 \arrow[d] \\
2n \arrow[r] & (2n)^2 = 4n^2 \\
10n \arrow[r] & (10n)^2 = 100n^2
\end{tikzcd}$$

K

$\binom{n}{K}$

for each triple $p_a\, p_b\, p_c\, p_d$

...

for i = ...

for j = ...

for k = ...

for l = ...

Claim: $\frac{n(n-1)}{2} \in \Theta(n^2)$

$$\lim_{n \to \infty} \frac{\frac{n(n-1)}{2}}{n^2} = \lim_{n \to \infty} \frac{1}{2} \frac{n^2}{n^2} + \lim_{n \to \infty} \frac{\cancel{n}}{2n^{\cancel{2}}}$$

$$= \frac{1}{2} + 0$$

$$= \frac{1}{2}$$

$$\Rightarrow \Theta$$

$\binom{n}{k} \rightarrow$ all subsets of size k

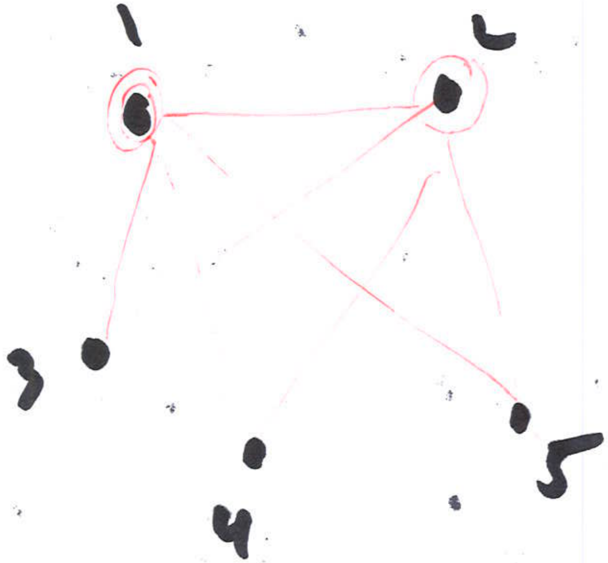
$k = 0, 1, 2, \ldots n$

$\binom{n}{n}$

$\left.\begin{matrix} 1\,2\,3 \\ 1\,3\,2 \\ 2\,1\,3 \\ 2\,3\,1 \\ 3\,1\,2 \end{matrix}\right\}$ permutations

$3\,2\,1$

$2^n < n!$



(P_1, P_1)
(P_1, P_2)
(P_2, P_1)

$$\left.\begin{array}{c}4\\3\\2\\1\end{array}\right\}10$$

$\underline{1}$

$\cancel{0}$

$$\cancel{\#}\;\left.\begin{array}{c}4\\4\\4\\4\end{array}\right\}16 \sim n^2 - n$$

$P_1\, P_2 \cdots P_n$

$\binom{n}{2}$ = # of pairs <u>without</u> order

$$= \frac{n(n-1)}{2}$$